ANOTHER PERVERSE EFFECT OF MONOPOLY POWER

Jean J. Gabszewicz* and Xavier Y. Wauthy†

November 2000

Abstract

We show that the simple fact that a monopolist sells a good in units which are indivisible may well induce him to select a quality for his product which is not the highest one, even if no cost of any sort is attached to quality improvement.

*CORE, université catholique de Louvain

†CEREC, Facultés universitaires Saint-Louis & CORE, université catholique de Louvain

1. Even leaving aside price discrimination, the list of perverse effects of monopoly power identified by economists is already impressive. To our surprise, the following note reveals that a new item should be added to it, in circumstances in which intuition would have predicted the contrary.

The first item on the list is certainly the fact that a monopolist sells too little an amount of the good he produces, and at a too higher price, compared with the amount and the price which would be optimal from the viewpoint of consumers. Also, when the monopolist sells several variants of the same product, the range of variants he decides to supply generally differs from the range which would maximise consumers welfare ; sometimes he offers too many variants, sometimes too few of them (Gabszewicz (1983)).

Several perverse effects have also been listed, which are related to the *quality* of the product which is selected by the monopolist (see Tirole (1988)). In particular, when the cost of producing a good depends on its quality, the monopolist selects a quality which differs from the quality which would be optimal to produce, given the relationship between cost and quality, and taking again consumers welfare as criterion. Yet, when the unit production cost of a product does *not* increase with its quality, it seems obvious that it is always optimal from the viewpoint of the monopolist to select the top quality among all the variants which are available to him. The intuition behind this statement is that the willingness to pay of consumers necessarly increases with quality, and thus the price at which the monopolist would maximise revenue. Since by assumption there is no restraint coming from cost increase related to quality improvement, the monopolist should always increase his profit when improving the quality of his product. Thus, at least in this extreme case, it seems that both the monopolist and consumers share parallel interests : what is optimal for the latter, is as well optimal for the former. Still in other words, monopoly product selection here appears as immune against the usual perverse effects which so frequently conduce to welfare disappointing results. Unfortunately, the present note reveals that, even in this most favourable case, our basic intuition must be subject to qualification. We show indeed that *the simple fact that the monopolist sells a good in units which are indivisible may well induce him to select a quality for his product which is not the highest one, even if no cost of any sort is attached to quality improvement.* Thus, as a by-product, our analysis also puts in evidence that the list of perverse effects of monopoly power is probably far from being closed !

2. A very popular model providing microfoundations to market demand analysis was proposed by Mussa and Rosen (1978), and afterwards extensively used by economic theorists interested in product differentiation and other aspects of industrial economics. The basic ingredients of the simplest version of this model can be briefly described as follows. Consider a market for some commodity which is bought in indivisible units by consumers represented as points θ in the unit interval $[0, 1]$ with unit density. If consumer θ buys one unit of the good at price p, his utility is given by $\theta u - p$, while, if he does not buy, his utility is given by 0 ; we call u the *utility index* of the product. To find market demand, it is then sufficient to identify the consumer θ_1 who is indifferent between the alternative of buying one unit of the product at price p or not buying at all, namely, the consumer for whom the equality

$$\theta u - p = 0$$

holds, or

$$\theta_1 = \frac{p}{u}.$$

Market demand is then given by

$$D(p, u) = 1 - \frac{p}{u},$$

which, combined with the unit density assumption, implies that consumers who purchase the good, buy a single unit of it. To analyse product differentiation, the model is slightly enriched in order to include the possibility for the consumer to select not only among the alternatives of buying or not, but, in the first case, to select also which variant of the product he wishes to buy, at the exclusion of the other(s). Again it is assumed that if a consumer decides to purchase a specific variant, he buys only a single unit of it. This way of defining the decision set open to consumers reduces the *quantity* decision to a binary choice : one or zero unit.

