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International R&D Collaboration Networks*

Huasheng Song
CORE,
Université catholique de Louvain.

Vincent Vannetelbosch†
FNRS and CORE,
Université catholique de Louvain.

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Abstract

We reconsider Goyal and Moraga-González [Rand J. of Econ. 32 (2001), 686-707] model of strategic networks in order to analyze how government policies (e.g. subsidies) will affect the stability and efficiency of networks of R&D collaboration among three firms located in different countries. A conflict between stability and efficiency is likely to occur. When governments cannot subsidize R&D, this conflict will occur if public spillovers are not very small. However, when governments can subsidize R&D, the likelihood of a conflict is considerably reduced. Indeed, a conflict will arise only if public spillovers are very small or quite large.

Key words: Networks, R&D collaboration, Subsidy.

JEL Classification: C70, F13, L13, L20.

*Vincent Vannetelbosch is Research Associate of the National Fund for Scientific Research (FNRS, Belgium), and Associate Fellow of CEREC, Facultés Universitaires Saint-Louis.

†Corresponding author address : Prof. Vincent Vannetelbosch, CORE, Université catholique de Louvain, Voie du Roman Pays 34, B-1348 Louvain-la-Neuve, Belgium. E-mail: vannetelbosch@core.ucl.ac.be, Tel: 0032-10-474142, Fax: 0032-10-474301.

1 Introduction

The number of agreements on international research and development (R&D) collaboration has been increasing at an unprecedented rate. For instance, Chesnais (1988) has reported that among inter-firm agreements in high technology industries in Italy, where the product markets are characterized by imperfect competition, a large portion were for R&D collaboration, and more than half were for international collaboration.[1] Government support for both domestic and international R&D collaboration, including public investment, subsidy, and antitrust law modification, has also become more frequent.

Goyal and Moraga-González (2001) have analyzed the incentives for R&D collaboration between horizontally related firms when governments cannot subsidize R&D. They have basically shown that a conflict between the incentives of firms to collaborate and social welfare is likely to occur, and will arise if public spillovers from research are not too small. The purpose of this paper is to go beyond their analysis by allowing (i) governments to subsidize R&D and (ii) for coalitional deviations in the formation of R&D collaboration networks. In this paper we address the following questions:

(i) When governments can subsidize R&D, what are the incentives of firms located in different countries to collaborate and what is the architecture of "stable" networks of collaboration?

(ii) Do subsidies reconcile individual incentives to collaborate and social welfare?

To answer these questions we develop a four-stage game. In the first stage, three firms located in different countries form pairwise collaboration links. The purpose of these collaboration links is to share R&D knowledge about a cost-reducing technology. The collection of pairwise links between the firms defines a network of international collaboration. In the second stage, each government (whose objective is to maximize social welfare) simultaneously announces its R&D subsidy rates. In the third stage, each firm chooses independently and simultaneously a level of effort in R&D. In the fourth stage, firms compete in the product market of a fourth country by setting quantities.[2] We will consider Goyal and Moraga-González model, where government cannot subsidize, as our benchmark.

[1] Hagedoorn (2002), who has provided a survey of empirical work on R&D collaboration among firms, has also reported that during the 1980s, on average there were an additional 100 collaborative agreements every year in biotechnology, and over 200 every year in information technologies.

[2] This assumption is standard in strategic trade policy models (see Brander, 1995) and represents situations where firms' home market is small or negligible relative to the size of the relevant market. Examples are Nokia in Finland, or Samsung in South-Korea.

R&D effort of a firm decreases its marginal cost of production. It has also positive knowledge spillovers on the costs of firms that are linked to the firm that undertakes R&D effort. It is assumed that the research knowledge of a "direct" collaboration is fully absorbed, while the research knowledge of a no direct collaboration (indirect collaboration or no collaboration at all) is partially absorbed (public spillovers).

A simple way to analyze the networks that one might expect to emerge in the long run is to examine a sort of equilibrium requirement that agents not benefit from altering the structure of the network. A weak version of such condition is the pairwise stability notion defined by Jackson and Wolinsky (1996). A network is pairwise stable if no agent benefits from severing one of their links and no other two agents benefit from adding a link between them, with one benefiting strictly and the other at least weakly. But, pairwise stability considers only deviations by at most a pair of agents at a time. It might be that some group of agents could all be made better off by some complicated reorganization of their links, which is not accounted for under pairwise stability. The definition of strong stable networks allows for larger coalitions than just pairs of agents to deviate, and is due to Jackson and van den Nouweland (2005). A strongly stable network is a network which is stable against changes in links by any coalition of agents.[3] In a model with three firms located in different countries, there are four possible network architectures: the complete network, the star network, the partially connected network, and the empty network. In the complete network every pair of firms is linked. The star network is a network in which there is a "hub" firm directly linked to every other firm, while none of the other firms have a direct link with each other. The partially connected network refers to a configuration in which two firms are linked while the third firm is isolated. In the empty network there are no collaboration links.

When governments cannot subsidize R&D, the complete network is always pairwise stable, while the partially connected network is pairwise stable only for very small public spillovers. Moreover, the partially connected network is the unique strongly stable network when spillovers are very small; otherwise, no strongly stable network exists. Indeed, the complete network is destabilized by coalitional deviations. We say that a network is strongly efficient if it maximizes the societal welfare defined as the sum of producing countries' social welfare. The partially connected network is the strongly efficient network for very small spillovers; otherwise, the star network is the strongly efficient network. Thus, a conflict between stability and efficiency is likely to occur. This conflict will occur if public spillovers are not very small.

However, we show that, once governments can subsidize R&D, the likelihood of a

[3] Jackson (2003, 2005) provides surveys of models of network formation.

conflict is considerably reduced. The complete network is always pairwise stable, but it is never strongly stable. The partially connected network is now pairwise stable when public spillovers are neither too small nor too large. Notice that the partially connected network is the strongly efficient network when public spillovers are not too large. Otherwise, the star network is the strongly efficient network. Thus, a conflict between stability and efficiency will "only" arise if public spillovers are very small or quite large.

In terms of societal welfare we find that, except if public spillovers are very small, governments should be allowed for R&D subsidies. Indeed, the societal welfare levels of stable networks when subsidies are forbidden are dominated by those of stable networks when subsidies are allowed. Thus, allowing governments to subsidize R&D will not only reduce the likelihood of a conflict between stability and efficiency, but it will also be superior in terms of maximizing the societal welfare.

Before presenting the model, it is worth to mention some related literature. Export subsidies can be used to shift rents strategically between rival firms. See e.g. Brander (1995). However, such outright subsidies on exports are strictly forbidden by the World Trade Organization (WTO). In contrast, subsidizing domestic R&D is allowed by the WTO and, as shown by Spencer and Brander (1983), via such R&D policy a government can achieve the same strategic outcomes otherwise obtained under direct export subsidies for firms engaging in international R&D competition. Qiu and Tao (1998) have gone beyond the analysis of Spencer and Brander (1983) by investigating the optimal government policy (subsidy or tax) towards international R&D collaboration. They have shown that, with linear demands, tax is never optimal. Moreover, the optimal policy is subsidy regardless of the strategic nature (substitute or complement) of the strategy variables. Thus, our analysis reinforces theories that have provided justification for such government policy interventions. For general background on R&D cooperation in oligopoly the reader is directed to Amir (2000), d'Aspremont and Jacquemin (1988), Kamien, Muller and Zang (1992), Katz (1986) and Suzumura (1992).[4]

The paper is organized as follows. The model is presented in Section 2. In Section 3 we analyze the stability of international R&D networks. In Section 4 we study the efficiency of international R&D collaboration, and we comment on the conflict between stability and

[4] Yi and Shin (2000) have analyzed the endogenous formation of research coalitions where coalition formation is modelled in terms of a coalition structure, which is a partition of the set of firms. But the restriction to partitions is a strong one indeed if our interest is in research collaborations, since it rules out situations in which, for example, firms 1 and 2 have a bilateral research agreement and firms 2 and 3 have a similar agreement but there is no agreement between 1 and 3. When this occurs, it is not appropriate to view firms 1, 2 and 3 as one coalition, and we cannot think of 1 and 2 and 2 and 3 being two distinct coalitions, since this violates the mutual exclusiveness property of coalitions. The theory of networks provides a natural way to think of such issues, since it allows for such intransitive relationships.

efficiency of networks. In Section 5 we conclude.

2 The model

The model is similar to Goyal and Moraga-González (2001) except that we consider R&D subsidies. There are three firms located in three different countries. The firms produce homogeneous goods and, as is standard in strategic trade policy models, it is assumed that all firms compete in a fourth country's market by setting quantity (Cournot competition). This allows us to examine only firm profits when analyzing welfare. We denote by $N = \{1, 2, 3\}$ the set of firms which are connected in a network of R&D collaboration. Let q_i denote the quantities of the good produced by firm $i \in N$. Let $P(Q) = a - Q$ be the market-clearing price when aggregate quantity on the market is $Q \equiv \sum_{i \in N} q_i$. More precisely, $P(Q) = a - Q$ for $Q < a$, and $P(Q) = 0$ otherwise, with $a > 0$. The firms can undertake R&D to look for cost reducing innovations. Moreover, the firms may engage in bilateral R&D collaboration. Finally, the government in each country, whose objective is to maximize welfare, has an R&D policy toward its firm's R&D activity. We consider R&D tax or subsidy proportional to the firm's R&D expenditure. Let s_i be country i's R&D subsidy (tax if negative) rate.

