

# Entry under Capacity Limitation and Vertical Differentiation: return of the Judo Economics\*

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## Abstract

Shaked and Sutton (1982) and Gelman and Salop (1983) are best remembered for their neat conclusions: a limited quality or limited capacity is an effective tool to relax competition and facilitate entry in a market. We aim at comparing the respective merits of these two strategic commitments. We claim that capacity limitation is more effective than quality reduction, mainly because it acts directly upon the incumbent to reduce his aggressiveness in the final price competition whereas quality tools works indirectly trough consumer's willingness to pay.

**Keywords:** Entry, Quality, Differentiation, Bertrand-Edgeworth Competition

**JEL Classification :**L13

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# 1 Introduction

The fact that many industries feature one or few dominant firms and a fringe of small competitors has been nicely formalized by Gelman and Salop (1983): in order to relax price competition and make entry profitable, an entrant can use a carrot and stick strategy. She voluntarily limits her production capacity to guarantee a large residual demand for the incumbent but she names a low price that would prove dear to undercut. In their discussion of possible means to achieve this credible commitment, the authors claim that “producing a product with limited consumer appeal is analogous to capacity limitation”.

It is indeed true that a similar strategic commitment is at work in the models of quality differentiation of Gabszewicz and Thisse (1979) and Shaked and Sutton (1982): the entrant optimally chooses a low quality and offers a substantial rebate on her product in order to induce the incumbent not to fight too aggressively in prices. The incumbent therefore prefers to accommodate entry although it is always possible for him to exclude the entrant from the market.

In this note we mix the two previous strands of literature by considering a game of entry where the entrant is allowed to choose the quality of its product and its production capacity. The question we raise is the following: does the entrant use product differentiation and capacity precommitment simultaneously? We show in Theorem 1 that under efficient rationing, quality imitation coupled with an optimal capacity limitation is more effective than having a large production capacity and a low quality. Even if differentiation occurs, it is limited and the product of quality by capacity remains equal to the optimal capacity limitation.

## 2 The model

We follow the Mussa and Rosen (1978) framework for modelling quality differentiation and consider a continuum of consumers identified by their type  $x$  which is uniformly distributed in  $[0; 1]$ . The utility of a consumer with type  $x$  is  $sx - p$  should he buy one unit of product of quality  $s$  at price  $p$  and 0 should he refrain from buying. Consumers maximize their utility and when indifferent between the two products, they select their purchase randomly.

We study the following 3 stages game  $G$ :

- At  $t = 0$ , an incumbent  $i$  enters the market and selects some top quality  $s_i = 1$  and a large capacity  $k_i = 1$ .
- At  $t = 1$ , an entrant  $e$  selects its quality  $s_e = s \leq 1$  and capacity  $k_e = k \leq 1$ .
- At  $t = 2$ , firms compete simultaneously in prices. Our solution concept for the game  $G(s, k)$  is Subgame Perfect Nash Equilibrium.

Quality cost is assumed nil as well as the cost of production up to the capacity limit and equal to  $+\infty$  otherwise. Observe that two classes of price subgames might be generated by choices made at  $t = 1$ : either  $k = 1$  and we face a standard game of vertical differentiation or  $k < 1$  and we face a Bertrand-Edgeworth game with (possibly) product differentiation.

Consumers make their choice at the last stage by comparing  $xs_i - p_i$ ,  $xs - p_e$  and 0. In the presence of differentiation ( $s < 1$ ), it is a straightforward exercise to show that demands are given by

$$D_i(p_i, p_e) = \begin{cases} 0 & \text{iff } p_i \geq p_e + 1 - s \\ 1 - \frac{p_i - p_e}{1 - s} & \text{iff } \frac{p_e}{s} \leq p_i \leq p_e + (1 - s) \\ 1 - \frac{p_i}{s_i} & \text{iff } p_i \leq \frac{p_e}{s} \end{cases} \quad (1)$$

$$D_e(p_i, p_e) = \begin{cases} 0 & \text{iff } p_e \geq p_i s \\ \frac{p_i - p_e}{1 - s} - \frac{p_e}{s} & \text{iff } p_i - 1 + s \leq p_e \leq p_i s \\ 1 - \frac{p_e}{s} & \text{iff } p_e \leq p_i - 1 + s \end{cases} \quad (2)$$

