# THE "EFFECTIVE NUMBER OF RELEVANT PARTIES": How Voting Power Improves Laakso-Taagepera's Index 

Patrick Dumont - Jean-François Caulier

December 11, 2003


#### Abstract

This paper proposes a new method to evaluate the number of relevant parties in an assembly. The most widespread indicator of fragmentation used in comparative politics is the 'Effective Number of Parties', designed in 1979 by M. Laakso and R.Taagepera. Taking both the number of parties and their relative weights into account, the ENP is arguably a good parsimonious operationalization of the number of 'relevant' parties. This index however produces misleading results in single-party majority situations as it still indicates that more than one party is relevant in terms of government formation. We propose to modify the ENP formula by replacing proportions of seats by voting power measures. This improved index behaves more in line with Sartori's definition of relevance, without requiring additional information (such as policy positions) in its construction. We thus advocate for the use of our 'Effective Number of Relevant Parties' in future comparative research.


Key words: Voting power indices - Effective Number of Parties - Party system fragmentation - Relevance - Coalition Formation

## 1 INTRODUCTION

Attempts at classifying or characterizing party systems for comparative purposes constitute a classical endeavour in the more general comparative study of democratic political systems. Ever since Duverger (1954) distinguished party systems on the simple basis of the counting of parties in competition, the numerical criterion has become a widely accepted basis for the comparative description of party systems (Lijphart 1994 and 1999; Mair 2002). By means of construction

[^0]of typologies, scholars have pointed to discriminating features in the comparison of democratic polities. But the measurement of how many parties compete and interact in parliament is needed because the shape of a party system may be the product of different determinants in a polity or, conversely, may have broader implications for this polity. Amongst others, scholars have shown that the shape of the party system is largely determined by the choice of an electoral system (Duverger 1954; Taagepera and Shugart 1989; Lijphart 1994 and 1999; Sartori 1997; Cox 1997), that it is also a function of the number of ideological cleavages present in a polity (Lipset and Rokkan 1967; Taagepera and Grofman 1985; Lijphart 1999), that an increase in the number of competing parties can be seen as a sign of democratization or, in advanced industrialised democracies, of some form of electoral change like the party dealignment phenomenon (Dalton, McAllister and Wattenberg 2000). Party systems, on the other hand, determine the degree of bargaining complexity that may affect government formation and maintenance (Warwick 1994; Van Roozendaal 1997; Lijphart 1999; Müller and Strom 2000; De Winter and Dumont 2004).

The Effective Number of parties (ENP) designed by Laakso and Taagepera in 1979 has become the standard numerical measure for the comparative analysis of party systems, as it takes both the number of parties and their relative weights into account to compute a unique variable. It has only recently been challenged by a number of scholars who showed that this index behaves inappropriately and thus gives counter-intuitive results under a number of circumstances (Molinar 1991; Taagepera 1999; Dunleavy and Boucek 2003). In this article, we propose to replace seat shares by voting power indices in the ENP formula. As argued by Gallagher et al., the power index approach highlights '. . . the ways in which the distribution of bargaining power can sometimes differ quite starkly from the distribution of seats in the legislature' (2001: 344), and indeed the power to influence a decision is not always proportional to a party's share of parliamentary seats. These authors, together with De Winter in his recent state of the art in government formation studies (2002: 186-187), argue that the power index approach is a promising avenue for research in the field. Through the introduction of voting power indices, that reflect the potential influence a party may have in the formation of a majority coalition, we also come closer to Sartori's definition of party 'relevance'. We nevertheless adopt Laakso-Taagepera's formula that allows us to give a degree of fragmentation of relevance in competition for government, rather than an absolute figure of parties considered as relevant for majority government formation. We also argue that with our Effective Number of Relevant Parties the most problematic counter-intuitive results of the ENP disappear. Moreover, there is no need to supplement our index with another variable to cope with specific circumstances. Finally, no other data on the party system than those needed for the construction of the ENP are requested.
ments) on the party system than the simple seat distribution amongst parliamentary parties, is not discussed in the present article.

## 2 THE MEASUREMENT OF PARTY SYSTEM SHAPE

Although the name of Sartori is foremost linked to his seminal typology of party systems that drew on both numerical and ideological criteria, his defence of the former type of measure in order to characterize party systems is worth being recalled:
... it does matter how many are the parties. For one thing, the number of parties immediately indicates, albeit roughly, an important feature of the political system: the extent to which political power is fragmented or non-fragmented, dispersed or concentrated. Likewise, simply by knowing how many parties there are, we are alerted to the number of possible 'interaction streams' that are involved... 2 parties allow for only 1 stream of reciprocal interaction, $\ldots 5$ parties for $10, \ldots 7$ parties for $21 \ldots$ the indication clearly is that the greater the number of parties (that have a say), the greater the complexity and probably the intricacy of the system. . . in particular, the tactics of party competition and opposition appear related to the number of parties; and this has, in turn, an important bearing on how governmental coalitions are formed and are able to perform. (Sartori 1976: 120)

Hence, a numerical criterion, per se, may capture important features of party systems. One has however to find what Sartori calls an 'intelligent' way of counting.

One way of characterizing a party system according to a numerical criterion is to simply count the number of parties that gained seats in parliament. Scholars may also set a threshold under which parties are not considered, but the problem with this technique is that the definition of level of the hurdle is essentially arbitrary. ${ }^{1}$ Moreover, even when theoretically justified criteria of 'party relevance' are applied, these methods treat each party -that clears the threshold or qualifies according to the criteria- equally, whatever their differences in size or influence in the competition for office.

The simple criterion of the number of parties present in parliament is nevertheless at the heart of the seminal distinction between two-party systems and multiparty systems made by Duverger (1954). In a refinement of this first classification, Blondel (1968) looked at both the number of parties and their relative size to identify two-party systems, two-and-a-half party systems and multiparty systems including (or not) a dominant party. The main weakness of this new

[^1]classification was that the relative size of 'half' and 'strong' or 'dominant' parties was arbitrarily set (around 10 percent for the former and 40 percent for the latter, in terms of popular vote). Moreover, the two criteria were treated separately. The first scholar who proposed a unique index based on both these variables was Douglas Rae (1967). The idea behind the design of such an index was that the universe of party system sizes is continuous and that scholars thus needed a continuous measure rather than setting numerical criteria in order to identify different classes of party systems. Rae's index of fractionalization can be computed using this formula :
$$
F=1-\sum\left(s_{i}\right)^{2}
$$
where $s_{i}$ is the proportion of parliamentary seats of party $i$, and $\sum$ stands for summation (in this case, the sum of all parties' squared proportions of seats is taken and subtracted from 1 to provide the fractionalization of parliamentary parties). ${ }^{2}$

The more the value of the Rae index comes closer to unit (its maximum value), the more fractionalized the system is. For instance, when 5 parties get each 20 percent of the available seats, that is a proportion of 0.2 each, the Rae index will amount to 0.80 , thus indicating a highly fractionalized party system, as we have to add all five squared proportions of seats and withdraw this result from 1 . If one party gets all the seats, thus getting a proportion of 1 , it is easy to see that the Rae index will point to 0 (its minimum value), indicating no fractionalization at all in an indeed completely concentrated party system. Thus, the fractionalization index designed by Rae does summarize important information about the number of parties and their relative size.

