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Constructing Multiregional Life Tables using Place-of-Birth-Specific Migration Data

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CONSTRUCTING MULTIREGIONAL LIFE TABLES USING PLACE-OF-BIRTH-SPECIFIC MIGRATION DATA

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FOREWORD

Declining rates of national population growth, continuing differential levels of regional economic activity, and shifts in the migration patterns of people and jobs are characteristic empirical aspects of many developed countries. In some regions they have combined to bring about relative (and in some cases absolute) population decline of highly urbanized areas; in others they have brought about rapid metropolitan growth.

The objective of the Urban Change Task in IIASA's Human Settlements and Services Area is to bring together and synthesize available empirical and theoretical information on the principal determinants and consequences of such urban growth and decline.

This paper focuses on two alternative multiregional life table construction methods. It demonstrates that a more disaggregated calculation procedure, based on probabilities that are specific to an individual's place of birth, yields more accurate estimates of regional allocations of life expectancies than does the more conventional Markovian-based solution.

A list of publications in the Urban Change series appears at the end of this paper.

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ABSTRACT

The usual approach to the construction of a multiregional life table involves the calculation of a multiradix increment-decrement life table based on commonly available data about interregional migration streams. However, if the migration data are cross-classified by place of birth, an alternative multiregional life table can be obtained by constructing a set of uniradix increment-decrement life tables, calculated separately for each regional share of the initial cohort.

This paper demonstrates this alternative method (place-of-birth-dependent approach) and contrasts it with the more traditional one (place-of-birth-independent approach) using data on the female population, observed between 1965 and 1970, for the system of the four US Census Regions. The main result is that the consideration of place-of-birth-specific migration data reduces somewhat the fraction of the regional expectation of life at birth to be spent outside the region of birth.

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INTRODUCTION

The ordinary life table is a device for following a closed group of people, born at the same time, as it decreases in size until the death of its last member. The emphasis is put on the nonreversible transition from one state (being alive) to another (being dead). A straightforward extension of this model is the multiple decrement life table which recognizes transitions to more than one final absorbing state (e.g., decrements due to various causes of death).

However, in the case of recurrent events, the latter model does not permit one to follow persons who have moved from one state to another and to analyze their subsequent experiences. Such a problem may be handled with the help of more complex life tables which recognize entries, or increments, as well as exits, or decrements. Because of their general nature, such life tables, denoted as increment-decrement life tables, are valuable in the analysis of marital status, labor force participation, birth parity, as well as interregional migration--in which case, they are often referred to as multiregional life tables (Rogers 1973).

Among such generalized life tables, a distinction is often made between uniradix increment-decrement life tables, for which the initial cohort is concentrated in a unique state, and multiradix increment-decrement life tables, for which the initial cohort is allocated to several, if not all, of the intercommunicating states.

The key feature of increment-decrement life tables--uniradix as well as multiradix ones--lies in their formulation as simple Markov chain models. As a consequence, such generalized life tables rely on stringent assumptions (population homogeneity and Markovian behavior) which are far from reflecting reality and thus often lead to faulty results (Ledent 1980a). This is especially true in the case of multiregional life tables since, as is well-known, individuals with identical demographic characteristics (age, sex, and race) can exhibit quite different propensities to move depending on past events in their lives.

In particular, consider the perhaps most interesting statistics to be drawn from a multiregional life table; namely, the number of years (total and distribution according to the regions in which they are to be spent) that an individual born in any of the regions can expect to live. They are likely to take on highly inaccurate values if they are derived from a multiregional life table calculated with the traditional approach (that is as a multiradix increment-decrement life table based on the type of migration data commonly available). This inaccuracy arises because the application, in the traditional approach, of the same mobility age schedules to the individuals of a given region (regardless of their region of birth) ignores the generally well-established fact that migration propensities are heavily dependent on the birthplace of the individuals concerned (for a quantitative observation of such a fact in the case of the United States, see Long and Hansen 1975; Ledent 1980b).

Therefore, to provide more acceptable values of the regional expectations of life at birth (total and regional shares), multiregional life tables should rely on interregional migration data cross-classified by place of birth. In view of this need, this paper demonstrates the construction of such multiregional life tables, which involves the calculation of a uniradix increment-decrement life table for each of the regional shares of the initial cohort. It also contrasts such an approach (hereafter called the place-of-birth-dependent approach) with the traditional approach based on commonly available migration data (the place-of-birth-independent approach). An illustration is provided with the help of an application to the system consisting of the four US Census Regions observed during the period 1965-70, for females only; the necessary migration data can be readily derived from published census information (US Bureau of the Census 1973).

This paper consists of four sections. Section 1, intended as a background section, presents a brief reminder on the theory and mathematical treatment of increment-decrement life tables. Then, section 2 proposes a discussion of the issue at hand, i.e., the influence of the population homogeneity assumption on the calculation of such tables: the discussion is centered around the particular role of the birthplace in migration decisions. Next, section 3 reports on the implementation of the place-of-birth-dependent approach and, finally, section 4 provides some perspectives on the contrast which this approach offers with regard to the usual place-of-birth-independent approach. The general calculation method for constructing the various increment-decrement life tables considered in this paper is described in the Appendix.

