H_∞ Fuzzy Control for Systems With Repeated Scalar Nonlinearities and Random Packet Losses

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***Abstract*—This paper is concerned with the H_∞ fuzzy control problem for a class of systems with repeated scalar nonlinearities and random packet losses. A modified Takagi–Sugeno (T–S) fuzzy model is proposed in which the consequent parts are composed of a set of discrete-time state equations containing a repeated scalar nonlinearity. Such a model can describe some well-known nonlinear systems such as recurrent neural networks. The measurement transmission between the plant and controller is assumed to be imperfect and a stochastic variable satisfying the Bernoulli random binary distribution is utilized to represent the phenomenon of random packet losses. Attention is focused on the analysis and design of H_∞ fuzzy controllers with the same repeated scalar nonlinearities such that the closed-loop T–S fuzzy control system is stochastically stable and preserves a guaranteed H_∞ performance. Sufficient conditions are obtained for the existence of admissible controllers, and the cone complementarity linearization procedure is employed to cast the controller design problem into a sequential minimization one subject to linear matrix inequalities, which can be readily solved by using standard numerical software. Two examples are given to illustrate the effectiveness of the proposed design method.**

***Index Terms*—Diagonally dominant matrix, fuzzy systems, H_∞ control, linear matrix inequality (LMI), random packet losses, repeated scalar nonlinearity.**

I. Introduction

SINCE the concept of fuzzy sets was introduced by Zadeh in 1965, fuzzy logic control has developed into one of the most important and successful branch of automation and control theory. In the past few decades, the fuzzy logic theory has been demonstrated to be effective in dealing with a variety of complex nonlinear systems, which has therefore received a rapidly growing interest in the literature. In particular, the control technique based on the so-called Takagi–Sugeno (T–S) fuzzy model has attracted much attention. The common practice based on this technique is as follows. Fuzzy models have provided an approach to represent complex nonlinear systems to a set of linear local models by using fuzzy sets and fuzzy reasoning. The overall fuzzy model of the system is achieved by smoothly blending these local models together through membership functions. It has a convenient and simple dynamic structure such that the existing results for linear systems theory can be readily extended for this class of nonlinear systems and, as a result, a great number of important results has been reported in the literature. For example, the problem of stability analysis has been investigated in [2], [3], [9], [10], and [25], and the stabilizing as well as H_∞ control designs have been reported in [1], [4], [8], [12], [19], [21], [22], [24] and [29].

The T–S fuzzy model has been widely employed to represent or approximate a nonlinear system, which is described by a family of fuzzy IF–THEN rules that represent local linear input–output relations of the system. Nevertheless, the local model is not necessarily a linear one but sometimes a "simple" or "slightly" nonlinear system whose dynamics can be thoroughly investigated. A good example of such a simple nonlinear system is the recurrent neural network that involves a nonlinear but known activation function. Therefore, there has appeared initial research interest focusing on the extended T–S model whose system dynamics is captured by a set of fuzzy implications which characterize local relations in the state space [15], [16], [30]. In this case, the local dynamics of each fuzzy rule is expressed by a well-studied nonlinear system, and the overall fuzzy model can be achieved by fuzzy "blending" of these simple nonlinear local systems. For example, a modified T–S fuzzy model has been proposed in [16] in which the consequent parts are composed of a set of stochastic Hopfield neural networks with time-varying delays, and a stability criterion has been derived in terms of linear matrix inequalities (LMIs). The results of [16] have then been extended in [15] to deal with the stability analysis problem for T–S fuzzy cellular neural networks with time-varying delays. Motivated by the works in [15], [16] and [30], in this paper, we will consider a more general yet well-known nonlinearity, namely repeated scalar nonlinearity [5], [7], [11] which covers some typical classes of nonlinearities such as the semilinear function, the hyperbolic tangent function that has been extensively used for activation function in neural networks, the sine function, etc.

On the other hand, we notice that, in almost all aforementioned literature concerning T–S fuzzy control systems, it has been implicitly assumed that the communication between the physical plant and controller is perfect, i.e., the signals transmitted from the plant always arrive at the controller without any information losses, and vice versa. Such an assumption, however, is not always true in practice. For example, due to the unreliability of the network links, a networked control

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system (NCS) typically exhibits significant communication delays and data loss across the network, which gives rise to considerable research attention on how to design control systems with the simultaneous consideration of the data loss issue (also called package dropout or missing measurements). For example, in [27] and [28], the control and filtering problems for uncertain discrete-time stochastic systems with missing measurements have been investigated when data travel along unreliable communication channels in a large, wireless, multihop sensor network. The problem of optimally controlling a linear discrete-time plant has been studied in [17] when some of the measurement and control packets were missing. However, to the best of the authors' knowledge, the T–S fuzzy control problem for *nonlinear* systems under *unreliable communication links* has not been fully investigated, which motivates our present research.

In this paper, the H_∞ fuzzy control problem is addressed for a class of nonlinear systems under unreliable communication links. The nonlinear system is described by a discrete-time state equation involving a repeated scalar nonlinearity that typically appears in recurrent neural networks. The communication links, existing between the plant and controller, are assumed to be imperfect, and the packet loss phenomena are modeled by a Bernoulli random binary distributed white sequence with a known conditional probability. The objective is to analyze and design a fuzzy controller such that the closed-loop fuzzy control system is stochastically stable while preserving a guaranteed H_∞ performance. Sufficient conditions that involve matrix equalities are obtained for the existence of admissible controllers, and the CCL procedure is employed to cast the nonconvex feasibility problem into a sequential minimization problem subject to LMIs, which can then be readily solved by using standard numerical software. Two numerical examples are given to illustrate the effectiveness of the proposed design method.

The rest of this paper is organized as follows. Section II formulates the problem under consideration. The stability condition and H_∞ performance of the closed-loop T–S fuzzy control system are given in Section III. The H_∞ fuzzy controller design problem is solved in Section IV. The validity of this approach is demonstrated by illustrative examples in Section V. Finally, in Section VI, the conclusion is given.

Notation. The notation used in the paper is fairly standard. The superscript "T" stands for matrix transposition, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all real matrices of dimension $m \times n$; I and 0 represent the identity matrix and zero matrix, respectively. The notation $P > 0$ means that P is real symmetric and positive definite; $\mathrm{tr}(M)$ refers to the trace of the matrix M; the notation $\|A\|$ refers to the norm of a matrix A defined by $\|A\| = \sqrt{\mathrm{tr}(A^T A)}$ and $\|\cdot\|_2$ stands for the usual l_2 norm. In symmetric block matrices or complex matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry, and $\mathrm{diag}\,\{\ldots\}$ stands for a block-diagonal matrix. In addition, $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|\,y\}$ will, respectively, mean expectation of x and expectation of x conditional on y. The set of all nonnegative integers is denoted by $\mathbb{I}^+$ and the set of all nonnegative real numbers is represented by $\mathbb{R}^+$. Ξ denotes the class of all continuous nondecreasing convex functions $\phi : \mathbb{R}^+ \to \mathbb{R}^+$ such that $\phi(0) = 0$ and $\phi(x) > 0$ for $x > 0$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

II. Problem Formulation

A. Physical Plant

In this paper, we consider the following discrete-time fuzzy systems with repeated scalar nonlinearities:

$\triangle$ *Plant rule i*: IF $\theta_1(k)$ is M_{i1} and $\theta_2(k)$ is M_{i2} and $\cdots$ and $\theta_p(k)$ is M_{ip} THEN

$$\left.\begin{aligned} x_{k+1} &= A_i f(x_k) + B_{2i}u_k + B_{1i}w_k \\ z_k &= C_i f(x_k) + D_{2i}u_k + D_{1i}w_k \\ i &= 1,\ldots,r \end{aligned}\right\} \tag{1}$$

where M_{ij} is the fuzzy set; $x_k \in \mathbb{R}^n$ represents the state vector; $u_k \in \mathbb{R}^m$ is the input vector; $w_k \in \mathbb{R}^p$ is the exogenous disturbance input which belongs to $l_2[0,\infty)$; $z_k \in \mathbb{R}^q$ is the controlled output; $A_i, B_{2i}, B_{1i}, C_i, D_{2i}$, and D_{1i} are all constant matrices with compatible dimensions; r is the number of IF-THEN rules; $\theta_k = [\theta_1(k), \theta_2(k), \ldots, \theta_p(k)]$ is the premise variable vector. It is assumed that the premise variables do not depend on the input variable u_k, which is needed to avoid a complicated defuzzification process of fuzzy controllers. f is a nonlinear function satisfying the following assumption as in [6].

