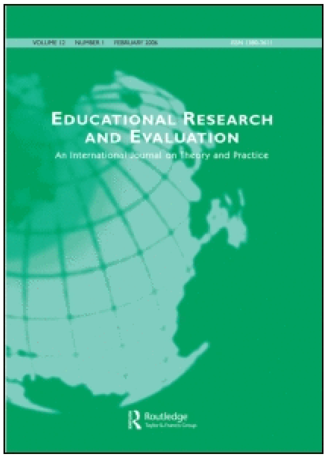

Educational Research and Evaluation

An International Journal on Theory and Practice

Voting Power on School Governing Boards: The Countenance of Proportionality

Anthony Kelly

To cite this Article: Kelly, Anthony , 'Voting Power on School Governing Boards: The Countenance of Proportionality', Educational Research and Evaluation, 8:3, 277 - 297
To link to this article: DOI: 10.1076/edre.8.3.277.3853
URL: http://dx.doi.org/10.1076/edre.8.3.277.3853

Educational Research and Evaluation
2002, Vol. 8, No. 3, pp. 277–297
1380-3611/02/0803–277$16.00

Voting Power on School Governing Boards: The Countenance of Proportionality

Anthony Kelly
Research and Graduate School of Education, University of Southampton, UK

ABSTRACT

Acknowledging the realities and responsibilities of power is a precondition to using it wisely. The claim that there has been a shift in power away from the formal providers of education towards the individual consumer is one that needs closer investigation.

This paper uses the mathematics of cooperative multiperson game theory to analyse the relative strengths of the various representative groupings on three different models of school governing bodies. Only a basic knowledge of mathematics is assumed as the various coalitions are analysed and compared, and conclusions drawn about the relative power of major and minor factions. Voting strategies, suggested payoffs for winning coalitions and implications for committee-forming are fully examined.

The paper is based on the author's direct experience of school amalgamations in the border region of Ireland. An extended consideration of the theory of voting in multiperson games can be found in his book *Decision-Making and Game Theory* (2002, Cambridge University Press).

BACKGROUND

There is a tendency among some researchers in education to analyse power only from the perspective of how participants perceive it. This is legitimate insofar as it represents one important aspect of governance, but it can obfuscate the reality of having and using power, however it is perceived.

Recently, some authors (Donnelly, 2000) have described a dramatic shift in the balance of power away from the formal providers of education towards the

Address correspondence to: Anthony Kelly, Research and Graduate School of Education, University of Southampton, Southampton SO17 1BJ, UK. Tel.: +44 23 8059 3475. Fax: +44 23 8059 3556. E-mail: a.kelly@soton.ac.uk

Manuscript submitted: February 27, 2001.
Accepted for publication: August 23, 2001.

individual consumer, although the extent to which parents and students are active participants in education, as opposed to passive recipients of it, has not been fully determined. It could be argued, for example, that although society has become more actively consumerist in economic terms, it remains doggedly passive in matters political and educational.

Certainly, ethnographic research on school governance provides a valuable insight into how different stakeholders perceive the extent of their power, particularly in relation to that of their counterparts in other institution types, but it does not address the question of how actual power is distributed. It is an important question. In Northern Ireland, for example, fair employment legislation does not apply to the appointment of teaching staff, so boards of governors have considerable power over school decision-making, and the only check on the exercise of this authority is the relative distribution of influence among the parties represented on the boards.

In 1993, following some years as a head teacher, the author led an amalgamation of two secondary schools from different traditions in a community on the Irish border, in the course of which a new model for school governance (in the Republic of Ireland) was developed. The exercise was widely acknowledged to have been a success and the author subsequently acted as consultant to similar rationalisations in other areas.

From first-hand experience of initial and developing reactions to these various restructurings, and from interviews with protagonists from boards at other schools and been granted access to minutes of meetings, it became apparent that different factions had different experiences of the extent to which their expectations of power had been met by their assigned voting strengths. For one political representative, the restructuring became:

> . . . . . a major disappointment. [As a] democratically elected representative, [he] came to feel like a spectator. [Participation on the board] became a complete waste of time.