1. REMINDER ON INCREMENT-DECREMENT LIFE TABLES

Although some of the issues underlying the construction of increment-decrement life tables were considered long ago, it is only recently that the thorough and systematic discussion of the methodological and empirical problems raised by such construction has appeared in the literature. Nevertheless, in less than a decade, the contribution of several researchers (Rogers 1973, 1975; Schoen and Nelson 1974; Rogers and Ledent 1975, 1976; Schoen 1975; Hoem and Fong 1976; Schoen and Land 1977; Ledent 1978, 1980a; Krishnamoorthy 1979) has led to the development of a formal mathematical treatment which now gives increment-decrement life tables a status comparable to that of the ordinary life table.

Perhaps, the single most important element responsible for such a development was the realization that an increment-decrement life table can be regarded as a generalized life table in which elements in matrix format are substituted for the scalar elements of the ordinary life table (Rogers and Ledent 1975, 1976; Rogers 1975).

In this section, we present an overview of a mathematical treatment of increment-decrement life tables that parallels the classical exposition of the ordinary life table: the correspondence between the formulas relevant to the ordinary and the increment-decrement life table is stressed in Table 1.

Suppose we have a system of r + 1 states (r intercommunicating states plus the state of death) in which the initial cohort is allocated among s states ($1 \leq s \leq r$): let $\ell^i(0)$ be the "radix" of state i. The principal problem here is one of estimating the state-specific curves of survivors $\ell^i(y)$ at each age y. Such estimation is centered around the differential equation in (3'); it presents a vector notation of the r scalar equations arrived at by substituting the equations (1') defining the instantaneous mobility rates into the accounting equations (2') showing the increments and decrements to each $\ell^i(y)$ group. [Note that equation (3') is a straightforward vector extension of the basic differential equation (3) of the ordinary life table.]

Table 1. A tabular comparison of the theoretical exposition of the ordinary and increment-decrement life tables.

Ordinary Life Table		Increment-Decrement Life Table	
$\mu(y) = \lim_{dy \to 0} \frac{d(y)}{\ell(y)dy}$	(1)	${}^{i}\mu^{j}(y) = \lim_{dy \to 0} \frac{{}^{i}d^{j}(y)}{\ell^{i}(y)dy}$	(1')
$\ell(y + dy) = \ell(y) - d(y)$	(2)	$\ell^{i}(y + dy) = \ell^{i}(y) - \sum_{\substack{j=1 \\ j \neq i}}^{r+1} {}^{i}d^{j}(y) + \sum_{\substack{j=1 \\ j \neq i}}^{r} {}^{j}d^{i}(y)$	(2')
$\frac{d}{dy}\ell(y) = -\mu(y)\ell(y)$	(3)	$\frac{d}{dy}\{\underset{\sim}{\ell}(y)\} = -\underset{\sim}{\mu}(y)\{\underset{\sim}{\ell}(y)\}$	(3')
$\ell(y) = \Omega(y)\ell(0)$	(4)	$\underset{\sim}{\ell}(y) = \underset{\sim}{\Omega}(y)\underset{\sim}{\ell}(0)$	(4')
$\Omega(y) = e^{-\int_0^y \mu(t)dt}$	(5)		
$\ell_{x+n} = p_x \ell_x$	(6)	$\underset{\sim}{\ell}_{x+n} = \underset{\sim}{p}_x \underset{\sim}{\ell}_x$	(6')
$p_x = \frac{\Omega(x+n)}{\Omega(x)}$	(7)	$\underset{\sim}{p}_x = \underset{\sim}{\Omega}(x+n)\underset{\sim}{\Omega}(x)^{-1}$	(7')
$L_x = \int_0^n \ell(x+t)dt$	(8)	$\underset{\sim}{L}_x = \int_0^n \underset{\sim}{\ell}(x+t)dt$	(8')
$T_x = \int_0^\infty \ell(x+t)dt$	(9)	$\underset{\sim}{T}_x = \int_0^\infty \underset{\sim}{\ell}(x+t)dt$	(9')
$e_x = \frac{T_x}{\ell_x}$	(10)	$\underset{\sim}{e}_x = \underset{\sim}{T}_x \underset{\sim}{\ell}_x^{-1}$	(10')

Source: Ledent (1980a, p. 536 and 542).

Equation (3') admits r linearly independent solutions, which can be expressed as equation (4'), a straightforward matrix extension of the ordinary life table solution (4). These independent solutions of (3') are the r multistate stationary populations that are generated by a unit (arbitrary) radix in each of the r states (regardless of whether some of the states are initially empty or not).

The matrix $\underset{\sim}{\Omega}(y)$ is a proper transition probability matrix showing the state specific survival probabilities at age y of the members of each radix. [Note that unlike its counterpart in the ordinary life table, this matrix cannot be simply expressed in terms of the instantaneous mobility rates but has to be determined by the infinitesimal calculus of Volterra (Schoen and Land 1977).] Then, it is readily possible to derive the number of survivors $\underset{\sim}{\ell}_x$, at fixed ages 0,n,2n,... by applying in succession as shown in equation (6') a set of age-specific transition probability matrices $\underset{\sim}{p}_x$ (generalizing the age-specific survival probabilities p_x of the ordinary life table).

