Incorporation of Socio-Economic Data into the Analysis of Remotely-Sensed Images: Basic Quantitative Strategy to Explore Functional Forms of Deforestation

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1. Introduction

This report reviews and straightens methodologies to explore functional forms of man-land-cover interrelations**. Tanaka and Nishii[7, 8, 9, 10, 12, 13], Nishii and Tanaka[4, 6], Nishii, Miyata, and Tanaka[5] have engaged in the analysis of deforestation by human population interactions. The basic strategy has been to use landcover data mashed up with cell-formed population density data.

Two factors – human population size (N) and relief energy (R: difference of minimum altitude from the maximum in a sampled area) – were picked up firstly to make elucidation of forest coverage ratio (F) on the same site. The functional forms with sigmoidal shape were suggested by step functions fitted to one-kilometer square high precision grid-cell data in Japan. By calculating relative appropriateness to data, Akaike’s Information Criterion (AIC) was applied. The two AIC values by step functions – one by independent model, the other by a model that takes neighbor effect – indicate baseline fitness to examine succeeding suitability of candidate functions.

To formulate world dataset, landcover dataset estimated from NOAA observations available at UNEP, Tsukuba for F, gridded population of the world (GPW) of CIESIN, US for N, and GTOPO30 of USGS for R, were used. The resolutions were matched by taking their common multiple of 20 minutes square.

How to refine models by adding minimal variables such as mean altitude of the site is examined also in this work.

2. Remotely-Sensed Images Mashed-up with Gridded Data

Table 1 shows data sources to make up F, N, and R, and Figure 1 depicts the data on map. Data on water surface as well as on missing cells are excluded in cell-aggregation process such as $F(s) = \frac{\sum_{c \in K} n_{c}}{25}$, in a coarsened larger cell of $20' \times 20'$ resolution, in case of calculating forest areal rate, where $K$ denotes the set of class codes (category identifier) of forests, and $c_i$ the identifier of $i^{th}$ original cell[3]. Similarly, $N(s) = \frac{\sum_{i} n_{org}}{16 \times 16}$, 

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Table 1 Data Source and Cell-Size Match (Coarsened)

<table>
<thead>
<tr>
<th>Source</th>
<th>Trimmed</th>
<th>Unit Area</th>
<th>Processed</th>
</tr>
</thead>
<tbody>
<tr>
<td>LCDS</td>
<td>1935×5400</td>
<td>5' × 5'</td>
<td></td>
</tr>
<tr>
<td>GPW</td>
<td>1548×4320</td>
<td>4' × 4'</td>
<td>20' × 20'</td>
</tr>
<tr>
<td>GTOPO30</td>
<td>15480×43200</td>
<td>30' × 30'</td>
<td>(~37×37 km)</td>
</tr>
</tbody>
</table>

LCDS: available at UNEP, Tsukuba
GPW: available at CIESIN, US
GTOPO30: available at USGS

Figure 1 Grid-cell based world dataset.

where \( n_{\text{org}} \) is of the population density of the original unit cell, and \( m \) denotes number of the missing values such as for water surfaces[1]. \( R(s) = \max(r_{\text{org}}; i = 1, ..., 1600) - \min(r_{\text{org}}; i = 1, ..., 1600) \), where \( r_{\text{org}} \) is the altitude of the original unit cell of GTOPO30 (Global TOPOgraphic data in 30 arc-second units)[14].

The above ‘s’ of \( F(s) \), \( N(s) \) and \( R(s) \) identifies the site. Note that the GPW is still the unique world data source that provides human population on a grid-cell basis.

In the case that resolutions do not match off by taking least common multiple, conventional spatial interpolation methods in general literature of remote sensing can be applied.

3. Quest for Additive Regression Models

Suppose that there are \( n \) sites numbered from \( i = 1 \) to \( n \). We partition the range of \( \log(N_i + 1) \) where \( N_i \) is population density as

\[
[a_0, a_1] \cup (a_1, a_2] \cup ... \cup (a_{s-1}, a_s]
\]

where \( a_0 = \min\{\log(N_i + 1) | i = 1, ..., n\} < a_1 < ... < a_{s-1} < a_s = \max\{\log(N_i + 1) | i = 1, ..., n\} \).

