

# Investment in Public Infrastructure and Tax Competition between Contiguous Regions

Carole Dembour\* and Xavier Wauthy<sup>†</sup>

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## Abstract

Two contiguous regions compete to attract a population of heterogeneous firms. They choose infrastructure levels in a first stage, then compete in tax. We compare the properties of subgame perfect nash equilibria in this stage-game depending on the intrinsic features of the infrastructure considered. Then we derive some implications regarding the scope for cooperation between the regions.

## 1 Introduction

The last decades have been marked by a sharp decrease of transportation costs, and more generally trade costs, together with an increased mobility for capital and to a lesser extent of the labour force caused by institutional factors. As a result, the location of firm's productive activities is more and more disconnected from the destination market of their final products. Because firms are more mobile, national or regional governments have become more and more concerned by tax competition issues. In particular, by the risk that firms actually bid up local authorities one against the other to obtain tax reliefs. Observations suggests that the risk is indeed present. For instance Sorensen (2000) presents evidence of a significant fall in capital nominal tax rate from the 80's to the end of the 90's.

A growing body of the literature deals with tax competition games. Fortunately this literature most often conclude to the development of a mitigated tax competition. In a sense, tax revenues may not decrease that much because of tax competition. At the same time, it is obvious the

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<sup>†</sup>CEREC, FUSL and CORE, UCL

fiscal motives are not the only reason why firms would delocalize production. The specific amenities of regions, be it exogenous or resulting from agglomeration externalities, enter the picture as well. Public authorities are not passive either in this respect. In particular they tend to attract firms by magnifying their local amenities, and/or stimulating the emergence of strong spatial externalities. Thus, local authorities may affect firms' location decisions in essentially two ways: by offering an attractive fiscal package, and by developing a favorable economic environment (enhancing the quality of their infrastructure, broadly defined). Head and al. (1999) conducted an empirical analysis revealing how sensitive firms can be to non-fiscal arguments.<sup>1</sup> As argued recently by Justman et al. (2003) it may actually be the case that by specializing their infrastructure packages, regions may in fact relax tax competition.<sup>2</sup>

In the literature dealing with regional competition, When local authorities compete one against the other at the level of taxes as well as at the level of infrastructure, it is most often assumed that the infrastructure offered is specific to each region. Justman and al. (2001) is a good example. Therefore, regional infrastructures are viewed as substitutes from the point of view of the firms. This is clearly reasonable when regions really differ in geographical locations, i.e. they are located at a significant distance from each other. Suppose by contrast that a well-defined economic activity area is actually divided in two (or several) political regions, each endowed with some fiscal autonomy.<sup>3</sup> In a sense firms actually contemplate the possibility of locating their activities in the economic area, as a whole. Then, if they chose to move to the area, they would have to address the question of where (i.e. in which political region) to locate *within the area*. The choice of a particular region will reflect the presence of tax differentials as well as possible differences in the infrastructure supplied by the regions. However, if regions are contiguous, it might be difficult to argue that the benefits of an infrastructure developed by one of the regional governments is entirely confined to its political frontiers. In many cases, a "local" infrastructure will inevitably see its "benefits" spillover across political entities to the whole economic basin. If this is the case, then an infrastructure located in one region might be viewed as a complement to the development strategy of the other entities. Think for instance of an airport terminal located in one region. Clearly enough, this infrastructure increases the attractiveness of the region. However, it is hard to see why it would not increase that

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<sup>1</sup>See also Dembour (2003) for a recent selective survey on theoretical models dealing with competition for business units.

<sup>2</sup>More generally, local authorities are very likely to transfer tax competition towards less direct fields. See Peralta and al. (2003).

<sup>3</sup>As is typically the case for the region of Bruxelles Capitale in Belgium

of the contiguous regions as well. Most reasonably, this infrastructure increases the attractiveness of the economic area as a whole. By contrast, the positive impact of a high speed telecommunication network could be more easily restricted to one region only. Similarly, it is reasonably easy to condition access to "administrative" support services on the fiscal location of firms.

The present note builds on this intuition. We consider a model where regions compete for firms by choose the "quality" of the infrastructure they will offer to the firms. They also compete in taxes. Regarding infrastructure, the critical issue is the extent to which the infrastructure proposed by one the region is truly specific to this region or spills over to the contiguous ones. We shall consider the two polar cases of a strictly regional-specific infrastructure and an infrastructure whose effects are equally distributed across regions. In the first case, infrastructure is a pure private good whereas in the second case, it is a pure public good.

We address two questions: to which extent does the economic nature of the infrastructure alter the equilibrium behaviour of regions? To which extent does the nature of the infrastructure enhances or hinders cooperation between regions?

Part of the regions' problem can then viewed as follows: regions could find interesting to cooperate at the level of infrastructure, and the more so the more they are closely connected, while being competitors at the fiscal level. Cooperation at the infrastructure level seems desirable if both regions benefit from the increased attractiveness of the area. However, since infrastructure as the attribute of a public good, each region is likely to adopt a free rider behavior, thereby inducing too low a level of investment.

In order to address these questions, we build a stylized model inspired by the canonical location model of Hotelling (1929). This model will allow to formalize regional competition as a two-stage between two contiguous, though different, regions. In a first stage regions choose infrastructure levels non-cooperatively, in a second stage they set taxes non-cooperatively. Then firms decide on locations. Our equilibrium concept is subgame perfect Nash equilibrium. Our analysis also rests on a 2 by 2 typology characterizing the infrastructure's type. On the one hand, infrastructure benefits may either be strictly localized or strictly non-localized. On the other hand the benefits might be either dependent or independent on firms' types. We show that the scope for regional cooperation is highly dependent on the characteristics of the infrastructure, because these characteristics impact differently the tax competition stage.

In the next section, we present the basic model. Section 3 is devoted to the analysis of infrastructures that affect firms symmetrically. We char-

acterize subgame perfect equilibria.<sup>4</sup> Then, in section 4 we draw some implications of our findings and discuss extensions of the basic model.

## 2 A Model

Let us denote by  $C$  a well-defined economic area (for simplicity we shall talk of the "country", when referring to the economic area  $C$ ) which is divided in two contiguous "regions" (in the following, "regions" designate local political entities):  $A$  and  $B$ .

### 2.1 The Firms

There is a continuum of heterogeneous mobile firms contemplating to relocate their activities in  $C$ . Each firm is identified by a type  $x$ . Types are uniformly distributed in the continuous  $[0, 1]$  interval. The density is normalized to 1 so that the total number of firms is also normalized to 1. These firms are supposed to be located somewhere outside  $C$  and if they decide not to move to  $C$ , they enjoy a reservation profit  $\bar{\pi}$ . Thus,  $\bar{\pi}$  defines their status quo option.

Firms' types can be understood as designating some technological specificities of the firm that will have to be matched to in area  $C$ . These types are for instance related to the particular industry in which the firm is active. It could consist of a specific know-how that has to be taught to new workers. The matching cost contributes to define the fixed cost the firm will have to bear if it chooses to locate in area  $C$ . Obviously, this cost may also depend on the region in which it chooses to locate. Indeed, regions themselves display a priori characteristics inherited from their own history. The key point at this step is to assume that regions and firms are heterogeneous. To capture this idea, we assume that each region is located at one point  $x_A, x_B$  respectively. For simplicity, we assume  $x_A = 0, x_B = 1$ .

The matching cost of a firm with type  $x \in [0, 1]$  depends positively on the "distance" from the region's location.<sup>5</sup> We may therefore define a matching cost function for each region:  $m_A(x), m_B(1 - x)$ . Finally, we denote by  $t_A$  and  $t_B$  the lump-sum fee each region levies on firms.

