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# Dual-Hop Cognitive Amplify-and-Forward Relaying Networks over $\eta-\mu$ Fading Channels 

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#### Abstract

This paper presents a thorough performance analysis of dual-hop cognitive amplify-and-forward (AF) relaying networks under spectrum-sharing mechanism over independent non-identically distributed (i.n.i.d.) $\eta-\mu$ fading channels. In order to guarantee the quality-of-service ( QoS ) of primary networks, both maximum tolerable peak interference power $\mathcal{Q}$ at the primary users (PUs) and maximum allowable transmit power $\mathcal{P}$ at secondary users (SUs) are considered to constrain transmit power at the cognitive transmitters. For integer-valued fading parameters, a closed-form lower bound for the outage probability (OP) of the considered networks is obtained. Moreover, assuming arbitrary-valued fading parameters, the lower bound in integral form for the OP is derived. In order to obtain further insights on the OP performance, asymptotic expressions for the OP at high SNRs are derived, from which the diversity/coding gains and the diversity-multiplexing gain tradeoff (DMT) of the secondary network can be readily deduced. It is shown that the diversity gain and also the DMT are solely determined by the fading parameters of the secondary network whereas the primary network only affects the coding gain. The derived results include several others available in previously published works as special cases, such as those for Nakagami- $m$ fading channels. In addition, performance evaluation results have been obtained by Monte Carlo computer simulations which have verified the accuracy of the theoretical analysis.


Index Terms-Outage probability (OP), amplify-and-forward (AF), cognitive relay network (CRN), $\eta-\mu$ fading, spectrum sharing.

## I. Introduction

Cognitive radio with spectrum sharing is regarded as a promising technique to improve spectral efficiency and solve the problem of spectrum scarcity in 5 G wireless communication networks [1]. In underlay cognitive networks, secondary

[^0]users (SUs) can simultaneously access the licensed spectrum of the primary user (PU) without causing harmful interference on PU. Thus, to ensure PU's quality of service (QoS), the transmit power constraint at SUs must be considered [2], [3]. Cooperative relaying can expand the coverage area and enhance the communication range of wireless networks without requiring additional powers at the transmitter [4]. To further improve the performance of cognitive networks, cooperative relaying has been incorporated into cognitive networks to form cognitive relay networks (CRNs).
Further research efforts have been focused on the performance analysis of CRNs employing amplify-and-forward (AF) and decode-and-forward (DF) protocols, including [5]-[11]. For example, the authors in [5], [6] have investigated the outage performance of AF CRNs with maximum interference power constraint of the primary network. In [7], tight lower bounds and asymptotic expressions for the outage probability (OP) of AF CRNs have been derived, where both interference and maximum allowable transmit power constraints are considered. In [9], an exact expression for the OP of a dual-hop DF CRN has been obtained, assuming that the transmit powers at SUs are governed by both the primary network and the secondary network. Bao et al. proposed cognitive multi-hop DF networks and analyzed the system performance with the interference limits in [10]. In [11], closed-form and asymptotic OP expressions are presented where the mutual interference between cognitive system and primary system is taken into account. Most recently, CRNs incorporating multiuser diversity and multiple-input multiple-output technologies with both AF and DF relaying have been considered in [12]. The authors in [13] considered a cognitive cooperative DF relaying network and investigated adaptive transmit power-allocation policies for the SUs. Recently, Zhang et al. considered an underlay cognitive DF relay network including one primary user receiver ( PU ) and a secondary system and investigated the performance of the considered network in [14].
In all the aforementioned works, small-scale fading is modelled by the Rayleigh [5], [8]-[10], [12], [13] or the Nakagami$m$ [6], [7], [11], [14] fading distributions. The Nakagami- $m$ distribution is a mathematically tractable model that well characterizes the wireless propagation channel in many practical cases. However, as it was pointed out in [15], this distribution cannot match well experimental data in many practical cases, particularly at the tail portion. A versatile fading distribution which generalizes many of the well known models for multipath fading is the so-called $\eta-\mu$ distribution [15]. This distribution better models small-scale fading under non-line-
of-sight (NLoS) conditions and includes the Rayleigh, the Hoyt and the Nakagami- $m$ distributions as special cases. In recent years, the $\eta-\mu$ fading model has gained increased interest in the field of performance analysis of single- and multi-hop systems over fading environment [16]-[19]. For example, the authors in [16] analyzed the error performance by using moment generating function (MGF) over $\eta-\mu$ fading channels, without investigating outage performance. Later, Peppas et al. analyzed the conventional dual-hop relaying network over mixed $\eta-\mu$ and $\kappa-\mu$ fading channels in [17]. More recently, a hexagonal cell layout was considered with the base stations located at the center of each cell in [20] and analytical evaluation of OP and capacity for $\kappa-\mu$ and $\eta-\mu$ fading channel distributions has been investigated. In [21], an analytical study of selection combining diversity under $\eta-\mu$ multi-path fading with integer-valued $\mu$ was presented.

Despite the wide applicability of the $\eta-\mu$ distribution, to the best of the authors' knowledge, the performance of cognitive AF relay networks in $\eta-\mu$ fading environment is still unexplored in the open technical literature. Motivated by the lack of such a generalized analytical framework, in this paper, a thorough OP analysis for dual-hop cognitive AF relay networks operating over independent non-identically distributed (i.n.i.d.) $\eta-\mu$ fading channels, is presented. In order to ensure the QoS of PU , both maximum tolerable peak interference at PU and maximum allowable transmit power at SUs are considered to constrain the transmit power at secondary source and relay. The main contributions of this paper are summarized as follows:

- For integer-valued fading parameters, a closed-form lower bound for the OP is presented which becomes tight for high signal-to-noise ratios (SNRs).
- For arbitrary-valued fading parameters, a generic frequency-domain approach is developed for the evaluation of the OP, in which the corresponding integral is transformed into the frequency domain. Moreover, such a transformation requires both the knowledge of the MGFs of the random variables and the incomplete MGF involved in the computation of the OP.
- In order to provide further insights as to the factors that affect system performance, simple asymptotic expressions for the OP are derived from which the diversity and coding gains as well as the diversity-multiplexing gain tradeoff (DMT) can be readily deduced.

The derived results include several others available in the open technical literature as special cases, namely those of Nakagami- $m$ cases.

The remainder of this paper is organized as follows. In Section II, the considered system model is provided. In Section III, tight lower bounds as well as high-SNRs asymptotic expressions for the OP are derived. In Section IV, the various performance evaluation results and their interpretations are presented. Finally, Section V concludes this paper. For the convenience of the reader, a comprehensive list of the mathematical operators and functions often used in this paper is presented in Table I.

## II. System Model

Consider a dual-hop cognitive AF relay network including one SU source (SU-S), one AF SU relay (SU-R), one SU destination (SU-D), and one PU destination (PU-D). All nodes are equipped with single antenna and operate in half-duplex mode. The communication from the SU-S to the SU-D is performed into two time slots. During the first time slot, the SU-S transmits signal $x$ to the SU-R with transmit power $P_{S}$. The received signal at the $S U-R$ is given by

$$
\begin{equation*}
y_{r}=g_{1} \sqrt{P_{S}} x+n_{r} \tag{1}
\end{equation*}
$$

where $g_{1}$ is the channel coefficient of the SU-S $\rightarrow$ SU-R link and $n_{r}$ is additive white Gaussian noise (AWGN) at the SU-R. During the second time slot, the received signal $y_{r}$ is amplified by $G$ and then forwarded to the SU-D with transmit power $P_{R}$. The received signal at the SU-D can be expressed as

$$
\begin{equation*}
y_{d}=G g_{1} g_{2} \sqrt{P_{S} P_{R}} x+G g_{2} \sqrt{P_{R}} n_{r}+n_{d} \tag{2}
\end{equation*}
$$

where $g_{2}$ is the channel coefficient of the link the SU-R $\rightarrow$ the SU-D and $n_{d}$ is AWGN at the SU-D. It is assumed that all AWGN components have zero mean and variance $N_{0}$.