But this index suffers from two major weaknesses. First, Rae's index does not follow a linear pattern when the number of tied parties increases, that is, doubling the number of parties of equal size does not trigger a fractionalization index twice bigger. For instance, a system with two equal sized parties (at 50 percent each) will amount to a F of 0.50 , whilst a system with four equal sized parties (at 25 percent each) will get a F of 0.75 , and, as mentioned above, a system with five equal sized parties will reach a F of 0.80 . This feature is problematic when we want a phenomenon and its operationalized measure to follow the same fashion, a quite obvious requirement in scientific inquiry. ${ }^{3}$ The second problem is linked to the first: even when we simply want to describe a party system, the Rae index, like most normalized indices, is far from being easy to interpret as a unique variable characterizing party system. ${ }^{4}$

[^2]Markku Laakso and Rein Taagepera (1979) transformed this too abstract fractionalization index into a more intuitive measure that will become widely accepted in comparative political science, the Effective Number of Parties (ENP). ${ }^{5}$ The following formula shows how the ENP is computed and its logical link with Rae's indices:

$$
E N P=\frac{1}{\sum\left(s_{i}\right)^{2}}=\frac{1}{1-F}
$$

using the same notation as above.
Although it simply uses the same information as Rae's index, the '... big advantage of ENP is that it can be visualized more easily as the number of parties than the Rae abstract index' (Lijphart, 1994: 69). Taking the example above, if all five parties of the system get each 20 percent of the votes or seats, the Effective Number of Parties is exactly $5.00(\mathrm{~F}=0.80)$. When seats are equally distributed among the parties, the ENP coincides with the raw number of parties (the maximum value of the index). This means that doubling the number of equal-sized parties provides an ENP value twice higher, the other requirement that Rae's index did not meet. If one party gets more seats than the others, the ENP will go down, approaching 4.00. This depicts well the situation. A party system consisting of five parties but with an ENP lower than 5.00 tells us that some parties are 'dominated' by others, that the chances of being necessary in a majority coalition formation are not equally allocated anymore.

The value given by the ENP can be, and is usually, interpreted in comparative political science as the number of hypothetical equal-sized parties competing or being influential for the building of a majority government. Taagepera and Shugart argue that the ENP has become widely used because it 'usually tends to agree with our average intuition about the number of serious parties (1989: 80)'. A decade later, one of its designers goes further by stating that the ENP

[^3]usually comes close to the estimates of Sartori (1976) of the number of 'relevant' parties, or at least '... as close as any operational index based on seat shares alone can come, without the detailed knowledge about the given country (Taagepera, 1999: 498)'. He nevertheless acknowledges that the ENP is not that useful when a single party has more than the majority of seats available in an assembly, '. . . meaning absolute dominance", as the ENP '...still indicates a multi-party constellation (Taagepera 1999: 497)'. In other words, in such situations, the ENP provides the counter-intuitive result that more than one party is relevant in terms of majority coalition formation, and thus gives a wrong picture of how coalition or blackmail potential is distributed. We will show in the subsequent sections of this article that, as size in seats is far from always synonymous to bargaining strength, it is possible to improve Laakso and Taagepera's index in its operationalization of Sartori's concept of party relevance.

Some refinements of the ENP were recently proposed. Taagepera (1999) himself suggested to 'supplement' the ENP in particular situations, through his 'largest component approach'. His advice is to look at another index when the ENP is deemed insufficient, that is, in particular when the largest party's share is bigger than 50 percent ' $\ldots$ and hence dominates absolutely a crowd of smaller parties (Taagepera 1999: 497)'. This secondary index, that we refer to here as the LC (Largest Component) index is the inverse of the share of the largest party. When this index is inferior to 2.00 , this party dominates the party system as its share is larger than 50 percent, a feature that one could see by simply looking at the share of the largest party. According to the author, looking at both indices provides not only an idea of how a party system is fragmented and the weight of the largest party, but also reduces the possible range of weights for the second party in the system (1999: 501). It is nevertheless clear that Taagepera is very cautious in defending the ENP, pointing out that : 'It should be stressed that for most purposes ENP alone will do... We should not clutter our data set by including the supplementary index unless it serves a purpose. However, the secondary index should be available when the need arises (Taagepera 1999: 499)'. We will see below that it is possible to create a unique index that has the property of taking the value 1.00 -meaning that only one party is relevant in majority coalition formation, whenever a single-party majority of configuration exists- thus triggering a more complete measure without loss in parsimony.

More recently, Dunleavy and Boucek (2003) argued that averaging the ENP score with the LC score yield a unique index that provides more realistic results than the ENP. Through a comprehensive experimental and empirical account of how different measures of fragmentation behave with changes in the level of support of the largest party and the number of parties in competition, they show that their $\mathrm{N}_{b}$ index gives lower scores than the ENP (their main critique against the ENP is its over-rating of fragmentation). Although the general technique they propose in order to evaluate the properties of any fragmentation measure is certainly worth considering, the gains in using $\mathrm{N}_{b}$ instead of the ENP (or the ENP and LC) when a party has a share superior to 50 percent are at best
minimal and in any case not satisfactory. Dunleavy and Boucek seem indeed content with a maximum Nb score more than half a party less than the ENP's maximum score when the largest party has 60 percent. But if what we are interested in is to correct the ENP when one party has more than a majority and thus 'wins' or dominates the others, having an index score of more than 1.00, even with a maximum possible value lower than the ENP remains misleading. As Taagepera himself noted, '... once a party has more than $50 \%$, how much does it matter whether it has 53 or $57 \%$ ? (Taagepera 1999: 502)', implicitly reckoning that in any such a situation a fragmentation index should probably be best set at 1.00 .

