VERTICAL DISPLACEMENT MEASUREMENT USING FIBER BRAGG GRATING (FBG) SENSORS FOR STRUCTURAL HEALTH MONITORING OF BRIDGES

Man Hong Yau
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Keywords

Vertical displacement, measurement, static characteristics, bridges, structural health monitoring (SHM), rating of bridges, curvature, slope, inclination sensor, inclinometer, tilt sensor, fiber Bragg grating (FBG), regression analysis
List of Abbreviations and Notations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>ArF</td>
<td>Argon fluoride</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge-coupled-device</td>
</tr>
<tr>
<td>CLF</td>
<td>Comprehensive likelihood factor</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary metal–oxide–semiconductor</td>
</tr>
<tr>
<td>CTE</td>
<td>Coefficient of thermal expansion</td>
</tr>
<tr>
<td>DBDD</td>
<td>Displacement based damage detection</td>
</tr>
<tr>
<td>El</td>
<td>Flexural rigidity of the cross section</td>
</tr>
<tr>
<td>EMI</td>
<td>Electromagnetic interference</td>
</tr>
<tr>
<td>FBG</td>
<td>Fiber Bragg grating</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite impulse response</td>
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<tr>
<td>FOS</td>
<td>Fiber optical sensors</td>
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<tr>
<td>GPS</td>
<td>Global positioning system</td>
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<tr>
<td>KrF</td>
<td>Krypton fluoride</td>
</tr>
<tr>
<td>LVDT</td>
<td>Linear variable differential transformer</td>
</tr>
<tr>
<td>MEMS</td>
<td>Micro-electro-mechanical systems</td>
</tr>
<tr>
<td>OPM</td>
<td>Optical power meter</td>
</tr>
<tr>
<td>OSA</td>
<td>Optical spectrum analyser</td>
</tr>
<tr>
<td>ROI</td>
<td>Region of interest</td>
</tr>
<tr>
<td>RTK</td>
<td>Real-time kinematic</td>
</tr>
<tr>
<td>RTS</td>
<td>Robotic total station</td>
</tr>
<tr>
<td>SHM</td>
<td>Structural health monitoring</td>
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<tr>
<td>UDL</td>
<td>Uniformly distributed load</td>
</tr>
<tr>
<td>UV</td>
<td>Ultraviolet</td>
</tr>
<tr>
<td>VBDD</td>
<td>Vibration based damage detection</td>
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Notations:

- $A$: cross section area of the fiber
- $a$: location of the point load, length of the arms
- $c$: viscous damping constant
- $D_2, D_3$: vertical displacement curve
- $d_0$: theoretical vertical displacement
- $E$: elastic modulus
- $F_R$: restoring force
- $F_I$: inertia force
- $G$: gravity force
- $g$: gravitational acceleration
- $h$: distance of sensors at the cross section
- $i$: $i^{th}$ longitudinal location
- $K_3$: curvature curve
- $K_g, K_T$ and $K_p$: coefficients of wavelength sensitivity to strain, temperature and pressure
- $k$: stiffness of the sensor system
- $k_0$: theoretical curvature
- $k_3$: measured curvature
- $l$: length of the vertical pendulum
- $m$: mass (Section 4)
- $m$: number of sensors along the span (Section 3)
- $n$: number of polynomial order
- $P$: pressure
- $p_e$: photoelastic coefficient of fiber
- $R$: radius of curvature
- $S_2$: slope curve
s0  theoretical slope  
s2  measured slope  
T  Period  
t  time  
v  vertical displacement, deflection  
X  distance along the length of the structure  
y  distance from neutral axis of cross section  
\( \bar{y} \)  neutral axis position from the bottom sensor at the cross section  
\( \alpha, \beta \)  integration constants, coefficients  
\( \alpha \)  reduction coefficient of the flexural rigidity (Eq.5.3)  
\( \alpha_f \)  coefficient of thermal expansion of fiber  
\( \alpha_s \)  coefficient of thermal expansion of the structure  
\( \gamma \)  damping coefficient  
\( \Delta T \)  change of temperature  
\( \Delta \varepsilon \)  change of strain  
\( \Delta \lambda \)  Bragg wavelength shift  
\( \Delta \nu \)  discrepancy of the vertical displacement  
\( \Delta \nu\% \)  percentage discrepancy of the vertical displacement  
\( \varepsilon \)  longitudinal strain  
\( \zeta \)  damping ratio  
\( \eta \)  refractive index  
\( \eta_{eff} \)  effective refractive index of the fiber core  
\( \theta \)  slope  
\( \dot{\theta} \)  angular velocity  
\( \ddot{\theta} \)  angular acceleration
\( \kappa \) curvature

\( \Lambda \) grating period

\( \lambda \) Bragg wavelength

\( \lambda \) roots of the quadratic equation

\( \xi \) thermo-optics coefficient

\( \rho_{11}, \rho_{12} \) the components of the fiber optics strain sensor

\( v \) Poisson’s ratio

\( \omega_d \) damped angular frequency

\( \omega_0 \) natural angular frequency of oscillation
Abstract

Structural health monitoring (SHM) has attracted much attention in both research and development in recent years. In general, a SHM system consists of measurements, a signal processing system and data interpretation. Measurements can be made under static or dynamic conditions. Most of the literature refers to dynamic measurements of parameters. However, the applications of these parameters are sometimes impractical because they require a large amount of sensors and data processing. Although most researchers have focused on these characteristics, research using vertical displacement is still on-going. Vertical displacements can also be applied to SHM, especially in performance monitoring, but it is difficult to obtain this measurement practically and accurately. However, the vertical displacements are one of the most relevant parameters for SHM of bridges in both the short and long term. Bridge managers around the globe are always looking for a simple way to measure vertical displacements of bridges. Conventional vertical displacement measurement methods such as using linear variable differential transformer (LVDT), land survey and Global Positioning System (GPS) are sometimes impractical, as specialised operators are required and measurements can be affected by weather. Thus, there is limited research on applying vertical displacements of bridges for SHM and damage identification, as it is difficult to carry out such measurements precisely.

In this thesis, two innovative approaches for measuring vertical displacements using curvature and slope measurements are proposed. For these measurements in bridges, sensors that enable repeatable measurements, provide immunity from electromagnetic interference (EMI) and have high accuracy, are required. Most conventional sensors used with bridges are based on electric signals. These signals are rarely distinguished from noise due to EMI. On the other hand, in recent years, with the advancement of fiber-optic technologies, fiber Bragg grating (FBG) sensors have been more commonly used in SHM due to their outstanding advantages, including multiplexing capability, immunity of EMI as well as high resolution and accuracy. For the slope measurement, only few FBG inclination sensors are available in the market. Their performances under ambient vibration and response time have
not been reported. The reliability and repeatability of the measurement are highly affected by the abovementioned issues. In this study, five FBG inclination sensors have been constructed and their static performances have been investigated. From the experience gained from the fabrication of these sensors, a novel frictionless FBG inclination sensor with extremely high sensitivity and resolution has been proposed and its performance under vibration is investigated. With the successful development of the FBG inclination sensor, it is proposed to use these sensors to develop a simple, inexpensive and practical method to measure vertical displacements of bridges.

In this study, a finite element model of a full-scale bridge has been established and a series of simulation tests have been conducted to implement the proposed methods. Further, a 3.9m beam has also been set up for loading tests to investigate the performances of the proposed methods using FBG sensors. The numerical and experimental verifications have proved these approaches can measure vertical displacement under various conditions. Hence, these approaches can be used to measure vertical displacements for most of the slab-on-girder and box-girder bridges. Moreover, FBG sensors have the advantage, as they can be implemented to monitor bridge behaviour remotely and in real time. The advantages and limitations of these approaches and suggestions for choosing between these approaches are discussed. Further applications of the proposed approaches are suggested at the end of the thesis.
Publication List

JOURNAL ARTICLES


CONFERENCE PAPERS


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Statement of Original Authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

Signature: QUT Verified Signature

Date: June 2014
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Chapter 1: Introduction

This chapter presents the background (Section 1.1) and objectives (Section 1.2) of the research carried out. Section 1.3 describes the significance and scope of this research and provides definitions of the terms used. Finally, Section 1.4 includes an outline of the remaining chapters of the thesis.

1.1 BACKGROUND

Structural health monitoring (SHM) has attracted much attention in both research and development in recent years. SHM involves collecting and analysing information obtained through measurements and analyses of a structure in order to evaluate its performance and to assess damage. Such assessment will enable timely retrofitting and prevention of collapse of the structure. In general, a structural health monitoring system consists of measurements, a signal processing system and data interpretation. Measurements can be made under static or dynamic conditions. Most of the literature refers to dynamic measurements of parameters such as natural frequencies, mode shapes and damping ratio. Natural frequency is one of the most convenient characteristics derived from a modal analysis and there are many applications using the natural frequency. However, these applications are restricted in detecting the existence of structural damage. Many mode shape-based methods for damage identification have been proposed. Mode shape measurements are uncertain in practical testing due to environmental noise. It is difficult to obtain accurate mode shapes using limited sensors. It is also difficult to obtain higher mode shapes in practice. A damping ratio is a measurement of the reduction in the amplitude of vibration of a structure. Only a limited amount of research has been published in this area as it is a challenging problem to obtain an accurate and reliable damping ratio in practice. Although most researchers have focused on these characteristics, research using strain and vertical displacement is still on-going. Strain and vertical displacement can be measured under static and dynamic conditions. Strain can only provide location specific information about a structure. Strain history measurement is
an important aspect in dynamic characteristics, e.g. fatigue analysis. Vertical displacements can also be applied to dynamic measurement, but it is difficult to obtain these measurements practically and accurately. However, the vertical displacements are one of the most relevant parameters for structural health monitoring of bridges, in both the short and long term. Vertical displacements can be used to evaluate performance via loading tests, rate the bridge capacity and predict service life of bridges, as well as being used for damage identification.