When capacity is not an issue ( $k = 1$ ) and products are differentiated ( $s < 1$ ), Lehmann-Grube (1997) shows that firms best replies are continuous and given by:

$$\phi_i(p_e) = \begin{cases} \frac{p_e + 1 - s}{2} & \text{iff } p_e \leq \frac{1 - s}{2 - s} s \\ \frac{p_e}{s} & \text{iff } \frac{1 - s}{2 - s} s \leq p_e \leq \frac{s}{2} \\ \frac{1}{2} & \text{iff } p_e \geq \frac{s}{2} \end{cases} \quad (3)$$

$$\phi_e(p_i) = \begin{cases} \frac{p_i s}{2} & \text{iff } p_i \leq \frac{2(1 - s)}{2 - s} \\ p_i - 1 + s & \text{iff } \frac{2(1 - s)}{2 - s} \leq p_i \leq 1 - \frac{s}{2} \\ \frac{s}{2} & \text{iff } p_i \geq 1 - \frac{s}{2} \end{cases} \quad (4)$$

These best replies are displayed on Figure 1 and the equilibrium is summarized in Lemma 1 below.

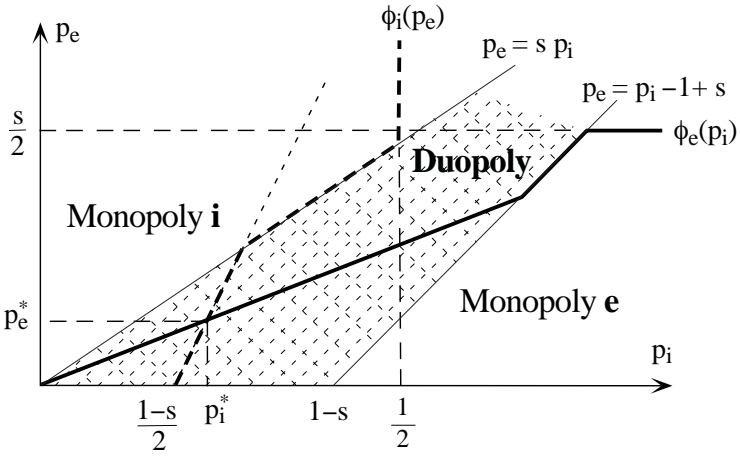


Figure 1: The price space with unlimited capacity

**Lemma 1** For  $s < 1$ , the game  $G(s, 1)$  has a unique pure strategy equilibrium:  $p_i^* = \frac{2(1-s)}{4-s}$  and  $p_e^* = \frac{s(1-s)}{4-s}$ .

**Corollary 1** The optimal quality for the entrant in the class of pricing games  $\{G(s, 1), s < 1\}$  is  $s^* = \frac{4}{7}$  yielding the profit  $\pi_e^* = \frac{1}{48}$ .

Notice that the pricing game  $G(1, 1)$  is a classical Bertrand game with linear demand  $D(p) = 1 - p$ . In case of a price tie, demand is shared equally by the two firms.

## 2.1 Rationing and Sales

Whenever the entrant has unlimited capacity ( $k = 1$ ), sales are equal to demand as characterized by equations (1) and (2). However, if the entrant has build a limited capacity ( $k < 1$ ), there will exist some pairs of prices  $(p_e, p_i)$  such that  $D_e(p_e, p_i) > k$ . In such cases, some consumers will be rationed and possibly report their purchase on firm the incumbent. In order to characterize firms' sales in that situation, we must specify the particular rationing rule that prevails in the market.

**H 1** *Efficient rationning is at work whenever  $k < D_e(\cdot)$ .*

Under (H1), consumers who are ultimately rationed are those exhibiting the lowest willingness to pay for the rationed good. The limited  $k$  units sold by the entrant will be contested by potential buyers,<sup>1</sup> the price  $p_e$  paid for them will rise to the level  $\bar{p}_e$  where the excess demand  $D_e(p_e, p_i) - k$  vanishes i.e.,

$$\frac{p_i - \bar{p}_e}{1-s} - \frac{\bar{p}_e}{s} = k \quad (5)$$

from which we obtain

$$\bar{p}_e = (p_i - k(1-s))s \quad (6)$$

Now, using  $D_i = 1 - \frac{p_i - p_e}{1-s}$  from (1) and (6), we obtain the residual demand addressed to the incumbent firm as

