THE AD-AS MODEL AND ITS SOLUTIONS: SOME NEW RESULTS IN A DISEQUILIBRIUM PERSPECTIVE*

ALESSANDRA CHIRCO
University of Bologna

and

CATERINA COLOMBO
University of Ferrara

Abstract

This paper extends the disequilibrium approach to aggregate demand and aggregate supply to situations in which the level of the fixed nominal wage is compatible with shortage of labour. We show that discontinuities arise in the AD-AS curves and that only a 'one sided', not bilaterally stable equilibrium exists, which corresponds to the underconsumption régime. In order to show its peculiarities, this extended AD-AS framework is then applied to a standard policy problem.

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I. INTRODUCTION

In recent years a fairly long list of papers appeared, discussing the internal consistency of the aggregate demand - aggregate supply (AD-AS) framework for the determination of the equilibrium output and price levels, in presence of nominal wage rigidity.[1] More precisely, the discussion has focussed on the possibility of describing the behaviour of the goods market, by employing simultaneously two schedules which are built under different hypotheses - the sticky price and quantity adjustment assumption of the IS-LM based AD curve, and the flexible price assumption implicit in an AS curve based on the marginal productivity condition (Rao, 1991). This inconsistency shows up immediately when one tries to define such concepts as excess demand or excess supply (Rabin and Birch, 1982). Indeed, rationing in the labour market implies that a quantity signal (income) enters as an argument of the demand function for goods expressed by wage and profit earners. However, while in equilibrium the level of income is unambiguously defined, the question arises of which notion of income should be embodied in the demand curve out of equilibrium. The traditional aggregate demand assumes for all prices the equality between income and aggregate demand; thus, a reduced form for the latter is derived, in which the price level affects (equilibrium) demand through real money balances. Following Rowan (1975), many authors (e.g. Hall and Treadgold, 1982; Fields and Hart, 1990) have suggested an alternative formulation of the demand schedule, such that the argument of the demand function is the planned supplied output (the so called Rowan demand function) - accordingly, an additional channel for aggregate price to affect demand is devised, through its effect on the firms' supply plans. Others have related demand to past income (Smyth, 1989) or to equilibrium income (Henry and Woodfield, 1985).

Some authors have tackled this consistency problem by applying to the disequilibrium demand and supply plans the so called short side rule - see Rao (1986), Dalziel (1993) and. in a different context, by Danthine and Peytrignet (1984). By confining their attention to situations in which the nominal wage is such that unemployment prevails at all prices, Rao and

Dalziel generate a demand curve which has the properties of the traditional equilibrium demand for prices above equilibrium, and the properties of the Rowan demand for prices below equilibrium - the two curves coinciding at equilibrium. On the other side, the existence of unemployment throughout ensures that the firm does not face any constraint on the labour market and that supply is determined by the standard marginal productivity condition.

The main purpose of this paper is to extend what may be called the 'disequilibrium approach' to the AD-AS system to situations in which the nominal wage is assumed to be fixed at a level compatible, at some prices, with shortage of labour: it is rigid both downwards and upwards. This extension is relevant for two main reasons. The first is related to the microfoundations of the assumed nominal rigidities. The by now most commonly accepted justification sees them as the outcome of costly pricing or negotiation procedures which generate a *symmetric* upwards and downwards inertia. These microfoundations are actually developed within a monopolistic competitive setup which mimic the properties of fix-price models: on the one side the fully flexible equilibrium exhibits properties similar to those of disequilibrium models with generalized excess supply (Benassy, 1990); but on the other side, in presence of menu costs large shocks may generate equilibria with excess demand both in the labour and the goods market (Dixon and Rankin, 1994, p.180). The second reason has to do with the market-clearing role of price, when price adjustment is driven by demand and supply functions defined under the disequilibrium hypotheses. Indeed, by allowing for excess demand for labour, we are able to prove that there is a whole range of wages (those lower than the Walrasian one) for which (*a*) the aggregate supply function is discontinuous; (*b*) the equilibrium in the goods market occurs at one extreme of the discontinuity and thus (*c*) a bilaterally stable AD-AS equilibrium does not exist - more precisely, a decrease in price does not drive excess supply to zero.

The paper is organized as follows. In section II those notions and properties of disequilibrium theory are recalled, which will be referred to in the sequel. In section III we develop a very simple microfounded model of aggregate demand and aggregate supply for all possible rationing structures in the labour and goods market, deriving the basic static results. The issue of stability is tackled in section IV, where some comparative static exercises are also performed. In section V very brief conclusions are presented.

II. UNCONSTRAINED AND CONSTRAINED DEMAND AND SUPPLY PLANS

Consider a very simple macroeconomic model with three goods - a composite consumption good Y, labour l, and a non-produced good, money m - and two agents: one household and one (competitive) firm. A third agent, government, exogenously determines the supply of the non-produced good. The household has an initial endowment of money, $\bar{m}$, and working time, $l_0 = 1$. We assume that his preferences are separable and represented by a well-behaved utility function:

$$U = U(Y, 1-l, m/p) \tag{1}$$

defined over consumption, leisure and real money balances, which are net substitutes and normal. The firm's production technology requires only labour and verifies the usual concavity property:

$$Y = F(l) \qquad F(0) = 0, \quad F'(l) > 0, \quad F''(l) < 0 \tag{2}$$

The firm is owned by the household who therefore receives both wage (w) and profit (π) income. Profits are instantaneously distributed.

For future reference we recall that, according to disequilibrium theory, on any given market an agent unconstrained on the other markets formulates a *notional* demand/supply plan which verifies the standard marginal conditions. Thus, from the marginal productivity condition we obtain the notional labour demand and notional output supply of an unrationed firm:

$$l_n^d = F'^{-1}(w/p) = l_n^d(w, p) \tag{3}$$

$$Y_n^s = F[F'^{-1}(w/p)] = Y_n^s(w, p) \qquad \frac{\partial Y_n^s}{\partial p} > 0 \tag{4}$$