Siaroff (2003) also presented recently his contribution to the field of party system characterization. He suggests to go back to the older agenda of the classification of party systems through the use of numerical indicators. ${ }^{6}$ Siaroff argues that adding the shares of the two largest parties provides an index of twoparty seat concentration that is instinctively clearer that the other measures and that loses very little information in the process of its computation. ${ }^{7} \mathrm{He}$ nevertheless acknowledges that this measure must be accompanied with another measure to distinguish between possibly quite different configurations displaying the same cumulative index result (Siaroff 2003: 271). He thus introduces two seat ratio measures as measures of competitiveness that complement his first measure (one indicating the ratio between the share of the largest party and the second and the other the same ratio but between the second and the third largest parties. ${ }^{8}$ From these measures, Siaroff proposes a classification of party systems where arbitrary scores of different measure discriminate between four types of party systems, and the combination of his cumulative index and the two ratio measures specifically helps pointing at two-and-a-half-party-systems. According to us, this contribution has the main disadvantages of setting arbitrary values to distinguish party systems and of using several numerical variables -not in a quite consistent way- instead of a unique indicator. ${ }^{9}$ As it would be preferable to compute a single measure for party system fragmentation if we are to use this indicator in large comparative datasets, we argue that the setting up of such a unique variable should be aimed at.

[^4]
## 3 INTRODUCING THE NORMALIZED BANZHAF INDEX

In this section, we define the most important notions and concepts related to the construction of indices in the voting power approach. As argued by Gallagher et al., this approach highlights '. . . the ways in which the distribution of bargaining power can sometimes differ quite starkly from the distribution of seats in the legislature' (2001: 344). Because the power to influence a decision is not always proportional to a party's share of parliamentary seats, we propose to replace seat shares by voting power indices in the ENP formula.

The language used in our subsequent presentation is intended to provide the most intuitive definition of the different notions necessary to understand what such indices measure and to indicate the type of contexts in which they can be applied. In particular, we will concentrate on the index designed by John F. Banzhaf in 1965, which is aimed at giving the relative power of each voter in an assembly. ${ }^{10}$

Assume an assembly with $n$ voters (parties), that may have a different weight in votes (seats), and a well-defined decision rule (also called quota). This decision rule determines the total number of votes needed for a proposal of collective action to be adopted. In general, the simple majority of the available votes is chosen $\left(\frac{n}{2}+1\right)$, but it may be set at a higher level (some form of qualified majority). If this quota is not reached, then the proposal is rejected. In such a weighted voting system where the number of voters is $n$, the assembly is denoted $N$, the weights of voters $[1,2, \ldots, n]$ are denoted $\left[w_{1}, w_{2}, \ldots, w_{n}\right]$. The voting situation can be summarized by the list :

$$
\left[q ; w_{1}, w_{2}, \ldots, w_{n}\right]
$$

with $q$ the quota and $w_{i}$ the number of votes of voter $i$ (which can be any of the parties present in the assembly, $1 \leq i \leq n$ ) has.

For a given proposal, the subset of voters in $N$ that cast a 'yes' vote is $S$, and those who cast a 'no' belong to $N \backslash S$, the complement of $S$ in $N$. A subset $S$ of voters voting in the same direction is called a coalition. Without loss of generality, we thus assume that the members of the coalition $S$ vote in favour of the proposal at hand. The set $2^{N}$ represents all the possible coalitions, including the empty set. The collection of subsets $W$ is the set of all winning coalitions $S$ such that $\sum_{i \in S} w_{i} \geq q$. The set $W$ contains all the coalitions ensuring acceptance of the proposal. It is common to assume the following :

1. $\varnothing \notin W$

To be winning, a coalition needs members.

[^5]2. $N \in W$

The Grand Coalition (that consists of all voters in $N$, in this case all voting in the same direction) is always winning.
3. $S$ and $T \subset N$; if $S \in W$ and $S \subset T$ then $T \in W$

Any coalition that contains a winning coalition (and is thus larger than the latter) must also be winning.
A last condition is usually added to guarantee efficiency of the decisionmaking :
4. if $S \in W$ then $N \backslash S \notin W$

This states that a proposal cannot be accepted and rejected at the same time. When this property is not respected, that is when a coalition (or a voter on its own) is not winning but can prevent its complement to win, we call it blocking. This type of coalition cannot enforce a decision on its own, but can prevent the acceptance of any proposal.

When none of the parties reaches the majority threshold on its own, and the formation of a coalition is thus needed to attain this quota, the configuration is called a minority situation. A coalition is said to be minimal winning (MWC), when all its members are necessary to reach the quota, and thus to enter the set of winning coalitions $W$. Stated otherwise, if one of its members leaves the coalition, the remaining coalition becomes a losing one. In a MWC, all the members are swings, that is each one can turn the coalition into a losing one by changing its vote (defecting). If it withdraws its support to the coalition, any member $i$ swings this MWC into a losing coalition.

$$
i \in S \in W \text { and } S \backslash\{i\} \notin W
$$

There are not only MWC's in the set of winning coalitions. ${ }^{11}$ These nonMWC's that are nevertheless winning are called oversized coalitions. In oversized coalitions, there is at least one member that is not necessary for the coalition to reach the threshold. In a minority situation, ${ }^{12}$ the Grand Coalition does not even contain any swing member (except in the special case where a voter is able, on its own, to block the formation of a winning coalition). ${ }^{13}$ Oversized coalitions are thus winning coalitions that either contain no swing member or only some its members are swings.

A dummy is a voter that is never a swing, or in other words, a voter that is never able to turn a winning coalition into a losing one. The opposite of the

[^6]dummy is the dictator, a voter that is a swing in all winning coalitions. When the weight of the largest party is bigger than the quota, we are thus dealing with a dictator, as this voter can enforce any decision on its own (all other voters are thus dummies). We call such type of configuration a single-party majority situation. If $i$ is the largest voter, then $S \in W$ for all $S \ni i$ and $S \notin W$ if $i \notin S$. Thereby, the dictator is the only swing voter, as it is the only one that can turn a winning coalition into a losing one by defecting, and the only possible MWC is thus the singleton $\{i\}$. Notice that in the special case of a blocking voter in a minority situation, this voter is also needed to form any winning coalition but cannot enforce a decision on its own. The blocking party must indeed find one or more partners to reach the quota, and at least one of these partners will also be a swing. Hence, it is not a dictator as it is not the only swing voter in all winning coalitions.

We denote the number of times voter $i$ is a swing in all winning coalitions by $\eta_{i}$. The normalized Banzhaf Index of voter $i$ is the number of times this voter $i$ is a swing divided by the total number of swing voters. Thus all individual voters' $\beta$ add up to one.

$$
\begin{aligned}
\frac{\eta_{i}}{\sum_{i=1}^{n} \eta_{i}} & =\beta_{i} \\
\sum_{i=1}^{n} \beta_{i} & =1
\end{aligned}
$$

Hence if $i$ is a dictator, $\beta_{i}=1$, and if $i$ is a dummy, then $\beta_{i}=0$.
In practical terms, in order to calculate the distribution of power in an assembly, one has to look at the list of all possible winning coalitions and record for each of these whether voter $1,2, \ldots n$ is a swing or not. Then the total number of swings of any voter $i$ is divided by the total number of swings for all voters in order to provide $\beta_{i}$. Summing the shares of power of all voters should give a result of one.