$$D_i^r(p_i) = 1 - ks - p_i. \quad (7)$$

The expressions for the sales functions are therefore:

$$S_e(p_i, p_e) = \begin{cases} 0 & \text{iff } p_e \geq p_i s \\ \frac{p_i - p_e}{1-s} - \frac{p_e}{s} & \text{iff } p_e \in [\max\{p_i - (1-s), \bar{p}_e\}; p_i s] \\ 1 - \frac{p_e}{s} & \text{iff } p_e \in [s(1-k); p_i - (1-s)] \\ k & \text{iff } p_e \leq \min\{\bar{p}_e, s(1-k)\} \end{cases} \quad (8)$$

$$S_i(p_i, p_e) = \begin{cases} 0 & \text{iff } p_i \geq p_e + (1-s) \\ 1 - ks - p_i & \text{iff } p_i \in [\frac{p_e}{s} + k(1-s); p_e + (1-s)] \\ 1 - \frac{p_i - p_e}{1-s} & \text{iff } p_i \in [\frac{p_e}{s}; \frac{p_e}{s} + k(1-s)] \\ 1 - \frac{p_i}{s} & \text{iff } p_i \leq \frac{p_e}{s} \end{cases} \quad (9)$$

where branch (9:b) is void if  $p_e > s(1-k)$ .

## 2.2 Price Best Responses

Whenever  $k < 1$ , the analysis of  $G(k, s)$  must take into account the possibility that firms sales are respectively given by equations (9:b) and (8:d). Suppose the entrant's capacity is binding, then it is immediate to see that the best she can do is to sell her capacity at the highest price, which is  $\bar{p}_e = (p_i - k(1-s))s$ . On the other hand, whenever the

<sup>1</sup>We implicitly assume that a secondary market opens where consumers may take advantage of the arbitrage possibilities at no cost.

incumbent plays along segment (9:b), he maximizes profits by setting  $\bar{p}_i \equiv \frac{1-ks}{2}$ , and obtains a minmax profit equal to  $\bar{\pi}_i \equiv \frac{(1-ks)^2}{4}$ .

Given the incumbent's price  $p_i$ , the entrant's payoff function remains concave in own prices (in the domain where  $D_e(\cdot) \geq 0$ ). The best response function is now given by

$$BR_e(p_i, k) = \begin{cases} \frac{p_i s}{2} & \text{iff } p_i \leq 2k(1-s) \\ \bar{p}_e & \text{iff } 2k(1-s) \leq p_i \leq \min\{1-\frac{s}{2}, 1-ks\} \\ p_i - 1 + s & \text{iff } 1-ks \leq p_i \leq 1-\frac{s}{2} \\ \max\{\frac{s}{2}, s(1-k)\} & \text{iff } p_i \geq \min\{1-\frac{s}{2}, 1-ks\} \end{cases} \quad (10)$$

On Figure 2 we illustrate the case  $k > \frac{1}{2}$  (in the other case, the third branch of (10) vanishes).