An unrationed household maximizes (1) with respect to Y, l and m/p, subject to

$$pY + m + w(1-l) = w + \pi^e + \bar{m}$$

where π^e are the profits expected by the household. The usual marginal conditions yield the *notional* output demand and labour supply functions:

$$Y_n^d = Y_n^d(w, p, w + \pi^e + \bar{m}) \tag{5}$$

$$l_n^s = l_n^s(w, p, w + \pi^e + \bar{m}) \tag{6}$$

which exhibit the standard homogeneity property. Notice that in a Walrasian framework the household believes that, for any announced (w,p) vector, his desired trades will be realized, $\pi^e = pY_n^d - wl_n^s$. By substituting for π^e in the above expressions, we obtain a system which can be solved for the *Walrasian* Y^d and l^s functions (see Negishi, 1979):

$$Y_W^d = Y_W^d\,(w, p, \bar{m})$$

$$l_W^s = l_W^s\,(w, p, \bar{m})$$

On the contrary, according to Benassy's criterion, if an agent perceives a quantity constraint on one market, he formulates an *effective* plan on the other markets, which differ from the notional one in that the additional quantity constraint is added to the standard optimization problem. The AD-AS debate has been developed in a framework in which the nominal wage is fixed and accordingly one side of the labour market is constrained. Therefore, effective plans are put forth on the goods market. We shall specify these plans in the next section. At this stage we simply notice that again the solution of the household's maximization problem will require that some conjectures be made about the transactions which define, *for any given wage and price*, the household's out-of-wage and the out-of-profit income. In a Walrasian framework they coincide with the notional household's desired transactions. But with rationing in the labour market, the household clearly perceives that his notional plans, as defined above, cannot be actually realized: he cannot rationally behave 'as if' the (fixed) wage and (flexible) announced price were the equilibrium ones. Which trades - if not the notional desired trades - should then be consistently included in the budget constraint? The disequilibrium approach answer to this question is to assume that for any price agents formulate their (effective) plans on the goods market, evaluating their budget constraint at the only realizable trades on the labour and goods market at that price. The household's choices then rest on conjectures on trades which are the only ones consistent with the actual trade opportunities at the given price. This is the route we shall follow in section III.

Notice that according to these considerations, with fixed nominal wage aggregate demand can be consistently defined at disequilibrium prices by assuming that even on the goods market trades take place at disequilibrium prices. This requires a careful interpretation of price flexibility within the disequilibrium AD-AS model. There are indeed two possible notions of price flexibility. The first is that implicit in the Walrasian general equilibrium model: prices adjust instantaneously, i.e. in 'virtual' time, so that disequilibrium is unobservable. The second is such that prices adjust in real time, allowing trades to take place in disequilibrium along a path converging - under stability - to the equilibrium. It is in this second perspective that price flexibility must be interpreted within the AD-AS scheme, when the nominal wage is fixed.

III. AGGREGATE DEMAND, AGGREGATE SUPPLY AND DISEQUILIBRIUM RÉGIMES

If the AD and AS curves are to be derived under the disequilibrium hypotheses, the most appropriate starting point is the well known partition of the $(w-p)$ plane into disequilibrium régimes, represented in Figure 1.[2] We recall that E is the Walrasian equilibrium vector, while C, K and R denote respectively the classical unemployment, Keynesian unemployment and repressed inflation areas. By means of Figure 1, a straightforward derivation of aggregate demand and supply schedules in the Y-p plane can be obtained by defining, for any given value of w, the household's and firm's behaviour for the whole range of possible values of p. The diagram makes it clear that a sharp distinction has to be made between situations in which the given w is higher or lower than the Walrasian wage, w^*.

Aggregate demand and supply for $w > w^*$

If $w > w^*$, only two regimes are observable, classical unemployment and Keynesian unemployment - the household is rationed in the labour market for all possible prices. In area C there is excess demand for goods while in area K there is excess supply. Since the firm is always unconstrained on the labour market, for all values of p its supply plan is the notional one, and aggregate supply is given by equation (4).

On the other side, in order to formulate his (effective) demand plan on the goods market the household maximizes the following Lagrangian function:

$$\mathcal{L} = U(Y, 1-l, m/p) + \lambda(w + \pi^e + \bar{m} - w(1-l) - pY - m) + \gamma(\bar{l} - l) \tag{7}$$

where λ and γ are the Lagrange multipliers of the budget constraint and the constraint on the labour market respectively. The constraint $\bar{l}$ being binding, the first order conditions are:

$$\frac{\partial U}{\partial Y} - \lambda p = 0 \qquad\qquad pY + m + w(1-l) = w + \pi^e + \bar{m}$$

$$\frac{\partial U}{\partial l} + \lambda w - \gamma = 0 \qquad\qquad \gamma(\bar{l} - l) = 0$$

$$\frac{\partial U}{\partial m/p} - \lambda p = 0 \qquad\qquad \gamma > 0,\ l = \bar{l}$$

Separability implies that the marginal rate of substitution between the non rationed goods is independent of the quantity constraint, thus the optimal solution yields the following effective demand for goods for all values of p:

$$Y_e^d = Y_e^d(p, w\bar{l} + \pi^e + \bar{m}) \tag{8}$$

which, for given p, w, and π^e is lower than the notional demand, $Y_e^d < Y_n^d$.[3]

By substituting for π^e the model consistent expectation based on realized transactions in disequilibrium, we get

$$Y_e^d = Y_e^d(p,\ pY + \bar{m}) \tag{9}$$

since realized profits, π, are equal to $pY - w\bar{l}$, where Y is the realized output transaction. An equilibrium price $\hat{p}_g = \hat{p}_g(w, \bar{m})$ can then be obtained which equates effective aggregate demand and notional supply:

$$\hat{Y} = Y_n^s(w, p) = Y_e^d(p,\ p\hat{Y} + \bar{m})$$

For any given $w > w^*$, the function $\hat{p}_g(w, \bar{m})$ defines the border between areas C and K in Figure 1. Along that border the goods market clears (in the sense that the *effective* demand (9) defined for all p equals the *notional* supply (4) defined for all p).

For $p < \hat{p}_g$, the realized transaction is equal to supply. Thus, in C equation (8) must be rewritten as

$$\tilde{Y}_{eC}^{d} = \tilde{Y}_{eC}^{d}(p,\ pY_n^s(w, p) + \bar{m}) = \tilde{Y}_{eC}^{d}(w, p, \bar{m}) \tag{10}$$

since in C realized profits are equal to $\pi_C = pY_n^s(w, p) - w\bar{l}$, with $\bar{l} = l_n^d(w, p)$, the notional demand for labour. Equation (10) has the properties of a Rowan demand function and homogeneity implies that $\partial\tilde{Y}_{eC}^{d}/\partial p > 0$ if $\partial Y_n^s/\partial p > \bar{m}/p^2$.