A basic result, concerning quality selection under monopoly, follows almost trivially from the above model of market demand. Suppose indeed that the product is sold by a monopolist *who produces the good at zero cost*, and selects the market price p. Then revenue writes as

$$R\ (p, u) = (1 - \frac{p}{u})\ p$$

and is maximal when $p^* = \frac{u}{2}$ with corresponding revenue $R\ (p^*, u)$ equal to $\frac{u}{4}$. Assume now that the monopolist is also allowed to choose the quality of his product, and selects it within a given range of variants leading to utility indices u in the interval $[u^-, u^+]$. Since no cost is attached to quality improvement, he will always select the top quality leading to the utility index u^+ : equilibrium revenue is increasing in u. Hence, at least in this model, consumers' welfare losses due to monopoly are restricted to those arising from a too higher selling price and a too smaller volume exchanged, but do not extend to the selection of the product itself : households will always be offered the best one. Besides this conclusion extends to the version of this model used to analyse price competition with vertically differentiated goods : when costs related to quality are uniformly equal to zero, the firm selling the high quality variant always selects at price equilibrium the highest quality.

3. Even though durable goods are generally bought in indivisible and single units, assuming 0 -1 quantity decisions about the purchase of these goods often constitutes a too drastic simplification. This becomes particularly true with the observed increase in living standards through the population, which allows many households to be equipped with several units of the same indivisible product. In rich countries today, it is far from seldom to observe households equipped with two or three different cars, or several TV-sets. Similarly, it is not difficult to identify consumers owning two or three different houses for their personal use only.[1] The above simplified model is not able to capture such situations, and cannot explain how a monopolist should revise his pricing or product quality decisions when the market also includes consumers who may be considering buying more than a single unit of the good.

It is not difficult, however, to adapt this model in order to take into account the fact that houdeholds may also be interested in consuming more than one unit of the good, while keeping the property that the good is still consumed in indivisible units. To this end, let us now assume that the quantity decision set of each household is extended to also include the possibility of buying *two* units of the good, and denote by $\phi(u)$ the utility index associated with this new option. Of course, the utility of buying two units at unit price p depends on the utility index u obtained from consuming already a single unit, and also on the fact that the consumer has now to pay twice the price p. Consequently, if consumer θ buys *two* units of the good at a unit price p, his utility is now assumed to be given

[1] Households selecting to buy several units of a same good often purchase different variants of that good : most consumers owning two cars, own two *different* models of cars. This cannot be taken into account in our case, since by assumption the industry is under monopoly and the monopolist sells a single variant only. See, however, Gabszewicz, Sonnac and Wauthy (2000)

by $\theta\phi(u) - 2p$. In order to introduce an assumption which is the analog of the assumption of decreasing marginal utility in the case of a perfectly divisible good, but formulated now for our present case of indivisible purchases, we shall assume throughout that

$$\phi(u) < 2u :$$

the utility index corresponding to the consumption of two units of the good is smaller than twice the utility index corresponding to the consumption of a single unit of it. Denote by θ_2 the consumer who is indifferent between buying at unit price p two units of the good and not buying at all, i.e. the value of θ satisfying the equality $\theta\phi(u) - 2p = 0$, or

$$\theta_2 = \frac{2p}{\phi(u)}.$$

Similarly, denote by θ_{12} the consumer who is indifferent between buying at unit price p one unit of the good and two units of it : the value of θ satisfies $\theta\phi(u) - 2p = \theta u - p$, or

$$\theta_{12} = \frac{p}{\phi(u) - u}.$$

Direct comparaisons show that, under the assumption $\phi(u) < 2u$, we have necessarly $\theta_1 < \theta_2 < \theta_{12}$.

Given a unit price p, consumers in the interval $[\theta_1, \theta_{12}]$ buy only a single unit while those in the interval $[\theta_{12}, 1]$ buy two units : only consumers with the higher willingness to pay are willing to buy two units. Notice that if p is too high, there may be no consumer who would like to buy two units ; this happens when the interval $[\theta_{12}, 1]$ is empty, or when $p \geq \phi(u) - u$. Consequently, when the quantity decision set of the household is extended to also include the possibility of buying *two* units of the good, the demand function of the monopolist now becomes

$$D(p, u) = 1 - \frac{p}{u}$$

if $p \geq \phi(u) - u$, and

$$D(p, u) = (1 - \frac{p}{u}) + (1 - \frac{p}{\phi(u) - u})$$

if $0 \leq p \leq \phi(u) - u$. Figure 1 depicts the demand function $D(p)$, which exhibits a corner at the value of p equal to the difference between the utility indices corresponding to the purchase of one or two units of the good.