Typically, parents and teachers expressed themselves content at the school-level restructuring, but became disappointed and perplexed at their lack of influence at board level:

> The school is good. The staff is happy enough and the principal is fair with the two staffs, but the Board is a washout. It's my fault in some ways because I pushed it. I thought we'd have an influence, but we don't. I don't know why. We just lose every important vote – early closings, the new posts, the 'Vpship', all that. [Teacher]

And:

> By the time the chairman lets us speak, it's all over. The others have stitched it up. I don't know if he does it deliberately . . . he knows how to manipulate a meeting. I suppose that's what comes of being a politician for thirty years! We tried different tactics, but he calls the speakers in the same way every month – when it's important anyway. [Parent]

Even a representative of a majority (church) faction had reservations:

> In terms of fulfilling the pastoral and spiritual needs of the community, I have no doubt that the school and its staff are doing their best and have been largely successful. But I would have concerns on account of our lack of influence on the board, to be frank. This is not to say . . . that I am unhappy.

Clearly, there is some disparity between the expectation of power among parties to a restructuring and what transpires to be the reality of voting strength. This paper is an attempt to explain that disappointment. It is a game theoretic exposition of the concepts that underpin the distribution of power in multiperson interactions. As such, it is of course a simplification of what practitioners (such as the author) know to be the reality of managing schools today. In the course of everyday headship, most decisions are made without recourse to formal voting. However, the decisions that *are* made in committee are often strategic ones and therefore of critical importance to the development of the school. For that reason it is important that the distribution of power in committee-like structures is properly understood. Awareness is often what separates a failing manager from a successful one.

Three different models of school governance are described below. They are loosely based on school types in Northern Ireland so that the reader has a sense of a situation in which voting conflict is realistic and pervasive, rather than artificial and occasional. However, the subsequent analysis is in no way peculiar to these models or to that setting. They merely serve to illustrate the principles involved.

Model A: Voluntary Maintained Secondary Schools

Voluntary Maintained schools in Northern Ireland are mostly non-selective Roman Catholic schools which receive funding from central government. The Roman Catholic church had originally refused to cede schools to the state, but accepted the "maintained" arrangement as a result of financial pressures in the 1970s. Voluntary Maintained schools typically have the following representations on their boards:

- 4 "trustees" (Roman Catholic church representatives);
- 2 Education and Library Board representatives (ELB) [An Education and Library Board is the equivalent of a local education authority];
- 1 parent representative;
- 1 teacher representative;
- 1 representative from the Department of Education, Northern Ireland (DENI).

Model B: Controlled Secondary Schools

Controlled Secondary schools in Northern Ireland are mostly non-selective Protestant state schools, ceded under the 1930 Education Act and financed by the state. Although not all controlled schools are Protestant (e.g., Controlled Integrated schools) and not all are non-selective (e.g., Controlled Grammar schools), the majority of them typically have the following representations on their boards:

- 4 "transferors" (Protestant church representatives);
- 2 Education and Library Board representatives (ELB);
- 2 parent representatives;
- 1 teacher representative.

Model C: The Rest: Out-Of-State, Integrated and Voluntary Grammar Schools

Finally, there are Voluntary Grammar schools (Roman Catholic and Protestant), Integrated schools (Grant Maintained, Independent and Controlled), and schools on the southern side of the Irish border attended by children from Northern Ireland. Naturally, no single model board exists for all of the above, even in one category (for example, Controlled Integrated schools have church nominees; Grant Maintained Integrated schools do not), but a typical management board for an out-of-state school might consist of:

- 3 Education and Library Board (or the equivalent) nominees;
- 3 parent representatives;
- 2 majority religious representatives;
- 1 minority religious representative;
- 2 teacher representatives.

ASSUMPTIONS ABOUT REPRESENTATION

Irrespective of religious affiliation or constitution, schools everywhere have become more complex organisations as a result of the increased participation of stakeholders. This has manifested itself in a proliferation of committee-like structures, such as boards of governors, which mediate between society and the organisation. Representation on these committees usually reflects some kind of proportional factional entitlement to power and many assumptions about the relative voting strengths of factions on such committees are frequently made. Take the case of the Model C school described above. At first sight, it appears that the voting power of the majority religious body is twice that of the minority religious body. Similarly, the ELB and parent factions are assumed to be 50% more "powerful" than the teaching or majority religious bodies. These casual assumptions are dangerous on two counts. Firstly, they are simply wrong; and secondly, they support an illusion of participative democracy and empowerment that distracts from the need to bring about fundamental change. The advantage of a game theoretic approach is that it can model majority voting situations such as exist on school governing boards, so that these inherent fallacies become apparent.

BACKGROUND IN GAME THEORY

Coalitions and Factions

A board of governors can be regarded as a cooperative multiperson game, similar to a weighted majority game (Colman, 1982), where coalitions are free to form and disintegrate as the agenda changes. A board of governors consists of "factions" (the term is not intended to indicate any belligerence) and a "coalition" is defined as one or more of these factions voting together, by agreement or by chance.

