Sliding Mode and Shaped Input Vibration Control of Flexible Systems

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In this paper, the vibration reduction problem is investigated for a flexible spacecraft during attitude maneuvering. A new control strategy is proposed, which integrates both the command input shaping and the sliding mode output feedback control (SMOFC) techniques. Specifically, the input shaper is designed for the reference model and implemented outside of the feedback loop in order to achieve the exact elimination of the residual vibration by modifying the existing command. The feedback controller, on the other hand, is designed based on the SMOFC such that the closed-loop system behaves like the reference model with input shaper, where the residual vibrations are eliminated in the presence of parametric uncertainties and external disturbances. An attractive feature of this SMOFC algorithm is that the parametric uncertainties or external disturbances of the system do not need to satisfy the so-called matching conditions or invariance conditions provided that certain bounds are known. In addition, a smoothed hyperbolic tangent function is introduced to eliminate the chattering phenomenon. Compared with the conventional methods, the proposed scheme guarantees not only the stability of the closed-loop system, but also the good performance as well as the robustness. Simulation results for the spacecraft model show that the precise attitudes control and vibration suppression are successfully achieved.

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I. INTRODUCTION

Modern spacecraft often employs large, complex and lightweight structures such as solar arrays in order to achieve increased functionality at a reduced launch cost and also provide sustainable energy during space flight. The combination of large and lightweight design results in these space structures being extremely flexible and having low-frequency fundamental vibration modes. These modes might be excited in a variety of tasks such as slewing and pointing maneuvers. To effectively suppress the induced vibrations poses a challenging task for the spacecraft designers.

Active control techniques have been increasingly used as the solutions for flexible spacecraft to achieve the degree of vibration suppression for required precision pointing. One special feed-forward control strategy, known as input shaping, has been studied widely since its first appearance in [1] for possessing the advantages of simplicity and effectiveness and requiring no additional sensors and actuators. In [2], Singhose et al. studied an input shaping controller for slewing a flexible spacecraft. In [3], Banerjee and Padereiro used the input shaping techniques to vibration reduction of a flexible spacecraft following momentum damping with/without slewing. The input shaping method was applied by Hillsley and Yurkovich [4, 5] in large angle movements of a two-link robot switched to feedback control when approaching the desired position. Using pulsewidth pulse FM technique, Song et al. [6] further treated the application of input shaping for vibration reduction of a flexible spacecraft. A modified input shaping scheme was presented by Shan et al. in [7] for multi-mode vibration suppression of a rotating single-link flexible manipulator. Nonlinear input shaping technique was considered by Gorinevsky and Vukovich in [8] for the flexible spacecraft reorientation maneuver. Furthermore, an adaptive input shaper was explored in [9] that provided robustness to parameter uncertainties by tuning the shaper to the flexible mode frequencies. Recently, the experimental testing of command shaping techniques was reported in [10], [11] on a flexible-link manipulator.

In addition, in the realistic environment, the knowledge about system parameters such as inertia matrix and modal frequencies is usually unknown, and various disturbances, including gravitational torque, aerodynamic torque, radiation torque, and other environmental and nonenvironmental torques, are also presented. Therefore, disturbance rejection control strategies that are also robust to parametric uncertainty and effectively suppress the induced vibration are of great interest in spacecraft applications. Sliding mode control (SMC), also known as variable structure control (VSC), has proven to be an effective approach to dealing with parametric uncertainties and external disturbances for dynamic systems due to its simplicity and effectiveness as well as its robustness [12–14]. A tutorial and survey on VSC can be found in [13], and a comprehensive guide on SMC for control engineers was given in [14]. SMC has also attracted great interests from engineers in the area of spacecraft attitude control research [15–18] because of its fast dynamic control, global asymptotic stability, and invariability for interference perturbation. Unfortunately, in most of the previous studies concerning SMC, it is assumed that the spacecraft is rigid and no flexible mode actions are considered. As we discussed already, for a highly flexible spacecraft model, the flexibility effect should be directly accommodated into the control law. There have been some research efforts on the attitude control problems for flexible spacecraft, see e.g. [19, 20], where the flexible spacecraft attitude dynamics are usually represented by hybrid coordinate systems, a combination of discrete attitude parameters and infinite-dimensional flexible motion parameters. However, these control strategies are limited to the systems with full-state feedback.

The SMC problem with respect to output feedback has recently drawn much attention. In Yallapragada et al. [21], a design method was proposed in order to obtain a controller satisfying the reaching conditions. In [22] Wang and Fan developed an interesting approach to designing a sliding mode output feedback control (SMOFC), where the definition of the sliding hyperplane includes an exponentially decaying term. Effectively, the initial system state starts from the sliding hyperplane. However, in [22], the issue of robustness against external disturbances was not addressed, and the uncertainties were assumed to satisfy the matching condition. Therefore, the results in [22] cannot be directly applied to the systems with mismatched uncertainties. To deal with such a limitation, in [23], [24], Kwan extended the results of [22] by eliminating the exponentially decaying term and formulating a time-varying upper bound of states. The robustness issue concerning mismatched disturbances was also examined where, unfortunately, the disturbance vector has a special structure that may limit its applicability to general systems. In [25], Lewis and Sinha continued to tackle the mismatched disturbance issue using output feedback and presented techniques for stability analysis, but the parametric uncertainties were not considered. In [26], a general SMOFC methodology was developed to cope with the uncertainties in the plant, control, and disturbance matrices provided that certain bounds are known. Up to now, to the best of the authors' knowledge, the study of SMC via output feedback for flexible spacecraft systems with mismatched parametric uncertainty and external disturbances has not received much attention and remains a challenging problem.

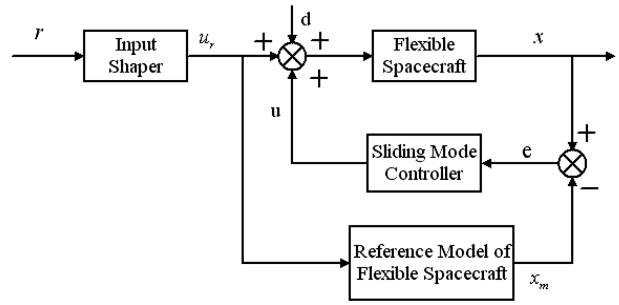

Fig. 1. Block diagram of sliding mode/input shaping algorithm for flexible spacecraft vibration reduction.

We aim here to deal with the active vibration reduction problem in flexible systems with mismatched uncertainties through sliding mode and shaped input control. The developed control strategy integrates the techniques of command input shaping and SMOFC techniques. The configuration of the proposed design scheme is shown in Fig. 1. The input shaper is implemented outside of the feedback loop, which is designed and achieves the exact elimination of residual vibration. The amplitudes and instances of the impulses application can be obtained for the natural frequency and damping ratio of the reference model, respectively. In the feedback loop, the SMOFC technique is employed to make the closed-loop system behave like the reference system with input shaper. The vibration of the flexible structures is suppressed in the presence of parametric uncertainty and external disturbances, which do not need to satisfy the traditional matching condition or invariance conditions. The chattering behavior can be eliminated using the smoothed hyperbolic tangent function. In order to verify the effectiveness of the designed controllers, a proportional derivative (PD) control is first developed for control of rigid body motion, and then an extended case is investigated for the control of vibration of the flexible structure where the input shaping is incorporated. The performances of the proposed control strategy are assessed in terms of the attitude pointing capability and vibration reduction as compared with response with the PD control and the extended case. Furthermore, a thorough comparison with the traditional SMOFC presented in [21] is also conducted. In addition, a near minimum-time control method in [27] is also involved for the comparison. Simulation results for the spacecraft model show that the precise trajectory control and vibration suppression are successfully achieved.

The rest of the paper is arranged as follows. Section II presents the model of the spacecraft with symmetric flexible appendages. The principle of input shaping is briefly described in Section III. Section IV describes the detailed control algorithm for the flexible spacecraft, and the simulation results

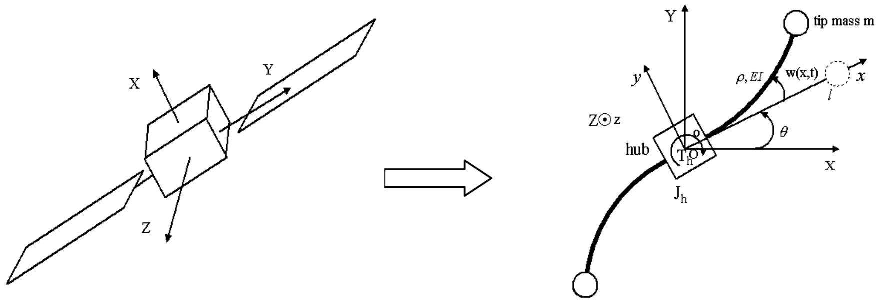

Fig. 2. Spacecraft model with single-axis rotation.

are given in Section V. Some concluding remarks are given in Section VI.