Assumption 1: The nonlinear function $f : \mathbb{R} \to \mathbb{R}$ in system (1) satisfies

$$\forall a, \quad b \in \mathbb{R} \quad |f(a) + f(b)| \le |a + b|. \tag{2}$$

In the sequel, for the vector $x = [x_1 \;\; x_2 \;\; \cdots \;\; x_n]^T$, we denote

$$f(x) \overset{\triangle}{=} [f(x_1) \quad f(x_2) \quad \cdots \quad f(x_n)]^T.$$

Remark 1: The model (1) is called a system with repeated scalar nonlinearity [5], [7], [11]. Note that f is odd (by putting $b = -a$) and 1-Lipschitz (by putting $b = -b$). Therefore, f encapsulates some typical classes of nonlinearities, such as

1) the semilinear function (i.e., the standard saturation $sat(s) := s$ if $|s| \le 1$ and $sat(s) := sgn(s)$ if $|s| > 1$);
2) the hyperbolic tangent function that has been extensively used for activation function in neural networks;
3) the sine function, etc.

Given a pair of (x_k, u_k), the final outputs of the fuzzy system are inferred as follows:

$$\left.\begin{aligned} x_{k+1} &= \sum_{i=1}^{r} h_i(\theta_k)[A_i f(x_k) + B_{2i}u_k + B_{1i}w_k] \\ z_k &= \sum_{i=1}^{r} h_i(\theta_k)[C_i f(x_k) + D_{2i}u_k + D_{1i}w_k] \end{aligned}\right\} \tag{3}$$

where the fuzzy basis functions are given by

$$h_i(\theta_k) = \frac{\vartheta_i(\theta_k)}{\sum_{i=1}^{r} \vartheta_i(\theta_k)}$$

with $\vartheta_i(\theta_k) = \Pi_{j=1}^{p} M_{ij}(\theta_j(k))$. $M_{ij}(\theta_j(k))$ represents the grade of membership of $\theta_j(k)$ in M_{ij}. Here, $\vartheta_i(\theta_k)$ has the following basic property:

$$\vartheta_i(\theta_k) \geq 0, \quad i = 1, 2, \ldots, r, \qquad \sum_{i=1}^{r} \vartheta_i(\theta_k) > 0, \qquad \forall k$$

and therefore

$$h_i(\theta_k) \geq 0, \quad i = 1, 2, \ldots, r, \qquad \sum_{i=1}^{r} h_i(\theta_k) = 1 \qquad \forall k.$$

B. Controller

In this paper, we consider the following fuzzy control law for the fuzzy system (3):

$\triangle$ *Controller rule i*: IF $\theta_1(k)$ is M_{i1} and $\theta_2(k)$ is M_{i2} and $\cdots$ and $\theta_p(k)$ is M_{ip} THEN

$$u_{ck} = K_i \hat{f}(x_k), \qquad i = 1, 2, \ldots, r.$$

Here, $\hat{f}(x_k) \in R^n$ is the input to the controller; $u_{ck} \in R^m$ is the output of the controller; K_i are the gain matrices of the controller to be designed. Hence, the controller can be represented by the following input–output form:

$$u_{ck} = \sum_{i=1}^{r} h_i(\theta_k) K_i \hat{f}(x_k). \tag{4}$$

C. Communication Links

Due to the existence of the packet losses between the physical plant and controller, the measurement of the plant is probably not equivalent to the input to the controller [i.e., $f(x_k) \neq \hat{f}(x_k)$], and the output of the controller is probably not equivalent to the input of the plant (i.e., $u_k \neq u_{ck}$). As a result, we model the packet losses phenomena via a stochastic approach as follows:

$$\hat{f}(x_k) = \alpha_k f(x_k) \quad u_k = \beta_k u_{ck}$$

where $\{\alpha_k\}$ and $\{\beta_k\}$ are two independent Bernoulli processes. $\{\alpha_k\}$ models the unreliable nature of the link from the sensor to the controller, and $\{\beta_k\}$ models that from the controller to the actuator. Obviously, $\alpha_k = 0$ holds when the communication link fails (i.e., data are lost), and $\alpha_k = 1$ means successful transmission. The same happens for β_k. A natural assumption on the sequence $\{\alpha_k\}$ and $\{\beta_k\}$ can be made as follows:

$$\text{Prob}\{\alpha_k = 1\} = \mathbb{E}\{\alpha_k\} = \bar{\alpha} \qquad \text{Prob}\{\alpha_k = 0\} = 1 - \bar{\alpha}$$
$$\text{Prob}\{\beta_k = 1\} = \mathbb{E}\{\beta_k\} = \bar{\beta} \qquad \text{Prob}\{\beta_k = 0\} = 1 - \bar{\beta}.$$

Based on this, we have

$$u_k = \sum_{i=1}^{r} h_i(\theta_k) \beta_k \alpha_k K_i f(x_k). \tag{5}$$

Note that such a stochastic Bernoulli approach has been extensively used for dealing with data missing problems; see, e.g., [13] and [26] and the references therein.

D. Closed-Loop System

In this paper, we introduce another Bernoulli process $\{\rho_k\}$ with $\rho_k \overset{\Delta}{=} \alpha_k \beta_k$. It is easy to know that $\rho_k = 1$ when both $\alpha_k = 1$ and $\beta_k = 1$ are true, and $\rho_k = 0$ otherwise. Then, we have

$$\text{Prob}\{\rho_k = 1\} = \mathbb{E}\{\rho_k\} := \bar{\rho} = \bar{\alpha}\bar{\beta}$$
$$\text{Prob}\{\rho_k = 0\} = 1 - \bar{\rho} = 1 - \bar{\alpha}\bar{\beta}.$$
$$u_k = \sum_{i=1}^{r} h_i(\theta_k) \rho_k K_i f(x_k). \tag{6}$$

The closed-loop T–S fuzzy control system can now be obtained from (3) and (6) that

$$\left. \begin{aligned} x_{k+1} &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\theta_k) h_j(\theta_k) [A_{ij} f(x_k) + B_{1i} w_k] \\ z_k &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\theta_k) h_j(\theta_k) [C_{ij} f(x_k) + D_{1i} w_k] \end{aligned} \right\} \tag{7}$$

where $A_{ij} = A_i + \bar{\rho} B_{2i} K_j + \tilde{\rho}_k B_{2i} K_j$, $C_{ij} = C_i + \bar{\rho} D_{2i} K_j + \tilde{\rho}_k D_{2i} K_j$, and $\tilde{\rho}_k = \rho_k - \bar{\rho}$. It is clear that $\mathbb{E}\{\tilde{\rho}_k\} = 0$ and $\mathbb{E}\{\tilde{\rho}_k^2\} = \bar{\rho}(1 - \bar{\rho})$.

Before formulating the problem to be investigated, we first introduce the following definitions and lemmas.

Definition 1 [23]: The solution $x_k = 0$ of the closed-loop T–S fuzzy control system in (7) with $w_k \equiv 0$ is said to be stochastically stable if, for any $\varepsilon > 0$, there exists a $\delta > 0$ such that

$$\mathbb{E}\{\|x_k\|\} < \varepsilon \tag{8}$$

whenever $k \in \mathbb{I}^+$ and $\|x_0\| < \delta$.

Definition 2: A square matrix $P \overset{\triangle}{=} [p_{ij}] \in \mathbb{R}^{n \times n}$ is called diagonally dominant if for all $i = 1, \ldots, n$

$$p_{ii} \geq \sum_{j \neq i} |p_{ij}|. \tag{9}$$

Lemma 1 [6]: If $P > 0$ is diagonally dominant, then for all nonlinear functions f satisfying (2), the following inequality holds for all $x_k \in \mathbb{R}^n$:

$$f^T(x_k) P f(x_k) \leq x_k^T P x_k. \tag{10}$$

Remark 2: It will be seen later that the purpose of requiring the matrix P to satisfy (10) is to admit the quadratic Lyapunov function $V(x_k) = x_k^T P x_k$.