Coalitions come in different sizes.

- The term "single" coalition is used to describe a coalition consisting of one faction only, although of course it is not strictly a partnership.
- A "grand" coalition is defined as one that contains all factions on the board.
- A "winning" coalition is one that commands a majority of members' votes.
- A "minimal" coalition is one that cannot suffer any defection without losing its majority.

- A faction is defined as "pivotal" to its coalition if it turns the coalition from a losing one into a winning one by virtue of *its* vote; and as "critical" if its withdrawal causes that coalition to change from a winning one to a losing one. (For example, in the winning coalition formed by the ELB, teacher and majority religious factions, all three groups are critical, but only the majority religious faction – the last one coalescing – is pivotal.)
- A "null" coalition is a coalition consisting of no faction and is ignored throughout this paper.

Underlying Assumptions: Sincerity, Completeness and Transitivity

By their very nature, committees often have to choose one course of action from several alternatives according to a set of formal principles designed to ensure "fair" outcomes, like majority voting (assumed in this paper). A tacit understanding underlying these principles is that voting is "sincere" and along self-interest lines, that is, that factions always vote for the choice they prefer, and that coalitions have not been pre-arranged. This corresponds to what is known in game theory as "maximax" strategy, where a player opts for the best of all possible outcomes, ignoring less favourable ones that are equally possible. Factions are also assumed to be aware of their own and each other's payoffs and to coalesce sequentially in full knowledge of what has gone before. Voting is therefore assumed to be by open declaration, not secret ballot.

Other axioms, like those of "completeness" and "transitivity," also underpin the principles of committee voting. Completeness refers to the assumption that a preference is real and irreversible, that is factions that prefer "Choice X" over "Choice Y" do not also prefer "Choice Y" over "Choice X." Transitivity assumes a consistent hierarchy of preferences, that is factions that prefer "Choice X" over "Choice Y" and "Choice Y" over "Choice Z," necessarily prefer "Choice X" over "Choice Z."

Two measurements of power, the Shapley value and the Shapley-Shubik index, will now be developed using the principles outlined above. These indices calculate the distribution of power among factions on committees and will be used to analyse in turn the three specific cases of Maintained (Model A), Controlled (Model B) and Out-of-state (Model C) boards of governors.

While only a basic level of mathematics is assumed for the derivation of the two power indices that follows, those readers with an aversion to mathematics may skip to the sections on analysis, noting only Equations (3) and (4) *en passant*.

The Shapley Value

The Shapley value, introduced by Lloyd Shapley in 1953, rates each faction according to its *a priori* power, that is in proportion to the value added to coalitions by that faction joining it. Suppose a board of governors, B, has *n* factions or "players" (groups of members) and some of them vote together to form a coalition C. Suppose an individual faction of C is denoted by f_i and the size of the coalition C is *s*. Then

$$B = \{f_1, f_2, f_3, \ldots\ldots f_n\}; C = \{f_1, f_2, \ldots\ldots f_i \ldots\ldots f_s\};$$
$$C \text{ is a subset of } B, \text{ not } \emptyset \text{ (the empty set).}$$

Clearly, f_i has (s − 1) partners, selected from (n − 1) factions. Therefore, there are

$$\frac{(n-1)!}{(s-1)![(n-1)-(s-1)]!}$$

ways of re-arranging the coalition partners of *i*. The reciprocal of this expression can be written

$$\frac{(s-1)!(n-s)!}{(n-1)!}$$

and is the probability of each such selection.

Assuming all sizes of coalition to be equally likely, a particular size occurs with a probability of (1/n). Therefore, the probability of any particular coalition of size *s* containing the individual faction *i*, from *n* factions, is given by the expression

$$\frac{(s-1)!(n-s)!}{n!} \tag{1}$$

Suppose that coalition C has a "security level" or "characteristic function" denoted by $\varpi\{C\}$, defined as the minimum benefit that coalition C can guarantee to its member factions (Von Neumann & Morgenstern, 1944). The security level that the remaining factions have if f_i is removed from C can then be denoted by $\varpi\{C - i\}$. Therefore, the contribution that f_i alone makes to C is:

$$\varpi\{C\} - \varpi\{C - i\} \tag{2}$$

The Shapley value is now defined as the product of expressions (1) and (2), summed over *s* from 1 to *n*:

$$\textbf{Shapley value, } \mathbf{S(f_i)} = \frac{\sum_s (\mathbf{s-1})!(\mathbf{n-s})!}{\mathbf{n!}} [\varpi\{\mathbf{C}\} - \varpi\{\mathrm{C} - \mathbf{i}\}] \tag{3}$$

A pivotal faction can now be more carefully defined as follows: f_i is pivotal if and only if f_i is the faction that changes C_i from a losing coalition to a winning one. The pivotal faction is the one that brings the coalition "past the winning post," therefore the order of voting (i.e., the order in which coalitions are formed) is crucially important.