Bridge managers around the globe are always looking for a simple way to measure vertical displacements of bridges. Conventional vertical displacement measurement methods such as using linear variable differential transformer (LVDT), land survey and Global Positioning System (GPS) are sometimes impractical as specialised operators are required and measurements can be affected by weather. Thus there is limited research on applying vertical displacements of bridges for structural health monitoring and damage identification as it is difficult to carry out such measurements precisely.

1.2 OBJECTIVES

This project aims to develop a practical, simple and inexpensive vertical displacement measurement method using optical sensors for structural health monitoring of bridges. The proposed method should be able to determine vertical displacement of a bridge’s span with accuracy and reliability. The research will be achieved upon completion of the following objectives:

- Due to geometry of beam type bridges such as slab-on-girder or box-girder bridges, their vertical displacements can be determined by curvature and slope measurements. Innovative methods will be proposed to measure vertical displacements of bridges using curvature and slope measurements.

- Due to the outstanding advantages of FBGs including multiplexing capability, immunity of electromagnetic interference (EMI) as well as high resolution and accuracy, using FBG sensors for curvature and slope measurement will be proposed. The applications of FBG strain sensors for curvature measurement will be investigated.
• For the slope measurement, a novel FBG inclination sensor will be proposed and its performance, including accuracy, resolution, repeatability, response time and response under vibration, will be investigated. Suggestions and recommendations for the FBG inclination sensors will also be provided.

• A series of simulation tests will be conducted to verify the performance of the proposed vertical displacement measurement methods under various conditions such as: different support conditions, varying flexural rigidities along the spans, different loadings and with no prior knowledge of the loading.

• An experimental validation will also be carried out to investigate the performances of the proposed methods using FBG sensors.

• Recommendations for the vertical displacement measurements in bridges will be provided.

1.3 SIGNIFICANCE AND SCOPE

The conventional vertical displacement measurement methods are often impractical for structural health monitoring, as specialised operators are required and measurements can be affected by weather. However, the vertical displacements are one of the most relevant parameters for structural health monitoring of bridges in both the short and long terms. They can be used to evaluate performance via loading tests, rating the bridge capacity and predicting service life of bridges. Besides, in the literature, only a limited number of researchers have investigated using vertical displacements for damage identification. This may be due to the lack of accurate and reliable vertical displacement measurement methods. As a consequence, the displacement-based structural health monitoring (including performance monitoring and damage identification) methods cannot compete with the vibration-based structural health monitoring methods.

The proposed vertical displacement measurement methods are simple, but have not attracted much attention in the literature. This is because the conventional
curvature and inclination sensors are based on electrical signals which are rarely distinguished from noise, due to electro-magnetic interference (EMI) in the field.

Due to FBG’s outstanding advantages including multiplexing capability, immunity of EMI as well as high resolution and accuracy, the use of FBG sensors for curvature and slope measurements are proposed and their performances are investigated. As there is a lack of suitable FBG inclination sensors in the market for fulfilling the requirements of the proposed vertical displacement methods, five FBG inclination sensors were fabricated and tested for the verification tests of the proposed vertical displacement measurement methods. Their performances, including accuracy, resolution and sensitivity, are reported. Based on the experience gained from the fabrication of the sensors, the friction at the joints of the sensors highly affect the accuracy of the measurement. To overcome this limitation, a novel, frictionless FBG inclination sensor with extremely high sensitivity and resolution is proposed. Its performances in static and dynamic measurements are investigated including accuracy, resolution and sensitivity, as well as response time and response under vibration.

Due to the proposed vertical displacement measurement methods with FBG sensing technology as well as the success of the FBG inclination sensor development, the vertical displacements of bridges can be measured practically and simply. The methods do not require a stationary reference, specialised person or large amount data processing. They are also not affected by weather, such as rain and low visibility in fog. The vertical displacement curve is determined instead of a point measurement. The proposed methods can be used for developing a fully automatic vertical displacement monitoring system for bridges.

This research focuses on the vertical displacement measurement methods for beam-like bridges and their applications. As well, this research focuses on the curvature and slope measurements of bridges using FBG sensors to implement the proposed methods. With respect to FBGs, this research focuses on the FBG technology that would make the measurement more practical.
1.4 THESIS OUTLINE

The thesis is organised into eight chapters as follows:

Chapter 1 introduces the background of this thesis and the research problem in the area of vertical displacement measurement and application to bridges. The significance and the scope are also presented.

Chapter 2 presents an overview of structural health monitoring of bridges. It also reviews the conventional and modern vertical displacement measurement methods of bridges as well as damage identification methods. Finally, a summary of findings and the research problem are then presented.

Chapter 3 presents the algorithm of the proposed vertical displacement measurement methods. The curvature and inclination approaches are presented.

Chapter 4 presents the overview of the FBG sensing technology including the principle, sensing system, fabrication methods, stripping methods, hydrogen loading, influence of exposure parameters on the mechanical strength of FBGs, characteristics and types of FBG sensors. The curvature measurement using FBG strain sensors and the development of the FBG inclination sensor are described. The performance of the developed FBG inclination sensor is investigated, including the accuracy, resolution, vibration response, oscillation effect, resonance and temperature variation.

Chapter 5 presents a series of simulation tests to verify the proposed vertical displacement measurement methods that can be implemented with various support conditions and various flexural rigidities along the spans, as well as under non-uniformly distributed loads. The self-compensation capacity of the measurement methods is also demonstrated by selectively deactivating various numbers of sensors in order to simulate that some sensors are faulty. The performance of the proposed methods with noise of 10% in the measurements is also studied.

Chapter 6 presents two beam loading tests in the laboratory. The test beam was installed with nine pairs of FBG strain sensors on the top surface and underneath surface and five FBG inclination sensors for curvature and slope measurements. The vertical displacements are measured using the proposed methods. The first test is to apply an incremental loading at the mid-span. Another test is to apply loadings at
different locations along the span. The performances of the proposed methods are presented.

Chapter 7 summarises the findings and the conclusions from this study. Recommendations of the proposed vertical displacement measurement methods, FBG inclination sensor development and further applications of vertical displacements for bridges are provided.
Chapter 2: Literature Review

2.1 Introduction

This literature review presents an introduction to structural health monitoring of bridges and summarises the techniques used in vertical displacement measurement of bridges. Section 2.2 introduces structural health monitoring. Section 2.3 reviews the conventional vertical displacement measurement methods of bridges including linear variable differential transform (LVDT) displacement transducers, survey methods and global positioning system (GPS). Section 2.4 reviews the image-based displacement measurement methods of bridges including close-range photogrammetry, wavelet edge detection technique, digital image correlation technique, terrestrial laser scanning and high speed camera. Section 2.5 reviews the modern indirect vertical displacement measurement methods of bridges using curvature and slope measurements. The damage detection techniques for structural health monitoring of bridges are described in Section 2.6. Finally, the conclusion and research problem are provided in 2.7.

2.2 STRUCTURAL HEALTH MONITORING (SHM)

Structural health monitoring has attracted much attention in both research and development in recent years. This reflects continuous deterioration conditions of important pieces of civil infrastructure; those especially in long-span bridges have alerted various authorities to the dangers of ignoring proper monitoring of these structures. Among them, many were built in the 1950s with a 40-to-50-year designed life span. The collapses and failures of these deficient structures cause increasing concern about structural integrity, durability and reliability, throughout the world (Li et al., 2004).

A number of definitions have been given in terms of SHM that refer to the use of in-situ, continuous or regular (routine) measurement and analyses of key structural and environmental parameters under operating conditions, for the purpose of warning impending abnormal states or accidents at an early stage to
avoid casualties as well as giving maintenance and rehabilitation advice. This tentatively proposed definition of SHM complements that given by Housner et al. (1997). This definition emphasises the essence of the advance alert ability of SHM. SHM is also defined by Aktan et al. (2000) as tracking the responses of a structure along with inputs, if possible, over a sufficiently long duration to determine anomalies, to detect deterioration and to identify damage for decision making. More specifically, measurements of the operating and loading environment and the critical responses of a structure are able to evaluate the symptoms of operational incidents, anomalies and/or deterioration or damage indicators that may affect operation, serviceability, safety and reliability.

Farrar and Worden (2007) defined SHM as the process of implementing a damage identification strategy for aerospace, civil and mechanical engineering infrastructure. In addition, damage is defined as changes to the material and/or geometric properties of these systems, including changes to the boundary conditions and system connectivity, which adversely affect the system’s performance.