In a network, firms are the nodes and each link indicates a pairwise R&D collaboration. Thus, a network g is simply a list of which pair of firms are linked to each other. If we are considering a pair of firms i and j, then $\{i, j\} \in g$ indicates that i and j are linked under the network g and that a R&D collaboration is established between firms i and j. For simplicity, write ij to represent the link $\{i, j\}$, so $ij \in g$ indicates that i and j are linked under the network g. The network obtained by adding link ij to an existing network g is denoted $g + ij$ and the network obtained by deleting link ij from an existing network g is denoted $g - ij$. For any network g, let $N(g) = \{i \in N \mid \exists\, j \text{ such that } ij \in g\}$ be the set of firms which have at least one link in the network g. Two firms i and j are connected if and only if there exists a sequence of firms $i_1, ..., i_K$ such that $i_k i_{k+1} \in g$ for each $k \in \{1, ..., K - 1\}$ with $i_1 = i$ and $i_K = j$. Let $N_i(g)$ be the set of firms with which firm i has a collaboration link. Let G be the set of all possible networks. In this three-firm market, there are four possible network architectures: (i) the complete network, g^c, in which every pair of firms is linked, (ii) the star network, g^s, in which there is one firm that is linked to the other two firms, (iii) the partially connected network, g^p, in which two firms have a link and the third firm is isolated, and (iv) the empty network, g^e, in which there are no collaboration links. In the star network, the firm which is linked to the other two firms is called the "hub" firm, while the other two firms are called the "spoke" firms.

Given a network g, every firm i chooses an R&D effort level x_i unilaterally. This effort

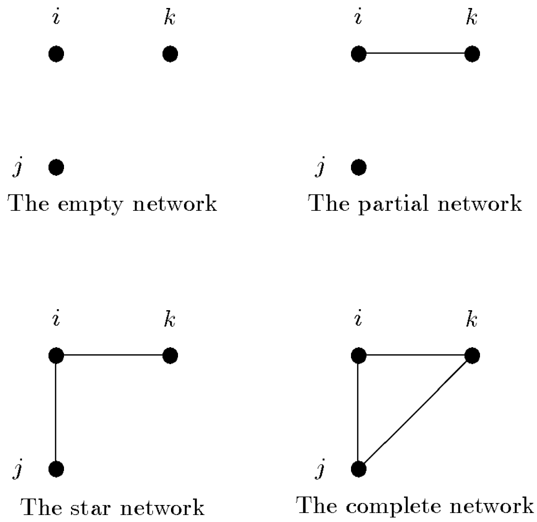

Figure 1: Four possible network architectures.

helps lower its own marginal cost of production. Given a network g and the collection of research outputs $\{x_i\}_{i \in N}$, the marginal cost of production for each firm $i \in N$ becomes

$$c_i(g) = \bar{c} - x_i - \sum_{k \in N_i(g)} x_k - \phi \sum_{l \notin N_i(g)} x_l. \tag{1}$$

Let

$$X_i \equiv x_i + \sum_{k \in N_i(g)} x_k + \phi \sum_{l \notin N_i(g)} x_l \tag{2}$$

be the total cost reduction for firm i obtained from its own research, x_i, from the research knowledge of firms that have a collaborative link with i which is fully absorbed, and from the research knowledge of firms that do not have a collaborative link with i which is partially absorbed depending on the spillover parameter $\phi \in [0, 1)$. We refer to this total cost reduction, X_i, as effective R&D output of firm i. Then, $c_i(g) = \bar{c} - X_i$. We assume that R&D effort is costly. Given a level $x_i \in [0, \bar{c}]$ of effort, the cost of effort is $y(x_i) = \gamma x_i^2$, $\gamma > 0$. We assume $\gamma = 1$ which suffices to ensure nonnegativity of all variables.

Thus, the profits of firm $i \in N$ in a collaboration network g are given by

$$\Pi_i(g) = \left[a - q_i(g) - \sum_{j \neq i} q_j(g) - c_i(g) \right] q_i(g) - (1 - s_i) \left[x_i(g) \right]^2. \tag{3}$$

For any network g, social welfare in each country i is defined as the profits of firm i minus the R&D subsidies. The objective function of each government is its social welfare. Let $W_i(g)$ denote social welfare of country i in network g.

$$W_i(g) = \Pi_i(g) - s_i \left[x_i(g) \right]^2. \tag{4}$$

Let $W(g) = \sum_{i \in N} W_i(g)$. We also define a concept of global social welfare, which is defined by the sum of producing country's welfare and importing country's consumer surplus:

$$V(g) = \sum_i W_i(g) + \frac{Q^2}{2}.$$

We describe the interaction between the firms and governments using a four-stage game. In the first stage, firms form pairwise collaboration links. In the second stage each government simultaneously announces its R&D subsidy (tax if negative) rates. In the third stage, each firm chooses independently a level of effort in R&D. In the fourth stage, firms compete in the product market of a fourth country by setting quantities. This multi-stage game is solved by backward induction.

Once we allow the government to subsidize R&D and to choose the subsidy rate (so adding a fourth stage to Goyal and Moraga-González' model), the solution of the whole game becomes much more complex, especially when we are solving for the asymmetric networks (partially connected and star networks). As a consequence, we cannot obtain closed-form solutions for the asymmetric networks when subsidies are allowed and endogenous. However, for each possible given value of public spillovers we are able to compute the equilibrium solution. Thus, we propose to focus on four different cases with respect to public spillovers ϕ: (i) no spillovers, $\phi = 0$, (ii) weak spillovers, $\phi = \frac{1}{4}$, (iii) medium spillovers $\phi = \frac{1}{2}$, (iv) strong spillovers, $\phi = \frac{3}{4}$; and we will analyze numerically the general case where $\phi \in [0, \frac{1}{2}]$ and we will show that the results obtained for the four different cases do not hide any irregularities.

3 Stability of international R&D networks

3.1 Pairwise and strong stability

A simple way to analyze the networks that one might expect to emerge in the long run is to examine a sort of equilibrium requirement that agents do not benefit from altering the structure of the network. A weak version of such condition is the pairwise stability notion defined by Jackson and Wolinsky (1996). A network is pairwise stable if no agent benefits from severing one of their links and no other two agents benefit from adding a link between them, with one benefiting strictly and the other at least weakly.

Definition 1 *A network g is pairwise stable if*

- *for all $ij \in g$, $\Pi_i(g) \geq \Pi_i(g - ij)$ and $\Pi_j(g) \geq \Pi_j(g - ij)$, and*
- *for all $ij \notin g$, if $\Pi_i(g) < \Pi_i(g + ij)$ then $\Pi_j(g) > \Pi_j(g + ij)$.*

Let us say that g' is adjacent to g if $g' = g + ij$ or $g' = g - ij$ for some ij. A network g' defeats g if either $g' = g - ij$ and $\Pi_i(g') \geq \Pi_i(g)$, or if $g' = g + ij$ with $\Pi_i(g') \geq \Pi_i(g)$ and $\Pi_j(g') \geq \Pi_j(g)$ with at least one inequality holding strictly. Pairwise stability is equivalent to saying that a network is pairwise stable if it is not defeated by another (necessarily adjacent) network. This definition of stability is quite weak and should be seen as a necessary condition for strategic stability.

While pairwise stability is natural and quite easy to work with, there are some limitations of the concept. First, it is a weak notion in that it only considers deviations on a single link at a time. For instance, it could be that an agent would not benefit from severing any single link but would benefit from severing several links simultaneously, and yet the network would still be pairwise stable. Second, pairwise stability considers only deviations by at most a pair of agents at a time. It might be that some group of agents could all be made better off by some complicated reorganization of their links, which is not accounted for under pairwise stability. The definition of strong stable networks is in that spirit, and is due to Jackson and van den Nouweland (2004). A strongly stable network is a network which is stable against changes in links by any coalition of agents.

A network $g' \in G$ is obtainable from $g \in G$ via deviations by S if

(i) $ij \in g'$ and $ij \notin g$ implies $ij \subset S$, and

(ii) $ij \in g$ and $ij \notin g'$ implies $ij \cap S \neq \varnothing$.

The above definition identifies changes in a network that can be made by a coalition S, without the need of consent of any agents outside of S. Part (i) requires that any new links that are added can only be between agents in S. This reflects the fact that consent of both agents is needed to add a link. Part (ii) requires that at least one agent of any deleted link be in S. This reflects the fact that either agent in a link can unilaterally sever the relationship.

Definition 2 *A network g is strongly stable if for any $S \subset N$, g' that is obtainable from g via deviations by S, and $i \in S$ such that $\Pi_i(g') > \Pi_i(g)$, there exists $j \in S$ such that $\Pi_j(g') < \Pi_j(g)$.*

Strong stability provides a powerful refinement of pairwise stability. The concept of strong stability mainly makes sense in smaller network situations where agents have substantial information about the overall structure and potential payoffs and can coordinate their actions. That is, it makes sense to model agreements between firms in an oligopoly.

We are interested in the networks of international R&D collaboration that emerge in two different settings: with subsidies or without subsidies ($s_i = 0$); and in four different situations: (i) no spillovers, $\phi = 0$, (ii) weak spillovers, $\phi = \frac{1}{4}$, (iii) medium spillovers $\phi = \frac{1}{2}$, (iv) strong spillovers, $\phi = \frac{3}{4}$.