For $p > \hat{p}_g$, the realized transaction is equal to demand. By substituting in (8) profits which are realized in area K, effective aggregate demand becomes

$$\check{Y}_{eK}^{d} = \check{Y}_{eK}^{d}\ (p, \bar{m}) \qquad \frac{\partial \check{Y}_{eK}^{d}}{\partial p} < 0 \tag{11}$$

since in K, $\pi_K = pY_{eK}^d - w\bar{l}$, with $\bar{l} = F^{-1}(Y_{eK}^d)$, the effective demand for labour. Equation (11) is the equivalent in our simplified framework of the textbook (IS-LM based) AD curve. Thus, for $w > w^*$ we obtain the configuration of the AD-AS schedules described in Figure 2:[4] the aggregate demand drawn there is, on its side, the equivalent of the 'hybrid' curve -

in Hall and Treadgold (1987) terminology - described by Rao (1986) and Dalziel (1993).

Aggregate demand and supply for $w < w^*$

If $w<w^*$, all three regions are observable. Areas C and K are separated by the repressed inflation area, R, where excess demand arises both in the goods and the labour market. Indeed, at very low prices, excess demand in the goods market may couple with household's rationing in the labour market - a portion of the classical unemployment area lies below w^* (see Figure 1) and aggregate demand and aggregate supply are there still defined by equations (10) and (4).

In areas C and R the household is rationed on the goods market. Thus for all $(w-p)$ pairs belonging to these regions he formulates his (effective) supply plan on the labour market by maximizing the following Lagrangian function:

$$\mathcal{L} = U(Y, 1-l, m/p) + \lambda(w + \pi^e + \bar{m} - w(1-l) - pY - m) + \mu(\bar{Y} - Y)$$

where λ and μ are the Lagrange multipliers. Given that the constraint on the goods market is binding, the following labour supply function is obtained for C and R:

$$l_e^s = l_e^s(w, p, \pi^e + \bar{m} - p\bar{Y}) \tag{12}$$

where $\bar{Y}$ is the constraint on the goods market.[5] By substituting for π^e the realized profits $p\bar{Y} - wl$, where l is the realized transaction, we get

$$l_e^s = l_e^s(w, p, \ \bar{m} - wl) \tag{13}$$

Notice now that in both C and R areas the demand for labour is the notional one given by equation (3). Since in C there is excess supply and in R excess demand for labour a price $\hat{p}_l(w,\bar{m})$ can be defined, at which the *effective* supply of labour (13) equals the *notional* demand (3):

$$\hat{l} = l_n^d(w,p) \;=\; l_e^s(w,p,\;\bar{m} - w\hat{l})$$

For $w < w^*$, the positive values of the function $\hat{p}_l(w,\bar{m})$ define the boundary between areas C and R in Figure 1.[6] As this boundary is defined by the equality between notional demand and effective supply of labour, this clearly implies that at $\hat{p}_l(w,\bar{m})$ excess demand for goods persists. Indeed for $p > \hat{p}_l(w,\bar{m})$, excess demand arises on both markets - we enter the repressed inflation region.

In area R realized transactions coincide with supply in both markets, and both labour supply plans and goods supply plans are effective plans. Indeed, the peculiarity of this region is that the firm supply plans become constrained by labour supply. The optimal output (effective) supply is thus obtained by maximizing the following Lagrangian function:

$$\mathcal{L} = pF(l) - wl + \nu(\bar{l} - l) \tag{14}$$

where ν is the Lagrange multiplier of the quantity constraint on the labour market. Given that the labour supply constraint is binding, the first order conditions are:

$$F'(l) = w/p + \nu$$

$$\nu(\bar{l} - l) = 0\,, \quad \nu > 0, \quad l = \bar{l}$$

which imply the following effective supply of goods:

$$Y^s_{eR} = F(\bar{l}\,) = F(\check{l}^s_{eR})$$

where $\check{l}^s_{eR}$ is effective supply of labour in repressed inflation (again, the subscript R indicates the relevant régime). The latter is obtained by substituting into (12) $\bar{Y} = F(\check{l}^s_{eR})$ and the value of realized profits in repressed inflation, $\pi_R = pF(\check{l}^s_{eR}) - w\check{l}^s_{eR}$:

$$\check{l}^s_{eR} = \check{l}^s_{eR}(w,p,\bar{m})$$

Thus the aggregate supply function relevant in R can be written as

$$Y^s_{eR} = F\Big(\check{l}^s_{eR}(w,p,\bar{m})\Big) = Y^s_{eR}(w,p,\bar{m}) \tag{15}$$

At this level of generality, the effect of price changes on the effective supply of labour - and accordingly on the effective supply of goods - cannot be signed unambiguously. When a log-linear specification of the utility function is adopted (as in Malinvaud, 1977; Negishi, 1979; Benassy, 1990), the effective supply of labour is independent of p and the aggregate supply turns out to be vertical.

Notice that at the boundary price $\hat{p}_l(w,\bar{m})$ between C and R the effective supply of goods (15) is equal to the notional supply (4). Indeed, if the quantity constraint $\bar{l}$ in the Lagrangian function (14) coincides with the notional plan l^d_n - which is actually the case at the boundary - the effective supply is the same as the notional supply: $\nu=0$ and the notional first order condition holds. Aggregate supply is continuous at $\hat{p}_l$.

The derivation of aggregate demand in R calls for a preliminary observation. The household is not rationed on the labour market; thus, according to Benassy's criterion, his demand for goods is a notional demand (equation (5) repeated here for convenience):

$$Y^d_n = Y^d_n(w,p,w+\pi^e+\bar{m})$$

However, expected profits do not coincide with notional profits: $\pi^e = \pi_R = pF(\check{l}^s_{eR}) - w\check{l}^s_{eR}$. Then, aggregate demand can be written as:[7,8]

$$\hat{Y}^d_{nR} = \hat{Y}^d_{nR}(w,p,\; w+pF[\check{l}^s_{eR}(w,p,\bar{m})] - w\check{l}^s_{eR}(w,p,\bar{m})+\bar{m}) = \hat{Y}^d_{nR}(w,p,\bar{m}) \qquad \frac{\partial \hat{Y}^d_{nR}}{\partial p} < 0 \tag{16}$$

While the aggregate supply is continuous at the C-R boundary, this is not the case for the aggregate demand curve. At $\hat{p}_l$ the notional demand (5) of the R region holds, for the household realizes is *effective* supply of labour and thus is unconstrained on the labour market. But for p tending to $\hat{p}_l$ from the C region, the limit value of the effective demand (8) does not coincide with the value of (5) at that price. For them to be equal, the labour market constraint $\bar{l}$ in the Lagrangian function (7) should coincide with the notional plan l^s_n (in which case $\gamma=0$ and the notional first order conditions hold). However, at $\hat{p}_l$ the effective - not the notional supply - of labour is realized: $\bar{l} = l^d_n = l^s_e < l^s_n$ and for $\bar{l} < l^s_n$, $Y^d_e < Y^d_n$.

