Predicting health program participation: a gravity-based, hierarchical modelling approach Supplementary Material

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Supplementary Figures

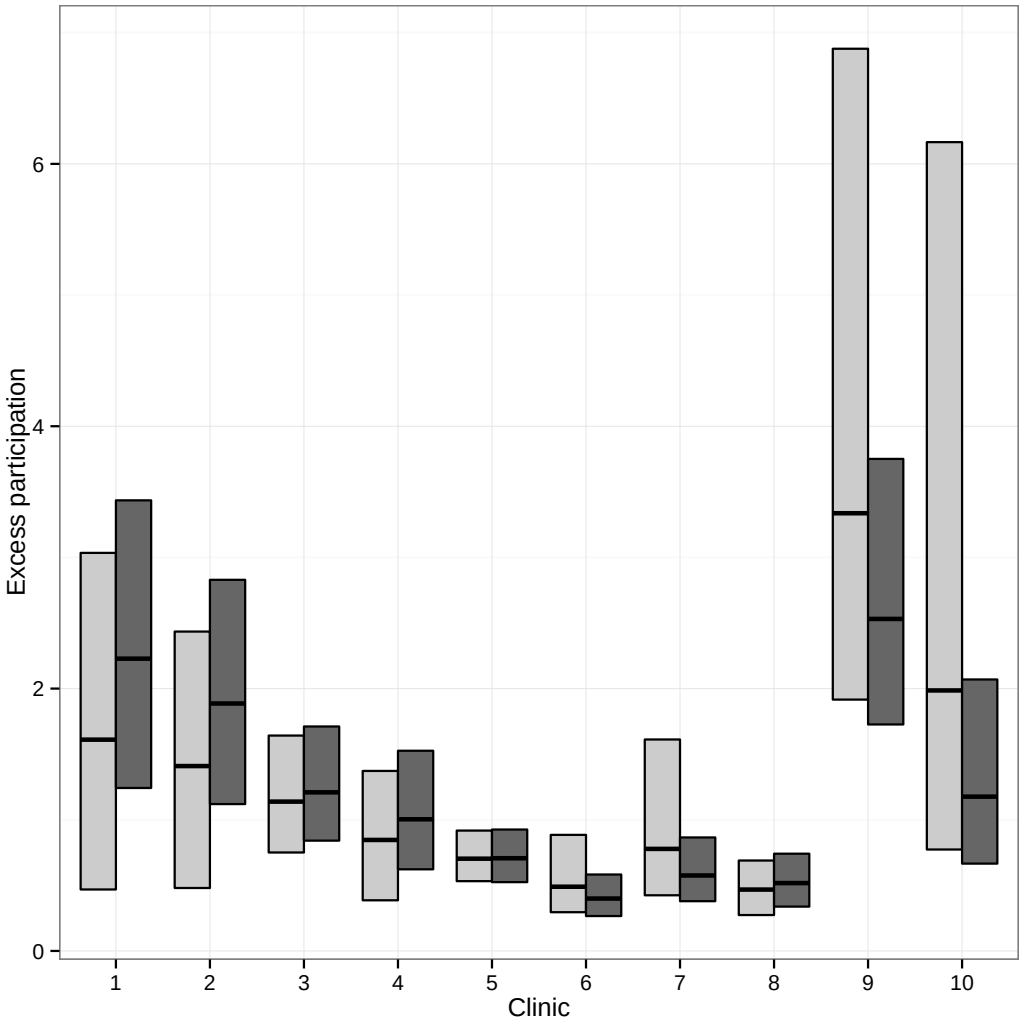

Fig. 1. Comparison of posterior means and 95% CIs for $\exp(v_i)$, for constrained and unconstrained gravity models with exponential decay distance function. Dark Grey = Constrained gravity model, Light Grey = Unconstrained gravity model