3.2 Stable R&D networks without subsidies

In order to characterize the strongly stable R&D networks we first derive the pairwise stable networks since a strongly stable network is pairwise stable while the reverse is not true. From Goyal and Moraga-González (2001), if there are no subsidies, the profits of the firm at equilibrium are given in Table 1 (see also the appendix). The next proposition is a corollary of Proposition 9 in Goyal and Moraga-González (2001) whose proof is given for completeness.

		g^c	g^s	g^p	g^e
$\phi = 0$	Π_i	$.089\,(a-\overline{c})^2$	$.161\,(a-\overline{c})^2$	$.188\,(a-\overline{c})^2$	$.042\,(a-\overline{c})^2$
	Π_j	$.089\,(a-\overline{c})^2$	$.057\,(a-\overline{c})^2$	0	$.042\,(a-\overline{c})^2$
$\phi = \frac{1}{4}$	Π_i	$.089\,(a-\overline{c})^2$	$.139\,(a-\overline{c})^2$	$.113\,(a-\overline{c})^2$	$.065\,(a-\overline{c})^2$
	Π_j	$.089\,(a-\overline{c})^2$	$.068\,(a-\overline{c})^2$	$.036\,(a-\overline{c})^2$	$.065\,(a-\overline{c})^2$
$\phi = \frac{1}{2}$	Π_i	$.089\,(a-\overline{c})^2$	$.119\,(a-\overline{c})^2$	$.104\,(a-\overline{c})^2$	$.083\,(a-\overline{c})^2$
	Π_j	$.089\,(a-\overline{c})^2$	$.078\,(a-\overline{c})^2$	$.063\,(a-\overline{c})^2$	$.083\,(a-\overline{c})^2$
$\phi = \frac{3}{4}$	Π_i	$.089\,(a-\overline{c})^2$	$.103\,(a-\overline{c})^2$	$.097\,(a-\overline{c})^2$	$.092\,(a-\overline{c})^2$
	Π_j	$.089\,(a-\overline{c})^2$	$.085\,(a-\overline{c})^2$	$.081\,(a-\overline{c})^2$	$.092\,(a-\overline{c})^2$

Table 1: Firm's profits when governments cannot subsidize R&D.

Proposition 1 *Suppose that governments cannot subsidize research and development. (i) The complete network g^c is always pairwise stable, (ii) the partially connected network g^p is pairwise stable only if there are no public spillovers ($\phi = 0$), (iii) the star and empty networks (respectively, g^s and g^e) are never pairwise stable.*

Proof. First we show that the complete network g^c is always pairwise stable. No pair of firms k and j have incentives to delete their link $kj \in g^c$. From Table 1 we have $\Pi_k^*(g^c) > \Pi_k^*(g^s)$ and $\Pi_j^*(g^c) > \Pi_j^*(g^s)$ with $kj \notin g^s$. Thus, g^c is pairwise stable. Obviously, the star network g^s cannot be pairwise stable since firms k and j have incentives to form the link $kj \notin g^s$. Second, the empty network g^e is never pairwise stable because we have $\Pi_i^*(g^p) > \Pi_i^*(g^e)$ and $\Pi_k^*(g^p) > \Pi_k^*(g^e)$ with $ik \in g^p$. Third, since the empty network is never pairwise stable, the network g^p is pairwise stable if and only if $\Pi_i^*(g^p) > \Pi_i^*(g^s)$ or $\Pi_j^*(g^p) > \Pi_j^*(g^s)$ with $ij \notin g^p$, $ij \in g^s$, and $j \notin N(g^p)$. From Table 1 the partially connected network g^p is pairwise stable only if there are no public spillovers ($\phi = 0$). ■

Network structures are more important when public spillovers are modest. When there are no public spillovers ($\phi = 0$), the partially connected network is pairwise stable. The isolated firm has a significant cost disadvantage and is driven out of the market. However, public spillovers destabilize the partially connected network. The intuition behind this remark is that the stability of the partially connected network relies on the great asymmetry existing between the linked firms and the isolated firm. It is this asymmetry that discourages a linked firm from forming a link with the isolated firm when public spillovers are absent. As spillovers are weak, medium or strong, this asymmetry reduces, and that destabilizes the partially connected network.

Proposition 2 *Suppose that governments cannot subsidize research and development. The partially connected network g^p is the unique strongly stable network if and only if there are no public spillovers ($\phi = 0$). Otherwise, no network $g \in G$ is strongly stable.*

Proof. First, since strong stability is a refinement of pairwise stability, we have that the empty and star networks are never strongly stable. Second, we show that the complete network g^c is never strongly stable. Indeed, from Table 1 we have $\Pi_i^*(g^p) > \Pi_i^*(g^c)$ and $\Pi_k^*(g^p) > \Pi_k^*(g^c)$ with $ik \in g^p$. Third, from Proposition 1 we know that the partially connected network is not pairwise stable if either public spillovers are weak, medium or strong; and so is not strongly stable. But, if there are no public spillovers ($\phi = 0$), then g^p is pairwise stable. Is g^p strongly stable too? Since g^p is pairwise stable, it suffices to show that no coalition has incentives to add links to form the complete network g^c. The answer is no since $\Pi_i^*(g^p) > \Pi_i^*(g^c)$ and $\Pi_k^*(g^p) > \Pi_k^*(g^c)$ with $ik \in g^p$ as shown above. Thus, if there are no public spillovers ($\phi = 0$), then g^p is the unique strongly stable network; otherwise no network is strongly stable. ■

Since a strongly stable network is a pairwise stable network, the only two candidates to be strongly stable are g^p and g^c when governments cannot subsidize R&D. We observe that in the four cases ($\phi = 0$, $\phi = \frac{1}{4}$, $\phi = \frac{1}{2}$, $\phi = \frac{3}{4}$) the complete network g^c is never strongly stable because two firms have incentives to form a coalition and to delete their links with the third firm; so moving to the partially connected network g^p. Thus, g^p is strongly stable if there are no public spillovers ($\phi = 0$), and it is the unique one. Otherwise, no network $g \in G$ is strongly stable.

Whenever no network is strongly stable we will observe a sequence of R&D networks due to continuously profitable deviations. In terms of competition policy, it would be interesting to know which networks are likely to be visited by such sequence of profitable deviations. In fact we will show that some R&D networks will be visited at most once, while others will belong to a closed cycle and will be visited regularly. We now define what is meant by a closed cycle. A network g' strongly defeats g if (i) g' is obtainable from g via deviations by $S \subset N$ and (ii) $\Pi_i(g') \geq \Pi_i(g)$ for all $i \in S$ and $\Pi_j(g') > \Pi_j(g)$ for some $j \in S$. An improving path from a network g to a network g' is a finite sequence of graphs $g_1, g_2, ..., g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, ..., K-1\}$ we have g_{k+1} strongly defeats g_k. A set of networks $\overline{G}$ form a cycle if for any $g \in \overline{G}$ and $g' \in \overline{G}$ there exists an improving path connecting g to g'. A cycle $\overline{G}$ is a closed cycle if no network in $\overline{G}$ lies on an improving path leading to a network that is not in $\overline{G}$. The characterization of the closed cycles follows immediately from the proofs of pairwise and strong stable R&D networks. When no strongly stable network exists we will observe a unique closed cycle of R&D networks where the star network will succeed to the partially connected network, the complete network will succeed to the star network, and the partially connected network will succeed to the complete network. The empty network which is the only network outside the closed cycle will be visited at most once. In fact, it will be visited only if it is the initial network.

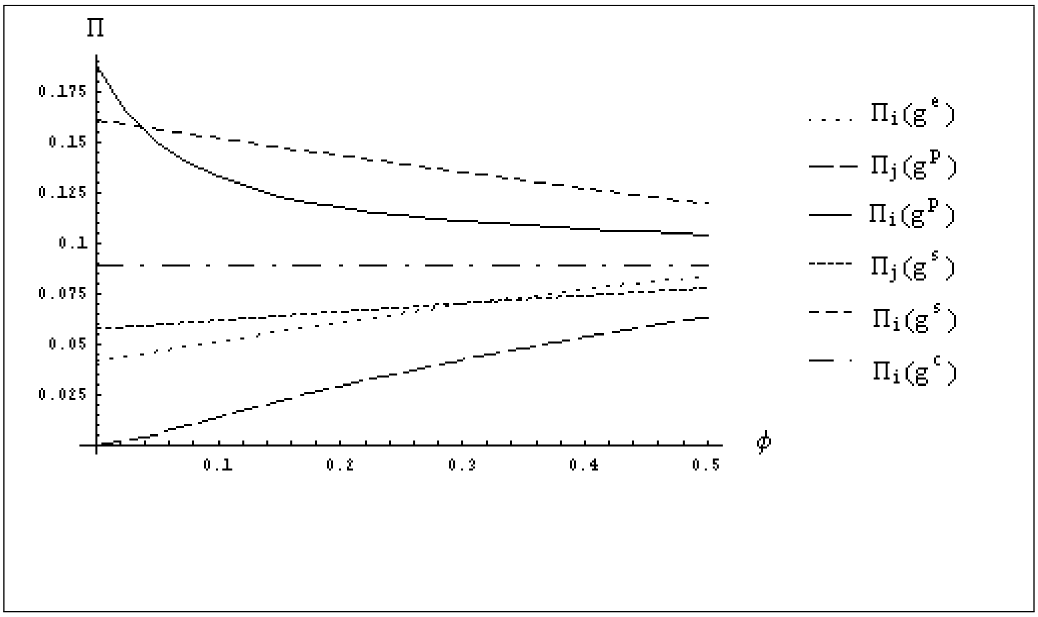

Figure 2: Profits of each firm when governments cannot subsidize R&D.