Under the preceding assumption, we formalize a firm's location decision as follows: a firm chooses the location where its profit is larger. This profit is defined as the operating profit (i.e. the profit resulting from the

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<sup>4</sup>The parallel analysis for type dependent benefits of infrastructure is developed in the Appendix.

<sup>5</sup>note that the term location should not be understood here in the geographical sense. Firms are located in the characteristics' space.

production activity)<sup>6</sup> minus the matching cost and the lump-sum tax. In case the firm does not move to country  $C$  (stays abroad), the "status quo" profit is defined by  $\bar{\pi}$ . Moving to area  $A$  yields a payoff:

$$\pi_A(x) = K_A - m_A(x) - t_A, \quad (1)$$

whereas moving to region  $B$  yields:

$$\pi_B(x) = K_B - m_B(1 - x) - t_B, \quad (2)$$

where  $K_i$  denotes operating profits in region  $i$ .

## 2.2 The Regions

Regional authorities choose the infrastructure they supply to the firms. They also choose tax levels non-cooperatively. Given the specification of firms' profits as a function of location (equations (1) and (2)), two types of infrastructure can be distinguished: either it affects the matching cost (negatively) or the operating profits (positively). In both cases, the attractiveness of the region is reinforced if the quality of infrastructure supplied to the firms is increased. Infrastructure levels are committed to by the regions in a first stage, than tax levels are set simultaneously in a second stage.

Two basic type of infrastructures must be distinguished because they have different implications in a world where tax competition takes place. We may consider first infrastructures whose benefits to a firm depends on the type of this firm. This is especially true of a training program aimed at matching local workers' qualifications to incoming firms' needs, or of a development office aimed at helping firms to install their administrative and sales network. On the other hand, infrastructures such as the supply of public transport or high speed communication networks benefit have their benefits more uniformly distributed accross firms. Moreover, they are recurrent benefits related to the daily activity of incoming firms whereas the previous examples were related to the installation costs of the incoming firm. We shall show later on that these two infrastructure types have different implications on the tax competition game.

A last feature of infrastructure expenses must be considered: the localization of its effects. Two polar cases are considered hereafter. Either the infrastructure is entirely *general*. This implies that any firm locating

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<sup>6</sup>For instance, Justman et al. (2000) assume the following operating profits structure: Firms produce according to a production function  $y(l) = l^\alpha$ , where  $l$  designates units of labor input and  $0 < \alpha < 1$ . Labor is homogeneous perfectly mobile within  $C$  so that the wage prevailing in the two regions is identical and given by  $w$ . Operating profits are thus defined as  $y(l) - wl$ .

in country  $C$  benefits from the infrastructure symmetrically. In this case, any infrastructure investment by one region increases the attractiveness of the whole area, without giving any specific advantage to this region. At the other extreme, we shall consider the case of a purely *specific*<sup>7</sup> infrastructure with  $i = A, B$ . In this case, the infrastructure benefits to a firm only if the firm locates in the specific region where the infrastructure has been installed. Accordingly, no region can benefit from the infrastructure installed by the other one. In this context, providing more infrastructure in the first stage is apt to give a competitive advantage in the second stage: being more attractive, the high infrastructure region can attract firms even if taxes are significantly higher than in the contiguous region.

The objective of the local authorities is to maximize regional Welfare, defined as the wage bill in the region minus the cost of infrastructure plus the tax revenue (or minus the subsidy expenses).

$$W_i = w * L_i + t_i M_i - c(K_i)$$

where  $L_i$  denotes the labour demand of the firms located in the region,  $M_i t_i$  is the tax revenue and  $c(K_i)$  is the cost of infrastructure. For simplicity, we assume  $t_i \geq 0$ , i.e. regions are not allowed to offer net subsidies to firms. We do not impose in this paper any explicit budget constraints. Obviously, this does not mean that the cost of public fund is zero in the model. Indeed lower taxes and infrastructure expenses affect the objective negatively.<sup>8</sup>

### 2.3 The Game

We solve the following stage game

- Stage 1: Regions decide simultaneously on infrastructure levels.
- Stage 2: Regions decide on tax levels with infrastructure levels being publicly observed, choosing  $t_i \geq 0$
- Stage 3: Observing infrastructure decisions and taxes, firms decide of their location

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<sup>7</sup>The specificvs general typology is inherited from Labour economics, where human capital is said to be "general" if transferable from one firm to another.

<sup>8</sup>In section 4, we nevertheless discuss the implications of specific budgetary constraints that could apply to local authorities. It is indeed often the case that regional authorities face constitutional constraints which limit their ability to display budget deficits. Obviously, this is likely to affect equilibrium behaviour.

### 3 Infrastructure with Uniform Benefits

In this section, we assume that decisions made regarding infrastructure alter the operating profits of firms, i.e. the term  $K_i$  in equations (1) and (2).<sup>9</sup> Note that a key feature of this type of infrastructure is that it affects equally all the firms that choose to locate within a given region.

#### 3.1 The Public Good Case

In the pure public good case, any investment by one of the region affects  $K_i$  symmetrically. We shall assume for simplicity that in (1) and (2), we have  $K_A = K_B = K$ , where  $K$  depends positively and symmetrically of the investments realized by either regions. Thus, the level of  $K$  will be used as a proxy for the quality of the infrastructure supplied by the regions.

- We first characterize *firms' equilibrium behaviour at stage 3*, i.e. firms' optimal choices given  $K$  and  $t_i$ .

Given its outside option  $\bar{\pi}$ , each firm compares  $\bar{\pi}$ ,  $\pi_A(x)$  and  $\pi_B(x)$ . Two types of configurations must be distinguished:

- $(K, t_i)$  are such that some firms are better off choosing their outside option. We shall denote such configurations as *non-covered* ones
- $(K, t_i)$  are such that all firms are attracted in country  $C$ . These configurations will be referred to as *covered* configurations.

For a non-covered configuration to prevail, there must exist some type  $x$  such that  $\max\{\pi_A(x), \pi_B(x)\} < \bar{\pi}$ . Focusing on the possibility of locating in, say, region  $A$ , or not moving at all, any firm  $x$  compares  $K - m_a(x) - t_A$  to  $\bar{\pi}$ . Solving

$$K - m_a(x) - t_A = \bar{\pi}$$

for  $x$ , we identify the subset of firms preferring to move to region  $A$  rather than enjoying the status quo. Let us denote this set of firms by  $M_A$ .

Performing a similar analysis for region  $B$  allows us to define a another set  $M_B$  which contains those firms willing to move to region  $B$  if the alternative is the status quo. Obviously, non-covered configurations prevail whenever require that  $[0, 1]/(M_A \cup M_B)$  defines a non empty set.

In order to obtain closed form solutions, we shall assume from now that

$$m_a(x) = mx \text{ and } m_b(x) = m(1 - x) \text{ with } m > 0 \quad (A1)$$

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<sup>9</sup>Think for instance of an investment increasing labour productivity, or a .

Moreover, we assume without loss of generality that  $\bar{\pi} = 0$ .<sup>10</sup>

Using (A.1), the fact that the distribution is uniform in  $[0, 1]$  and the density is 1, we compute the number of firms locating in both regions. We solve  $K - mx - t_A = 0$  and  $K - m(1 - x) - t_B = 0$  to obtain<sup>11</sup>

$$M_A^n = \frac{K - t_A}{m} \quad (3)$$

$$M_B^n = 1 - \frac{K - t_B}{m}. \quad (4)$$

Notice that  $M_A^n > 1$  whenever  $K - t_A > m$  (resp.  $M_B^n < 0$  whenever  $K - t_B > m$ ). This condition therefore identifies the constellations where all firms prefer region  $A$  (resp.  $B$ ) to the status quo.