The Shapley–Shubik Index

The Shapley–Shubik index is a friendlier variation of the Shapley value (Cowen & Fisher, 1998; Shapley & Shubik, 1954). Suppose that a board of governors B has *n* factions, which form themselves into various coalitions C for voting purposes. Then, as before,

$$\mathrm{B} = \{\mathrm{f}_1, \mathrm{f}_2, \mathrm{f}_3, \ldots\ldots \mathrm{f}_\mathrm{n}\}; \mathrm{C} = \{\mathrm{f}_1, \mathrm{f}_2 \ldots\ldots \mathrm{f}_\mathrm{i} \ldots\ldots \mathrm{f}_\mathrm{s}\};$$
$$\text{C is a subset of B, not } \emptyset$$

The Shapley–Shubik index is defined as follows:

$$\mathbf{SS\ of\ f_i} = \frac{\sum_i \mathbf{C_i}\ \textbf{where}\ \mathbf{f_i}\ \textbf{is pivotal}}{\sum_i \mathbf{C_i}} \tag{4}$$

The Shapley–Shubik index is normalised, since $0 \leq \mathrm{SS(f_i)} \leq 1$, and 1 represents absolute power.

Previous Use

Shapley and Shubik (1969) famously used the index in an analysis of power in the United Nations Security Council. Up to 1965, there were five permanent members of the Security Council (USA, USSR, UK, France and China) and six non-permanent members. Analysis showed that the permanent members, who had (and still have) power of veto, had 98.7% of the power. In 1965, in an attempt to increase the power of non-permanent members, their number was increased to 10, but the Shapley–Shubik analysis showed that the power of the same five permanent members had only decreased marginally, to 98.1%, proving that membership ratios are not true reflections of actual power.

Both the Shapley value and the Shapley–Shubik index will now be used to evaluate the relative power of each faction on each of the three types of school governing boards.

AN ANALYSIS OF POWER ON MODEL A: VOLUNTARY MAINTAINED BOARDS

Let the five factions on the board of governors be denoted as follows:

Church trustees (4 votes):	C
Education and Library Board (2 votes):	L
Parent body (1 vote):	P
Teaching staff (1 vote):	T
Department of Education (1 vote):	D

For any one particular order of C, L, P, T and D, there are:

5	One-faction or "single" coalitions.
20	Two-faction coalitions.
60	Three-faction coalitions.
120	Four-faction coalitions.
120	Five-faction or "grand" coalitions.

A little consideration reveals the following:

- None of the single coalitions is a winning one.
- 8 two-faction coalitions are winning ones (they all include C).
- 36 three-faction coalitions are winning (they all include C).
- All four- and grand coalitions are winning.

Consequently, there are 284 winning coalitions out of a possible 325.

The three-, four- and grand coalitions require further investigation.

Three-Faction Coalitions

There are 36 winning three-faction coalitions. If C is first, the second voter is pivotal. If C is second or third (12 occasions each) then C itself is pivotal. Therefore, C is pivotal 24 times; L, P, T and D are pivotal on 3 occasions each.

Four-Faction Coalitions

There are 120 four-faction coalitions (all winning). If C is first in the voting (i.e., the first faction to take a position), then the second faction to vote is pivotal (6 times each for L, P, T and D). If C is second, third or fourth in the voting, then C itself is pivotal. In the case of the 24 coalitions that do not include C, the last faction will always be the pivotal one.

Therefore, C is pivotal 72 times; L, P, T and D are pivotal on 12 occasions each.

Grand Coalitions

In grand coalitions, the last faction to vote (i.e., to coalesce) can never be pivotal, even if it is C. Therefore these cases reduce to the four-faction coalition analysis outlined already.

Summary Table

Table 1 is a summary table of the extent to which each faction is pivotal in each of the five possible coalition sizes. The two power measurements – the Shapley value and the Shapley–Shubik index – can now be calculated for each of the five participating factions.

The Shapley Value for Each Faction

For the Shapley value, $n = 5$ and $s = \{1, 2, 3, 4, 5\}$. We assume that the contribution of faction f_i to each coalition in which it is pivotal, namely $\varpi\{C\} - \varpi\{C - i\}$, is unity; and that the contribution of f_i to each unsuccessful coalition is zero.