In order to ensure the QoS provision at PU, i.e., total accumulated interference at PU cannot exceed the maximum tolerable interference power $\mathcal{Q}$, and considering the maximum transmit power at SU-S and SU-R as $\mathcal{P}$, the transmit powers at SU-S and SU-R are strictly governed by

$$
P_{S}=\min \left(\mathcal{Q} /\left|h_{1}\right|^{2}, \mathcal{P}\right), \text { and } P_{R}=\min \left(\mathcal{Q} /\left|h_{2}\right|^{2}, \mathcal{P}\right)
$$

respectively, where $h_{1}$ and $h_{2}$ are the channel coefficients of the interference link SU-S $\rightarrow$ PU-D and SU-R $\rightarrow$ PU-D. Consequently, the end-to-end instantaneous SNR at SU-D can be deduced as [7]

$$
\begin{equation*}
\gamma_{\mathrm{end}}=\frac{\gamma_{1} \gamma_{2}}{\gamma_{1}+\gamma_{2}+1} \tag{3}
\end{equation*}
$$

where

$$
\begin{align*}
& \gamma_{1}=\min \left(\bar{\gamma}_{\mathcal{Q}} \frac{\left|g_{1}\right|^{2}}{\left|h_{1}\right|^{2}}, \bar{\gamma}_{\mathcal{P}}\left|g_{1}\right|^{2}\right) \\
& \gamma_{2}=\min \left(\bar{\gamma}_{\mathcal{Q}} \frac{\left|g_{2}\right|^{2}}{\left|h_{2}\right|^{2}}, \bar{\gamma}_{\mathcal{P}}\left|g_{2}\right|^{2}\right) \tag{4}
\end{align*}
$$

with $\bar{\gamma}_{\mathcal{Q}}=\mathcal{Q} / N_{0}$ and $\bar{\gamma}_{\mathcal{P}}=\mathcal{P} / N_{0}$.
Throughout this analysis, it is assumed that all links are subject to i.n.i.d. $\eta-\mu$ fading. Thus, $\left|g_{\ell}\right|^{2}$ and $\left|h_{\ell}\right|^{2}$ follow the $\eta-\mu$ distribution with parameters $\mu_{g_{\ell}}, \eta_{g_{\ell}}$, and $\mu_{h_{\ell}}$, $\eta_{h_{\ell}}$, respectively, where $\ell \in\{1,2\}$. Let $\mathbb{E}\left\{\left|g_{1}\right|^{2}\right\}=\Omega_{1}$, $\mathbb{E}\left\{\left|g_{2}\right|^{2}\right\}=\Omega_{2}, \mathbb{E}\left\{\left|h_{1}\right|^{2}\right\}=\Omega_{3}$ and $\mathbb{E}\left\{\left|h_{2}\right|^{2}\right\}=\Omega_{4}$.

The PDF of $X$, where $X \in\left\{\left|g_{\ell}\right|^{2},\left|h_{\ell}\right|^{2}\right\}$, can be expressed as [15]

$$
\begin{align*}
f_{X}(x)= & \frac{2 \sqrt{\pi} \mu^{\mu+0.5} h^{\mu} x^{\mu-0.5}}{\Gamma(\mu) H^{\mu-0.5} \bar{X}^{\mu+0.5}} \\
& \quad \times \exp \left(-\frac{2 \mu h x}{\bar{X}}\right) I_{\mu-0.5}\left(\frac{2 \mu H x}{\bar{X}}\right), \tag{5}
\end{align*}
$$

where $\bar{X}=\mathbb{E}(X), \mu>0, \mu \in\left\{\mu_{g_{\ell}}, \mu_{h_{\ell}}\right\}$ and $\eta \in\left\{\eta_{g_{\ell}}, \eta_{h_{\ell}}\right\}$.
The parameters $h$ and $H$ are given by $h=\left(2+\eta^{-1}+\right.$ $\eta) / 4, H=\left(\eta^{-1}-\eta\right) / 4$ with $0<\eta<\infty$, with $h \in\left\{h_{g_{\ell}}, h_{h_{\ell}}\right\}$

TABLE I: Mathematical Operators and Functions

| $\imath=\sqrt{-1}$ | imaginary unit |
| :---: | :--- |
| $\bar{z}$ | conjugate of the complex number $z$ |
| $\mathcal{F}\{g(t) ; t ; \omega\}$ | Fourier transform of the function $g(t)$ |
| $\mathbb{E}\{\cdot\}$ | expectation operator |
| $\operatorname{Pr}\{\cdot\}$ | probability operator |
| $f_{X}(\cdot)$ | probability density function (PDF) of the random variable (RV) $X$ |
| $F_{X}(\cdot)$ | cumulative distribution function (CDF) of RV $X$ |
| $\mathcal{M}_{X}(\cdot)$ | moment generating function (MGF) of RV $X$ |
| $\mathcal{M}_{X}(t, s)$ | incomplete moment generating function (MGF) of RV $X$ |
| $I_{a}(\cdot)$ | modified Bessel function of the first kind and order $a$ [22, eq. (8.431)] |
| $\Gamma(\cdot)$ | Gamma function [22, eq. (8.310.1)] |
| $\Gamma(\cdot, \cdot)$ | incomplete Gamma function [22, eq. (8.350.2)] |
| ${ }_{p} F_{q}(\cdot)$ | generalized hypergeometric function [22, eq. (9.14.1)] |
| $\delta(\cdot)$ | Dirac's delta function |

and $H \in\left\{H_{g_{\ell}}, H_{h_{\ell}}\right\}$ [15]. Assuming integer values of $\mu$, the CDF of $X$ can be obtained as follows [17],

$$
\begin{align*}
F_{X}(x)=1-\frac{1}{\Gamma(\mu)}\left(\frac{h}{H}\right)^{\mu} & \left\{\sum _ { k = 0 } ^ { \mu - 1 } \sum _ { p = 0 } ^ { \mu - k - 1 } \frac { 1 } { p ! } \left[A^{p} x^{p} a^{(k)} \exp (-A x)\right.\right. \\
+ & \left.\left.(-1)^{\mu} B^{p} x^{p} b^{(k)} \exp (-B x)\right]\right\} \tag{6}
\end{align*}
$$

where

$$
\begin{aligned}
A & =\frac{2 \mu(h-H)}{\bar{X}}, \quad B=\frac{2 \mu(h+H)}{\bar{X}} \\
a^{(k)} & =\frac{(-1)^{k}(\mu+k-1)!H^{-k}}{2^{\mu+k} k!(h-H)^{\mu-k}}, b^{(k)}=\frac{(\mu+k-1)!H^{-k}}{2^{\mu+k} k!(h+H)^{\mu-k}},
\end{aligned}
$$

and $a^{(k)} \in\left\{a_{g_{\ell}}^{(k)}, a_{h_{\ell}}^{(k)}\right\}, b^{(k)} \in\left\{b_{g_{\ell}}^{(k)}, b_{h_{\ell}}^{(k)}\right\}, A \in\left\{A_{g_{\ell}}, A_{h_{\ell}}\right\}$, $B \in\left\{B_{g_{\ell}}, B_{h_{\ell}}\right\}$.

For arbitrary values of $\mu$, the CDF of $X$ can be expressed as

$$
\begin{equation*}
F_{X}(x)=1-Y_{\mu}\left(\frac{H}{h}, \sqrt{\frac{2 h \mu x}{\bar{X}}}\right) \tag{7}
\end{equation*}
$$

where
$Y_{\mu}(x, y)=\frac{\sqrt{\pi} 2^{1.5-\mu}\left(1-x^{2}\right)^{\mu}}{\Gamma(\mu) x^{\mu-0.5}} \int_{y}^{\infty} e^{-t^{2}} t^{2 \mu} I_{\mu-0.5}\left(t^{2} x\right) \mathrm{d} t$
denotes the Yacoub integral [15, eq. (20)]. It is noted that $Y_{\mu}(x, y)$ can be expressed in terms of tabulated functions for integer or half-integer values of $\mu$ only. For arbitrary values of $\mu$, an expression of $Y_{\mu}(x, y)$ in terms of the bivariate confluent hypergeometric functions is available in [23, eq. (2)].

The MGF of $X$ can be deduced in closed form as [24, eq. (3)]

$$
\begin{equation*}
\mathcal{M}(s)=[(1+s / A)(1+s / B)]^{-\mu} \tag{9}
\end{equation*}
$$

Finally, by employing an infinite series representation for the modified Bessel function, [22, eq. (8.447)] as well as the definition of the incomplete gamma function [22, eq.
(8.350.2)], the incomplete MGF of $X$, defined as $\mathcal{M}(x, s) \triangleq$ $\int_{t}^{\infty} \exp (-s x) f_{X}(x) \mathrm{d} x$, can be deduced as

$$
\begin{align*}
& \mathcal{M}(x, s)=\frac{2 \sqrt{\pi} h^{\mu}}{\Gamma(\mu)} \\
& \times \sum_{k=0}^{\infty} \frac{H^{2 k}(\mu / \bar{X})^{2 \mu+2 k} \Gamma(2 \mu+2 k, 2 \mu h t / \bar{X})+s t}{k!\Gamma(\mu+k+1 / 2)(2 \mu h t / \bar{X}+s)^{2 \mu+2 k}} . \tag{10}
\end{align*}
$$

## III. Outage Performance Analysis

In this section, the OP of cognitive AF relaying system over i.n.i.d. $\eta-\mu$ fading will be obtained. The OP, i.e., $P_{\text {out }}\left(\gamma_{\mathrm{th}}\right)$, is defined as the probability that the instantaneous SNR $\gamma_{\text {end }}$ at SU-D is below a specified SNR threshold $\gamma_{\mathrm{th}}$, i.e., $P_{\text {out }}\left(\gamma_{\mathrm{th}}\right)=$ $\operatorname{Pr}\left\{\gamma_{\text {end }} \leqslant \gamma_{\text {th }}\right\}$.