In general, a typical SHM system includes three major components: a sensor system, a data processing system (including data acquisition, transmission and storage), and a health evaluation system (including diagnostic algorithms and information management). The sensors utilized in SHM are required to monitor not only the structural status, for instance stress, displacement, acceleration etc., but also influential environmental parameters, such as wind speed, temperature and the quality of its foundation.

2.3 CONVENTIONAL VERTICAL DISPLACEMENT MEASUREMENT METHODS OF BRIDGES

Bridge managers all over the world are always looking for simple ways to measure bridge vertical displacements for structural health monitoring. This section reviews the vertical displacement measurement methods on bridges including linear variable differential transform (LVDT) displacement transducers, land surveying methods and global positioning system (GPS).
2.3.1 Linear Variable Differential Transform (LVDT) Displacement Transducers

Traditional sensors for vertical displacement measurement of bridges, such as the linear variable differential transform (LVDT) displacement transducers, provide a precise displacement measurement. The operation principle of LVDT is to convert the displacement of an object to a corresponding electric signal. They provide only one dimensional displacement. They are only suitable to be used in a laboratory because their electrical signal may be affected by the electromagnetic interference (EMI). Besides, their measurement range is limited, which restricts the measurement. Furthermore, they require a stationary reference and most often the mechanical extensometers are impractical for over-water bridges.

2.3.2 Survey

Land Surveying is a science of accurately determining the terrestrial or three-dimensional position of points; and the distances and angles between them. Levelling and trigonometrical levelling are the surveying methods to obtain the elevation and three-dimensional position of points respectively. Total station, a survey instrument used for spatial measurement, has been used to monitor the long term movement of structures. Coordinates of an unknown point are determined by the relative angle and distance of a known coordinate point using trigonometry. The vertical displacement of a bridge can be determined by the elevation change. However, the measurement is required to be conducted manually by a skilled surveyor over a period of an hour. It is unsuitable for vertical displacement measurement of bridges under motion, due to traffic or wind gusts. Robotic Total Station (RTS) is a modern survey instrument that can automatically track a moving target and record the coordinates. However, the distance between the RTS and the target is restricted within a few hundred metres for an accurate measurement (Psimoulis & Stiros, 2007). It is sometimes impractical to set up a stationary reference for the measurements. Besides, Total Stations require a clear line of sight to two or more points with known coordinates and targets. In the other words, the measurements are affected by the weather, such as rain conditions and low visibility in fog.
2.3.3 Global Positioning System (GPS)

Global Positioning System (GPS) is designed as a navigation system for real-time positioning, by military and civilian users. Now, it is also an emerging tool for measuring and monitoring both static and dynamic displacement responses of large civil engineering structures (Nakamura, 2000; Wong et al., 2001). The accuracies of dynamic displacement measurements are at a sub-centimetre to millimetre level (Ashkenazi & Roberts, 1997; Nakamura, 2000; Ogundipe et al., 2012).

Ashkenazi & Roberts (1997) conducted a feasibility study of the kinematic GPS to measure displacement responses of the Humber Bridge in the UK. It is a long-span cable supported bridge with a total span of 2220 m. The system has a resolution of ±1 mm horizontally and ±3 mm vertically.

Nakamura (2000) conducted field measurements using kinematic GPS on a Japanese suspension bridge with a main span of 720 m and two side spans of 330 m each. The system is schematically shown in Figure 2-1. The displacements at mid-span induced by wind forces were measured. The measured error of the GPS system was estimated to be within 1.6 cm in the horizontal direction and within 2.1 cm in the vertical direction.

![Figure 2-1 GPS measurement system (Nakamura, 2000)](image)

Watson et al. (2007) used GPS to monitor the movement of a cable-stayed steel truss bridge, Batman Bridge, with a main span of 206 m, in Australia. The results show the system’s resolution of 2 mm level in the horizontal and 3.5 mm in the vertical.
The measurements in the early days of GPS were always in the static mode. A number of survey methods were developed to enhance the precision of GPS measurements, but required post-processing of the data. Real-time kinematic (RTK) equipment can provide survey accuracy on the spot but requires a radio-link (or equivalent) between a base station and a rover. A typical RTK (carrier phase and dual frequency) precision of the line between the base station and the rover is the ± (10 mm + 1 ppm) quoted by Leica in 2001 for their SR530 'Geodetic RTK Receiver' (Rueger, 2006).

GPS measurements require at least four satellites to measure accurate coordinates (Kaplan & Hegarty, 2006). The measurement will discontinue when the required number of satellites is not available. A reference GPS antenna is required to fix on a rigid point as a benchmark. (Nakamura, 2000)

The reference GPS receiver is required to fix at a rigid point as a benchmark where the signal path between the reference receivers and the receivers will not be blocked, and as such, a high vehicle and multipath effect will not occur. Moreover, when a high vehicle blocks the signal path between the reference receiver and the receiver, that causes a signal loss (Ashkenazi & Roberts, 1997).

Although the GPS can measure a three dimensional coordinate, it is difficult to determine the relation steady wind forces and the displacements of bridges (Nakamura, 2000). As well, each GPS antenna can only obtain a point measurement. It can only measure displacement in a selected point. The deflection profile of the span cannot be obtained unless many GPS are used for the measurement. It is thus that the cost is highly increased. The accuracy of dynamic displacement measurement using GPS at a sub-centimetre to millimetre level, however, depends on many factors, such as data sampling rate, satellite coverage, atmospheric effect, multipath effect, and GPS data processing methods (Chan et al., 2006b).

The application of a GPS for bridge monitoring has two limitations: (Ko & Ni, 2004) (i) the measurement accuracy of a GPS is not good enough to completely meet bridge health monitoring requirements; and (ii) a GPS does not work well for
monitoring the displacement of piers beneath the bridge deck (caused by ships colliding, settlement, etc.).

2.4 IMAGE BASED DISPLACEMENT MEASUREMENT METHODS OF BRIDGES

2.4.1 Close-Range Photogrammetry for Bridge Measurements

Photogrammetric surveying that has a long history of development and application is a method to determine the geometric information of an object from photographs, where three-dimensional measurements are made from two-dimensional photographs taken of an object. In general, photographs are taken of an object from at least two camera positions. It is used in different fields such as topographic mapping and geology. However, it is not economical for monitoring bridges.

From each camera position, there is a line of sight that runs from each point on the object to the perspective centre of the camera (Jauregui et al., 2003). When the object size and the camera-to-object distance are both less than 100m, terrestrial photogrammetry is further defined as close-range photogrammetry (Jiang et al., 2008). Using the principle of triangulation, the point of intersection between the different lines of sight for a particular point is determined mathematically to identify the spatial or three-dimensional location of the object point (Jauregui et al., 2003).

**3D Close-Range Photogrammetry**

Whiteman et al. (2002) used a two-video camera system using close-range digital photogrammetry for measurement of 3D deflections in concrete beams during destructive testing with a precision of ±0.25mm for vertical direction. The object space points were also mounted on some known stable points for reference (Figure 2-2). Sub-millimetre object point precision was achieved in all three coordinate dimensions, giving complete information about the mode of deformation that the contact sensors could not deliver.
Jáuregui et al. (2003) used digital close-range terrestrial photogrammetry to measure vertical static deflection of bridges using a high-end semi-metric digital camera. The designed double-sided photogrammetric targets with framing are attached to the bridge. A set of control points with known 3D coordinates was used to establish the geometrical relationship between targets and their images. Results from laboratory testing of a steel beam showed an accuracy ranging from 0.51 mm to 1.3 mm. Field evaluation of a prestressed concrete bridge showed an average difference of approximately 3.2 mm as compared with elevation measurements made with a total station.

2D of Close-Range Photogrammetry

Photogrammetric techniques offer the potential of 3-D measurement of deformations of a large number of signalised points or natural texture on structural buildings by using at least two cameras or images taken from at least two different positions of one camera. If primarily one-dimensional or two-dimensional deformations are expected and the dimensions of the monitoring building are acceptable, a single camera can be sufficient (Albert et al., 2002).

Olaszek (1999) developed a method that incorporated the photogrammetric principle to investigate the dynamic characteristics of bridges. Results of two field
tests revealed that the method was reliable with an accuracy of 0.1–1 mm for camera-pattern distance ranges between 10 and 100 m and vibration frequency under 5 Hz.

Albert et al. (2002) presented studies on the deflection measurement of a laboratory beam and an existing bridge using a single camera as shown in Figure 2-4. The basic principle is to measure the image projection that is directly proportional to the dimension of the object if the camera and the object planes are parallel, as shown in Figure 2-3. The accuracies were approximately 0.1 mm and 2 mm in laboratory and field measurements, respectively.