The normalized Banzhaf index, as well as a number of other well-known measures of voting power, is part of the class of a priori power indices. This means that the measure only takes the distribution of resources (generally seat weights) and the decision rule (the quota, usually set at the absolute majority of seats available) into account to evaluate each party's probability of affecting the outcome of a vote in an assembly. All other potential factors that may influence of such an outcome (such as preferences, persuasion or negotiation skills, history of previous interactions, institutional setting, etc.) are abstracted away and thus all combinations of parties that clear the threshold of the decision rule -those that form the set of winning coalitions- are considered as equally probable. This may be either because we do not have reason to believe that certain coalitions are more likely to form than others or because we do not have accurate data for inferring a distribution of probabilities. The assumption of equiprobability is desirable in the former case, and this is what scholars who engage in normative
studies of constitutional design argue: when the goal is to (re-)design a voting system, ${ }^{14}$ it would indeed be inappropriate to look at empirical patterns of roll calls in order to favour or punish countries that happened to vote more often with others in the former institutional setting. On the other hand, if we have an intuition that some factors may well make some coalitions more likely than others, and that information on those factors is both available and trustworthy, positive research on the power each party has in an assembly may need the construction of actual, or a posteriori power indices. More specifically, it is possible to feed the a priori indices with this pertinent information in order to weight the probability of a certain type of coalition to form with regard to another, or simply restrict the number of feasible coalitions according to such information. ${ }^{15}$ In any case, as Felsenthal and Machover (2003: 474) recently argued, a priori voting power is a component of this actual or a posteriori voting power.

There is another distinction made amongst classes of indices in the voting power literature. This one concerns the object of the decision taken by the assembly, and thus relates somehow to the motivation that drives parties in majority building. The Banzhaf indices are part of the indices that reflect $I$ Power, that is 'power as influence' over the outcome of a decision. ${ }^{16}$ These indices are indeed correctly interpreted as the probability each party has, under a specified decision rule, in influencing such an outcome (the passing or the rejection of a vote). This is computed by simply counting the number of times it is crucial to the building of a majority. As this outcome is not related to a value, or a structure of payoffs, Felsenthal and Machover (1998; 2001) argue that indices of I-Power reflect power in a context of competition amongst policyseeking parties. ${ }^{17}$ On the other hand, when a fixed total payoff is specified, majority building can be seen as a simple cooperative game with transferable utility and indices reflecting P-Power, or 'power as a price' should be used. These indices, such as the one designed by Shapley and Shubik (1954), reflect the expectations of parties in terms of share of the payoff to be distributed (amongst the components of a winning coalition only). ${ }^{18}$ In this context, parties

[^7]are essentially seen as office-driven as they are expected to seek to maximize their office payoffs.

In the following section, we argue that knowledge of policy preferences is not necessary for the operationalization of Sartori's concept of 'relevant' party, ${ }^{19}$ and that what we need is a reflection of the potential influence of individual parties on the building of a majority. We thus justify the choice of the normalized Banzhaf index, an a priori I-Power index, for this purpose.

## 4 TWO WORLDS COLLIDE: PARTY SYSTEM LITERATURE AND THE VOTING POWER APPROACH

### 4.1 The normalized Banzhaf index and Sartori's concept of 'relevance'

We have seen that the normalized Banzhaf index is the number of times each voter is a 'swing' when all winning coalitions are considered, divided by the sum of the total number of swings (thus each time any voter is a swing). In this section, we show that this index, despite its a priori character, gives an appropriate measure of what Sartori (1976) calls the 'relevance' of parties in a parliamentary assembly.

The concept of relevance at the heart of Sartori's typology of party systems refers to the place a party has in the competition for office in a given system. Mair (2002: 93) indeed points out that Sartori's approach '.. . helps to focus attention directly on what is perhaps the most important aspect of party systems, and on what distinguishes most clearly between different party systems: the structure of inter-party competition, and especially the competition for government'. This focus on the competition for government is clear in his definition of a two-party system: 'We have a two-party format whenever the existence of third parties does not prevent the two major parties from governing alone, i.e. when coalitions are unnecessary. . ' and implies that one has to count parties on the basis of their seat shares because '...governments are formed, and perform, on the basis of their strength in parliament (Sartori 1976: 186)'.

[^8]According to Sartori, one should look at both the parties that are considered as having 'coalition potential' -are willing to compete for access to government and are accepted as such by their competitors- and those that have a 'blackmail potential' -those that are either anti-system or not considered fit for government by other parties but that are able to block the formation of certain coalitions because of their weight in parliament- in order to assess their relevance in the competition for office in a given party system. Given that the normalized Banzhaf index of a party is correctly interpreted as 'what is that party's probability to influence the outcome, relative to the other parties present in a given assembly', political scientists should immediately see the connection between Sartori's relevance criteria and this measure aimed at quantifying potential influence. ${ }^{20}$

Sartori however argued that the relevance of a party '... is a function not only of the relative distribution of power -as is obvious- but also, and especially, of its position value, that is, of its positioning along the left-right dimension' (1976: 121). Hence, comparative politics scholars like Lane and Ersson (1999: 136) interpret the coalition potential criterion as applying to all, thus even small parties, of the political centre, and the blackmail potential criterion as applying to large extremist parties. What the voting power approach teaches us however is that size does not always equal power, and that smaller parties, whatever their ideological position, may influence the composition of governments by preventing some formulas to reach a majority: Gallagher et al. (2003: 344) take the classic example of a legislature in which two parties win 45 percent of the seats and a third party the remaining 10 percent. Because of the presence of a third party, none of the big parties can govern on its own and a coalition is needed to pass the majority threshold in parliament. If we assume that parties are more concerned about getting into office than about implementing a specific policy, any coalition consisting of two parties can form, regardless of parties' policy positions. Hence, the smaller party has the same bargaining power as the two larger ones. Gallagher et al. point to the fact that this is the type of situation in which Germany was before re-unification. They thus show that the presence of the smaller Free Democrat Party in most governments can also be explained without the express recourse to its (median) policy position in the German left-right party competition. Indeed, what the voting power approach shows is that the FDP had a disproportionate share of bargaining power (the same as the two larger parties' despite its sheer size in seats). Even

[^9]if we had reliable party placements on the main dimensions of competition and we assumed that parties are policy-seekers, isn't it the case that even a small extremist party can constrain the process of cabinet formation? In the example taken above, not only an extremist party with 10 percent of the seats prevents, by its mere presence, any single-party majority to form, but it also forces the two larger parties to form a coalition, as any coalition in which the extremist party would participate should be erased in the list of feasible coalitions on ideological grounds. In this case, the presence of a small extremist party in parliament indeed determines the composition of the coalition, as the coalition of the two larger parties is the only possible majority formula. ${ }^{17}$ We argue that parties that exert such an influence on the process of cabinet formation should not be discarded, as they display a clear blackmail potential. Hence, if the blackmail potential ${ }^{18}$ of a party (that does not qualify under the coalition potential definition) is not related to its absolute but to its relative strength in seats, taking the whole distribution of seats into account in order to evaluate each party's bargaining power, even small extremist parties qualify for relevance.