## A. The Lower Bound Analysis for $O P$

Theorem 1. For integer values of $\mu_{g_{\ell}}$ and $\mu_{h_{\ell}}, \forall \ell=\{1,2\}$, a tight lower bound for OP is given as (11) on the top of the next page, where $F_{\left|g_{1}\right|^{2}}\left(\frac{\gamma}{\bar{\gamma}_{\mathcal{P}}}\right)$ is given by (6) and

$$
\begin{aligned}
& \times \sum_{k_{1}=0}^{\mu_{g_{1}}-1} \sum_{p_{1}=0} \sum_{k_{2}=0}^{\mu_{g_{1}}-k_{1}-1} \sum_{p_{2}=0}^{\mu_{h_{1}}-1} \frac{1}{p_{1}!p_{2}!}\left(\frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)^{p_{2}}, \\
& \widehat{\substack{k_{1}, p_{1} \\
k_{2}, p_{2}}} \left\lvert\,=\frac{1}{\Gamma\left(\mu_{g_{2}}\right) \Gamma\left(\mu_{h_{2}}\right)}\left(\frac{h_{g_{2}}}{H_{g_{2}}}\right)^{\mu_{g_{2}}}\left(\frac{h_{h_{2}}}{H_{h_{2}}}\right)^{\mu_{h_{2}}}\right. \\
& \times \sum_{k_{1}=0}^{\mu_{g_{2}}-1} \sum_{p_{1}=0} \sum_{k_{2}=0}^{\mu_{g_{2}}-k_{1}-1} \sum_{p_{2}=0}^{\mu_{h_{2}}-1} \frac{1}{p_{1}!p_{2}!}\left(\frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)^{p_{2}} .
\end{aligned}
$$

Proof: See Appendix A.
It is noted that, for the special case of Nakagami- $m$ fading channels, i.e., $\mu_{g_{l}}=m_{g_{l}}, \mu_{h_{l}}=m_{h_{l}}$ and $\eta_{g_{l}}=\eta_{h_{l}}=$

$$
\begin{align*}
& P_{\text {out }}\left(\gamma_{\mathrm{th}}\right) \geq 1-\left\{1-F_{\left|g_{1}\right|^{2}}\left(\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}\right)-\widetilde{\sum_{\substack{k_{1}, p_{1} \\
k_{2}, p_{2}}}\left(\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}\right)^{p_{1}}\left[a_{g_{1}}^{\left(k_{1}\right)} A_{g_{1}}^{m_{1}}\left(a_{h_{1}}^{\left(k_{2}\right)} A_{h_{1}}^{p_{2}} \exp \left[-\left(A_{g_{1}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+A_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)\right] .\right] .\right] .}\right. \\
& \left.+(-1)^{\mu_{h_{1}}} b_{h_{1}}^{\left(k_{2}\right)} B_{h_{1}}^{p_{2}} \exp \left[-\left(A_{g_{1}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+B_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)\right]\right)+(-1)^{\mu_{g_{1}}} b_{g_{1}}^{\left(k_{1}\right)} B_{g_{1}}^{p_{1}} \\
& \left.\times\left(a_{h_{1}}^{\left(k_{2}\right)} A_{h_{1}}^{p_{2}} \exp \left[-\left(B_{g_{1}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+A_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)\right]+(-1)^{\mu_{h_{1}}} b_{h_{1}}^{\left(k_{2}\right)} B_{h_{1}}^{p_{2}} \exp \left[-\left(B_{g_{1}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+B_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)\right]\right)\right] \\
& +\widetilde{\sum_{k, p, q}}\left(\frac{\gamma_{\text {th }}}{\bar{\gamma}_{\mathcal{Q}}}\right)^{p} \frac{a_{g_{1}}^{(k)} A_{g_{1}}^{p}}{\left(\mu_{h_{1}}+p-q-1\right)!}\left[\frac{(-1)^{q} \Gamma\left(\mu_{h_{1}}+p-q, A_{g_{1}} \frac{\gamma_{\text {th }}}{\bar{\gamma}_{\mathcal{P}}}+A_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)}{\left(A_{g_{1}} \frac{\gamma_{\mathrm{\gamma}_{\mathrm{Q}}}}{\bar{\gamma}_{\mathcal{Q}}}+A_{h_{1}}\right)^{\mu_{h_{1}}+p-q}}\right. \\
& \left.+\frac{(-1)^{\mu_{h_{1}}} \Gamma\left(\mu_{h_{1}}+p-q, A_{g_{1}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+B_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)}{\left(A_{g_{1}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{Q}}}+B_{h_{1}}\right)^{\mu_{h_{1}}+p-q}}\right]+(-1)^{\mu_{g_{1}}} b_{g_{1}}^{(k)} B_{g_{1}}^{p} \\
& \left.\times\left[\frac{(-1)^{q} \Gamma\left(\mu_{h_{1}}+p-q, B_{g_{1}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+A_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)}{\left(B_{g_{1}}{\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{Q}}}}+A_{h_{1}}\right)^{\mu_{h_{1}}+p-q}}+\frac{(-1)^{\mu_{h_{1}} \Gamma\left(\mu_{h_{1}}+p-q, B_{g_{1}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+B_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)}}{\left(B_{g_{1}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{Q}}}+B_{h_{1}}\right)^{\mu_{h_{1}}+p-q}}\right]\right\} \\
& \times\left\{1-F_{\left|g_{2}\right|^{2}}\left(\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}\right)-\widehat{\sum_{\substack{k_{1}, p_{1} \\
k_{2}, p_{2}}}\left(\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}\right)^{p_{1}}\left[a _ { g _ { 2 } } ^ { ( k _ { 1 } ) } A _ { g _ { 2 } } ^ { p _ { 1 } } \left(a_{h_{2}}^{\left(k_{2}\right)} A_{h_{2}}^{p_{2}} \exp \left[-\left(A_{g_{2}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+A_{h_{2}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)\right]\right.\right.}\right. \\
& \left.+(-1)^{\mu_{h_{2}}} b_{h_{2}}^{\left(k_{2}\right)} B_{h_{2}}^{p_{2}} \exp \left[-\left(A_{g_{2}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+B_{h_{2}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)\right]\right)+(-1)^{\mu_{g_{2}}} b_{g_{2}}^{\left(k_{1}\right)} B_{g_{2}}^{p_{1}} \\
& \left.\times\left(a_{h_{2}}^{\left(k_{2}\right)} A_{h_{2}}^{p_{2}} \exp \left[-\left(B_{g_{2}} \frac{\gamma_{t h}}{\bar{\gamma}_{\mathcal{P}}}+A_{h_{2}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)\right]+(-1)^{\mu_{h_{2}}} b_{h_{2}}^{\left(k_{2}\right)} B_{h_{2}}^{p_{2}} \exp \left[-\left(B_{g_{2}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+B_{h_{2}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)\right]\right)\right] \\
& +\widehat{\sum_{k, p, q}}\left(\frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{Q}}}\right)^{p} \frac{a_{g_{2}}^{(k)} A_{g_{2}}^{p}}{\left(\mu_{h_{2}}+p-q-1\right)!}\left[\frac{(-1)^{q} \Gamma\left(\mu_{h_{2}}+p-q, A_{g_{2}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+A_{h_{2}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)}{\left(A_{g_{2}} \frac{\gamma_{\mathrm{\gamma}_{\mathrm{Q}}}}{\bar{\gamma}_{\mathcal{Q}}}+A_{h_{2}}\right)^{\mu_{h_{2}}+p-q}}\right. \\
& \left.+\frac{(-1)^{\mu_{h_{2}}} \Gamma\left(\mu_{h_{2}}+p-q, A_{g_{2}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+B_{h_{2}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)}{\left(A_{g_{2}} \frac{\gamma}{\mathrm{th}}^{\bar{\gamma}_{\mathcal{Q}}}+B_{h_{2}}\right)^{\mu_{h_{2}}+p-q}}\right]+(-1)^{\mu_{g_{2}}} b_{g_{2}}^{(k)} B_{g_{2}}^{p} \\
& \left.\times\left[\frac{(-1)^{q} \Gamma\left(\mu_{h_{2}}+p-q, B_{g_{2}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+A_{h_{2}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)}{\left(B_{g_{2}} \frac{\gamma_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{Q}}}+A_{h_{2}}\right)^{\mu_{h_{2}}+p-q}}+\frac{(-1)^{\mu_{h_{2}} \Gamma\left(\mu_{h_{2}}+p-q, B_{g_{2}} \frac{\bar{\gamma}_{\mathrm{th}}}{\bar{\gamma}_{\mathcal{P}}}+B_{h_{2}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)}}{\left(B_{g_{2}} \bar{\gamma}_{\bar{\gamma}_{\mathcal{Q}}}+B_{h_{2}}\right)^{\mu_{h_{2}}+p-q}}\right]\right\} . \tag{11}
\end{align*}
$$

