Backstepping based power control of a three-phase single-stage grid-connected PV system

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ABSTRACT
In order to reduce costs while maintaining superior performance, this paper presents a new control methodology of a three-phase grid connected photovoltaic system without using the intermediary DC/DC converter. Based on the synchronized nonlinear model of the whole photovoltaic system, two controllers have been proposed for the three-phase inverter in order to ensure the operation of the PV system at the maximum power point with unity power factor and minimum grid disturbance. Grid synchronization has been ensured by a three-phase 2nd order PLL (Phase-Locked Loop). The stability of each controller is demonstrated by means of Lyapunov analysis and evaluated under changing atmospheric conditions using the Matlab/Simulink environment, the simulation results clearly demonstrate the performance provided by each controller.

Keywords:
Backstepping controller
Grid connection
Maximum power point tracking
PV systems
Unity power factor

1. INTRODUCTION
Nuclear energy is not likely to be a major source of world energy consumption because of the relative risks associated with unleashing the power of the atom and we have no choice but to invest heavily in renewable energy production. Solar energy is one of the most promising renewable electricity sources in the world. Moreover, the coordinated inter-connection of solar energy sources between global electricity markets can supply more flexibility and balancing to the grid. This is why the grid-connected photovoltaic systems have a great bright future.

Achieving the reduction of PV system manufacturing costs remains a major task. Among the trends to achieve such a reduction is the elimination of the DC/DC converter stage. Conventionally, the first converter stage, which is usually placed between the PV arrays and the inverter [1-3], achieves the MPPT whereas the inverter stage delivers and controls the energy injected into the grid. Therefore, to achieve this cost reduction, the three-phase inverter must also take care of the maximum power point tracking.

Maximum power point tracking is mandatory to maximize photovoltaic systems efficiency. To this end, several MPPT control strategies have been largely published in the last few years [1-13]. Perturb and observe (P&O) and incremental conductance (IC) algorithms are the most widely proposed in literature. They are the simplest algorithms to implement [3-5], but the dilemma -the choice between convergence speed and output fluctuations [5, 6], has led in recent years to several research aimed at improving these two techniques [14-17]. The nonlinearity of photovoltaic characteristics makes the control of PV systems by conventional control strategies a complex task. However, the recent involvement of robust and nonlinear controls has enriched the field of research and has proved most appropriate for the control of nonlinear systems [7-12]. In particular, [6, 18] are comparative studies between the backstepping control and other conventional controls which clearly showed the performance of the backstepping control.

In this paper, two controllers for the three-phase inverter, based on backstepping approach, are proposed and discussed. The first control objective is to track the maximum power point \( (\partial P_p / \partial V_p = 0) \), and the second objective is to ensure a grid connection with unity power factor (output current must be in phase with the grid voltage). The proposed controllers are developed in the synchronous orthogonal frame and the voltage phase angle of the grid utility is detected using a three-phase PLL (Phase-Locked Loop).

This paper is organized as follows. In section two, the system description and the mathematical model are presented. Section three develops the control strategies of proposed system. Simulation results are presented and compared in section four. In the last section, a conclusion followed by the reference list.

2. DESCRIPTIONS AND MODELING OF PV SYSTEM

The PV cell is basically a p-n semiconductor junction that converts light energy into electrical energy. The PV cell can be represented by a current source controlled by voltage, sensitive to temperature and solar radiation, in parallel with one diode and a shunt resistor \( R_{sh} \) and the whole is in series with a resistor \( R_s \) as shown in Figure 1. Although it is a simplified model, this equivalent circuit is sufficiently accurate to represent the different types of photovoltaic cells [19]. The mathematical model of solar cells is detailed in [3].

![Figure 1. Equivalent circuit of PV cell](image1)

The photovoltaic array is a multiple associations, in series and parallel, of PV cells. The PV generator considered in this paper is composed by thirty-three SM55 Siemens panels connected in series. The electrical specifications for one panel are enlisted in Table 1. The modeling of the SM55 panel, using Matlab/Simulink, allowed tracing its characteristics for different values of irradiance and temperature which are shown in Figure 2 and Figure 3. Figure 4 shows the power-voltage characteristics of the PV generator considered in this paper under changing climatic conditions. The coordinates of the Maximum Power Points (MPP1, MPP2 and MPP3) summarized in Table 2 will be used for verification of the simulation results.