II. MATHEMATICAL MODEL OF THE SPACECRAFT

The model of a flexible spacecraft under consideration is shown in Fig. 2. The model consists of a rigid central hub that represents the spacecraft body, and two flexible appendages that represent antennas, solar arrays, or any other flexible structures. This model is representative of a relatively large class of spacecrafts employed for communication, remote sensing or numerous other applications [28, 29]. Define the OXYZ and oxyz as the inertial frame and the frame fixed on the hub, respectively. Denote $w(x,t)$ as the flexible deformation at point x with respect to the oxy frame, and l is the distance of a point chosen on the appendage from the center of the hub.

In this study, control of rotational motion from the fixed frame oxyz to the inertial frame OXYZ is considered. Let θ denote the rotation angle that is to be controlled using a torque T_h generating device located at the center of the structure. The flexible appendages tend to vibrate due to the coupling effect with the rigid body rotation. It is assumed that the appendages undergo elastic transverse bending only in the orbital plane x-y. They are simplified as flexible beams with tip masses, and the frequencies of oscillation are tuned by the tip masses. It is assumed that two solar arrays are identical in geometric and material properties. Under the torque control input only to the center body, the deflection of each solar array should be identical, that is, the deflection takes place in an antisymmetric fashion.

The equations of motion are derived using the Lagrangian approach. Although the vibration of the appendages is described by partial differential equations, spatial discretization method is used to obtain a set of ordinary differential equations to describe the motion of the spacecraft. For spatial discretization using assumed modes method, the transverse elastic deflection of the appendage along y in the oxyz plane is expressed as

$$w(x,t) = \sum_{i=1}^{N} \phi_i(x) q_i(t) \tag{1}$$

where $\phi_i(x)$ $(i = 1,2,\ldots N)$ are the chosen admissible functions satisfying the geometric and physical boundary conditions, and $q_i(t)$ are the generalized coordinates for the flexible deflection. We suppose that the N modes are sufficient for the computation of elastic deformation.

The nonlinear differential equations describing the rotational and elastic dynamics are given by (see [29])

$$J\ddot{\theta} + M_{\theta q}\ddot{q} = T_h + d \tag{2a}$$

$$M_{\theta q}^T \ddot{\theta} + M_{qq}\ddot{q} + C_{qq}\dot{q} + K_{qq}q = 0 \tag{2b}$$

where $q = [q_1, q_2, \ldots q_N]^T$, the element mass, stiffness matrices, and the nonlinear terms are governed by $J = J_h + 2\int_b^l \rho x^2 dx + 2m_t l^2$, $[M_{\theta q}]_i = 2\int_b^l \rho x \phi_i(x) dx + 2m_t l \phi_i(l)$, $[M_{qq}]_{ij} = 2\int_b^l \rho \phi_i(x)\phi_j(x) dx + 2m_t \phi_i(l)\phi_j(l)$, $[K_{qq}]_{ij} = 2\int_b^l E\mathbf{I}\phi_i''(x)\phi_j''(x)dx$, $[C_{qq}]_{ij} = 2\int_b^l \mathbf{C}\mathbf{I}\phi_i''(x)\phi_j''(x)dx$. Here, $\mathbf{C}$ and E are the damping coefficient and modulus of elasticity, respectively, for the appendages, and $\mathbf{I}$ are the damping coefficient and modulus of elasticity, respectively, for the appendages, and $\mathbf{I}$ is the sectional area moment of inertia with respect to the appendage bending axis. $d(t) \in R$ is the external disturbance belonging to $L_2[0,\infty)$ and $\|d(t)\| \leq \delta_d$ where δ_d is a known positive constant.

Equation (2) can be written in a compact form as

$$\bar{M}\ddot{\bar{Z}} + \bar{C}\dot{\bar{Z}} + \bar{K}\bar{Z} = \bar{B}(T_h + d(t)) \tag{3}$$

where

$$\bar{Z} = [\theta, q^T]^T \in R^{N+1}, \qquad \bar{M} = \begin{bmatrix} J & M_{\theta q} \\ M_{\theta q}^T & M_{qq} \end{bmatrix}$$

$$\bar{C} = \mathrm{diag}\{0, C_{qq}\}, \qquad \bar{K} = \mathrm{diag}\{0, K_{qq}\}, \qquad \bar{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and 0 denotes a null vector of appropriate dimension.

The system of $\bar{Z}$ second-order differential equations, (3), can be transformed into the state-space form

$$\dot{x} = Ax + B\bar{u}(t) + Bd(t) \tag{4}$$

where

$$x = \begin{bmatrix} \bar{Z} \\ \dot{\bar{Z}} \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & I \\ -\bar{M}^{-1}\bar{K} & -\bar{M}^{-1}\bar{C} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \bar{M}^{-1}\bar{B} \end{bmatrix}, \qquad \bar{u}(t) = T_h.$$

Considering the flexible space structure as shown in Fig. 2, the objective of the slew maneuver of this study is a rest-to-rest maneuver from a rest state to another rest state in the shorts time possible. The angle $\theta(t)$ is rotated from initial state to $\theta_d \in [0, 2\pi]$, for example, setting to 60 deg throughout this study. The sensor output available for the output feedback is hub angle θ and angular rate $\dot{\theta}$. Based on the customary requirements of flight task of actual spacecraft, the control scheme should also satisfy the following dominating demands: 1) short transient time, no or less overshooting, high precision, and less vibration stirred; 2) strong capability to resist disturbance of vibrations in both transient process and steady state.

III. INPUT SHAPING

For the input shaping method, an input command is convolved with a sequence of impulses, an input shaper, designed to produce a resulting input command that causes less residual vibration than the original unshaped command. That is, any vibration induced by the first part of the command is canceled by vibration induced by a later portion of the command, and the result of the convolution is then used to derive the system. The convolution can be precomputed if the entire unshaped input is known, or more likely, it can be computed in real time from the input command generator. The impulses that constitute the shaper must have appropriate amplitudes and time locations, which are determined by solving a set of constraint equations. Most constraints can be categorized as residual vibrations constraints, robustness constraints, constraints on the impulse amplitudes, or time optimality requirements. This method is briefly described in this section. Relevant research on this subject can be found in, e.g., [6], [10].

For an undamped second-order linear system with a mode of natural frequency ω and damping ratio ξ, its response to an impulse input at t_0 can be obtained as

$$Y(t) = \frac{X\omega}{\sqrt{1-\xi^2}} e^{\xi\omega(t-t_0)} \sin\left[\omega\sqrt{1-\xi^2}(t-t_0)\right] \tag{5}$$

where X and t_0 are the amplitude and time of the impulse, respectively. Furthermore, the response to a sequence of impulses can be obtained by superposition of the impulse responses. With $\omega_d = \omega(\sqrt{1-\xi^2})$, for $\bar{N}$ impulses, the impulse response can then be expressed as

$$Y(t) = \Gamma \sin(\omega_d t + \beta) \tag{6}$$

where

$$\Gamma = \sqrt{\left(\sum_{i=1}^{\bar{N}} \Pi_i \cos\phi_i\right)^2 + \left(\sum_{i=1}^{\bar{N}} \Pi_i \sin\phi_i\right)^2}$$

$$\Pi_i = \frac{X_i\omega}{\sqrt{1-\xi^2}} e^{\xi\omega(t-t_0)}$$

$$\beta = \tan^{-1}\left(\sum_{i=1}^{\bar{N}} \frac{\Pi_i \cos\phi_i}{\Pi_i \sin\phi_i}\right), \qquad \phi_i = \omega_d t_i$$

and X_i and t_i are the magnitudes and times at which the impulses occur.

The residual single mode vibration amplitude of the impulse response is obtained at time of the last impulse, $t_{\bar{N}}$ as follows

$$V = \sqrt{V_1^2 + V_2^2} \tag{7}$$

where

$$V_1 = \sum_{i=1}^{\bar{N}} \frac{X_i\omega}{\sqrt{1-\xi^2}} e^{\xi\omega(t_{\bar{N}}-t_0)\cos(\omega_d t_i)}$$

$$V_2 = \sum_{i=1}^{\bar{N}} \frac{X_i\omega}{\sqrt{1-\xi^2}} e^{\xi\omega(t_{\bar{N}}-t_0)} \sin(\omega_d t_i).$$

To achieve zero vibration after the last impulse, it is required that both V_1 and V_2 in (7) are independently zero. Furthermore, to ensure that the shaped command input produces the same rigid body motion as the unshaped command, it is required that the sum of strengths of the impulses is unity. To avoid response delay, the first impulse is selected as time $t_1 = 0$. Hence by setting V_1 and V_2 in (7) to zero, $\sum_{i=1}^{\bar{N}} X_i = 1$, we have a two-impulse sequence with parameters as

$$t_1 = 0, \qquad t_2 = \frac{\pi}{\omega_d} \tag{8a}$$

$$X_1 = \frac{1}{1+K}, \qquad X_2 = \frac{K}{1+K} \tag{8b}$$

where $K = e^{-\xi\pi/\sqrt{1-\xi^2}}$.