Effects of population in the intervals are denoted by \( \beta_0, \beta_1, ..., \beta_{s-1} \) where the effect in \( [a_0, a_1] \) is set to zero as a base line. Similarly, we divide the range of relief energy \( \log(R_i + 1) \) as

\[
[b_0, b_1] \cup (b_1, b_2] \cup ... \cup (b_{t-1}, b_t]
\]

where \( b_0 = \min\{\log(R_i + 1) | i = 1, ..., n\} < b_1 < ... < b_{t-1} < b_t = \max\{\log(R_i + 1) | i = 1, ..., n\} \). Effects of the relief energy in the intervals are denoted by \( \beta_0, ..., \beta_{s+t-2} \) where the effect of \([b_0, b_1]\) is also set to zero as a base line. Now, we set \( \hat{\beta} = (\beta_1, ..., \beta_{s+t-2})' : (s+t-2) \times 1 \), \( \beta = (\beta_0, \hat{\beta})' : (s+t-1) \times 1 \), where \( \beta_0 \) is the general mean. We define \( p = 1 + (s-1) + (t-1) = s + t - 1 \) as a number of all regression coefficients.

3.1 Estimation of Step Functions by Independent Models

Our linear model is as follows

\[
y_i = \beta_0 + \beta_u + \beta_{v+1} + \epsilon_i \tag{1}
\]

where \( i \) denotes a site number, \( a_u < \log(N_i + 1) \leq a_{u+1} \) and \( b_v < \log(R_i + 1) \leq b_{v+1} \) with \( u \geq 1 \) and \( v \geq 1 \). For \( u = 0 \) or \( v = 0 \), the effect is regarded as zero.

Let \( \tilde{X} = (x_{ik}) : n \times (p-1) \) be a design matrix with zero or one entries, whose \( i \)th row describes which intervals of \( (a_u, a_{u+1}] \) and \( (b_v, b_{v+1}] \) include \( N_i \) and \( R_i \). The linear model (1) is denoted by
\[ y = \beta_0 + \bar{X}\tilde{\beta} + \epsilon = X\beta + \epsilon, \]  
where \( \mathbf{1} = (1, ..., 1)^T: n \times 1, \beta = (\beta_0, \tilde{\beta})^T: n \times p \) and \( X = [\mathbf{1}, \bar{X}]: n \times p \) with \( p = 1 + s + t - 2. \) Then, AIC of \( y \) is given by

\[
\text{AIC}(y) = n \log(2\pi \sigma^2) + n \log \hat{\sigma}^2 + 2(p + 1)
\]

with \( \hat{\sigma}^2 = y' \left[ I - X(XX)^{-1}X' \right] y/n \)

where the penalty term \( 1 \) is for \( \sigma^2. \)

If \( y = \logit(F) \equiv \log(F/(1 - F)) \) for forest areal ratio \( 0 < F < 1, \) we have the following relation between densities.

\[
g(y)|dy = g(\logit(F)) \left| \frac{dy}{dF} \right| dF
\]

\[
= g(\logit(F)) \left( \frac{1}{F} + \frac{1}{(1 - F)} \right) dF
\]

\[
= g(\logit(F)) \left( F(1 - F)^{-1} \right) dF.
\]

Therefore, AIC of \( F \) based on the transformed vector \( y = (\logit(F_1), ..., \logit(F_n))^T \) is given by

\[
\text{AIC}(F) = n \log(2\pi \sigma^2) + n \log \hat{\sigma}^2
\]

\[
+ 2 \sum_{i=1}^{n} \log \left[ F_i(1 - F_i) \right] + 2(p + 1).
\]

### 3.2 Estimation of Step Functions by Spatially-Dependent Models

Let \( i_2, i_3, i_4 \) and \( i_4 \) be neighbors in the first-order neighborhood of site \( i. \) The effect from the neighbor \( i_1 \) is given by the \( i_1 \)th entry of \( X\tilde{\beta}. \) Then, we suppose that this effect is degraded by multiplication of a unknown constant \( \phi. \) Intuitively, the unknown constant would meet inequality \( |\phi| < 1/4. \)

Let \( H = \left( h_{ij} \right): n \times n \) be an adjacency matrix of the first-order neighborhood. Then, it holds that

\[ h_{ij} \times \{ \text{effect from site } j \} \]

\[ = \sum_{j=1}^{n} h_{ij} \times \{ \text{effect from site } j \} \]

\[ = \text{sum of effects from sites } i_1, i_2, i_3, i_4. \]