![Figure 2-3  Single camera deflection measurement (Albert et al., 2002)](image)
Lee et al. (2007) developed a vision-based real-time displacement measurement system at multiple locations, using a digital image processing technique for health monitoring of bridges. A target is designed with four white spots with known geometry and black background. The horizontal and vertical length are predetermined, considering the expected maximum displacement to be measured and the performance of hardware including a digital video camcorder and a...
telescopic lens. As the system used only one camera, it could only measure in-plan 2D displacement of a specific target. In a static loading test carried on a steel-plate girder bridge, the error is less than 1%. In a shaking table test, the measured displacement by the synchronized vision-based system was compared with the data from a contact-type sensor, a linear variable differential transformer (LVDT), and showed close results to conventional sensors with less than 3% error in maximum values. The advantages of the vision-based system are as below (Lee et al., 2007):

- Dynamic measurement with high resolution and cost-effectiveness
- Multi-points measurement
- Real-time measurement and visualization
- Easy installation and easy operation

2.4.2 Wavelet Edge Detection Technique

Patsias and Staszewski (2002) used image-based on the wavelet edge detection technique and the use of the orthogonal wavelet transform to measure vibration and identify the presence and location of damage. The advantage of this vibration measurement method is that it offers more ‘measurement points’ in vibration mode shapes than a traditional transducer.

Poudel et al. (2005) proposed a video imaging technique for damage detection of a prismatic steel beam. A high speed, complementary metal–oxide–semiconductor (CMOS) camera was employed to capture the dynamic response of the beam. Mode shapes of the beam were extracted by the wavelet transform and were used for damage localisation.

2.4.3 Digital Image Correlation Technique

Digital image correlation technique is widely used for displacement measurement of an object surface in the field of experimental solid mechanics. Because digital image correlation technique does not need a complicated optical system, the measurement can be performed simply and easily (Yoneyama et al., 2007).
Yoneyama et al. (2007) used digital image correlation technique to evaluate the bridge deflection distribution using digital single-lens reflex cameras. A 15.4 meter single span steel girder bridge was tested to study the method. The test results show that the measured deflections agree well with those obtained by the transducers. In the loading test, the standard deviation of the difference between the deflection by digital image correlation and the approximated curve is estimated as 0.31 mm.

Fu and Moosa (2002) proposed an optical approach using a monochrome charge-coupled-device (CCD) cameras to measure vertical displacements by sub-pixel edge detection technique, and detect structural damage. The advantage of this approach is that it is not necessary to install the target object on the bridge. In addition, a sub-pixel edge detection technique was used, together with curve fitting, to locate damage in the beam. The laboratory tests indicate that the approach can identify the location of damage as small as a 3% loss in the section stiffness.

Chan et al. (2009) used CCD camera for vertical displacement measurement that utilises image processing techniques for pixel identification and subsequent edge detection. The tests include applying the methods to determine the vertical displacements separately for a concrete beam and a steel beam under various loadings. The degree of accuracy is larger than 95% for measuring bridges with deflection larger than 10mm.

2.4.4 Terrestrial Laser Scanning (TLS)

Terrestrial Laser Scanning (TLS) can obtain the 3D spatial locations of sampling points with a high sampling rate, which allows measurement of the 3D displacement of any particular point in a structure, as well as the static deformed shape of the structure (Park et al., 2007). The collected data can be used to reconstruct the model of the object. This technology has been widely used in ancient architecture building surveying, heritage conservation, urban three-dimensional model, geological engineering, and measurement fields (QIU & DING, 2010).

However, 3D coordinates of a target structure acquired using TLS can have maximum errors of about 10 mm, which is insufficient for the purpose of health monitoring of structures (Park et al., 2007). To overcome this shortcoming, Park et
al. (2007) proposed to use a displacement measurement model which construct a baseline of the object before deformation. A 4 m simply supported steel beam subjected to a concentrated load was tested using this method. The results show a good agreement compared to the references.

However, this method has the common disadvantages of the image based displacement measurement methods such as requiring a stationary reference for locating the scanner and being unable to be used in bad weather.

2.4.5 High Speed Camera

Moore et al. (2013a; 2011) used a laser source and a high speed camera to measure short span timber bridge girder deflections. The laser source, which was mounted on a stable support, produced an image of the laser on a graduated scale, which was mounted on a mid-span of the bridge girder. The scale movements were recorded by video. This recording was analysed to determine the peak deflection. The data can be interpreted to identify details of traffic flow, maximum bridge loading, dynamic behaviour of bridge girders and enable the probability of structural failure to be computed.

Moore et al. (2013b) used a staff, vernier and high speed camera to measure dynamic movement of a timber bridge. The staff, with a graduated scale, was attached to the mid-span of the bridge girder (Figure 2-5). The movement of the staff was compared to the vernier, which was placed on the ground and recorded by the high speed camera.
Figure 2-5  Staff with graduated scale and vernier (Moore et al., 2013b)

This method can be set up easily and quickly for field measurement. However, it is impractical for bridges where operators are unable to access the area under the girder, such as over-water bridges. In addition, this method is based on comparison with a graduated scale and a reference. The resolution is limited in millimeter level unless a

2.5 INDIRECT VERTICAL DISPLACEMENT MEASUREMENT METHODS OF BRIDGES

2.5.1 Curvature Measurement

Vurpillot et al. (1996) developed a mathematical model for the determination of the vertical displacement from internal horizontal measurements of a bridge. Then, Vurpillot et al. (1998) applied this vertical displacement measurement method on a timer beam and a pre-stressed concrete bridge, using deformation sensors and inclination sensors.
Perregaux et al. (1998) presented an approach for monitoring bridges by vertical displacement using SOFO\(^1\) system. The SOFO system is a fiber optical displacement sensor with a resolution in the micrometer range and an excellent long-term stability. The measurement setup uses low-coherence interferometry to measure the length difference between two optical fibers installed on the structure to be monitored. The measurement fiber is pre-tensioned and mechanically coupled to the structure at two anchorage points in order to follow its deformations, while the reference fiber is free and acts as temperature reference. Both fibers are installed inside the same pipe and the measurement basis can be chosen between 200 mm and 10 m. The resolution of the system is of 2 \(\mu\)m independently from the measurement basis and its precision is of 0.2\% of the measured deformation, even over years of operation. The sensors are used to calculate the vertical curvature of each span. By double-integration of the curvature measurement, the exact function of the vertical displacement is calculated.

Kim and Cho (2004) proposed using FBG sensors to estimate deflection of a simple beam model. The theory is similar to others, but the curvature function is assumed as a linear function \(\kappa(x) = Ax + B\) where \(\kappa\) is curvature, and \(x\) is longitudinal location of the beam, and \(A\) and \(B\) are integration constants. The determination of curvature is based on longitudinal strain and assuming known neutral axis distance. This method assumes the maximum vertical deflection relates linearly with the applied load and the sample beam has an axisymmetric behaviour in the longitudinal direction. This method showed good correlated to the test results for the steel beams in the linear elastic region. The curvature measurements assume that the neutral axis keep in a constant. However, if the material of a beam is degraded or damaged, the neutral axis shifts. The curvature measurements will fail. Hence, this method is not applicable if the neutral axis changes.

To overcome this limitation, Chung et al. (2008) utilised mean curvatures to extrapolate the vertical deflection using long fibre gauge fibre sensors that are based

\(^{1}\) SOFO in French : < Surveillance d’Ouvrage par Fiber Optique>, Monitoring of structures by Fiber Optic Sensors (Perregaux et al., 1998)
on the principle of low-coherence interferometry to estimate prestressed deflection. A full-scale test has been performed on the prestressed concrete girder compositely fabricated with reinforced concrete deck. This method is able to estimate the deflection of the prestressed concrete girder after cracking because the neutral axis position is directly measured rather than calculated. It is also possible to extend the method to keep trace of the neutral axis position, which indicates the degradation in concrete structures due to the formation and propagation of cracks.

Chan et al. (2009) presented a preliminary study to validate the vertical displacement measurement methods in the laboratory. The tests include applying the methods to determine the vertical displacements using fiber Bragg grating (FBG) sensors. The method is based on the measured horizontal strains together with the identified curvature functions obtained by a self-developed FBG inclination sensor. A simply supported concrete beam and a steel beam under a point load were tested separately with optical fiber sensors in the laboratory. The results showed a high degree of accuracy.

The above methods considered the loadings as a uniformly distributed load along the span or a concentrated load at the mid span. The performances of these methods under other forms of loading have not been investigated, such as a non-uniformly distributed load and concentrated load at different locations. Further, the situations for bridges under different support conditions, varying loadings and varying flexural rigidities along the spans have not been reported.

2.5.2 Slope Measurement

Limited research has utilised slope measurement for vertical displacement measurements. One of the reasons is the limitation of an available sensor for the bridge environment, with which to obtain reliable measurements. Most of the sensors in the markets are electrical signal based and acceleration based. Electrical signal based means a lot of wiring and data acquisition systems are required. Also, electromagnetic interference may increase the uncertain of data in accuracy. Acceleration based means the sensors are sensitive to the vibration. This limits the
application of the sensors for bridges, because they are always under ambient vibration.

Hou et al. (2005) utilised ultralow frequency accelerometer to measure slope of bridges in order to estimate the deflection of a bridge in static and dynamic conditions. This method reduces the installation time of the sensors because they are easily installed on bridges directly. However, the sensors are used in electrical based. The influence by electromagnetic interference has not been reported. Besides, lots of wiring and data acquisition systems are required, which increases the cost of installation and instruments.