Now, it is possible to define multistate life table functions generalizing the usual statistics found in a life table. Equation (8') defines the multistate life table function $\underset{\sim}{L}_x$ whose (i,j)-th element represents the number of people born in state j and alive in state i of the life table between ages x and x + n or, alternatively, the number of person-years lived in state i between those ages by the members of the j-th radix. From there, it is possible to define generalized T-statistics [equation (9')] and, finally, generalized e-statistics [equation (10')]: the (i,j)-th element of $\underset{\sim}{e}_x$ denotes the number of years in prospect that an x-year old individual present in state j can expect to spend in state i.

Another generalization of interest is that of the mortality rates m_x and survivorship proportions s_x of the ordinary life table, because the feasibility of calculating applied increment-decrement life tables is centered around the equalization of the life table values of the generalized m- and s-statistics with their observed counterparts. The relevant approaches are known as the movement and transition approaches devised by Schoen (1975) and Rogers (1973, 1975), respectively.

On the one hand, interstate "passage" can be observed as a move, i.e., an instantaneous event similar to a death. This leads to the movement approach--consistent with the approach taken in the ordinary life table--in which the linkage with the observed population is ensured through an equalization of the life table mortality and mobility rates with their observed counterparts. On the other hand, interstate "passage" can be observed as a change in an individual's state of presence between two points of time (regardless of the numbers of eventual moves made in the meantime). This characterizes the essence of the transition approach, in which the linkage with the observed population is ensured through an equalization of the life table survivorship proportions with their observed counterparts.

These two alternative approaches are not competitive but complementary in that the choice of either is dictated by the type of data at hand (for a detailed contrast, see Ledent 1980a). In fact, in most applications of increment-decrement life tables to real situations, the movement approach is the relevant one. The major exception requiring the use of the transition approach occurs in the field of interregional migration when data come from population censuses that describe changes of residence between two points in time.

2. THE ISSUE ADDRESSED IN THIS PAPER

The most important feature of increment-decrement life tables is the formulation of their underlying model as a simple Markov chain model. It follows that all the individuals of a given age present at the same time in a given state have identical propensities of moving out of that state (population homogeneity assumption) and that these propensities are independent of the past history of the individuals concerned (Markovian assumption).

Clearly, in some instances, such an assumption is far from being realistic. Take, for example, the case of interregional migration in which the place of birth of the prospective migrant heavily influences his decision to move and his choice of a destination. For example, in their study of migration flows to the South from the rest of the US, Long and Hansen (1975) present convincing evidence of the fact that the probability of moving to the South is considerably higher for the Southern-born than for the non-Southern born. Also, in another paper, the author (Ledent 1980b) presents some more general evidence of the influence of the place of birth on migration patterns with reference to a four-region system of the United States.

The migration data set used for that paper was obtained by rearranging in a suitable format data taken from the volume entitled *Lifetime and Recent Migration* published by the US Bureau of the Census (1973). The lengthy Table 11 of this volume provides estimates of the numbers of 1970 residents in each state cross-classified by place of birth and place of residence in 1965 (ten geographical units are retained: the state of presence in 1970 and the nine US Census Divisions). These estimates are provided for each sex as well as for each race for ten age groups: 0-4, 5-9, 10-14, 15-19, 20-24, 25-29, 30-39, 40-49, 50-50, 60 and over in 1965. The data concerning the ten female age groups were aggregated and rearranged to show the changes of residence (cross-classified by place of birth) which were made between 1965 and 1970 in the US Census four-region system. The interregional migration streams thus obtained for the highly migratory group of women, aged 20 to 24 in 1965, were shown in Table 2. For example, 73,703 women in that age group moved from the South region to the North Central region. Among them, 43,047 were born in the South, and 30,656 elsewhere. Interestingly enough, most of the non-South born--24,847 or 81 percent of them--were born in the North Central.

Table 2. 1965-70 place-of-birth-specific interregional migration streams for females aged 20-24 in 1965.

From	To Northeast	North Central	South	West
	Born in the Northeast			
Northeast	1110763	18637	36184	24299
North Central	10491	33482	3597	3256
South	21675	4051	59628	4808
West	9562	2331	3798	46020
	Born in the North Central			
Northeast	21364	7887	3297	2887
North Central	16550	1285304	38998	48157
South	3546	24847	64224	7117
West	2833	23586	7282	133306
	Born in the South			
Northeast	83292	3112	3847	2905
North Central	2919	159435	28663	6217
South	23652	43047	1553583	23064
West	2218	4876	33119	108464
	Born in the West			
Northeast	7088	660	891	3595
North Central	834	22878	2134	8216
South	976	1758	23212	9554
West	5611	10563	13470	579719
	Born anywhere in the US			
Northeast	1222507	30296	54219	33686
North Central	30794	1501099	73392	65846
South	49849	73703	1700647	54598
West	20224	41356	47614	867509

Source: Calculated by aggregating data from US Bureau of the Census (1973, Table 11).