$\eta \rightarrow 0(l=1,2)$ [15], where $m_{g_{l}}$ and $m_{h_{l}}$ denote Nakagami fading parameters, (11) becomes identical to a previously known result, i.e. [7, eq. (13)]. This can be easily proved as follows. By letting $\mu_{g_{l}}=m_{g_{l}}, \mu_{h_{l}}=m_{h_{l}}$ and $\eta \rightarrow 0$, we have $h-H \rightarrow 1 / 2, h+H \rightarrow \infty$ and $h / H \rightarrow 1$; hence, $A \rightarrow m / \bar{X}$ and $B \rightarrow \infty$. Utilizing these results and after some mathematical manipulations yield [7, eq. (13)]. For arbitrary values of $\mu_{g_{\ell}}$ and $\mu_{h_{\ell}}$, a lower bound for OP can be obtained as follows:

Theorem 2. For arbitrary values of $\mu_{g_{\ell}}$ and $\mu_{h_{\ell}}, \forall \ell=\{1,2\}$,
a lower bound for OP can be deduced as

$$
\begin{align*}
P_{\mathrm{out}}\left(\gamma_{\mathrm{th}}\right) \geq & \sum_{\ell=1}^{2}\left[Y_{\mu_{g_{\ell}}}\left(\frac{H_{g_{\ell}}}{h_{g_{\ell}}}, \sqrt{\frac{2 h_{g_{\ell}} \mu_{g_{\ell}} \gamma_{\mathrm{th}}}{\bar{X} \bar{\gamma}_{\mathcal{P}}}}\right)\right. \\
& \left.\times Y_{\mu_{h_{\ell}}}\left(\frac{H_{h_{\ell}}}{h_{h_{\ell}}}, \sqrt{\frac{2 h_{h_{\ell}} \mu_{h_{\ell}} \bar{\gamma}_{\mathcal{Q}}}{\bar{X} \bar{\gamma}_{\mathcal{P}}}}\right)\right] \\
+ & \sum_{\ell=1}^{2}\left[\mathcal{J}\left(\mu_{g_{\ell}}, \mu_{h_{\ell}}, \eta_{g_{\ell}}, \eta_{h_{\ell}}, \gamma_{\mathrm{th}}, \bar{\gamma}_{\mathcal{P}}, \bar{\gamma}_{\mathcal{Q}}\right)\right] \tag{12}
\end{align*}
$$

where $\mathcal{J}\left(\mu_{g_{\ell}}, \mu_{h_{\ell}}, \eta_{g_{\ell}}, \eta_{h_{\ell}}, \gamma_{\mathrm{th}}, \bar{\gamma}_{\mathcal{P}}, \bar{\gamma}_{\mathcal{Q}}\right)$ is given as

$$
\begin{align*}
& \mathcal{J}\left(\mu_{g_{\ell}}, \mu_{h_{\ell}}, \eta_{g_{\ell}}, \eta_{h_{\ell}}, \gamma_{\mathrm{th}}, \bar{\gamma}_{\mathcal{Q}}\right)=\frac{1}{2}+\frac{F_{\left|g_{\ell}\right|^{2}}(\Lambda)}{2} \\
& +\frac{1}{\pi} \int_{0}^{\infty} \frac{1}{\omega} \operatorname{Im}\left\{\mathcal{M}_{\left|g_{\ell}\right|^{2}}(\Lambda, \jmath \omega) \mathcal{M}_{\left|h_{\ell}\right|^{2}}(-\jmath q \omega)\right\} \mathrm{d} \omega \tag{13}
\end{align*}
$$

where $\Lambda=\bar{\gamma}_{\mathcal{Q}} / \bar{\gamma}_{\mathcal{P}}$, and $q=\gamma_{\mathrm{th}} / \bar{\gamma}_{\mathcal{Q}}$.

## Proof: See Appendix B.

## B. The Asymptotic Analysis for $O P$

In order to obtain further insights on the system performance, high-SNRs asymptotic expressions for OP will be derived, wherefrom the diversity and coding gains can be deduced. Without loss of generality, it is assumed that $\bar{\gamma}_{\mathcal{P}}=\xi \bar{\gamma}_{\mathcal{Q}}$, where $\xi$ is a positive constant. An asymptotic OP expression for arbitrary-valued fading parameters is deduced in the following theorem.
Theorem 3. For arbitrary values of $\mu_{g_{\ell}}$ and $\mu_{h_{\ell}}, \forall \ell=\{1,2\}$, when $\bar{\gamma}_{\mathcal{Q}} \rightarrow \infty$, the asymptotic approximation for $O P$ is obtained as

$$
\begin{equation*}
P_{\text {out }}\left(\gamma_{\text {th }}\right) \stackrel{\bar{\gamma}_{\mathcal{Q}} \rightarrow \infty}{=} \Theta \cdot\left(\frac{\gamma_{\text {th }}}{\bar{\gamma}_{\mathcal{Q}}}\right)^{\min \left(2 \mu_{g_{1}}, 2 \mu_{g_{2}}\right)} \tag{14}
\end{equation*}
$$

where

$$
\Theta= \begin{cases}\Theta_{1}, & \text { if } \mu_{g_{1}}<\mu_{g_{2}}  \tag{15}\\ \Theta_{1}+\Theta_{2}, & \text { if } \mu_{g_{1}}=\mu_{g_{2}} \\ \Theta_{2}, & \text { if } \mu_{g_{1}}>\mu_{g_{2}}\end{cases}
$$

and $\Theta_{1}, \Theta_{2}$ are given by

$$
\begin{aligned}
\Theta_{1}= & \frac{h_{g_{1}}^{\mu_{g_{1}}} h_{h_{1}}^{\mu_{h_{1}}}}{4 \mu_{g_{1}} \mu_{h_{1}} \Gamma\left(2 \mu_{g_{1}}\right) \Gamma\left(2 \mu_{h_{1}}\right)}\left(\frac{2 \mu_{g_{1}}}{\xi \Omega_{1}}\right)^{2 \mu_{g_{1}}}\left(\frac{2 \mu_{h_{1}}}{\xi \Omega_{3}}\right)^{2 \mu_{h_{1}}} \\
+ & \frac{\sqrt{\pi} \mu_{h_{1}}^{\mu_{h_{1}}+0.5} h_{g_{1}}^{\mu_{g_{1}}} h_{h_{1}}^{\mu_{h_{1}}}}{\mu_{g_{1}} \Gamma\left(2 \mu_{g_{1}}\right) \Gamma\left(\mu_{h_{1}}\right) H_{h_{1}}^{\mu_{h_{1}}-0.5} \Omega_{3}^{\mu_{h_{1}+0.5}}} \\
& \times\left(\frac{2 \mu_{g_{1}}}{\Omega_{1}}\right)^{2 \mu_{g_{1}}} \mathcal{L}\left(\mu_{g_{1}}, \mu_{h_{1}}, h_{h_{1}}, \Omega_{3}, y\right)
\end{aligned}
$$

and

$$
\begin{aligned}
\Theta_{2}= & \frac{h_{g_{2}}^{\mu_{g_{2}}} h_{h_{2}}^{\mu_{h_{2}}}}{4 \mu_{g_{2}} \mu_{h_{2}} \Gamma\left(2 \mu_{g_{2}}\right) \Gamma\left(2 \mu_{h_{2}}\right)}\left(\frac{2 \mu_{g_{2}}}{\xi \Omega_{2}}\right)^{2 \mu_{g_{2}}}\left(\frac{2 \mu_{h_{2}}}{\xi \Omega_{4}}\right)^{2 \mu_{h_{2}}} \\
+ & \frac{\sqrt{\pi} \mu_{h_{2}}^{\mu_{h_{2}}+0.5} h_{g_{2}}^{\mu_{g_{2}}} h_{h_{2}}^{\mu_{h_{2}}}}{\mu_{g_{2}} \Gamma\left(2 \mu_{g_{2}}\right) \Gamma\left(\mu_{h_{2}}\right) H_{h_{2}}^{\mu_{h_{2}}-0.5} \Omega_{4}^{\mu_{h_{2}+0.5}}} \\
& \times\left(\frac{2 \mu_{g_{2}}}{\Omega_{2}}\right)^{2 \mu_{g_{2}}} \mathcal{L}\left(\mu_{g_{2}}, \mu_{h_{2}}, h_{h_{2}}, \Omega_{4}, y\right)
\end{aligned}
$$

respectively, where $\mathcal{L}\left(\mu_{g_{\ell}}, \mu_{h_{\ell}}, h_{h_{\ell}}, \Omega_{i}, y\right)$ is given as

$$
\begin{aligned}
& \mathcal{L}\left(\mu_{g_{\ell}}, \mu_{h_{\ell}}, h_{h_{\ell}}, \Omega_{i}, y\right)=\int_{\frac{1}{\xi}}^{\infty} y^{2 \mu_{g_{\ell}}+\mu_{h_{\ell}}-0.5} \\
& \quad \times \exp \left(-\frac{2 \mu_{h_{\ell}} h_{h_{\ell}}}{\Omega_{i}} y\right) I_{\mu_{h_{\ell}}-0.5}\left(\frac{2 \mu_{h_{\ell}} H_{h_{\ell}}}{\Omega_{i}} y\right) d y
\end{aligned}
$$

which cannot be derived in closed form.
Proof: See Appendix C.