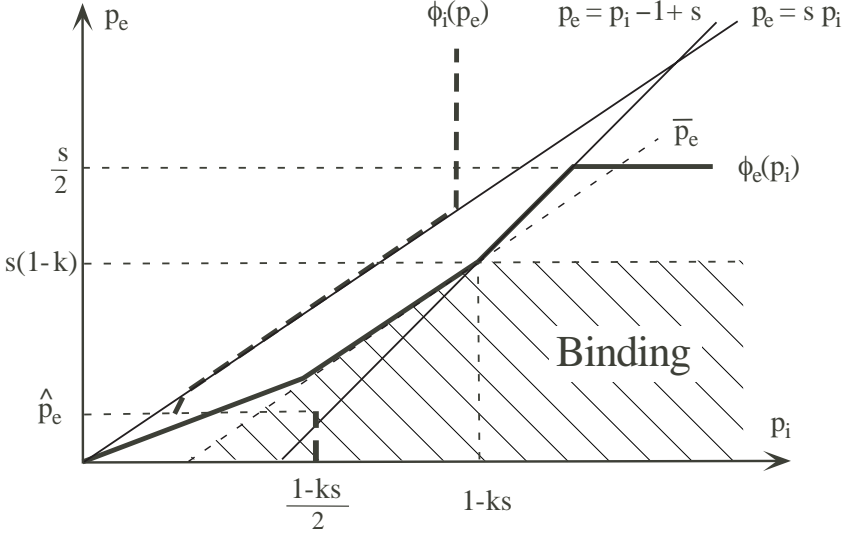


Figure 2: The price space with binding capacity

As should appear from the inspection of  $S_i(p_e)$ , the payoff of firm  $i$  is likely to be non-concave when we pass from segment (9:b) to (9:c). Accordingly, the best response to  $p_e$  might be non-unique. Solving  $\pi_i\left(\frac{p_e+1-s}{2}, p_e\right) = \bar{\pi}_i$  for  $p_e$ , we obtain:

$$\hat{p}_e(s, k) \equiv \sqrt{1-s}\left(1-ks-\sqrt{1-s}\right) \quad (11)$$

which is represented on Figure 2. Yet, it might also be the case that  $\bar{\pi}_i > \pi_i(\frac{p_e+1-s}{2}, p_e)$  over the whole domain where  $\phi_i(p_e)$  is defined by equation (3:a). In this case we must compute firm  $i$ 's payoff along segment (3:b). Solving  $\frac{p_e}{s}(1 - \frac{p_e}{s}) = \bar{\pi}_i$  for  $p_e$ , we obtain:

$$\tilde{p}_e(s, k) \equiv \frac{s}{2} \left( 1 - \sqrt{ks(2 - ks)} \right) \quad (12)$$

Last, to know when one case or the other applies, we solve  $\hat{p}_e = \tilde{p}_e$  to obtain:

$$h(s) \equiv \frac{1}{s} \left( 1 - \frac{2\sqrt{1-s}}{2-s} \right) \quad (13)$$

Depending on the value of the capacity  $k$ , we might therefore obtain two different shapes for the best response of the incumbent firm in the pricing game:

- if  $k \geq h(s)$ , then

$$BR_i(p_e) = \begin{cases} \frac{1-ks}{2} & \text{iff } p_e \leq \hat{p}_e \\ \frac{p_e+1-s}{2} & \text{iff } \hat{p}_e < p_e \leq \frac{1-s}{2-s}s \\ \frac{p_e}{s} & \text{iff } \frac{1-s}{2-s}s \leq p_e \leq \frac{s}{2} \\ \frac{1}{2} & \text{iff } p_e \geq \frac{s}{2} \end{cases} \quad (14)$$

- if  $k \leq h(s)$ , then

$$BR_i(p_e) = \begin{cases} \frac{1-ks}{2} & \text{iff } p_e \leq \tilde{p}_e \\ \frac{p_e}{s} & \text{iff } \tilde{p}_e < p_e \leq \frac{s}{2} \\ \frac{1}{2} & \text{iff } p_e \geq \frac{s}{2} \end{cases} \quad (15)$$

The critical values  $\hat{p}_e$  and  $\tilde{p}_e$  therefore identify the price level at which firm  $i$  is indifferent between naming the security price  $\bar{p}_i = \frac{1-ks}{2}$  or naming a lower price which ensures a larger market share. The resulting discontinuity is likely to destroy the existence of a pure strategy equilibrium.

### 2.3 Price Equilibrium

We analyze the Nash equilibria for each price subgame  $G(s, k)$ . Let us first deal with a particular class of subgames where  $s = 1$ . In this case, the vertical differentiation model degenerates into a Bertrand-Edgeworth competition for an homogenous product. Levitan and Shubik (1972) analyze this game under the efficient rationing hypothesis H1 and derive the following result whose proof is given in appendix.

**Lemma 2** For  $s = 1$  and under H1, there always exists a unique price equilibrium in which the entrant earns exactly  $k\hat{p}_e(1, k)$ . Furthermore the maximum of this payoff is  $\pi_e^\dagger \equiv \frac{3}{4} - \frac{1}{\sqrt{2}} \simeq 0.043$  and is reached for  $k^\dagger \equiv 1 - \frac{1}{\sqrt{2}} \simeq 0.293$ .

