# An improved modal strain energy method for structural dam age detection, 2D simulation

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**Abstract.** Structural damage detection using modal strain energy (MSE) is one of the most efficient and reliable structural health monitoring techniques. However some of the existing MSE methods have been validated for special types of structures such as beams or steel truss bridges which demands improving the available methods. The purpose of this study is to improve an efficient modal strain energy method to detect and quantify the damage in complex structures at early stage of formation. In this paper, a modal strain energy method was mathematically developed and then numerically applied to a fixed-end beam and a three-story frame including single and multiple damage scenarios in absence and presence of up to five per cent noise. For each damage scenario, all mode shapes and elemental stiffness of intact structures and the first five mode shapes of assumed damaged structures were obtained using STRAND7. The derived mode shapes of each intact and damaged structure at any damage scenario were then separately used in the improved formulation using MATLAB to detect the location and quantify the severity of damage as compared to those obtained from previous method. It was found that the improved method is more accurate, efficient and convergent than its predecessors. The outcomes of this study can be safely and inexpensively used for structural health monitoring to minimize the loss of lives and property by identifying the unforeseen structural damages.

**Keywords:** Modal strain energy; degree-of-freedom (DOF); finite element method (FEM); vibration based damage detection; structural damage

## 1. Introduction

Increasing the importance and complexity of infrastructures demands more reliable and precise techniques to detect structural damage. Structural health monitoring (SHM) as a new emerging technology has delivered some effective techniques that are successfully used to detect the damage of structures (Chan and Thambiratnam, 2011). Vibration based damage detection (VBDD) methods such as MSE methods are a significant group of SHM techniques.

Shi et al. (2000) established an MSE based method to detect the damage using the change in

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MSE in each element. Shi's approach is simple and capable of detecting single or multiple structural damages accurately. The sensitivity of the MSE was also derived as a function of the analytical mode shape and stiffness matrix. The results also showed that the proposed approach is capable of locating single and multiple damages that are contaminated with some percentage of noise effect. Although in this approach only the incomplete measured mode shapes and analytical system matrices are used for damage detection, there is a need to improve the method to more accurately detect the damage and quantify its severity.

Kisa and Gurel (2005) developed a numerical model to investigate the vibration analysis in cracked cantilever composite beams. The model employs finite element analysis and component mode synthesis method which is based on total strain energy of the system. Having modal data, the method was capable of identifying the location and dimension of the defect (crack) in the beam. However, the method is unique for detecting crack in cantilever composite beams with special cross section. Also it requires more studies for the same type of structure with other boundary conditions.

Asgarian et al. (2009) numerically applied an MSE method on a 3D four-story frame of a jacket offshore platform for damage detection. Modal strain energy change ratio (MSECR) and cross modal strain energy (CMSE) were used for locating and quantifying the damage respectively. Although this method performs well for this kind of structures, it is not capable of detecting the damage in all directions of vertical bracings of the case study demonstrated. Also it needs experimental studies to be applicable for this type of structures in reality. Shih et al. (Shih et al., 2009) blended a multi-criteria procedure incorporating modal flexibility and modal strain energy methods that was applied to a plate and a beam structures. The purpose was to identify single and multi-damages via a structural model simulation technique. Nine damage scenarios were considered in each element. For single damage it was found that modal flexibility changes (MFC) and MSE changes provided similar results with no locating error. Although for multiple-damage scenarios MSE changes increased the accuracy of the damage locating in the plate, the simulation of multiple-damage needs more investigation.

Brehm et al. (2010) enhanced a purely mathematical modal assurance criteria (MAC) called energy-based modal assurance criteria (EMAC) in terms of MSE. A numerical model and a benchmark study (cantilever truss) were presented to show the efficiency of the proposed method. The method sufficiently reduces uncertainties about mode shapes particularly when limited spatial information is available. However, this methodology cannot replace a cautious preparation of modal tests. Srinivas et al. (2010) proposed a multi-stage approach to detect structural damage using MSE and genetic algorithm (GA)-based optimization technique. The method was successfully applied to a simply supported beam and a plane truss. Although it is mentioned that the method can be used for damage detection in large-scale structures, no case study for this type of structure has been reported.

Yan et al. (2010) combined an CMSE with the niche genetic algorithms (GMs). The method was numerically used to detect the damage of an airfoil with composite materials. However, experimental works have not been reported in order to detect the structural damage in bridges or buildings. Wu and Sun (2011) compared and improved two damage identification methods which are based on MSE. Numerical studies showed that Shi's MSECR method is more accurate than Stubbs damage index method (SDIM). Even though both methods are noise sensitive and have limited robustness in damage identification. To improve these concerns and also modal expansion method, more studies are required. Hu et al. (2011) presented the surface crack detection in an aluminum circular hollow cylinder using MSE and scanning damage index methods. The

experimental results indicate the accuracy of the method. However, this method still needs to be more simplified for large structures and be applicable for different type of structures and different size of damages.

Wang et al. (2010) improved a modal strain energy correlation (MSEC) method using a theoretically derived MSE-to-damage sensitivity variable. Although this method was more efficient, noise contamination might give false alarms. Wang's method was further developed and validated for complicated steel truss bridges using multi-layer GA which became more efficient and feasible even in presence of noise (Wang et al. 2012). However, it is yet to be verified for other type of bridges or buildings. Wahalathantri et al. (2012) validated a damage index based MSE method for a simply supported and a two-span beam. This method was capable of locating and quantifying damage at any one of the measured modes. It was also found that the method was inexpensive and less time-consuming. Although this method is efficient enough, it is applicable only for simple beams.

Yan (2012) formulated a damage detection method based on element MSE sensitivity. Yan's method that is adapted a closed form of elemental MSE sensitivity, was numerically applied to some two-dimensional structures and high efficiency results were noted. Seyedpoor (2012) proposed a two-stage modal strain energy based index (MSEBI) to locate and quantify the structural damage. The numerical results of two samples showed the reliability of the method in damage identification. However, convergence achieves after some iterations which usually demands high computations. Also the effect of noise for the first case study has not been reported.