Direct computations using the above expressions indicate that  $M_A^n < M_B^n$  whenever  $2K - m > t$ . When  $2K - m < t$ , no firm prefers the status quo to locating in at least one region, i.e. we have a covered configuration. For such configurations it then remains to characterize in which of the regions  $A$  or  $B$  will a firm with type  $x$  locate. To answer this question we solve  $K - mx - t_A = K - m(1 - x) - t_B$  to obtain:

$$\tilde{x}(t_A, t_B, K) = \frac{m - t_A + t_B}{2m} \quad (5)$$

Any firm with type  $x < \tilde{x}$  locates in region  $A$  whereas firms with types  $x > \tilde{x}$  locate in region  $B$ . We have<sup>12</sup>

$$M_A^c = \frac{m - t_A + t_B}{2m} \quad (6)$$

$$M_B^c = 1 - \frac{m - t_A + t_B}{2m}. \quad (7)$$

It then remains to check for the boundary conditions, i.e. the conditions which ensure that the number of firms locating in each region is non-negative. Using equation (6) and (7) we obtain:

$$0 < \tilde{x} < 1 \iff -m < t_A - t_B < m$$

**Proposition 1.** *The equilibrium partition of the firms in stage 3 is defined as follows:*

<sup>10</sup>Notice that under (A1), our model is formally equivalent to the generic Hotelling model with linear transportation costs and endogenous market coverage.

<sup>11</sup>The upperscript  $n$  denotes the non-covered configuration.

<sup>12</sup>The upperscript  $c$  denotes covered configurations.



- Whenever  $2K - m \geq t_A + t_B$  a non-covered configuration prevails. The number of firms locating in region  $A$  is given by  $\max \left\{ 0, \min \left\{ 1, M_A^n \right\} \right\}$ . The number of firms locating in region  $B$  is given by  $\max \left\{ 0, \min \left\{ 1, M_B^n \right\} \right\}$ .
- Whenever  $2K - m < t_A + t_B$ , a covered configuration prevails. The number of firms locating in region  $A$  is given by  $\max \left\{ 0, \min \left\{ 1, M_A^c \right\} \right\}$ . The number of firms locating in region  $B$  is given by  $\max \left\{ 0, \min \left\{ 1, M_B^c \right\} \right\}$ .

Figure 1 illustrates Proposition 1 by partitioning the  $(t_A, t_B)$  space according to the firms' optimal choices.

*Insert figure 1 about here*

We may restrict attention to the sub-domain where  $t_i \leq K$ . Indeed,  $t_i > K$  is clearly a dominated strategy. There are then 4 areas of interest. In Area  $I$ , each region benefits from a local monopoly: regions are not competing among themselves but each are separately competing with the status quo option. In Area  $II^A$ , they are truly competing with each other. In Area  $II^C$  and  $II^B$ , tax differentials are so large that only one region attracts all the firms.

- We are now equipped to solve for *the second stage of the game where regions compete in taxes*.

In order to simplify the exposition, let us first neglect the wage bill component of the Regions' objective function<sup>13</sup> so that we are left, in stage 2, with two regions wishing to maximize tax revenues.<sup>14</sup>

Figure 1 provides a useful benchmark to understand the nature of tax competition in this game. In Area  $I$ , region's payoffs are independent. We may then characterize a region's optimal behaviour by maximizing  $t_A M_A^n$  over  $t_A$ . We obtain

$$t_A^n = t_B^n = \frac{K}{2} \quad (8)$$

The corresponding partition of firms is given by  $M_A^n = \frac{K}{2m}$  and  $M_B^n = 1 - \frac{K}{2m}$ . This solution is feasible if only we are indeed located in Area  $I$ . Solving  $2K - m < t_A^n + t_B^n$  we obtain the necessary and sufficient condition

$$K < m. \quad (C1)$$

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<sup>13</sup>We discuss the implication of this assumption in the last section of the paper.

<sup>14</sup>Recall that since we do not consider for the moment any explicit budget constraint, infrastructure expenses made in stage 1 are totally irrelevant in stage 2, i.e. they are pure sunk costs.

Turning to Area *II* we note that there cannot be an equilibrium in area *IIB* or *IIC*. Indeed, in these areas, one region enjoys a zero payoff since no firm locates in the region<sup>15</sup> On the other hand, it is always possible for this firm to name a lower tax, that leads to Area *II<sup>A</sup>* where its payoff is positive. Accordingly, we concentrate on the payoffs in Area *II<sup>A</sup>*. Observe that these payoffs are now interdependent through the definition of  $\tilde{x}(t_A, t_B)$ . We solve for a Nash equilibrium. Maximizing  $t_i M_i^c$  over  $t_i$ , we obtain the following specification for regions' best reply functions:

$$\varphi_i(t_j) = \frac{m + t_j}{2} \quad (9)$$

Straightforward computations lead to the following characterization of Nash equilibrium taxes:

$$t_A^c = t_B^c = m \quad (10)$$

All firms on the left of type  $x = \frac{1}{2}$  locate in region A whereas the complement locates in region B. It then remains to verify that this solution is indeed defined in Area *II<sup>A</sup>*. To this end we check that  $2K - m > t_A^c + t_B^c$ . We obtain the necessary and sufficient condition

$$K \geq \frac{3m}{2}. \quad (C2)$$

Combining *C1* (the feasibility condition for the non-covered interior equilibrium with that of the interior covered equilibrium (*C2*), we observe that none of them is satisfied in the sub-domain  $k \in [m, \frac{3m}{2}]$ . In such cases, a continuum of corner solutions exists. It is defined by:

$$t_b^{cor} \in [\min \left\{ \frac{K}{2}, \frac{4K}{3} - m \right\}, 2K - m - t_A^-] \quad (11)$$

with  $t_A^- = \min \left\{ \frac{K}{2}, \frac{4K}{3} - m \right\}$ . We do not develop the characterization of these corner solutions. The intuition underlying their existence is best summarized referring to figure 2.

*Inser Figure 2 about here*

For intermediate values of  $K$  relative to  $m$ , each region's best reply consist of three segments: first there is the segment  $\varphi_i$ , up to the frontier between Area *I* and *II*. Then there is the frontier itself, down to  $t_i^n$  then  $t_i^n$ . Corner solutions are thus located along the frontier.

We may now summarize the characterization of equilibrium tax rates as a function of the values of  $K$ , that result from first stage choices.

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<sup>15</sup>One could think here of a variant of the model where the payoffs in these areas might be positive, and increasing in the number of firms located in the other region. This could be the case with a spillover effect. Typically, attracting firms might require a tax decrease which may not compensate for the gain.

**Proposition 2a.** *Nash equilibrium tax rates in stage 2 are given by:*

- $t_A^c = t_B^c = m$  whenever  $K \geq \frac{3m}{2}$
- $(t_A^{cor}, t_B^{cor})$  whenever  $K \in [m, \frac{3m}{2}]$
- $t_A^n = t_B^n = \frac{K}{2}$  whenever  $K \leq m$ .

• Using the above proposition, we analyze now *the first stage of the game where region choose infrastructure levels.*

In order to capture the idea that the regions' respective infrastructure are public goods, we simply assume that the aggregate level  $K$  is defined by the addition of regional infrastructure levels. Stated differently, we assume  $K = K_A + K_B$ . In the first stage of the game, regions are assumed to choose  $(K_A, K_B)$  simultaneously and non-cooperatively.

*Insert Figure 3 about here*

Figure 3 partitions the strategy space according to the nature of tax equilibrium that will follow the corresponding infrastructure choices. There are obviously three areas of interests. A key feature of Area  $a$  is that regions' payoffs in the tax game do not depend on infrastructure levels. Accordingly, even when infrastructures are almost not costly, no  $(K_A, K_B)$  pair in the interior of Area  $a$  can be part of a subgame perfect equilibrium. In Area  $b$ , the corner solution prevails. It is again easy to show that no subgame perfect equilibrium can exist in this Area.<sup>16</sup>

The following Proposition summarizes the previous finding.