The above effect is degraded by multiplication of \( \phi, \) which captures the effect from the neighbors to the center. Thus, we obtain the following conditionally-independent model:

\[
y = \beta_0 + \bar{X}\tilde{\beta} + \phi HX\tilde{\beta} + \epsilon
\]

\[
= \beta_0 + (1 + \phi H)\bar{X}\tilde{\beta} + \epsilon
\]

\[
= X_\phi\tilde{\beta} + \epsilon, \quad \epsilon - N(0, \sigma^2 I)
\]

where \( X_\phi = [\mathbf{1}; (1 + \phi H)\bar{X}]: n \times p. \) If \( \phi \) is given, the maximum likelihood estimates of \( \beta \) and \( \sigma^2 \) given \( \phi \) are calculated by the usual least square method as

\[
\hat{\beta}(\phi) = \left( X_\phi^T X_\phi \right)^{-1} X_\phi^T y,
\]

\[
\hat{\sigma}^2(\phi) = y'[I - X_\phi(X_\phi X_\phi)^{-1} X_\phi^T] y / n.
\]

Then, the profile log likelihood is given by

\[
\log L(\phi) = -(n/2) \log(2\pi \sigma^2) - (n/2) \log \hat{\sigma}^2(\phi)
\]

Thus, \( \phi \) can be estimated by maximizing the formula (4) for \(-1/\text{max eigenvalue of } H < \phi < -1/\text{min eigenvalue of } H. \) Let \( \hat{\phi} \) be the optimal solution. Then, the maximum likelihood estimates are given by \( \hat{\beta}(\hat{\phi}) \) and \( \hat{\sigma}^2(\hat{\phi}) \) using formulas (3) and (4).

Thus, AIC for \( y \) is given by

\[
\text{AIC}(y) = n \log(2\pi \sigma^2) + n \log \hat{\sigma}^2(\hat{\phi}) + 2(p + 1 + 1)
\]

where the penalty terms \( 1 \) for \( \sigma^2 \) and \( 1 \) for \( \phi. \)

If \( y = \logit(F) \equiv \log(F/(1 - F)), \) AIC of the original vector \( F \) is given by

\[
\text{AIC}(F) = n \log(2\pi \sigma^2) + n \log \hat{\sigma}^2(\hat{\phi})
\]

\[
+ 2 \sum_{i=1}^{n} \log \left[ F_i(1 - F_i) \right] + 2(p + 1 + 1)
\]

### 3.3 Estimation of Non-linear Regression Functions by Independent Models

We fit nonlinear regression functions specified unknown parameters \( \theta: s \times 1 \) and \( \tau: t \times 1 \) as
\[ y_i = \beta_0 + g(N_i; \theta) + h(R_i; \tau) + \epsilon_i, \quad \epsilon_i \sim iid N(0, \sigma^2) \]  \hspace{1cm} (5)

where \( i \) denotes a site number. The functions are supposed to satisfy \( g(0; \theta) = h(0; \tau) = 0 \) as similar to the step functions. Then, a target function to be minimized is given by

\[ \sum_{i=1}^{n} \left( y_i - \beta_0 - g(N_i; \theta) - h(R_i; \tau) \right)^2. \]  \hspace{1cm} (6)

The function should be minimized w.r.t. \((\beta_0, \theta, \tau)\). We define the estimated parameters as \((\hat{\beta}_0, \hat{\theta}, \hat{\tau})\). The variance is estimated by

\[ \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} \left( y_i - \hat{\beta}_0 - g(N_i; \hat{\theta}) - h(R_i; \hat{\tau}) \right)^2 \]  \hspace{1cm} (7)

Then, AIC of \( y \) is given by

\[ \text{AIC}(y) = n \log(2\pi e) + n \log \hat{\sigma}^2 + 2(n + s + t + 1) \]

where the penalty term is for \( \beta_0, \theta, \tau \) and \( \sigma^2 \). If \( y = \log(F) \equiv \log(F/(1 - F)) \) for forest areal ratio \( 0 < F < 1 \), AIC of \( F \) based on the transformed vector \( y = (\log(F_1), ..., \log(F_n))^\top \) is given by

\[ \text{AIC}(F) = n \log(2\pi e) + n \log \hat{\sigma}^2 + 2 \sum_{i=1}^{n} \log \left| F_i(1 - F_i) \right| + 2(s + t + 3). \]  \hspace{1cm} (8)