Li et al. (2013) calculated the sensitivity of element MSE of three structures: a fixed–fixed beam, an automobile frame and a two-bar truss structure using the methods available in the literature and the new method they proposed. The results of three numerical examples done from different methods were compared together. It was resulted that for large numbers of degrees of freedom (DOFs) and when the number of design variables exceeds the number of individual element stiffness matrices of interest, the proposed method has a good preferability. However, the storage capacity issue needs to be improved more.

Ding et al. (2013) proposed a damage index based MSE method for girder road and bridge structures. Numerically applying the method to a bridge using a continuous beam model was resulted in a good agreement with assuming damages at different locations with various quantities. Wang (2013) developed an iterative modal strain energy (IMSE) method using frequency measurements to estimate the structural damage severity. Unlike the other MSE methods, this method requires only few modal frequencies from damaged structure. The result of the experimental data from a clamped-free beam indicates the capability of the method in quantifying the damage extent accurately. Wang et al. (2013) developed an CMSE method to estimate the connection stiffness of the semi-rigid joints. The numerical study was successfully performed for a four-story frame structure considering different connection type of beam and column in presence of noise. The outcome of this method can be directly used to create an accurate model for structural damage detection.

From the literature reviewed, it is observed that MSE has been usually used for structural damage detection. However, the existing MSE methods have often been validated for some specific type of structures such as beam like structures, airfoil, offshore platforms, plane truss or steel truss bridges. Therefore it is essential to enhance/improve the available MSE methods in order to provide a more applicable and reliable approach for damage detection and quantification in any structure. This study aims to develop an MSE scheme in order to increase the accuracy of locating and quantifying the damage in any structure. The research contribution will result in

decreasing the loss of lives and property by preventing the unexpected structural damages and finally providing the safety of structures.

In this paper an MSE method is mathematically improved for detecting the structural damage of elements. The improved method is numerically applied to two 2D structural samples. Single and multiple damage scenarios with 3% and 5% noise in each scenario are also considered. The results of noise-free and noise-polluted cases are compared with a previous MSE based method (Shi et al., 2000), concluded and reported.

## 2. Traditional MSE Theory

Occurrence of damage in one or more elements of a structure results in changing in some of the structural parameters such as mode shapes, natural frequencies and stiffness (Shi et al., 2000) as follows;

The change in mode shape can be written;

$$\{\phi_i^d\} = \{\phi_i\} + \{\Delta\phi_i\} = \{\phi_i\} + \sum_{r=1}^{md} c_{ir}\{\phi_r\} \tag{1}$$

where $c_{ir} = \frac{\{\phi_r\}^T[\Delta K]\{\phi_i\}}{\lambda_i - \lambda_r}$ $(i \neq r)$,

md= number of analytical modes and

$\{\phi_i^d\}$ and $\{\phi_i\}$ are damaged and undamaged mode shapes at mode *i* respectively.

The natural frequencies change as follow Eq.

$$\lambda_i^d = \lambda_i + \Delta\lambda_i \tag{2}$$

where $\lambda_i^d$ and $\lambda_i$ are the damaged and undamaged eigenvalues at mode *i*.

Also it can be derived;

$$[K_m^d] = [K_m] + [\Delta K_m] = [K_m] + \alpha_m[K_m] \qquad (-1 < \alpha_m \leq 0) \tag{3}$$

where $[K_m^d]$ and $[K_m]$ are damaged and undamaged stiffness matrix of element *m* and $\alpha_m$ is the fractional reduction coefficient of $m^{th}$ elemental stiffness matrix.

Extending the Eq. (3) for all elements and accumulating;

$$\sum_{m=1}^{L}[K_m^d] = \sum_{m=1}^{L}[K_m] + \sum_{m=1}^{L}[\Delta K_m] = \sum_{m=1}^{L}[K_m] + \sum_{m=1}^{L}\alpha_m[K_m] \tag{4}$$

Simplifying (Shi et al. 2000);

$$[K^d] = [K] + [\Delta K] = [K] + \sum_{m=1}^{L}[\Delta K_m] = [K] + \sum_{m=1}^{L}\alpha_m[K_m] \tag{5}$$

where $K^d$ and $K$ are global damaged and undamaged stiffness of the structure respectively.

## 3. Improved MSE method formulation

In this paper, the previous study performed by Shi et al. (2000) has been improved in order to increase the accuracy of damage detection (Moradipour et al., 2013). Initially, unlike the previous study, the structural damaged stiffness matrix was used for establishing a more accurate MSE equation. It is expected that using the new MSE formulated can get more accurate strain energy which is stored in structural elements and finally provides a proper damage detection model as well as having less computation and iteration efforts.

Strain energy stored in the $j^{th}$ element at mode *i* before and after damage are as follow respectively (Shi et al. 2000);

$$\mathrm{MSE}_{i,j} = \frac{1}{2}\{\phi_i\}^T[K_j]\{\phi_i\} \tag{6}$$

$$\mathrm{MSE}^d_{i,j} = \frac{1}{2}\{\phi^d_i\}^T[K^d_j]\{\phi^d_i\} \tag{7}$$

The change in MSE is;

$$\Delta\mathrm{MSE}_{i,j} = \mathrm{MSE}^d_{i,j} - \mathrm{MSE}_{i,j} = \frac{1}{2}\{\phi^d_i\}^T[K^d_j]\{\phi^d_i\} - \frac{1}{2}\{\phi_i\}^T[K_j]\{\phi_i\} \tag{8}$$

Substituting for $\{\phi^d_i\}$ and $[K^d_j]$ in Eq. (8) from Eqs. (1) and (3) respectively;

$$\Delta\mathrm{MSE}_{i,j} = \frac{1}{2}\{\phi_i + \Delta\phi_i\}^T([K_j] + \alpha_j[K_j])\{\phi_i + \Delta\phi_i\} - \frac{1}{2}\{\phi_i\}^T[K_j]\{\phi_i\} \tag{9}$$

Simplifying and neglecting the higher order term leads to;

$$\Delta\mathrm{MSE}_{i,j} = \frac{1}{2}\alpha_j\{\phi_i\}^T[K_j]\{\phi_i\} + \frac{1}{2}(1+\alpha_j)\left[\{\phi_i\}^T[K_j]\{\Delta\phi_i\} + \{\Delta\phi_i\}^T[K_j]\{\phi_i\}\right] \tag{10}$$