Lemma 2 [23]: If there exist a Lyapunov function $V(x_k)$ and a function $\phi(x) \in \Xi$ satisfying the following conditions

$$V(0) = 0 \tag{11}$$

$$\phi(\|x_k\|) \leq V(x_k) \tag{12}$$

$$\mathbb{E}\{V(x_{k+1})\} - \mathbb{E}\{V(x_k)\} < 0, \qquad k \in \mathbb{I}^+ \tag{13}$$

then the solution $x_k = 0$ of the closed-loop T–S fuzzy control system in (7) with $w_k \equiv 0$ is stochastically stable.

Consider the fuzzy control problem in the presence of packet losses phenomena and suppose the parameter $\bar{\rho}$ describing intermittent transmission is known. We are now in a position to state the problem of H_∞ fuzzy control for systems with repeated scalar nonlinearities and random packet losses as follows.

Problem H_∞ fuzzy control with data loss (*HFCDL*): Given a scalar $\gamma > 0$, design a controller in the form of (4) such that

1) (stochastic stability) the closed-loop T–S fuzzy control system in (7) is stochastically stable in the sense of Definition 1;
2) (H_∞ performance) under zero initial condition, the controlled output z_k satisfies

$$\|z\|_{\mathbb{E}} \le \gamma \|w\|_2 \tag{14}$$

where

$$\|z\|_{\mathbb{E}} \triangleq \mathbb{E}\left\{ \sqrt{\sum_{k=0}^{\infty} z_k^T z_k} \right\}$$

and $\|\cdot\|_2$ stands for the usual l_2 norm.

If the earlier two conditions are satisfied, the closed-loop T–S fuzzy control system is said to be stochastically stable with a guaranteed H_∞ performance γ, and the problem *HFCDL* is solved.

III. H_∞ Fuzzy Control Performance Analysis

In this section, the problem *HFCDL* formulated in the previous section will be tackled via a quadratic approach described in the following theorem.

Theorem 1: Consider the fuzzy system in (3) and suppose the gain matrices K_i $(i = 1, \ldots, r)$ of the controllers in (4) are given. The closed-loop fuzzy system in (7) is stochastically stable with a guaranteed H_∞ performance γ if there exists a positive diagonally dominant matrix P satisfying

$$\Psi_{ii}^T \bar{P} \Psi_{ii} + \Lambda_{ii}^T \Lambda_{ii} - \bar{L} < 0, \qquad i = 1, 2, \ldots, r \tag{15}$$

$$\begin{aligned}(\Psi_{ij} + \Psi_{ji})^T \bar{P} (\Psi_{ij} + \Psi_{ji}) + (\Lambda_{ij} + \Lambda_{ji})^T (\Lambda_{ij} + \Lambda_{ji}) \\ - 4\bar{L} < 0, \qquad 1 \le i < j \le r\end{aligned} \tag{16}$$

where

$$\Psi_{ij} = \begin{bmatrix} A_i + \bar{\rho} B_{2i} K_j & B_{1i} \\ q B_{2i} K_j & 0 \end{bmatrix} \quad \bar{L} = \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix}$$

$$\Lambda_{ij} = \begin{bmatrix} C_i + \bar{\rho} D_{2i} K_j & D_{1i} \\ q D_{2i} K_j & 0 \end{bmatrix} \quad \bar{P} = \begin{bmatrix} P & 0 \\ 0 & P \end{bmatrix}$$

$$q = \sqrt{\bar{\rho}(1 - \bar{\rho})}.$$

Proof: In order to show that the fuzzy system in (7) is stochastically stable with a guaranteed H_∞ performance γ under conditions (15) and (16), we define the following Lyapunov function candidate

$$V(x_k) = x_k^T P x_k. \tag{17}$$

When $w_k \equiv 0$, the difference of the Lyapunov function is calculated as

$$\begin{aligned}
\mathbb{E}\{\Delta V(x_k)\} &= \mathbb{E}\{ V(x_{k+1}) | x_k \} - V(x_k) \\
&= \mathbb{E}\left\{ \left[\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\theta_k) h_j(\theta_k) A_{ij} f(x_k) \right]^T P \right. \\
&\quad \left. \times \left[\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\theta_k) h_j(\theta_k) A_{ij} f(x_k) \right] \Bigg| x_k \right\} \\
&\quad - x_k^T P x_k \\
&= \mathbb{E}\Bigg\{ \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{s=1}^{r} \sum_{t=1}^{r} h_i(\theta_k) h_j(\theta_k) h_s(\theta_k) h_t(\theta_k) \\
&\quad \times f^T(x_k) A_{ij}^T P A_{st} f(x_k) | x_k \Bigg\} - x_k^T P x_k \\
&= \mathbb{E}\Bigg\{ \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{s=1}^{r} \sum_{t=1}^{r} h_i(\theta_k) h_j(\theta_k) h_s(\theta_k) h_t(\theta_k) \\
&\quad \times f^T(x_k) \left(\frac{A_{ij} + A_{ji}}{2} \right)^T P \left(\frac{A_{st} + A_{ts}}{2} \right) \\
&\quad \times f(x_k) | x_k \Bigg\} - x_k^T P x_k \\
&\le \mathbb{E}\Bigg\{ \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\theta_k) h_j(\theta_k) f^T(x_k) \\
&\quad \times \left(\frac{A_{ij} + A_{ji}}{2} \right)^T P \left(\frac{A_{ij} + A_{ji}}{2} \right) \\
&\quad \times f(x_k) | x_k \Bigg\} - x_k^T P x_k.
\end{aligned} \tag{18}$$

Note that in the earlier inequality, the elementary inequality of $2a^T b \le a^T a + b^T b$ for $a, b \in \mathbb{R}^n$ has been used.

According to Lemma 1, we have

$$\begin{aligned}
\mathbb{E}\{\Delta V(x_k)\} &\le \mathbb{E}\Bigg\{ \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\theta_k) h_j(\theta_k) f^T(x_k) \\
&\quad \times \left[\left(\frac{A_{ij} + A_{ji}}{2} \right)^T P \left(\frac{A_{ij} + A_{ji}}{2} \right) - P \right] \\
&\quad \times f(x_k) | x_k \Bigg\} \\
&= f^T(x_k) \sum_{i=1}^{r} h_i^2(\theta_k) (\Pi_{ii}^T \bar{P} \Pi_{ii} - P) f(x_k)
\end{aligned}$$

$$+\frac{1}{2}f^T(x_k)\sum_{i,j=1,i<j}^{r}h_i(\theta_k)h_j(\theta_k)$$
$$\times\left[(\Pi_{ij}+\Pi_{ji})^T\bar{P}(\Pi_{ij}+\Pi_{ji})-4P\right]f(x_k)$$

where

$$\Pi_{ij}=\begin{bmatrix}A_i+\bar{\rho}B_{2i}K_j\\ qB_{2i}K_j\end{bmatrix}.$$

By the Schur complement lemma, we know from inequalities (15) and (16) that

$$\Pi_{ii}^T\bar{P}\Pi_{ii}-P<0, \qquad i=1,2,\ldots,r$$
$$(\Pi_{ij}+\Pi_{ji})^T\bar{P}(\Pi_{ij}+\Pi_{ji})-4P<0, \qquad 1\le i<j\le r.$$

Thus, we have

$$\mathbb{E}\{V(x_{k+1})\}-\mathbb{E}\{V(x_k)\}<0$$

which satisfies (13). Taking $\phi(\|x_k\|)=\lambda_{\min}(P)x_k^2$ such that $\phi(\cdot)\in\Xi$, we obtain

$$\phi(\|x_k\|)=\lambda_{\min}(P)\|x_k\|^2=\lambda_{\min}(P)x_k^Tx_k$$
$$\le x_k^TPx_k=V(x_k)$$

which satisfies (12). Considering $V(0)=0$, it follows readily from Lemma 2 that the closed-loop system in (7) with $w_k\equiv 0$ is stochastically stable.

