A Passivity Study of the Classical Position-Force Teleoperation controller

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1 Introduction
This work rigorously analyzes the stability properties of the popular Position-Force bilateral teleoperation controller. Firstly, the existing concepts of two-port passivity and absolute stability are discussed, after which a new method for stability analysis is presented, referred to as one-port Passivity.

In the basic configuration of the Position-Force controller, the control inputs for the motors are:

\[ \tau_m = -\lambda F_e, \quad \tau_s = (K_v s + K_p) (\mu x_m - x_s) \]

with \( \mu \) and \( \lambda \) the position and force scaling factor. The analysis is based on simple mass-damper models for the master \((M_m, B_m)\) and the slave \((M_s, B_s)\). \( x_m \) and \( x_s \) are the position of the master and the slave respectively. \( F_e \) is the measured interaction force with the environment.

2 Two-port Passivity and Absolute Stability
Two-port Passivity is a sufficient condition for stable interaction. Absolute stability is a less conservative sufficient condition incorporating the structural knowledge that no direct interaction between operator and environment occurs. However, the authors proved analytically that the Position-Force controller as defined in the introduction is never passive nor absolutely stable for non trivial parameters. These proofs are based on the Raisbeck passivity criterion and on Llewellyn’s absolute stability criterion respectively.

3 One-port Passivity
The new method is based on combining the dynamics of the master, slave, controller and environment into a one-port network \( Y_{MS(K_v)} \). The coupled stability between any operator and the \( Y_{MS(K_v)} \) one-port is now discussed (Figure 1). Coupled stability can be checked by verifying positive realness of the admittance \( Y_{MS(K_v)} \) [11]. In order to determine \( Y_{MS(K_v)} \), an assumption has to be made about the environment. As stated in [1], pure springs and pure masses can be considered as the worst case environments since their admittance is not strictly passive. Since for teleoperation research displaying stiffness is in most cases more relevant, the environment considered here is a pure spring \((K_v)\).

![Figure 1: A one-port network \( Y_{MS(K_v)} \) representation of a combined teleoperator-environment system.](image)

Through this assumption the admittance of the one-port can be written as:

\[ Y_{MS(K_v)} = \frac{\tau_m}{\tau_s - \tau_e} = \frac{\tau_m}{(K_v s + K_p) (\mu x_m - x_s)} \]

The \( R(Y_{MS(K_v)}(j\omega)) \geq 0 \) condition results in a rather complicated set of analytic conditions on the parameters of the system. However, the authors found that in most practical cases, the conditions simplify to the following:

\[ \mu \lambda \leq \frac{B_m (B_s + K_v)}{M_m K_v} (B_s + K_v)^2 - 2 M_s K_v + 2 \lambda K_v (K_v s + K_p) \]

Figure 2 shows the effect on the maximum allowed \( \mu \lambda \) as a function of the selected \( K_v^{\text{max}} \) and the parameters \( M_s, B_m \) and \( K_p \), every time for all other parameters fixed.

![Figure 2: Effect \( K_v^{\text{max}} \), \( M_s \), \( B_m \) and \( K_p \) on \( \mu \lambda \). The nominal parameters are based on an experimental setup: \( M_s = 0.6 \text{ kg} \), \( K_v = 4000 \text{ N/m} \), \( K_v = 80 \text{ N/m} \), \( B_m = 3.4 \text{ Ns/m} \), \( B_s = 11 \text{ Ns/m} \). \( K_v^{\text{max}} \) is set at 1000 N/m.](image)

4 Discussion and Conclusions
The inequality (3) shows the incapability of the Position-Force controller to display arbitrarily large environment stiffnesses. Other fundamental insights obtained from (3) are: the mass of the slave strongly limits the achievable \( \mu \lambda \). An increase of master damping \((B_m)\) allows larger \( \mu \lambda \) and also a tighter position loop at the slave allows larger \( \mu \lambda \) (when the damping ratio is kept constant). In future work the same systematic approach will be followed to analyze other more general teleoperation controllers.

References

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