Substituting for $\{\Delta\phi_i\}$ from Eq. (1) in Eq. (10) yields;

$$\begin{gathered}\Delta\mathrm{MSE}_{i,j} = \\ \frac{1}{2}\alpha_j\{\phi_i\}^T[K_j]\{\phi_i\} + \\ \frac{1}{2}(1+\alpha_j)\left[\{\phi_i\}^T[K_j]\sum_{r=1}^{md}\frac{\{\phi_r\}^T[\Delta K]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \right. \\ \left.\sum_{r=1}^{md}\frac{\{\phi_r\}^T[\Delta K]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_j]\{\phi_i\}\right] \qquad (i \neq r)\end{gathered} \tag{11}$$

where *i* is normally in the range of 1 to 5
and *r* is the number of analytical modes under consideration ($r \leq \text{no. of DOFs}$)

Substituting for $[\Delta K]$ from Eq. (4) into Eq. (11) ( $[\Delta K] = \sum_{i=1}^{L}\alpha_i[K_i]$) and simplifying

$$\Delta\mathrm{MSE}_{i,j} =$$

$$\frac{1}{2}\alpha_j\{\phi_i\}^T[K_j]\{\phi_i\} + \frac{1}{2}\{\phi_i\}^T[K_j]\sum_{i=1}^{L}\alpha_i\sum_{r=1}^{md}\frac{\{\phi_r\}^T[K_i]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \frac{1}{2}\sum_{i=1}^{L}\alpha_i\sum_{r=1}^{md}\frac{\{\phi_r\}^T[K_i]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_j]\{\phi_i\} + \frac{1}{2}\alpha_j\left[\{\phi_i\}^T[K_j]\sum_{r=1}^{md}\frac{\{\phi_r\}^T[\Delta K]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \sum_{r=1}^{md}\frac{\{\phi_r\}^T[\Delta K]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_j]\{\phi_i\}\right] \quad (i \neq r) \tag{12}$$

Ignoring the higher order terms leads to final equation of changing in MSE of the element *j* of the structure at mode *i* as follow;

$$\Delta MSE_{i,j} = \frac{1}{2}\alpha_j\{\phi_i\}^T[K_j]\{\phi_i\} + \frac{1}{2}\left[\{\phi_i\}^T[K_j]\sum_{i=1}^{L}\alpha_i\sum_{r=1}^{md}\frac{\{\phi_r\}^T[K_i]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \sum_{i=1}^{L}\alpha_i\sum_{r=1}^{md}\frac{\{\phi_r\}^T[K_i]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_j]\{\phi_i\}\right] \quad (i \neq r) \tag{13}$$

### *3.1 Locating the damage*

The proposed technique by Shi et al. (2000) is used for locating the damage. The change in MSE is upgraded with the improved $\Delta MSE$ from Eq. (13) to locate the damage more accurately. In this technique a damage location indicator called MSECR obtained from Eq. (14) is used. MSECR can be either derived for a single mode such as mode *i* and element *j* as given in Eq. 14(a) or normalized for the first five mode shapes of element *j* as given in Eq. 14(b). Since the improved $\Delta MSE$ is used for calculating the MSECR, it is expected the recent damage indicator to be more accurate in locating the damage. When MSECR is plotted versus element numbers, the elements with higher amounts of MSECR are the probably damaged elements.

$$MSECR_{ij} = \frac{\left|MSE_{i,j}^{d} - MSE_{i,j}\right|}{MSE_{i,j}} \tag{14.a}$$

$$MSECR_{j} = \frac{1}{m}\sum_{i=1}^{5}\frac{MSECR_{ij}}{MSECR_{i,max}} \tag{14.b}$$

where $MSECR_j$ is the average of $MSECR_j^i$ summation for the first five mode shapes normalized with respect to the largest value $MSECR_{max}^i$ of each mode.

Therefore, to locate the damage, any of Eq. (14.a) or (14.b) can be separately used to calculate the MSECR indicator. In case of using Eq. (14.a) any of the first five modes can be used i.e. *i* = any of 1 to 5. Though, the number of modes of damaged structure selected should be necessarily associated with that of undamaged one. However, using the Eq. (14.b) which mostly gives better results, requires the first five modes of both damaged and undamaged structures i.e. *i* =5.

### *3.2 Quantifying the damage*

The second attempt in the present study is derivation of a sensitivity matrix using the improved

MSE equation. When the damaged element/s is/are located among the most probably suspected elements from the previous section, damage quantifying process is conducted within those elements seeking for their α values. It is trying to find the amount of α's as the fractional reduction coefficient of elemental stiffness. The amount of α for true damaged elements will converge to their real damage percentage while for other suspected elements converge to zero. However, the exact value of each set of α's may be obtained through a number of iterations. The improved procedure is as follows;

From Eq. (13) ignoring the coefficient $\frac{1}{2}$, it can be derived;

$$[\beta]\{\alpha\} = \{MSEC'\} \tag{15}$$

Where $MSEC'$ is obtained from difference of damage and undamaged cases as Eq. (18) and $\beta$ is;

$$\beta_{s,t} = \frac{\partial MSE}{\partial \alpha} = \{\phi_i\}^T[K_j]\{\phi_i\} + \sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \sum_{r=1}^{n}\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_s]\{\phi_i\} \tag{16}$$

where *s* is a selected element for computation of the MSEC and
*t* is a suspected damaged element.