The results are summarised on Table 2.

The Shapley–Shubik Index for Each Faction

The results are summarised on Table 3.

Table 1. Summary Table for Voluntary Maintained Boards.

	C pivotal	L pivotal	P pivotal	T pivotal	D pivotal	Totals
Single	0	0	0	0	0	0
Two-faction	4	1	1	1	1	8
Three-faction	24	3	3	3	3	36
Four-faction	72	12	12	12	12	120
Grand	72	12	12	12	12	120
Totals	172	28	28	28	28	284

Table 2. Shapley Values for Voluntary Maintained Boards.

	$\frac{(s-1)!(n-s)!}{n!}$	C	L	P	T	D	Total
Single	1/5	0	0	0	0	0	0
Two-faction	1/20	4	1	1	1	1	8
Three-faction	1/30	24	3	3	3	3	36
Four-faction	1/20	72	12	12	12	12	120
Grand	1/5	72	12	12	12	12	120
Shapley values		19.0	3.15	3.15	3.15	3.15	

Table 3. Shapley–Shubik Index for Voluntary Maintained Boards.

	C pivotal	L pivotal	P pivotal	T pivotal	D pivotal
Number coalitions where pivotal	172	28	28	28	28
Number of possible winning coalitions	284	284	284	284	284
Shapley–Shubik	0.61	0.099	0.099	0.099	0.099

AN ANALYSIS OF POWER ON MODEL B: CONTROLLED SECONDARY BOARDS

Let the four factions on the board of governors be denoted as follows:

Church transferors (4 votes):	C
Education and Library Board (2 votes):	L
Parent body (2 votes):	P
Teaching staff (1 vote):	T

For any one particular order of C, L, P, and T there are:

4	One-faction or “single” coalitions.
12	Two-faction coalitions.
24	Three-faction coalitions.
24	Four-faction or “grand” coalitions.

- None of the single coalitions is a winning one.
- 6 two-faction coalitions are winning ones (the ones which include C).
- All three- and grand coalitions are winning.

Consequently, there are 54 winning coalitions out of a possible 64.

Two-Faction Coalitions
Three have C voting last and pivotal and the other three have C voting first.

Three-Faction Coalitions
There are 24 winning three-faction coalitions. If C is first to vote, the second faction becomes pivotal (twice each for L, P and T). If C is second (6 times) or third (6 times) in the voting, then C itself is pivotal. In the other 6 coalitions without C, the last one to vote becomes pivotal.

Therefore, C is pivotal 12 times; L, P, T are pivotal on 4 occasions each.

Grand Coalitions
These cases reduce to the three-faction coalition analysis outlined above.

Summary Table
Table 4 is a summary table of the extent to which each faction is pivotal in each of the five possible coalition sizes. The two actual power measurements for each of the five participating factions can now be calculated: the Shapley value and the Shapley–Shubik index.

The Shapley Value for Each Faction
For the Shapley value, $n = 4$ and $s = \{1, 2, 3, 4\}$. We assume that the contribution of f_i to each coalition in which it is pivotal, namely $\varpi\{C\} - \varpi\{C - i\}$, is unity; and that the contribution of f_i to each unsuccessful coalition is zero.

The results are summarised on Table 5.

The Shapley–Shubik Index for Each Faction
The results are summarised on Table 6.

Table 4. Summary Table for Controlled Secondary Boards.

	C pivotal	L pivotal	P pivotal	T pivotal	Totals
Single	0	0	0	0	0
Two-faction	3	1	1	1	6
Three-faction	12	4	4	4	24
Grand	12	4	4	4	24
Totals	27	9	9	9	54

Table 5. Shapley Values for Controlled Secondary Boards.

	$\frac{(s-1)!(n-s)!}{n!}$	C	L	P	T	Total
Single	1/4	0	0	0	0	0
Two-faction	1/12	3	1	1	1	6
Three-faction	1/12	12	4	4	4	24
Grand	1/4	12	4	4	4	24
Shapley values		4.25	1.42	1.42	1.42	

Table 6. Shapley–Shubik Index for Controlled Secondary Boards.

	C pivotal	L pivotal	P pivotal	T pivotal
Number coalitions where pivotal	27	9	9	9
Number of possible winning coalitions	54	54	54	54
Shapley–Shubik	0.50	0.167	0.167	0.167

AN ANALYSIS OF POWER ON MODEL C: A SAMPLE OUT-OF-STATE/INTEGRATED/GRAMMAR SCHOOL BOARD

Let the five factions on the board of governors be denoted as follows:

Education and Library Board (3 votes):	L
Parent body (3 votes):	P
Majority religious body (2 votes):	R
Minority religious body (1 vote):	r
Teaching staff (2 votes):	T

For any one particular order of L, P, R, r and T, there are:

5	One-faction or “single” coalitions.
20	Two-faction coalitions.
60	Three-faction coalitions.
120	Four-faction coalitions.
120	Five-faction or “grand” coalitions.