**Proposition 3.** *When the infrastructure is a public good, there exists no SPE involving  $K_A + K_B > m$ .*

We are thus left with candidates SPE in the interior or at the frontier of Area  $c$ . Suppose that infrastructure costs are zero, then we cannot have an equilibrium in the interior of Area  $c$ . Since only the aggregate level matters, each region is willing to complement the other's investment up to  $K_i = m - K_j$ . Accordingly, we expect to end up with a continuum of subgame perfect equilibria characterized by  $K_A^* + K_B^* = m$  whenever infrastructure costs are low enough. Notice that the marginal value of investing in infrastructure is constant and in particular does not depend on the possible difference  $K_A - K_B$ . Accordingly, regarding investment levels, each region is actually willing to invest up to the level of infrastructure it would invest for itself, should it be alone.

<sup>16</sup>Notice that we face here an additional problem: the existence of multiple equilibria

In Area  $c$  region A's payoff in stage 2 is given by  $\frac{K^2}{4m}$ . The marginal benefit of increasing  $K$  is therefore given by  $\frac{K}{2m}$ . Denoting the cost of investing up to an infrastructure level  $K_i$  by  $C(K_i)$ , the optimal level of infrastructure in the aggregate is the level for which  $\frac{K}{2m} = \frac{\partial c}{\partial K}$ . Let us denote this level by  $K^* < m$ . Then, we may claim that against any  $K_B < K^*$  region A will complement region B's investment in order to ensure that  $K_A + K_B = m$  as long as  $K_A < k^*$ . It follows that equilibrium in the first stage can be summarized as follows:

**Proposition 4.** *When infrastructure is a pure public good, there exists a continuum of SPE with the following features: any pair  $(K_A, K_B)$  such that  $K_A + K_B = m$  and  $K_i \leq K^*$  for  $i = A, B$  is part of an equilibrium. In the ensuing subgame, regions announce the equilibrium taxes  $t_A^n = t_B^n$ . All the firms are attracted in country  $C$  and each region hosts half of the firms.*

This Proposition may seem surprising. Indeed, it essentially states that in a SPE, i.e. when regions behave non-cooperatively, they jointly invest so as to achieve the efficient level of infrastructure. This is especially surprising if one recalls that the infrastructure is a public good. In such cases indeed, it is traditionally accepted that under-investment should prevail in a Nash equilibrium. Should we conclude that in the present framework, players manage to get rid of the free-riding problem that occurs when contributing to a public good? The answer is no. The problem regions face here can be summarized as follows: If they choose infrastructure simultaneously, the realization is likely to be "too much" or "too few" investment, which in both cases are problematic. Either because too few firms are attracted in the region or because too much money is spent on attracting them. However, from an individual viewpoint, the marginal value of complementing the other's investment is only related to the number of firms that will end up in the specific region. Thus given that the other has invested too little, i.e. some firm would not be attracted, it is a best response to contribute up to the required joint level. Actually the free-riding issue is at work. Indeed, in order to avoid inefficient realization of the equilibrium, regions could play in sequence. But then a very clear first mover advantage appears. If it can indeed commit not to revise his decision, the first mover will invest only  $m - K^*$  because it is then a best reply for the follower to contribute for the remaining. Thus, free-riding will take the form of competing for leadership. If they both act as leaders, only  $2(m - K^*)$  is invested.

### 3.2 The Private Good Case

The previous analysis has been performed under the assumption that infrastructures developed in some region was equally beneficial to any firm locating in the country. In other words, the benefits of the infrastructure spilled over throughout the whole country. We now turn to the case where these regional investments benefit to the firms if and only if they locate within the region. In other words, access to the infrastructure can be denied on a location basis. Infrastructure can then be viewed as a local private good: a firm "buys" the good by locating in the corresponding region. In this context, investing in infrastructure increases the attractiveness of country  $C$  only to the extent that firms are willing to locate in the region that initiates the investment.

We shall not develop the analysis of the three stage game in full details. Indeed, the formal derivation is similar to that developed in the previous subsection. Rather, we focus on the key differences that emerge in the game when we switch from the public good infrastructure to a private good one.

The key difference between the two approaches is simple to understand: *because they are exclusive attributes of each region, infrastructure levels now affect the relative attractiveness of a region (i.e. in comparison to the contiguous one) in addition to its absolute attractiveness (which refers the comparison with the status quo).*

Firms now compare  $K_A - mx - t_A$ ,  $K_B - m(1 - x) - t_B$  and the status quo (which we normalize again to zero). Focusing on the last stage of the game, we may adapt Figure 1 to depict the possible distribution of the firms on the  $(t_A, t_B)$  space. This is done in Figure 4.

*Insert Figure 4 about here*

With respect to non-covered configurations the structure is essentially identical to the previous case. However the equation of the frontier between Are  $I$  and  $II$  is now given by  $K_A + K_B - m = t_A + t_B$ . Obviously, different infrastructure levels induce an asymmetry between regions that will result in asymmetric market coverage.

The formal specifications of firms' distribution across regions is  $M_i^n = \frac{K_i - t_i}{m}$  with  $i = A, B$  for non-covered configuration. For covered configurations, we may identify the firm which is indifferent between the two regions. The type of this firm, which we denote by  $\tilde{x}(\cdot)$  is now equal to

$$\tilde{x}(t_A, t_B) = \frac{m + K_A - K_B - (t_A - t_B)}{2m}. \quad (12)$$

The above equation indicates that when infrastructure are private goods, they will matter in the covered configurations only to the extent that they

exhibit different levels.<sup>17</sup> This is also materialized by the fact that the frontiers separating areas  $II^B$  and  $II^c$  from  $II^a$  may not be symmetrically positioned.<sup>18</sup>

We may then turn to the analysis of the tax competition game. Optimal behaviour in non-covered configurations is now directly related to each regions' investment levels:

$$t_A^n = \frac{K_A}{2} \text{ with corresponding } M_A^n = \frac{K_A}{2m}. \quad (13)$$

As for covered configurations direct computations yield the following best reply function:

$$\chi_i = \frac{t_j + m + K_i - K_j}{2}. \quad (14)$$

This expression has to be compared with  $\varphi$  in the public good case (see equation (5)). The comparison illustrates the key difference between the two models: *when infrastructure are purely private goods, investing more than the other has a strategic value in the tax competition game.* Taking this into account we may characterize Nash equilibrium in the tax game for covered configuration. Last, checking the interiority conditions for the above candidate equilibria in their respective domains we identify again a domain for  $(K_A, K_B)$  values where the equilibrium consists of a corner solution, which we denote  $t_i^{cor}$ . Summing up we obtain the following proposition, which parallels proposition 2a for the private good case

**Proposition 2b.** *In the private good case, the Nash equilibrium tax rates in stage 2 is given by:*

- $t_i^c = m + \frac{k_i - k_j}{3}$  with  $\tilde{x} = \frac{1}{2} + \frac{K_A - K_B}{6m}$  whenever  $K_A + K_B \geq 3m$
- $(t_A^{cor}, t_B^{cor})$  whenever  $K_A + K_B \in [2m, 3m]$
- $t_i^n = \frac{K_i}{2}$  whenever  $K \leq 2m$ .

Relying on the above proposition, we go backward in the game tree and analyze the first stage. There are three configurations of interest in the  $(K_A, K_B)$  space. Either both infrastructure levels are small and the uncovered configuration prevails, or we have a domain of intermediate values where the ensuing tax game exhibits corner equilibria. Last, for high levels of  $K_i$ , a covered configuration prevails in the ensuing tax game. In order to study optimal infrastructure choices in each case we assume

$$C(K_i) = \frac{K_i^2}{F}, \quad (A2)$$

<sup>17</sup>This expression is best understood when compared to equation (5) which applied in the public good case.