In order to enhance the robustness, the shaper needs to satisfy some additional constraints. One such constraint is that the derivative of the residual vibration (7) with respect to frequency is zero, i.e.,

$$dV/d\omega = 0. \tag{9}$$

Imposing the first derivatives of V_1 and V_2 with respect to ω in (7) and simplifying them yield

$$\frac{dV_1}{d\omega} = \sum_{i=1}^{\bar{N}} X_i t_i e^{-\xi\omega(t_{\bar{N}}-t_0)} \sin(\omega_d t_i) \tag{10a}$$

$$\frac{dV_2}{d\omega} = \sum_{i=1}^{\bar{N}} X_i t_i e^{-\xi\omega(t_{\bar{N}}-t_0)} \cos(\omega_d t_i). \tag{10b}$$

Hence, by setting (10a) and (10b) to zero, we obtain a three-impulse sequence with parameters given by

$$t_1 = 0, \qquad t_2 = \frac{\pi}{\omega_d}, \qquad t_3 = 2t_2 \tag{11a}$$

$$\begin{aligned} X_1 &= \frac{1}{1+2K+K^2} \\ X_2 &= \frac{2K}{1+2K+K^2} \\ X_3 &= \frac{K^2}{1+2K+K^2} \end{aligned} \tag{11b}$$

where K is as in (8). Such a procedure of considering robustness could be extended further. By including the second derivative of the zero residual vibration constraints, the parameters of a four impulses train can be calculated as follows

$$\begin{aligned} t_1 &= 0, \qquad t_2 = \frac{\pi}{\omega_d} \\ t_3 &= 2t_2, \qquad t_4 = 3t_2 \end{aligned} \tag{12a}$$

$$\begin{aligned} X_1 &= \frac{1}{1+3K+3K^2+K^3} \\ X_2 &= \frac{3K}{1+3K+3K^2+K^3} \\ X_3 &= \frac{3K^2}{1+3K+3K^2+K^3} \\ X_4 &= \frac{K^3}{1+3K+3K^2+K^3} \end{aligned} \tag{12b}$$

where K is given in (8).

To simplify the notations, for the single mode case, the corresponding amplitudes and the constants of time in (8), (11), or (12) of the $\bar{N}$ ($\bar{N} = 2, 3$, or 4) impulses can also be expressed in a general form as follows:

$$X_i = \frac{\begin{pmatrix} \bar{N}-1 \\ i-1 \end{pmatrix} K^{i-1}}{\sum_{j=0}^{\bar{N}-1} \begin{pmatrix} \bar{N}-1 \\ j \end{pmatrix} K^j}, \qquad t_i = (i-1)\frac{\pi}{\omega_d}, \qquad i = 1, \ldots, \bar{N} \tag{13}$$

where X_i is the amplitude of the ith impulse, t_i is the time of the ith impulse, and K is given in (8).

The input shaper impulse sequences can be generalized to accommodate more than one vibration mode, by convolving the impulse sequences for each individual mode with one another. Let the input with $\bar{N}_j$ ($j = 1, \ldots, n$) ($n > 1$) impulses be used in the jth mode. After necessary convolutions, the input impulse sequences X_{mult} can be expressed by

$$X_{\text{mult}} = X_{1s} * X_{2s} \cdots * X_{ns} \tag{14}$$

where X_{js} is the impulse sequences of the jth mode of the system with $\bar{N}_j$ impulses, and $*$ is the convolution operator.

In this manner, for a vibratory system, the described impulse sequence can be convolved to an arbitrary input, so as to obtain the same vibration-reducing properties of the impulsive input case. In addition, the same expressions that guarantee the vibration-reducing properties with constraints on frequency also guarantee the vibration-reducing properties with respective to damping ratio. Moreover, high variations in damping ratio can be tolerated; see [1]. However, the major drawback of the input shaping via open-loop controllers is its limitation in coping with parameter changes and disturbances, because this technique requires relatively precise knowledge of the dynamics of the system. If the models have parametric uncertainties, system performances would not result in zero residual vibration. Although several design approaches have been proposed in the literature to improve the robustness of input shaping for the damping factors and natural frequencies of the flexible structure [6, 7, 9], it should be pointed out that a linear plant is essential for the input shaping technique to work. However, the model of the flexible spacecraft considered in Section II includes nonlinear terms, such as unstructured uncertainties and coupling effect on the right-hand side of (4). Therefore, we proposed to employ the input shaping technique in conjunction with SMC method for maneuvers of flexible spacecraft. In the remainder of this paper, we show that such a new architecture provides very good control performance. In the next section, we give systematic design procedure for the desired sliding mode controller.

IV. SLIDING MODE/INPUT SHAPING CONTROL

In order to improve the robustness and performance of the input shaping method, a sliding mode controller combined with input shaping is presented, as shown in Fig. 1. The impulse shaper is implemented outside of the feedback loop, which is designed for the reference model and achieves the exact elimination of residual vibration. The amplitudes and instances of impulse application can be obtained from (13) with the natural frequency ω_{jm} and the damping ratio ς_{jm} of the reference model, respectively. The feedback controller based on SMC is designed to make the closed-loop system behave like the reference model with shaper so as to eliminate the residual vibrations. This is an effective method for implementing input shaping while achieving satisfactory performance and robustness even when parameter variations and external disturbance occur simultaneously in the process.

From Fig. 1, the system (4) can be rewritten as

$$\dot{x} = Ax + Bu(t) + Bu_r + Bd(t) \tag{15}$$

where $\bar{u}(t) = u(t) + u_r(t)$.

Here, the reference model is selected as the nominal system. The combination of the input shaper convolving with the reference model dynamics can be described by

$$\dot{x}_m = A_m x_m + B_m u_r \tag{16}$$

where A_m and B_m are the known matrices of the nominal system.

According to the principle of the input shaping technique, the shaped input u_r can be expressed as

$$u_r = r(t) * X_{\text{mult}} \tag{17}$$

where $*$ is the convolution operator.

Applying the convolution operator, (17) can be rewritten as

$$u_r = \left(\prod_{j=1}^{n} X_{j,1}\right) r(t) + X_{1,2}\left(\prod_{j=2}^{n} X_{j,1}\right) r(t - t_{1,2}) + \cdots + \left(\prod_{j=1}^{n} X_{j,\bar{N}_j}\right) r\left(t - \sum_{j=1}^{n} t_{j,\bar{N}_j}\right) \tag{18}$$

For the sake of simplification, (18) can be written as

$$u_r = \sum_{k=0}^{2n-1} a_k r(t - t_k) \tag{19}$$

where

$$a_0 = \prod_{j=1}^{n} X_{j,1}, \qquad a_1 = X_{1,2}\prod_{j=2}^{n} X_{j,1}, \ldots, a_{2n-1} = \prod_{j=1}^{n} X_{j,\bar{N}_j}$$

$$t_0 = 0, \qquad t_1 = t_{1,2}, \ldots, t_{2n-1} = \sum_{j=1}^{n} t_{j,\bar{N}_j}.$$

Substituting (19) into (16) and (17) yields the following equations:

$$\dot{x} = Ax + Bu(t) + B\left[\sum_{k=0}^{2n-1} a_k r(t - t_k)\right] + Bd(t) \tag{20}$$

$$\dot{x}_m = A_m x_m + B_m\left[\sum_{k=0}^{2n-1} a_k r(t - t_k)\right]. \tag{21}$$

The parameter variations of the system considered here are defined as follows. The parameter uncertainties of the system ΔA can be expressed as

$$\Delta A = A - A_m. \tag{22}$$

It is noted that in our paper the uncertainty matrix ΔA does not need to satisfy the so-called matching conditions. However, the uncertainty in the input is assumed to satisfy the matching condition and can be expressed as

$$\Delta B = B - B_m = B_m D_B. \tag{23}$$

Letting the error be $e(t) = x(t) - x_m(t)$, it follows from (20) and (21) that the error dynamics obey

$$\dot{e} = (A_m + \Delta A)e + B_m(I + D_B)u + \Delta A x_m + B_m D_f \tag{24}$$

where $D_f = D_B \sum_{k=0}^{2n-1} a_k r(t - t_k) + (I + D_B)d$.

Suppose that the attitude angle and angular velocity are measurable and that the elastic modes are unavailable. The measurement available for the controller design can be expressed in the output form as

$$y = Ce \tag{25}$$

where $y \in R^p$ and C are matrices of appropriate dimensions.

Throughout the rest of this paper, the following assumptions are taken to be valid.