We would like to examine more deeply the relation between stable networks and public spillovers; that is, for $\phi \in [0, \frac{1}{2}]$. The equilibrium values of the profits in the different networks are given in the appendix and are plotted (for $a - \overline{c} = 1$) in Figure 2. Using Figure 2 we can study the stability of different networks with respect to public spillovers when governments cannot subsidize R&D as in Goyal and Moraga-González (2001). The complete network g^c is pairwise stable for all $\phi \in [0, \frac{1}{2}]$, while the partially connected network g^p is pairwise stable for $\phi \in [0, \overline{\phi}]$ where $\overline{\phi} \simeq 0.04$ is the solution to equation $\Pi_i(g^s) = \Pi_i(g^p)$ with firm i being a hub in g^s and a linked firm in g^p. The star network and the empty network are never pairwise stable. As already mentioned, greater spillovers destabilize the partially connected network rapidly. We have that the complete network g^c is never strongly stable because two firms have incentives to form a coalition and to delete their links with the third firm; so moving to the partially connected network g^p. Thus, g^p is strongly stable if $\phi \leq \overline{\phi}$ where $\overline{\phi} \simeq 0.04$ is the solution to equation $\Pi_i(g^s) = \Pi_i(g^p)$ with firm i being a hub in g^s and a linked firm in g^p. So, if $\phi \leq \overline{\phi} \simeq 0.04$ the partially connected network g^p is the unique strongly stable network; otherwise, no network $g \in G$ is strongly stable (a formal proof is given in the appendix).

3.3 Stable R&D networks with subsidies

Whenever governments can subsidize R&D, the subsidies and profits of the firm at equilibrium are given in Table 2 and Table 3, respectively. Some observations are made: (i) in the complete network g^c, the equilibrium subsidy rate does not depend on the public spillovers; (ii) in the star network g^s, the equilibrium subsidy rates are decreasing with the public spillovers; (iii) in the empty network g^e, the subsidy rate first decreases with public spillovers, then it increases with spillovers; (iv) in the partially connected network g^p, there is a continuum of optimal subsidy rates for the government of the isolated firm when there are no public spillovers. The reason is that the governments of the linked firms do not need to subsidize R&D to keep the isolated firm out of the market. Thus, the continuum of optimal subsidy rates for the government of the isolated firm are the levels of subsidies such that the isolated firm does not find profitable to produce. For instance, if $s_j > 1.375$ then it would be profitable to produce and to enter the market. However, the welfare of the country of the isolated firm would be negative.

		g^c	g^s	g^p	g^e
$\phi = 0$	s_i	.146	.5	0	.25
	s_j	.146	.5	$[0, 1.375]$	.25
$\phi = \frac{1}{4}$	s_i	.146	.181	.344	.046
	s_j	.146	.334	.406	.046
$\phi = \frac{1}{2}$	s_i	.146	.132	.169	0
	s_j	.146	.257	0	0
$\phi = \frac{3}{4}$	s_i	.146	.086	.019	.039
	s_j	.146	.197	.122	.039

Table 2: Governments R&D subsidy rates.

		g^c	g^s	g^p	g^e
$\phi = 0$	Π_i	$.095\,(a-\overline{c})^2$	$.389\,(a-\overline{c})^2$	$.188\,(a-\overline{c})^2$	$.028\,(a-\overline{c})^2$
	Π_j	$.095\,(a-\overline{c})^2$	$.056\,(a-\overline{c})^2$	0	$.028\,(a-\overline{c})^2$
$\phi = \frac{1}{4}$	Π_i	$.095\,(a-\overline{c})^2$	$.173\,(a-\overline{c})^2$	$.255\,(a-\overline{c})^2$	$.065\,(a-\overline{c})^2$
	Π_j	$.095\,(a-\overline{c})^2$	$.075\,(a-\overline{c})^2$	0	$.065\,(a-\overline{c})^2$
$\phi = \frac{1}{2}$	Π_i	$.095\,(a-\overline{c})^2$	$.132\,(a-\overline{c})^2$	$.119\,(a-\overline{c})^2$	$.083\,(a-\overline{c})^2$
	Π_j	$.095\,(a-\overline{c})^2$	$.084\,(a-\overline{c})^2$	$.057\,(a-\overline{c})^2$	$.083\,(a-\overline{c})^2$
$\phi = \frac{3}{4}$	Π_i	$.095\,(a-\overline{c})^2$	$.110\,(a-\overline{c})^2$	$.103\,(a-\overline{c})^2$	$.093\,(a-\overline{c})^2$
	Π_j	$.095\,(a-\overline{c})^2$	$.090\,(a-\overline{c})^2$	$.082\,(a-\overline{c})^2$	$.093\,(a-\overline{c})^2$

Table 3: Firm's profits when governments can subsidize R&D.

Comparing Table 1 with Table 3 we observe that allowing governments to subsidize R&D does not necessarily increase firms' profits, except in the complete network g^c. For instance, in the empty network g^e without public spillovers, subsidies will push firms to overinvest even more in R&D in order to gain market shares from their rivals.[5] In the complete networks g^c, this effect is absent because each firm benefits entirely from the R&D effort of each other firm. We now study pairwise and strongly stable networks when governments can subsidize R&D.

Proposition 3 *Suppose that governments can subsidize research and development. (i) The complete network g^c is always pairwise stable, (ii) the partially connected network g^p is pairwise stable if only if there are weak public spillovers ($\phi = \frac{1}{4}$), (iii) the star and empty networks (respectively, g^s and g^e) are never pairwise stable.*

Proof. First we show that the complete network g^c is always pairwise stable. No pair of firms k and j have incentives to delete their link $kj \in g^c$. From Table 3 we have $\Pi_k^*(g^c) > \Pi_k^*(g^s)$ and $\Pi_j^*(g^c) > \Pi_j^*(g^s)$ with $kj \notin g^s$. Thus, g^c is pairwise stable. Obviously, the star network g^s cannot be pairwise stable since firms k and j have incentives to form the link $kj \notin g^s$. Second, the empty network g^e is never pairwise stable because we have $\Pi_i^*(g^p) > \Pi_i^*(g^e)$ and $\Pi_k^*(g^p) > \Pi_k^*(g^e)$ with $ik \in g^p$. Third, since the empty network is never pairwise stable, the network g^p is pairwise stable if and only if $\Pi_i^*(g^p) > \Pi_i^*(g^s)$ or $\Pi_j^*(g^p) > \Pi_j^*(g^s)$ with $ij \notin g^p$, $ij \in g^s$, and $j \notin N(g^p)$. From Table 3 the partially connected network g^p is pairwise stable only if there are weak public spillovers ($\phi = \frac{1}{4}$). ■

[5] An R&D subsidy by the "domestic" government enhances the firm's cost advantage through discouraging the "foreign" rival's R&D investment, which in turn enables the "domestic" firm to snatch a larger market share at the expense of the other "foreign" firms.

Proposition 4 *Suppose that governments can subsidize research and development. The partially connected network g^p is the unique strongly stable network if and only if there are weak public spillovers ($\phi = \frac{1}{4}$). Otherwise, no network $g \in G$ is strongly stable.*

Proof. First, since strong stability is a refinement of pairwise stability, we have that the empty and star networks are never strongly stable. Second, we show that the complete network g^c is never strongly stable. Indeed, from Table 3 we have $\Pi_i^*(g^p) > \Pi_i^*(g^c)$ and $\Pi_k^*(g^p) > \Pi_k^*(g^c)$ with $ik \in g^p$. Third, from Proposition 3 we know that the partially connected network is not pairwise stable if either public spillovers are absent, medium or strong; and so is not strongly stable. But, if there are weak public spillovers ($\phi = \frac{1}{4}$), then g^p is pairwise stable. Is g^p strongly stable too? Since g^p is pairwise stable, it suffices to show that no coalition has incentives to add links to form the complete network g^c. The answer is no since $\Pi_i^*(g^p) > \Pi_i^*(g^c)$ and $\Pi_k^*(g^p) > \Pi_k^*(g^c)$ with $ik \in g^p$ as shown above. Thus, if there are weak public spillovers ($\phi = \frac{1}{4}$), then g^p is the unique strongly stable network; otherwise no network is strongly stable. ■

We observe again that the complete network g^c is never strongly stable because two firms have incentives to move to the partially connected network g^p by deleting their link with the third firm. Thus, g^p is strongly stable for $\phi = \frac{1}{4}$; otherwise, no network is strongly stable. When no network is strongly stable we will observe a sequence of R&D networks due to continuously profitable deviations. In fact, we will again observe a unique closed cycle of R&D networks where the star network will succeed to the partially connected network, the complete network will succeed to the star network, and the partially connected network will succeed to the complete network.