- None of the single coalitions is a winning one.
- Only two of the two-faction coalitions are winning ones (LP and PL).
- The only three-faction coalitions that are not winning are the six variations of TrR.

- All four- and grand coalitions are winning.

Consequently, there are 296 winning coalitions out of a possible 325.

The three-, four- and grand coalitions require further investigation.

Three-Faction Coalitions

There are 54 winning three-faction coalitions. Twelve of these finish with voting from L; 12 with voting from P; and 10 with voting from each of R, T and r. Of these last 30, 6 start with PL or LP. Therefore, L and P are each pivotal in 15 three-faction coalitions; and R, T and r are each pivotal in 8.

Four-Faction Coalitions

There are 120 four-faction coalitions (all winning) and in one-fifth of them, L votes first. One-quarter of those 24 times, P votes next, so P will be pivotal in these 6 coalitions. R, T and r (in any order) will vote first on a further 12 occasions and half that time, P will be pivotal.

In all other coalitions, the pivotal position will be third in the voting, and P will be in this position on 24 occasions. In total then, P will be pivotal for 36 coalitions.

A similar analysis reveals that L is also pivotal for 36 four-faction coalitions; and R, T and r are each pivotal in 16.

Grand Coalitions

Since grand coalitions have 11 votes and the largest faction commands only 3, the last faction voting can never be pivotal. Therefore, these cases reduce to the four-faction coalition analysis outlined above.

Summary Table

Table 7 is a summary table of the extent to which each faction is pivotal in each of the five possible coalition sizes. The two actual power measurements – the Shapley value and the Shapley–Shubik index – for each of the five participating factions can now be calculated.

The Shapley Value for Each Faction

For the Shapley value, $n = 5$ and $s = \{1, 2, 3, 4, 5\}$. We assume that the contribution of f_i to each coalition in which it is pivotal, namely $\varpi\{C\} - \varpi\{C - i\}$, is unity; and that the contribution of f_i to each unsuccessful coalition is zero.

The results are summarised on Table 8.

The Shapley–Shubik Index for Each Faction

The results are summarised on Table 9.

Table 7. Summary Table for Integrated/Grammar/Out-Of-State Boards.

	L pivotal	P pivotal	R pivotal	T pivotal	r pivotal	Totals
Single	0	0	0	0	0	0
Two-faction	1	1	0	0	0	2
Three-faction	15	15	8	8	8	54
Four-faction	36	36	16	16	16	120
Grand	36	36	16	16	16	120
Totals	88	88	40	40	40	296

Table 8. Shapley Values for Integrated/Grammar/Out-Of-State Boards.

	$\frac{(s-1)!(n-s)!}{n!}$	L	P	R	T	r	Total
Single	1/5	0	0	0	0	0	0
Two-faction	1/20	1	1	0	0	0	2
Three-faction	1/30	15	15	8	8	8	54
Four-faction	1/20	36	36	16	16	16	120
Grand	1/5	36	36	16	16	16	120
Shapley values		9.55	9.55	4.27	4.27	4.27	

Table 9. Shapley–Shubik Index for Integrated/Grammar/Out-Of-State Boards.

	L pivotal	P pivotal	R pivotal	T pivotal	r pivotal
Number coalitions where pivotal	88	88	40	40	40
Number of possible winning coalitions	296	296	296	296	296
Shapley–Shubik	0.297	0.297	0.135	0.135	0.135

SUMMARY

The Relative Power of Major and Minor Factions

Voluntary Maintained School Boards

The church nominees have four seats on the board, and one each for parent, teacher and Department of Education (DENI) representatives. The Education and Library Board (ELB) has two seats. However, analysis reveals that church nominees have (more than) *six times* the power of any of the other factions!

Controlled Secondary School Boards
The church nominees have four seats on the board, to two each for parent and ELB representatives. There is one teacher seat. Analysis from both indices reveals that church nominees have *three times* the power of any of the other factions.

A Sample Out-Of-State/Integrated/Grammar School Board
The Shapley value and the Shapley–Shubik index both reveal that the power of the ELB faction and the parent body is (approximately) 2.2 times that of each of the other three factions. This is a truer reflection of power than the ratio of memberships; 3:2 in the case of teachers and majority religious factions; 3:1 in the case of minority religious body.