Assumption 1 The triplet (A_m, B_m, C) is controllable and observable.

Assumption 2 There exist known constants δ_A and δ_B such that the uncertainties ΔA and ΔB in (22) and (23) are known but bounded by $\|\Delta A\| \le \delta_A$ and $\|D_B\| \le \delta_B$.

REMARK 1 If it is possible to design a control law that makes the error dynamics (24) have a stable zero steady state solution, i.e., $e(t) \to 0 \Rightarrow x(t) \to x_m(t)$, then the closed-loop system will exactly eliminate the residual vibration like the reference model. Note that SMC provides good ability to reject disturbances and remains robust to parameter perturbations while tracking a desired trajectory, and SMC can be suitably used to deal with the problem addressed in this paper.

In order to simplify the development of the control design scheme, the following state transformation is applied:

$$\bar{e}(t) = Te(t) \tag{26}$$

with $T^{-1} = T^T$.

Then, the transformed equations with $\bar{e}^T = [\bar{e}_1^T \ \bar{e}_2^T]^T$, $\bar{e}_1 \in R^{n-m}$ and $\bar{e}_2 \in R^m$ are given as follows

$$\dot{\bar{e}}_1 = (A_{m11} + \Delta A_{11})\bar{e}_1 + (A_{m12} + \Delta A_{12})\bar{e}_2 + [\Delta\bar{A}_{11} \quad \Delta\bar{A}_{12}]x_m(t) \tag{27a}$$

$$\dot{\bar{e}}_2 = (A_{m21} + \Delta A_{21})\bar{e}_1 + (A_{m22} + \Delta A_{22})\bar{e}_2 + [\Delta\bar{A}_{21} \quad \Delta\bar{A}_{22}]x_m(t) + B_2(I + D_B)u + B_2 D_f \tag{27b}$$

$$y = CT^T\bar{e} = C_1\bar{e}_1 + C_2\bar{e}_2 \tag{27c}$$

where

$$\begin{bmatrix} A_{m11} & A_{m12} \\ A_{m21} & A_{m22} \end{bmatrix} = TA_m T^T, \qquad \begin{bmatrix} \Delta A_{11} & \Delta A_{12} \\ \Delta A_{21} & \Delta A_{22} \end{bmatrix} = T\Delta A T^T$$

$$\begin{bmatrix} \Delta\bar{A}_{11} & \Delta\bar{A}_{12} \\ \Delta\bar{A}_{21} & \Delta\bar{A}_{22} \end{bmatrix} = T\Delta A, \qquad \text{and} \qquad TB_m = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$$

and B_2 is invertible. Note that the variables $\bar{e}_1$ and $\bar{e}_2$ are introduced just to design the controller conveniently and there is no practical meaning for this definition.

A. Sliding Hyperplanes

In sequel, output feedback SMC law is designed to drive the maneuver and guarantee the stability of the compound system. It is well known that there are two major steps involved in designing an SMC system. The first step is the determination of switching hyperplanes such that the system would have the desired behavior once the state enters the switching hyperplanes. The second is the design of control law such that the sliding mode occurs on the switching surface. The design procedure of sliding mode controller is stated as follows.

The sliding hyperplanes are introduced as [12, 26]

$$S = (GC_2)^{-1}Gy = (GC_2)^{-1}GC_1\bar{e}_1 + \bar{e}_2 \tag{28}$$

where $S \in R^m$ and (GC_2) is assumed to be invertible. The matrix $G \in R^{m\times p}$ is selected by the designer. Equation (28) can be rewritten to express $\bar{e}_2$ in terms of $\bar{e}_1$ and S as

$$\bar{e}_2 = S - (GC_2)^{-1}GC_1\bar{e}_1. \tag{29}$$

Substituting (29) into (27) gives

$$\dot{\bar{e}}_1 = A_r\bar{e}_1 + (A_{12m} + \Delta A_{12})S + [\Delta\bar{A}_{11} \quad \Delta\bar{A}_{12}]x_m(t) \tag{30a}$$

$$A_r = A_{11m} - A_{12m}(GC_2)^{-1}GC_1 + \Delta A_{11} - \Delta A_{12}(GC_2)^{-1}GC_1 \tag{30b}$$

where G must be chosen to ensure that the real parts of the eigenvalues of the reduced-order matrix A_r are negative, i.e., A_r is stable. There are bounded uncertainties associated with the nominal matrix A_r. In the absence of uncertainties, conditions were given in [12], [26] for how to choose G such that $(n-m)$ prescribed non-zero and complex eigenvalues $\{-\lambda_1, -\lambda_2, \ldots, -\lambda_{n-m}\}$ with $\mathrm{Re}(\lambda_i) > 0$ $(i = 1,2,\ldots, n-m)$ can be assigned, namely, arbitrary pole placement is possible if

$$\mathrm{rank}[C_2(GC_2)^{-1}G - I] \le p - m. \tag{31}$$

It is easy to see that, if this condition (31) is not satisfied, it may still be possible to achieve stable poles even in the presence of uncertainty, but such a pole assignment may not be arbitrary. Having determined the stable A_r matrix, the eigenvalues of A_r can be grouped as $\{-\lambda_1, -\lambda_2, \ldots, -\lambda_{n-m}\}$. It can be shown that

$$\|\exp(A_r t)\| \le \gamma \exp(-\lambda_{\min} t) \tag{32}$$

where $\lambda_{\min} > 0$ is the minimum real part of the λ_i and $\gamma > 0$. Thus, what remains to deal with is to determine $\lambda_{\min}$ given a stable set of eigenvalues.

LEMMA 1 *Consider* (30a). *Let* $\lambda_{\min} > 0$ *be the minimum real part of* $\{-\lambda_1, \lambda_2, \ldots, \lambda_{n-m}\}$. *Then we have the following.*

1) $\|\exp(A_r t)\| \le \gamma\exp(-\lambda_{\min} t)$ *for some* $\gamma > 0$.

2) $\|\bar{e}_1\|$ *is bounded by* $w(t)$ *after a finite period of time with* $w(t)$ *being the solution of*

$$\dot{w}(t) = -\lambda_m w(t) + \gamma[\|(A_{12m} + \Delta A_{12})\|\|S\| + d^*] \tag{33}$$

where $w(0) > 0$, $\lambda_w < \lambda_{\min}$ *and* $\|[\Delta\bar{A}_{11} \quad \Delta\bar{A}_{12}]x_m(t)\| < d^*$.

PROOF The proof is straightforward and follows a similar line of those in [23], [24].

REMARK 2 Since $\Delta\bar{A}_{11}$ and $\Delta\bar{A}_{12}$ are bounded, and $x_m \to 0$ as $t \to \infty$, the last term in (30a) will decay to zero as $t \to \infty$, and therefore the equilibrium point of the system (30a) will not be affected. Subsequently, the term $\|[\Delta\bar{A}_{11} \ \ \Delta\bar{A}_{12}]x_m(t)\|$ is bounded.

B. Sliding Mode Output Feedback Controller Design

Once a proper switch hyperplane has been chosen, it is time to design a SMOFC such that the system state is driven to the sliding hyperplanes described by (28). To guarantee the sliding mode condition $S \equiv 0$, we define the following Lyapunov function

$$V = \frac{1}{2}S^T S. \tag{34}$$

Differentiating the Lyapunov function V gives

$$\dot{V} = S^T\dot{S} = S^T[P\bar{e}_1 + QS + R + B_2(I + D_B)u] \tag{35}$$

where

$$P = (GC_2)^{-1}GC_1[A_{11m} + \Delta A_{11} - (A_{12m} + \Delta A_{12})(GC_2)^{-1}GC_1] + A_{21m} + \Delta A_{21} - (A_{22m} + \Delta A_{22})(GC_2)^{-1}GC_1$$

$$Q = (GC_2)^{-1}GC_1(A_{12m} + \Delta A_{12}) + (A_{22m} + \Delta A_{22})$$

$$R = (GC_2)^{-1}GC_1[\Delta\bar{A}_{11} \quad \Delta\bar{A}_{12}]x_m(t) + [\Delta\bar{A}_{21} \quad \Delta\bar{A}_{22}]x_m(t) + B_2 D_f.$$

From the above analysis, it is observed that the matrix $B_2(I + D_B)$ in front of u in (35) poses a challenge for sliding mode controller design for the system given by (27). To solve the problem, here, we suppose that B_2 is invertible without loss of any generality, and a swapping technique similar to that of

Khan [24] is proposed here by rearranging $B_2(I + D_B)$ to the form of $(I + \bar{D}_B)B_2$ with $\bar{D}_B = (B_2 D_B)(B_2)^{-1}$. To this end, (35) becomes

$$\dot{V} = S^T \dot{S} = S^T[P\bar{e}_1 + QS + R + (I + \bar{D}_B)B_2 u]. \quad (36)$$

In order to make sure that $\dot{V} < 0$ holds and account for the uncertainties, in the following theorem, we propose a control law that drives the uncertain system onto the sliding mode $S = 0$.