Note that the integral in (14) can be easily evaluated numerically by employing available in popular mathematical software packages such as Matlab, Maple or Mathematica. An asymptotic OP expression in closed-form for integer-valued fading parameters is deduced in the following theorem.
Theorem 4. For integer values of $\mu_{g_{\ell}}$ and $\mu_{h_{\ell}}, \forall \ell=\{1,2\}$, when $\bar{\gamma}_{\mathcal{Q}} \rightarrow \infty$, the asymptotic approximation for $O P$ is obtained as

$$
\begin{equation*}
P_{\text {out }}\left(\gamma_{\text {th }}\right) \stackrel{\bar{\gamma}_{\mathcal{Q}} \rightarrow \infty}{=} \Theta^{\prime} \cdot\left(\frac{\gamma_{\text {th }}}{\bar{\gamma}_{\mathcal{Q}}}\right)^{\min \left(2 \mu_{g_{1}}, 2 \mu_{g_{2}}\right)} \tag{16}
\end{equation*}
$$

where

$$
\Theta^{\prime}= \begin{cases}\Theta_{1^{\prime}}, & \text { if } \mu_{g_{1}}<\mu_{g_{2}}  \tag{17}\\ \Theta_{1^{\prime}}+\Theta_{2^{\prime}}, & \text { if } \mu_{g_{1}}=\mu_{g_{2}} \\ \Theta_{2^{\prime}}, & \text { if } \mu_{g_{1}}>\mu_{g_{2}}\end{cases}
$$

and $\Theta_{1^{\prime}}, \Theta_{2^{\prime}}$ are given by

$$
\begin{aligned}
& \Theta_{1^{\prime}}=\frac{h_{g_{1}}^{\mu_{g_{1}}} h_{h_{1}}^{\mu_{h_{1}}}}{4 \mu_{g_{1}} \mu_{h_{1}} \Gamma\left(2 \mu_{g_{1}}\right) \Gamma\left(2 \mu_{h_{1}}\right)}\left(\frac{2 \mu_{g_{1}}}{\xi \Omega_{1}}\right)^{2 \mu_{g_{1}}}\left(\frac{2 \mu_{h_{1}}}{\xi \Omega_{3}}\right)^{2 \mu_{h_{1}}} \\
&+\frac{h_{g_{1}}^{\mu_{g_{1}}}}{2 \mu_{g_{1}} \Gamma\left(2 \mu_{g_{1}}\right) \Gamma\left(\mu_{h_{1}}\right)}\left(\frac{h_{h_{1}}}{H_{h_{1}}}\right)^{\mu_{h_{1}}}\left(\frac{2 \mu_{g_{1}}}{\Omega_{1}}\right)^{2 \mu_{g_{1}}} \\
& \quad \times \sum_{k=0}^{\mu_{h_{1}}-1} \frac{\left(\mu_{h_{1}}+k-1\right)!}{k!\left(\mu_{h_{1}}-k-1\right)!\left(4 H_{h_{1}}\right)^{k}}\left(\frac{\mu_{h_{1}}}{\Omega_{3}}\right)^{\mu_{h_{1}}-k} \\
& \quad \times\left[(-1)^{k} \frac{\Gamma\left(2 \mu_{g_{1}}+\mu_{h_{1}}-k, A_{\left.h_{1} / \xi\right)}\right.}{A_{h_{1}}^{2 \mu_{g_{1}}+\mu_{h_{1}}-k}}\right. \\
&\left.\quad+(-1)^{\mu_{h_{1}}} \frac{\Gamma\left(2 \mu_{g_{1}}+\mu_{h_{1}}-k, B_{h_{1}} / \xi\right)}{B_{h_{1}}^{2 \mu_{g_{1}}+\mu_{h_{1}}-k}}\right]
\end{aligned}
$$

and

$$
\begin{gathered}
\Theta_{2^{\prime}}=\frac{h_{g_{2}}^{\mu_{g_{2}}} h_{h_{2}}^{\mu_{h_{2}}}}{4 \mu_{g_{2}} \mu_{h_{2}} \Gamma\left(2 \mu_{g_{2}}\right) \Gamma\left(2 \mu_{h_{2}}\right)}\left(\frac{2 \mu_{g_{2}}}{\xi \Omega_{2}}\right)^{2 \mu_{g_{2}}}\left(\frac{2 \mu_{h_{2}}}{\xi \Omega_{4}}\right)^{2 \mu_{h_{2}}} \\
+\frac{h_{g_{2}}^{\mu_{g_{2}}}}{2 \mu_{g_{2}} \Gamma\left(2 \mu_{g_{2}}\right) \Gamma\left(\mu_{h_{2}}\right)}\left(\frac{h_{h_{2}}}{H_{h_{2}}}\right)^{\mu_{h_{2}}}\left(\frac{2 \mu_{g_{2}}}{\Omega_{2}}\right)^{2 \mu_{g_{2}}} \\
\quad \times \sum_{k=0}^{\mu_{h_{2}}-1} \frac{\left(\mu_{h_{2}}+k-1\right)!}{k!\left(\mu_{h_{2}}-k-1\right)!\left(4 H_{h_{2}}\right)^{k}}\left(\frac{\mu_{h_{2}}}{\Omega_{3}}\right)^{\mu_{h_{2}}-k} \\
\quad \times\left[(-1)^{k} \frac{\Gamma\left(2 \mu_{g_{2}}+\mu_{h_{2}}-k, A_{h_{2}} / \xi\right)}{A_{h_{2}}^{2 \mu_{g_{2}}+\mu_{h_{2}}-k}}\right]
\end{gathered}
$$

respectively.
Proof: See Appendix D.
According to (14) and (16), the diversity gain $G_{d}$ and the coding gain $G_{c}$ can be given by

$$
\begin{equation*}
G_{d}=\min \left(2 \mu_{g_{1}}, 2 \mu_{g_{2}}\right) \tag{18}
\end{equation*}
$$

and

$$
\begin{equation*}
G_{c}=\frac{1}{\gamma_{\mathrm{th}}} \Theta^{-1 / \min \left(2 \mu_{g_{1}}, 2 \mu_{g_{2}}\right)} \tag{19}
\end{equation*}
$$

respectively. Specially, for integer values of $\mu_{g_{\ell}}$ and $\mu_{h_{\ell}}, \forall \ell=$ $\{1,2\}, \Theta$ in (19) can be replaced by $\Theta^{\prime}$ in (17).

As it can be observed from Theorem 3 and Theorem 4, the diversity gain only depends on the more severe fading channel between two hops of the secondary network, whereas the primary network only affects its coding gain.

## C. Diversity-Multiplexing Tradeoff

According to [4], the diversity-multiplexing tradeoff can be formulated as

$$
\begin{equation*}
d(r)=\lim _{\bar{\gamma}_{\mathcal{Q}} \rightarrow \infty} \frac{-\log P_{\mathrm{out}}\left(\bar{\gamma}_{\mathcal{Q}}, r\right)}{\log \bar{\gamma}_{\mathcal{Q}}} \tag{20}
\end{equation*}
$$

where $r$ is the normalized spectral efficiency. The outage threshold $\gamma_{\text {th }}$ can be expressed in terms of the spectral efficiency $R(\mathrm{bit} / \mathrm{s} / \mathrm{Hz})$ as $\gamma_{\text {th }}=2^{2 R}-1$. Furthermore, $R$ can be expressed in terms of $r$ as $R=r \log _{2}\left(1+\bar{\gamma}_{\mathcal{Q}}\right)$ [4]. Consequently, $\gamma_{\text {th }}$ can be deduced as

$$
\begin{equation*}
\gamma_{\mathrm{th}}=\left(1+\bar{\gamma}_{\mathcal{Q}}\right)^{2 r}-1 \tag{21}
\end{equation*}
$$

By substituting (21) into (14) or (16), $P_{\text {out }}\left(\bar{\gamma}_{\mathcal{Q}}, r\right)$ can be readily obtained. Finally, plugging it into (20), $d(r)$ can be deduced as

$$
\begin{equation*}
d(r)=\min \left(2 \mu_{g_{1}}, 2 \mu_{g_{2}}\right)(1-2 r) \tag{22}
\end{equation*}
$$

From (22), it is evident that the maximum diversity order, i.e., $\min \left(2 \mu_{g_{1}}, 2 \mu_{g_{2}}\right)$, can be achieved as $r \rightarrow 0$, while the maximum normalized spectral efficiency, i.e., $1 / 2$, can be achieved as $d \rightarrow 0$. In addition, the DMT only depends on the more severe fading channel of the secondary network and is independent of the primary network.