In previous studies MSEC has been considered as following terms (Shi et al., 2000 and Wang et al., 2012) to be used in the right side of Eq. (15);

$$MSEC_{ij} = \{\phi_i^d\}^T[K_j]\{\phi_i^d\} - \{\phi_i\}^T[K_j]\{\phi_i\} \tag{17}$$

As the value of $MSE_{i,j}^d$ theoretically is a function of $[K_j^d]$, definitely it is expected by using $K_j^d$ instead of $K_j$ get more exact value for $MSEC_{ij}$, therefore;

$$MSEC'_{ij} = \{\phi_i^d\}^T[K_j^d]\{\phi_i^d\} - \{\phi_i\}^T[K_j]\{\phi_i\} \tag{18}$$

Substituting for $K_j^d$ from Eq. (3) into Eq. (18), simplifying and then arranging:

$$MSEC'_{ij} = \alpha_j\{\phi_i^d\}^T[K_j]\{\phi_i^d\} + \{\phi_i^d\}^T[K_j]\{\phi_i^d\} - \{\phi_i\}^T[K_j]\{\phi_i\} \tag{19}$$

Substituting Eq. (17) into Eq. (19) gives;

$$MSEC'_{ij} = \alpha_j\{\phi_i^d\}^T[K_j]\{\phi_i^d\} + MSEC_{ij} \tag{20}$$

Substituting Eqs. (16) and (20) into Eq. (15)

$$\Big[\{\phi_i\}^T[K_j]\{\phi_i\} + \sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} +$$

$$\sum_{r=1}^{n}\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_s]\{\phi_i\}\Big]\{\alpha\} = \alpha_j\{\phi_i^d\}^T[K_j]\{\phi_i^d\} + \{MSEC\} \tag{21}$$

Simplifying

$$\Big[-[\{\phi_i^d\}^T[K_j]\{\phi_i^d\} - \{\phi_i\}^T[K_j]\{\phi_i\}] + \sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \sum_{r=1}^{n}\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_s]\{\phi_i\}\Big]\{\alpha\} = \{MSEC\} \tag{22}$$

Substituting Eq. (17) into Eq. (22)

$$\Big[-[MSEC] + \sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \sum_{r=1}^{n}\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_s]\{\phi_i\}\Big]\{\alpha\} = \{MSEC\} \tag{23}$$

Denoting $\beta_{s,t}^* = -MSEC_{ij}$ and

$$\beta'_{s,t} = \sum_{r=1}^{n}\{\phi_i\}^T[K_s]\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\} + \sum_{r=1}^{n}\frac{\{\phi_r\}^T[K_t]\{\phi_i\}}{\lambda_i-\lambda_r}\{\phi_r\}^T[K_s]\{\phi_i\},$$

Then, $\beta_{s,t}$ can be written in the following form;

$$\beta_{s,t} = \beta_{s,t}^* + \beta'_{s,t} \tag{24}$$

Reconstructing the Eq. (15) in matrix notation,

$$([\beta^*] + [\beta'])\{\alpha\} = \{MSEC\} \tag{25}$$

Or

$$\left(\begin{bmatrix} \beta^*{}_{11} & 0 & \dots & 0 \\ 0 & \beta^*{}_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \beta^*{}_{qq} \end{bmatrix} + \begin{bmatrix} \beta'_{11} & \beta'_{12} & \dots & \beta'_{1q} \\ \beta'_{21} & \beta'_{22} & \dots & \beta'_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \beta'_{q1} & \beta'_{q2} & \dots & \beta'_{qq} \end{bmatrix}\right)\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_q \end{bmatrix} = \begin{bmatrix} MSEC_{i1} \\ MSEC_{i2} \\ \vdots \\ MSEC_{iJ} \end{bmatrix} \tag{26}$$

where $[\beta^*]$ is a diagonal matrix that is proposed in this study in order to increase the accuracy of $\{\alpha\}$'s. Each array of $[\beta^*]$ is a function of MSEC of the associated element in a specific mode. Finally from Eq. (26), $\{\alpha\}$'s are obtained in the following expanded form;

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_q \end{bmatrix} = \left(\begin{bmatrix} \beta^*{}_{11} & 0 & \dots & 0 \\ 0 & \beta^*{}_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \beta^*{}_{qq} \end{bmatrix} + \begin{bmatrix} \beta'_{11} & \beta'_{12} & \dots & \beta'_{1q} \\ \beta'_{21} & \beta'_{22} & \dots & \beta'_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \beta'_{q1} & \beta'_{q2} & \dots & \beta'_{qq} \end{bmatrix}\right)^{-1}\begin{bmatrix} MSEC_{i1} \\ MSEC_{i2} \\ \vdots \\ MSEC_{iJ} \end{bmatrix} \tag{27}$$

Accordingly, to calculate the alpha coefficients, $\alpha_i$, in order to quantify the damage using Eq. (27), two sets of mode shapes are required. According to Eq. (24), the number of analytical mode

shapes required from undamaged case is $r$ which equals to or is less than the number of DOFs of the structure under consideration ($r \leq$ no. of DOFs). However, the alpha calculation process can be stopped at a very lower mode number than the nominated number of DOFs when it converges. Whereas, from damaged case the number of required mode/s is $i$ that equals to any of modes from 1 to 5. Since mode one and three normally give better solution, then $i$=1 or 3.

### *3.3 Noise effect*

The effect of noise is applied using Eq. (28) (Shi et al. (2000));

$$\overline{\varphi}_{ij} = \varphi_{ij}(1 + \gamma_i^{\varphi}\rho^{\varphi}\left|\varphi_{max,j}\right|) \tag{28}$$

where $\overline{\varphi}_{ij}$ and $\varphi_{ij}$ are the mode shape components of the $j^{th}$ mode at $i^{th}$ DOF
$\gamma_i^{\varphi}$ are the random numbers with the mean of zero and a variance of one
$\rho^{\varphi}$ is the noise level (per cent)
$\varphi_{max,j}$ is the largest component of the $j^{th}$ mode shape

## 4. Verification

To verify the improved method in this study, an attempt is made to validate it for 2D structures. For this purpose, two structures including an 2D steel beam and an three-story steel frame with frame elements of three DOFs at each end have been selected to apply the improved method and compare the results with those from previous method (Shi et al., 2000).