These figures indeed suggest large differences in the propensity to migrate according to the place of birth: whereas a 20-24 year-old Southern resident has--regardless of her place of birth--a 0.0392 probability of moving to the North Central over a five-year period, she has a smaller probability (0.0260) or a higher probability depending on whether she was born within or outside the South. In the latter case, the probability reaches 0.0449 and 0.0495 for women born in the North Central and the West, respectively, but increases to almost 25 percent (0.2491) for those born in the North Central. More generally, someone living outside his or her region of birth appears to have a high probability of returning there (for a detailed analysis of this subject, see Ledent 1980c).

Clearly the large mobility differentials, according to the place of birth, just described sharply contradict the population homogeneity assumption which underlies the calculation of a multiregional life table from migration data relating to the whole of a nation's population. Thus, we can reasonably expect that the expectations-of-life-at-birth statistics coming out of such multiregional life tables to take on inaccurate values because they are based on average mobility propensities rather than on mobility propensities specific to the regional shares of the initial cohort.

However, the availability of interregional migration data cross-classified by place of birth, such as shown in Table 3, immediately suggests the possibility of circumventing or, more exactly, attenuating the stringent character of the population homogeneity assumption that underlies the calculation of a multiregional life table from aggregate (place-of-birth-independent) migration data. The leading idea here is to construct separate uniradix increment-decrement life tables for each of the radices, that is the regional shares of the (arbitrary) initial cohort. In this way, multistate life table statistics can be obtained which no longer relate to a single homogeneous population but to a population divided into r homogeneous groups (as many as there are regions), defined by their place of birth.

3. THE PLACE-OF-BIRTH-DEPENDENT APPROACH ILLUSTRATED

Methodologically, the implementation of the approach just suggested does not raise any problem: it simply requires the calculation of r increment-decrement life tables instead of one (the fact that they are uniradix rather than multiradix increment-decrement life tables does not have any bearing on the actual calculation of the multistate life table functions). Thus, in this section, we simply demonstrate this new approach with the help of an application to the set of US place-of-birth-specific migration data discussed in section 2.

The migration information available here coming evidently in the count of migrants or transitions, suggests that the relevant perspective from the implementation of the various increment-decrement life tables to be calculated in this paper is the transition perspective mentioned earlier in section 1. The actual calculation method used, an overview of which is presented in the Appendix, combines the estimation of the age-specific survival methods from a method developed elsewhere by this author (Ledent 1980a) and the calculation of the number of person-years lived $\underset{\sim}{L}_x$ from a linear integration approach (Rogers 1973, 1975).

Note that, because no mortality information cross-classified by place of birth is available, we simply use the same set of age-specific mortality rates: those observed for the population of each region regardless of the region of birth (source: US Department of Health, Education, and Welfare, selected years). Actually, this treatment hardly constitutes a problem. In effect, although it does not yield the most precise values for the multistate statistics referring to each regional cohort, the consideration of identical mortality rates for the calculation of the four uniradix increment-decrement life tables appears to be quite acceptable: the dependence of mortality on the place of birth is probably minimal as long as the spatial units considered are broad geographical areas (this is certainly less true in the case of rural-urban systems, especially in the case of developing countries). Then, what the contrast of the multistate life table statistics relating to each radix offers is an assessment of the influence of differential mobility according to the place of birth with the effect of mortality differentials removed.

Let us now turn to the examination of the results obtained in reference with the application announced above. Table 3 which sets out the transition probabilities for women exactly 20 years old according to their region of birth typifies the general observation that the probability of moving from region i to region j is smaller for those born in region i and much higher for those born in region j than for those which were born neither in region i nor in region j.

Table 4 displays the expected numbers of remaining years --disaggregated into figures specific to the regions in which they are spent--that 20 year-old residents of each region can expect to live according to their place of birth. For example, a resident of the South region, born in the same region, is expected to survive 56.55 years of which 49.29 years (about 87.2 percent) will be spent in the South. However, if this Southern resident was born in another region, a much smaller part of her remaining lifetime (from 56.27 to 57.53 years according to the region of birth) is expected to be spent in the South: 22.08 years if born in the Northeast, 20.09 years if born in the North Central and 16.45 years if born in the West.

Observe the regional variations in the total expectations of remaining life according to the place of birth in spite of the independence of the mortality pattern vis-a-vis the place of birth. For example, the total expectation of remaining life for a Southern resident is much higher (smaller) if she is born in the West (Northeast) than if born in the South: this is indeed a consequence of the assumption underlying a multi-regional life table that an inmigrant adopts the mortality regime of that region.

Table 3. Place-of-birth-dependent approach: transition probabilities for females exactly 20 years old.