THEOREM 1 *Consider the uncertain system* (27) *subject to Assumption* 1 *and* 2. *Let the elements of* $\bar{D}_B$ *in* (27b) *satisfy* $|(\bar{D}_B)_{ij}| \le (\bar{\bar{D}}_B)_{ij}$ $i,j = 1,2,\ldots,m$. *Assume that the magnitude of the largest eigenvalue of* $\bar{\bar{D}}_B$ *is less than* 1. *If the sliding mode controller is chosen to be*

$$u = (B_2)^{-1}(I - \bar{\bar{D}}_B)^{-1}[-\|P\|\bar{e}_1(t) - \|Q\|\|S\| - \|R\| - \eta]\mathrm{sgn}((S)) \quad (37)$$

with $\eta_i > 0$, $i = 1,2,\ldots,m$, *where* $\mathrm{sgn}(S) = \mathrm{sgn}[s_1 \cdots s_m]^T$

$$\mathrm{sgn}(XX) = \begin{cases} 1 & if \quad (XX) > 0 \\ 0 & if \quad (XX) = 0 \\ -1 & if \quad (XX) < 0 \end{cases} \quad (38)$$

then $\dot{V} \le -\sum_{i=1}^m \eta_i |s_i| < 0$ *holds and the system reaches sliding mode in finite time. Finally, the closed-loop system is globally stable.*

PROOF Defining matrices $K \stackrel{\Delta}{=} (I - \bar{D}_B)^{-1}[\|P\bar{e}_1(t)\| + \|Q\|\|S\| + \|R\| + \eta]$, we can simplify the controller (37) to

$$u = -B_2^{-1}K(t)\mathrm{sgn}(S). \quad (39)$$

Substituting control u in (39) into (36) yields

$$\dot{V} = S^T[P\bar{e}_1 + QS + R - (I + \bar{D}_B)K(t)\mathrm{sgn}(S)]. \quad (40)$$

Form the definition of K, the reaching conditions of $s_i \dot{s}_i < -\eta_i |s_i|$ $(i = 1,2,\ldots,m)$ are satisfied.

However, for the control law (37), the vector $\bar{e}_1$ is not available from measurement. To circumvent this problem, an auxiliary variable is needed to avoid the measurement of $\bar{e}_1$ in the final sliding mode controller. Hence, we propose the following SMOFC law

$$u = (B_2)^{-1}(I - \bar{\bar{D}}_B)^{-1}[-Hw(t) - \Pi\|S\| - \delta - \eta]\mathrm{sgn}((S)) \quad (41)$$

where $H \ge \|P\|$, $\Pi \ge \|Q\|$, $\delta_i \ge \sup(R)_i$, $\eta_i > 0$, $i = 1,2,\ldots,m$, and $\bar{\bar{D}}_B$ satisfies the assumptions given in Theorem 1. The following theorem shows that the proposed control law in (41) drives the mismatched uncertain system onto the sliding mode $S(t) = 0$.

THEOREM 2 *Consider the uncertain system* (27) *subjected to Assumption* 1 *and* 2. *If the input control* $u(t)$ *in* (27) *is given as that indicated by* (41), *then* $\dot{V} \le -\sum_{i=1}^m \eta_i |s_i| < 0$ *holds and the system reaches sliding mode in finite time. Finally, the closed-loop system is globally stable.*

PROOF It can be seen from $S = (GC_2)^{-1}Gy$ that the controller (41) does not require the state measurement. Defining vectors $f \stackrel{\Delta}{=} P\bar{e}_1 + QS + R$ and $\bar{K} \stackrel{\Delta}{=} (I - \bar{\bar{D}}_B)^{-1} \cdot [Hw(t) + \Pi\|S\| + \delta + \eta]$, (41) can be written as

$$u = -B_2^{-1}\bar{K}(t)\mathrm{sgn}(S). \quad (42)$$

Substituting the control u in (42) into (36) yields

$$\begin{aligned}\dot{V} &= S^T[f - (I + \bar{D}_B)\bar{K}(t)\mathrm{sgn}(S)] \\ &= \sum_{i=1}^m s_i \Bigg[f_i - \sum_{j \ne i}(\bar{D}_B)_{ij}\bar{K}_j(t)\mathrm{sgn}(s_j) \\ &\qquad - [1 + (\bar{D}_B)_{ii}]\bar{K}_i(t)\mathrm{sgn}(s_i) \Bigg]. \end{aligned} \quad (43)$$

It follows that the sliding condition holds if

$$[1 - (\bar{\bar{D}}_B)_{ii}]\bar{K}_i(t) \ge L_i + \eta_i + \sum_{j \ne i}(\bar{\bar{D}}_B)_{ij}\bar{K}_j(t), \quad i = 1,2,\ldots,m \quad (44)$$

with $\bar{\bar{D}}_B$ defined in (37) and $\|f_i\| \le Hw(t) + \Pi\|S\| + \delta_i \stackrel{\Delta}{=} L_i$.

Note that the sliding condition is also true if the vector $\bar{K}(t)$ is chosen such that

$$[1 - (\bar{\bar{D}}_B)_{ii}]\bar{K}_i(t) = L_i + \eta_i + \sum_{j \ne i}(\bar{\bar{D}}_B)_{ij}\bar{K}_j(t), \quad i = 1,2,\ldots,m. \quad (45)$$

Equation (45) contains a set of m equalities with m switching gains $\bar{K}_i$. Using matrix notation, (45) is equivalent to

$$[I - \bar{\bar{D}}_B]\bar{K}(t) = L + \eta. \quad (46)$$

Under the assumption that the magnitude of the largest eigenvalue of $\bar{\bar{D}}_B$ is less than 1, by using the Frrobenius-Perron theorem [30], we can see that a unique and positive solution for $\bar{K}(t)$ exists and is given by

$$\bar{K}(t) = [I - \bar{\bar{D}}_B]^{-1}(L + \eta). \quad (47)$$

Hence, controller (42) follows from (47). Substituting (47), or equivalently (45), into (43) and noticing

$w(t) \geq \|\bar{e}_1(t)\|$ from Lemma 1, it can be verified that

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{m} s_i \left\{ f_i - \sum_{j\neq i} (\bar{D}_B)_{ij} \bar{K}_j(t)\operatorname{sgn}(s_j) - \frac{[1+(\bar{D}_B)_{ii}]}{[1-(\bar{\bar{D}}_B)_{ii}]} \left[L_i + \eta_i + \sum_{j\neq i} (\bar{\bar{D}}_B)_{ij} \bar{K}_j(t) \right] \operatorname{sgn}(s_i) \right\} \\
&= \sum_{i=1}^{m} s_i \left\{ f_i - \frac{[1+(\bar{D}_B)_{ii}]}{[1-(\bar{\bar{D}}_B)_{ii}]} [L_i + \eta_i] \operatorname{sgn}(s_i) - \sum_{j\neq i} \left\{ (\bar{D}_B)_{ij} \bar{K}_j(t)\operatorname{sgn}(s_j) + \frac{[1+(\bar{D}_B)_{ii}]}{[1-(\bar{\bar{D}}_B)_{ii}]} (\bar{\bar{D}}_B)_{ij} \bar{K}_j(t)\operatorname{sgn}(s_i) \right\} \right\} \\
&\leq -\sum_{i=1}^{m} \eta_i |s_i|.
\end{aligned} \tag{48}$$

Hence, the sliding mode can be reached in finite time. The time to reach sliding mode is determined by η_i. Obviously, the larger the η_i, the shorter the time will be to reach the sliding mode.

In practice, a discontinuous control law such as (41) usually leads to an undesirable chattering of system variables because of the non-zero time required for control switching. However, the chattering can be eliminated, which is of great practical importance, by the use of a continuous approximation of the control law (19) instead. One such approximation is given as follows (see [31])

$$\tanh(\beta\sigma) = \frac{e^{\beta\sigma} - e^{-\beta\sigma}}{e^{\beta\sigma} + e^{-\beta\sigma}}. \tag{49}$$

Using this approximation, the control law in (41) can be modified as

$$u = (B_2)^{-1}(I - \bar{\bar{D}}_B)^{-1}[-\|H\|w(t) - \Pi\|S\| - \delta - \eta]\tanh(\beta S). \tag{50}$$

Note that the hyperbolic tangent function is continuously differentiable (with respect to S) and, as $\beta \to \infty$, we have $\tanh(XX) \to \operatorname{sgn}(XX)$, so (50) tends to (41) in the limit. It is being used here to illustrate that a continuous approximation to the discontinuous SMC law can alleviate undesirable chattering, without incurring a significant loss of the performance achieved by the original designed control law. Other approaches to chattering attenuation, such as those described by Edwards and Spurgeon [30], can also be considered.