### *4.1 Illustrative example 1*

The first numerical example is a fixed-end steel beam consisting of 12 elements and 13 nodes with 33 DOFs as shown in Fig. 1. The material properties and geometric data are as follow;
Length of each element =L= 0.60 m
Modulus of elasticity = E=$207 \times 10^9$ $N/m^2$
Cross-sectional area = A=0.0016 $m^2$
Second moment of area = I= $3.4133 \times 10^{-9}$ $m^4$
Mass density = 7870 kg/$m^3$

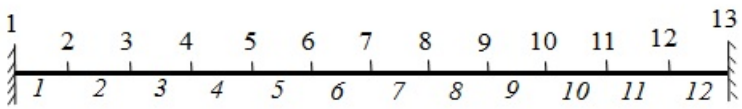
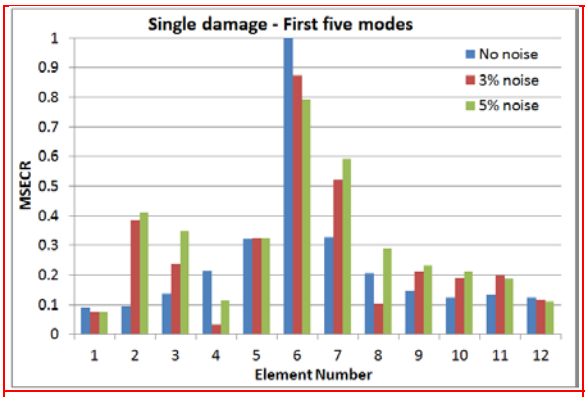



Fig. 1 FEM of the fixed-supported beam

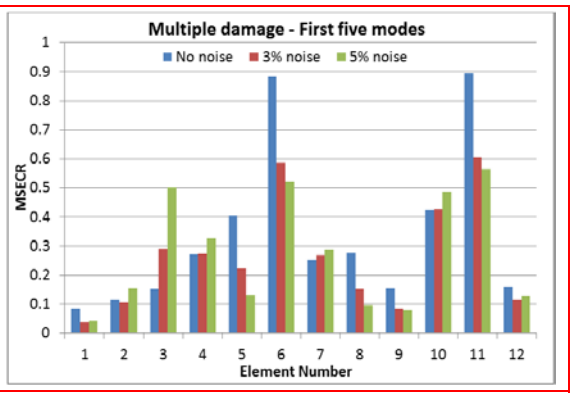
Two damage cases are assumed to occur in the beam. Case 1 is a single-damage that occurs in element 6 with a stiffness loss of 15% and case 2 is a multiple-damage case with damage in elements 6 and 11 with stiffness loss of 10% in each element. Three and five per cent of noises are also considered in each damage scenario respectively. The results of noise contamination will be compared with the case with no noise (zero per cent noise).

To detect the single and multiple damage locations, the MSCER parameters are calculated and shown in Fig. 2 using Eq. 14(b). For this purpose, the first five mode shapes of both damaged and undamaged cases are used i.e. $i$ =5.

The second calculation is to find the alpha coefficients to quantify the damage. According to Eq. (11 or 13), the number of analytical modes required from undamaged case is $r$ which equals to or is less than the number of DOFs of the structure ($r \leq$ no. of DOFs). While, from damaged case the number of required mode/s is $i$ that equals to any of modes from 1 to 5. Since mode one gives more exact solution, then $i$=1. So in this example, $r$ =33 and $i$ =1. Finally, the α's of the improved method are calculated from Eq. (27) while in previous study (Shi et al., 2000) its own equation is used. The single and multiple damage coefficients (α's) using the improved method and previous study quantified with the first mode are shown in Figs. 3 and 4 respectively. It is seen that the horizontal axis of Figs. 3 and 4 is $r$ that should be started from 1 and continued to the maximum possibly analytical mode which is the number of DOFs of the structure. However, the procedure of alpha calculation can be stopped at a much lower mode than the number of nominated DOFs once it converges. In this example, although the convergence has been achieved after mode 21, still the calculation has been continued. For the first few modes also, alpha coefficients may get large values that cannot be shown in the figures with same scale. That is why some of the (first) modes are missing in the figures while they have been considered in calculation.

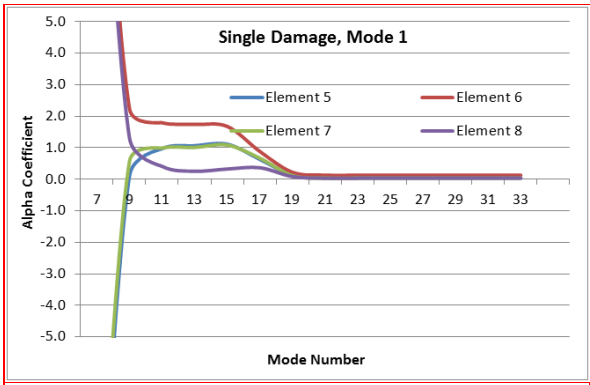



(a) Single damage, element 6

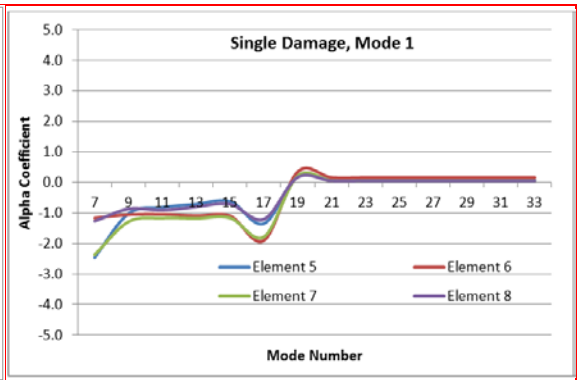



(b) Multiple damage elements 6 and 11

Fig. 2 Elemental damage located with first five modes

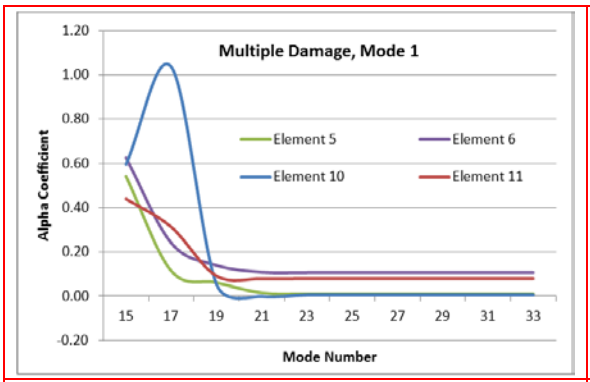



(a) Previous study (Shi et al., 2000)