Once we allow the government to subsidize R&D and to choose the subsidy rate, the solution of the whole game becomes much more complex, especially when we are solving for the partially connected and star networks; and, we cannot obtain closed-form solutions for those asymmetric networks. However, we can analyze numerically the general case where $\phi \in [0, \frac{1}{2}]$. The equilibrium values (for $a - \overline{c} = 1$) of the profits in the different networks are plotted in Figure 3. Using Figure 3 we observe that the complete network g^c is pairwise stable for all $\phi \in [0, \frac{1}{2}]$. The profits of the hub firm in the star network g^s are still decreasing with ϕ. But, the profits of a linked firm in the partially connected network g^p are now first increasing with ϕ until it becomes profitable for the isolated firm to enter the market, then the profits start to decrease with ϕ. As a consequence, we have that the partially connected network g^p is pairwise stable when ϕ is not too small nor too large. Precisely, g^p is pairwise stable for $\phi \in [\underline{\phi}, \overline{\phi}]$ where $\underline{\phi} \simeq 0.125$ and $\overline{\phi} \simeq 0.36$ are the solutions to the equation $\Pi_i(g^s) = \Pi_i(g^p)$ with firm i being a hub in g^s and a linked firm in g^p. The

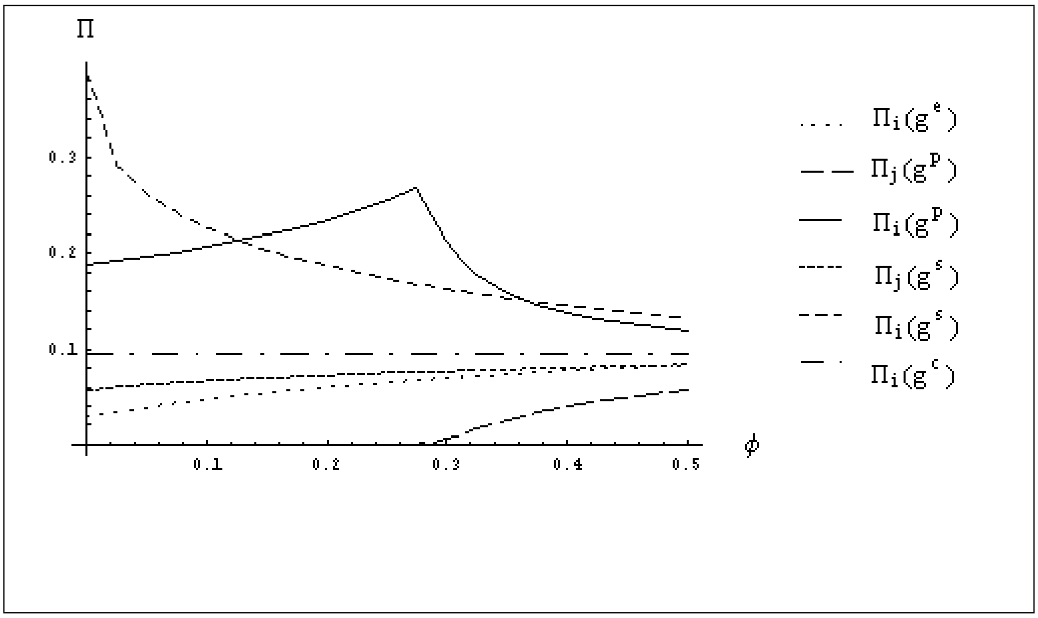

Figure 3: Profits of each firm when governments can subsidize R&D.

star network and the empty network are never pairwise stable. With respect to strong stability, we observe again that the complete network g^c is never strongly stable because two firms have incentives to move to the partially connected network g^p by deleting their link with the third firm. Thus, g^p is strongly stable for $\phi \in [\underline{\phi}, \overline{\phi}]$ where $\underline{\phi} \simeq 0.125$ and $\overline{\phi} \simeq 0.36$ are the solutions to the equation $\Pi_i(g^s) = \Pi_i(g^p)$ with firm i being a hub in g^s and a linked firm in g^p. So, if $\phi \in [\underline{\phi}, \overline{\phi}]$ the partially connected network g^p is the unique strongly stable network. Otherwise, no network $g \in G$ is strongly stable.

We now provide some intuition for the fact that, once governments can subsidize R&D, the partially connected network g^p is no more stable for very small public spillovers. For very small spillovers we have: (i) $\Pi_i(g^s) < \Pi_i(g^p)$ when governments cannot subsidize and (ii) $\Pi_i(g^s) > \Pi_i(g^p)$ when governments can subsidize, with i being a hub in g^s and a linked firm in g^p. Subsidies have a double effect on the profits of linked firms in the partially connected network g^p. First, there is a bigger overinvestment in R&D due to subsidies that implies a stronger competition between the two linked firms and tends to reduce profits. Second, there is a cost advantage for the linked firms with respect to the isolated firm that tends to increase profits. The first effect (which is much weaker when government cannot subsidize) dominates the second effect for low values of spillovers when government can subsidize R&D. Notice that overinvestment in R&D is decreasing with ϕ.

4 Efficiency of international R&D networks

In evaluating societal welfare, we may take various perspectives. A network g is *Pareto efficient* if there does not exist any $g' \subset G$ such that $W_i(g') \geq W_i(g)$ for each country i with strict inequality for some country i. This definition of efficiency of a network can be thought of as applying to situations where no intervention is possible. A network $g \subset G$ is *strongly efficient* if $W(g) = \sum_i W_i(g) \geq \sum_i W_i(g') = W(g')$ for all $g' \subset G$. This is a strong notion of efficiency as it takes the perspective that value is fully transferable.

If there are no subsidies, the social welfare of each country at equilibrium is simply the profit of its firm. Thus, social welfare of each country at equilibrium is given in Table 1 and some observations can be made. Suppose that governments cannot subsidize research and development:

(i) The partially connected network g^p is the strongly efficient network if and only if public spillovers are absent ($\phi = 0$) or weak ($\phi = \frac{1}{4}$) or strong ($\phi = \frac{3}{4}$). Otherwise, the star network g^s is the strongly efficient network (i.e. for $\phi = \frac{1}{2}$).

(ii) The partially connected network g^p and the star network g^s are always Pareto efficient, the complete network g^c is always Pareto efficient except if public spillovers are strong ($\phi = \frac{3}{4}$), and the empty network g^e is Pareto efficient only if public spillovers are strong ($\phi = \frac{3}{4}$).

Once governments can subsidize R&D, the equilibrium social welfare of each country is given in Table 4 and some observations can be made. Suppose that governments can subsidize research and development:

(i) The partially connected network g^p is the strongly efficient network if and only if public spillovers are absent ($\phi = 0$) or weak ($\phi = \frac{1}{4}$), and the star network g^s is the strongly efficient network if and only if public spillovers are medium ($\phi = \frac{1}{2}$) or strong ($\phi = \frac{3}{4}$).

(ii) The partially connected network g^p and the star network g^s are always Pareto efficient, the complete network g^c is always Pareto efficient except if public spillovers are strong ($\phi = \frac{3}{4}$), and the empty network g^e is Pareto efficient only if public spillovers are strong ($\phi = \frac{3}{4}$).

A question we would like to answer is whether allowing for subsidies is superior in terms of societal welfare. In order to answer this question we compare the societal welfare levels of the different pairwise stable networks when governments cannot subsidize R&D

		g^c	g^s	g^p	g^e
$\phi = 0$	W_i	$.094\,(a-\overline{c})^2$	$.333\,(a-\overline{c})^2$	$.188\,(a-\overline{c})^2$	0
	W_j	$.094\,(a-\overline{c})^2$	0	0	0
$\phi = \frac{1}{4}$	W_i	$.094\,(a-\overline{c})^2$	$.164\,(a-\overline{c})^2$	$.200\,(a-\overline{c})^2$	$.063\,(a-\overline{c})^2$
	W_j	$.094\,(a-\overline{c})^2$	$.070\,(a-\overline{c})^2$	0	$.063\,(a-\overline{c})^2$
$\phi = \frac{1}{2}$	W_i	$.094\,(a-\overline{c})^2$	$.128\,(a-\overline{c})^2$	$.114\,(a-\overline{c})^2$	$.083\,(a-\overline{c})^2$
	W_j	$.094\,(a-\overline{c})^2$	$.082\,(a-\overline{c})^2$	$.057\,(a-\overline{c})^2$	$.083\,(a-\overline{c})^2$
$\phi = \frac{3}{4}$	W_i	$.094\,(a-\overline{c})^2$	$.107\,(a-\overline{c})^2$	$.101\,(a-\overline{c})^2$	$.093\,(a-\overline{c})^2$
	W_j	$.094\,(a-\overline{c})^2$	$.089\,(a-\overline{c})^2$	$.082\,(a-\overline{c})^2$	$.093\,(a-\overline{c})^2$

Table 4: Social welfare of each country when governments can subsidize R&D.

and when governments can do it. Using Table 1 and Table 4, as well as the results on pairwise stable networks, we get the following proposition.

Proposition 5 *Except if there are no public spillovers* ($\phi = 0$)*, societal welfare is higher when governments can subsidize research and development.*

From the above observations we have that a conflict between stability and strong efficiency may occur when governments cannot subsidize R&D as well as when they can do it. When governments cannot subsidize R&D, the conflict will occur except if spillovers are absent ($\phi = 0$). When governments can subsidize R&D, the conflict will occur except if spillovers are weak ($\phi = \frac{1}{4}$). Thus, a conflict is likely to arise but we do not know whether it is more likely when governments can subsidize or when governments cannot.