<sup>18</sup>The frontier between  $II^a$  and  $II^b$  is for instance given by  $t_A = t_B + m + K_A - K_B$

with  $F > 0$ .

Figure 5 depicts the relevant domains in the  $(K_A, K_B)$  space.

*insert Figure 5 about here*

Under non-covered configurations, regions' equilibrium welfare in stage 2 is equal to  $\frac{K_A^2}{4m} - \frac{K_A^2}{F}$ . This expression is strictly increasing and convex in the domain where  $K_A > 0$  whenever  $F > 4m$ . In this case, a region's optimal investment decision is to maximize  $K_i$ . Otherwise the optimal investment is zero. Therefore, whenever  $F > 4m$ , the best reply of region  $i$  to any  $K_j < 2m$  is  $K_i = 2m - K_j$ .

When a corner solution prevails in the tax game, i.e. in the domain where  $K_A + K_B \in [2m, 3m]$ , there is a continuum of tax equilibria. In any of these equilibria, at least one region obtains a payoff which is strictly increasing in  $K_i$  whenever  $F > 4m$ . Therefore, this region is better off deviating upwards, to the boundary  $K_A + K_B = 3m$ . Proposition 5 summarizes our analysis of the two first configurations.

**Proposition 5.** *Suppose investment cost is defined by (A2). Then, in the private good case, there exist no subgame perfect equilibrium in the non-covered domain, nor in the interior of the corner solution domain.*

Notice that this proposition can be viewed as the exact opposite to Proposition 3: indeed, it implies that, contrarily to the public good case, a subgame perfect equilibrium must belong to the sub-domain where covered configurations prevail in the tax subgames.

Contrarily to the case of a public infrastructure, each region's payoffs are dependent on both  $K_A$  and  $K_B$  in covered configurations. More precisely, a region's equilibrium gross welfare, i.e. neglecting investment costs, is given by:

$$\left(m + \frac{K_i - K_j}{3}\right) \left(\frac{1}{2} + \frac{K_i - K_j}{6m}\right) \quad (15)$$

This expression is clearly convex in  $K_i$  and reaches a minimum for  $K_i = 3m - K_j$ , i.e. precisely along the lower bound of the domain where covered configurations prevail. Accordingly, as far as tax revenues are concerned, regions are always willing to increase  $K_i$  in the covered configuration domain. The upper limit to investments should come from costs. Using equation (15), we express a region's net welfare as:

$$W_i^*(K_i, K_j) = \left(m + \frac{K_i - K_j}{3}\right) \left(\frac{1}{2} + \frac{K_i - K_j}{6m}\right) - \frac{K_i^2}{F} \quad (16)$$

This expression is globally concave whenever  $F < 18m$ <sup>19</sup>. In this domain, region  $i$ 's best reply in the first stage is given by:

$$K_i(K_j) = (3m - K_j) \frac{F}{18m - F} \quad (17)$$

Straightforward algebra yields the subgame perfect equilibrium candidate:

$$K_A^* = K_B^* = \frac{F}{6}. \quad (18)$$

It then remains to check for interiority conditions, i.e.  $K_A^* + K_B^* \geq 3m$ . This condition is satisfied whenever  $F \geq 9m$ . We have thus established the following proposition:

**Proposition 6.** *Suppose costs is defined by (A2) and infrastructure are private goods. Then, whenever  $F \in [9m, 18, m]$ , there exists a unique symmetric subgame perfect equilibrium. In this equilibrium, regions invest in infrastructure up to  $K_A^* = K_B^* = \frac{F}{6}$ ; equilibrium taxes are  $t_A^* = t_B^* = m$  and each region hosts half of the firms.*

Proposition 6 Should be contrasted with Proposition 4. It shows indeed that for a non-trivial domain of the parameters, where investments costs take intermediate levels, the non-cooperative behaviour of the region induces too much investments. Indeed, part the amount invested is strictly unvaluable to the regions (although it increases firms's rent).

## 4 Comments and Extensions

In this section, we discuss some obvious limitations of the present model as well as possible extensions. Last, we identify the scope for regional cooperation as it is revealed by the outcome of the non-cooperative behaviour of the regional governments.

### 4.1 Budget Constraints

The analysis has been performed without any explicit budget constraint. As already mentioned, we assumed that public funds were costly since expenses or lost taxes affect a region's welfare negatively. However, in reality, local or local public authorities may face binding constraint, for instance because the constitution does not allow for too large a deficit, or imposes strict budget balances. A possibly important implication of such constitutional aspects is that regions may actually be heterogeneous with respect to fiscal issues.

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<sup>19</sup>Should  $F \geq 18m$  the function would be convex and the game has no solution.



One easy way to introduce such an asymmetry in the model is to put some explicit weight  $\lambda_i$  in the objective function of each region, in order to reflect the fact that the cost of public funds differ, depending on each region's fiscal global balance. As long as the  $\lambda_i$  is assumed to be constant, our results will not be qualitatively affected. However, assuming that  $\lambda_i$  depends on the budget balance as it results from the model (i.e.  $\lambda_i$  is a function of  $K_i$  and  $t_i$ ) severely complicate the picture. The following limit case may provide some intuition about the nature of the problem at stake here.

Suppose that infrastructure investments must be financed "within the model", i.e. we assume that at the beginning of the game, regions are already budget constrained while the constitution imposes strict budget balance. Thus, any investment aimed at attracting firms must be financed by the tax levied on the incoming firms. This obviously affects the rule of the game since any investment committed to in period 1 fixes a lower bound on the minimum tax revenue a region must secure in stage 2. Moreover, tax revenues of a region depends on the tax *pair*  $(t_A, t_B)$ , and not only on this region's actions. We plan to pursue the analysis of such situations in future research.

## 4.2 Reducing Firms' Matching Costs

Up to now we have considered the case where regional infrastructure aimed at decreasing sunk costs uniformly accross firms. However, it seems reasonable to assume that depending on their specific type, firms value regional policies differently. This is for instance the case of training programs aimed at matching workers' qualifications to the firms requirements. The actual value of such a program is typically dependent on the firms' types. To what extent do the implications of such policies differ from those emphasized in the case of  $K_i$  infrastructure?

In our model, such policies can be captured by assuming that they decrease  $m$ . A region is more likely to host some given firm if it commits to take part of its matching cost in charge. Formally, we alter our basic model as follows:

- First we define matching costs  $m_A(x) = \frac{1-x}{q_A}$  and  $m_Bx = \frac{1-x}{q_B}$
- In the public good case, we assume  $q_A = q_B = q_C$  whereas for the private good case, each region is characterized by its specific  $q_i$
- Then we solve the model using the same methodology than in the previous section.

The analytical developments have been relegated to the appendix. The analysis reveals contrarily to the case where investments affect  $K_i$ , the fact that infrastructure are private goods is not sufficient to remove the multiplicity of subgame perfect equilibria. Actually, when investments are aimed at decreasing  $m_i$  regions always end up on the frontier which separates non-covered configurations from covered ones. The intuition for this result is to be found in the strategic value of the infrastructure investment. As long as we consider non-covered configurations, the strategic value of  $K_i$  and  $q_i$  is strictly positive, irrespective on the public or private good nature of the infrastructure. However, for covered configurations, this is no longer true. Looking at Proposition 2b, we observe that  $K_i$  alters positively the level of  $t_i^*$  as well as the number of firms actually attracted in region  $i$  in equilibrium. In other words, the strategic value of  $K_i$  is positive. By contrast, looking at Proposition A1 or equation (a18) in the appendix, we observe that larger  $q_i$  decrease equilibrium taxes. Therefore, in the public good case, investing beyond the level that ensures coverage is clearly purposeless. In the private good case, increasing  $q_i$  decreases  $t_i^*$  but increases the number of firms attracted in region  $i$ . However, in equilibrium, the negative tax effect dominates. Again, investing beyond the coverage threshold is not profitable.