![Figure 2. Power-voltage characteristics for the SM55 panel at 1000W/m2 with varying temperature levels](image2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum power</td>
<td>55W</td>
</tr>
<tr>
<td>Current at the maximum power point</td>
<td>3.15A</td>
</tr>
<tr>
<td>Voltage at the maximum power point</td>
<td>17.4V</td>
</tr>
<tr>
<td>Maximum current (short circuit output)</td>
<td>3.45A</td>
</tr>
<tr>
<td>Maximum voltage (open circuit)</td>
<td>21.7V</td>
</tr>
<tr>
<td>Current temperature coefficient</td>
<td>1.2 mA/°C</td>
</tr>
<tr>
<td>Number of series cells ( N_s )</td>
<td>36</td>
</tr>
<tr>
<td>Number of parallel modules ( N_p )</td>
<td>1</td>
</tr>
</tbody>
</table>

![Figure 3. Power-voltage characteristics for the SM55 panel at 1000W/m2 with varying temperature levels](image3)

Backstepping based power control of a three-phase single-stage grid-connected PV system (M. El malah)
Figure 3. Power-voltage characteristics for the SM55 panel at 1000W/m² with varying temperature levels

Figure 4. Power-voltage characteristics for the SM55 panel at 1000W/m² with varying temperature levels

Table 2. Maximum power points (MPP) in Figure 4

<table>
<thead>
<tr>
<th>Maximum power point</th>
<th>Voltage [V]</th>
<th>Power [W]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPP1</td>
<td>584</td>
<td>1847</td>
</tr>
<tr>
<td>MPP2</td>
<td>531.6</td>
<td>1669</td>
</tr>
<tr>
<td>MPP3</td>
<td>573.2</td>
<td>1447.8</td>
</tr>
</tbody>
</table>

The basic power circuit of proposed single stage PV-system as shown in Figure 5 consists of a DC-link capacitor which is directly connected to the three-phase inverter. Grid connection was performed through a low-pass filter used to reduce the ripple components due to the switching actions in PWM inverter. It is assumed that all three phase inverter switches are ideal, the low-pass filter phases are identical and the grid voltage is symmetric. The three-phase model is detailed in [7]. A simplified model can be obtained in the synchronous frame rotating at the angular frequency of the grid voltage. For this purpose, the power-invariant dq-transformation, from balanced three phase electrical quantities to balanced two phase quadrature quantities, has been used:

\[
\begin{align*}
[K_d & \quad K_q]^T = T(\theta)[k_1 \quad k_2 \quad k_3]^T \\
[I_d & \quad I_q]^T = T(\theta)[i_1 \quad i_2 \quad i_3]^T \\
[E_d & \quad E_q]^T = T(\theta)[e_1 \quad e_2 \quad e_3]^T
\end{align*}
\]

\( (1) \)

and

\[
T(\theta) = \sqrt{3} \begin{bmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \end{bmatrix}
\]

where \((k_1; k_2; k_3)\): Inverter control inputs; \((i_1; i_2; i_3)\): Injected currents; \((e_1; e_2; e_3)\): Grid voltages, and \(\theta\) is the angular position of the dq-frame. The state-space model can be re-written in the new frame, as \((1)\):
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\[
\begin{align*}
\frac{di_d}{dt} &= \frac{-R}{L} i_d + \omega i_q - \frac{E_d}{L} + \frac{V_p}{L} K_d \\
\frac{di_q}{dt} &= -\omega i_d + \frac{-R}{L} i_q - \frac{E_q}{L} + \frac{V_p}{L} K_q \\
\frac{dv_p}{dt} &= \frac{1}{C} (i_p - i_d K_d - i_q K_q)
\end{align*}
\]  
(2)

where \( i_p, v_p \) : Are the PV array current and voltage

3. CONTROLLER DESIGN

The injected currents have to be synchronized with the grid voltages. To this end, the grid voltage phase angle is detected using a 2nd order phase locked loop (PLL). The structure of the PLL implemented in this work as shown in Figure 6 uses the grid voltage abc-dq transformation to track the grid voltages phase angle. A PI regulator was used to generate a corrective phase angle (\( \theta_{est} \)), that is fed back into the grid voltage abc-dq transformation module, from the quadrature voltage error [20]. The three-phase grid currents and voltages are transformed into direct and quadrature axis components, the controller outputs (\( K_d \) & \( K_q \)) are then transformed into three-phase components using the inverse dq-abc transformation, and then a PWM control is used to make them suitable for switching the inverter switches as shown in Figure 5.