2.6 DAMAGE EVALUATION

As bridges are an important component of a transport network, the safety of bridges is of concern to the bridge authorities. To maintain safe and reliable of bridges, maintenance work should be carried out to ensure bridges are operating safety. During the operation of bridges, damage will be caused by different issues such as corrosion, fatigue, environment attack and artificial damage. A structural health monitoring (SHM) system can prevent failure of a structure. The SHM system uses non-destructive techniques to provide continuous information about the state of a structure. This section describes the definition of damage and damage detection techniques.

2.6.1 Definition of Damage

As bridges cannot endure forever, the states of them are an unknown after operation. Damage is caused by different issues such as corrosion, extra loading, and unexpected impact. Worden & Dulieu-Barton (2004) defined three states of damage as fault, damage and defect.

- A fault is when the structure can no longer operate satisfactorily. If one defines the quality of a structure or system as its fitness for purpose or its ability to meet customer or user requirements, it suffices to define a fault as a change in the system that produces an unacceptable reduction in quality.
• Damage is when the structure is no longer operating in its ideal condition but can still function satisfactorily, i.e. in a sub-optimal manner.

• A defect is inherent in the material and statistically all materials will contain some unknown amount of defect; this means that the structure can operate at its design condition even if the constituent materials contain defects.

In the most general terms, damage can be defined as changes introduced into a system that adversely affects its current or future performance. Implicit in this definition is the concept that damage is not meaningful without a comparison between two different states of the system, one of which is assumed to represent the initial, and often undamaged, state (Farrar & Worden, 2007). Worden and Barton (2004) stated that detection of damage is a facet of the broader problem of damage awareness or damage identification. The objective of a monitoring system must be to accumulate sufficient information about the damage for appropriate remedial action to be taken to restore the structure or system to high quality operation or at least to ensure safety. A hierarchical structure in five levels to think about the damage identification problem has been classified as

• **Detection**: the method gives a qualitative indication that damage might be present in the structure.

• **Localisation**: the method gives information about the probable position of the damage.

• **Classification**: the method gives information about the type of damage.

• **Assessment**: the method gives an estimate of the extent of the damage.

• **Prediction**: the method offers information about the safety of the structure for example, estimates a residual life.

### 2.6.2 Visual Inspection and Non-Destructive Testing (NDT)

The visual inspection and non-destructive testing (NDT) are conventional methods to evaluate the structural deficiencies of bridges that are manually identified
and classified by experienced inspectors and engineers. However, it is sometimes impractical to inspect a whole structure, as some parts are either covered or inaccessible (Huston, 2009). These methods are required in previously known damage locations and where they are readily accessible (Doebling et al., 1998). Hence, these methods are hardly applicable for damage at an inaccessible location (Wang, 2012). Although both an in-depth fracture-critical and a routine inspection of the I-35W bridge over the Mississippi River in Minneapolis, USA, were conducted in June 2006, the bridge collapsed after about a year, in August 2007. In the inspection reports, the bridge had been classified ‘Structurally deficient’ since 1991 (MDOT, 2007). Unfortunately, the disaster had not been prevented.

Due to the high maintenance cost and the limit of the identified damage state, these methods may not fulfil today’s maintenance needs (Wang, 2012). The damage evaluation of bridges requires a more scientific method for indicating damages and monitoring their long-term performances.

In the literature, a lot of advanced methods are presented to help bridge authorities to identify the damages of bridges. They can be classified into two main categories, which are vibration based damage detection method (VBDD) (Section 2.6.3) and displacement based damage detection method (DBDD) (Section 2.6.4).

### 2.6.3 Vibration Based Damage Detection (VDBB)

Dynamic characteristics are sensitive to changes in the mass and stiffness of the structure. Therefore, the changes in the dynamic characteristics can be used to locate the reduction in the stiffness, which indicates structural damage. The vibration based damage detection (VBDD) technique is commonly used in structural health monitoring of bridges. Several different approaches of VBDD are briefly described as following:

**Natural Frequency Based Methods**

Natural frequency is a conventional characteristic in modal analysis. Natural frequency based methods basically use the natural frequency change for damage identification. There are many applications of natural frequency, such as a concrete
cantilever beam (Gudmundson, 1982), a concrete cantilever plate (Friswell et al., 1998) and a truss bridge (Juneja et al., 1997).

In general, the natural frequency based method can be applied to simple structures with small cracks in a controlled laboratory condition. However, the applications for real complex structures or multiple damage detection are limited (Fan & Qiao, 2010). These applications are restricted in detecting the existence of structure damage, and are not for providing damage location information.

**Mode Shape Based methods**

As changes in mode shape are very sensitive to local damage, damage detection methods based directly on mode shape have been developed. Measurement of the mode shapes of a structure requires either a single excitation point and many sensors or a roving exciter with one or more fixed sensors (Carden & Fanning, 2004).

Two commonly used methods to compare two sets of mode shapes are the Modal Assurance Criterion (MAC) and the Coordinate Modal Assurance Criterion (COMAC). The MAC value can be considered as a measure of the similarity of two mode shapes. A MAC value of 1 is a perfect match and a value of 0 means they are completely dissimilar. Thus, the reduction of a MAC value may be an indication of damage. The COMAC is a pointwise measure of the difference between two sets of mode shapes and takes a value between 1 and 0. A low COMAC value would indicate discordance at a point and thus is also a possible damage location indicator (Carden & Fanning, 2004). Although it is sensitive to damage, mode shape based methods require a large number of location measurements. It is difficult to obtain accurate mode shapes using limited sensors. It is also difficult to obtain higher mode shapes in practice.

**Curvature Mode Shape Based methods**

The curvature is often calculated from the measured displacement mode shapes using a central difference approximation as

\[
\phi_j^c = \frac{\phi_{j+1} - 2\phi_j + \phi_{j-1}}{L^2}
\]  

(2-1)
where $\Phi = \text{mode shape}$, $\Phi^* = \text{curvature mode shape}$, $i = \text{mode shape number}$; $j = \text{node number}$; $L = \text{distance between the nodes}$.

Pandey et al. (1991) demonstrated to locate damage using curvature mode shape. The change of curvature mode shapes, shows usefulness in detecting and locating the region of damage.

Catbas et al. (2008) constructed a base line of displacement and curvature of the grid model test before damage. Then, they compared the calculated flexibility-based displacement and curvature after damage and the base line to indicate damage location.

2.6.4 Displacement Based Damage Detection (DBDD)

Method Based on Curvature, Slope and Displacement

Lee & Eun (2008) utilized static deflection measurement for detecting and locating damage. The damage is found at the location to exhibit the largest difference between the displacement curvatures of the damaged and undamaged structures as:

$$
\Delta \Phi^* = \Phi_d^* - \Phi^*
$$

where $\Delta \Phi^*$ is largest difference, $\Phi_d^*$ and $\Phi^*$ are the displacements of the damaged and of the undamaged structures, respectively. The results of a simple cantilever beam test showed that the deflection is increased with single and/or multi damage but these are not any indication with which to localize the damage. Utilizing the measured displacements as displacement constraints to describe a damaged beam, minimizing the change of the displacement vector between undamaged and damaged states, and neglecting the variation between the test data and the analytical results, the deflection shape and displacement curvature of the damaged beam can be estimated, and the damage can be detected by the curvature.

Stimac and Kozar (2005) proposed the damage detection method from displacement in-time function to avoid the need for many measurement points. Displacement-in-time function can be measured at one point in the structure. The deflection slope difference and the deflection curvature difference are reliable
indicators for damage location, except the damage is located near the first or the last support.

Zeng (2009) has developed an influence line type inspection technique that measures mid-span displacement under a moving point load. However, due to the measurement noise, the capability of damage identification from experimental data is not as impressive as it was in the numerical and theoretical investigations. Hence, an accurate vertical displacement measurement method could improve this method.

**Comprehensive Likelihood Factor**

Fu and Moosa (2002) developed a new method for damage detection using a predefined grid of data points. These data points and their corresponding slopes, curvatures, and deflections were used to obtain condition maps based on the comprehensive likelihood factor (CLF). The method was demonstrated using a simple I-beam, in which the location of the pre-cut points (induced damages), could easily be identified using such condition maps.

**Evaluation of flexural rigidities**

When a loading is applied to the structure, internal reaction is induced and the corresponding response of the structure is induced as deflections and rotations. Hence, the response of a bridge subjected to a known loading can be used to evaluate the flexural rigidities of bridges using an inverse problem.

Considering an external load applied to a structural element, the internal reaction can be established using application of a unit deflection and unit rotation to keep the equilibrium (Lakshmanan* et al.*, 2008). The free body diagram of a structural element, showing the reactions developed due to application of a unit deflection and unit rotation under an external force, is given in Figure 2-6. The force and moment equilibrium relationship between the applied load at node i+1 to the deformations at the same node and two adjoining nodes (i and i+2) are given in Eq. 2-3 and 2-4 where $\theta$, $\delta$, $l$, $EI$ are rotation, deflection, length and flexural rigidity. The flexural rigidities along the span can be evaluated by deflection and slope measurement. This method can be used to determine flexural rigidities using measured deflection and rotation. However, due to the presence of instrumental noise.
and resolution errors, the method may not provide a confidence solution to determine the exact flexural rigidities of bridges.