Given that in C and R there is excess demand for goods, while in K there is excess supply, one would expect that an AD-AS equilibrium exists at the boundary between R and K (see Figure 1). We recall that that boundary represents the one-dimension limit of a so-called underconsumption area separating R and K - an area characterized by excess demand for labour and excess supply of goods, the former (but not the latter) vanishing at the right border with K, the latter (but not the former) vanishing at the left border with R. Since that area, in which the household is unrationed on both markets, collapses (in absence of inventory accumulation) to a one dimensional space, the R-K boundary is characterized as follows:

(*a*) it marks the border between a region of *generalized* excess demand and a region of *generalized* excess supply, so that the goods excess demand function is differently defined on its left and on its right: while in repressed inflation it is given by $Y^d_n - Y^s_e$, in the Keynesian region the excess supply is $Y^s_n - Y^d_e$.

(*b*) it can be defined interchangeably by the condition holding at the left and right borders of the underconsumption area, namely (a) that the notional demand for goods and the effective supply of goods prevailing in the *R* region are equal or (b) that the notional supply and the effective demand for labour prevailing in the *K* region are equal.

Thus, for a given wage the boundary price $\hat{p}_{gR} = \hat{p}_{gR}(w, \bar{m})$ can be obtained by setting,

$$Y^s_{eR}(w, p, \bar{m}) = \hat{Y}^d_{nR}(w, p, \bar{m})$$

Notice that the above definition of the boundary implies that neither the excess supply of goods of K, nor the excess demand for labour of R vanish at that boundary. Indeed, these properties of the boundary result in the following configuration of aggregate demand and aggregate supply:

(*i*) The aggregate demand function is continuous at the boundary. For p tending to $\hat{p}_{gR}$ from the K region, the effective demand for goods prevailing in that region becomes a notional demand, for by definition (b) of the boundary, the household realizes his notional supply of labour when hitting the border from the right. Thus, in the Lagrangian function (7) the perceived constraint $\bar{l}$ coincides with the notional plan, $\gamma = 0$, and the effective demand (8) and the notional demand (5) coincide (both obviously evaluated at the realized profits $pF(l^s_n) - wl^s_n$). Therefore, the demand curve is continuous at the R-K boundary price.

(ii) The aggregate supply supply function is discontinuous at the boundary. For p tending to $\hat{p}_{gR}$ from the K region, the limit value of aggregate supply is that of the notional supply prevailing in the K region: indeed, when hitting the border from the right, the firm is still unconstrained in the labour market. For p tending to $\hat{p}_{gR}$ from the R region, the limit value of aggregate supply - coinciding with the value of this function at $\hat{p}_{gR}$ - is that of the effective supply of the R region. The latter is lower than that of the notional supply. Indeed, for them to be equal, the value of the quantity constraint $\bar{l}$ in the Lagrangian function (14) should coincide (in the limit) with the notional labour demand at that price, l^d_n - in which case $\nu = 0$ and the notional first order condition would hold. But when hitting the border from the left, the firm is still constrained on the labour market. Thus, $\nu > 0$, $F'(l^s_n) = (w + \nu)/p > w/p$ and $Y^s_{eR}\ (w, \hat{p}_{gR}, \bar{m}) < \lim_{p \to \hat{p}_{gR}} Y^s_{nK}$: there is a discontinuity in the aggregate supply function.

Thus, the goods market equilibrium condition which defines the underconsumption boundary $Y^s_{eR} = \hat{Y}^d_{nR}$ does not imply that at $\hat{p}_{gR}$ the excess supply prevailing for $p > \hat{p}_{gR}$ vanishes. At the R-K boundary a 'one-sided' equilibrium arises, whose graphical representation is given in Figure 3 (drawn under the hypothesis that the utility function is log-linear). The economic interpretation of this configuration of the AS-AD system is easy. In a situation of Keynesian excess supply of goods, a decrease in p induces a reduction of the notional supply of goods and an increases of both the effective demand for goods and the effective demand for labour. The latter matches the notional supply of labour at a price higher than that which would equate the notional supply and effective demand for goods. For $w < w^*$ the firm's notional plan is never realized as an equilibrium in the goods market due to a supply constraint on the labour market. It is precisely in this sense that the firm is still said to be rationed in the goods market at the underconsumption boundary.

By way of conclusion, if the AD-AS system is derived from the disequilibrium hypotheses, for $w < w^*$ a very peculiar equilibrium is obtained, at which the excess demand prevailing at lower than equilibrium prices vanishes, but the excess supply prevailing at higher than equilibrium prices persists. Clearly, this has a bearing on dynamics, when a competitive price adjustment rule is invoked as clearing mechanism of the goods market. We briefly tackle this issue in the following section.

IV. STABILITY AND COMPARATIVE STATICS

In the former section we derived a complete AD-AS model under the hypotheses that realized transactions enter the budget constraint and trades consistently occur in disequilibrium. This does not imply that prices are fixed; rather it implies that they are predetermined: they do not take step changes to clear the market, but move 'slowly' (in real time and not instantaneously) to equate demand and supply.