Prior constraints for distance-based function F4; $f\left(d_{ij};\boldsymbol{\theta}\right)=\left(1+\alpha_2+d_{ij}\right)^{-\alpha_1}$

The unknown parameters in function F4 (see Section 3.2 of main manuscript), α_1 and α_2, are difficult to identify without imposing constraints. To resolve this issue, the constraint $\alpha_1 > \alpha_2$ was applied and was motivated as follows. Setting $\delta = \alpha_2/\alpha_1$ with $\delta \in (0,1)$, one can write,

$$f\left(d_{ij};\boldsymbol{\theta}\right) \quad = \quad \left(1+\delta\alpha_1+d_{ij}\right)^{-\alpha_1}$$

such that, as $\delta \to 1$, $f(d_{ij};\theta) \to \left(1+\alpha_1+d_{ij}\right)^{-\alpha_1}$. Conversely, as $\delta \to 0$ corresponding to small α_2, $f(d_{ij};\theta) \to \left(1+d_{ij}\right)^{-\alpha_1}$. Therefore, this function can be viewed as a generalisation of distance-based function F2, reducing to the latter as $\alpha_2 \to 0$.

In contrast, the behaviour of $f\left(d_{ij};\boldsymbol{\theta}\right)$ under the constraint $\alpha_1 < \alpha_2$ is less desirable. This is explored by setting $\delta^* = \alpha_1/\alpha_2$ with $\delta^* \in (0,1)$, to give,

$$f\left(d_{ij};\boldsymbol{\theta}\right) \quad = \quad \left(1+\alpha_2+d_{ij}\right)^{-\alpha_2/\delta^*}. \tag{1}$$

In this case, as $\delta^* \to 0$ in line with small α_1, $f\left(d_{ij};\boldsymbol{\theta}\right) \to 0$ and therefore, does not reduce to a function of a single parameter. Furthermore, this constraint is no longer a generalisation of function F2, since α_1 will tend to zero before α_2.

Table 1. Posterior means and 95% credible intervals (CIs) for model variance components $(\sigma^2, \sigma_v^2, \sigma_s^2)$, for each stated choice of prior distribution. For each prior distribution, the corresponding posterior contribution of each variance component to total variation is also compared. Prior distributions marked with an asterisk denote distributions assumed for the variance in place of the standard deviation.

Model	*Prior distribution*	*Parameter*	*Posterior estimate*		*% of total variation*
			Mean	95% *CI*	$(\sigma^2, \sigma_v^2, \sigma_s^2)$
Constrained F1	Inverse Gamma*	σ^2	0.87	(0.72,1.04)	44
		σ_v^2	0.44	(0.15,1.10)	22
		σ_s^2	0.69	(0.32,1.27)	34
	Left-truncated Normal	σ^2	0.87	(0.72,1.04)	40
		σ_v^2	0.60	(0.19,1.54)	28
		σ_s^2	0.69	(0.33,1.25)	32
	Uniform	σ^2	0.87	(0.72,1.05)	39
		σ_v^2	0.67	(0.21,1.90)	30
		σ_s^2	0.69	(0.33,1.26)	31

Sensitivity analysis

In light of the relevance of the random effects estimates to the objectives of this study, a sensitivity analysis was performed on the prior distributions assumed for the variance components, $\left(\sigma^2, \sigma_v^2, \sigma_s^2\right)$. Specifically, three choices of prior distribution were considered: the original $U(0, 100)$ distribution, a left-truncated Normal distribution $\mathcal{N}(0, 1)_+$ and a proper Inverse Gamma distribution, $\mathcal{IG}(1, 0.01)$. The first two prior distributions were assumed for the standard deviation, whereas the third prior was placed on the variance. For the sake of brevity, the presentation of results is restricted to each choice of prior distribution, when it is assumed to be the same for all three variance components.