The combination of the two criteria for Sartori's concept of party relevance and the insights of the voting power approach has an important implication: as even smaller extremist parties can qualify for blackmail potential, counting the number of parties without knowledge of ideological positions becomes indeed a fair operationalization of the number of relevant parties. ${ }^{19}$ In any case, we argue that the index we present below comes closer to the number of what Sartori refers as 'relevant' parties than any operational index created without such a detailed knowledge of a polity.

However, the question of parties that are considered as having 'coalition potential' but are not as strong as to be a swing in any winning coalition (what we refer to as dummies) can be raised, as these will have a normalized Banzhaf index of zero, indicating that they have no relevance at all. To this potential objection, we would respond both theoretically and empirically: first, we would

[^10]argue that the concept of relevance, at least in its coalition potential component, essentially refers to an 'active' role in the competition for office. If a dummy party eventually ends up being part of a government, it is through the will of other actors that are -contrary to the dummies- crucial to the building of the coalition, not through its own bargaining strength. It may be because of its ideological leaning that other parties request its presence, turning a minimal winning coalition into an oversized one that can then become an ideologically minimal connected coalition (Axelrod 1970). But this type of governmental participation for a dummy party owes more to 'luck' more than to real 'power' (Barry 1980). Moreover, we cannot evaluate a party's coalition potential unless we look at other information than the distribution of seats in an assembly. If we want our indicator to stick as much as possible to the principle of parsimony, we are thus better off computing relevance through the numerical blackmail potential parties have. Second, dummies almost never participate in governments: in a dataset consisting of all the distributions of seats following elections in twelve west-European parliaments over the full post-war period (1945-1999), and excluding cases of single-party majorities, there were no dummies included in the cabinets that were formed directly after these elections $(\mathrm{N}=163) .{ }^{20}$ Hence, the alleged coalition potential of dummies almost never translate in effective participation to government (this is mainly due to their sheer seat strength, 2.5 percent on average).

Finally, let us note that the normalized Banzhaf index can also be seen as operationalizing the concept of 'credible exit threats' ${ }^{21}$ or of 'walk-away values' ${ }^{22}$ that parties may or may not have when bargaining in coalition formation. The number of swings parties have can indeed be seen as important resources in the process of coalition formation, as they can be used to exert threats on the other actors. If a party is a swing in the coalition formula that is being negotiated, but has a number of other alternatives in which it can exert the same influence whilst the other members of that coalition have not, it can use these resources by credibly threatening to leave the negotiation table if its partners do not give in to its demands. If the partners have no alternatives (they may be a swing in the coalition being formed, but not in other winning combinations), they either give in or join the opposition by letting the powerful party implement is threat. Having more credible exit threats than others, or, in other words, having a larger amount of alternatives to turn to when walking away from the

[^11]negotiation table is thus an important asset in a bargaining situation. If the normalized Banzhaf index can be seen as operationalizing these concepts, the type of information it provides is the relative bargaining power of an individual party. By applying the formula proposed by Laakso and Taagepera to these values instead of parties' seat shares, we argue that we give a more appropriate picture of how 'coalition and blackmail potential', 'credible exit threats' or 'walkaway values' are distributed. We will come back to the potential applications of the Banzhaf index and our ENRP in terms of bargaining complexity in the discussion at the end of this article, but in the following section we first indicate how exactly to measure the new index and illustrate how it behaves.

### 4.2 Measuring the Effective Number of Relevant Parties: formula and examples

The Effective Number of Relevant Parties present in an assembly is computed using the following formula :

$$
E N R P=\frac{1}{\sum\left[\beta_{i}\right]^{2}}
$$

where $\beta_{i}$ is the normalized Banzhaf index for party $i$, and $\sum$ stands for the summation over all parties' normalized Banzhaf indices.

Table 1 shows how our ENRP behaves in a number of party constellations that are characterized by an Effective Number of Parties of 3.00. These examples are drawn from the article in which Taagepera suggests to supplement the ENP with a measure of the largest component (LC) in order to tackle the issue of single-party majority situations. The first column provides the distribution of seat shares amongst parties, the second provides the crudest numerical indicator of party systems, that is the number of parties represented, the third and fourth show respectively Laakso-Taagepera's Effective Number of Parties and Taagepera's Largest Component indicator. The fifth column indicates the number of 'relevant' parties according to Sartori's definition, as evaluated by Taagepera (1999). Finally, the last column provides our Effective Number of Relevant Parties.

| Party constellations (from P1 to Px) | N | ENP | LC* | Relevant <br> parties | ENRP |
| :--- | :---: | :---: | :---: | :---: | :---: |
| A: $0.33-0.33-0.33$ |  |  |  | 3.00 | 3.00 |
| 3 | 3.00 |  |  |  |  |
| B: $0.35-0.33-0.32$ | 3 | $3.00^{*}$ | 2.86 | 3 | 3.00 |
| C: 0.39-0.32-0.28-0.01 | 4 | 3.00 | 2.56 | 3 | 3.00 |
| D: 0.45-0.29-0.21-0.05 | 4 | 3.00 | 2.22 | 3 | 3.27 |
| E: 0.47-0.24-0.22-0.07 | 4 | 3.01 | 2.13 | 4 | 2.97 |
| F: 0.48-0.23-0.21-0.08 | 4 | 3.00 | 2.08 | 4 | 2.97 |
| G: $0.48-0.32-20$ parties at 0.01 | 22 | 2.99 | 2.08 | 1 or 2 | $1.00^{* *}$ |
| H: 0.53-0.15-0.10-0.10-0.10-0.02 | 6 | 3.00 | 1.89 | 1 | 1.00 |
| I: 0.55-6 parties at $0.07-0.03$ | 8 | 3.00 | 1.82 | 1 | 1.00 |
| J: 0.57-21 parties at $0.02-0.01$ | 23 | 3.00 | 1.75 | 1 | 1.00 |

Effective Number of Relevant Parties for configurations with constant ENP
(Adapted from Taagepera 1999: 498)

* LC=Largest component approach (Taagepera 1999). The value of the index is the inverse of the largest party's share of votes or seats.
**ENP $=2.996$
***ENRP $=1.004$

Configuration A is the classical example that shows that in an assembly consisting in a number of equally strong (in terms of seats) parties, the ENP takes the value of this number. Hence, with three parties sharing equally the number of available seats, the ENP will be 3.00, just like in the case of two parties owing 50 percent of the seats, the ENP would be 2.00 . These cases of extreme fragmentation constitute the upper bounds of the ENP, but as is shown by all configurations in the table, many different distributions of seats amongst three or more parties may give the same ENP value. As soon as the distribution of seats favours slightly one party over the others, the ENP takes a value inferior to that of the number of parties receiving seats, just like in configuration B. The ENRP shares the same desirable (for the sake of its interpretation) constraint of taking the number of parties as its maximum value when these parties share power equally (in terms of swings).