Both types of infrastructure make regions more attractive to firms. However, they have very different implications for the tax competition game in a covered configuration. Essentially, improving matching can be viewed as making firms more mobile from one region to the other. This has the unhappy consequence of reinforcing tax competition, so that in equilibrium, tax levels are lower.

### 4.3 Scope for Cooperation

As mentioned in the introduction, a key feature of the regional competition we envisage in this paper is that competition takes place between contiguous regions. Within our model this is marked first in the fact that regions face firms with the same status quo option. This assumption was meant to capture the idea that firms put our two regions "on a par", except for taxes (when they differ) and/or infrastructure. More importantly, regions' contiguity translates into the public nature of infrastructure decisions. Indeed, if regions are truly contiguous, physical location in region  $A$  rather than  $B$  does not actually matter as far as spatial externalities are concerned. This is exactly what happens when infrastructure is public.

Regions are in a co-opetition context (see Brandenburger and Nalebuff (1996) for a non formal treatment of co-opetition theory.). Because of their contiguity, the spatial externalities are magnified when the country has a

whole becomes larger. Thus, as far as the *total* number of firms to attract is concerned, regions share a common interest: it is best for both to attract as many firm as possible. Still, they might compete for the associated fiscal revenues. Intuition suggests that regions should at least cooperate as far as infrastructures are concerned, even if they do not manage to refrain from competing in taxes. In the present model we have considered a purely non-cooperative context. This allows us now to identify more clearly the scope and necessity of cooperation. More precisely, it turns out that the interest of cooperation, and the problem it involves differ according to the type of infrastructure.

Essentially, Proposition 4 reveals that when the infrastructure has the attribute of a public good, reaching the efficient level from the point of view of regions (i.e. the minimum level ensuring that all firms move to the country, given the ensuing tax game) may result from equilibrium behaviour. However, the mutliplicity of SPE is problematic. In this context, the scope for cooperation comes from the benefits of coordination: once an equilibrium is selected, it is self-enforcing. Of course, coordination is not that easy because all equilibria are Pareto efficient. Therefore, the key issue for regional governments in the present context is to allow for bargaining and communication. In this respect, the rules of the game, i.e. the institutional framework, is likely to be determinant in enhancing cooperation.

By contrast, the case of a private good infrastructure calls for a very different form of cooperation. Indeed, it follows from Proposition 6 that unless infrastructure are very high, regions will overinvest in infrastructure in a SPE. They are actually caught in a prisoner's dilemma where they both end up investing in a totally unproductive manner. A cooperative solution in this case is apt to improve regions' welfare but is not self-enforcing. If regions wish to implement this solution, it is crucial that they can make credible commitments on the cooperative actions. This requires additional cooperation in the design of institutional rules.

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## 5 Appendix: Matching Infrastructure

- *The Public Good Case*<sup>20</sup>

Under the assumption that infrastructure is a pure public good, it is only the aggregate infrastructure  $q_C = q_A + q_B$  that matters for the firms. The third stage of the game is then solved as follows.

Given  $(q_A, q_B, t_A, t_B)$ , each firm compares

$$\left\{ K - \frac{x}{q_C} - t_A, K - \frac{1-x}{q_C} - t_B, \bar{\pi} \right\}$$

It decides on its location by maximizing profits.

Let us first identify the potential market share of region  $A$ . The potential market is defined by the subset of firms who prefers to locate in region  $A$  than to stay abroad. To this end, we identify the type  $x_A$  which by definition obtains the same profit in the two alternatives. Formally, we solve  $K - \frac{x}{q_C} - t_A = \bar{\pi}$  for  $x$  and obtain

$$\bar{x}_A = (K - \bar{\pi} - t_A)q_C \quad (a1)$$

The potential market of region  $A$  is then defined by the interval  $[0, \bar{x}_A]$ . Under our normalizations, this implies that this potential market consists of  $x_A$  firms.

Solving  $K - \frac{x}{q_C} - t_B = \bar{\pi}$  for  $x$  we obtain

$$\bar{x}_B = q_C(K - \bar{\pi} - t_B) \quad (a2)$$

The potential market of region  $B$  is then defined by the interval  $[\bar{x}_B, 1]$ , in which there are thus  $1 - \bar{x}_B$  firms.

Assuming that firms are willing to move to country  $C$ , either in region  $A$  or  $B$ , we identify the firm which is indifferent between locating in any of the two regions. We denote this indifferent firm by  $\tilde{x}$ . By definition  $\tilde{x}$  solves

$$K - \frac{x}{q_C} - t_A = K - \frac{1-x}{q_C} - t_B$$

Accordingly, we obtain:

$$\tilde{x}(t_A, t_B, q_C) = \frac{1}{2} + \frac{q_C(t_B - t_A)}{2} \quad (a3).$$

Clearly enough, the number of firms locating in region  $A$  is given by  $\tilde{x}(\cdot)$  whereas the corresponding number going to region  $B$  is  $1 - \tilde{x}(\cdot)$

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<sup>20</sup>In the appendix, we do not normalize  $\bar{\pi}$  to zero and we keep the wage bill component in the regions' objective functions.

Using equations (1), (2) and (3), we may partition the tax space according to the distributions of firms between the two regions and "abroad". Using equation (1), we first observe that for region  $A$  to attract the firm located at 0, we need  $t_A \leq K - \bar{\pi} = t_A^+$ . Obviously,  $t_A^+$  defines the upper bound of region  $A$  relevant strategy space. On the other hand, whenever  $t_A < K - \bar{\pi} - \frac{1}{q_C} = t_A^-$ , the potential market of region  $A$  coincides with the whole interval. A symmetric analysis for region  $B$  defines  $(t_B^+, t_B^-)$ .

Second, we may partition the strategy space according to whether all firms choose to locate in area  $C$ , irrespective of whether they locate in  $A$ , or  $B$ , or some of them stay "abroad". To this end, we simply have to compare  $\bar{x}_A$  and  $1 - \bar{x}_B$ . Whenever  $\bar{x}_A < 1 - \bar{x}_B$ , potential markets do not overlap. Accordingly, all of the firms located in  $[\bar{x}_A, 1 - \bar{x}_B]$  stay abroad (a non-covered configuration). On the other hand, whenever  $\bar{x}_A \geq 1 - \bar{x}_B$  potential markets overlap. In this case, no firm stays "abroad". Notice that it is only when potential market overlap that we regions will compete for firms' location in a well-defined sense (a covered configuration).

Solving  $\bar{x}_A = \bar{x}_B$ , we obtain the equation of the frontier between covered and uncovered configurations:

$$t_A + t_B = 2(K - \bar{\pi}) - \frac{1}{q_C} \quad (a4)$$

Suppose then that  $(t_A, t_B)$  are low enough to ensure a covered configuration. Markets shares are then defined by  $\tilde{x}(t_A, t_B)$  provided it belongs to  $[0, 1]$ . Indeed, should  $t_A$  be low enough relative to  $t_B$ , even the firm with type  $x = 1$  could prefer to locate in region  $A$ : the larger matching cost being more than compensated by a lower tax. Formally, we may thus identify the tax differentials which are compatible with a true market sharing by the regions. To this end we solve  $0 < \tilde{x}(t_A, t_B) < 1$  to obtain

$$t_B - \frac{1}{q_C} < t_A < t_B + \frac{1}{q_C} \quad (a5)$$

Whenever  $t_B - \frac{1}{q_C} > t_A$  region  $A$  "preempts" the market whereas the contrary prevails whenever  $t_A > t_B + \frac{1}{q_C}$

The resulting partition can be depicted by a Figure which is similar to Figure 1 in the text.