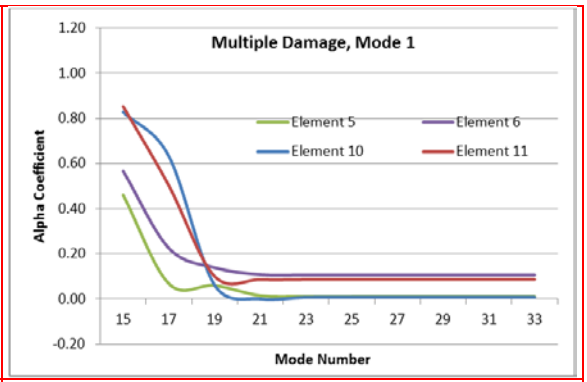



(b) Proposed method in current study

Fig. 3 Coefficients of single-damages quantified with first mode




(a) Previous study (Shi et al., 2000)




(b) Proposed method in current study

Fig. 4 Coefficients of multiple-damages quantified with first mode

### *4.2 Illustrative example 2*

The second example is a three-story steel frame with frame elements of three DOFs at each end consisting of nine elements and eight nodes with 18 DOFs as shown in Fig. 5. The material properties and geometric data are as follow;

Length =L= 3.0 m
Modulus of elasticity = E=$207 \times 10^9$ $N/m^2$
Cross-sectional area = A=0.0015 $m^2$
Second moment of area = I= $1.125 \times 10^{-7}$ $m^4$
Mass density = 7870 kg/$m^3$

Similarly, two damage scenarios are assumed to occur in the frame. Case 1 is a single-damage that occurs in element 8 with a stiffness loss of 15% and case 2 is a multiple-damage case with damages in elements 4 and 8 with stiffness loss of 10% in each element. Three and five per cent noises are also considered in each damage scenario respectively. The MSCER parameters are

calculated and shown in Fig. 6 using Eq. 14(b) and the first five mode shapes of both damaged and undamaged cases indicating $i$ =5. Similar to example 1, to find the alpha coefficients, the same equations are used instead here $r$ =18 and $i$ =1.

The single and multiple damage coefficients (α's) using the improved method and previous study quantified with the first mode are shown in Figs. 7 and 8 respectively. The same procedure applies to the number of analytical mode shapes to be shown on horizontal axis or to the last mode shape that is associated with convergence. Although in this example, the convergence has been achieved after mode 10, the calculation has been continued also.

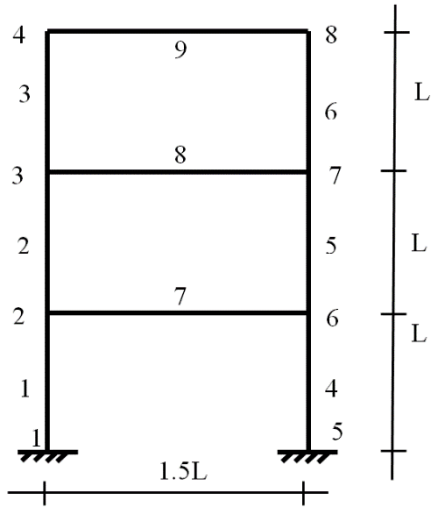
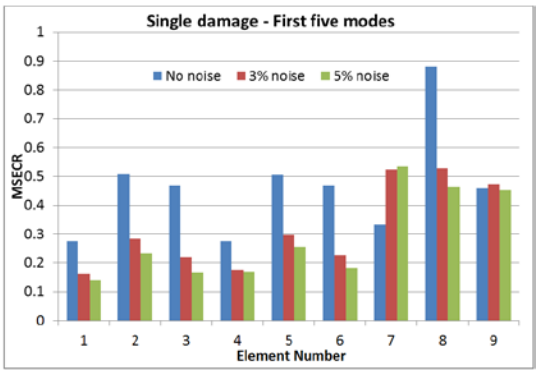
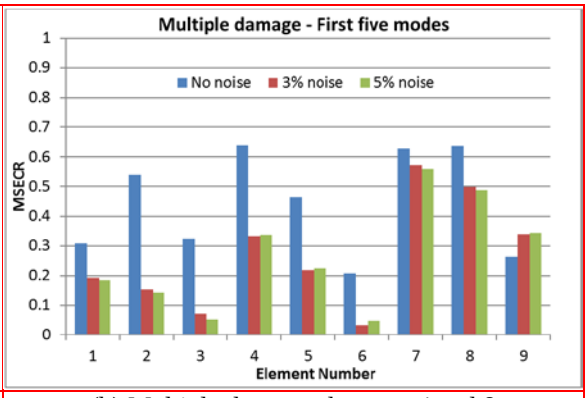