Let us now add to our system the following dynamic equations for the price level, in which a competitive adjustment is assumed for all prices, given the nominal wage.[9] For $w > w^*$:

$$\dot{p}_C = \phi_C \left(\widetilde{Y}^{\,d}_{eC}(w, p, \bar{m}) - Y^s_n(w, p)\right) \qquad \phi'_C > 0 \qquad \text{for } p \in [0, \widehat{p}_g]$$

$$\dot{p}_K = \phi_K \left(\check{Y}^d_{eK}\ (p, \bar{m}) - Y^s_n(w, p)\right) \qquad \phi'_K > 0 \qquad \text{for } p \in [\widehat{p}_g, \infty]$$

For $w < w^*$

$$\dot{p}_C = \phi_C \left(\widetilde{Y}^{\,d}_{eC}(w, p, \bar{m}) - Y^s_n(w, p)\right) \qquad \phi'_C > 0 \qquad \text{for } p \in [0, \widehat{p}_l[$$

$$\dot{p}_R = \phi_R \left(\widehat{Y}^d_{nR}(w, p, \bar{m}) - \ Y^s_{eR}(w, p, \bar{m})\right) \qquad \phi'_R > 0 \qquad \text{for } p \in [\widehat{p}_l, \widehat{p}_{gR}]$$

$$\dot{p}_K = \phi_K \left(\check{Y}^d_{eK}\ (p, \bar{m}) - Y^s_n(w, p)\right) \qquad \phi'_K > 0 \qquad \text{for } p \in]\widehat{p}_{gR}, \infty]$$

The main implication of the static analysis developed above, is that price flexibility (with fixed nominal wage) ensures that an unemployment equilibrium can be reached for $w > w^*$ - $\dot{p}_C = \dot{p}_K = 0$ at $\widehat{p}_g$. By contrast, if $w < w^*$, then downward price flexibility does not ensure consistency of the firm's and the household's choices in presence of excess supply of goods.

The dynamic counterpart of the equilibrium being 'one-sided' at the boundary between the R and K regions is that the $\dot{p}_K$ function does not tend to zero at that boundary, while $\dot{p}_R = 0$. For $w < w^*$ we cannot define a $\dot{p}$ function continuous in zero. The price adjustment rule relevant in excess supply does not allow that excess supply to vanish, while the rule prevailing in excess demand drives excess demand to zero. The AD-AS equilibrium is not bilaterally stable.

This stability problem disappears, however, once a competitive wage adjustment rule is imposed in each régime:

$$\dot{w}_K = \lambda_K \left(F^{-1}(Y^d_{eK}(p,\bar{m})) - \hat{l}^s_{nK}\ (w,p,\bar{m})\right) \qquad \lambda'_K > 0$$
$$\dot{w}_C = \lambda_C \left(l^d_n(w,p) - \tilde{l}^{\,s}_{eC}(w,p,\bar{m})\right) \qquad \lambda'_C > 0$$
$$\dot{w}_R = \lambda_R \left(l^d_n(w,p) - \tilde{l}^s_{eR}(w,p,\bar{m})\right) \qquad \lambda'_R > 0$$

where $\hat{l}^s_{nK}$ is the notional supply of labour relevant in the K region.

It has been shown that if both the price and the wage adjust according to the rules specified above, then the complete system converges to the Walrasian equilibrium provided that two basic sets of conditions are met (see Cuddington, Johansson and Löfgren, 1984, pp. 213-235). The first set refers to the stability of the three regions - in both markets the gap between demand and supply tend to decrease through the price and wage rules specified above - a condition we have assumed to hold at the beginning.[10] The second set deals with the behaviour of the system at the boundary between the repressed inflation and Keynesian unemployment regions. Starting from any (w,p) point on that boundary, a sliding trajectory towards the Walrasian equilibrium exists provided that in an arbitrarily small neighbourhood of that point - excluding that point itself - the following inequality on the limit adjustment speeds holds (Honkapohja and Ito, 1983):

$$\dot{w}_R(\dot{p}_R - \dot{p}_K) > \dot{p}_R(\dot{w}_R - \dot{w}_K)$$

The dynamic trajectories in the (w,p) plane, either converge directly to the Walrasian equilibrium (which may, but not necessarily, happen if at the starting point $w > w^*$), or 'hit' the boundary between the R and K regions and, provided that the above condition is verified, slide along that border towards the Walrasian equilibrium.

Once stability has been checked, we can now proceed to the representation of the comparative statics of this disequilibrium model of aggregate demand and supply. Starting from a Walrasian equilibrium, we want to show the effects of negative and positive monetary shocks both in the short and in the long run. We recall that we have defined three demand schedules: $\check{Y}^d_{eK} = \check{Y}^d_{eK}(p, \bar{m})$, $\widetilde{Y}^d_{eC} = \widetilde{Y}^d_{eC}(w, p, \bar{m})$ and $\widehat{Y}^d_{nR} = \widehat{Y}^d_{nR}(w, p, \bar{m})$. For $w \geq w^*$, the disequilibrium AD function (Figure 2) is defined, for each price level, as

$$Y^d = \min\ (\check{Y}^d_{eK},\ \widetilde{Y}^d_{eC}) \tag{17}$$

$\widehat{Y}^d_{nR}$ being immaterial. In particular for $w = w^*$, $\widehat{Y}^d_{nR}$ is tangent to Y^d at the Walrasian equilibrium price, p^*. For $w < w^*$, the disequilibrium AD is:

$$\begin{aligned} Y^d &= \min\ (\check{Y}^d_{eK},\ \widehat{Y}^d_{nR}) && \text{for } p \in [\widehat{p}_l, \infty] \\ Y^d &= \widetilde{Y}^d_{eC} && \text{for } p \in [0, \widehat{p}_l[\end{aligned} \tag{18}$$

Figure 4 shows the effects of a negative shock. The initial Walrasian equilibrium for $\bar{m} = m_0$ is described in Figure 4a. For $w = w^*_0$, the aggregate demand curve is given by (17): it is of the Keynesian type for $p \geq p^*_0$ and of the classical unemployment type for $p \leq p^*_0$ - the two curves coinciding at the equilibrium point E_0; the aggregate supply is the notional, upward sloping one. If a negative shock occurs reducing m to $m_1 < m_0$, both the $\check{Y}^d_{eK}$ and the $\widetilde{Y}^d_{eC}$ portions of the AD curve shift inwards, still intersecting along the AS curve (Figure 4b). The fixed wage and price equilibrium is in E_1: a situation of Keynesian unemployment occurs, where price and wage tend to decrease. The adjustment may entail either a monotonic convergence to the new Walrasian equilibrium - E_2 in Figure 4c - or a more complex, cyclical, behaviour of prices and wages. However, stability implies that E_2 is eventually reached with a decrease of the wage and price equilibrium levels equiproportional to that of the money stock. In E_2 output is again at its (unchanged) Walrasian level Y^*. Notice that in the new long run equilibrium position, the aggregate supply and the $\tilde{Y}^d_{eC}$ curves have shifted outwards due to the reduction in the wage level.