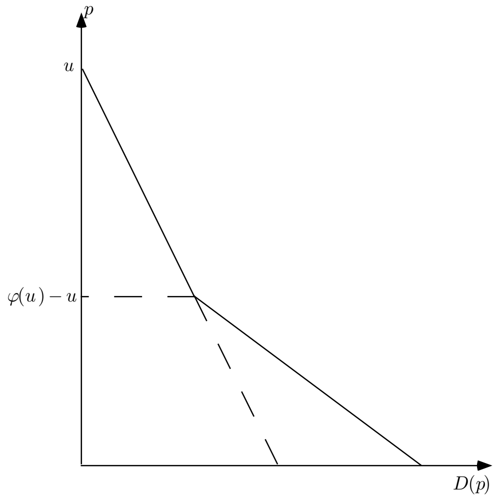

Figure 1:

Keeping the assumption that production takes place at zero cost, the revenue function then writes as ,

$$R(p,u) = \left(1 - \frac{p}{u}\right) p$$

when $p \geq \phi(u) - u$, and

$$R(p,u) = \left[(1 - \frac{p}{u}) + (1 - \frac{p}{\phi(u) - u}) \right] p$$

when $0 \leq p \leq \phi(u)$.

It is now easy to examine how, given the utility index u, the monopolist should revise his pricing decision when the market also includes consumers who may be considering buying more than a single unit of the good. When the price maximising revenue lies above $\phi(u) - u$, it must satisfy the first-order necessary condition

$$\frac{\partial R}{\partial p} \, |p \geq \phi(u) - u| = 1 - \frac{2p}{u}$$

in which case this price p^* obtains as

$$p^* = \frac{u}{2},$$

with a corresponding revenue $R(p^*, u)$ equal to $\frac{u}{4}$. When, on the contrary, revenue is maximal at a price p^{**} which is less than $\phi(u) - u$, the first-order necessary condition then implies that

$$\frac{\partial R}{\partial p} |p \leq \phi(u) - u| = 2 - \frac{2p}{u} - \frac{2p}{\phi(u) - u} = 0$$

which in turn implies

$$p^{**} = \frac{u\left[\phi(u) - u\right]}{\phi(u)}.$$

In that case, it is easy to check that the corresponding demand $D(p^{**}, u)$ is equal to 1, so that the resulting revenue $R(p^{**}, u)$ is equal to p^{**}. From a direct comparison of $R(p^*, u)$ and $R(p^{**}, u)$, we see that the monopolist will select a price which induces some customers to buy two units of the good whenever

$$\phi(u) \geq \frac{4u}{3}.$$

Otherwise he behaves as if this alternative would not exist. In the admissible domain $D = \{(u, \phi(u)) : u \leq \phi(u) \leq 2u\}$, two subdomains must be distinguished, namely,

$$D_1 = \left\{(u, \phi(u)) : u \leq \phi(u) \leq \frac{4u}{3}\right\}$$

and

$$D_2 = \left\{(u, \phi(u)) : \frac{4u}{3} \leq \phi(u) \leq 2u\right\},$$

with p^* as the monopoly solution in D_1, and p^{**} in D_2.

As a by-product, the above analysis reveals that the assumption according to which consumers buy only a single unit whenever they buy, is far from being innocuous. In a large range of situations, -namely, those corresponding to the domain D_2-, this assumption leads to truncate the "true" demand function since, for any price p smaller than $\phi(u) - u$, demand is taken as equal to $1 - \frac{p}{u}$, while it is, in fact, equal to $(1 - \frac{p}{u}) + (1 - \frac{p}{\phi(u)-u})$. This would not matter, inasmuch as it would not affect the market solution ; but, as seen above, it does it whenever the pair $(u, \phi(u))$ belongs to the domain D_2, a large domain indeed. In the next section, we show that it may affect not only the pricing decision of the firm but also the manner in which the monopolist selects the quality of his product.