Fig. 5 FEM of the three-story steel frame

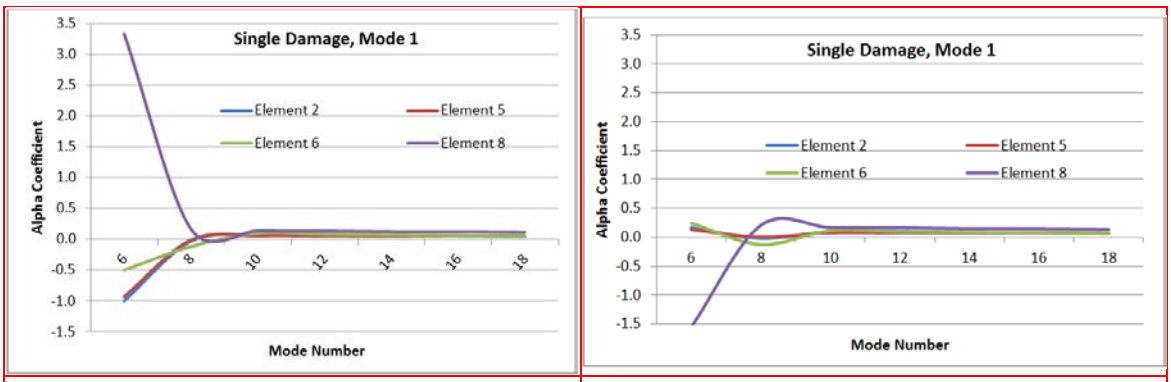



(a) Single damage, element 8



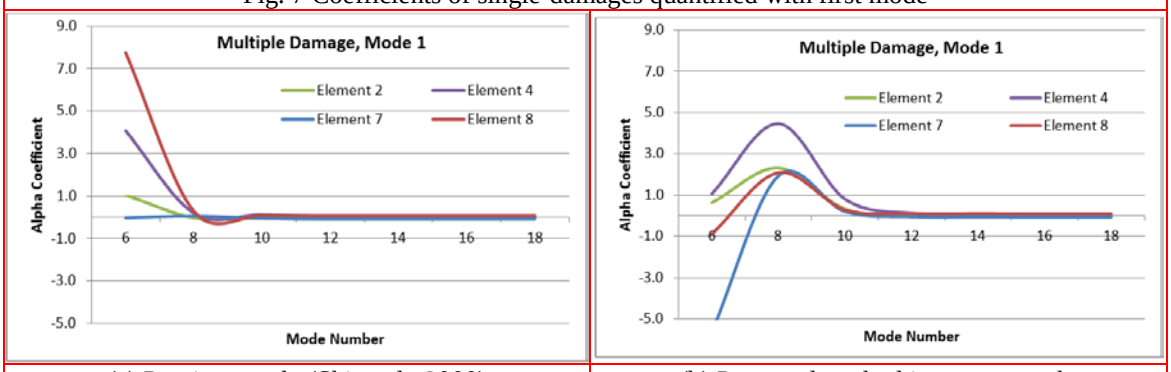

(b) Multiple damage elements 4 and 8

Fig. 6 Elemental damage located with first five modes




(a) Previous study (Shi et al., 2000) (b) Proposed method in current study

Fig. 7 Coefficients of single-damages quantified with first mode




(a) Previous study (Shi et al., 2000) (b) Proposed method in current study

Fig. 8 Coefficients of multiple-damages quantified with first mode

## 5. Results and discussion

For the beam structure, in single-damage scenario, shown in Fig. 2(a), it is seen that the MSECR crests at element 6 which represents it is the highly suspected element to damage. Even though, elements 4, 5, 7, 8, 9 and 11 are also likely exposure to damage because of high value of MSECR. However, to decrease the computation volume, few suspected elements such as 5, 6, 7, and 8 are selected for next stage to quantify their $\alpha$ coefficients. The calculation of $\alpha$ coefficient for selected elements versus analytical mode is depicted in Figs. 3(a) and 3(b) for previous study (Shi et al., 2000) and current study respectively. From this figure, it is seen that the amount of all $\alpha$'s converge to zero except $\alpha_6$ which converges to around 0.16 in current study and 0.13 in previous study. Compared to the assumed damage of 15 per cent in element 6, current study gives a more exact value for $\alpha_6$. It also indicates that the improved method is more convergence at any mode shape than previous study as shown in Figs. 3 to 4 and 7 to 8. In other words, the rate of convergence in the improved method is faster.

Similarly, in multiple-damage scenario, shown in Fig. 2(b), the MSECR peaks at elements 6 and 11 which shows their highly possibility to damage. Elements 4, 5, 7, 8 and 10 are also probably damaged elements but among these suspected elements, only elements 5, 6, 10 and 11 are selected to their $\alpha$ coefficients be quantified. Fig. 4 shows that the amount of $\alpha_5$ and $\alpha_{10}$

converge to zero but $\alpha_6$ and $\alpha_{11}$ converge to around 0.102 & .088 in current study and 0.106 & 0.079 in previous study respectively. Compared to the assumed damage of 10 per cent in both elements 6 and 11, in this case it is also seen that the improved method performs better.
The effect of 3 and 5 per cent noises are also included in calculation as shown in Fig. 2. It is seen that the MSECR parameter of damaged elements is reduced in presence of noise (Shi et al., 2000) and also it decreases with increasing the noise percentage. However, the MSECR parameter of undamaged elements such as elements 3, 4, 7 and 10 has mostly increased a few percent. The procedure of quantifying damage for contaminated cases with noise is same and almost the same results are obtained with more computational cycles.

In a similar way, for the second structure, in single-damage scenario, according to Fig. 6(a), element numbers 2, 5, 6 and 8 are selected as the suspected damaged elements. The obtained coefficient of $\alpha_8$ are 0.1366 and 0.099 for current and previous study respectively as shown in Fig. 7 in which the performance of current study is more exact compared to the assumed damage of 15 per cent. In multiple-damage scenario also based on Fig. 6(b) among the selected suspected elements 2, 4, 7 and 8, the amount of $\alpha_4$ and $\alpha_8$ are calculated 0.086 & 0.1018 and 0.067 & 0.085 in current and previous studies respectively as shown in Fig. 8. The more vicinity of alpha coefficient in current study to assumed damage of 10 percent for elements 4 and 8 indicates the more accuracy of the current study. The effect of 3 and 5 per cent noises are also included in calculation for this example as shown in Fig. 6. The trend is similar to previous example explained for Fig. 2.

It should mention that for selecting the suspected damaged elements, there is no limitation neither in the number nor order of elements. It is because of that only the true damaged elements will finally get a non-zero coefficient of damage. However, selecting the large numbers of suspected elements could increase the computational cycles particularly for complex structures but it does not affect identifying the true damaged elements.

Additionally, it is seen that in this method, the required numbers of mode shapes are as follow;

i. For locating the damage

   a. The first five modes of damaged structure
   b. The first five modes of undamaged structure

ii. For quantifying the damage

   a. Only one mode from damaged structure is required, usually mode one or three
   b. From undamaged structure, as much mode as possible, the more the better (at least the first five modes that were used in damage localization)

In practice, incomplete mode measuring may occur because of some parameters such as less number of sensors, improper placement of sensors, difficulty in measuring the rotational DOFs, effect of noise and error in processing the data. Though, normally at least the first five modes can be obtained. So, in this method, there is no difficulty for locating the damage. However, having less number of modes from undamaged structure may decrease the damage quantification accuracy.

To overcome this issue, the mode expansion method proposed by Shi et al. (1995) can be used to expand the inadequate number of DOFs measured to the full dimension of FEM. Also according to Hu (1987), when the stiffness of the structure changes, each perturbed mode shape can be linearly expressed as a combination of the original mode shapes.