When products are differentiated and one firm faces a capacity constraint, the existence of a price equilibrium is not problematic since payoffs are continuous (the Nash existence theorem applies). Moreover, there exists quality-capacity constellations where a pure strategy equilibrium exists. More precisely, the pure strategy equilibrium prevailing in the limiting case where  $k = 1$  is preserved. Let us define  $g(s) \equiv 1 - \frac{4\sqrt{1-s}}{4-s}$  and notice for later use that  $g(s) > h(s) \Leftrightarrow 16s^2(1-s) + s^4(3+s) > 0$  which is always true over the relevant domain.

**Lemma 3** For  $s < 1$  and under H1,  $p_i^* = \frac{2(1-s)}{4-s}$  and  $p_e^* = \frac{s(1-s)}{4-s}$  is a pure strategy equilibrium in the pricing subgames whenever  $k \geq g(s)$ .

*Proof* The well known candidate equilibrium  $(p_i^*, p_i^*)$  is found using equations (9) and (8) (it is illustrated on Figure 1). Next, we identify the conditions under which  $p_i^*$  is indeed a best response to  $p_e^*$ . To this end we only have to solve  $p_e^* \geq \hat{p}_e$ ; straightforward computations yield the condition  $k \geq g(s)$  and since  $g(s) > h(s)$ ,  $\hat{p}_e$  was indeed the benchmark to use. ■

Whenever  $k < g(s)$ , a pure strategy equilibrium fails to exist. For intermediate capacities, it is easy to identify a particular equilibrium in which the incumbent randomizes over two atoms while the entrant plays the pure strategy  $\hat{p}_e$ . However, there also exists a domain of small capacities where even this equilibrium fails to exist. When this is the case, both firms use non-degenerate mixed strategy in equilibrium. The equilibrium strategy used by firm  $j$  in equilibrium of  $G(k, s)$  is denoted  $F_j$ ; the lower bound and upper bound of the support of  $F_j$  are denoted respectively by  $p_j^-$  and  $p_j^+$ .

**Lemma 4** Let  $k < g(s)$  and  $s < 1$ . In equilibrium of  $G(k, s)$ ,  $p_i^+ \leq \frac{1-ks}{2}$  and  $p_e^+ \leq BR_e\left(\frac{1-ks}{2}\right)$ .

*Proof:* The proof proceeds by iteration. Observe firstly that the monopoly price  $\frac{1}{2}$  is an upper bound for  $p_i^+$  because at any  $p_i > \frac{1}{2}$ ,  $\pi_i(p_i, p_e)$  is decreasing in  $p_i$ , thus the average  $\pi_i(p_i, F_e)$  is also decreasing in  $p_i$  which proves that such a price cannot belong



to the support of  $F_i$ . Using this result and the expression of firm  $e$ 's best response, we may eliminate the range of prices  $p_e$  which lie above the best response to  $p_i = \frac{1}{2}$  (check on Figure 2). Within the remaining range of price  $p_e$ , we may further restrict the domain of prices used by firm  $i$  in equilibrium by using the expression of  $\phi_i(\cdot)$ . Reiterating the process, we end up with  $p_i^+ \leq \frac{1-ks}{2}$  and  $p_e^+ \leq BR_e\left(\frac{1-ks}{2}\right)$ . ■

**Lemma 5** *Let  $k < g(s)$ . In equilibrium of  $G(k, s)$ ,  $p_i^+ = \frac{1-ks}{2}$  and the equilibrium payoff is the minimax  $\bar{\pi}_i$ .*

*Proof* When  $k < g(s)$  it is true that  $2k(1-s) < \bar{p}_i = \frac{1-ks}{2}$ . This implies that  $BR_e(\bar{p}_i) = \bar{p}_e$  so that  $\pi_i(p_i, F_e) = p_i(1-ks-p_i)$  in a neighborhood of  $\bar{p}_i$ . Now, if  $p_i^+ < \bar{p}_i$ , then  $\pi_i(p_i, F_e)$  is strictly increasing over  $]p_i^+, \bar{p}_i[$  which implies that  $p_i^+$  cannot be part of an equilibrium strategy for the incumbent.<sup>2</sup> Hence, it must be true that  $p_i^+ = \frac{1-ks}{2}$  and since the equilibrium payoff can be computed at any price in the support of  $F_i$ , we have  $\pi_i(p_i^+, F_e) = p_i^+(1-ks-p_i^+) = \frac{(1-ks)^2}{4} = \bar{\pi}_i$ . ■