On the other hand, there is only one case in which the ENP would be exactly 1.00, that is the lower bound of the index, and that is when one party receives all the seats. As long as there is more than one party in an assembly, the ENP will be greater than this overall minimum. Hence Taagepera's own critique of the ENP (this critique is furthered at length by Dunleavy and Boucek 2003) in situations of single-party majority. For instance, although in configurations from H to J one party has a majority of seats on its own, and can thus enforce all its preferred decisions, the ENP still indicates a value of 3.00 whereas the number of relevant parties for majority coalition formation would be 1 . This is a rather
embarrassing result for an index that is supposedly the best operationalization of Sartori's concept of relevance without knowledge of party policy positions. Taagepera's solution of supplementing the ENP with another indicator for such types of configurations is not really convincing either, as the LC index only shows that a party has a majority of seats when it takes a value inferior to 2.00 (again, this index only gives a value of 1.00 when one party gets all the seats available). Having a continuous measure between 1.00 and 2.00 is however not appropriate either in such a context, as Taagepera himself implicitly recognized by remarking that '... once a party has more than $50 \%$, how much does it matter whether it has 53 or $57 \%$ ? (1999: 502, also quoted above)'. Sartori already had made a major warning against the use continuous measures in the comparison of party systems. ${ }^{23}$ According to him, nominal measures have the advantage of taking thresholds, or turning points, and thus better reflect jumps from one situation to another. More specifically, Sartori refers to the 'jump, or the all-or-none threshold... established by the majority principle (1976: 315)' in the world of voting that is the one assemblies evolve in. He argues that continuous measures are blind to these jumps present in democratic politics and thus that they should be accompanied by nominal measures that indicate whether only one or more parties are needed to reach the majority of seats.

By replacing seat shares by normalized Banzhaf indices in the formula of the ENP designed by Laakso and Taagepera, we are capable of responding to this critique on continuous measures, as voting power indices depend on both the distribution of seats and a decision rule (usually set at the absolute majority of seats in the context of parliamentary assemblies). They thus reflect this qualitative jump from a minority configuration to a majority situation. Specifically, the ENRP we designed has the interesting property of taking the value 1.00 whenever a single-party majority exists. Hence, the lower bound of the ENRP will be reached any time a party is a dictator in terms of swings and not only when one party is a dictator in terms of seats (when no other party is represented). These situations occur whenever one party has a majority of seats in an assembly, as it becomes a swing in all majority formula and renders all parties powerless (dummies), whatever the number of parties owning seats. Our index thus displays the feature implicitly suggested as desirable by Taagepera (1999), that is that whatever the strength of the majority in seats for the largest party, the fragmentation of relevance for majority building is set at 1.00 . Whatever how much the party is majoritarian, the probability of this party letting a minority forming a government and enforcing decisions is indeed very small. In case it nevertheless decides to form a surplus coalition by inviting a dummy party to join government, it retains the absolute power to revoke this unnecessary partner at any time, leaving the latter a negligible potential influence on decisions. Hence, such a feature seems pretty reasonable and makes our index come closer Sartori's concept of relevance, as shown in

[^12]configurations H to J in Table $1 .{ }^{24}$
How do the different measures reflect how coalition and blackmail potential are distributed amongst parties indices in the rest of the configurations listed in Table 1? In configuration C, the fourth party (P4) is a dummy (too small to be a swing for any winning coalition), leaving all bargaining power to the three larger parties. As the latter three, despite some differences in seats, have the same number of swings, the ENRP equals exactly 3.00. From configuration D to configuration E, the largest party and the smallest party gain seats from the two middle-sized parties. These increases in seats for P1 and P4 render possible the formation of a minimal winning coalition P1-P4 that was not feasible in configuration D. Both parties thus gain one swing. But this change in relative strengths also affects P2 and P3, as in configuration E they are not swings anymore in the respective winning coalitions P1-P2-P4 and P1-P3-P4. Only P1 remains a swing in these coalitions, and thus the gap between the most powerful party and the following widens, a change towards more power concentration that is reflected in the ENRP but not in the ENP. It is unclear however why Taagepera estimates that the number of relevant parties is 4 for both configurations E and F and not for D. He argues that '...in F, even the smallest of the four parties has coalition potential. Ideology permitting, it could be the largest party's preferred partner, or it could clinch a majority coalition that excludes the largest party (1999: 501)'. But the balance of power tips more in favour of the largest party in F (and E) than in configuration D , indicating less fragmentation than in D (where the smallest of the four parties already had coalition potential, although less than in D, as it was a swing in the coalition excluding the largest party). Hence, more potential coalitions does not trigger an automatic increase in fragmentation, as the balance of the number of swings may change dramatically and the largest party may benefit from it. Thereby our index should -and does- indicate more concentration instead. The example of D also shows that our ENRP result is not always lower than the value of ENP. ${ }^{25}$

The comparison of configurations F and G is more appealing, as the size of the largest party is the same, and thus so is the LC index. As Taagepera points out (1999: 501) ' $\ldots$. when political coalition building enters, then cases F and G look quite different, despite having the same ENP and also the same LC...' as with a highly fragmented number of small parties, the largest party that is very close to the majority threshold can form a coalition and be a swing in a very large number of coalitions. This is much less the case for the second

[^13]largest party that has to convince no less than 19 of the 20 small parties to exclude P1 from government. We are thus confident in the result given by our ENRP, that is very close to 1.00 (1.004), as the power of majority building is highly concentrated in the hands of the largest party. ${ }^{21}$ In any case, none of the measures proposed by Taagepera manages to reflect this major difference between the two configurations, as he himself acknowledges:

The disappointing surprise is that the same combination of ENP and LC can hide coalition-building implications as different as those of cases F and G. What it means is that even the two indicators... jointly cannot always convey all the information we would like to have (1999: 502).