V. SIMULATION RESULTS

In order to demonstrate the effectiveness of the proposed control schemes, numerical simulations have been performed and presented in this section. The key technical indexes of flexible spacecraft used in the simulation are given in [33]. In this paper, the reference model is the normal system with the first two low-order modes of five, $\omega_{1m} = 3.161$ and $\omega_{2m} = 16.954$ rad/s, and the damping ratio $\varsigma_{1m} = \varsigma_{2m} = 0.001$, respectively. The relations between the parametric uncertainty, the actual natural frequencies ω_i, and the normal natural frequencies ω_{im} can be expressed as follows:

$$|\omega_i^2 - \omega_{im}^2| \leq \delta_A \omega_{im}^2, \qquad i = 1,2. \tag{51}$$

Assume that $\Delta B = B_m \cdot 0.5\sin(4t)$, the uncertainty input $D_B = 0.1$ is less than the upper bound of input variation $\delta_B = 1$, and the external disturbance $d(t)$ is a random disturbance torque given by

$$d(t) = d_{\max}\mathcal{N}(XX) \tag{52}$$

where the maximum absolute $d_{\max}$ is fixed to 0.1 Nm; $\mathcal{N}(XX)$ denotes the normal distribution with mean zero and standard deviation one.

In this simulation, the flexible spacecraft is commanded to perform a 60 deg slew. The vibration energy level is described by

$$E = \dot{q}^T\dot{q} + q^T K_{qq} q. \tag{53}$$

In the numerical simulation, the input shaper is the convolved four impulses zero-vibration-derivative-derivative (ZVDD)-shaper for the first mode and two impulses zero-vibration (ZV)-shaper for the second mode. For the purpose of comparison, four cases are considered as follows:

1) attitude maneuver control using only PD controllers; and PD control+input shaping (IS);
2) attitude maneuver control using only proposed variable structure output feedback controllers (41) or (50), and the proposed SMOFC+IS;
3) attitude maneuver control using the traditional sliding mode output feedback controller given in [21], and also traditional SMOFC+IS;
4) attitude maneuver control using the near minimum-time maneuvering control [27].

All computations and plots shown in the paper are performed using MATLAB/Simulink software package.

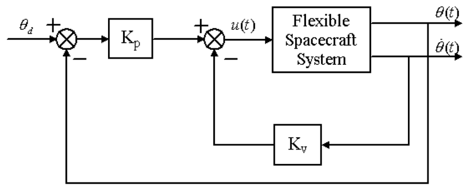

Fig. 3. PD control structure.

A. PD Control and PD+IS

To demonstrate the performance of the vibration control schemes, a PD feedback control of collocated sensor signals is adopted for control of rigid body motion of the spacecraft A block diagram of the PD controller is shown in Fig. 3, where K_p and K_v are the proportional and derivative gains, respectively, θ is the hub angle, $\dot{\theta}$ is the hub velocity, and θ_d is the reference hub angle. Essentially, the task of the controller design is to maneuver the flexible spacecraft to the specified angle of demand. The hub angle and hub velocity signals are fed back and used to control the hub angle for the spacecraft.

The control signal $U(s)$ in Fig. 3 can be obtains as

$$U(s) = K_p[\theta_d(s) - \theta(s)] - K_v\dot{\theta} \tag{54}$$

where s is the Laplace variable. The closed-loop transfer function is obtained as

$$\frac{\theta(s)}{\theta_d(s)} = \frac{K_pG(s)}{1 + K_v[s + (K_p/K_v)]G(s)} \tag{55}$$

where $G(s)$ is the open-loop transfer function from the input torque to the hub angle of the system. Thus, the closed-loop poles of the system satisfy the characteristic equation

$$1 + K_v(s + Z)G(s) = 0 \tag{56}$$

where $Z = K_p/K_v$ represents the compensator zero that determines the control performance of the closed-loop system. In this study, a root locus approach is utilized to design the PD controller. The proportional gain and the derivative gain of the PD controller for attitude control are 15 and 50, respectively. The corresponding hub angle and velocity of the spacecraft, modal vibrations, and the required control torque of response using the PD control are shown in Fig. 4. It is noted that an acceptable hub angle response is achieved. The spacecraft reached the demanded angle with a settling time about 30 s without overshoot. However, a significant amount of vibration occurred during the maneuvering of the flexible spacecraft as demonstrated in the vibration energy's plot in Fig. 4. It should be noted that the effect of disturbance is not considered in this case (when involving the disturbance, the system is not easy to be stabilized or has big overshooting, the time response is not given for space limitation).

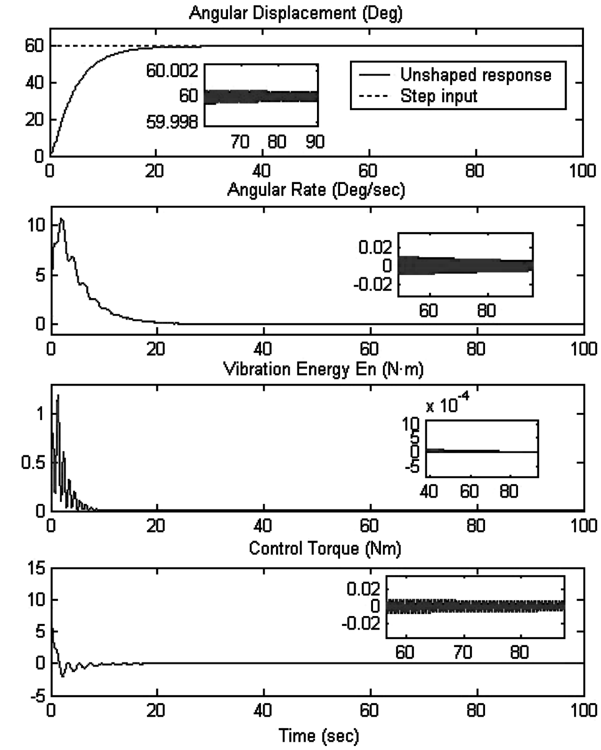

Fig. 4. Time response for using PD control case.

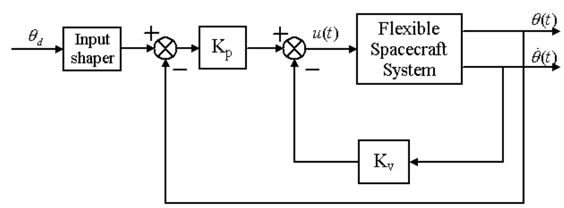

Fig. 5. PD with input shaper structure.

In order to actively suppress the modal vibration, a hybrid control structure for control of rigid body motion and vibration suppression of the flexible appendages using PD control with active vibration reduction technique based on shaping is presented here. A block diagram of the hybrid control scheme is shown in Fig. 5. In this case, the PD controller parameters for the attitude control remain the same for a fair comparison, and four impulses ZVDD-shaper for the first mode and two impulses ZV-shaper for the second mode are implemented. Fig. 6 shows the results of employing the PD controller with input shaper. It is clear from the top plot of Fig. 6 that the imposed desired angular displacement is accurately achieved by employing the hybrid law. The relatively large amplitude vibrations excited by rapid maneuvers can be actively suppressed, as shown in the third plot from the top of Fig. 6. This reflects the effectiveness of the input shaping for active vibrations suppression. It should be noted in this case that the disturbance is also not involved (when considering the disturbance, the system can hardly be stabilized or has big overshooting).

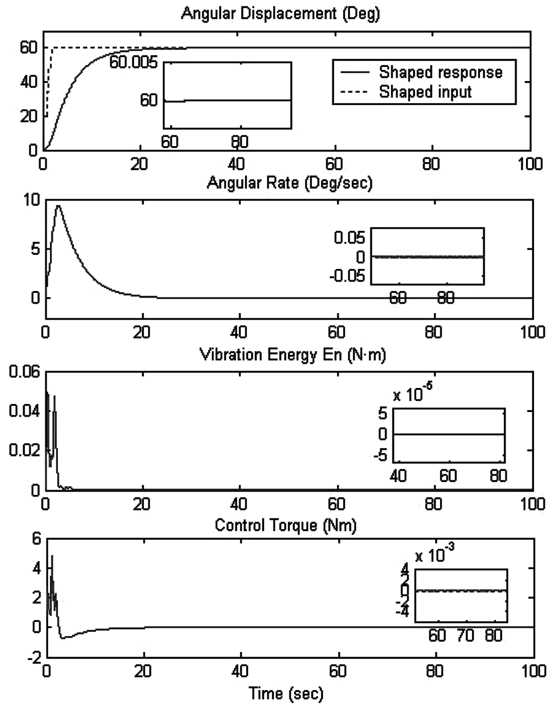

Fig. 6. Time response of using PD+IS technique case.