**Lemma 6** *Let  $k < g(s)$ . In equilibrium of  $G(k, s)$ ,  $p_e^- \leq \hat{p}_e$  if  $k \geq h(s)$  and  $p_e^- \leq \bar{p}_e$  if  $k \leq h(s)$ . The entrant's equilibrium payoff is bounded from above by  $k\hat{p}_e(s, k)$  if  $k > \hat{k}_e(s)$  and by  $k\bar{p}_e(s, k)$  if  $k \leq h(s)$ .*

*Proof:* Let us consider first the case  $k < h(s)$ . If  $p_e^- > \bar{p}_e$  then for any  $p_i < \frac{p_e^-}{s}$ , the incumbent's demand is monopolistic whatever  $p_e \geq p_e^-$ . Hence,  $\pi_i(p_i, F_e) = p_i(1-p_i)$  is strictly increasing, which means the lower of the mixed strategy  $F_i$  cannot belong to this area, necessarily  $p_i^- \geq \frac{p_e^-}{s}$ . If  $p_i^- = \frac{p_e^-}{s}$ , then at  $p_i^-$ , the incumbent is a monopoly whatever  $p_e \geq p_e^-$ , thus  $\pi_i(p_i^-, F_e) = p_i^-(1-p_i^-) = \frac{p_e^-}{s} \left(1 - \frac{p_e^-}{s}\right) > \frac{\bar{p}_e}{s} \left(1 - \frac{\bar{p}_e}{s}\right) = \frac{(1-ks)^2}{4} = \bar{\pi}_i$  by definition of  $\bar{p}_e$  and by the previous lemma. This inequality is a contradiction with  $p_i^-$  being in the support of  $F_i$ . If  $p_i^- > \frac{p_e^-}{s}$ , then  $\pi_i(p_i^-, F_e) \geq \pi_i(p_e^-/s, F_e)$  since  $p_i^-$  is an

<sup>2</sup>If  $p_i^+ < 2k(1-s)$ , then the previous argument does not apply because  $S_i$  is not always  $1-ks-p_i$ . However, if this case occurs then the entrant's demand, when facing  $F_i$ , is always of the duopolistic kind without capacity constraint, hence his best reply is the pure strategy  $\phi_e$  computed at the average of  $p_i$ . Since the pure strategy equilibrium does not exist over the present domain, the incumbent must be playing a mixed strategy and the only candidate when the entrant plays a pure strategy involves playing the security price  $\bar{p}_i$ , a contradiction with  $p_i^+ < \bar{p}_i$ .

optimal price and  $p_e^-/s$  is not; observing that  $\pi_i(p_e^-/s, F_e) = \frac{p_e^-}{s} \left(1 - \frac{p_e^-}{s}\right)$ , the previous argument applies.

The second claim is a simple consequence of the fact that the equilibrium payoff can be computed at any price in the support of  $F_e$ , hence

$$\pi_e(p_e^-, F_i) = p_e^- \int S_e(p_e^-, p_i) dF_i(p_i) \leq k p_e^-$$

since sales are bounded by the capacity. The case for  $k \geq h(s)$  is identical since the benchmark  $\tilde{p}_e$  and  $\hat{p}_e$  play a symmetric role. ■

Although we do not have a full characterization of the mixed strategy equilibrium in all possible subgames, we have derived enough to state:

**Theorem 1** *An optimal quality-capacity pair is  $s = 1$  and  $k = k^\dagger$ . Other optimal pairs necessarily satisfy  $s \geq \bar{s} \equiv 2(\sqrt{2} - 1) \simeq 0.83$  and satisfy  $sk = k^\dagger$ .*

*Proof* For  $k < h(s)$ ,  $\pi_e(F_e, F_i) \leq k \tilde{p}_e(s, k) = \frac{ks}{2} (1 - \sqrt{ks(2 - ks)})$  which is a function of the product  $x = ks$ , whose maximum is reached for  $x = k^\dagger$ , yields an overall maximum  $\pi_e^\dagger$ . It then remains to observe that this is precisely the optimal quality and the maximum entrant's payoff for  $s = 1$  and  $k = k^\dagger$  as shown in Lemma 2. The corresponding point is indicated on Figure 3 on the right border of the box. The maximum payoff over the domain  $s < 1$  and  $k < h(s)$  is therefore dominated by that in  $G(k^\dagger, 1)$ .