Next, the H_∞ performance criteria for the closed-loop system in (7) will be established. Assuming zero initial conditions, an index is introduced as follows:

$$\bar{J}=\mathbb{E}\{V(x_{k+1})|\,\xi_k\}+\mathbb{E}\{z_k^Tz_k\,|\,\xi_k\}$$
$$-\gamma^2w_k^Tw_k-f^T(x_k)Pf(x_k).$$

Defining

$$\xi_k=[\,x_k^T\quad w_k^T\,]^T\qquad \eta_k=[\,f^T(x_k)\quad w_k^T\,]^T$$
$$\bar{G}_{ij}=[\,A_{ij}\quad B_{1i}\,]\qquad \bar{H}_{ij}=[\,C_{ij}\quad D_{1i}\,]$$

similar to the derivation of (18), we have

$$\bar{J}=\mathbb{E}\left\{\left[\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta_k)h_j(\theta_k)\bar{G}_{ij}\eta_k\right]^TP\left[\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta_k)\right.\right.$$
$$\left.\left.\times\,h_j(\theta_k)\bar{G}_{ij}\eta_k\right]\Bigg|\,\xi_k\right\}$$
$$+\mathbb{E}\left\{\left[\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta_k)h_j(\theta_k)\bar{H}_{ij}\eta_k\right]^T\right.$$
$$\left.\times\left[\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta_k)h_j(\theta_k)\bar{H}_{ij}\eta_k\right]\Bigg|\,\xi_k\right\}-\eta_k^T\bar{L}\eta_k$$
$$\le\mathbb{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta_k)h_j(\theta_k)\eta_k^T\left(\frac{\bar{G}_{ij}+\bar{G}_{ji}}{2}\right)^TP\right.$$
$$\left.\times\left(\frac{\bar{G}_{ij}+\bar{G}_{ji}}{2}\right)\eta_k\,\Bigg|\,\xi_k\right\}$$
$$+\mathbb{E}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta_k)h_j(\theta_k)\eta_k^T\left(\frac{\bar{H}_{ij}+\bar{H}_{ji}}{2}\right)^T\right.$$
$$\left.\times\left(\frac{\bar{H}_{ij}+\bar{H}_{ji}}{2}\right)\eta_k\,\Bigg|\,\xi_k\right\}-\eta_k^T\bar{L}\eta_k$$
$$=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta_k)h_j(\theta_k)\eta_k^T\left(\frac{\Psi_{ij}+\Psi_{ji}}{2}\right)^T\bar{P}$$
$$\times\left(\frac{\Psi_{ij}+\Psi_{ji}}{2}\right)\eta_k+\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta_k)h_j(\theta_k)\eta_k^T$$
$$\times\left(\frac{\Lambda_{ij}+\Lambda_{ji}}{2}\right)^T\left(\frac{\Lambda_{ij}+\Lambda_{ji}}{2}\right)\eta_k-\eta_k^T\bar{L}\eta_k$$
$$=\eta_k^T\sum_{i=1}^{r}h_i^2(\theta_k)(\Psi_{ii}^T\bar{P}\Psi_{ii}+\Lambda_{ii}^T\Lambda_{ii}-\bar{L})\eta_k+\frac{1}{2}\eta_k^T$$
$$\times\sum_{i,j=1,i<j}^{r}h_i(\theta_k)h_j(\theta_k)$$
$$\times\left[(\Psi_{ij}+\Psi_{ji})^T\bar{P}(\Psi_{ij}+\Psi_{ji})\right.$$
$$\left.+(\Lambda_{ij}+\Lambda_{ji})^T(\Lambda_{ij}+\Lambda_{ji})-4\bar{L}\right]\eta_k$$

From inequalities (15) and (16), we know that $\bar{J}\le 0$ and, according to Lemma 1, we have

$$\mathbb{E}\{x_{k+1}^TPx_{k+1}\,|\,\xi_k\}+\mathbb{E}\{z_k^Tz_k\,|\,\xi_k\}-\gamma^2w_k^Tw_k-x_k^TPx_k\le 0.$$

Taking mathematical expectation on both sides, we obtain

$$\mathbb{E}\{x_{k+1}^TPx_{k+1}\,|\,\xi_k\}-\mathbb{E}\{x_k^TPx_k\,|\,\xi_k\}+\mathbb{E}\{z_k^Tz_k\}$$
$$-\gamma^2w_k^Tw_k\le 0.$$

For $k=0,1,2,\ldots,$ summing up both sides under zero initial condition and considering $\mathbb{E}\{x_\infty^TPx_\infty\}\ge 0$, we arrive at

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}z_k^Tz_k\right\}-\sum_{k=0}^{\infty}\gamma^2w_k^Tw_k\le 0$$

which is equivalent to (14). The proof is now completed. ■

Remark 3: In Theorem 1, with given controller gain and disturbance attenuation level γ, we obtain the stochastic stability conditions of the nominal fuzzy system (7), which are represented via a set of matrix inequalities in (15) and (16). We will show later in the next section that such inequalities can be converted into LMIs when designing the actual controllers. Note that the feasibility of LMIs can be easily checked by using the MATLAB LMI toolbox.

Remark 4: Let us now consider the *standard* H_∞ performance criterion for a discrete-time fuzzy closed-loop system with perfect communication links between the plant and controller. In this case, we have $\bar{\rho}=1$ in (7) and then the inequalities (15) and (16) reduce to

$$X_{ii}^T \bar{P} X_{ii} + Y_{ii}^T Y_{ii} - \bar{L} < 0, \qquad i = 1, 2, \ldots, r \tag{19}$$

$$(X_{ij} + X_{ji})^T \bar{P}(X_{ij} + X_{ji}) + (Y_{ij} + Y_{ji})^T (Y_{ij} + Y_{ji}) - 4\bar{L} < 0, \qquad 1 \le i < j \le r \tag{20}$$

where

$$X_{ij} = [\, A_i + B_{2i}K_j \quad B_{1i} \,] \qquad Y_{ij} = [\, C_i + D_{2i}K_j \quad D_{1i} \,].$$

Later, we will show via simulation that, in the case there *indeed* exist random packet losses, the main results given by Theorem 1 will provide much improved performance over the standard H_∞ approach that does not take into account the data missing problem.

IV. H_∞ Fuzzy Controller Design

In this section, we aim at designing a controller in the form of (4) based on Theorem 1, i.e., we are interested in determining the controller parameters such that the closed-loop fuzzy system in (7) is stochastically stable with a guaranteed H_∞ performance. The following theorem provides sufficient conditions for the existence of such H_∞ fuzzy controller for system (7).

Theorem 2: Consider the fuzzy system in (3). There exists a state-feedback controller in the form of (4) such that the closed-loop system in (7) is stochastically stable with a guaranteed H_∞ performance γ, if there exist matrices $0 < P \overset{\triangle}{=} [p_{ij}]$, $L > 0$, $M_i, i = 1, \ldots, r$, $R = R^T \overset{\triangle}{=} [r_{ij}]$ satisfying

$$\begin{bmatrix} -L & * & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * & * \\ A_i L + \bar{\rho} B_{2i} M_i & B_{1i} & -L & * & * & * \\ q B_{2i} M_i & 0 & 0 & -L & * & * \\ C_i L + \bar{\rho} D_{2i} M_i & D_{1i} & 0 & 0 & -I & * \\ q D_{2i} M_i & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad i = 1, 2, \ldots, r \tag{21}$$

$$\begin{bmatrix} \Upsilon_{11} & * \\ \Upsilon_{21} & \Upsilon_{22} \end{bmatrix} < 0 \tag{22}$$

$$p_{ii} - \sum_{j \ne i} (p_{ij} + 2r_{ij}) \ge 0 \tag{23}$$

$$r_{ij} \ge 0 \qquad \forall i \ne j \tag{24}$$

$$p_{ij} + r_{ij} \ge 0 \qquad \forall i \ne j \tag{25}$$

$$PL = I \tag{26}$$

where

$$\Upsilon_{11} = \begin{bmatrix} -4L & * \\ 0 & -4\gamma^2 I \end{bmatrix}$$

$$\Upsilon_{21} = \begin{bmatrix} (A_i + A_j)L + \bar{\rho}\bar{B}_2 & B_{1i} + B_{1j} \\ q\bar{B}_2 & 0 \\ (C_i + C_j)L + \bar{\rho}\bar{D}_2 & D_{1i} + D_{1j} \\ q\bar{D}_2 & 0 \end{bmatrix}$$

$$\Upsilon_{22} = \mathrm{diag}\{-L, -L, -I, -I\}$$

$$\bar{B}_2 = B_{2i}M_j + B_{2j}M_i \qquad \bar{D}_2 = D_{2i}M_j + D_{2j}M_i$$

$$1 \le i < j \le r.$$

Furthermore, if the earlier conditions have feasible solutions, the gain K_i of the subsystem controller in (4) is given by