Figure 5 describes the effects of a positive monetary shock. In Figure 5a the initial Walrasian equilibrium is represented again. Notice that we add to the disequilibrium curve $Y^d = \min(\check{Y}^d_{eK}, \tilde{Y}^d_{eC})$, the (irrelevant at the Walrasian equilibrium) $\hat{Y}^d_{nR}$ one. The impact effect of a positive monetary shock is depicted in Figure 5b. If money increases from m_0 to $m_2 > m_0$, the curves $\check{Y}^d_{eK}$, $\tilde{Y}^d_{eC}$ and $\hat{Y}^d_{nR}$ shift outwards. Since w_0^* is now lower than the new Walrasian wage, aggregate demand is given by (18). While $\hat{p}_l$ is univocally determined on the labour market, by construction the intersection between $\check{Y}^d_{eK}$ and $\hat{Y}^d_{nR}$ occurs at $\hat{p}_{gR}$. The aggregate supply is now defined by:

$$Y^s = Y^s_n(w, p) \qquad \text{for } p \in [0, \hat{p}_l] \text{ and } p \in]\hat{p}_{gR}, \infty]$$

$$Y^s = Y^s_{eR}(w, p, \bar{m}) \qquad \text{for } p \in [\hat{p}_l, \hat{p}_{gR}]$$

The fixed price equilibrium occurs in E_1, which corresponds to a repressed inflation situation. Output decreases due to the well-known supply multiplier effect (e.g. Barro and Grossman, 1976). As the wage increases, $\hat{Y}^d_{nR}$ moves outwards, while $\tilde{Y}^d_{eC}$ shifts inwards. At the same time $Y^s_n(w, p)$ moves leftwards, while $Y^s_{eR}(w, p, \bar{m})$ shifts outwards. The two boundary prices $\hat{p}_l$ and $\hat{p}_{gR}$ progressively converge to the new Walrasian price, p_2^*, so that the effective supply portion of the AS curve progressively reduces in length. In the new Walrasian equilibrium, E_2, output is again at its original level, while the wage and the price has increased equiproportionally to money. The boundary prices $\hat{p}_l$ and $\hat{p}_{gR}$ both coincide with p_2^*, so that Y^s_{eR} and $\hat{Y}^d_{nR}$ become irrelevant. As a final remark we notice that along the adjustment path, w monotonically increases, while p is likely to overshoot its Walrasian level. If the wage and price trajectory hits the boundary between the Keynesian and Repressed inflation regions, then from that point on a sequence of 'boundary' equilibria at $p = \hat{p}_{gR}$ occurs with a decrease of the price level towards p_2^*.

V. CONCLUSIONS

In all models in which the nominal wage and the price level are predetermined, the case cannot be ruled out that the impact effect of positive demand shocks be shortage of labour, with supply decisions constrained on the labour market. This may happen with $w < w^*$ in traditional fix-price models, and may happen as well within menu costs models *à la* Blanchard and Kiyotaki, when the output expansion due to a positive monetary shock becomes constrained by the competitive labour supply before reaching its competitive level.(Blanchard and Kiyotaki, 1987, p.660).

The extension in this direction of the AD-AS disequilibrium model developed in this paper allows to assess the properties of this class of equilibria through a simple and manageable theoretical apparatus. When AD and AS disequilibrium curves are derived for a low wage level, interesting discontinuities arise, which reflect the different marginal conditions relevant for notional and effective plans - a question which does not arise when unemployment is assumed throughout so that demand is effective and supply is notional for all prices. The AD-AS equilibrium occurs at one extreme of the discontinuity of the supply curve, where it is defined with respect to the effective but not the notional plans of the firm. The dynamic counterpart of this result is that the equilibrium is unstable from above: excess supply cannot be driven to zero through the price mechanism. If the economy is in a situation of Keynesian unemployment, and both real and nominal wages are low with respect to their Walrasian levels, neither a price reduction nor a money expansion allows the firm to realize its notional supply plan: the labour supply constraint replace the effective demand constraint. For a given low nominal wage that plan could rather be realized as an unemployment AD-AS equilibrium via a money contraction, while a price *and* wage competitive adjustment can lead to the Walrasian equilibrium, provided that the Honkapohja and Ito conditions hold at the R-K boundary.

FOOTNOTES

1. For a recent assessment of this debate, see Allen and Stone (1993).

2. See, for example, Malinvaud (1977). Notice that Figure 1 embodies the assumption that all regions are stable.

3. Since the quantity of leisure does not influence the marginal rate of substitution between consumption and money, the constraint on labour supply has only an income effect on consumption demand. This explains why the term $\overline{l}$ does not enter as a separate argument of equation (8). Moreover, notice that if the constraint is binding, $\gamma > 0$ and both consumption and money are actually substituted for leisure at a shadow price $(w+\gamma)/p$, higher than the real wage. The household would like to exchange leisure for both consumption and money. Thus, if the quantity constrained is relaxed, under separability both consumption and money demand increase.

4. We are assuming that the condition for the demand curve to be positively sloped in the relevant price range is satisfied.

5. Following the same reasoning developed in fn. 3, one can establish that given p, w and π^e, $l^s_e < l^s_n$.

6. Notice that this labour market equilibrium defined in presence of excess demand on the goods market resembles that of the goods market equilibrium at the C-K border.

7. In this simple macroeconomic model, where the household receives both wage and profit income, the derivation of notional curves in disequilibrium is somewhat puzzling. The two requirements that (a) notional plans are formulated whenever the agent is unconstrained on the other market, and that (b) realized transactions are included in the budget constraint through realized profits, give rise to solutions whose interpretation is somewhat ambiguous. Indeed, the demand plan (16) is a notional one for given profits, but it embodies realized profits, $pF(\bar{l}^s_{eR}) - w\bar{l}^s_{eR}$, which do not coincide with notional profits, $pY^d_n - wl^s_n$. Negishi (1979, pp. 64-65) explicitly recognizes that a solution based on notional profits is unsatisfactory, but suggests it as a shortcut. Many contributions in disequilibrium analysis escaped from this problem, by assuming either that profits do not affect current expenditure decisions - *e.g.* Picard (1993) - or that labour supply always coincide with notional supply (Sneessens, 1984).

8. In (16) a price increase might in principle increase profits; but it is unlikely that this effect, if observed, dominates the substitution and the real balance effect.

9. The existence of a *competitive* price and wage adjustment can be questioned. Rationed agents might find it profitable to reduce their own price to relax the perceived constraints; but this calls for a non competitive interpretation of price adjustment. On this issue, see Benassi, Chirco and Colombo (1994, Chapters 2 and 9).