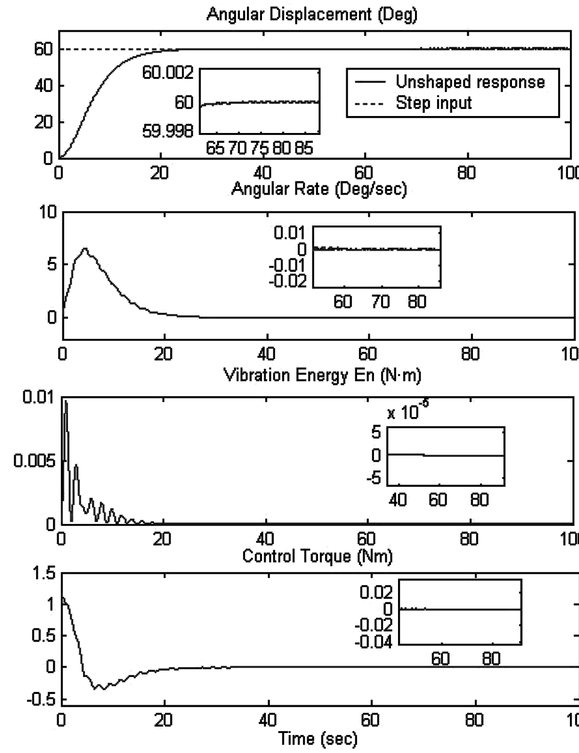

Fig. 7. Time response of using proposed SMOFC case.

B. Sliding Mode Output Feedback Control and SMOFC+IS

Fig. 7 shows the results of implementing only the proposed sliding mode output feedback controller acting on the rigid hub in the presence of mismatched uncertainty. It is clear from the plot of the top of Fig. 7 that the imposed desired angular displacement is accurately achieved by employing the SMOFC law. From the comparison between Fig. 4 and Fig. 7, it can be observed that the relatively large amplitude vibrations excited by rapid maneuvers can be passively suppressed. Nevertheless, from Fig. 7, we can see that the inner torque of each flywheel approaches zero at the time of 30 s, but with some little chattering.

Even though the sliding mode controller can suppress the relatively large amplitude vibrations induced by rapid maneuvers, some residual micro-vibrations may be present. It is sometimes desirable to further suppress the residual micro-vibrations for precision pointing/targeting of advanced spacecraft. It also has been demonstrated that input shaping technique can provide such a control effort by modifying the input command form. Therefore, it is suggested that the technique of active vibration control using input shaping should be used in conjunction with attitude controller in order to further improve and finely tune the system performance. The block diagram for this case is shown in Fig. 1.

In this case, the SMOFC parameters for the attitude control remain the same for a fair comparison, and four impulses ZVDD-shaper for the first mode and two impulses ZV-shaper for the second mode are also implemented. Fig. 8 shows the results of employing this combination. It is clear from the top plot of Fig. 8 that the imposed desired angular displacement is accurately achieved by employing the hybrid law in the presence of the external disturbances. The relatively large amplitude vibrations excited by rapid maneuvers can be actively suppressed, as shown in the third plot from the top of Fig. 8. This further demonstrates the validity of active vibration reduction base on the input shaping technique. Moreover, the chattering can also be reduced in some sense.

For the smoothed control case in (50), the above two tests are also repeated with the same control parameters, and the simulation results are shown in Figs. 9 and 10. Here $\beta = 0.5$ is selected. From the comparison of the case using the unsmoothed control in (41) and the smoothed control in (50), the vibration energy in the latter case is less than the first and the control torque magnitude is also reduced. The smoothed case also generates smoother control action than the case of the nonsmoothed. This reflects the advantages of smooth control over the non-time-varying case. On the other hand, when the input shaping technique is implemented, the vibration energy can be further reduced, as shown in Fig. 10. And also the control chattering can be significantly eliminated.

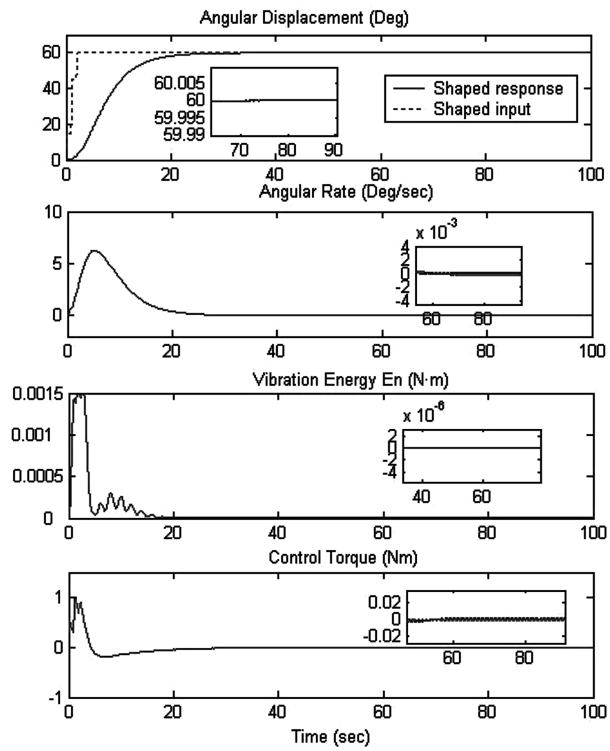

Fig. 8. Time response of using proposed SMOFC+IS case.

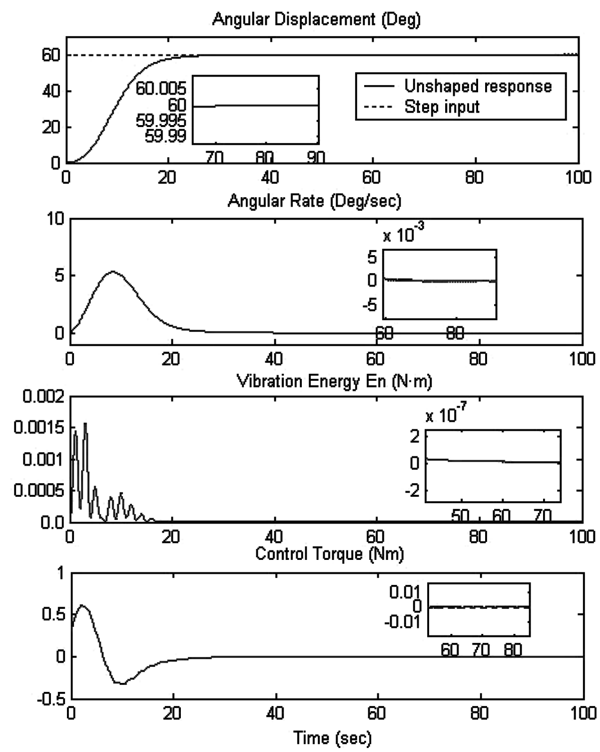

Fig. 9. Time response of using proposed SMOFC with smoothed control case.

C. Traditional Sliding Mode Output Feedback Control and TSMOFC+IS

For comparison, the system is also controlled by using the SMOFC designed in [21]. The same

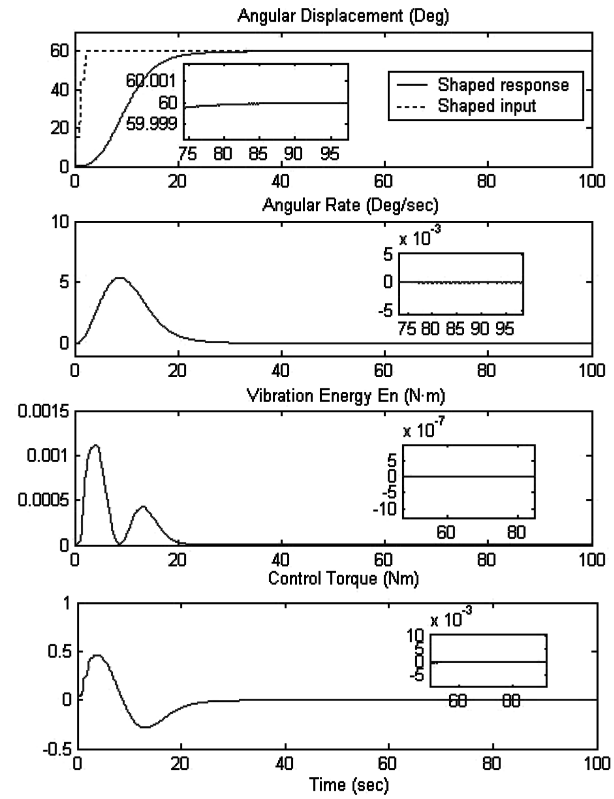

Fig. 10. Time response of using proposed SMOFC+IS with smoothed control case.

simulation case is repeated with the traditional SMOFC (TSMOFC), where the smoothed control compensation replaces the proposed sliding mode feedback control for a fair complexion, and the results of simulation are shown in Fig. 11. For this case, the imposed desired angular displacement can be achieved, but excessive control chattering can be observed in the time response plot of the inner torque of each flywheel even using the smoothed compensation. At the same time, in order to eliminate the vibration corresponding to the flexible appendage, the convolved firstm-mode ZVDD-shaper and second-mode ZV-shaper is also used. The plots of this case are shown in Fig. 12. In this case, the vibration observed in Fig. 12 is more severe even though the active vibration control approach which is adopted than the proceeded cases, and also drastic chattering is observed. This further shows the effectiveness of the proposed SMOFC for the attitude maneuver and the vibration reduction.