## IV. Numerical and Computer Simulation Results

In this section, various performance evaluation results obtained using the OP expressions presented in Sections III are presented. In order to validate the accuracy of the proposed analytical framework, all numerical results are accompanied by equivalent ones obtained via Monte-Carlo. Without loss of generality, for the simulations in Figs. 1 and 2, it is assumed that the average channel powers of all links are given by $\Omega_{i}=\bar{\gamma}_{\mathcal{Q}}, i=\{1,2,3,4\}$, whereas in Fig. 3, the average channel power is written as $\Omega_{i}=1 / d_{i}^{4}$, where $d_{i}$ denotes the distance between the transceivers. The outage threshold $\gamma_{\text {th }}$ is set to 3 dB for all considered analysis. Moreover, it is assumed that $\xi=1$, i.e., $\bar{\gamma}_{\mathcal{P}}=\bar{\gamma}_{\mathcal{Q}}$. From Figs. $1-4$, it can be observed that the derived OP lower bounds are very tight and the asymptotic results predict well the diversity and coding gains, thus validating the correctness of the proposed analysis.

Fig. 1 depicts the OP of the considered network, assuming $\eta_{h_{1}}=\eta_{h_{2}}=0.7, \mu_{g_{1}}=2$ and $\mu_{h_{1}}=\mu_{h_{2}}=1$, respectively. In order to investigate the impact of parameters $\mu_{g_{2}}, \eta_{g_{1}}$ and $\eta_{g_{2}}$, several different cases are considered. As it can be observed, the outage performance improves as $\mu_{g_{2}}$ increases and/or $\eta_{g_{1}}, \eta_{g_{2}}$ increase. In addition, it is obvious that, there is a significant increase in diversity gain when $\mu_{g_{2}}$ increases from 1 to 3 , however, the same diversity gain can be achieved when $\mu_{g_{2}}=3$. This is because the diversity gain depends on the


Fig. 1: OP of the considered network over $\eta-\mu$ fading channels with parameters $\eta_{h_{1}}=\eta_{h_{2}}=0.7, \mu_{g_{1}}=2, \mu_{h_{1}}=$ $\mu_{h_{2}}=1$.


Fig. 2: OP of the considered network over $\eta-\mu$ fading channels with parameters $\eta_{g_{1}}=\eta_{g_{2}}=0.7, \mu_{g_{1}}=2, \mu_{g_{2}}=3$.
more severe fading channel between two hops of the secondary network.

Fig. 2 shows the impact of the primary network on the OP performance for different values of $\mu_{h_{1}}, \mu_{h_{2}}$ and $\eta_{h_{1}}, \eta_{h_{2}}$, assuming $\eta_{g_{1}}=\eta_{g_{2}}=0.7, \mu_{g_{1}}=2$ and $\mu_{g_{2}}=3$. By keeping the parameters of secondary network fixed, several different schemes are considered. It can be observed that the OP performance improves when $\mu_{h_{1}}$ and $\mu_{h_{2}}$ increase from $\mu_{h_{1}}=\mu_{h_{2}}=1$ to $\mu_{h_{1}}=\mu_{h_{2}}=5$ and/or $\eta_{h_{1}}, \eta_{h_{2}}$ increase. Moreover, as expected, the fading parameters of interference links only affect the coding gain, without affecting the diversity gain, just as our preceding analysis.

To evaluate the effect of PU's position on the considered network, Fig. 3 portrays the OP of cognitive AF relay network


Fig. 3: OP of the considered network over $\eta-\mu$ fading channels with parameters $\eta=0.7, \mu_{g_{1}}=2, \mu_{g_{2}}=3, \mu_{h_{1}}=$ $\mu_{h_{2}}=1$.


Fig. 4: Diversity order $d(r)$ in (22) versus normalized spectral efficiency $r$ for different fading severity parameters.
for different PU's position with $\eta=0.7, \mu_{g_{1}}=2, \mu_{g_{2}}=$ 3 and $\mu_{h_{1}}=\mu_{h_{2}}=1$, respectively. It is assumed that all SUs are located in a straight line. The coordinates of SUS, SU-R and SU-D are $(0,0),(1 / 2,0)$ and $(1,0)$, respectively. Moreover, PU-D can be located in three different positions, namely $(0.44,0.44),(0.55,0.55)$ and $(0.66,0.66)$. From Fig. 3, it can be observed that the position of PU-D significantly affects the OP performance of the secondary network. Interestingly, when PU-D is located at $(0.66,0.66)$, the best performance can be obtained.

Finally, Fig. 4 depicts the diversity order $d(r)$ versus the normalized spectral efficiency $r$, for different fading severity parameters. It is obvious that the maximum diversity order can be achieved as $r \rightarrow 0$, whereas the maximum normalized
spectral efficiency, i.e., $1 / 2$, can be achieved as $d \rightarrow 0$, thus validating the proposed theoretical analysis.

## V. Conclusion

In this paper, a comprehensive analytical framework for the performance evaluation CRN operating over $\eta-\mu$ fading channels has been presented. To ensure the QoS provision at the primary network, both maximum tolerable interference power at PU and maximum allowable transmit power at SU have been taken into account. Tight lower bounds as well as asymptotic expressions of OP for cognitive AF relay network have been obtained. Moreover, a concise frequency-domain approach for evaluating the OP with arbitrary-valued fading parameters was presented. Our findings reveal that diversity gain and the DMT of secondary networks are independent of the primary networks. More specifically, they are solely determined by the more severe fading hop among the two hops of the secondary network. In addition, the only impact from primary network that can be observed is the coding gain which is severely degraded when the PU is located nearby the secondary network. The generality and computational efficiency of these results render themselves as efficient tools for both theoretical analysis and practical applications.

## Appendix A <br> Proof of Theorem 1

It can be observed that $\gamma_{\text {end }}$ in (3) is upper bounded by $\gamma_{\text {end }} \leqslant \min \left\{\gamma_{1}, \gamma_{2}\right\}$ [25], yielding

$$
\begin{align*}
& P_{\text {out }}\left(\gamma_{\text {th }}\right) \geqslant \operatorname{Pr}\left\{\min \left(\gamma_{1}, \gamma_{2}\right) \leqslant \gamma_{\text {th }}\right\} \\
& \quad=1-\left(1-F_{\gamma_{1}}\left(\gamma_{\text {th }}\right)\right)\left(1-F_{\gamma_{2}}\left(\gamma_{\text {th }}\right)\right) \\
& \quad=F_{\gamma_{1}}\left(\gamma_{\text {th }}\right)+F_{\gamma_{2}}\left(\gamma_{\text {th }}\right)-F_{\gamma_{1}}\left(\gamma_{\text {th }}\right) F_{\gamma_{2}}\left(\gamma_{\text {th }}\right) \tag{A-1}
\end{align*}
$$

In order to obtain a lower bound for OP, the CDFs of $\gamma_{1}$ and $\gamma_{2}$, i.e., $F_{\gamma_{1}}(\gamma)$ and $F_{\gamma_{2}}(\gamma)$, are required. Specifically, $F_{\gamma_{1}}(\gamma)$ is given as

$$
\begin{align*}
F_{\gamma_{1}}(\gamma)= & \operatorname{Pr}\left\{\min \left(\bar{\gamma}_{\mathcal{Q}} \frac{\left|g_{1}\right|^{2}}{\left|h_{1}\right|^{2}}, \bar{\gamma}_{\mathcal{P}}\left|g_{1}\right|^{2}\right) \leqslant \gamma\right\} \\
= & \underbrace{\operatorname{Pr}\left\{\bar{\gamma}_{\mathcal{P}}\left|g_{1}\right|^{2} \leqslant \gamma, \frac{\bar{\gamma}_{\mathcal{Q}}}{\left|h_{1}\right|^{2}} \geqslant \bar{\gamma}_{\mathcal{P}}\right\}}_{\mathcal{F}_{1}} \\
& +\underbrace{\operatorname{Pr}\left\{\bar{\gamma}_{\mathcal{Q}} \frac{\left|g_{1}\right|^{2}}{\left|h_{1}\right|^{2}} \leqslant \gamma, \frac{\bar{\gamma}_{\mathcal{Q}}}{\left|h_{1}\right|^{2}} \leqslant \bar{\gamma}_{\mathcal{P}}\right\}}_{\mathcal{F}_{2}} \tag{A-2}
\end{align*}
$$

Using [22, eq. (8.467)], the modified Bessel function $I_{\mu-0.5}(z)$ in (6), with $\mu>0$ being an integer, is expressed in closed form as

$$
\begin{align*}
& I_{\mu-0.5}(z)=\frac{1}{\sqrt{\pi}} \sum_{k=0}^{\mu-1} \frac{(\mu-1+k)!}{k!(\mu-1-k)!} \\
& \quad \times\left[\frac{(-1)^{k} \exp (z)-(-1)^{\mu-1} \exp (-z)}{(2 z)^{k+0.5}}\right] \tag{A-3}
\end{align*}
$$