From	To Northeast	North Central	South	West	Death
	Born in the Northeast				
Northeast	0.9138	0.0206	0.0397	0.0228	0.003060
North Central	0.2114	0.6429	0.0726	0.0696	0.003389
South	0.2359	0.0475	0.6611	0.0517	0.003738
West	0.1264	0.0352	0.0538	0.7812	0.003390
	Born in the North Central				
Northeast	0.5914	0.2639	0.0839	0.0490	0.003133
North Central	0.0137	0.9064	0.0357	0.0480	0.003418
South	0.0339	0.2438	0.6527	0.0659	0.003774
West	0.0173	0.1292	0.0468	0.8033	0.003409
	Born in the South				
Northeast	0.7795	0.0372	0.1499	0.0303	0.003119
North Central	0.0164	0.7954	0.1496	0.0352	0.003443
South	0.0184	0.0332	0.9212	0.0234	0.003837
West	0.0167	0.0370	0.1613	0.7815	0.003435
	Born in the West				
Northeast	0.5943	0.0583	0.0876	0.2568	0.003119
North Central	0.0271	0.6707	0.0673	0.2315	0.003421
South	0.0299	0.0498	0.6720	0.2446	0.003778
West	0.0104	0.0207	0.0277	0.9378	0.003406

Table 4. Place-of-birth-dependent approach: expectations of remaining life for females exactly 20 years old.

	Number of years spent in				
Region of residence	Northeast	North Central	South	West	Total
	Born in the Northeast				
Northeast	47.31	1.66	4.48	2.67	56.11
North Central	23.53	19.25	7.49	6.15	56.42
South	25.26	3.45	22.08	5.48	56.27
West	18.76	3.20	6.97	27.85	56.78
	Born in the North Central				
Northeast	14.86	25.17	7.62	9.18	56.82
North Central	0.99	46.73	3.80	5.30	56.83
South	2.15	25.76	20.09	8.78	56.78
West	1.29	17.48	4.79	33.72	57.27
	Born in the South				
Northeast	27.88	3.73	21.47	3.41	56.49
North Central	1.58	30.21	21.10	3.87	56.76
South	1.51	3.17	49.29	2.58	56.55
West	1.66	4.01	23.04	28.37	57.08
	Born in the West				
Northeast	12.59	3.64	4.99	36.37	57.60
North Central	1.50	17.41	4.38	34.31	57.60
South	1.64	3.39	16.45	36.05	57.53
West	0.66	1.53	1.99	53.66	57.83

4. THE PLACE-OF-BIRTH-DEPENDENT AND PLACE-OF-BIRTH-INDEPENDENT APPROACHES CONTRASTED

The purpose of this section is to provide a meaningful contrast of the place-of-birth-dependent and place-of-birth-independent approaches to the construction of a multiregional life table. In principle, this requires the consolidation of the separate uniradix increment-decrement life tables previously calculated and then the comparison of the results obtained with those of the multiradix increment-decrement life table based on the same data set, however aggregated over the alternative birthplaces.

There is, however, an interesting conclusion which we can derive even before aggregating the various uniradix life tables. It relates to the life expectancies at birth and their regional distributions. In effect, instead of focusing on expectations of life at age 20, let us consider the similar expectations for age zero. In this case, only the life expectancies concerning identical regions of birth and residence are indeed meaningful. In this way each of the uniradix increment-decrement life tables calculated provides a figure of life expectancy at birth for females born in the relevant region, broken down into several numbers indicating the decomposition of this figure according to the time spent in each region. The figures obtained from each uniradix life table can then be grouped into a single matrix, such as the one shown at the top of Table 5. It appears that an American woman has a life expectancy greater than 74 years (from 74.20 years if born in the South to 75.85 years if born in the West) of which more than 60 years will be spent in the region of birth (60.21 years if born in the North Central to 68.51 years if born in the West).

How do these life expectancies compare with those obtained with the place-of-birth-independent approach, i.e., from the multiradix increment-decrement life table based on the same data set aggregated over birthplaces? The matrix of expectation of life at birth which the latter leads to is shown at the bottom of Table 5; it indicates a much smaller proportion of total lifetime spent in the region of birth than with the place-of-birth-dependent approach: from 48.70 years in the case of the Western born female to 52.64 years in the case of the Southern born female.

Table 5. Expectations of life at birth, totals and regional distributions (in years): place-of-birth-dependent and place-of-birth-independent approaches contrasted.

	Number of years spent in				
Region of birth	Northeast	North Central	South	West	Total
	A - place-of-birth-dependent-approach				
Northeast	61.78	2.53	6.06	3.87	74.24
North Central	1.44	60.21	5.22	7.43	74.30
South	2.49	5.10	62.63	3.99	74.20
West	1.10	2.71	3.52	68.51	75.85
	B - place-of-birth-independent approach				
Northeast	52.09	5.80	10.99	5.56	74.43
North Central	4.10	50.43	11.38	8.36	74.26
South	5.55	8.83	52.64	7.23	75.25
West	4.45	9.25	12.88	48.70	75.27

Thus, the substitution of place-of-birth-specific migration data for the traditional migration data reduces the expected numbers of years to be spent in the region of birth by about ten years (9.69 years in the case of the Northeast, 9.78 years in the case of the North Central, 9.99 years in the case of the South) except in the case of the West where the decrease is about double in size (19.81 years exactly). This result is consistent with the above observation that, once an American woman--and a Western born even more than another--has moved out of her region of birth, she is very likely to come back. In addition, note that the use of place-of-birth-migration data implies increased differentials between the total regional life expectancies, which take on values nearing those they would have if migration was ignored.