4. Now that we have characterised the *price* monopoly solution when allowing households to buy also two units of the good, we can examine the problem of *quality* selection by the monopolist under the same circumstance. To this end let us assume that the domain of variants which can be selected by the monopolist leads to the set of utility indices $[u^-, u^+]$. If, for all values of u in this domain, the pair $(u, \phi(u)) \in D_1$, it is not difficult to see that the optimal quality selection for the monopolist again consists in choosing the highest quality leading to the utility index u^+ : equilibrium revenue is then equal to $\frac{u}{4}$ in D_1 and is thus monotone increasing in u. Now let us consider the other possibility in which, for all values of u in this domain, the pair $(u, \phi(u)) \in D_2$. Surprisingly enough, we show that, *in spite of the fact that no cost is attached to quality, no similar statement can be formulated in that case.* Consider indeed the following example. Let the domain $[u^-, u^+]$ be the interval $\left[\frac{2}{3}, \frac{6}{5}\right]$ and assume that, in this domain, the relationship between u and $\phi(u)$ is given by

$$\phi(u) = 1 + \frac{u}{2}.$$

Figure 2 provides a geometric representation of $\phi(u)$ and the corresponding admissible domain of utility indices.

It is easy to check that, for all u in the admissible domain $\left[\frac{2}{3}, \frac{6}{5}\right]$, the corresponding pair $(u, \phi(u)) \in D_2$: in this admissible domain, it is always optimal to select the price p^{**}, so that $R(p^{**}, u)$ is then defined in D_2 by $\frac{u[\phi(u)-u]}{\phi(u)}$, that is, in our case,

$$R(p^{**}, u) = \frac{u\,(2-u)}{2+u}.$$

Since $\frac{\partial^2 R(p^{**},u)}{\partial u^2} < 0$, the necessary and sufficient condition for $R(p^{**}, u)$ to reach a maximum in u is that $\frac{\partial R(p^{**},u)}{\partial u} = 0$, or, denoting by $\hat{u}$ the solution of this equation,

$$\hat{u} = 2\left(\sqrt{2} - 1\right).$$

Since $\hat{u}$ is in the admissible domain of utility indices $\left[\frac{2}{3}, \frac{6}{5}\right]$, but strictly smaller than the utility index corresponding to the top quality $u^+ = \frac{6}{5}$, *we conclude that the monopolist does not select the highest quality in it, in spite of the fact that*

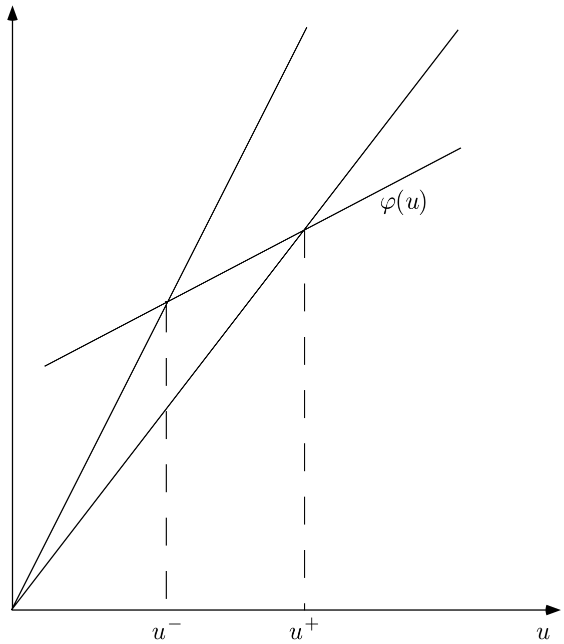

Figure 2:

we have assumed that cost does not increase with quality ! Hence, a new perverse effect appears since, in our context, efficiency requires to choose u^+.