$$K_i = M_i L^{-1}. \tag{27}$$

Proof: From Theorem 1, we know that the closed-loop system in (7) is stochastically stable with a guaranteed H_∞ performance γ if there exists a diagonally dominant matrix $P > 0$ satisfying (15) and (16). By the Schur complement, the following inequalities are obtained:

$$\begin{bmatrix} -P & * & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * & * \\ A_i + \bar{\rho} B_{2i} K_i & B_{1i} & -P^{-1} & * & * & * \\ q B_{2i} K_i & 0 & 0 & -P^{-1} & * & * \\ C_i + \bar{\rho} D_{2i} K_i & D_{1i} & 0 & 0 & -I & * \\ q D_{2i} K_i & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{28}$$

$$\begin{bmatrix} \hat{\Upsilon}_{11} & * \\ \hat{\Upsilon}_{21} & \hat{\Upsilon}_{22} \end{bmatrix} < 0 \tag{29}$$

where

$$\hat{\Upsilon}_{11} = \begin{bmatrix} -4P & * \\ 0 & -4\gamma^2 I \end{bmatrix}$$

$$\hat{\Upsilon}_{21} = \begin{bmatrix} (A_i + A_j) + \bar{\rho}\hat{B}_2 & B_{1i} + B_{1j} \\ q\hat{B}_2 & 0 \\ (C_i + C_j) + \bar{\rho}\hat{D}_2 & D_{1i} + D_{1j} \\ q\hat{D}_2 & 0 \end{bmatrix}$$

$$\hat{\Upsilon}_{22} = \mathrm{diag}\left\{-P^{-1}, -P^{-1}, -I, -I\right\}$$

$$\hat{B}_2 = B_{2i}K_j + B_{2j}K_i \qquad \hat{D}_2 = D_{2i}K_j + D_{2j}K_i.$$

Performing congruence transformations to inequalities (28) and (29) by $\mathrm{diag}\left\{P^{-1}, I, I, I, I, I\right\}$, we have

$$\begin{bmatrix} -P^{-1} & * & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * & * \\ (A_i + \bar{\rho} B_{2i} K_i)P^{-1} & B_{1i} & -P^{-1} & * & * & * \\ q B_{2i} K_i P^{-1} & 0 & 0 & -P^{-1} & * & * \\ (C_i + \bar{\rho} D_{2i} K_i)P^{-1} & D_{1i} & 0 & 0 & -I & * \\ q D_{2i} K_i P^{-1} & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0$$

$$\begin{bmatrix} \check{\Upsilon}_{11} & * \\ \check{\Upsilon}_{21} & \check{\Upsilon}_{22} \end{bmatrix} < 0$$

where

$$\check{\Upsilon}_{11} = \begin{bmatrix} -4P^{-1} & * \\ 0 & -4\gamma^2 I \end{bmatrix}$$

$$\check{\Upsilon}_{21} = \begin{bmatrix} (A_i + A_j)P^{-1} + \bar{\rho}\check{B}_2 & B_{1i} + B_{1j} \\ q\check{B}_2 & 0 \\ (C_i + C_j)P^{-1} + \bar{\rho}\check{D}_2 & D_{1i} + D_{1j} \\ q\check{D}_2 & 0 \end{bmatrix}$$
$$\check{\Upsilon}_{22} = \mathrm{diag}\left\{-P^{-1}, -P^{-1}, -I, -I\right\}$$
$$\check{B}_2 = (B_{2i}K_j + B_{2j}K_i)P^{-1}$$
$$\check{D}_2 = (D_{2i}K_j + D_{2j}K_i)P^{-1}.$$

Defining $L = P^{-1}$ and $M_i = K_i P^{-1}$, we can obtain (21) and (22) readily. Furthermore, from (23) to (25), we have

$$p_{ii} \geq \sum_{j\neq i}(p_{ij} + 2r_{ij}) = \sum_{j\neq i}(|p_{ij} + r_{ij}| + |-r_{ij}|) \geq \sum_{j\neq i}|p_{ij}|$$

which guarantees the positive definite matrix P to be diagonally dominant, and the proof is then complete. ∎

It is worth noting that, by far, we are unable to apply the LMI approach in the design of controller because of the matrix equality in Theorem 2. Fortunately, this problem can be addressed with help from the cone complementarity linearization (CCL) algorithm proposed in [14]. The basic idea behind the CCL algorithm is that if the LMI

$$\begin{bmatrix} P & I \\ I & L \end{bmatrix} \geq 0$$

is feasible in the $n \times n$ matrix variables $L > 0$ and $P > 0$, then $tr(PL) \geq n$; and $tr(PL) = n$ if and only if $PL = I$. Based on this, it is likely to solve the equalities in (26) by using the CCL algorithm. In view of this observation, we put forward the following nonlinear minimization problem involving LMI conditions instead of the original nonconvex feasibility problem formulated in Theorem 2.

The nonlinear minimization problem: $\min tr(PL)$ subject to (21)–(25) and

$$\begin{bmatrix} P & I \\ I & L \end{bmatrix} \geq 0. \tag{30}$$

If the solution of $\min\ tr(PL)$ subject to (21)–(25) exists and $\min tr(PL) = n$, then the conditions in Theorem 2 are solvable.

Finally, the following algorithm is suggested to solve the earlier problem.

Algorithm HinfFC (HinfFC: H_∞ Fuzzy Control)

Step 1: Find a feasible set $(P_{(0)}, L_{(0)}, M_{i(0)}, R_{(0)})$ satisfying (21)–(25) and (30). Set $q = 0$.

Step 2: According to (21)–(25) and (30), solve the LMI problem: $\min tr(PL_{(q)} + P_{(q)}L)$.

Step 3: Substitute the obtained matrix variables (P, L, M_i, R) into (15) and (16). If conditions (15) and (16) are satisfied with $|tr(PL) - n| < \tau$ for some sufficiently small scalar $\tau > 0$, then output the feasible solutions. Exit.

Step 4: If $K > N$, where N is the maximum number of iterations allowed. Exit. Else, set $q = q + 1$ and go to *Step 2*.

Remark 5: As is well known, the packet dropout problem may occurs in the area of networked fault detection and isolation (FDI); see, e.g., [18] and [20] and the references therein. One of the future research topics would be the extension of our main results to network-based FDI and filtering problems.

Remark 6: Our main results are based on the LMI conditions. The LMI control toolbox implements state-of-the-art interior-point LMI solvers. While these solvers are significantly faster than classical convex optimization algorithms, it should be kept in mind that the complexity of LMI computations remains higher than that of solving, say, a Riccati equation. For instance, problems with a thousand design variables typically take over an hour on today's workstations. However, research on LMI optimization is a very active area in the applied math, optimization and the operations research community, and substantial speedups can be expected in the future.

V. Illustrative Examples

In this section, two simulation examples are presented to illustrate the fuzzy controller design method developed in this paper.

Example 1: Consider a T–S fuzzy model (1) with repeated scalar nonlinearities and random packet losses. The rules are given as follows:

`Plant rule` 1: IF $f_1(x_k)$ is $h_1(f_1(x_k))$ THEN

$$\begin{aligned} x_{k+1} &= A_1 f(x_k) + B_{21}u_k + B_{11}w_k \\ z_k &= C_1 f(x_k) + D_{21}u_k + D_{11}w_k. \end{aligned} \tag{31}$$

`Plant rule` 2: IF $f_1(x_k)$ is $h_2(f_1(x_k))$ THEN

$$\begin{aligned} x_{k+1} &= A_2 f(x_k) + B_{22}u_k + B_{12}w_k \\ z_k &= C_2 f(x_k) + D_{22}u_k + D_{12}w_k. \end{aligned} \tag{32}$$

`Controller rule` 1: IF $f_1(x_k)$ is $h_1(f_1(x_k))$ THEN $u_{ck} = K_1\hat{f}(x_k)$.

`Controller rule` 2: IF $f_1(x_k)$ is $h_2(f_1(x_k))$ THEN $u_{ck} = K_2\hat{f}(x_k)$.