Perhaps, a better way of assessing the impact of using place-of-birth-specific migration data on the calculation of a multiregional life table is to look at the changes in the regional distributions of the life expectancies at birth that the consideration of such data causes. From the figures shown in Table 6, it can be readily established that the introduction of such disaggregate data cuts the proportion of years to be spent outside the region of birth by about half except in the case of the Western born women for which the cut amounts to slightly more than 70 percent: this proportion decreases from 30.0 to 16.8 percent for women born in the Northeast, 32.1 to 19.0 percent for women born in the North Central, 29.1 to 15.6 percent for Southern born women and 35.3 to 9.7 percent for Western born women.

We now turn to the consolidation of the four uniradix increment-decrement life tables--calculated for each regional share of the initial cohort--into a multiregional life table directly comparable to that obtained from the traditional approach. Fundamentally, the possibility of implementing such a consolidation raises an important issue relating to the appropriate choice of the regional shares of the initial cohort needed to perform such aggregation.

We suggest here, that since the mobility and mortality patterns studied in our US illustration is that of a given point in time (period 1965-70), the radices or regional shares of the initial cohort ought to be in proportion to the numbers of female births observed in each region at this same point in time (i.e., over the period 1965-70). Thus the initial cohort should be allocated as follows: 22735 (Northeast), 27791 (North Central), 32245 (South), and 17229 (West).

The consolidated transition probabilities (relating to females aged 20) which result from such a choice of the regional allocation are shown at the top of Table 7 whose bottom part shows the corresponding transition probabilities obtained with the place-of-birth-independent approach.

Interestingly enough, the two corresponding sets of transition probabilities are roughly identical with significant discrepancies arising only in the case of the migration probabilities out of the West: the retention probability is .9106 in the case of the place-of-birth-dependent approach versus .8907 in the case of the place-of-birth-independent approach, hence a 19.9 per thousand absolute difference versus a maximum 3.0 per thousand difference observed in the case of the other regions. A similar result can also be observed for all the other age groups.

By contrast, the consolidated expectations of life which we receive by aggregation of the four place-of-birth-specific increment-decrement life tables calculated for each radix present large discrepancies with those derived from the place-of-birth-independent approach. For example, the consolidated expectations of life for females aged 20 indicate that the number of remaining years to be spent in the region of residence are: 45.5 years (if present in the Northeast), 44.3 (if present in the North Central), 45.8 (if present in the South), 49.4 (if present in the West) versus 42.0, 41.1, 43.3, and 40.6, respectively, as obtained with the place-of-birth-independent approach (Table 8).

The contrast of the figures shown in Tables 7 and 8 thus indicates that the place-of-birth-dependent approach leads to aggregate multistate life table functions which are little or largely different from those obtained from the place-of-birth-independent approach, depending on whether they relate to events occurring over a single age interval or over a long interval.

Let us recall that the results just derived rely on a consolidation of the four uniradix increment-decrement life tables calculated for each regional share of the initial cohort using justifiable weights but questionable ones nevertheless.

Table 7. Transition probabilities for females exactly 20 years old: place-of-birth-dependent and place-of-birth-independent approaches.

From	To Northeast	North Central	South	West	Death
	Place-of-birth-dependent approach				
Northeast	0.8975	0.0262	0.0461	0.0272	0.003064
North Central	0.0191	0.8869	0.0448	0.0458	0.003418
South	0.0285	0.0444	0.8901	0.3320	0.003828
West	0.0156	0.0312	0.0392	0.9106	0.003408
	Place-of-birth-independent approach				
Northeast	0.8959	0.0265	0.0480	0.0265	0.003065
North Central	0.0194	0.8839	0.0485	0.0448	0.003420
South	0.0289	0.0443	0.8921	0.0309	0.003829
West	0.0187	0.0395	0.0476	0.8907	0.003409

Table 8. Expectations of remaining life for females exactly 20 years old: totals and regional distributions in years): place-of-birth-dependent and place-of-birth-independent approaches.

Region of residence	Number of years spent in Northeast	North Central	South	West	Total
	A - place-of-birth-dependent approach				
Northeast	45.39	2.25	5.26	3.26	56.16
North Central	1.60	44.27	5.01	5.94	56.83
South	2.53	4.27	45.79	4.00	56.58
West	1.43	3.07	3.81	49.39	57.70
	B - place-of-birth-independent approach				
Northeast	41.95	3.44	7.26	3.61	56.26
North Central	2.57	41.05	7.52	5.66	56.80
South	3.46	5.38	43.32	4.40	56.56
West	2.80	5.87	8.10	40.62	57.39

This raises the problem as to whether alternative allocations among the regions of the initial cohort would lead to quite different aggregated multiregional life tables. In view of this, we have performed an alternative consolidation of the four uniradix increment-decrement life tables using identical weights (radices). The multistate life table functions thus obtained, which we do not show here, did not appear to differ very significantly from those calculated earlier. Thus, as long as the state allocation of the initial cohort consists of radices which more or less reflect the weights of the four regions with regard to a meaningful socioeconomic factor--these weights are expected to represent an allocation which does not depart too much from an equal allocation--very similar estimates of the aggregate multistate life table functions are to be expected.