## 6. Conclusions

In this article an MSE method was mathematically improved and formulated to precisely detect and quantify the structural damage in complex structures. The improvement was conducted in two stages; firstly, the equation of MSE was more exactly formulated considering damaged elemental stiffness. The improved MSE was then used to get an accurate sensitivity matrix to perfectly detect and quantify the damage. Verification of the improved method was performed by applying the method to two plane structures with frame elements of three DOFs at each end as the comprehensive 2D samples. Single and multiple damage scenarios were considered for each structure. The mode shapes associated with assumed damages also were contaminated by 3 and 5 per cent noises. After getting the results and analyzing them, it was observed that;

- The current method is more accurately capable of detecting and quantifying the structural damage than previous study (Shi et al. 2000).
- The improved method converges faster with higher rate i.e. converges with less number of modes. This feature considerably decreases the number of iterations especially for complex structures that makes it more inexpensive.
- Although the method is slightly noise sensitive, in presence of some usual percentage of noise performs well and is capable of both detecting and quantifying the damage accurately.

In a similar way, this study can be numerically extended for 3D structures. Also it can be practically tested for any laboratory model or real structure by measuring the first five mode shapes of the model or prototype instead of mode shapes calculated from assumed damage/s. These studies currently are under investigation and will be subsequently reported.

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## References

Asgarian, B., Amiri, M. & Ghafooripour, A. (2009), “Damage detection in jacket type offshore platforms using modal strain energy”, *Struct. Eng. Mech.*, **33**(2), 325-337.

Brehm, M., Zabel, V. & Bucher, C. (2010), “An automatic mode pairing strategy using an enhanced modal

assurance criterion based on modal strain energies", *J. Sound Vib.*, **329**, 5375-5392.
Chan, T. H. T. & Thambiratnam, D. P. (2011), *Structural Health Monitoring In Australia*, Nova Science Publishers, New York, NY, USA.
Ding Q., Zou C., Tang Y. & Zhang S. (2013), "Damage detection in road and bridge based on modal strain energy method", *ICTE © American Society of Civil Engineers (ASCE)*, 1753-1758.
Hu, H. C. (1987). *The vibration theory of multiple freedom system*, Science Press, China.
Hu, H., Wu, C. & Lu, W.-J. (2011), "Damage detection of circular hollow cylinder using modal strain energy and scanning damage index methods", *Comput. Struct.*, **89**, 149-160.
Kisa, M. & Gurel, M. A. (2005), "Modal analysis of cracked cantilever composite beams", *Struct. Eng. Mech.*, **20**(2), 143-160.
Li L., Hun Y. & Wang X. (2013), "Numerical methods for evaluating the sensitivity of element modal strain energy", *Finite Elem. Analy. Design*, **64**, 13–23.
Moradipour, P., Chan T. H. T & Gallage C. (2013), "Health monitoring of short- and medium-span bridges using an improved modal strain energy method", *Proceedings of the 5th International Workshop on Civil Structural Health Monitoring(CSHM-5), The International Society for Structural Health Monitoring of Intelligent Infrastructure (ISHMII)*, Ube, Yamaguchi, Japan, Oct..
Seyedpoor, S. M. (2012), "A two stage method for structural damage detection using a modal strain energy based index and particle swarm optimization", *Int. J. Non-Lin. Mech.*, **47**, 1-8.
Shi, Z. Y., Ding, X. H., and Gu, H. Z. (1995). ''A new model reduction and expansion method.'' *Proc., Int. Conf. on Struct. Dyn., Vibration, Noise and Control*, 847–852.
Shi, Z. Y., Law, S. S. & Zhang, L. M. (2000), "Structural damage detection from modal strain energy change", *J. Eng. Mech.*, **126**, 1216-1223.
Shih, H. W., Thambiratnam, D. P. & Chan, T. H. T. (2009), "Vibration based structural damage detection in flexural members using multi-criteria approach", *J. Sound Vib.*, **323**, 645-661.
Srinivas, V., Ramanjaneyulu, K. & Jeyasehar, C. A. (2010), "Multi-stage approach for structural damage identification using modal strain energy and evolutionary optimization techniques", *Struct. Health Monit.*, **10**(2), 219-230.
Wahalthantri, B., Thambiratnam, D. P., Chan, T. H. T. & Fawzia, S. (2012), "An Improved method to detect damage using modal strain energy based damage index", *Adv. Struct. Eng.*, **15**, 727-742.
Wang, F. L., Chan, T. H. T., Thambiratnam, D. P., Tan, A. C. & Cowled, C. J. (2012), "Correlation-based damage detection for complicated truss bridges using multi-layer genetic algorithm", *Adv. Struct. Eng.*, **15**, 693-706.
Wang, L., Chan, T. H. T., Thambiratnam, D. P. & Tan, A. (2010), "Damage detection for truss bridge structures using correlation-based structural modal strain energy", *Proceedings of the 8th International Conference on Short and Medium Span Bridges, The Canadian Society for Civil Engineering*, Niagara Falls, Ontario, Canada, Aug..
Wang S. (2013), "Iterative modal strain energy method for damage severity estimation using frequency measurements", *Struct. Control Health Monit.*, **20**, 230–240.
Wang, S., Zhang, M. & Liu, F. (2013), "Estimation of semi-rigid joints by cross modal strain energy method", *Struct. Eng. Mech.*, **47**(6), 757-771.
Wu, H. S. & Sun, L. M. (2011), "Comparison and improvements of two damage identification methods based on modal strain energy", *Proceedings of the Sensors and Smart Structures Technologies for Civil, Mechanical, and Aerospace Systems, SPIE 7981*, San Diego, California, USA, April.
Yan, W.-J., Ren, W.-X. & Huang, T.-L. (2012), "Statistic structural damage detection based on the closed-form of element modal strain energy sensitivity", *Mechanical Systems and Signal Processing*, **28**, 183-194.
Yan, Y. J., Yang, H. F., Wu Z. Y. & Ge, X. (2010), "Damage detection method for composite structures based on a combined technique of cross modal strain energy and niche genetic algorithms", *J. Vib. Cont.*, **16**(11), 1673-1683.