The final outputs of the fuzzy system are inferred as follows:

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{2} h_i(f_1(x_k))[A_i f(x_k) + B_{2i}u_k + B_{1i}w_k] \\ z_k &= \sum_{i=1}^{2} h_i(f_1(x_k))[C_i f(x_k) + D_{2i}u_k + D_{1i}w_k]. \end{aligned} \tag{33}$$

The model parameters are given as follows:

$$A_1 = \begin{bmatrix} 1.0 & 0.31 & 0 \\ 0 & 0.33 & 0.21 \\ 0 & 0 & -0.52 \end{bmatrix} \quad C_1 = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0.8 & -0.38 & 0 \\ -0.2 & 0 & 0.21 \\ 0.1 & 0 & -0.55 \end{bmatrix} \quad B_{21} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$B_{11}=\begin{bmatrix}0.1\\0\\0\end{bmatrix}\quad C_2=\begin{bmatrix}-0.12&0&0.1\\0&0&0\\0&0&0.1\end{bmatrix}$$

$$B_{12}=\begin{bmatrix}0\\0.12\\0\end{bmatrix}\quad D_{21}=\begin{bmatrix}1&1\\0&1\\0&1\end{bmatrix}\quad B_{22}=\begin{bmatrix}1&0\\0&1\\0&1\end{bmatrix}$$

$$D_{12}=\begin{bmatrix}0\\0\\0.22\end{bmatrix}\quad D_{11}=\begin{bmatrix}0.15\\0\\0\end{bmatrix}\quad D_{22}=\begin{bmatrix}1&1\\0&1\\0&1\end{bmatrix}$$

and $\gamma = 0.8$. The membership function is assumed to be

$$h_1(f_1(x_k))=\begin{cases}1, & f_1(x_0)=0\\ |\sin(f_1(x_0))|/f_1(x_0), & \text{else}\end{cases}\tag{34}$$

$$h_2(f_1(x_k))=1-h_1(f_1(x_k)).\tag{35}$$

Earlier, the nonlinear function $f(x_k)=\sin(x_k)$ satisfies (2). Our aim is to design a state-feedback paralleled controller in the form of (4) such that the system (33) is stochastically stable with a guaranteed H_∞ norm bound γ.

Let $\bar{\rho}=0.8$. By applying Theorem 2 with help from Algorithm *HinfFC*, we can obtain admissible solutions as follows:

$$L=\begin{bmatrix}3.0596&-0.4887&0.0629\\-0.4887&0.7434&0.3136\\0.0629&0.3136&0.8091\end{bmatrix}$$

$$P=\begin{bmatrix}0.3874&0.2799&-0.1065\\0.2799&2.0222&-0.9528\\-0.1065&-0.9528&1.8081\end{bmatrix}$$

$$K_1=\begin{bmatrix}-0.4004&-0.0364&-0.1353\\-0.0157&-0.1077&0.0209\end{bmatrix}$$

$$K_2=\begin{bmatrix}-0.1291&0.1054&-0.2146\\0.0513&-0.0160&0.0166\end{bmatrix}.$$

First, we assume the initial condition to be

$$x_0=[0\quad 0.01\quad 0]^T\tag{36}$$

and the external disturbance $w_k\equiv 0$. Fig. 1 gives the state responses for the uncontrolled fuzzy systems, which are apparently unstable. Fig. 2 gives the state simulation results of the closed-loop fuzzy system, from which we can see that the closed-loop system is stochastically stable.

Next, to illustrate the disturbance attenuation performance, we choose the initial condition $x_0\equiv 0$ and the external disturbance w_k as follows:

$$w_k=\begin{cases}0.2, & 20\le k\le 30\\ -0.2, & 40\le k\le 50\\ 0, & \text{else}\end{cases}\tag{37}$$

Fig. 3 shows the controller output, and Fig. 4 shows the evolution of the state variables. The disturbance input w_k and controlled output z_k are depicted in Fig. 5. It can be calculated that

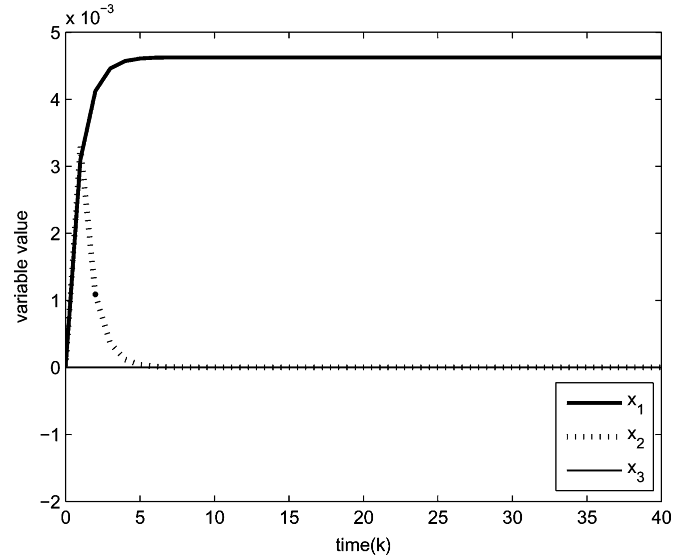

Fig. 1. State evolution x_k of uncontrolled systems.

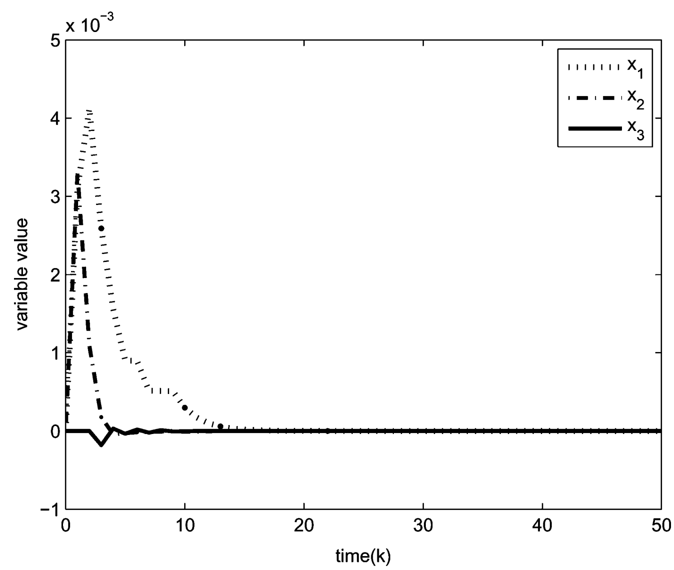

Fig. 2. State evolution x_k of controlled systems.

$\|z\|_2=0.1669$ and $\|w\|_2=0.9381$, and therefore, $\gamma=0.4218$, which stays below the prescribed upper bound $\gamma^*=0.8$.

Example 2: In this example, we aim to show the advantage of considering the probabilistic packet losses. We demonstrate through numerical simulation that, when the measurement transmission between the plant and controller is indeed imperfect, the design method proposed in this paper gives better performance than the standard H_∞ approach without taking into account the packet loss problem. For this purpose, we let

$$A_1=\begin{bmatrix}1.0&0.31&0\\0&0.33&0.21\\0&0&-1.5\end{bmatrix}$$

and the other system data of (7) be the same as those in Example 1 with $\bar{\rho}=0.6$. First, we assume the initial condition to be $x_0=[0\quad 0.01\quad 0.2]^T$, and the external disturbance $w_k\equiv 0$.

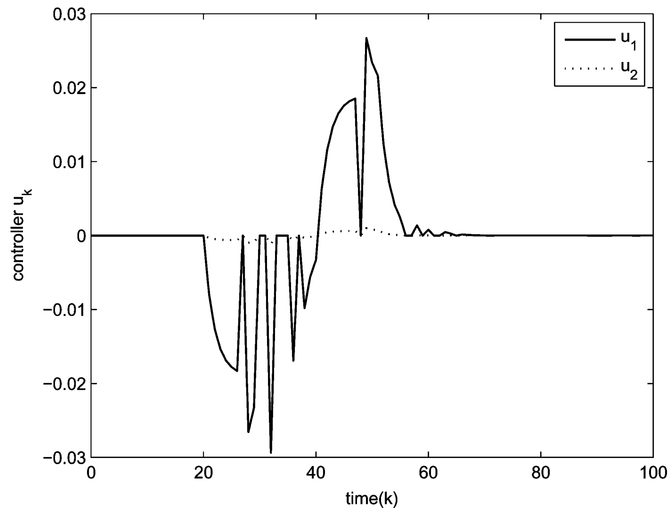

Fig. 3. Controllers u_k.