A likewise analysis applies for  $s < 1$  and  $k \in [h(s); g(s)]$ . The upper bound  $k \hat{p}_e(s, k) = k\sqrt{1-s}(1 - ks - \sqrt{1-s})$  when analyzed as a function defined over  $[0; 1] \times [0; 1]$  reaches its maximum for  $k = \frac{1 - \sqrt{1-s}}{2s}$ . Replacing by the optimal value and simplifying, the objective is now  $\frac{\sqrt{1-s}(1 - \sqrt{1-s})^2}{4s}$  and its maximum is achieved for  $s = \bar{s}$  which leads to the optimal capacity  $k = k^\dagger/s \simeq 0.35$  and profit  $\pi_e^\dagger$ . This pair satisfies  $k = h(s)$  i.e., is on the  $h$  curve as indicated on Figure 3. The entrant's profit for  $k \in [h(s); g(s)]$  is thus lesser than the upper bound whose maximum is  $\pi_e^\dagger$ .

Finally, for  $s < 1$  and  $k \geq g(s)$ , the optimum strategy is to differentiate with  $s^* = \frac{4}{7}$  to earn  $\pi_e^* = \frac{1}{48} \simeq 0.021 < \pi_e^\dagger \simeq 0.043$ . Overall,  $s = 1$  and  $k = k^\dagger$  is an optimal strategy pair; there might other optimal strategies satisfying  $ks = k^\dagger$  but they all give the same final payoff. ■

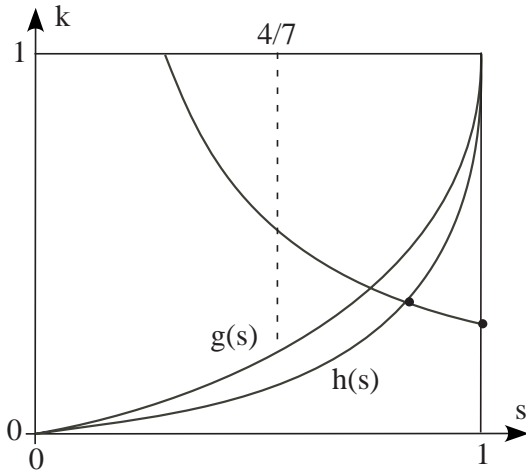


Figure 3: Strategy Space

## Appendix

Proof of Lemma 2: Let  $F_e$  and  $F_i$  be the equilibrium cumulative distributions, assuming no mass except at the end points. Due to the nature of demand, the entrant gets all demand if her price  $p$  is the lowest i.e., with probability  $F_i(p)$ , her payoff is thus  $\pi_e = (1 - F_i)p \min\{k, 1 - p\}$ . Likewise the incumbent's is  $\pi_i = p(1 - p - F_e \min\{k, 1 - p\})$ . Bottom prices have to be the same because otherwise one profit would be strictly increasing in between (all prices are lesser than the monopoly one) and this would contradict the equilibrium definition.

At the common bottom price  $p_l$ ,  $F_i = 0$  and  $1 - p_l > k$ , thus  $\pi_e = kp_l$ . The entrant's top price cannot be greater than the incumbent's one because  $\pi_e$  would be zero, hence at the top price  $p_h$ ,  $F_e = 1$ . If there was no rationing at  $p_h$  then  $\pi_i$  would be zero, thus  $1 - p_h > k$  and  $\pi_i = p_h(1 - p_h - k)$ . Furthermore the right derivative must be negative to make sure than no other greater price is better, hence  $p_h \geq \frac{1-k}{2}$ . We also have  $F_e(p) = \frac{1-p-\pi_i/p}{k}$  (recall that  $1 - p > k$  over the whole interval) thus the density must be  $f_e(p) = \frac{1}{k}(\pi_i/p^2 - 1)$ . Being positive, we derive  $p^2 \leq \pi_i = p_h(1 - p_h - k)$  and applying this inequality at the top price, we get  $p_h \leq \frac{1-k}{2}$ . combining with the reverse inequality, we obtain  $p_h = \frac{1-k}{2}$ , so that  $\pi_i = \frac{(1-k)^2}{4}$ . Now, at the bottom price  $\pi_i = p_l(1 - p_l)$ , thus  $p_l = \frac{1}{2}(1 - \sqrt{k(2-k)})$  and  $\pi_e$ . ■

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