We are now in a position to analyze the second stage of the game, i.e. the tax competition stage. In non-covered configurations, each region's payoff is independent of the other's tax. Payoffs functions are defined by

$$U_i = q_C(W + t_i)(K - \bar{\pi} - t_i)$$

with  $i = A, B$  and  $W$  defining the wage bill in a firm. First order condition yields the following equilibrium tax candidate, which we denote  $t_i^m$ :

$$t_i^m = \frac{K - \bar{\pi} - W}{2} \quad (a6)$$

Corresponding market shares are therefore given by

$$x_i^m = \left( \frac{K - \bar{\pi} + W}{2} \right) q \quad (a7)$$

Equilibrium payoffs are therefore defined by  $U_i^m = q_C \frac{(K - \bar{\pi} + W)^2}{4}$ . Notice then that for this tax pair to be an equilibrium, it must indeed define a non-covered configuration, i.e.  $t_A^m + t_B^m \geq 2(K - \bar{\pi}) - \frac{1}{q_C}$  must be satisfied. Direct computations yield the following condition:

$$q_C \leq \frac{1}{K - \bar{\pi} + W} \quad (a8)$$

Consider now covered configurations. As before, no region can preempt the whole set of firms in equilibrium. Thus, the only configuration of interest is the configuration where regions share the firms. Regions' payoffs are defined as follows:

$$U_A = (W + t_A)\tilde{x}(t_A, t_B); U_B = (W + t_B)(1 - \tilde{x}(t_A, t_B)) \quad (a9)$$

First order conditions yield the following best reply functions:

$$t_i = \frac{1}{2q_C} - \frac{W}{2} + \frac{t_j}{2} \quad (a10)$$

The Nash equilibrium candidate is therefore given by

$$t_A^* = t_B^* = \frac{1}{q_C} - W \quad (a11)$$

Obviously, the symmetry of the Nash equilibrium candidate implies that regions share the set of firms equally, i.e.  $\tilde{x}(t_A, t_B)^* = \frac{1}{2}$ . Interiority conditions for this equilibrium candidate yield the following condition:

$$t_A^* + t_B^* \leq 2(K - \bar{\pi}) - \frac{1}{q_C} \iff q_C \geq \frac{3}{2(K - \bar{\pi} + W)} \quad (12)$$

Comparing equation (8a) and (12a), it is immediate to see that there exists a non-empty parameter constellation in which neither the non-covered candidate nor the covered one are valid candidates. Indeed we have  $\frac{1}{K - \bar{\pi} + W} < \frac{3}{2(K - \bar{\pi} + W)}$ .

Whenever  $q \in ]\frac{1}{K-\bar{\pi}+W} < \frac{3}{2(K-\bar{\pi}+W)}[$ , a Nash equilibrium is defined as a corner solution  $t_A + t_b = 2(K - \bar{\pi} + W)$ . Combining the covering condition with the definition of best replies in the covered configuration, we identify the upper bound for taxes in the corner solution as  $t_i^+ = \frac{2(K-\bar{\pi})-W}{3}$ . Accordingly, regions' market shares are defined by  $(x_A, 1 - x_A)$ . Notice that the equilibrium is not unique in this case.

Proposition A1 summarizes our findings regarding the tax competition game

**Proposition A1.**

1. case 1: Whenever  $q_C \geq \frac{3}{2(K-\bar{\pi}+W)}$ , there exists a unique equilibrium in the tax competition game, given by  $t_A^* = t_B^* = \frac{1}{q_C} - W$
2. case 2: Whenever  $q_C \in ]\frac{1}{K-\bar{\pi}+W}, \frac{3}{2(K-\bar{\pi}+W)}[$ , there exists a continuum of equilibria defined by  $t_A + t_B = 2(K - \bar{\pi}) - \frac{1}{q_C}$  with  $t_A, t_B \geq \frac{2(K-\bar{\pi})-W}{3}$
3. case 3: Whenever  $q_C \leq \frac{1}{K-\bar{\pi}+W}$ , there exists a unique equilibrium given by  $t_A^m = t_B^m = \frac{K-\bar{\pi}-W}{2}$ .

With the help of Proposition A1, we now turn to the analysis of the first stage of the game where regional governments decide on infrastructure.

At this step we assume that infrastructure is not costly. Notice first that whenever case 1 prevails, regions equilibrium utilities are given  $U_i^* = \frac{1}{2} \frac{1}{q_C}$  and is thus decreasing in  $q_C$ . It is therefore immediate to see that no pair  $(q_A, q_B)$  such that  $q_A + q_B > \frac{3}{2(K-\bar{\pi}+W)}$  can be part of a subgame perfect equilibrium. Consider indeed that the contrary prevails. Each of the two region benefits from a downward deviation to  $q_c = \frac{3}{2(K-\bar{\pi}+W)}$ . This result is not surprising. Since infrastructure is a public good, no region gains from increasing the level of  $q_C$  beyond the level which ensures full market coverage. Indeed, the only impact of such a strategy is to intensify tax competition, which is detrimental to both firms.

It therefore remains to consider subgame perfect equilibrium candidates inducing subgames with equilibrium in case 2 or 3. From equation (a1) and (a2), it is obvious that if infrastructure is not costly, there is no interior subgame perfect equilibrium inducing a subgame exhibiting a non-covered configuration in equilibrium.<sup>21</sup> Candidate equilibria inducing case 3 subgame equilibrium are therefore defined by  $q_A + q_B = \frac{1}{K-\bar{\pi}+W}$ .

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<sup>21</sup>Clearly enough, introducing a cost to infrastructure may alter this conclusion. However, the general conclusion remains valid: for many parameter constellations, regions are willing to invest in infrastructure up to levels such that the resulting  $q_C$  level does not allow for a tax equilibrium corresponding to case 1.



We turn then to the analysis of infrastructure choices inducing case 2 equilibria. Note first, there is a multiplicity of equilibria in the tax subgames and they cannot be pareto ranked. However, region  $i$  is better off at the  $t_i^-$  equilibrium candidate (because, if not at its monopoly equilibrium, it is closest to it). Take then any other realization among the possible equilibria and consider the point of view of region  $A$ . The utility of region  $A$  in such an equilibrium is given by

$$U_a^c = \left( \left( \frac{3}{2}(K - \bar{\pi} + W) - \frac{1}{q} \right) \left( \frac{1}{q} - \frac{K - \bar{\pi} + W}{2} \right) \right)$$

Direct computations show that a necessary and sufficient condition for  $\frac{\partial U_a^c}{\partial q} < 0$  is that  $q > \frac{1}{K - \bar{\pi} + W}$ . This argument applies whatever the corner solution equilibrium we select to any of the two regions which does not enjoy its monopoly payoff in this equilibrium. Therefore, there cannot be an infrastructure equilibrium that would lead us in the interior of  $\left] \frac{1}{K - \bar{\pi} + W}, \frac{3}{2(K - \bar{\pi} + W)} \right[$

According to the above analysis, we can summarize our results in the following proposition:

**Proposition A2.** *When the infrastructure is a pure public good, there exists a continuum of Subgame perfect equilibria defined as follows:*

$$q_A^* + q_B^* = \frac{1}{K - \bar{\pi} + W}$$

$$t_A^m = t_B^m = \frac{K - \bar{\pi} - W}{2}$$

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• *Specific Infrastructures*

We consider now the case where infrastructure is purely specific to the region where the firm locates. Now, infrastructure investment has a strategic value for each region. Suppose indeed that  $q_A > q_B$ , then for  $t_A = t_B$  region  $A$  attracts more firms than region  $B$ . In other words, a better infrastructure ensures a competitive advantage in the tax competition game.