D. Near Minimum-Time Maneuvering Control [27]

For the purpose of further comparison, a minimum-time controller design [27] based on a priori reference trajectory is also employed for the system. Note that here the flywheel is used as the actuator in the simulation. The same simulation case is repeated with the near minimum-time controller and the results

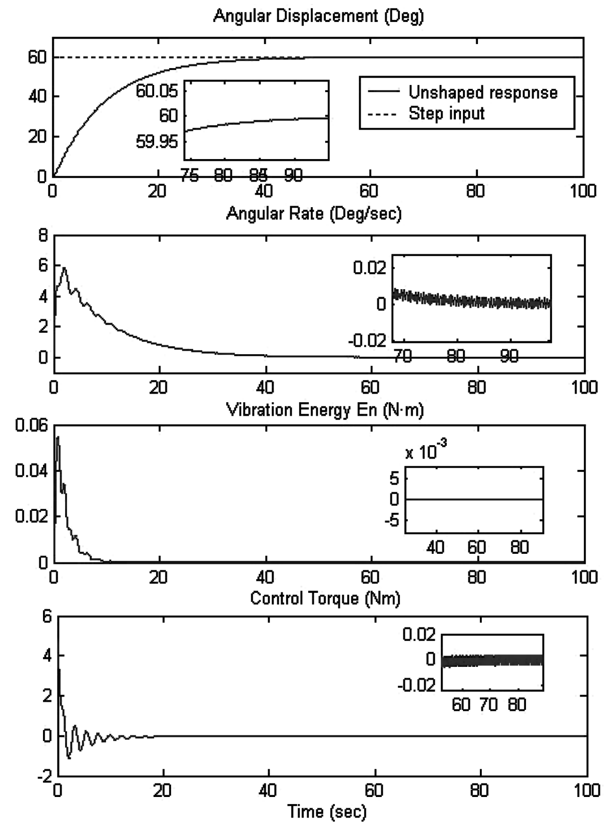

Fig. 11. Time response of using TSMOFC with smoothed control case.

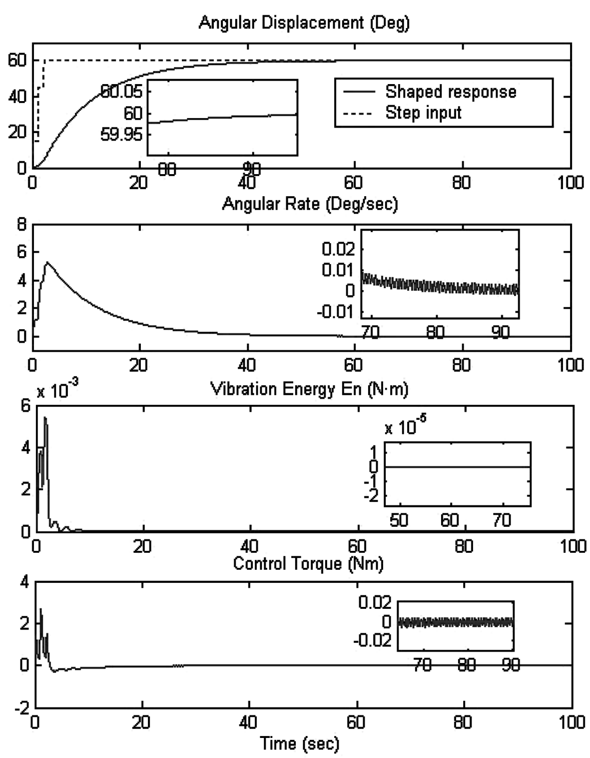

Fig. 12. Time response of using TSMOFC+IS with smoothed control case.

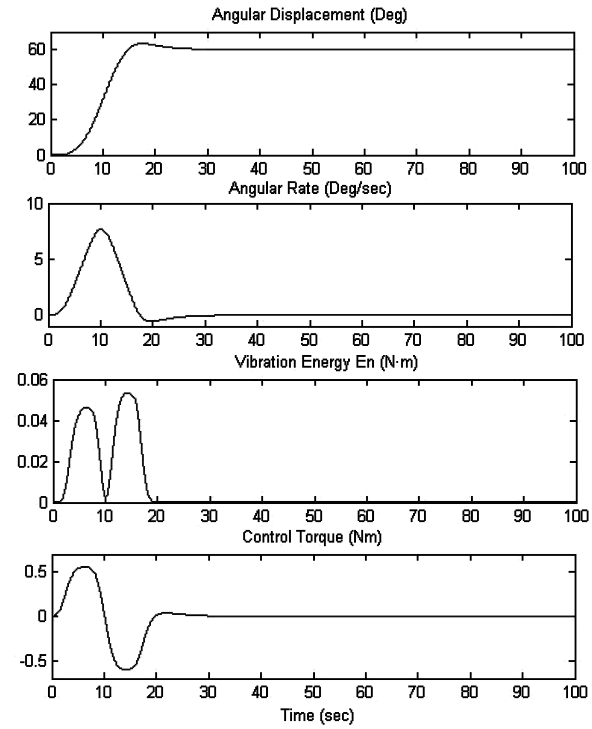

Fig. 13. Time response of using near minimum-time control case.

of simulation are shown in Fig. 13. As one can see in Fig. 13, the maneuvering time is no less than 30 s, but an overshoot in the angle is a result of the control law. Despite the fact that there is not much improvement in the angle response, there still exists some room for improvement with different design parameter sets. It should be noted in this case that the disturbance is also not involved (when the disturbance is involved, the system can hardly be stabilized or the results can further deteriorate).

For the several different control cases, the overall results on settling time, peak vibration energy, peak control torque, and pointing accuracy are approximately summarized in Table I. From the comparison of the above cases, it is shown that the proposed approach can not only accomplish the attitude control during maneuvers, but also simultaneously suppress the undesired vibrations of the flexible appendages even though the uncertainties and disturbances are explicitly considered. This control approach also provides the theoretical basis for the practical application of the advanced control theory to flexible spacecraft attitude control system.

VI. CONCLUSION

In this paper, a new approach for vibration reduction of flexible spacecraft during attitude maneuver operations has been presented. This approach integrates the method of command input shaping and the theory of SMOFC that takes into account parameters uncertainties and external

TABLE I
Performance Comparison on Settling Time, Peak Vibration Energy, Peak Control Torque and Pointing Accuracy

	Settling Time (s)	Peak Vibration Energy (Nm)	Peak Control Torque (Nm)	Pointing Accuracy (deg)
PD	28	1.3	10	0.0015
PD+IS	30	0.05	4.8	0.001
SMOFC	24	0.01	1.2	0.0005
SMOFC+IS	32	0.0015	1.0	0.0001
Smoothed SMOFC	26	0.0015	0.52	0.0001
Smoothed SMOFC+IS	32	0.0012	0.48	0.0001
TSMOFC	54	0.058	5	0.03
TSMOFC+IS	58	0.0058	2.8	0.02
NMTMC	28	0.06	0.52	0.06

Note: NMTMC is near minimum-time maneuvering control.

disturbances provided that the bounded are known. The method of command input shaping has been implemented outside of the feedback loop to modify the existing command so that less vibration will be caused by the command itself. The amplitudes and instances of the impulses application have been obtained, for the natural frequency and damping ratio of the reference model (normal plant), respectively, such that the exact elimination of the residual vibration is achieved for the reference model. Since measurement of elastic modes is not available in the practical case, synthesis of attitude controller using only the attitude and angular rate information feedback has been conducted. The feedback controller based on SMOFC has been designed to make the closed-loop system behave like the reference system with input shaper and also suppress the vibration of the flexible structures in the presence of parametric uncertainty and external disturbances. Simulation results of slew operation of a spacecraft with flexible appendage has demonstrated that, with the command input shaper and the sliding mode output feedback controller, the proposed new approach can significantly reduce the vibration of the flexible beam during slew operations.

Our future research directions include the following: 1) extensions of the proposed algorithms to the case of tracking; 2) combination of these algorithms with some active vibration suppression techniques, such as using piezoelectric materials for further reducing the vibration during and after the maneuver operations; 3) digital implementation of the control scheme on hardware platforms for attitude control experimentation.

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