Figure 5. Control scheme of the proposed photovoltaic system

\[
\begin{align*}
P &= E_d i_d + E_q i_q \\
Q &= E_q i_d - E_d i_q \\
P &= E_d i_d \\
Q &= -E_d i_q
\end{align*}
\]  
(3)

Figure 6. Basic principle of PLL

In the synchronous d-q reference frame (\( E_q = 0 \)), the injected powers are simplified, the active and reactive powers can be controlled by \( i_d \) and \( i_q \) respectively:
In order to achieve a minimum injection of reactive power the quadrature current reference $I_{qref}$ must be set to zero. If we neglect ohmic loss, the power conservation principle gives:

$$I_{dref} = \frac{V_p l_p}{E_d} = \frac{P_p}{E_d}$$  \hspace{1cm} (4)

Finally, when the PV-generator power is at its maximum state, Figure 3, its derivative with respect to PV-voltage is zero. Table 3 summarizes the selected dynamic outputs and their references for each controller.

Table 3. Dynamic outputs and their references

<table>
<thead>
<tr>
<th>The dynamic output considered</th>
<th>Output Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Backstepping Controller</td>
<td>$y = \begin{bmatrix} y_1 \ y_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_p}{\partial V_p} \ e_d^2 + e_q^2 \end{bmatrix}$</td>
</tr>
<tr>
<td>Second Backstepping Controller</td>
<td>$y = \begin{bmatrix} y_1 \ y_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial P_p}{\partial V_p} \ I_q \end{bmatrix}$</td>
</tr>
</tbody>
</table>

where: $e_d = I_d - I_{dref}$ is the direct current error, $e_q = I_q - I_{qref}$ is the quadrature current error.

The output chosen for the second controller is simpler than the first, but it is a challenging and time-consuming task to tune the appropriate backstepping parameters in this controller. This is largely due to the fact that establishing stability for switched systems is difficult. Therefore, in the first controller, we have chosen to also involve the direct current component $I_d$ in the control of the output power.

### 3.1. Formulation of MPPT control law (controller 1&2)

Let us define the first tracking error $\epsilon_1$ and its LFC (Lyapunov Function Candidate) $V_1$, using (2), it is possible to deduce:

$$\epsilon_1 = y_1 - y_{1ref} = l_p + P_p \frac{\partial l_p}{\partial P_p} \quad \Rightarrow \quad \dot{\epsilon}_1 = \left( V_p \frac{\partial^2 l_p}{\partial V_p^2} + 2 \frac{\partial l_p}{\partial V_p} \right) \dot{V}_p = \frac{f}{C} (l_p - l_d K_d - l_q K_q)$$  \hspace{1cm} (5)

$$V_1 = \frac{1}{2} \epsilon_1^2 \quad \Rightarrow \quad \dot{V}_1 = \epsilon_1 \dot{\epsilon}_1 = \frac{\epsilon_1 f}{C} (l_p - l_d K_d - l_q K_q)$$  \hspace{1cm} (6)

The stabilizing controls $K_d$ and $K_q$ is chosen as follows:

$$B_1 + A_1 K_d + A_2 K_q = 0 \quad \text{where} \quad \begin{cases} B_1 = \frac{l_p f}{C} + c_1 \epsilon_1 ; A_1 = -l_d f \frac{C}{C} ; A_2 = -l_q f \frac{C}{C} ; \\
 f = V_p \frac{\partial^2 l_p}{\partial V_p^2} + 2 \frac{\partial l_p}{\partial V_p} \text{ and } c_1 \text{ is a positive parameter} \end{cases}$$  \hspace{1cm} (7)

With the above choice $\dot{V}_1 = -c_1 \epsilon_1^2$ becomes defined negative, $\epsilon_1$ is proved to be asymptotically stable and converge to zero by the Lyapunov design. This means that $y_1 \rightarrow y_{1ref}$, so $P_p \rightarrow (P_p)_{MPP}$.