Regarding the general case $\phi \in [0, \frac{1}{2}]$, the equilibrium values (for $a - \overline{c} = 1$) of the societal welfare in the different networks are plotted in Figure 4 and Figure 5. Remember that societal welfare is simply the sum of the welfare of producing countries. When governments are not allowed to subsidize R&D (see Figure 4), the partially connected network g^p is the strongly efficient network for $\phi \in [0, \widehat{\phi}]$ where $\widehat{\phi} \simeq 0.113$ is the solution to equation $W(g^s) = W(g^p)$. For $\phi \in [\widehat{\phi}, \frac{1}{2}]$, the star network g^s is the strongly efficient network. When governments can subsidize R&D (see Figure 5), the partially connected network g^p is the strongly efficient network for $\phi \in [0, \widehat{\phi}]$ where $\widehat{\phi} \simeq 0.375$ is the solution to equation $W(g^s) = W(g^p)$. For $\phi \in [\widehat{\phi}, \frac{1}{2}]$, the star network g^s is the strongly efficient network.

Figure 6 and Figure 7 contrast the strongly efficient and stable networks with respect to public spillovers. We observe that, when governments cannot subsidize R&D, the likelihood of a conflict is 92%. This conflict will arise if spillovers are not very small. However, when governments can subsidize R&D, the likelihood of a conflict is considerably

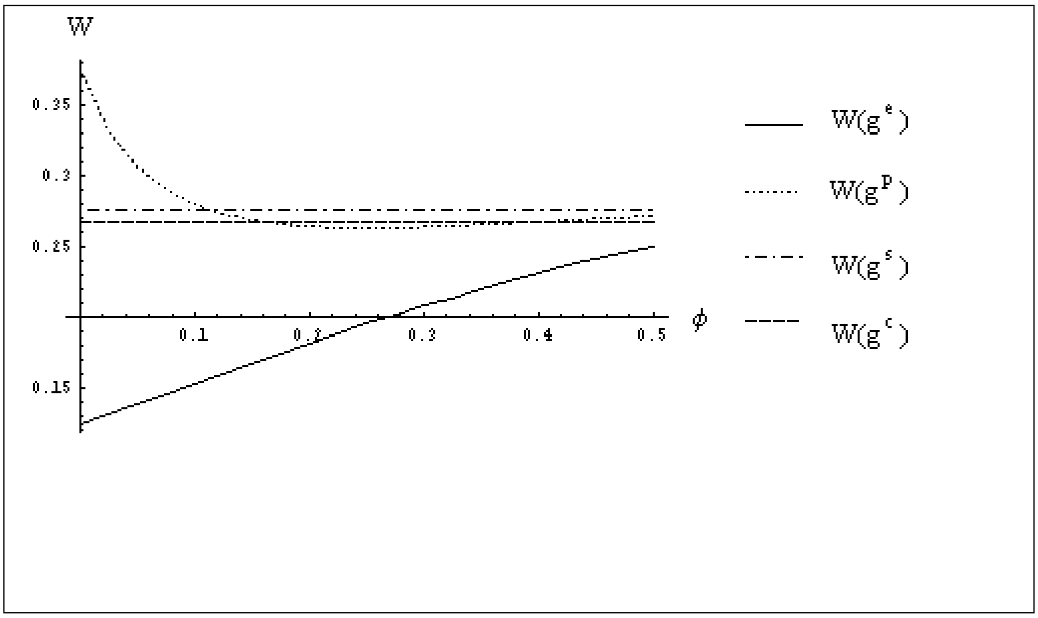

Figure 4: Societal welfare when government cannot subsidize R&D.

reduced to 53%. Indeed, a conflict will arise "only" if spillovers are very small or quite large. Notice that the same conflict exists if we consider the global welfare (which includes the consumer surplus of the fourth country) instead of the sum of the welfare of producing countries (see Figure 9 and Figure 11 of the appendix).

Finally, using Figure 4 and Figure 5 (and the results on pairwise stable networks), we compare the societal welfare levels of the different pairwise stable networks when governments cannot subsidize R&D and when governments can do it, and we confirm that, except if public spillovers are very small ($\phi \leq 0.04$), one should definitely allow for R&D subsidies. As public spillovers grows, it becomes always superior to allow governments to subsidize R&D.

5 Conclusion

It has become increasingly prevalent that rival firms of different countries engage in R&D collaboration. Thus, it is important to understand how government policies affect the stability and efficiency of international R&D collaboration networks. We have shown that a conflict between stability and efficiency is likely to occur. When governments cannot subsidize R&D, this conflict occurs if public spillovers are not too small. However, when governments can subsidize R&D, the likelihood of a conflict is considerably reduced. Indeed, a conflict arises only if public spillovers are very small or quite large.

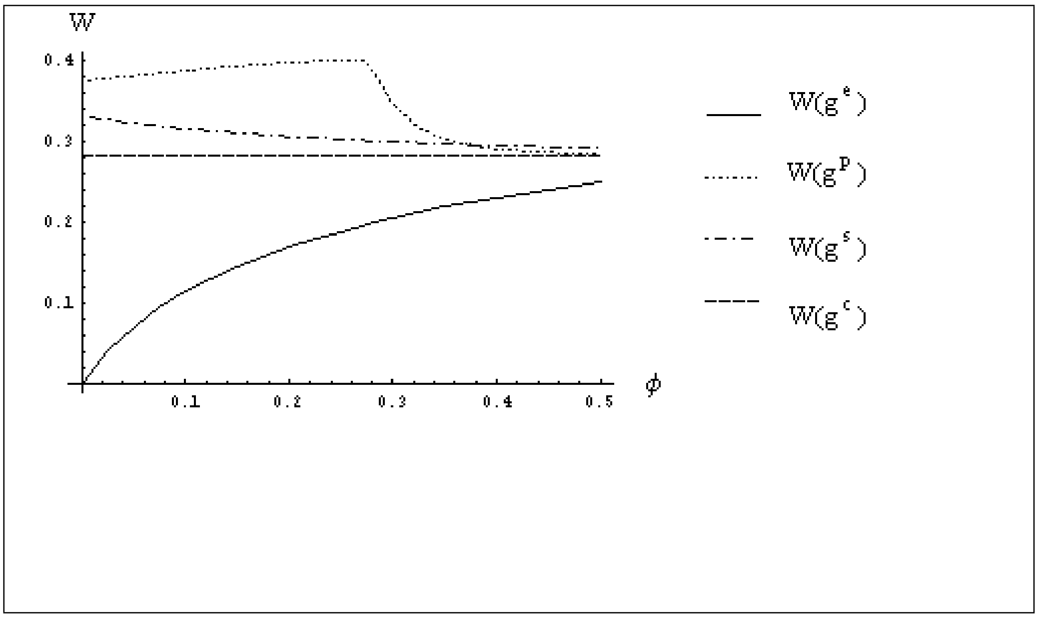

Figure 5: Societal welfare when government can subsidize R&D.

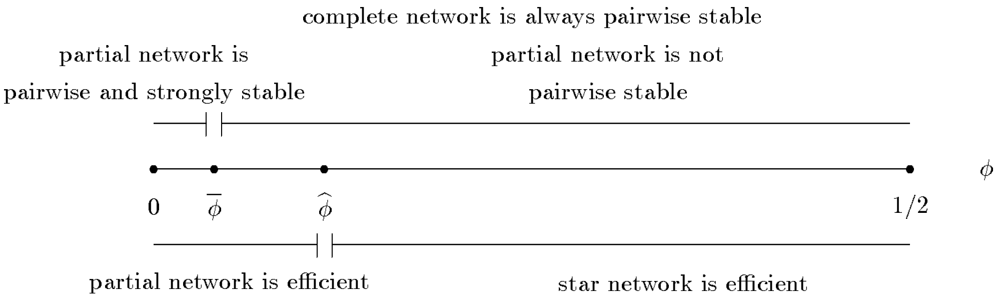

Figure 6: The conflict between stability and strong efficiency without R&D subsidies.

Acknowledgments

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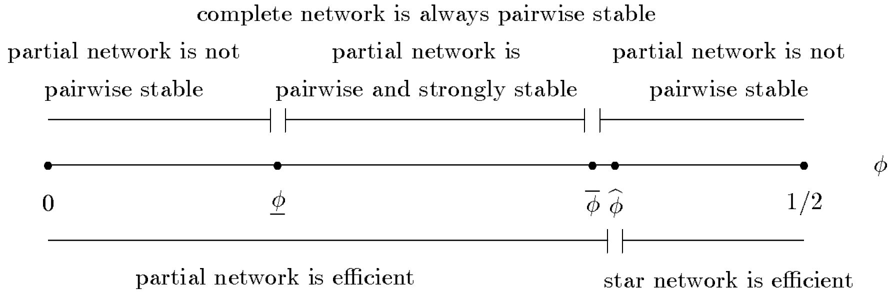

Figure 7: The conflict between stability and strong efficiency with R&D subsidies.