In brief, in contrast to the place-of-birth-independent approach, the more desirable place-of-birth-dependent approach leads to aggregate multistate life table functions which depend on the choice of the regional allocation of the initial cohort. However, as long as the radices are reasonably chosen, this "radix problem" does not bear a large influence on the values of the aggregate multistate life table functions obtained by consolidation of the place-of-birth-specific increment-decrement life tables.

SUMMARY AND CONCLUSION

An important assumption common to all life table models is the population homogeneity assumption stemming from their Markovian formulation. Such an assumption is in sharp contrast with the observation that, in the real world, equally aged individuals of a given status category (i.e., belonging to a given state of the system) generally exhibit quite different propensities to move out of their current status category.

First, these mobility differentials can be related to different personal characteristics (sex, race) or socioeconomic characteristics (occupation) which affect the level of mobility at a given instant. Then, if one is striving for more accurate estimates of increment-decrement life tables, one can simply calculate separate life tables for those groups of people which can be easily differentiated, such as men and women or whites and non-whites, etc.

Second and more importantly, in the case of increment-decrement life tables, the aforementioned mobility differentials can also appear in the form of a difference in the repetitive nature of the phenomenon considered. Unfortunately, such differentials cannot generally be attributed to an easily identifiable characteristic and one will not be able, as above, to calculate separate life tables for more homogeneous groups. An exception to this occurs in the analysis of migration when data on adequate census information allows one to distinguish among interregional migrants and groups of migrants characterized by similar birth places. Then, as shown in this paper, an alternative multiregional life table can be constructed as a set of uniradix increment-decrement life tables relating to each of the regional shares of the initial cohort.

With regard to the traditional approach to the calculation of a multiregional life table, this alternative approach appears to provide more detail (in the case of the transition probabilities) but also more accuracy (in the case of the life expectancies at birth): such an improvement follows from the consideration of a more realistic migration pattern, one which explicitly accounts for return migration to the birthplace (a demographic phenomenon of importance as shown in section 2).

However, note that the amelioration thus brought to the calculation of a multiregional life table represents only a partial step toward the total removal of the population homogeneity and Markovian assumptions implicit to the traditional approach: within each stationary population associated with each radix, the two aforementioned assumptions remain valid.

APPENDIX: A BRIEF DESCRIPTION OF THE CALCULATION METHOD

The general calculation procedure used to calculate the uniradix as well as multiradix increment-decrement life tables considered in this paper consists of

a) estimating a set of transition probabilities $\bar{\underset{\sim}{p}}_x$ conditional on survival, by equalizing life table transition proportions conditional on survival with their observed counterparts (obtained from the matrices shown in Table 3 by dividing each element by the sum of the elements in the corresponding row and
b) transforming them into the requested set of transition probabilities by introducing independent information ensuring the equalization of the life table and observed (conventional mortality rates).

More specifically, such a procedure means that $\underset{\sim}{p}_x$ is obtained from

$$\underset{\sim}{p}_x = \bar{\underset{\sim}{p}}_x \underset{\sim}{p}_x^{\delta} \qquad (A1)$$

where $\bar{\underset{\sim}{p}}_x$ is a matrix of transition probabilities conditional on survival evaluated in terms of the observed transition proportions conditional on survival and $\underset{\sim}{p}_x^{\delta}$ is a diagonal matrix of survival probabilities.

In the first approximation, $\bar{\underset{\sim}{p}}_x$ can be estimated using the averaging formula proposed by Rees and Wilson (1977)

$$\underset{\sim}{p}_x = \frac{1}{2}\left(\bar{\underset{\sim}{S}}_{x-n} + \bar{\underset{\sim}{S}}_x\right) \tag{A2}$$

However, a better estimation can be performed by interpolating between the transition proportions conditional on survival in a less crude fashion: as suggested by Ledent (1980a), one can, for each pair of states i and j (j ≠ i), interpolate between the transition proportions ${}^{i}\bar{S}_x^{j}$ conditional on survival by using cubic spline functions which are increasingly used in the field of demography (McNeil, Trussell, and Turner 1977). Since we deal here with a five-year time interval (1965-70), the ordinate --for age y--of the continuous curve thus obtained represents the probability for an individual present at age y in region i to be present in region j five years later. In this way, it is readily possible, for each region i, to estimate at evenly spaced ages 0,5,10,15,... migration probabilities ${}^{i}\bar{p}_x^{j}$ (j = 1, ...,r; j ≠ i) from which retention probabilities ${}^{i}\bar{p}_x^{i} = 1 - \sum_{j\neq i} {}^{i}\bar{p}_x^{j}$ immediately follows.