### 3.4 Estimation of Non-Linear Regression Functions by Spatially-Dependent Models

Let \( i_1, i_2, i_3 \) and \( i_4 \) be neighbors of site \( i \) in the first-order neighborhood. The total values for the neighbor \( i_j \) is given by

\[ \sum_{j=1}^{4} \left\{ g(N_{i_j}; \theta) + h(R_{i_j}; \tau) \right\} \]  \hspace{1cm} (9)

We suppose that multiplication to the above quantity has an effect to the mean value at the site \( i \). So, our conditionally-independent model would be

\[ y_i = g(N_i; \theta) + h(R_i; \tau) + \phi \sum_{j \in U_i} \left\{ g(N_j; \theta) + h(R_j; \tau) \right\} + \epsilon_i, \quad \epsilon_i \sim iid N(0, \sigma^2) \]  \hspace{1cm} (10)

where \( U_i \) denotes a first-order neighborhood of site \( i \). Note that a number of neighbors in \( U_i \) is ranging from 0 to 4. Then, the target function to be minimized is given by

\[ \sum_{i=1}^{n} \left[ y_i - \beta_0 - g(N_i; \hat{\theta}) - h(R_i; \hat{\tau}) \right]^2 - \phi \sum_{j \in U_i} \left\{ g(N_j; \hat{\theta}) + h(R_j; \hat{\tau}) \right\}^2 \]

The function should be minimized w.r.t. \((\phi, \beta_0, \theta, \tau)\). We define the estimated parameters as \((\hat{\phi}, \hat{\beta}_0, \hat{\theta}, \hat{\tau})\). The variance is estimated by

\[ \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} \left[ y_i - \hat{\beta}_0 - g(N_i; \hat{\theta}) - h(R_i; \hat{\tau}) \right]^2 \]

Then, AIC of \( y \) is given by

\[ \text{AIC}(y) = n \log(2\pi e) + n \log \hat{\sigma}^2 + 2(n + s + t + 1) \]

where the penalty term is for \( \phi, \beta_0, \theta, \tau \) and \( \sigma^2 \). If \( y = \log(F) \equiv \log(F/(1 - F)) \) for forest areal ratio \( 0 < F < 1 \), AIC of \( F \) based on the transformed vector \( y = (\log(F_1), ..., \log(F_n))^\top \) is given by

\[ \text{AIC}(F) = n \log(2\pi e) + n \log \hat{\sigma}^2 + 2 \sum_{i=1}^{n} \log \left| F_i(1 - F_i) \right| + 2(s + t + 3) \]
4. Estimation of Spatially-Dependent Models by Pseud-Maximum Likelihood

In Section 3, estimations by maximum-likelihood methods are streamlined in terms of acquiring additive regression models of deforestation. Here, we newly introduce much easier and faster ways to estimate the relative appropriateness to data of the models by AIC as well as Bayesian Information Criterion (BIC).

From the relation (9), the following is derived
\[ y_i = \phi\{g(N_i; \hat{\theta}) + h(R_i; \hat{\tau})\} + \psi \sum_{j \in U_i} \{g(N_j; \hat{\theta}) + h(R_j; \hat{\tau})\} + \epsilon_i \] (11)

The variance is estimated by
\[ \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} \left[ y_i - \phi\{g(N_i; \hat{\theta}) + h(R_i; \hat{\tau})\} - \psi \sum_{j \in U_i} \{g(N_j; \hat{\theta}) + h(R_j; \hat{\tau})\} \right]^2 \] (12)

The function should be minimized w.r.t. \( \phi \) snd \( \psi \).

We define the estimated parameters as \( (\hat{\phi}, \hat{\psi}) \).

Then, AIC and BIC of \( y \) are given by
\[
\text{AIC}(y) = n \log(2\pi e) + n \log \hat{\sigma}^2 + 2 \, p
\]
\[
\text{BIC}(y) = n \log(2\pi e) + n \log \hat{\sigma}^2 + \log(n) \, p
\]
where \( p \) is the total number of estimated parameters.

Let call us this pseudo-maximum likelihood (PML) estimation.