Table 1 summarises the posterior distribution of each variance component and its percentage contribution to total residual variation, for each choice of prior distribution. In this case, the greatest sensitivity concerned the variance associated with the clinic level random effects. In particular, under the $IG(1, 0.01)$ prior, a percentage reduction of $6 - 8\%$ in the contribution of σ_v^2 to total variation was observed in conjunction with a relatively narrow credible interval. These changes were also observed, albeit to a much lesser extent, for σ_s^2.

Differences in the posterior distributions for σ_v^2 indicated that the Inverse Gamma prior resulted in an unwanted level of shrinkage in random effects estimates, which in turn could possibly affect inferences related to excess participation. To explore this further, posterior estimates for each v_i and their uncertainty were compared under each prior distribution against an independent prior distribution, namely $v_i \sim \mathcal{N}(0, 1000)$, $i = 1, \ldots, L$. These results are summarised in Figure 2. For each prior distribution assumed for σ_v^2, the percentage change in estimates of $\exp(v_i)$ is taken to be relative to the same estimate under the independent prior distribution. The absolute value of this change is used as a measure of hierarchical shrinkage towards zero.

Following on from the results of Table 1, the greatest differences in random effects estimates were seen under the Inverse Gamma prior, with larger levels of shrinkage observed across all available clinics. For the remaining two prior distribution (left-truncated Normal and Uniform), results were generally consistent, with moderate shrinkage towards zero,

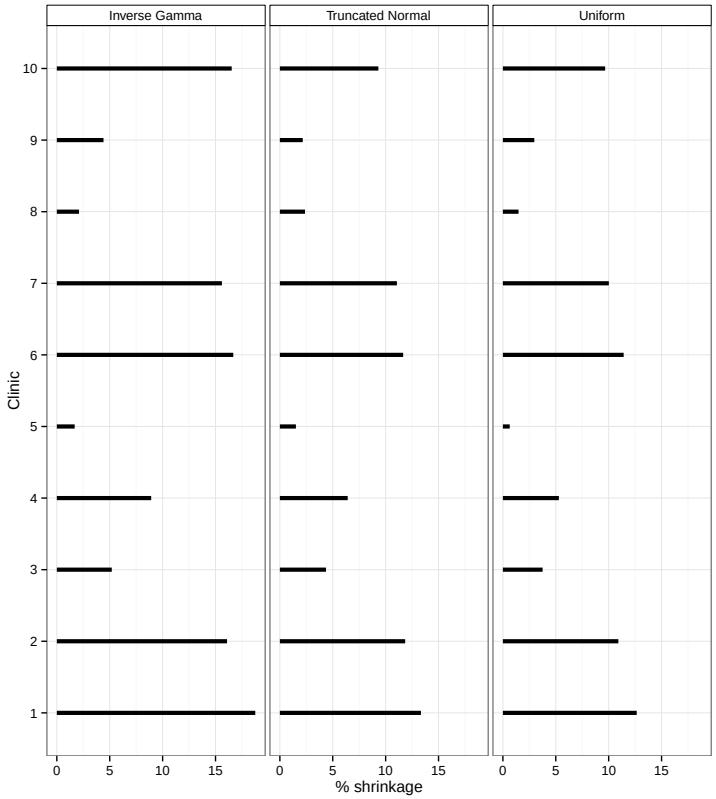

Fig. 2. Comparison of three prior distributions with respect to hierarchical shrinkage. In each plot, the absolute value of the percentage change in the posterior mean of each $\exp(v_i)$ relative to the estimate of excess participation under the independent prior $v_i \sim \mathcal{N}(0, 1000)$ is provided. Higher values indicate greater shrinkage towards zero.

which was seen to range between 5 and 13%.

Despite these discrepancies, inferences at the clinic level with respect to excess participation and the corresponding ranking of clinics were largely unaffected. This was confirmed in Table 2, where each clinic is summarised in terms of its rank distribution (also see Section 4.4 of main manuscript) and posterior probability of its associated random effect exceeding zero. Given our primary interest in inferences of this nature, as indicated in the objectives of the case study, it was concluded that they were generally robust to the choice of prior distribution.