## 5 SUMMARY and DISCUSSION

The Effective Number of Relevant Parties operationalizes Sartori's concept party relevance by taking advantage of the insights from the voting power literature and the formula of the fragmentation index designed by Laakso and Taagepera back in 1979. The latter was up to now rightly considered as the best unique operational index for comparing party systems without a detailed knowledge of the polities under study. We showed that the ENP was problematic in singleparty majority situations, in that it still indicates that more than one party is relevant in terms of government formation, whilst it is clear that such a party has the power to ensure the enforcement of all its preferred policies without the others being able to prevent such an outcome. Our ENRP gives a more accurate picture of the number of relevant parties for building a majority in this type of configuration. We also explained why and how Sartori's definition of relevance might be operationalized without requiring additional information such as ideological positions of the parties. Moreover, our ENRP provides a degree of fragmentation of Sartorian party 'relevance', and not simply absolute figures of relevant parties. The two first features derive from our use of the voting power approach, and the third one from our recourse to a fragmentation formula. As these improvements of the ENP are not detrimental to the principle of parsimony in the construction of our index -it does not require new information nor the adjunction of a second indicator to supplement it in special cases- and because of its easiness to compute, ${ }^{27}$ we advocate for its use in future positive research in the field of comparative politics.

[^14]Contrary to Albert's recent critique (2003) ${ }^{28}$ we thus argue that voting power indices can become descriptive and even predictive tools in the context of positive research. We even showed that a priori power indices may be used to reflect theoretical concepts more adequately than measures widely endorsed by empirical scholars in political science. In so doing, they render such concepts amenable to better empirical tests of hypotheses. We thus have good news for List, who argues that
...voting power might plausibly serve as a regressor in models of certain empirical phenomena... it is conceivable (though still an untested hypothesis) that voting power might affect decision outcomes: policies preferred by agents with greater voting power might prevail more often than ones preferred by agents with less voting power. Similarly, the distribution of voting power might conceivably affect the dynamic of decision processes and perhaps the nature of deliberation in a collectivity (2003: 490).

The index we propose in this article is not only theoretically promising: it has already been used in empirical studies as reflecting bargaining complexity in political assemblies and proved to be an important determinant in government formation (Bäck and Dumont 2003; De Winter and Dumont 2004; Mitchell 2004). It is also a useful indicator for research on the relationship between electoral and party systems (Dumont, Indridason and Caulier 2004), and one could think of fruitful applications in public policy analysis as well. Finally, individual a priori indices have also been recently used by scholars to explain and predict parliamentary behaviour (Laver and Gianetti 2001; Laver and Kato 2001; Laver and Benoit 2003). Hence, as long as we are cautious in both the validity of our operationalizations of theoretical concepts and the interpretation of our results, ${ }^{22}$ we are convinced of the potential of the voting power approach for positive research in political science.

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[^0]:    ${ }^{0}$ The second property that emerged in the literature devoted to the comparison of party systems is the ideological distance that separates the most extreme parliamentary parties (the degree of polarization, that correlates with the type of party competition -centripetal or centrifugal- Sartori 1976). This property, that requires more information (ideological place-

[^1]:    ${ }^{1}$ Ware (1996) for instance only considers parties that have at least 3 percent of the seats available in an assembly. The number of parties sharing this property is then called the number of 'relevant' parties, a very liberal understanding indeed of Sartori's concept of 'relevance' (see infra).

[^2]:    ${ }^{2}$ Although most of the classical fragmentation indices may be applied to both measures at the electoral level (vote shares) and measures at the parliamentary level (seat shares), we stick to the latter application.
    ${ }^{3}$ Using such an index in bivariate or multivariate analyses would trigger flawed results in terms of relationships between a dependent and one or several independent variables.
    ${ }^{4}$ The Rae fractionalization index is nothing else than the complement to unit of the Herfindhal-Hirschman concentration index (abbreviated HH here), that is not as widely known

[^3]:    in political science. The basic idea is the same, that is to give a specific weight -that is not arbitrary- to the parties according to their size when counting them: the seat shares of parties determine their own weight, an operation that is done by squaring these proportions (a party with half the seats will receive a weight of 0.5 , so that its value is $0.5 \times 0.5=0.25$, whilst a party with 10 percent will have a weight of 10 percent and a value of $0.1 \times 0.1=0.01$ ). The concentration index is then calculated as follows :

    $$
    H H=\sum\left(s_{i}\right)^{2}
    $$

    where $s_{i}$ is the proportion of seats for party $i$.
    ${ }^{5}$ Two other indices are sometimes cited in theoretical discussions about indicators of party system fragmentation, but they were not taken over in many empirical studies. Whereas in Laakso and Taagepera's index the shares of votes (or seats) of a party are self-weighting (by squaring these values), Wildgen's (1971) index of 'hyperfractionalization' accords special weight to small parties and Molinar (1991) gives more weight to the largest party. Quite logically, the ENP usually generates values that are smaller than the Wildgen index and greater than Molinar's. Although the latter seems at first sight quite attractive as it gives more intuitive results than the ENP in certain circumstances, it triggers more problematic counter-intuitive results in others, and is more difficult to compute (Lijphart 1994: 69-70; Taagepera 1999: 499; Dunleavy and Boucek 2003: 308-13).

[^4]:    ${ }^{6}$ Since the creation of the fractionalization indices and the classic contribution of Sartori on the basis of both numerical and ideological criteria, the identification of classes of party systems according to the sole numerical criterion had been almost abandoned.
    ${ }^{7}$ Cumulative indices of the like were already introduced by Lijphart (1968) but at the electoral level (percentage of votes) only. Blondel (1968) even specifically suggested the use of a cumulative index of the support for the two strongest parties in order to identify different classes. Sartori (1976: 306-7) however showed that when seat shares were computed instead of vote shares, the cutting points Blondel had found discriminating for groups of countries disappeared.
    ${ }^{8}$ Intervals between the first and the second party strengths were already used by Sartori (1976) in order to identify predominant party systems.
    ${ }^{9}$ It is worth pointing out that the author's main concern was to find a way to characterize 'two-and-a-half-party-systems' and thus customized his research indicators for this specific end.

[^5]:    ${ }^{10}$ As Felsenthal and Machover (1998) argue, the presentation of this index does not require recourse to the formal apparatus of cooperative game theory, as this brand of measure was first designed by Penrose in statistical theory.

[^6]:    ${ }^{11}$ Unless we are in a 2-party assembly with equal seat shares.
    ${ }^{12}$ If there is a swing member in the grand coalition, we are dealing with a single-party situation and this player is a dictator (see infra).
    ${ }^{13}$ For instance, in an assembly where 100 seats are distributed amongst n parties but the largest of these parties has 50 seats, it is a swing in all possible winning coalitions, including the Grand Coalition. In a 2-party assembly displaying such a feature, both parties are blocking (because of the 50-50 distribution) and both are thus swings in the Grand Coalition.