The basic problem of firm in the third stage of the game is now defined as choosing the best of the three following alternatives

$$\left\{ K - \frac{x}{q_A} - t_A, K - \frac{1-x}{q_B} - t_B, \bar{\pi} \right\}$$

Let us first define the partition of the tax space according to the nature of firms' optimal locations. Notice first that the set a firms enjoying a

positive surplus in region  $i$  is defined as follows :

$$\bar{x}_i = q_i(K - \bar{\pi} - t_i) \quad (a13),$$

for  $i = a, b$ .

Accordingly, all firms choose to locate in one of the two regions (covered configuration) whenever  $\bar{x}_A + \bar{x}_B \geq 1$ . Solving this expression for tax rates, we define the frontier between covered and uncovered markets in the tax space by the following relation:

$$t_A q_A + t_B q_B = (K - \bar{\pi})(q_A + q_B) - 1 \quad (14)$$

Notice that the upper bound in the relevant tax domain for region  $i$  is still given by  $K - \bar{\pi}$  whereas the tax level ensuring that the potential market coincides with the full market is given by  $K - \bar{\pi} - \frac{1}{q_i}$ .

Let us then assume that regions' markets overlap. The firm being indifferent between the two regions, which we denote by  $\hat{x}(t_A, t_B)$  solves by definition

$$K - \bar{\pi} - \frac{x}{q_A} = K - \bar{\pi} - \frac{1 - x}{q_B}.$$

We therefore obtain

$$\hat{x}(t_A, t_B) = \frac{q_A q_B}{q_A + q_B} \left( \frac{1}{q_B} = t_B - t_A \right). \quad (a15)$$

It then remains to check for the interiority conditions of market sharing, i.e. identify the conditions under which  $\hat{x}(t_A, t_B) \in [0, 1]$ . Direct computations yield:

$$\begin{aligned} \hat{x}(t_A, t_B) \leq 0 &\iff t_A \geq t_B + \frac{1}{q_B} \\ \hat{x}(t_A, t_B) \geq 1 &\iff t_B \geq t_A + \frac{1}{q_A} \end{aligned}$$

We may now characterize the equilibrium distribution of the firms as a function of the tax pairs. Replicating the analysis of the previous section, it is immediate to derive Monopoly equilibrium candidates for  $i = a, b$  as:

$$t_i^m = \frac{K - \bar{\pi} - W}{2} \text{ with } x_i^m = q_i \left( \frac{K - \bar{\pi} + W}{2} \right) \quad (a16)$$

We may then derive the feasibility condition for such an equilibrium by using equation (a16). More precisely, we solve  $t_A^m q_A + t_B^m q_B \leq (K - \bar{\pi})(q_A + q_B) - 1$  for  $q_A$  and obtain the interiority condition for the two regions to behave as monopolist as:

$$q_A \leq \frac{2}{K - \bar{\pi} + W} \quad (a17)$$

We characterize now a covered market equilibrium configuration. Using the definition of  $\hat{x}(t_A, t_B)$  and equation (9), we characterize regions' best replies as follows:

$$t_i = \frac{t_j - W}{2} + \frac{1}{2q_j} ; i, j = A, B$$

Accordingly, the unique Nash equilibrium is given

$$t_i^* = \frac{1}{3q_i} + \frac{2}{3q_j} - W \quad (a18)$$

Notice that markets shares in this equilibrium are defined by

$$\hat{x}(t_A, t_B)^* = \frac{q_A + 2q_B}{3(q_A + q_B)} \quad (a19)$$

We now have to check for interiority conditions, i.e. solving

$$t_A^* q_A + t_B^* q_B \leq (K - \bar{\pi})(q_A + q_B) - 1 \quad (a20)$$

This equation can be re-expressed as

$$q_A^2(2 - 3Zq_B) + q_A(q_B(5 - 3Zq_B)) < 0 \quad (a21)$$

where  $Z = (K - \bar{\pi} + W)$ .

Solving this expression for  $q_A$  is not straightforward. However, in the relevant domain of parameters, we obtain after some algebraic manipulations the following condition:

$$q_A \geq \frac{5 - 3q_B Z + \sqrt{9 - 6q_B Z + 9q_B^2 Z^2}}{-4 + 6Z} = f(q_B) \quad (a22)$$

It is again a matter of computations to show that equations (a17) and (a22) are mutually exclusive. Accordingly, these two conditions define three mutually exclusive regions in the  $(q_A, q_B)$  space. Notice also that whenever neither (a17) nor (a22) hold, the equilibrium is defined as continuum of tax pairs such that condition (a14) is satisfied, i.e. we have corner equilibria. These equilibria can be characterized as follows:

$$t_A^c = (t_B^c q_B + (K - \bar{\pi})(q_A + q_B) - 1) \frac{1}{q_A} \quad (a23)$$

$$x_A^c = 1 - (K - \bar{\pi})q_B - t_B^c q_B \quad (a24)$$

Nash equilibrium in the tax game can thus be summarized through the following Proposition:

**Proposition A3.** *Suppose infrastructure are region's specific, then the equilibria are given by:*

1. *Monopoly tax rates as defined by (a16) whenever  $q_A \leq \frac{2}{K-\bar{\pi}+W}$*
2. *Corner solutions as defined by (a23) whenever  $q_A \in [\frac{2}{K-\bar{\pi}+W}, f(q_B)]$*
3. *Duopoly tax rates as defined by (a18) whenever  $q_A \geq f(q_B)$*

Having characterized tax equilibria, we may now go backward in the game tree to consider infrastructure choices. Using equations (a19) and (a20), we define equilibrium utilities in the covered equilibrium configurations as follows:

$$U_i^*(q_i, q_j) = \left( \frac{1}{3q_i} + \frac{2}{3q_j} \right) \frac{q_j + 2q_i}{3(q_i + q_j)} \quad (a26)$$

A sufficient condition for  $\frac{\partial U_i}{\partial q_i} < 0$  is  $q_i, q_j > 0$ . In other words, whatever the other regions' infrastructure, each region's wishes to minimize its own infrastructure. Notice that this implies that a pair  $(q_A, q_B)$  such that (a22) holds with strict inequality cannot be part of a subgame perfect equilibrium. This result may be surprising at first sight because by increasing  $q_i$ , this region will manage to capture a larger share of the firm. However, this turn out to be very costly in terms of equilibrium tax levels. Accordingly, the region prefers to keep  $q_i$  "small" to relax tax competition.

A similar analysis can be performed in the case of corner solutions, leading to the same conclusion: a region always wishes to reach the lower bound of the domain defining corner solutions, whatever the equilibrium which is selected in the tax game. Accordingly, if a subgame perfect equilibrium exists it must be located in the domain where monopoly tax rate equilibria are defined. Using equations (a15), it is immediate to check that any region's utility is monotonically increasing in  $q_i$  within the Monopoly domain. Accordingly with end up with multiple equilibria that take the following form:

**Proposition A4.** *When the infrastructure is a pure private good, there exists a continuum of Subgame perfect equilibria defined as follows:*

$$q_A^* + q_B^* = \frac{2}{K - \bar{\pi} + W}$$

$$t_A^m = t_B^m = \frac{K - \bar{\pi} - W}{2}$$

Notice that in any of these subgame perfect equilibria, regions name the same tax levels. However, they enjoy different market shares depending on

the levels of their infrastructure. Remark also that we may identify bounds on the admissible domain of  $(q_a^*, q_B^*)$ . Specifically, arbitrarily small levels for  $q_i$  cannot be part of an interior. Indeed, the market share captured by firm  $i$  in this equilibrium is so small that it will find it profitable to deviate to a larger  $q_i$  that would enforce an equilibrium with covered configuration in the ensuing tax subgame.