10. The instability problem might arise in the R region. If a price increase reduced the effective labour supply, then the supply of goods itself - not only demand - would decrease and an increase in p could then widen and not reduce the excess demand in the goods market.

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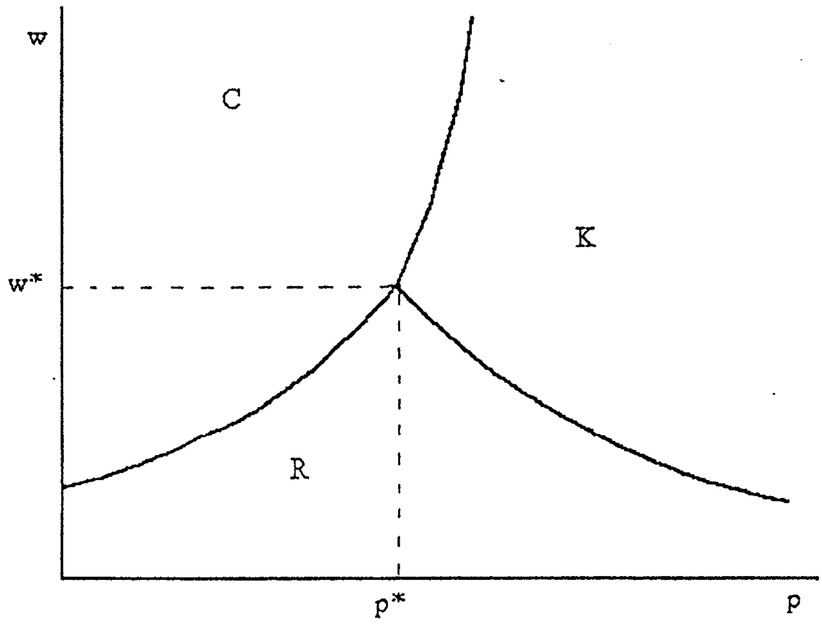


Figure 1

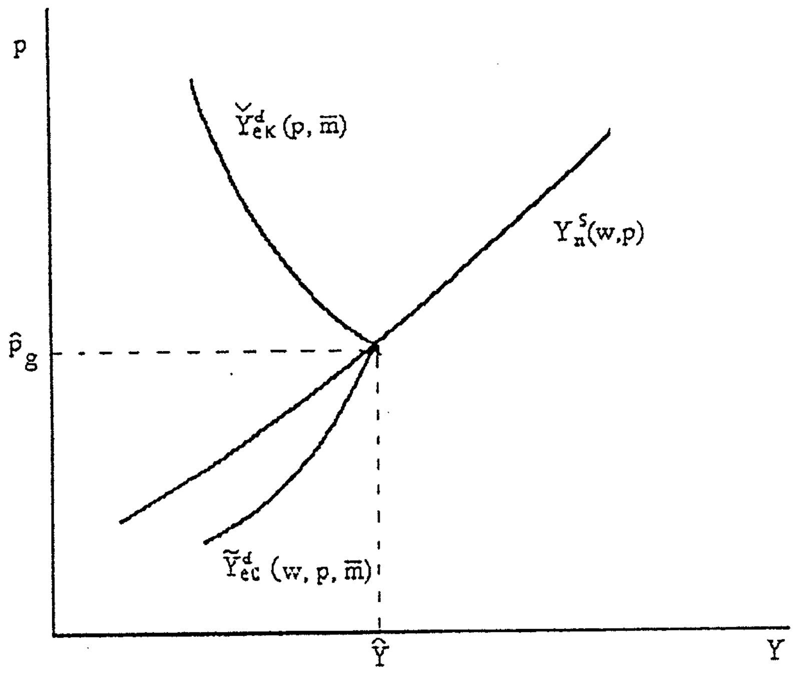
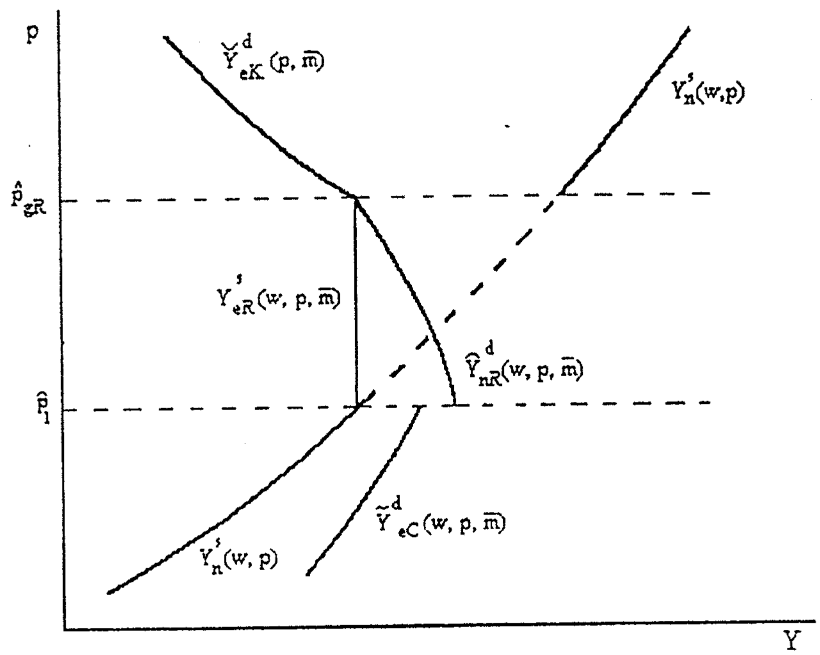

p
$\check{Y}^d_{eK}(p, \overline{m})$
$Y^s_n(w,p)$
$\hat{p}_g$
$\tilde{Y}^d_{ec}(w, p, \overline{m})$
$\hat{Y}$
Y

Figure 2



p
$\breve{Y}^d_{eK}(p, \bar{m})$
$Y^s_n(w,p)$
$\hat{p}_{gR}$
$Y^s_{eR}(w, p, \bar{m})$
$\hat{Y}^d_{nR}(w, p, \bar{m})$
$\hat{p}_1$
$\tilde{Y}^d_{eC}(w, p, \bar{m})$
$Y^s_n(w, p)$
Y