Table 2. Posterior summary of clinic level random effects under the constrained gravity model with exponential decay function (F1), for different choices of prior distribution. For each clinic, the distribution of ranks (smallest to largest v_i) are described by the proportion of MCMC iterations that the estimated random effect falls into each quantile (see Section 4.4). The posterior probability of each random effect exceeding zero, as a measure of excess over-participation, is also provided. Inverse Gamma: $\mathcal{IG}(1, 0.01)$; Left-truncated Normal: $\mathcal{N}(0, 1)_+$; Uniform: $U(0, 100)$; Independent: fixed $\sigma_v^2 = 1000$. Prior distributions marked with an asterisk denote distributions assumed for the variance in place of the standard deviation.

Clinic	*Prior distribution*	*Distribution of ranks*				$Pr(v_l > 0 \mid \mathbf{y})$
		$0-25\%$	$26-50\%$	$51-75\%$	$76-100\%$	
1	Inverse Gamma*	0.00	0.00	0.08	0.92	0.99
	Left-truncated Normal	0.00	0.00	0.05	0.95	1.00
	Uniform	0.00	0.00	0.06	0.94	1.00
	Independent	0.00	0.00	0.03	0.97	1.00
2	Inverse Gamma*	0.00	0.01	0.19	0.80	0.98
	Left-truncated Normal	0.00	0.00	0.14	0.86	1.00
	Uniform	0	0.01	0.15	0.84	0.99
	Independent	0.00	0.01	0.07	0.92	0.99
3	Inverse Gamma*	0.00	0.15	0.79	0.06	0.81
	Left-truncated Normal	0.00	0.14	0.81	0.05	0.83
	Uniform	0.00	0.13	0.81	0.05	0.83
	Independent	0.00	0.10	0.86	0.04	0.87
4	Inverse Gamma*	0.05	0.50	0.44	0.01	0.44
	Left-truncated Normal	0.04	0.48	0.48	0.01	0.49
	Uniform	0.05	0.47	0.46	0.01	0.50
	Independent	0.03	0.34	0.62	0.01	0.59
5	Inverse Gamma*	0.28	0.70	0.02	0.00	0.01
	Left-truncated Normal	0.25	0.73	0.02	0.00	0.00
	Uniform	0.25	0.73	0.02	0.00	0.01
	Independent	0.20	0.79	0.01	0.00	0.00
6	Inverse Gamma*	0.99	0.01	0.00	0.00	0.00
	Left-truncated Normal	1.00	0.00	0.00	0.00	0.00
	Uniform	1.00	0.00	0.00	0.00	0.00
	Independent	1.00	0.00	0.00	0.00	0.00
7	Inverse Gamma*	0.76	0.22	0.01	0.01	0.01
	Left-truncated Normal	0.80	0.20	0.00	0.00	0.00
	Uniform	0.79	0.20	0.01	0.00	0.00
	Independent	0.87	0.12	0.01	0.00	0.00
8	Inverse Gamma*	0.90	0.10	0.00	0.00	0.00
	Left-truncated Normal	0.90	0.10	0.00	0.00	0.00
	Uniform	0.90	0.10	0.00	0.00	0.00
	Independent	0.88	0.12	0.00	0.00	0.00
9	Inverse Gamma*	0.00	0.00	0.00	1.00	1.00
	Left-truncated Normal	0.00	0.00	0.01	0.99	1.00
	Uniform	0.00	0.00	0.01	0.99	1.00
	Independent	0.00	0.00	0.02	0.98	1.00
10	Inverse Gamma*	0.01	0.31	0.47	0.21	0.69
	Left-truncated Normal	0.01	0.35	0.50	0.14	0.63
	Uniform	0.01	0.35	0.47	0.17	0.60
	Independent	0.02	0.52	0.38	0.08	0.47