The distribution of power is most equal on the sample Out-of-state/Integrated/Grammar school board and most skewed in favour of the majority on (Roman Catholic) Voluntary Maintained boards – twice as much as the skew towards the majority in (Protestant) Controlled Secondary boards. Both churches, but particularly the church representatives on Voluntary Maintained boards, retain considerable power despite the illusion of empowerment.

The Relative Power of the Minor Factions

Voluntary Maintained School Boards
The minor factions all have equal power. The ELB does not have twice that of parents, teachers or DENI, as might be assumed. This is a reflection of the fact that each of the four minor factions is equally "useful" in forming winning coalitions.

Controlled Secondary School Boards
The minor factions all have equal power on Controlled Secondary boards too – a minor pleasant surprise for teachers, but a disappointment to parents and the ELB.

A Sample Out-Of-State/Integrated/Grammar School Board
Both indices reveal that the power of the minority religious body is the same as that of the majority religious body, although first impressions would suggest

that the latter was twice as powerful. This may be significant in a Northern Ireland context, as it may encourage cooperation while maintaining a countenance of proportionality for the participants.

The Payoff for Winning Coalitions

If the Shapley value was used to award payoffs commensurate with contribution, as suggested by ‘‘minimum resource theory’’ (Gamson, 1961), then it would be a way of achieving a market-like outcome where a market did not exist *per se* – such as with school boards. However, members of school boards are not rewarded like that, if at all, and the motivational payoff for the pivotal factions can best be understood in terms of political influence and control. Experience suggests that the payoff for a winning coalition is simply that its member factions are perceived to be influential and share, to a greater or lesser extent, the power and control associated with winning.

The Order of Voting or Coalescence

Voluntary Maintained School Boards

Table 10 reveals that minority factions on Voluntary Maintained boards are least powerful when voting third, that is when coalescing as the third party to a coalition. Voting second is slightly better than voting third (57% of the time the faction will be pivotal, compared to 43%). It does not matter so much for the four church trustees.

Voting first or last (i.e., being the first faction to take a position or the last to coalesce) means that the faction cannot be pivotal, so there is a commensurate loss of power in so doing.

Controlled Secondary School Boards

Table 11 reveals that, even for the majority faction (the church transferors), there is no great advantage to coalescing second as opposed to third (56 to 44%), but again, factions should avoid voting first or last.

Table 10. Most Pivotal Position in the Voting Sequence for Voluntary Maintained Board Factions.

	% times pivotal in 2nd	% times pivotal in 3rd	% times pivotal in 4th
C	37	35	28
L	57	0	43
P	57	0	43
T	57	0	43
D	57	0	43

Table 11. Most Pivotal Position in the Voting Sequence for Controlled Secondary Board Factions.

	% times pivotal in 2nd	% times pivotal in 3rd
C	56	44
L	56	44
P	56	44
T	56	44

A Sample Out-Of-State/Integrated/Grammar School Board

Analysis reveals that a faction is most powerfully placed when it is the third faction to make its position known. This is particularly so for the three minor factions, as Table 12 shows. For these to be pivotal, they *must* coalesce third in the winning coalition. Even for the two majority factions, there is considerable advantage to voting third in the order, and second is slightly better than fourth. As always, factions should avoid voting first or last.

Minimal Winning Coalitions

Coalescing with the pivotally positioned faction is the next-best thing to being pivotal oneself. If it is assumed that there is no political reward for losing, and that all factions want to share the spoils of winning, then the strategy should always be to end up on the winning side. Unfortunately, this is opposed by an equal and opposite desire on the part of those who have already formed a winning coalition not to accept superfluous members! Just as there is no incentive for the last voting faction to dissent, there is no incentive for the first three or four factions to form grand coalitions, since the last-joining faction is never "important."

This idea is analogous to Riker's "minimal winning coalition theory" (1962), which states that if a coalition is large enough to win it should avoid

Table 12. Most Pivotal Position in the Voting Sequence for Integrated/Grammar/Out-Of-State Board Factions.

	% times pivotal in 2nd	% times pivotal in 3rd	% times pivotal in 4th
L	18	68	14
P	18	68	14
R	0	100	0
T	0	100	0
r	0	100	0

accepting additional factions, since these new members will demand a share in the payoff. The concept of minimal winning coalitions forms the basis for a power index not used in this paper, the Deegan–Packel index, but which may be of interest to some readers (Deegan & Packel, 1978).