The estimation of the set of survival probabilities $\underset{\sim}{p}_x^{\delta}$, assuming the availability of conventional mortality rates in each region, is not so straightforward, because the two-step estimation procedure suggested by (A1) causes the mortality pattern to be a characteristic of the place of residence at the exact age x = 0,n,2n,..., immediately below the age at which death occurs (rather than the characteristic of the place of occurrence).

The methodology used for such an estimation relies on an iterative procedure centered around the estimation of mortality rates dependent on the place of residence at age x from the conventionally observed mortality rates (for details of the iterative procedure, see Ledent 1980a). Then, the requested set of survival probabilities is obtained by analogy with the survival probability p_x of an ordinary life table from

$$\underset{\sim}{p}_x^{\delta} = \left(\underset{\sim}{I} + \frac{n}{2}\,\hat{\underset{\sim}{M}}_x^{\delta}\right)^{-1}\left(\underset{\sim}{I} - \frac{n}{2}\,\hat{\underset{\sim}{M}}_x^{\delta}\right) \qquad \text{(A3)}$$

where $\hat{\underset{\sim}{M}}_x^{\delta}$ is a diagonal matrix containing the mortality rates previously estimated.

In the end, combining the estimates of $\bar{\underset{\sim}{p}}_x$ and $\underset{\sim}{p}_x^{\delta}$ as required by (A1) leads to the required estimates of p_x and then, by using (6'), to the estimates of $\underset{\sim}{\ell}_x$. From there, the numbers of person-years lived $\underset{\sim}{L}_x$ are simply obtained from the usual linear integration method (Rogers 1975)

$$\underset{\sim}{L}_x = \frac{n}{2}\left(\underset{\sim}{\ell}_x + \underset{\sim}{\ell}_{x+n}\right) \qquad \text{(A4)}$$

and the T-statistics are then calculated from

$$\underset{\sim}{T}_x = \sum_k \underset{\sim}{L}_{x+kn} \qquad \text{(A5)}$$

As for the expectations of life and survivorship proportions, they are calculated using relationships (10') and (12').

REFERENCES

Hoem, J.M., and M.S. Fong (1976) *A Markovian Chain Model of Working Life Tables.* Working Paper No. 2. Copenhagen: Laboratory of Actuarial Mathematics, University of Copenhagen.

Krishnamoorthy, S. (1979) Classical approach to increment-decrement life tables: an application to the study of marital status of United States females, 1970. *Mathematical Biosciences* 44(1-2):139-154.

Ledent, J. (1978) *Some Methodological and Empirical Considerations in the Construction of Increment-Decrement Life Tables.* RM-78-25. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Ledent, J. (1980a) Multistate Life Tables: Movement versus Transition Perspectives. *Environment and Planning A* 12: 533-562.

Ledent, J. (1980b) *An Improved Methodology for Constructing Increment-Decrement Life Tables from the Transition Perspective.* WP-80-00. Laxenburg, Austria: International Institute for Applied Systems Analysis (forthcoming).

Ledent, J. (1980c) *The Influence of the Birthplace on Geographical Mobility in the US, II: Analysis of Migration Propensities.* WP-80-00. Laxenburg, Austria: International Institute for Applied Systems Analysis (forthcoming).

Long, L.H., and K.A. Hansen (1975) Trends in Return Migration to the South. *Demography* 12(4):601-614.

McNeil, D.R., T.J. Trussell, and J.C. Turner (1977) Spline Interpolation of Demographic Data. *Demography* 14:245-252.

Rees, P.H., and A.G. Wilson (1977) *Spatial Population Analysis.* London: Edward Arnold.

Rogers, A. (1973) The multiregional life table. *Journal of Mathematical Sociology* 3:127-137.

Rogers, A. (1975) *Introduction to Multiregional Mathematical Demography.* New York: John Wiley.

Rogers, A., and J. Ledent (1975) *Multiregional Population Projection.* Discussion paper No. 128. Evanston, Illinois: The Center for Mathematical Studies in Economics and Management Science, Northwestern University. Published in revised form in *Optimization Techniques Modeling and Optimization in the Service of Man.* Part I, edited by Jean Cea (pages 31-58), Springen Verlag, Belgium 1976.

Rogers, A., and J. Ledent (1976) Increment-decrement life tables: a comment. *Demography* 13(2):287-290.

Schoen, R. (1975) Constructing increment-decrement life tables. *Demography* 12(2):313-324.

Schoen, R., and K.C. Land (1977) *A General Algorithm in Estimating a Markov-Generated Increment-Decrement Life Table with Applications to Marital Status Patterns.* WP-77-15. Champaign, Illinois: Program in Applied Social Statistics, Department of Sociology, University of Illinois at Urbana. (Published in revised form in the Journal of the American Statistical Association, 1979).

Schoen, R., and V.E. Nelson (1974) Marriage, divorce and mortality: a life table analysis. *Demography* 11:267-290.

US Bureau of the Census (1973) *Census of Population: 1970: Lifetime and Recent Migration.* Final Report PC(2)-2D, US Government Printing Office, Washington, D.C.

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