Figure 3

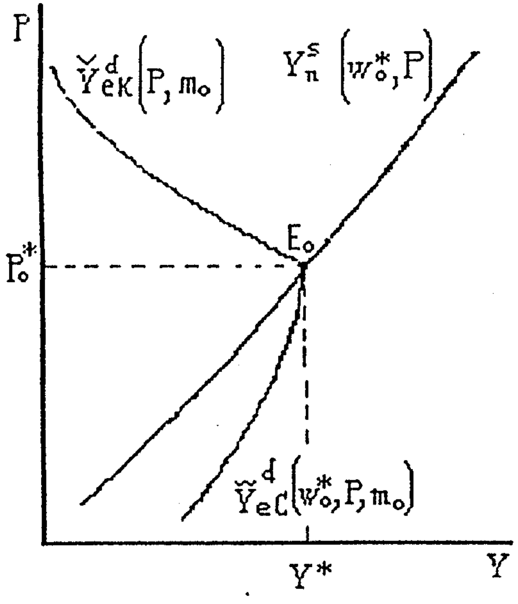


FIGURE 4a

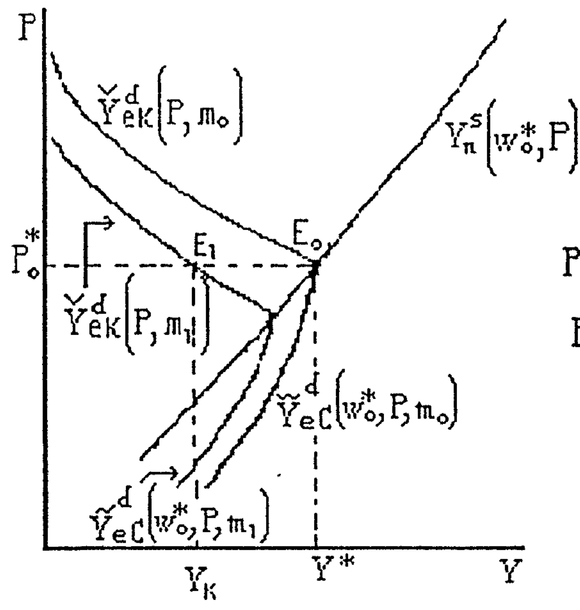


FIGURE 4b

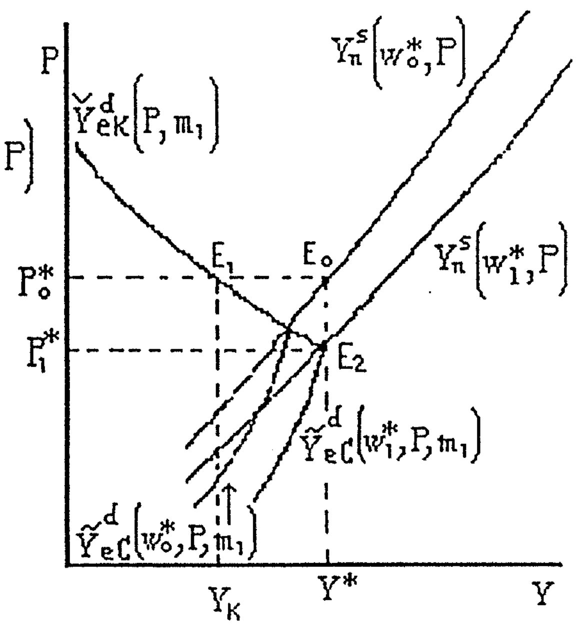


FIGURE 4c

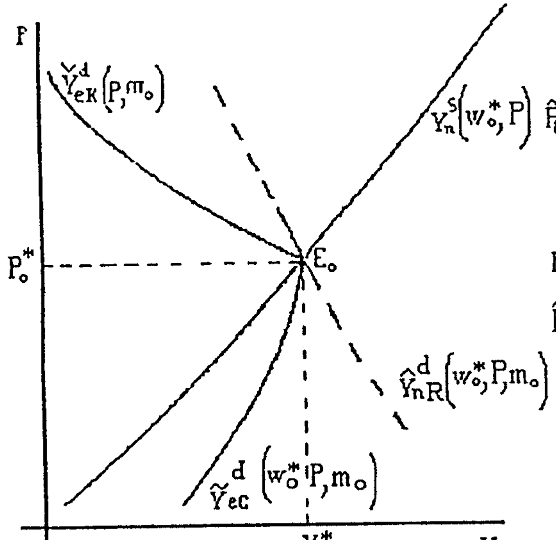


FIGURE 5a

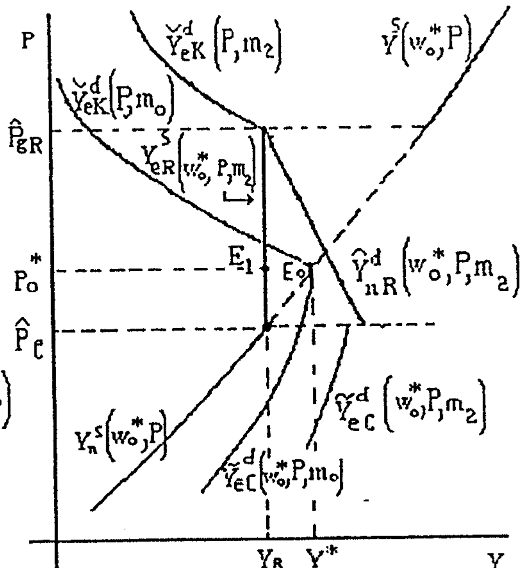


FIGURE 5b

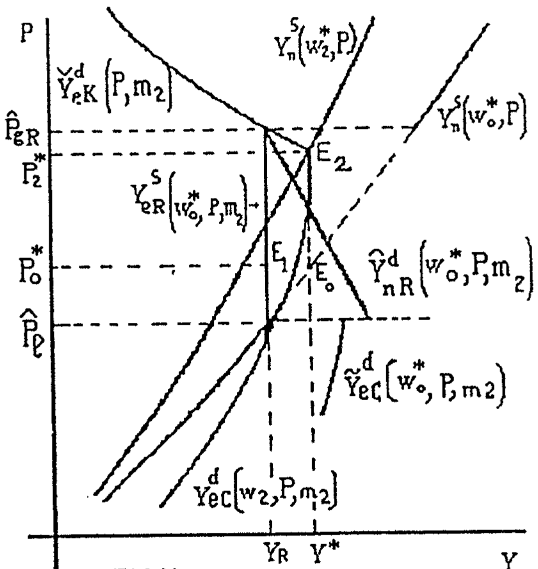


FIGURE 5c