Appendix A: Networks without subsidies

From Goyal and Moraga-González (2001) we have that, in the complete network g^c, the equilibrium effort, effective R&D output, quantity, and profits are given by

$$x_i^*(g^c) = \frac{a-\overline{c}}{13};\ X_i^*(g^c) = \frac{3(a-\overline{c})}{13};\ q_i^*(g^c) = \frac{4(a-\overline{c})}{13};\ \Pi_i^*(g^c) = \frac{15(a-\overline{c})^2}{169}.$$

Consider the star network g^s and let firm i be the hub and firm j be a spoke. Then, the equilibrium effort levels of the different firms are

$$x_j^*(g^s) = \frac{4(a-\overline{c})(2-\phi)}{7\phi^2-13\phi+58};\ x_i^*(g^s) = \frac{(a-\overline{c})(\phi^2-3\phi+6)}{7\phi^2-13\phi+58}.$$

and the effective R&D outputs are

$$X_j^*(g^s) = \frac{(a-\overline{c})(14+\phi-3\phi^2)}{7\phi^2-13\phi+58};\ X_i^*(g^s) = \frac{(a-\overline{c})(22+(-11+\phi)\phi)}{7\phi^2-13\phi+58}.$$

The equilibrium quantities and profits are

$$\begin{aligned} q_j^*(g^s) &= \frac{16(a-\overline{c})}{7\phi^2-13\phi+58};\ \Pi_j^*(g^s) = \frac{16(a-\overline{c})^2(12-\phi^2+4\phi)}{(7\phi^2-13\phi+58)^2}; \\ q_i^*(g^s) &= \frac{4(a-\overline{c})(\phi^2-3\phi+6)}{7\phi^2-13\phi+58};\ \Pi_i^*(g^s) = \frac{15(a-\overline{c})^2(\phi^2-3\phi+6)^2}{(7\phi^2-13\phi+58)^2}. \end{aligned}$$

Consider the partially connected network g^p and let firms i and k be the linked firms and firm j be the isolated firm. Then, the equilibrium effort levels of the different firms are

$$x_i^*(g^p) = \frac{(a-\overline{c})(2+9\phi-9\phi^2+2\phi^3)}{2(4+33\phi-16\phi^2+7\phi^3-2\phi^4)};\ x_j^*(g^p) = \frac{\phi(a-\overline{c})(9-9\phi+2\phi^2)}{4+33\phi-16\phi^2+7\phi^3-2\phi^4}.$$

and the effective R&D outputs are

$$X_i^*(g^p) = \frac{(a-\overline{c})(2+9\phi-7\phi^3+2\phi^4)}{2(4+33\phi-16\phi^2+7\phi^3-2\phi^4)};\ X_j^*(g^p) = \frac{\phi(a-\overline{c})(11+\phi^2(-7+2\phi))}{4+33\phi-16\phi^2+7\phi^3-2\phi^4}.$$

The equilibrium quantities and profits are

$$\begin{aligned} q_i^*(g^p) &= \frac{2(a-\overline{c})(1+5\phi-2\phi^2)}{4+33\phi-16\phi^2+7\phi^3-2\phi^4};\ \Pi_i^*(g^p) = \frac{(a-\overline{c})^2(1+5\phi-2\phi^2)^2(12-\phi^2+4\phi)}{4(4+33\phi-16\phi^2+7\phi^3-2\phi^4)^2}; \\ q_j^*(g^p) &= \frac{4(a-\overline{c})\phi(3-\phi)}{4+33\phi-16\phi^2+7\phi^3-2\phi^4};\ \Pi_j^*(g^p) = \frac{(a-\overline{c})^2\phi^2(3-\phi)^2(7+12\phi-4\phi^2)}{(4+33\phi-16\phi^2+7\phi^3-2\phi^4)^2}. \end{aligned}$$

In the empty network g^e, the equilibrium effort, effective R&D output, quantity, and profits are given by

$$\begin{aligned} x_i^*(g^e) &= \frac{(a-\overline{c})(3-2\phi)}{13-4\phi+4\phi^2};\ X_i^*(g^e) = \frac{(a-\overline{c})(3-2\phi)(1+2\phi)}{13-4\phi+4\phi^2}; \\ q_i^*(g^e) &= \frac{4(a-\overline{c})}{13-4\phi+4\phi^2};\ \Pi_i^*(g^e) = \frac{(a-\overline{c})^2(7+12\phi-4\phi^2)}{(13-4\phi+4\phi^2)^2}. \end{aligned}$$

The societal welfare under the different networks is given by $W(g) = \sum_{i\in N} W_i(g) = \sum_{i\in N} \Pi_i(g)$. The global welfare is given by $V(g)$ which is simply $\sum_i \Pi_i(g) + \frac{Q^2}{2}$. Then, we have

$$\begin{aligned} V^*(g^c) &= \frac{9(a-\overline{c})^2}{13};\ V^*(g^s) = \frac{(a-\overline{c})^2(2492-1084\phi+579\phi^2-138\phi^3+23\phi^4)}{(7\phi^2-13\phi+58)^2}; \\ V^*(g^p) &= \frac{(a-\overline{c})^2(28+380\phi+1345\phi^2-802\phi^3-111\phi^4+108\phi^5-12\phi^6)}{2(4+33\phi-16\phi^2+7\phi^3-2\phi^4)^2}; \\ V^*(g^e) &= \frac{3(a-\overline{c})^2(31+12\phi-4\phi^2)}{(13-4\phi+4\phi^2)^2}. \end{aligned}$$

Proposition 6 (Goyal and Moraga-González, 2001) *Suppose that governments cannot subsidize research and development and $\phi \in [0,1]$. The complete network g^c is pairwise stable for all $\phi \in [0,1]$, while the partially connected network g^p is pairwise stable for $\phi \in [0,\overline{\phi}]$ where $\overline{\phi} \simeq 0.04$ is the solution to equation $\Pi_i(g^s) = \Pi_i(g^p)$ with firm i being a hub in g^s and a linked firm in g^p. The star network and the empty network are never pairwise stable.*

Proposition 7 *Suppose that governments cannot subsidize research and development and $\phi \in [0,1]$. The partially connected network g^p is the unique strongly stable network for $\phi \in [0,\overline{\phi}]$ where $\overline{\phi} \simeq 0.04$ is the solution to equation $\Pi_i(g^s) = \Pi_i(g^p)$ with firm i being a hub in g^s and a linked firm in g^p. The complete network, the star network and the empty network are never strongly stable.*

Proof. First, since strong stability is a refinement of pairwise stability, we have that the empty and star networks are never strongly stable. Second, we show that the complete network g^c is never strongly stable. Indeed, from we have $\Pi_i^*(g^p) > \Pi_i^*(g^c)$ and $\Pi_k^*(g^p) > \Pi_k^*(g^c)$ with $ik \in g^p$. Third, from Proposition 6 we know that the partially connected network is not pairwise stable if $\phi > \overline{\phi} \simeq 0.04$; and so is not strongly stable. But, if $\phi \in [0, \overline{\phi}]$, then g^p is pairwise stable. Is g^p strongly stable too? Since g^p is pairwise stable, it suffices to show that no coalition has incentives to add links to form the complete network g^c. The answer is no since $\Pi_i^*(g^p) > \Pi_i^*(g^c)$ and $\Pi_k^*(g^p) > \Pi_k^*(g^c)$ with $ik \in g^p$. Thus, if $\phi \in [0, \overline{\phi}]$, then g^p is the unique strongly stable network; otherwise no network is strongly stable. ■

In Figure 8 and Figure 9 we have plotted, respectively, the effective R&D outputs and the global welfare for each possible network whenever governments cannot subsidize R&D.

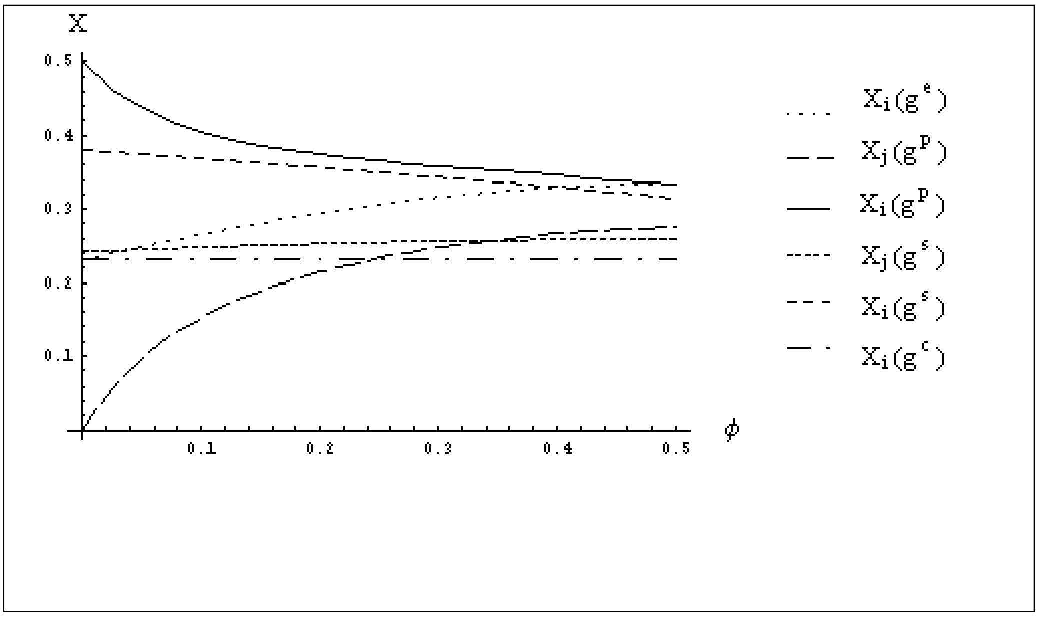

Figure 8: Effective R&D when governments cannot subsidize R&D.