Figure 2-6  Free body diagram of the reactions in a structural node due to (a) unit deflection and (b) unit rotation (Lakshmanan et al., 2008)

\[
\left(-\frac{6}{l_i^2} \theta_i - \frac{12}{l_i^2} \delta_i - \frac{6}{l_i^2} \theta_{i+1} + \frac{12}{l_i^2} \delta_{i+1}\right)EI_i
\]

\[+ \left(\frac{6}{l_{i+1}^2} \theta_{i+1} + \frac{12}{l_{i+1}^2} \delta_{i+1} + \frac{6}{l_{i+1}^2} \theta_{i+2} - \frac{12}{l_{i+1}^2} \delta_{i+2}\right)EI_{i+1} = F_{i+1} \quad (2-3)
\]

\[
\left(\frac{2}{l_i} \theta_i - \frac{6}{l_i^2} \delta_i + \frac{4}{l_i^2} \theta_{i+1} - \frac{6}{l_i^2} \delta_{i+1}\right)EI_i
\]

\[+ \left(\frac{4}{l_{i+1}^2} \theta_{i+1} + \frac{6}{l_{i+1}^2} \delta_{i+1} + \frac{2}{l_{i+1}^2} \theta_{i+2} - \frac{6}{l_{i+1}^2} \delta_{i+2}\right)EI_{i+1} = M_{i+1} \quad (2-4)
\]

**Damage Factor Using Change in Deflection Ratio**

Based on the flexibility method, ratio of deflection change is proportional to the damage. Hence, ratio of deflection change can be used to indicate the damage using inverse analysis.

Total strain energy of the undamaged beam, \( U_{ud} \) due to flexure is expressed as

\[
U_{ud} = \int_A \frac{M^2 \, dx}{2EI} + \int_B \frac{M^2 \, dx}{2EI} + \int_C \frac{M^2 \, dx}{2EI} + \int_D \frac{M^2 \, dx}{2EI} + \int_E \frac{M^2 \, dx}{2EI}
\]
where $M$ and $EI$ are moment and flexural rigidity. For the damaged beam, the $EI$ is replaced by $\alpha EI$ in the damaged element. Total strain energy of the damaged beam, $U_d$ is expressed as

$$U_d = \int_a^b \frac{M^2}{2EI} + \int_c^d \frac{M^2}{2EI} + \int_{d+\Delta}^{d+2\Delta} \frac{M^2}{2EI}$$

where $\alpha$ is remaining ratio of $EI$. The formulation can be expressed as

$$U_d = U_{ud} - \int_b^d \frac{M^2}{2EI} + \int_b^{d-\Delta} \frac{M^2}{2\alpha EI} = U_{ud} + \frac{1-\alpha}{\alpha} \int_b^d \frac{M^2}{2EI}$$

For a simply supported structure described in Figure 2-7, the ratio of deflection change for the same loading can be expressed as (Lakshmanan et al., 2008)

$$\frac{\Delta v_i}{v_i} = \sum_j^N \beta_j \frac{2b_0 j}{b_i^2} (3l_0^2 + b_0^2)$$

where $v$ is deflection, $i$ is $i^{th}$ node, $j$ is $j^{th}$ element and $\beta = 1 - \alpha$ is damage factor. The ratio of deflection change that is proportional to the damage can be used to indicate damage.

![Figure 2-7](image-url)  
A simply supported beam with damage occurred at element D for system identification
2.7 CONCLUSION AND RESEARCH PROBLEM

In many bridges, the vertical displacements are the most relevant parameters for monitoring the structural health in both the short and long term. However, as is well known amongst bridge authorities, it is difficult to measure the vertical displacements of a bridge. According to the literature review, research gaps are identified as:

- Conventional vertical displacement measurement methods, such as using LVDT and land surveying, are sometimes impractical because a specialised operator is required and measurements can be affected by weather. They are not applicable for measuring vertical displacements for the long-term and for real time monitoring. GPS is limited to an accuracy that depends on many factors such as data sampling rate, satellite coverage, atmospheric effect, multipath effect, and GPS data processing methods. Photogrammetry measurement has a long history of development and application. However, it is not affordable for monitoring bridges.

- Recent, close-range photogrammetry and other image based measurement methods have been developed using a single or a stereo camera that highly reduces the cost of the measurements. However, their applications are limited in low visibility and they require a wide insight angle and reference point for locating the cameras. The atmospheric effects also increase the uncertainties of accuracies.

- Several researchers have studied how to utilise curvature measurements to estimate deflection of bridges. However, their applications under different conditions such as loading, supports, varying flexural rigidities of structures and number of sensors have not been reported.

- According to the theory of the curvature measurement method, slope measurement of bridges can also be employed to obtain vertical displacements. However, most of the inclination sensors in the markets are electrical signal based and acceleration based. Electrical signal sensors require a large amount of wiring and a data acquirement system. Using
FBG sensors can overcome these issues due to their distinct advantages such as multiplexing capability and immunity from EMI. Besides, acceleration based inclination sensors are sensitive to the vibration and hence have limited application to bridges, which are always under ambient vibration. The available FBG inclination sensors in the markets, however, have not been investigated in order to address this problem.

- Vibration based damage detection (VBDD) technique is commonly used in SHM. Natural frequency based methods can only detect the existence of damage. Mode shape and curvature mode shape based methods can be used to locate damage. However, they require a large amount of sensors and data processing.

- On the other hand, due to difficulties in the measurement of vertical displacements of bridges, most researchers have been reluctant to consider their use for damage detection. Only a limited number of researchers have treated displacement based damage detection (DBDD) methods to demonstrate that vertical displacements are correlated to damage location and damage extent. Due to lack of simple, reliable and practical methods to measure vertical displacements, the displacement based damage detection methods have not received much attention. Deflection measurements can be used to evaluate the flexural righties of simple bridge structures. Further, variation in displacement between the current and original values can also be used to indicate the damage in the bridge.
3.1 INTRODUCTION

This chapter describes the methodology of vertical displacement measurement of bridges using curvature and slope measurements. Section 3.2 describes the relationships between vertical displacement, slope and curvature. Sections 3.3 and 3.4 propose the curvature and inclination approaches for vertical displacement measurement, respectively. Section 3.5 describes the regression analysis to enhance the accuracies of the approaches. Section 3.6 concludes this chapter.

3.2 RELATIONSHIPS BETWEEN VERTICAL DISPLACEMENT, SLOPE AND CURVATURE

A simple bridge such as a slab-on-girder bridge or a box-girder bridge can be considered as a beam structure. For a simply supported beam under a uniformly distributed load, the beam deforms into a curve as described in Figure 3-1.
According to the geometric relationship, the curvature and slope can be expressed respectively as

\[ \kappa = \frac{1}{R} = \frac{d\theta}{ds} \quad (3-1) \]

\[ v' = \frac{dv}{dx} = \tan\theta \quad (3-2) \]

Since the vertical displacement curve has a very small displacement and angle of rotation, \( ds \approx dx \) and \( \tan\theta \approx \theta \), the curvature and slope in Eq. 3-1 and Eq. 3-2 can be expressed respectively as

\[ \kappa = \frac{1}{R} = \frac{d\theta}{dx} \quad (3-3) \]
\[ \theta = \frac{dv}{dx} \]  \hspace{1cm} (3-4) 

The first derivative of \( \theta \) with respect to \( x \) in Eq. 3-3 is expressed as

\[ \frac{d\theta}{dx} = \frac{d^2v}{dx^2} \]  \hspace{1cm} (3-5) 

Combining it with Eq. 3-3, the relationships between the curvature, slope and vertical displacement can be expressed as Eq. 3-6 and the summary of their relationships is given in Figure 3-2.

\[ \kappa = \frac{d\theta}{dx} = \frac{d^2v}{dx^2} \]  \hspace{1cm} (3-6) 

Figure 3-2 Relationships between vertical displacement, slope and curvature
3.3 CURVATURE APPROACH

This section presents an approach to determine vertical displacements using curvature measurements based on the geometric relationship between curvature and vertical displacement. The procedure of the curvature approach is given in Figure 3-3a. The procedure of the inclination approach is given in Figure 3-3b and will be described in Section 3.4.

\[
\kappa_i = \frac{-\varepsilon_i}{y}
\]  

(3-7)

Figure 3-3 Flow chart of the (a) curvature approach and (b) inclination approach of vertical displacement measurements

The relationship between curvature and strain can be expressed as
where \( i \) is \( i \)th longitudinal location, \( \varepsilon \) is longitudinal strain and \( y \) is distance from neutral axis of cross section. The curvature can be determined by longitudinal strain measurements parallel to the neutral axis. However, the neutral axis may be shifted if damage of structure or temperature variation occurs. Two strain sensors placed at different distances parallel to the neutral axis can be used to eliminate the effect of the shift of the neutral axis. For two sensors at corresponding longitudinal locations, the curvature is expressed as

\[
\kappa_i = \frac{\varepsilon^b_i - \varepsilon^t_i}{h}
\]  

(3-8)

where \( \varepsilon^b \) and \( \varepsilon^t \) are the bottom and top strain and \( h \) is the distance between the sensors.