Since $\left|g_{1}\right|^{2}$ and $\left|h_{1}\right|^{2}$ are independent, the first term of (A-2), i.e., $\mathcal{F}_{1}$, can be expressed as

$$
\begin{array}{r}
\mathcal{F}_{1}=\operatorname{Pr}\left\{\left|g_{1}\right|^{2} \leqslant \frac{\gamma}{\bar{\gamma}_{\mathcal{P}}}\right\} \cdot \operatorname{Pr}\left\{\left|h_{1}\right|^{2} \leqslant \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right\} \\
=F_{\left|g_{1}\right|^{2}}\left(\frac{\gamma}{\bar{\gamma}_{\mathcal{P}}}\right) \cdot F_{\left|h_{1}\right|^{2}}\left(\frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right) \tag{A-4}
\end{array}
$$

By employing (6), $F_{\left|g_{1}\right|^{2}}(\cdot)$ and $F_{\left|h_{1}\right|^{2}}(\cdot)$ can be readily obtained, then substituting the resulting expressions into (A-4), $\mathcal{F}_{1}$ can be easily deduced. As far as $\mathcal{F}_{2}$ is concerned, one obtains:

$$
\begin{align*}
\mathcal{F}_{2}= & \operatorname{Pr}\left\{\left|g_{1}\right|^{2} \leqslant \frac{\gamma}{\bar{\gamma}_{\mathcal{Q}}}\left|h_{1}\right|^{2},\left|h_{1}\right|^{2} \geqslant \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right\} \\
= & \int_{\frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}}^{\infty} f_{\left|h_{1}\right|^{2}}(y) \int_{0}^{\frac{\gamma}{\bar{\gamma}_{\mathcal{Q}}} y} f_{\left|g_{1}\right|^{2}}(x) \mathrm{d} x \mathrm{~d} y \\
& =\int_{\frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}}^{\infty} f_{\left|h_{1}\right|^{2}}(y) F_{\left|g_{1}\right|^{2}}\left(\frac{\gamma}{\bar{\gamma}_{\mathcal{Q}}} y\right) \mathrm{d} y \tag{A-5}
\end{align*}
$$

From (5) and (6), $f_{\left|h_{1}\right|^{2}}(\cdot)$ and $F_{\left|g_{1}\right|^{2}}(\cdot)$ can be readily deduced, respectively. By substituting the resulting expressions into (A-5) and employing [22, eq. (3.351.2)], $\mathcal{F}_{2}$ can be obtained. Moreover, substituting $\mathcal{F}_{1}$ and $\mathcal{F}_{2}$ into (A-2), the CDF of $\gamma_{1}$ can be obtained as (A-6) on the top of the next page.

Following a similar line of arguments, $F_{\gamma_{2}}(\gamma)$ can be directly derived from (A-6) by substituting the respective parameters by their counterparts (i.e., $\mu_{g_{1}} \rightarrow \mu_{g_{2}}, \mu_{h_{1}} \rightarrow \mu_{h_{2}}$, $h_{g_{1}} \rightarrow h_{g_{2}}, h_{h_{1}} \rightarrow h_{h_{2}}, H_{g_{1}} \rightarrow H_{g_{2}}, H_{h_{1}} \rightarrow H_{h_{2}}$, $\underset{g_{1}}{a_{(k)}^{\left(k_{1}\right)}} \rightarrow a_{g_{2}}^{\left(k_{1}\right)}, a_{h_{1}}^{\left(k_{2}\right)} \rightarrow a_{h_{2}(k)}^{\left(k_{2}\right)}, b_{g_{1}}^{\left(k_{1}\right)} \rightarrow b_{g_{2}}^{\left(k_{1_{2}}\right)}, b_{h_{1}}^{\left(k_{2}\right)} \rightarrow b_{h_{2}}^{\left(k_{2}\right)}$, $a_{g_{1}}^{(k)} \rightarrow a_{g_{2}}^{(k)}, b_{g_{1}}^{(k)} \rightarrow b_{g_{2}}^{(k)}, A_{g_{1}} \rightarrow A_{g_{2}}, A_{h_{1}} \rightarrow A_{h_{2}}$, $B_{g_{1}} \rightarrow B_{g_{2}}, B_{h_{1}} \rightarrow B_{h_{2}}$ and $\left.\Omega_{3} \rightarrow \Omega_{4}\right)$. Finally, utilizing the CDFs of $\gamma_{1}$ and $\gamma_{2}$, a tight lower bound for OP can be deduced as $P_{\text {out }}\left(\gamma_{\text {th }}\right)=1-\left[1-F_{\gamma_{1}}\left(\gamma_{\text {th }}\right)\right]\left[1-F_{\gamma_{2}}\left(\gamma_{\text {th }}\right)\right]$, thus concluding the proof.

## Appendix B <br> Proof of Theorem 2

Following a similar line of arguments as in the proof of Theorem 1, in order to obtain the required bound, $\mathcal{F}_{1}$ and $\mathcal{F}_{2}$ defined in (A-4) and (A-5) need to be evaluated for arbitrary values of fading parameters. In order to deduce an analytical expression for (A-5), integrals of the form

$$
\begin{align*}
\mathcal{J}\left(\mu_{g_{\ell}}, \mu_{h_{\ell}},\right. & \left.\eta_{g_{\ell}}, \eta_{h_{\ell}}, \gamma_{\mathrm{th}}, \bar{\gamma}_{\mathcal{P}}, \bar{\gamma}_{\mathcal{Q}}\right) \\
& =\int_{\frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}}^{\infty} f_{\left|h_{1}\right|^{2}}(y) F_{\left|g_{1}\right|^{2}}\left(\frac{\gamma}{\bar{\gamma}_{\mathcal{Q}}} y\right) \mathrm{d} y . \tag{B-1}
\end{align*}
$$

need to be evaluated. For arbitrary values of the $\mu$ fading parameters, the computation of (B-1) is very difficult. In order to circumvent this problem, $\mathcal{J}\left(\mu_{g_{\ell}}, \mu_{h_{\ell}}, \eta_{g_{\ell}}, \eta_{h_{\ell}}, \gamma_{\mathrm{th}}, \bar{\gamma}_{\mathcal{P}}, \bar{\gamma}_{\mathcal{Q}}\right)$ can be reformulated as

$$
\begin{align*}
& \mathcal{J}\left(\mu_{g_{\ell}}, \mu_{h_{\ell}}, \eta_{g_{\ell}}, \eta_{h_{\ell}}, \gamma_{\mathrm{th}}, \bar{\gamma}_{\mathcal{P}}, \bar{\gamma}_{\mathcal{Q}}\right) \\
& \quad=\int_{\Lambda}^{\infty} \operatorname{Pr}\left\{\left|g_{\ell}\right|^{2} \leqslant\left. q y| | h_{\ell}\right|^{2}=y\right\} f_{\left|h_{\ell}\right|}(y) \mathrm{d} y \tag{B-2}
\end{align*}
$$

where $q=\frac{\gamma}{\bar{\gamma}_{\mathcal{Q}}}$ and $\Lambda=\bar{\gamma}_{\mathcal{Q}} / \bar{\gamma}_{\mathcal{P}}$. By invoking the GilPelaez theorem [26], $\mathcal{J}\left(\mu_{g_{\ell}}, \mu_{h_{\ell}}, \eta_{g_{\ell}}, \eta_{h_{\ell}}, \gamma_{\mathrm{th}}, \bar{\gamma}_{\mathcal{P}}, \bar{\gamma}_{\mathcal{Q}}\right)$ can be expressed as (13), thus concluding the proof.

## Appendix C <br> Proof of Theorem 3

Using the lower bound $P_{\text {out }}(\gamma)=\operatorname{Pr}\left\{\min \left(\gamma_{1}, \gamma_{2}\right) \leqslant \gamma\right\}$, $P_{\text {out }}(\gamma)$ can be approximated at high SNRs as $P_{\text {out }}(\gamma)=$ $F_{\gamma_{1}}(\gamma)+F_{\gamma_{2}}(\gamma)-F_{\gamma_{1}}(\gamma) F_{\gamma_{2}}(\gamma) \simeq F_{\gamma_{1}}(\gamma)+F_{\gamma_{2}}(\gamma)$. From [17, eqs. (14),(15)], for $x \rightarrow 0^{+}$, the asymptotic approximation for $f_{X}(x)$ can be expressed as

$$
\begin{equation*}
f_{X}(x) \simeq \frac{h^{\mu}}{\Gamma(2 \mu)}\left(\frac{2 \mu}{\bar{X}}\right)^{2 \mu} x^{2 \mu-1} \tag{C-1}
\end{equation*}
$$

where $\mu \in\left\{\mu_{g_{1}}, \mu_{g_{2}}, \mu_{h_{1}}, \mu_{h_{2}}\right\}$ and $h \in\left\{h_{g_{1}}, h_{g_{2}}, h_{h_{1}}\right.$, $\left.h_{h_{2}}\right\}$. Employing ( $\mathrm{C}-1$ ), one can finally obtain the asymptotic approximation for $F_{X}(x)$ as

$$
\begin{equation*}
F_{X}(x) \simeq \frac{h^{\mu}}{2 \mu \Gamma(2 \mu)}\left(\frac{2 \mu}{\bar{X}}\right)^{2 \mu} x^{2 \mu} \tag{C-2}
\end{equation*}
$$