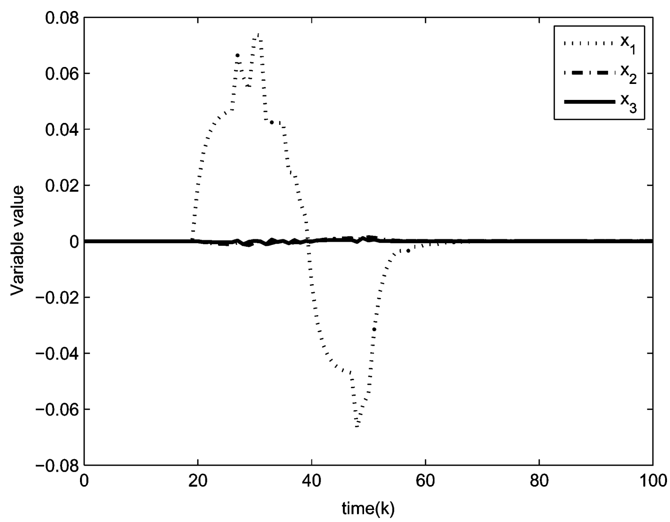

Fig. 4. State evolution x_k of controlled systems when $w(k) \neq 0$.

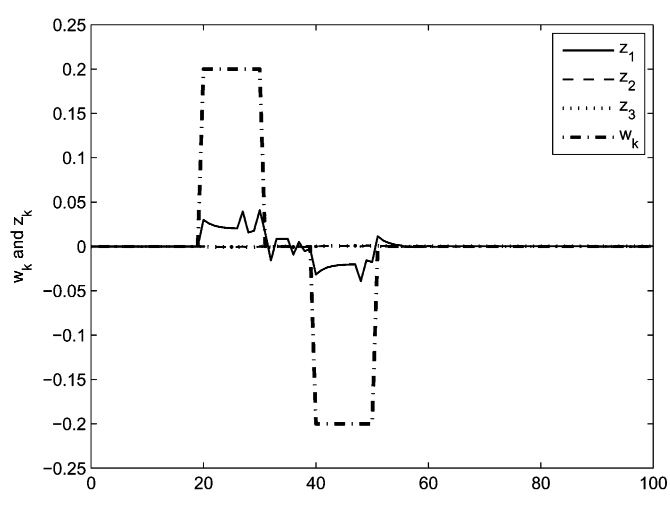

Fig. 5. Controlled output z_k and disturbance input w_k.

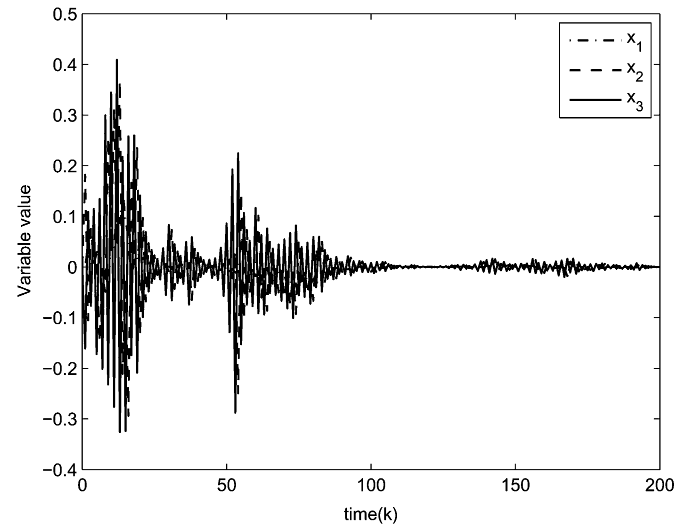

Fig. 6. Closed-loop state variables by taking the standard H_∞ performance criterion.

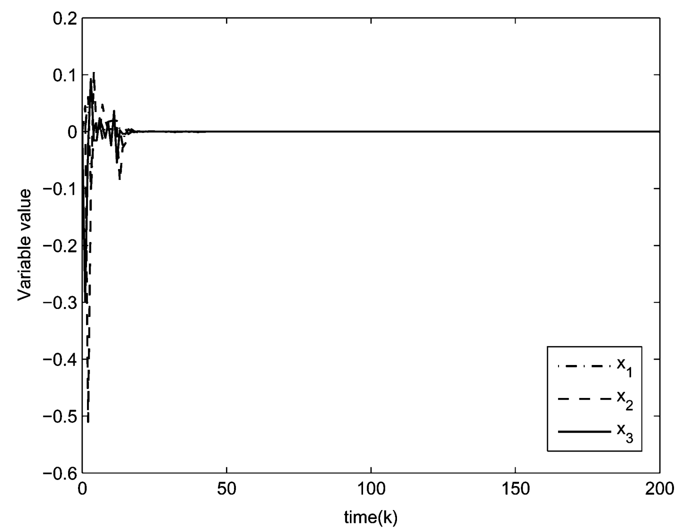

Fig. 7. Closed-loop state variables by taking the Theorem 2.

In such a case, if we use the *standard* H_∞ performance criterion described in (19) and (20), the controller gain matrices can be obtained as follows:

$$K_1 = \begin{bmatrix} -0.3231 & 0.0460 & -0.7181 \\ -0.0076 & -0.1092 & 0.6953 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -0.3231 & 0.0597 & -0.2481 \\ 0.0129 & -0.0018 & 0.2066 \end{bmatrix}$$

and the evolution of the corresponding state variables is given in Fig. 6. If we use Theorem 2, which accounts for the packet losses, the controller gain matrices are obtained as

$$K_1 = \begin{bmatrix} -1.7265 & 0.0129 & -1.7443 \\ 0.3543 & -0.1478 & 1.5592 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -0.8813 & 0.4437 & -0.1694 \\ 0.1682 & -0.1474 & 0.9820 \end{bmatrix}$$

with the evolution of the state variables depicted in Fig. 7.

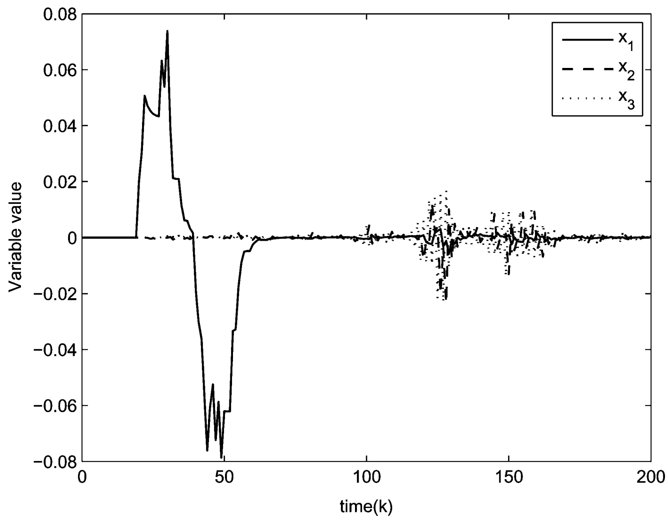

Fig. 8. Closed-loop state variables by taking the standard H_∞ performance criterion when $w(k) \neq 0$.

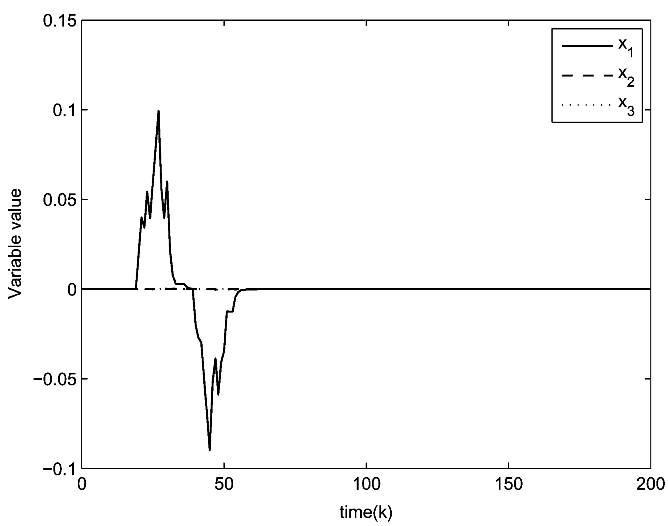

Fig. 9. Closed-loop state variables by taking the Theorem 2 when $w(k) \neq 0$.