If there are more than two sensors applied at the corresponding longitudinal location of the beam, curvature can be found by drawing the strain diagram, as shown in Figure 3-4. The unknown \( \alpha \) and \( \beta \) are solved using the least square error method.

![Strain diagram](image)

Figure 3-4  Strain distribution diagram

\[
[\varepsilon_i = -\alpha y_i + \beta]_{i=1to4}
\]  

(3-9)

The neutral axis is the axis through the beam where the stress and strain are zero. When two sensors are installed at the tension and compression locations, the neutral axis can be found as
\[ \bar{y} = \frac{e_B}{e_e + e_B} h \]  

(3-10)

where \( \bar{y} \) is the neutral axis position from the bottom sensor and \( h \) is the distance of sensors at the cross section as shown in Figure 3-4. If there are more than two sensors, the neutral axis is determined by using the strain distribution diagram shown in Figure 3-4.

For measuring vertical displacements of bridges in real time, it is necessary to consider whether the curvature curve is influenced by varied loading conditions. As the applied load is unknown, conducting a regression analysis is necessary to retrieve the exact curvature function that can be expressed in an \( n \)th order polynomial as

\[ \kappa_i = c_n x_i^n + c_{n-1} x_i^{n-1} + \cdots + c_1 x_i + c_0 \]  

(3-11)

where \( c_n, c_{n-1}, \ldots, c_1, c_0 \) are the coefficients of the curvature function that are obtained by the curvature measurements; \( x \) is curvilinear abscissa along the beam. The regression analysis will be described in detail in Section 3.5. If more than three sensors are used along the span, the curvature coefficient in Eq. 3-12 can be solved using the least square method in Eq. 3-15.

\[
\begin{bmatrix}
K_1 \\
K_2 \\
\vdots \\
K_m
\end{bmatrix} = \begin{bmatrix}
x_1^n & \cdots & x_1^2 & x_1 & 1 \\
x_2^n & \cdots & x_2^2 & x_2 & 1 \\
\vdots & \ddots & \vdots & \vdots & \vdots \\
x_m^n & \cdots & x_m^2 & x_m & 1
\end{bmatrix} \begin{bmatrix}
c_n \\
c_{n-1} \\
\vdots \\
c_0
\end{bmatrix}
\]  

(3-12)

where \( m \) is the number of sensors.

In matrix notation, Eq. 3-12 expresses as:

\[ K = XC \]  

(3-13)

Multiplying \( X^T \) in both sides,

\[ X^T K = X^T XC \]  

(3-14)

This matrix equation can be solved by inverse \( X^T X \),
\[ C = (X^T X)^{-1} X^T K \]  

(3-15)

The deflected shape function can be determined by double integrating the curvature in Eq. 3-6 as

\[ v(x) = \int \int \kappa(x) \, dx \, dx \]  

(3-16)

For a simply supported beam under a uniformly distributed load, the curvature function is a second degree polynomial as

\[ \kappa_i = c_2 x_i^2 + c_1 x_i + c_0 \]  

(3-17)

Substituting it into Eq. 3-16,

\[ v(x) = \int (c_2 x^2 + c_1 x + c_0) \, dx \, dx \]  

(3-18)

\[ v(x) = \frac{c_2 x^4}{12} + \frac{c_1 x^3}{6} + \frac{c_0 x^2}{2} + \alpha x + \beta \]  

(3-19)

where \( \alpha \) and \( \beta \) are integration constants that can be obtained by applying boundary conditions such as zero displacement at supports or using slope measurements. For example assuming zero displacement at both the supports \( v(0) = 0 \) and \( v(L) = 0 \) where \( L \) is the length of the span), gives \( \beta = 0 \) and

\[ \alpha = -\left(\frac{c_2 L^3}{12} + \frac{c_1 L^2}{6} + \frac{c_0 L}{2}\right) \]  

(3-20)

The deflected shape function is then determined as

\[ v(x) = \frac{c_2 x^4}{12} + \frac{c_1 x^3}{6} + \frac{c_0 x^2}{2} - \left(\frac{c_2 L^3}{12} + \frac{c_1 L^2}{6} + \frac{c_0 L}{2}\right) x \]  

(3-21)

### 3.4 INCLINATION APPROACH

With the advancement of FBG inclination sensors (Section 4.10.2), which have all advantages attributed to FBG sensors, high accuracy slope measurements can be implemented for bridge vertical displacement measurements. As the measurements are only relevant to the geometry of the deformed shapes, they will not be affected by the changes of internal deformations. This can increase the practicability of the
vertical displacement measurements for bridges, and therefore an inclination approach for vertical displacement measurements is proposed.

The procedure of the inclination approach is given in Figure 3-3b. From Eq. 3-6, the slope function can be integrated with respect to $x$ as

$$\theta = \int \kappa \, dx$$

Combining it with Eq. 3-11, the slope function is expressed as

$$\theta_i = \frac{c_n x_i^{n+1}}{n+1} + \frac{c_{n-1} x_i^n}{n} + \cdots + c_0 x + \alpha$$

where $\alpha$ is a integration constant.

The slope of the deflection curve ($\theta$) is the first derivative of the deflected curve (Eq. 3-3). The deflected shape function can be determined by integrating the slope function as

$$v = \int \theta \, dx$$

From the example of a simply supported beam under a uniformly distributed load, the curvature function is a second degree polynomial (Eq.3-17). Combining Eq. 3-20 and Eq. 3-23 the slope function can be expressed as

$$\theta_i = \frac{c_2 x_i^3}{3} + \frac{c_1 x_i^2}{2} + c_0 x - \left( \frac{c_2 x_i^3}{12} + \frac{c_1 L^2}{6} + \frac{c_0 L}{2} \right)$$

Then, the vertical displacement function is expressed in the same manner as in Eq. 3-21.

### 3.5 Regression Analysis

In most situations, the loading is unknown. The loading of bridges may be non-uniformly distributed load or multi-axles point load. A regression analysis is required to retrieve an optimum curvature and slope functions. When a bridge is in service, the loading can be assumed as a uniformly distributed load. The curvature and slope functions are second and third order polynomial, respectively. To enhance the
accuracies of the approaches, a regression analysis is required to retrieve the exact curvature (Eq. 3-11) and slope (Eq. 3-23) functions.

To find the optimized degree of polynomial for the curve fitting, the coefficient of determination $R^2$ can be compared for different order polynomials.

3.5.1 Linear Curvature Function

When a loading is concentrated at a location, the curvature and slope functions are linear and second order polynomials, respectively. The linear curvature function cannot be expressed in a single equation. It is required to be presented as two equations to show the whole curvature function along the span.

When a concentrated load is added on the span, the curvature function is divided into two segments as

$$\kappa^L(x) = c_1^L x + c_0^L \quad (0 < x \leq a)$$  \hspace{1cm} (3-26)

$$\kappa^R(x) = c_1^R x + c_0^R \quad (a \leq x < L)$$  \hspace{1cm} (3-27)

where $a$ is the location of the point load, $\kappa^L$ and $\kappa^R$ are the curvature of the left and right segments, respectively. $c_1^L$, $c_0^L$, $c_1^R$ and $c_0^R$ are the coefficients of the curvature function of the left and right segments, respectively.

According to Eq. 3-16, the deflected shape function of the left and right segments can be determined as

$$v^L(x) = \frac{c_1^L x^3}{6} + \frac{c_0^L x^2}{2} + \alpha^L x + \beta^L \quad (0 < x \leq a)$$  \hspace{1cm} (3-28)

$$v^R(x) = \frac{c_1^R x^3}{6} + \frac{c_0^R x^2}{2} + \alpha^R x + \beta^R \quad (a \leq x < L)$$  \hspace{1cm} (3-29)

The integration constants $\alpha^L$, $\alpha^R$, $\beta^L$ and $\beta^R$ can be obtained by applying boundary conditions.