Other Applications

While this discussion has concentrated on school governance, analysis of this sort can easily be applied to other competitive voting situations, both within education and without. For example, similar theory has been used to analyse voting power in the European Union. When its forerunner was set up by the Treaty of Rome in 1958, there were five member states, of which Luxembourg was one. Luxembourg had 1 vote out of a total of 17, but it has subsequently been shown that no coalition of member states ever needed Luxembourg in order to achieve a majority. In effect, Luxembourg was a powerless bystander in terms of competitive voting, though no doubt it benefited in other ways from membership of the "grand coalition."

Implications for Forming Committees

There are some additional practical implications for how committees are constituted, whether they are dissemination forums or statutory decision-making bodies.

- The numerical voting strength of a faction on a committee is not a reflection of its real power. This can lead to frustration, but it can also be a source of stability.
- Committees should be constituted so as to reflect accurately the desired (or entitled) proportional representation.
- School managers need to be aware of the possibility of disproportionate voting power, particularly when setting up structures for staff involvement in decision-making. Staff committees, which appear to reflect the relative sizes of different subject groupings, for example, may be dangerously skewed.

CONCLUSION

This paper considered two measurements of power; the Shapley value and the Shapley–Shubik index. There are others, such as the Johnston Index (Johnston, 1978), which looks at the (reciprocal of the) number of *critical* factions; the Deegan–Packel index (1978), which looks at the (reciprocal of the) number of *minimal* factions; and the Banzhaf index (1965), which looks at the number of coalitions in which a faction is both *critical and pivotal*. None is any better suited to the circumstances of school governance than the two used in this paper, but they make interesting reading for those who wish to apply game theory to more complicated situations.

All power indices are limited, in a small way, by the axioms and assumptions already noted in the text. These include the assumption that factions always vote sincerely and along rational self-interest lines; that voting is open; that coalitions have not been pre-arranged; that all coalitions are equally likely to appear; and that there is a reward for being part of a winning coalition. The appropriateness of these assumptions is, of course, a matter for judgement. Each faction judges the properties of a particular solution according to the favourableness of its outcomes, not by its innate attractiveness. Therefore, power is ultimately judged by its actual exercise, rather than its perceived distribution. Perceptions can be mistaken, as this paper has hopefully shown.

Power is the exercise of authority and influence, and in schools this is largely the remit of the governing body, with the head as chief executive. The constitution of these powerful committees is sometimes taken as a reflection of something deeper happening in society generally, as notions about democratisation and empowerment are transferred to and from school settings. However, the perception of how power is distributed is often flattering to deceive. The reality is often disappointing. Despite minority stakeholder representation on boards of governors and the like, the formal providers of education still retain power out of all proportion to their membership. It is for others to decide whether this is desirable or not, but it is unlikely that power has been fundamentally redistributed in recent times, as is sometimes claimed.

REFERENCES

Banzhaf, J. (1965). Weighted voting doesn't work: A mathematical analysis. *Rutgers Law Review, 19*, 317–343.

Colman, A. (1982). *Game theory and experimental games: The study of strategic interaction.* Oxford: Pergamon.

Cowen, R., & Fisher, P. (1998). Security council reform: A game theoretic analysis. *Mathematics Today, 34*(4), 100–104.

Deegan, J., & Packel, E. (1978). A new index for simple n-person games. *International Journal of Game Theory, 7*, 113–123.

Donnelly, C. (2000). Churches and the governing of schools in Northern Ireland: A redistribution of power? *Educational Management and Administration, 28*(2), 161–172.

Gamson, W. (1961). A theory of coalition formation. *American Sociological Review, 26*, 373–382.

Johnston, R. (1978). On the measurement of power: Some reactions to Laver. *Environment and Planning, 10A*, 907–914.

Kelly, A. (2002). *Decision-making and game theory.* Cambridge: Cambridge University Press.

Riker, W.H. (1962). *The theory of political coalitions.* New Haven, CT: Yale University Press.

Shapley, L.S. (1953). A value for n-person games. In H.W. Kuhn & A.W. Tucker (Eds.), *Contributions to the theory of games Vol. II, Annals of mathematics studies number 28* (pp. 307–318). Princeton, NJ: Princeton University Press.

Shapley, L.S., & Shubik, M. (1954). A method for evaluating the distribution of power in a committee system. *American Political Science Review, 48*, 787–792.

Shapley, L.S., & Shubik, M. (1969). Pure competition, coalitional power and fair division. *International Economic Review, 10*(3), 337–362.

Von Neumann, J., & Morgenstern, O. (1944). *Theory of games and economic behaviour.* Princeton, NJ: Princeton University Press.