It is not easy to explain in intuitive terms why this surprising paradox arises in our example. In fact, it follows from a simultaneous displacement of the kink and of the slope of the lower branch of the demand function, when we consider it as a function of u in the domain $D_{2.}$. When u increases, the difference $\phi(u) - u$ monotonically decreases so that the kink of the demand function takes place for a smaller and smaller price. On the other hand, the slope of the lower branch of the demand function does not vary monotonically with u, which entails that the value p^{**} is a rather complicated function of u (namely the function $\frac{u[\phi(u)-u]}{\phi(u)}$). It is not difficult, then, to build the paradox.

5. An objection which could be raised against the above example is that the monopolist could still reach a higher revenue if he would refrain from selling the good one unit at a time, but would allow consumers to buy only *two* units as a bundle. Denote by π the price proposed for the bundle. Monopolist's revenue

then writes as

$$R(\pi,\phi(u)) = (1-\frac{\pi}{\phi(u)})\pi$$

and reaches its maximal value at $\pi^* = \frac{\phi(u)}{2}$, with a corresponding equilibrium revenue $R(\pi^*,\phi(u))$ equal to $\frac{\phi(u)}{4}$. It is easy to see that, if $(u,\phi(u)) \in D_2$, the difference $R(\pi^*,\phi(u)) - R(p^{**},u)$ is positive if and only if $(\phi(u)-2u)^2$ is positive, which is always the case, so that it is clearly more profitable to sell only bundles . Since in that case revenue $R(\pi^*,\phi(u))$ is again monotone increasing in the quality u, the example covered by section 4 would not be really meaningful.

Is this a valid objection against our above example ? We do not think so because, if the monopolist would adopt the policy consisting in selling bundles only, *he would expose himself to the arbitrage of some consumers willing to buy two units of the good in order to resell one unit of them to other consumers who are interested in buying a single unit, but not two units of it.* In our context, this standard arbitrage argument takes the following form. Consider the consumer $\theta = \frac{1}{2}$ who is indifferent between buying a bundle of two units at price π^* and not buying at all. This consumer would be willing to resell one of the two units in the bundle at any price which would exceed the price p^0 defined by the condition$\frac{u}{2} - \frac{\phi(u)}{2} + p^0 = 0$: indeed, the left-hand term of this equality is his utility level after buying the bundle at price π^* and reselling one unit of it at price p^0, while the second term is his utility level after the purchase of the bundle at price π^*. On the other hand, any consumer θ in the interval $\left[\frac{\phi(u)-u}{2u}, \frac{1}{2}\right]$ would be willing to buy such a unit at price $p^0 = \frac{1}{2}\left[\phi(u)-u\right]$, because he would then reach a utility level equal to $\theta u - \frac{1}{2}\left[\phi(u)-u\right]$, which is positive if $\theta \in \left[\frac{\phi(u)-u}{2u}, \frac{1}{2}\right]$. Since $\frac{\phi(u)-u}{2u} < \frac{1}{2}$ due to the assumption $\phi(u) < 2u$, the set of consumers willing to buy a unit of the good at price $p^0$is non-empty, giving rise to an advantageous transaction between consumer $\theta = \frac{1}{2}$ and any consumer θ in the interval $\left[\frac{\phi(u)-u}{2u}, \frac{1}{2}\right]$. Certainly the monopolist is willing to avoid such a threatening competition ; this should prevent him to restrict the choice of consumers to the sole purchase of bundles, thus opening also the faculty of buying a single unit of the good as well. This is probably why the sale of bundles consisting of two identical units of the same consumers' durable good is never observed in practice.

References

[1] Gabszewicz, J. (1983). Blue and Red Cars, or Blue Cars Only. *Economica*, 50, 203-206

[2] Gabszewicz, J., N. Sonnac & X.Y.Wauthy (2000). Price Competition with Complementary Goods. *Economics Letters*, forthcoming.

[3] Mussa, M. & S. Rosen (1978). Monopoly and Product Quality, *Journal of Economic Theory*, 18, 301-317.

[4] Tirole, J. (1988). *The Theory of Industrial Organization*. MIT Press, Cambridge, Massachusetts