### 3.2. Formulation of output power control law

#### 3.2.1. First controller

According to Table 3 we define the error $\epsilon_2$ and its derivative, deduced from (2), as follows:

$$\epsilon_2 = y_2 - y_{2ref} \quad \Rightarrow \quad \dot{\epsilon}_2 = 2 \frac{\partial e_d}{\partial \frac{\partial l_p}{\partial e_d}} \left[ -\frac{V_p}{L} l_d + \omega l_q - \frac{V_p}{L} K_d - l_{dref} \right] + 2 \frac{\partial e_q}{\partial \frac{\partial l_p}{\partial e_q}} \left[ -\frac{V_p}{L} l_d + \omega l_q + \frac{V_p}{L} K_q \right]$$  \hspace{1cm} (8)

where: $l_{dref} = \frac{P_p}{V_p} = \frac{\epsilon_1}{E_d} \dot{V}_p = \frac{\epsilon_1}{E_d} (l_p - l_d K_d - l_q K_q)$.
Let us consider the following new LFC, its derivative is deduced from (8) as follows:

\[
\begin{align*}
V_2 &= V_1 + \frac{1}{2} e_2^2 \\
\dot{V}_2 &= -c_1 e_1^2 + 2e_2 [e_d \left( \frac{-R}{L} I_d + \omega I_q - \frac{E_d}{L} + \frac{V_p}{L} K_d - \hat{I}_{dref} \right) + e_q \left( -\omega I_d + \frac{R}{L} I_q + \frac{V_p}{L} K_q \right)]
\end{align*}
\]

Then the stabilizing controls \((K_d \text{ and } K_q)\) is chosen as follows:

\[
B_2 + A_3 K_d + A_4 K_q = 0 \begin{bmatrix} B_1 + \frac{1}{2} c_d e_2^2 + e_d \left( \frac{-R}{L} I_d + \omega I_q - \frac{E_d}{L} + \frac{E_q}{L} \right) + e_q \left(-\omega I_d + \frac{R}{L} I_q \right) \\
A_3 &= e_d \frac{V_p}{L} + e_1 e_d I_d; \quad A_4 = e_q \frac{V_p}{L} + e_1 e_d I_q; \quad c_2 \text{ is a positive parameter.}
\end{bmatrix}
\]

With the above choice \(\dot{V}_2 = -c_1 e_1^2 - c_2 e_2^2 < 0, e_1 \text{ and } e_2\) are proved to be stable and converge to zero by the Lyapunov design. This means that \(y\) converge to \(y_{ref}\). We are finally in a position to determine the control signals \(K_d \text{ & } K_q\). From (7) and (10), we deduce (11):

\[
\begin{bmatrix} A_1 & A_2 \\
A_3 & A_4 \end{bmatrix} \begin{bmatrix} K_d \\
K_q \end{bmatrix} = - \begin{bmatrix} B_1 \\
B_2 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} [A_1 & A_2]^{-1} [B_1] \\
[A_3 & A_4]^{-1} [B_2] \end{bmatrix}
\]

### 3.2.2. Second controller

The tracking error \(e_2\) is reduced to (12):

\[
\dot{e}_2 = I_q - I_{qref} = I_q \quad \text{From (2),} \quad \dot{e}_2 = -\omega I_d + \frac{R}{L} I_q - \frac{E_d}{L} + \frac{V_p}{L} K_q
\]

and the control low \(K_q\) can be extracted directly from the second equation of mathematical model (2), by choosing \(K_d\) such that:

\[
K_q = \frac{L}{V_p} \left( -c_1 I_d + \omega I_q + \frac{E_d}{L} I_q \right) \quad \Rightarrow \dot{e}_2 = -c_2 e_2 \quad \Rightarrow \dot{V}_2 = \dot{V}_1 + e_2 \dot{e}_2 = -c_1 e_1^2 - c_2 e_2^2
\]