For arbitrary values of fading parameters, when $\bar{\gamma}_{\mathcal{Q}} \rightarrow \infty$, by substituting (5) and ( $\mathrm{C}-2$ ) into (A-4) and (A-5), the CDF of $\gamma_{1}$ can be approximated as

$$
\begin{align*}
F_{\gamma_{1}}(\gamma)^{\bar{\gamma}_{\mathcal{Q}} \rightarrow \infty} & \stackrel{h_{g_{1}}^{\mu_{g_{1}}} h_{h_{1}}^{\mu_{h_{1}}}}{4 \mu_{g_{1}} \mu_{h_{1}} \Gamma\left(2 \mu_{g_{1}}\right) \Gamma\left(2 \mu_{h_{1}}\right)}\left(\frac{2 \mu_{g_{1}} \gamma}{\xi \Omega_{1} \bar{\gamma}_{\mathcal{Q}}}\right)^{2 \mu_{g_{1}}}\left(\frac{2 \mu_{h_{1}}}{\xi \Omega_{3}}\right)^{2 \mu_{h_{1}}} \\
+ & \frac{\sqrt{\pi} \mu_{h_{1}}^{\mu_{h_{1}}+0.5} h_{g_{1}}^{\mu_{g_{1}}} h_{h_{1}}^{\mu_{h_{1}}}}{\mu_{g_{1}} \Gamma\left(2 \mu_{g_{1}}\right) \Gamma\left(\mu_{h_{1}}\right) H_{h_{1}}^{\mu_{h_{1}}-0.5} \Omega_{3}^{\mu_{h_{1}+0.5}}} \\
& \times\left(\frac{2 \mu_{g_{1}} \gamma}{\Omega_{1} \bar{\gamma}_{\mathcal{Q}}}\right)^{2 \mu_{g_{1}}} \mathcal{L}\left(\mu_{g_{1}}, \mu_{h_{1}}, h_{h_{1}}, \Omega_{3}, y\right) . \quad \text { (C-3) } \tag{C-3}
\end{align*}
$$

Similarly, the asymptotic expression for $F_{\gamma_{2}}(\gamma)$ can be directly obtained from (C-3) after replacing the parameters by their counterparts. Finally, utilizing $P_{\text {out }}(\gamma) \simeq F_{\gamma_{1}}(\gamma)+F_{\gamma_{2}}(\gamma)$, Theorem 3 can be readily deduced.

## APPENDIX D <br> Proof of Theorem 4

For integer values of fading parameters, when $\bar{\gamma}_{\mathcal{Q}} \rightarrow \infty$, substituting the modified Bessel function in closed form (A-3) into (6), and then substituting (5) and (C-2) into (A-4) and (A-5), the CDF of $\gamma_{1}$ can be approximated as

$$
\begin{gather*}
F_{\gamma_{1}}(\gamma)^{\stackrel{\bar{\gamma}_{\mathcal{Q}} \rightarrow \infty}{ }} \stackrel{h_{g_{1}}^{\mu_{g_{1}}} h_{h_{1}}^{\mu_{h_{1}}}}{4 \mu_{g_{1}} \mu_{h_{1}} \Gamma\left(2 \mu_{g_{1}}\right) \Gamma\left(2 \mu_{h_{1}}\right)}\left(\frac{2 \mu_{g_{1}} \gamma}{\xi \Omega_{1} \bar{\gamma}_{\mathcal{Q}}}\right)^{2 \mu_{g_{1}}}\left(\frac{2 \mu_{h_{1}}}{\xi \Omega_{3}}\right)^{2 \mu_{h_{1}}} \\
+\frac{h_{h_{1}}}{2 \mu_{g_{1}} \Gamma\left(2 \mu_{g_{1}}\right) \Gamma\left(\mu_{h_{1}}\right)}\left(\frac{h_{h_{1}}}{\mu_{h_{1}}}\right)^{\mu_{h_{1}}-1} \sum_{k=0}^{2 \mu_{1}} \\
\times \frac{\left(\mu_{h_{1}}+k-1\right)!}{k!\left(\mu_{h_{1}}-k-1\right)!\left(4 H_{h_{1}}\right)^{k}}\left(\frac{\mu_{h_{1}}}{\Omega_{3}}\right)^{\mu_{h_{1}-k}}\left(\frac{2 \mu_{g_{1}} \gamma}{\Omega_{1} \bar{\gamma}_{\mathcal{Q}}}\right)^{2 \mu_{g_{1}}} \\
\quad \times\left[(-1)^{k} \frac{\Gamma\left(2 \mu_{g_{1}}+\mu_{h_{1}}-k, A_{h_{1}} / \xi\right)}{A_{h_{1}}^{2 \mu_{g_{1}}+\mu_{h_{1}}-k}}\right] .
\end{gather*}
$$

$$
\begin{align*}
& \left.+(-1)^{\mu_{h_{1}}} b_{h_{1}}^{\left(k_{2}\right)} B_{h_{1}}^{p_{2}} \exp \left[-\left(A_{g_{1}} \frac{\gamma}{\bar{\gamma}_{\mathcal{P}}}+B_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)\right]\right)+(-1)^{\mu_{g_{1}}} b_{g_{1}}^{\left(k_{1}\right)} B_{g_{1}}^{p_{1}} \\
& \left.\times\left(a_{h_{1}}^{\left(k_{2}\right)} A_{h_{1}}^{p_{2}} \exp \left[-\left(B_{g_{1}} \frac{\gamma}{\bar{\gamma}_{\mathcal{P}}}+A_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)\right]+(-1)^{\mu_{h_{1}}} b_{h_{1}}^{\left(k_{2}\right)} B_{h_{1}}^{p_{2}} \exp \left[-\left(B_{g_{1}} \frac{\gamma}{\bar{\gamma}_{\mathcal{P}}}+B_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)\right]\right)\right\} \\
& -\widetilde{\sum_{k, p, q}}\left(\frac{\gamma}{\bar{\gamma}_{\mathcal{Q}}}\right)^{p}\left\{\frac { a _ { g _ { 1 } } ^ { ( k ) } A _ { g _ { 1 } } ^ { p } } { ( \mu _ { h _ { 1 } } + p - q - 1 ) ! } \left(\frac{(-1)^{q} \Gamma\left(\mu_{h_{1}}+p-q, A_{g_{1}} \frac{\gamma}{\bar{\gamma}_{\mathcal{P}}}+A_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)}{\left(A_{g_{1}} \frac{\gamma}{\bar{\gamma}_{\mathcal{Q}}}+A_{h_{1}}\right)^{\mu_{h_{1}}+p-q}}\right.\right. \\
& \left.+\frac{(-1)^{\mu_{h_{1}}} \Gamma\left(\mu_{h_{1}}+p-q, A_{g_{1}} \frac{\gamma}{\bar{\gamma}_{\mathcal{P}}}+B_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)}{\left(A_{g_{1}} \frac{\gamma}{\bar{\gamma}_{\mathcal{Q}}}+B_{h_{1}}\right)^{\mu_{h_{1}}+p-q}}\right)+\frac{(-1)^{\mu_{g_{1}}} b_{g_{1}}^{(k)} B_{g_{1}}^{p}}{\left(\mu_{h_{1}}+p-q-1\right)!} \\
& \left.\times\left(\frac{(-1)^{q} \Gamma\left(\mu_{h_{1}}+p-q, B_{g_{1}} \frac{\gamma}{\bar{\gamma}_{\mathcal{P}}}+A_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)}{\left(B_{g_{1}} \frac{\gamma}{\bar{\gamma}_{\mathcal{Q}}}+A_{h_{1}}\right)^{\mu_{h_{1}}+p-q}}+\frac{(-1)^{\mu_{h_{1}} \Gamma\left(\mu_{h_{1}}+p-q, B_{g_{1}} \frac{\gamma}{\bar{\gamma}_{\mathcal{P}}}+B_{h_{1}} \frac{\bar{\gamma}_{\mathcal{Q}}}{\bar{\gamma}_{\mathcal{P}}}\right)}}{\left(B_{g_{1}} \frac{\gamma}{\bar{\gamma}_{\mathcal{Q}}}+B_{h_{1}}\right)^{\mu_{h_{1}}+p-q}}\right)\right\} . \tag{A-6}
\end{align*}
$$

Similarly, the asymptotic expression for $F_{\gamma_{2}}(\gamma)$ can be directly obtained from (D-1) after replacing the parameters by their counterparts. Finally, utilizing $P_{\text {out }}(\gamma) \simeq F_{\gamma_{1}}(\gamma)+F_{\gamma_{2}}(\gamma)$, thus concluding the proof.

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