Appendix B: Networks with subsidies

Standard computations show that, in the complete network g^c, the equilibrium subsidy, effort, effective R&D output, quantity, and profits are given by

$$\begin{aligned} s_i^*(g^c) &= 0.146;\ x_i^*(g^c) = \frac{a-\overline{c}}{5+4\sqrt{2}};\ X_i^*(g^c) = \frac{3(a-\overline{c})}{5+4\sqrt{2}};\ q_i^*(g^c) = \frac{(3\sqrt{2}-2)(a-\overline{c})}{7}; \\ \Pi_i^*(g^c) &= \frac{(54-25\sqrt{2})\,(a-\overline{c})^2}{196}. \end{aligned}$$

The social welfare and global welfare are, respectively, given by

$$W_i^*(g^c) = \frac{(a-\overline{c})^2}{5+4\sqrt{2}};\ V^*(g^c) = \frac{6(5\sqrt{2}-1)\,(a-\overline{c})^2}{49}.$$

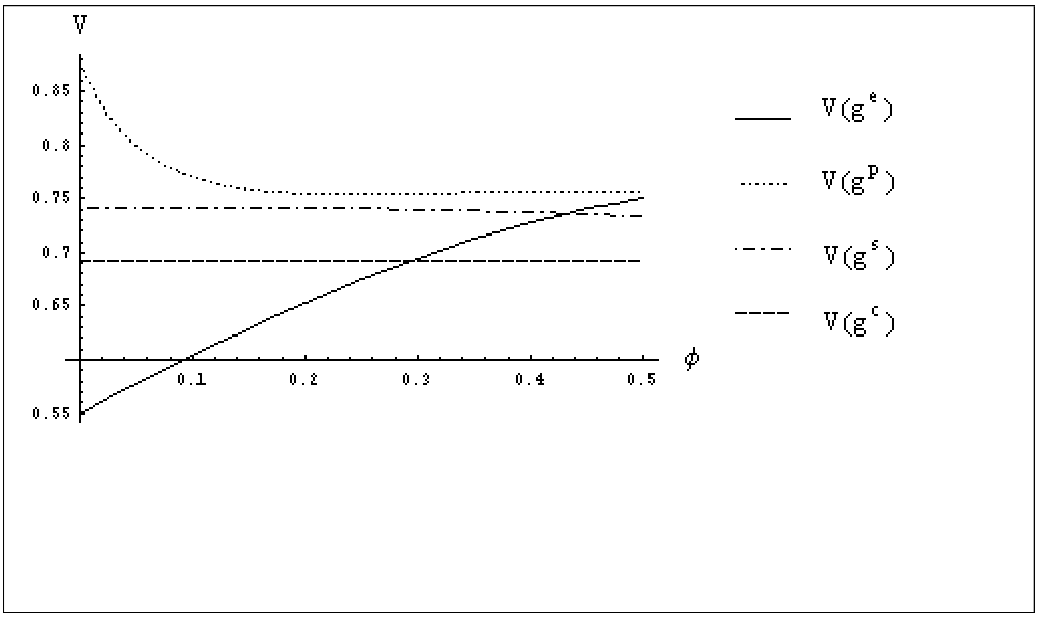

Figure 9: Global welfare when governments cannot subsidize R&D.

Let A be given by $A = \sqrt{1 + \phi(26 + \phi(-19 + 4(-1 + \phi)\phi))}$. In the empty network g^e, the equilibrium subsidy, effort, effective R&D output, quantity, and profits are given by

$$
\begin{aligned}
s_i^*(g^e) &= \frac{3 - \phi + 2\phi^2 - A}{8};\ x_i^*(g^e) = \frac{(a-\overline{c})(3-2\phi)}{7 - 2\phi + 2A};\\
X_i^*(g^e) &= \frac{(a-\overline{c})(3 + 4(1-\phi)\phi)}{7 - 2\phi + 2A};\ q_i^*(g^e) = \frac{4(a-\overline{c})}{11 - 11\phi + 4\phi^3 + (1+2\phi)A};\\
\Pi_i^*(g^e) &= \frac{(a-\overline{c})^2(7 + 123\phi - 66\phi^2 - 44\phi^3 + 24\phi^4 + (11 + 4(4-3\phi)\phi)A}{8(7 - 2\phi + 2A)^2}.
\end{aligned}
$$

The social welfare and global welfare are, respectively, given by

$$
\begin{aligned}
W_i^*(g^e) &= \frac{2(2-\phi)\phi\,(a-\overline{c})^2}{3 + 18\phi - 11\phi^2 - 4\phi^3 + 4\phi^4 + (3 + \phi(-1+2\phi))A};\\
V^*(g^e) &= \frac{3(29 + 138\phi - 111\phi^2 - 20\phi^3 + 20\phi^4 - 5(-5 + \phi(-1+2\phi))A)\,(a-\overline{c})^2}{4(7 - 2\phi + 2A)^2}.
\end{aligned}
$$

Unfortunately, for the star network g^s and the partially connected network g^p we cannot obtain closed-form solutions. However, for each possible given value of $\phi \in [0, \frac{1}{2}]$ (public spillovers) we are able to compute the equilibrium solutions. The equilibrium values (for $a - \overline{c} = 1$) of the profits, the societal welfare, the effective R&D outputs and the global welfare are plotted in Figure 3, Figure 5, Figure 10 and Figure 11, respectively.

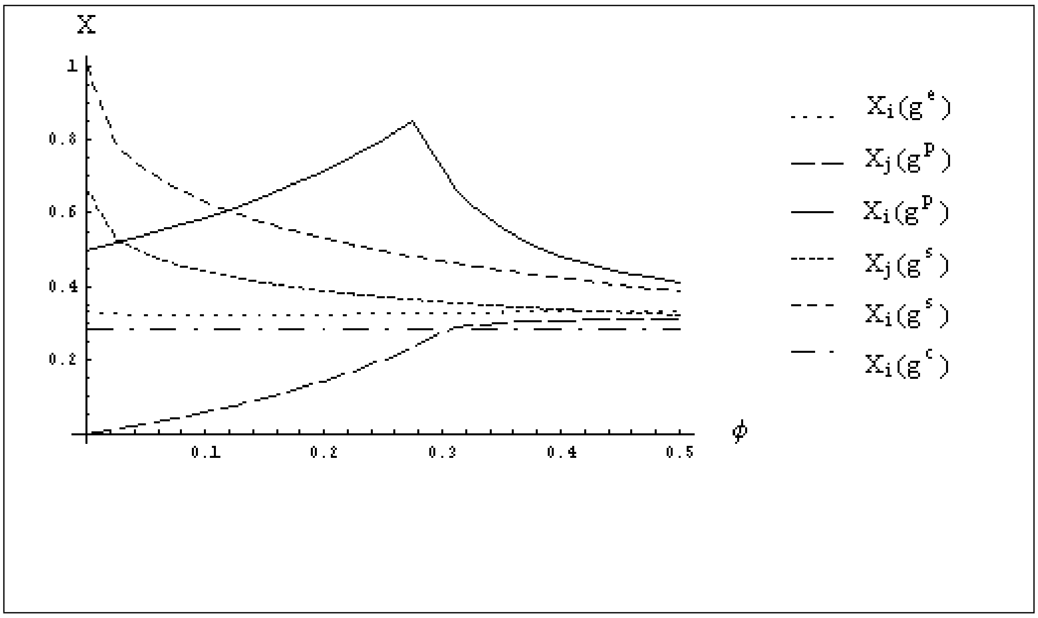

Figure 10: Effective R&D when governments can subsidize R&D.

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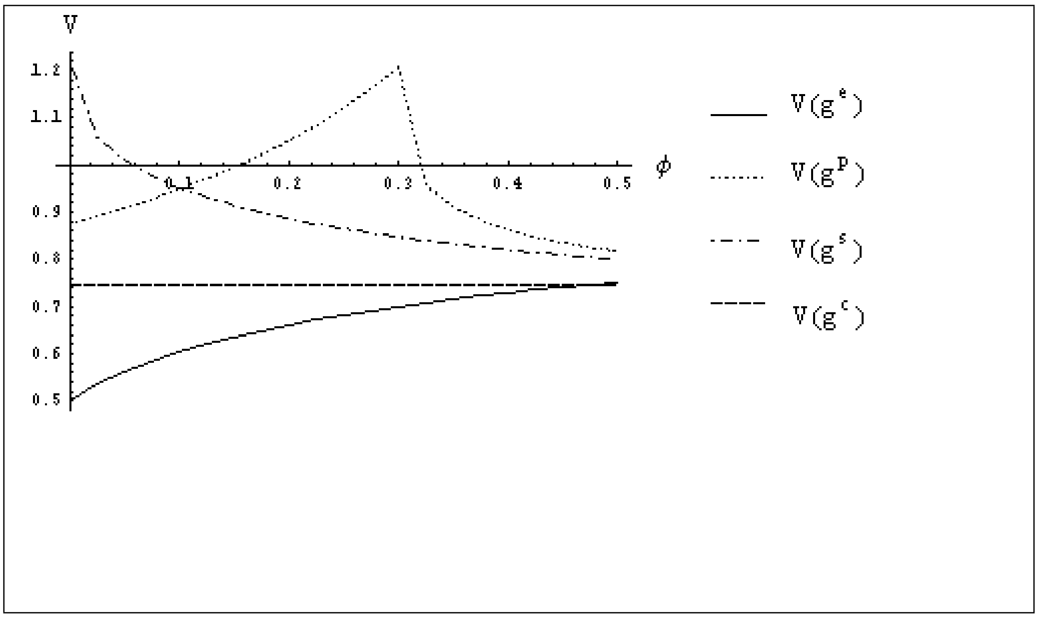

Figure 11: Global welfare when governments can subsidize R&D.

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