5. Modeling for Partly Deforested Coverage Ratios

Actual forest coverage conforms to the 0,1-inflated distribution. In deserts or in the sea, there are no forests. In the climax forest, usually the land is covered fully by forest. Therefore divide the observed area into three classes = \( D_0 = \{s \in \mathcal{D} \mid F_s = 0\}, D_1 = \{s \in \mathcal{D} \mid F_s = 1\}, \) and \( D_2 = \{s \in \mathcal{D} \mid 0 < F_s < 1\} \). See Nishii and Tanaka[4, 5, 6] for details.

\( D_2 \) is the class to explore the functional forms of deforestation by human population interactions here. Asymptotic lower and upper bounds with homosedastic error assumption give anomaly in analysis. We suppose therefore that the logit transformation of forest areal rate \( IF(N, R) = \log \frac{F}{1-F} \) is expressed by the following additive model:
\[ IF(N, R) = \alpha_0 + g(N) + h(R) + \epsilon, \quad (-\infty < IF < \infty) \] (13)

where \( \alpha_0 \) is intercept.

Figures 2 and 3 of the step functions described in Section 3.1 in Japanese test area (data size: 6762) strongly indicate that the shapes should be sigmoidal. Thus we investigated functions in Table 2. As the result, Tanaka and Nishii[9] obtained \( IF(N, R) = \alpha_0 + g_2(N) + h_2(R) \) showed the best result among the models both in Hiroshima with 1 km\(^2\) cell resolution of high-precision, and Chinese test areas with resolution of Table 1.
Next, to get better fitness to real data, variable addition is examined. So far we used relief energy as a rough measure of slope steepness, but more careful consideration should be taken into account.

As the candidate additional models for regression functions $h(D_s(1), ..., D_s(M))$ we considered are: (a) functions $h_i(R_i)$ of the relief energy, $R_i = \max\{D_s(i)\} - \min\{D_s(i)\}$. (b) Functions $h_i(D_s)$ of the averaged altitude, $D_s = \sum_{i=1}^M D_s(i) / M$. (c) Functions $h_s(V_s)$ of the sample variance, $V_s = \sum_{i=1}^N (D_s(i) - D_s)^2 / M$ at the site ‘$s$’. (d) Sum of two or all regression functions in the above, where $h_i(.)$ may be parametric or non-parametric functions.

The best parametric model was of the form $h_2(R_s)$. So, the power transform to the arguments of the functions like $h_i(\sqrt{D_s})$ or $h_i(\log V_s)$ were examined[12, 13].

It had been known already that the term $g_2(n)$ give the best fitness.

The spatial model $\logit(F) = \alpha_0 + g_2(N_s) + h_2(R_s) + D_s$ showed the best result so far, which can be interpreted as: the higher the altitude, the more human effort required to remove forests, thus forests are kept. Detailed results will appear in the near future with respect to the efficacy of newly introduced PML with use of BIC.

### Table 2  Regression Functions

$$g_1(N) = -\beta \log(N + 1)$$
$$g_2(N) = -\beta_2 \log(N + 1) + \frac{\beta_1}{1 + \exp[\beta_2 - \beta_3 \log(N + 1) + \exp(\beta_2)]}$$
$$g_3(N) = \beta \left[1 - \exp[\alpha \log(N + 1)]\right]$$

$h_1(x) = 1(x > \theta) \cdot \delta \log(x - \theta + 1)$

$h_2(x) = 1(x > \theta) \cdot \delta \log(\frac{1}{2})$

$h_3(x) = \gamma_2 \exp(-\gamma_1 e^{\gamma_2 x}) - \gamma_2 \exp(-\gamma_1)$

$h_4(x) = \frac{\gamma_1}{1 + \exp(\gamma_2 - \gamma_2 x)} - \frac{\gamma_1}{1 + \exp(\gamma_2)}$

6. Discussion

It is reported that adjacency to developed land and proximity to transportation networks and major human settlements are important factors that determine regional patterns of land development[2], as the fragmentation and dispersion of forests should be taken into account. It would be more accurate to take up cells that are in a buffer region along a timber transport alley for example, but before going too far, to know neighbor cell effect should be the primer, and our work verifies the bold drastic efficacy. This work is based on cross-sectional data, but it is vital to design spatio-temporal models[11]. GPWv4 from CIESIN, Columbia University will make it possible to analyze the spatial changes in time-series.

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References

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