Furthermore, to illustrate the disturbance attenuation performance, we set the initial condition $x_0 \equiv 0$ and the external disturbance w_k be the same as that in (37). Again, we apply the standard H_∞ performance criterion as described in (19) and (20) and Theorem 2, and display the evolutions of the state variables under these two situations in Figs. 8 and 9, respectively. The disturbance attenuation performance indexes can also be calculated as $\gamma = 0.4670$ and $\gamma = 0.4445$, respectively. By comparing Fig. 6 with Fig. 7 and noticing the H_∞ performance indexes, we can conclude that the controller design by Theorem 2 has given better dynamical behavior as well as a better disturbance rejection attenuation level, which confirms our theoretical analysis for the problem of H_∞ fuzzy control for systems with repeated scalar nonlinearities and random packet losses.

VI. Conclusion

In this paper, we have investigated the H_∞ fuzzy control problem for systems with repeated scalar nonlinearities and random packet losses. The nonlinear system is described by a discrete-time state equation containing a repeated scalar nonlinearity and the control strategy takes the form of parallel distributed compensation. The missing measurements are modeled by a stochastic variable satisfying the Bernoulli random binary distribution. The quadratic Lyapunov function has been used to design H_∞ fuzzy controllers such that, for the admissible random measurement missing and repeated scalar nonlinearities, the closed-loop T–S fuzzy control system is stochastically stable and preserves a guaranteed H_∞ performance. By using the CCL algorithm, sufficient conditions have been established that ensure the stochastic stability of the closed-loop system, and the controller gains have been obtained by the solution of a set of LMIs. Two illustrative simulation examples have been given to illustrate the effectiveness of the proposed design method.

We would like to point out that our main results can be extended to more general/practical systems such as Itô-type stochastic systems, Markovian jumping systems and time-delay systems, and the corresponding results will appear in the near future.

References

[1] W. Assawinchaichote, S. K. Nguang, P. Shi, and E. Boukas, "H_∞ fuzzy state-feedback control design for nonlinear systems with D-stability constraints: An LMI approach," *Math. Comput. Simul.*, vol. 78, pp. 514–531, 2008.

[2] Y. Y. Cao and P. M. Frank, "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 2, pp. 200–211, Apr. 2000.

[3] Y. Cao and P. M. Frank, "Stability analysis and synthesis of nonlinear time-delay systems via linear Takagi–Sugeno fuzzy models," *Fuzzy Sets Syst.*, vol. 124, pp. 213–229, 2001.

[4] B. Chen, X. Liu, and S. Tong, "Robust fuzzy control of nonlinear systems with input delay," *Chaos Solitons Fractals*, vol. 37, pp. 894–901, 2006.

[5] Y. Chu, "Further results for systems with repeated scalar nonlinearities," *IEEE Trans. Autom. Control*, vol. 44, no. 12, pp. 2031–2035, Dec. 2001.

[6] Y. Chu and K. Glover, "Bounds of the induced norm and model reduction errors for systems with repeated scalar nonlinearities," *IEEE Trans. Autom. Control*, vol. 44, no. 3, pp. 471–483, Mar. 1999.

[7] Y. Chu and K. Glover, "Stabilization and performance synthesis for systems with repeated scalar nonlinearities," *IEEE Trans. Autom. Control*, vol. 44, no. 3, pp. 484–496, Mar. 1999.

[8] G. Feng, "Controller synthesis of fuzzy dynamic systems based on piecewise Lyapunov function," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 5, pp. 605–612, Oct. 2003.

[9] G. Feng and J. Ma, "Quadratic stabilization of uncertain discrete-time fuzzy dynamic systems," *IEEE Trans. Circuits Syst.-I: Fundam. Theory Appl.*, vol. 48, no. 11, pp. 1337–1344, Nov. 2001.

[10] G. Feng, "Stability analysis of discrete-time fuzzy dynamic systems based on piecewise Lyapunov functions," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 1, pp. 22–28, Feb. 2004.

[11] H. Gao, J. Lam, and C. Wang, "Induced l_2 and generalized H_2 filtering for systems with repeated scalar nonlinearities," *IEEE Trans. Signal Process.*, vol. 53, no. 11, pp. 4215–4226, Nov. 2005.

[12] H. Gao, Z. Wang, and C. Wang, "Improved H_∞ control of discrete-time fuzzy systems: A cone complementarity linearization approach," *Inf. Sci.*, vol. 175, no. 1–2, pp. 57–77, 2005.

[13] H. Gao, Y. Zhao, J. Lam, and K. Chen, "H_∞ fuzzy filtering of nonlinear systems with intermittent measurements," *IEEE Trans. Fuzzy Syst.*, to be published.

[14] L. El, Ghaoui, F. Oustry, and M. A. Rami, "A cone complementarity linearization algorithm for static output-feedback and related problems," *IEEE Trans. Autom. Control*, vol. 42, no. 8, pp. 1171–1176, Aug. 1997.

[15] Y.-Y. Hou, T.-L. Liao, and J.-J. Yan, “Stability analysis of Takagi–Sugeno fuzzy cellular neural networks with time-varying delays,” *IEEE Trans. Syst., Man, Cybern.: B, Cybern*, vol. 37, no. 3, pp. 720–726, Jun. 2007.

[16] H. Huang, D. W. C. Ho, and J. Lam, “Stochastic stability analysis of fuzzy hopfield neural networks with time-varying delays,” *IEEE Trans. Circuits Syst. II: Exp. Briefs*, vol. 52, no. 5, pp. 251–255, May 2005.

[17] O. C. Imer, S. Yuksel, and T. Basar, “Optimal control of LTI systems over unreliable communication links,” *Automatica*, vol. 42, no. 9, pp. 1429–1439, 2006.

[18] B. Jiang, M. Staroswiecki, and V. Cocquempot, “Fault accommodation for nonlinear dynamic systems,” *IEEE Trans. Autom. Control*, vol. 51, no. 9, pp. 1578–1583, Sep. 2006.

[19] X. Liu, “Delay-dependent H_∞ control for uncertain fuzzy systems with time-varying delays,” *J. Comput. Appl. Math.*, vol. 68, no. 5, pp. 1352–1361, 2008.

[20] Z. H. Mao and B. Jiang, “Fault identification and fault-tolerant control for a class of networked control systems,” *Int. J. Innovative Comput., Inf. Control*, vol. 3, no. 5, pp. 1121–1130, 2007.

[21] S. K. Nguang and P. Shi, “Fuzzy H_∞ output feedback control of nonlinear systems under sampled measurements,” *Automatica*, vol. 39, pp. 2169–2174, 2003.

[22] S. K. Nguang and P. Shi, “H_∞ fuzzy output feedback control design for nonlinear systems: An LMI approach,” *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 3, pp. 331–340, Jun. 2003.

[23] B. Shen, Z. Wang, H. Shu, and G. Wei, “On nonlinear H_∞ filtering for discrete-time stochastic systems with missing measurements,” *IEEE Trans. Autom. Control*, vol. 53, no. 9, pp. 2170–2180, Oct. 2008.

[24] P. Shi and S. K. Nguang, “H_∞ output feedback control of fuzzy system models under sampled measurements,” *Comput. Math. Appl.*, vol. 46, no. 5–6, pp. 705–717, 2003.

[25] K. Tanaka, T. Hori, and H. O. Wang, “A multiple Lyapunov function approach to stabilization of fuzzy control systems,” *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 4, pp. 582–589, Aug. 2003.

[26] Z. Wang, D. W. C. Ho, and X. Liu, “Variance-constrained filtering for uncertain stochastic systems with missing measurements,” *IEEE Trans. Autom. Control*, vol. 48, no. 7, pp. 1254–1258, Jul. 2003.

[27] Z. Wang, D. W. C. Ho, and X. Liu, “Variance-constrained control for uncertain stochastic systems with missing measurements,” *IEEE Trans. Syst., Man Cybern. A, Syst., Humans*, vol. 35, no. 5, pp. 746–753, Sep. 2005.

[28] Z. Wang, F. Yang, D. W. C. Ho, and X. Liu, “Robust H_∞ control for networked systems with random packet losses,” *IEEE Trans. Syst., Man Cybern. B, Cybern.*, vol. 37, no. 4, pp. 916–924, Aug. 2007.

[29] H. Wu, “Delay-dependent H_∞ fuzzy observer-based control for discrete-time nonlinear systems with state delay,” *Fuzzy Sets Syst*, vol. 159, pp. 2696–2712, 2008.

[30] K. Yuan, J. Cao, and J. Deng, “Exponential stability and periodic solutions of fuzzy cellular neural networks with time-varying delays,” *Neurocomputing*, vol. 69, no. 13–15, pp. 1619–1627, 2006.

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