Using (7) and (13), we obtain the control signals (14):

\[
\begin{bmatrix} K_d \\
K_q \end{bmatrix} = - \begin{bmatrix} A_1 & A_2 \\
0 & \frac{V_p}{L} \end{bmatrix}^{-1} \begin{bmatrix} B_1 \\
B_2 \end{bmatrix}
\]

### 4. SIMULATION RESULTS

The theoretical performances of the proposed controllers discussed in section three will be illustrated by simulation in this section. The PV system as shown in Figure 5 is simulated jointly with each controller, using the instantaneous three phase model, in Matlab/Simulink environment as shown in Figure 7. The model in d-q axis (2) is only used in the controllers design. Important simulation parameters are given in the Table 4. Controllers’ parameters have been selected using a ‘trial-and-error’ search method. In order to prove the robustness of the control algorithms, the simulation is performed with the following scenario:

- A temperature increase from T=25°C (298.15K) to T=45°C (318.15K) after 1 sec of start of simulation, then returns to T=25°C at 1.6sec, as shown in Figure 8.
- A solar irradiation drop from 1000W / m² to 800W / m² after 0.4 sec of start of simulation, then returns to 1000W / m² at 0.3 s as shown in Figure 9.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV-array power</td>
<td>(P_p)</td>
<td>1847W</td>
</tr>
<tr>
<td>DC bus capacitor</td>
<td>(C)</td>
<td>100µF</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>(f_s)</td>
<td>108Hz</td>
</tr>
<tr>
<td>AC source</td>
<td>(V_g)</td>
<td>110V</td>
</tr>
<tr>
<td>Line frequency</td>
<td>(f)</td>
<td>50Hz</td>
</tr>
<tr>
<td>Filter parameters</td>
<td>(L)</td>
<td>12.5mH</td>
</tr>
<tr>
<td>(R)</td>
<td>0.5655 (\Omega)</td>
<td></td>
</tr>
<tr>
<td>controller parameters</td>
<td>(c_1)</td>
<td>(8 \times 10^4)</td>
</tr>
<tr>
<td>(c_2)</td>
<td>(9.7 \times 10^4)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4. Characteristics of controlled photovoltaic system**
The first and second controllers are based respectively on (11) and (14), and use the same backstepping parameters as shown in Table 4. Simulation results of the PV array power and voltage under transient condition are shown in Figure 10, which correspond very well to the MPP coordinates summarized in Table 2.

Figure 10 illustrates the change of direct and quadrature current components. It can be clearly seen that the second controller can track the reference values with a fast transient response, but the first controller is the most accurate and the least disruptive of the grid. Figure 12 shows the grid voltage and the injected...
current obtained with each one of the proposed controllers (the grid current scale was multiplied by 10). It can easily be seen in the zoomed portions as shown in Figure 13 that the grid current is sinusoidal and in phase with the grid voltage which proving that the power factor unit is well achieved.

Remark: It should be noted that it is very difficult to tune backstepping parameters of the second controller to establish the stability, unlike the first controller whose parameters influence precisely on the precision and settling time.

Figure 11. Current components behavior

Figure 12. Voltage and the injected current of one phase
5. CONCLUSION

In this paper, two Backstepping controllers have been developed for a three phase single stage grid connected photovoltaic system, without using the conventional DC/DC converter for MPPT control, in order to ensure the operation of the PV system at maximum power point with unity power factor. From the simulation results, it has been proven that both algorithms, although they require more computation, are able to work under various levels of irradiation and temperature, it has also been deduced that the correct selection of system outputs helps to facilitate control tasks. Although the current of the first controller has fewer ripples compared to the second, the future work will deal with the mitigation of switching noise and the practical implementation.

REFERENCES


BIOGRAPHIES OF AUTHORS

Mohammed El Malah was born in Morocco on November 02, 1980. He received the Master in Automatic, Signal Processing and Industrial Computing from Science and Technical Faculty, Hassan 1st University, Settat, Morocco in 2015. His research consists in the control of the linear and nonlinear systems with use of the advanced controller. Currently, he is preparing his PhD titled “Nonlinear control of renewable energy generation systems” in the Laboratory of System Analysis and Information Processing at Hassan 1st University.

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