[^7]:    ${ }^{14}$ Or an aspect of this voting system. Take for instance the EU Council of ministers in the wake of the coming enlargement: given the distribution of votes bargained in Nice in 2000, at what level the decision rule should be set in order to keep the same individual distribution of voting power amongst incumbent member states before and after this enlargement.
    ${ }^{15}$ Different methods that plug preferences in have been presented. See Bilal and Hösli (1999) or Laruelle and Valenciano (2002) for attributing different probabilities to potential outcomes and Pajala (2002) for the restriction of the set of winning coalitions to connected winning coalitions.
    ${ }^{16}$ The absolute Banzhaf index, also referred to in the literature as the Penrose-Banzhaf measure, divides the number of swings a party has by the total number of coalitions (winning or losing) that party may be part of (thus $2^{n}-1$ ).
    ${ }^{17}$ This policy-seeking assumption is quite different than the one commonly understood in coalition theories drawn from political science. Whilst in the latter it refers to the maximisation of policy preferences in spatial models, it is here devoid of any reference to expectations in terms of payoffs based on known policy positions.
    ${ }^{18}$ Instead of looking at all the theoretically possible winning combinations (as in the com-

[^8]:    putation of Banzhaf indices), the Shapley-Shubik index looks at all theoretically possible permutations of voters. When one party is a swing and thus turns a losing coalition into a winning one in a specific sequence of votes, it gets all the payoffs. By computing the swings of all possible sequences of votes, the index averages out the share of payoffs each party could expect in a given assembly.
    ${ }^{19}$ Without even entering the debate over the quality of existing party policy placements (see Laver 2001) or pointing at the possible overestimation of the explanatory power of policyseeking motivations of parties in coalition formation (see for instance the exclusion of centre parties in Belgian, Dutch or Finnish governments and even the participation to power of populist and extreme parties in Italy, Austria or the Netherlands in the last decade).

[^9]:    ${ }^{20}$ Such a distinction between individual parties' power to contribute positively to the formation of a majority coalition and their power to destroy it (or in other words, block its formation) is clearer in the indices designed by Coleman in 1971. The Power to Initiate action $\left(I_{i}\right)$ is the number of positive swings of a party (the number of times a party is able to turn a losing coalition into a winning one), whereas the Power to Prevent action $\left(P_{i}\right)$ is its number of negative swings (the number of times a party is able to turn a winning coalition into a losing one). These indices measure parties' decisiveness conditionally to the outcome (respectively the formation of a majority coalition or its failure). But both the Coleman indices, when normalized in order to have the sum of individual powers equal to 1 , give the normalized Banzhaf index (and the absolute Banzhaf index of party $i$ is the harmonic mean of $I_{i}$ and $P_{i}$ ).

[^10]:    ${ }^{17}$ To be sure, take also the example of a parliament with four parties, ordered alphabetically from left to right: A has 41 percent of the seats, B has 10 , C has 39 and D, which is an extremeright party considered unfit for government, has the remaining 10 seats. Only coalitions AB , AC and ABC are feasible in terms of majority building. Hence, the presence and weight of $D$ in such a distribution of seats erase any possibility to form a coalition without A (BC does not reach a majority and BCD is not feasible as D cannot take part in government).
    ${ }^{18}$ This example of a system consisting of two large parties and a smaller one, but the latter considered by the others as unfit for government, closely resembles the Austrian case in the 1990s. Because of its blocking power, the FPÖ largely determined the composition of governments, as the SPÖ and ÖVP had to govern together in grand coalitions. In such a paralyzed system, the populist stances of the FPÖ and the ever more negative incumbency effect for outgoing coalition partners paved the way for electoral victories of the pariah party and finally its participation to government.
    ${ }^{19}$ If we accept this demonstration, then no information on policy positions are needed in order to operationalize the concept of party relevance, as the influence any party may have in restricting or expanding the number of feasible alternatives for parties competing for office should be accounted for. Hence, using a voting power index that considers all potential winning combinations of parties as equiprobable does not seem inappropriate.

[^11]:    ${ }^{20}$ In a larger dataset consisting of all cabinets formed in these twelve countries ( $\mathrm{N}=300$ ), thus accounting for cabinets that form during the inter-election period, only 3 dummy players out of 101 entered a cabinet. The authors wish to thank Wolfgang Müller, Kaare Strom and Torbjorn Bergman for allowing them to use this subset of the Coalitions Governance dataset.
    ${ }^{21}$ Van Roozendaal 1997 introduces this concept in his study of government duration, and operationalizes it with the 'dominance' certain parties exert on others. In a nutshell, a party system is dominated whenever the second largest party is unable to form a majority coalition with the partner(s) that the largest party could choose to form a the smallest majority coalition in seats it can be part of. In that case, the second party has less evidently at least one alternative less than the largest in terms of majority building.
    ${ }^{22}$ Lupia and Ström 2004 argue that walk-away values are amongst the most important determinants of the life and death of coalition governments.

[^12]:    ${ }^{23}$ At the time (1976), it was directed at Rae's index of fractionalization but the critique encompasses all indices based on '... mathematics in actual usage in the social science [as the latter] is not suitable for handling thresholds (Sartori 1976: 315)'.

[^13]:    ${ }^{24}$ Although calculated on these 10 examples only, the strength of correlation coefficients between the different indices and the number of relevant parties as evaluated by Taagepera (and depending on taking 1 or 2 as a value for this variable in configuration $G$ ) is around 0.9 for the ENRP, around 0.5 for the ENP and the LC (the latter being more correlated than the former if the value 2 is taken for configuration $G$, and the other way round if the value 1 is taken) and between -0.9 and -0.5 (thus negative) for the crude number of parties.
    ${ }^{25}$ In the comparative dataset referred to above (that excludes single-party majority configurations), the value of the ENRP is higher than the ENP in one third of all cases.

[^14]:    ${ }^{21}$ Indeed, we would argue that the value 1 (suggested in Table 1 together with the value 2) should be chosen for the number of relevant parties as estimated by Taagepera according to Sartori's definition.
    ${ }^{27}$ It is possible to compute individual power indices as well as fragmentation indices such as the ENP on line at http://powerslave.val.utu.fi/, a website maintained by Antti Pajala from the University of Turku, Finland.

[^15]:    ${ }^{28}$ And to Garrett and Tsebelis' (1999; 2001) critiques on the specific use of a priori indices in empirical analyses of European Union institutions.
    ${ }^{22}$ In non-normative research, scholars have so far mostly used and sometimes abused power indices (in the interpretation of results) in descriptions of power distributions in EU institutions or national assemblies.