For the left segment, assuming $v^L(0) = 0$,

$$\beta^L = 0$$  \hspace{1cm} (3-30)
According to Eq. 3-22, the slope is given by

$$\theta_i = \frac{c_i^l x_i^2}{2} + c_0^l x_i + \alpha^l$$  \hspace{1cm} (3-31)

Hence,

$$\alpha^l = \theta_i - \left(\frac{c_i^l x_i^2}{2} + c_0^l x_i\right)$$  \hspace{1cm} (3-32)

The deflection shape function of the left segment is determined as

$$v^l(x) = \frac{c_i^l x^3}{6} + \frac{c_0^l x^2}{2} + \alpha^l x$$  \hspace{1cm} (3-33)

For the right segment, assuming \(v(L) = 0\),

$$\beta = -\left(\frac{c_i^R L^3}{6} + \frac{c_0^R L^2}{2} + \alpha^R L\right)$$  \hspace{1cm} (3-34)

Applying \(v^L(a) = v^R(a)\) as continuity conditions,

$$v^L(a) = \frac{c_i^l a^3}{6} + \frac{c_0^l a^2}{2} + \alpha^R a - \left(\frac{c_i^R L^3}{6} + \frac{c_0^R L^2}{2} + \alpha^R L\right)$$  \hspace{1cm} (3-35)

$$\alpha^R = \left[v^L(a) + c_1^R \frac{L^3-a^3}{6} + c_0^R \frac{L^2-a^2}{2}\right]/(a - L)$$  \hspace{1cm} (3-36)

The deflection shape function of the right segment is determined as

$$v^R(x) = \frac{c_i^R x^3}{6} + \frac{c_0^R x^2}{2} + \alpha^R x - \left(\frac{c_i^R L^3}{6} + \frac{c_0^R L^2}{2} + \alpha^R L\right)$$  \hspace{1cm} (3-37)

### 3.6 CONCLUDING REMARKS

This chapter describes the methodology of vertical displacement measurement of bridges using curvature and slope measurements. Curvature and inclination approaches are proposed using curvature and slope measurement along the spans of bridges. The support conditions and the flexural rigidities of the bridges are independent to the vertical displacement determination. They can be applied to simply supported and continuously supported bridges. The flexural rigidities are not required in the equations and hence the member size and its elastic modulus are not
required as a known priori. Further, the theory is based on the span geometry. Hence, support settlement or suspension bearing at the support would not affect the measurement. The methods can be applied to different materials and most of the beam type bridge structures, such as slab-on-girder and box-girder bridges made of steel and concrete. Slab-on-girder bridges are one of the common types of structure forms for bridges. The girder is used for a primary load bearing structure and the slab is connected to the girders to provide a plane surface for traffic. The girders can be considered as a beam structure. The sensors can be installed on the bridge along the span. For a slab-on-girder bridge with multi-girders, the sensors can be installed to each girder along the span. The approaches should be implemented separately to obtain the displacement function of each girder.

Box-girder bridges comprising girders of a hollow section can also be considered as a beam structure form. For the curvature approach, the sensors should be installed around the section at the corresponding longitudinal location to obtain the curvature.

The two proposed approaches can be implemented individually to determine vertical displacements. The choice of the particular approach will be determined by the budget, access of bridges and installation methods. As the proposed methods are simple, a large amount of data processing time is not required. A fully automatic real time monitoring system can be further developed.

The curvature is a relative parameter which is induced by the bending of the span under loading. The curvature measurement depends on the tension and compression change in the section of the span. As bridges are subjected to certain loading such as self-weight, traffic load and wind force, the initial value for long term measurement history record needs to be determined carefully. When one of the sensors is faulty and requires replacement, its initial value may change. The replacement of the sensor requires a calibration in order to consist of the initial value. The curvature sensors can be embedded into a concrete structure or attached on the surface of a structure.
Alternatively, the inclination approach utilises slope measurements of bridges with respect to gravity, which is an absolute parameter. The inclinometers can be easily placed on the bridge and hence maintenance or replacement of the sensors will be easy.

If the loading, such as non-uniformly distributed load and multi-axle loads on the bridge is unknown, a regression analysis is required to retrieve the exact curvature and slope function in order to enhance the accuracies of the approaches. When a point load is expected, the curvature function is required to be divided into two segments, as shown in Section 3.5.1. The regression analysis will be demonstrated in numerical and experimental tests in Chapters 5 and 6.

These approaches utilise curvature and slope measurements to determine vertical displacements of bridges. The requirements of the sensors are very important such as high accuracy, resolution and reliability. In the market, most of the sensors for curvature and slope measurements are electrical based, which will suffer from EMI and this will highly affect the reliability of the measurement. Fiber Bragg grating (FBG) sensors have been widely applied to the measurement systems for structural health monitoring of bridges. FBGs have many advantages including multiplexing capability, immunity to EMI as well as high resolution and accuracy. To implement these approaches, FBG sensors are proposed for the measurements and will be described in Chapter 4.

Although the conventional methods such as LVDT, surveying and GPS allow displacement measurements at a point, the proposed methods allow displacement measurements of bridges’ deformed shape. The photogrammetry methods can measure displacement of a bridge at several points depending on the number of targets. However, their applications are limited in low visibility and they require a wide insight angle and reference point for locating the cameras. The atmospheric effects also increase the uncertainties of accuracies. However, the proposed methods are not affected by the weather.

Several research studies have been investigated using curvature measurement to estimate vertical displacement of bridges. However, the studies have not reported
the applications of those methods under different conditions. In the current study, the proposed methods are validated under different conditions such as loading, supports, varying flexural rigidities of structures and number of sensors (Chapters 5 and 6).
Chapter 4: FBG Sensing Technology

4.1 INTRODUCTION

In this study, vertical displacement measurement methods using curvature and inclination approaches were proposed in Chapter 3. Their implementations require high accuracy, resolution and reliability of sensors. Most of the sensors for curvature and slope measurements in the market are electrical based. These signals are rarely distinguishable from noise, due to electromagnetic interference (EMI). On the other hand, fiber optical sensors (FOS) are widely used in structural health monitoring of bridges (Li et al., 2004; Mehrani & Ayoub, 2009), especially fiber Bragg grating (FBG) sensors (Chan et al., 2006a; Majumder et al., 2008), because they have all the advantages normally attributed to fiber optical sensors including multiplexing capability, immunity of EMI as well as high resolution and accuracy. They are simple to fabricate and interrogate/demodulate as well as easy to install. Hence, FBG sensors have become one of the most prominent sensors used for structural health monitoring.

FBGs is one type of fiber grating that is defined as a region of an optical fiber with a periodic structure of the refractive index with the period $\lambda$ induced in the fiber core, which has a certain spatial distribution (Vasil'ev et al., 2005). Fiber grating includes fiber Bragg grating, long-period fiber grating, chirped grating, tilted fiber grating and sampled fiber grating (Lee, 2003). Fiber Bragg gratings have been widely used as sensors for the measurement of strain (James et al., 1996; Jin et al., 1997), temperature (James et al., 1996; Jin et al., 1997), pressure (Xu et al., 1993), inclination (Chen et al., 2008; Guan et al., 2004), displacement (Yu et al., 2000) and acceleration (Au et al., 2008; Theriault et al., 1997).

Only few FBG inclination sensors are available in the market. Their performance under ambient vibration, response time and temperature effect have not been reported. The reliability and repeatability of the measurement are highly affected by issues abovementioned. One of the objectives of this thesis is to develop
an FBG inclination sensor for vertical displacement measurements of bridges. FBGs used in this sensor are self-fabricated in the laboratory. The fabrication of FBGs involves many procedures such as hydrogen loading, stripping, cleaving and splicing. The reflectivity of the FBGs is influenced by the photosensitivity of fiber core, fabrication methods, as well as UV laser source parameters such as wavelength and intensity. Those procedures and fabrication methods will also influence the mechanical strength and the performance of the FBGs. These factors influence the reliability and performance of the FBG sensors.

This chapter overviews the FBG technology for the purposes of sensor development and application including: principle (Section 4.2), FBG sensing system (Section 4.3), fabrication methods (Section 4.4), stripping methods (Section 4.5), adhesive and epoxy (Section 4.6), hydrogen loading (Section 4.7), influence of exposure parameters on the mechanical strength of FBGs (Section 4.8), characteristics (Section 4.9), types of FBG sensors (Section 4.10). Section 4.11 describes the curvature measurement using FBG sensors. Section 4.12 describes the development of FBG inclination sensors including the theory, fabrication and testing. Section 4.13 proposes a novel frictionless FBG inclination sensor with extremely high sensitivity and resolution. This sensor’s performance under static and vibration are reported. The concluding remarks of this chapter are given in Section 4.14.

### 4.2 PRINCIPLE OF FBG SENSORS

Bragg grating is a periodic structure of refractive index along with the fiber core, which is fabricated by exposing photosensitized fiber core to ultraviolet (UV) light. The formation of refractive index gratings in an optical fiber by exposing the photosensitivity core of fiber to a 488nm or 514.5nm argon-ion laser was first demonstrated by Hill et al. (1978).

The basic principle of FBG sensing is to monitor the Bragg wavelength that is induced by temperature, strain and pressure. When a broadband source transmits to the Bragg grating, the Bragg wavelength is reflected and the other wavelengths transmit as described as Figure 4-1. The Bragg wavelength, \( \lambda \) is given by

\[
\lambda = 2\Lambda n_{eff}
\]  

(4-1)
where $\Lambda$ is the grating period, and $\eta_{\text{eff}}$ is the effective refractive index of the fiber core.

The changes in strain, $\varepsilon$, temperature, $T$, and pressure, $P$ of fiber grating, alter both the effective refractive index and the grating period. When strain and pressure are exerted, the expansion or contraction of the grating periodicity and the photoelastic effect will shift the Bragg wavelength. The temperature change causes the Bragg wavelength shift through thermal expansion and contraction of the grating periodicity and through thermal dependence of the refractive index (Meltz et al., 1988).

As the Bragg wavelengths are cross sensitivities to strain, pressure and temperature, the total of Bragg wavelength shift is given by:

$$\Delta \lambda = K_\varepsilon \varepsilon + K_T \Delta T + K_P P$$

(4-2)

where $K_\varepsilon$, $K_T$ and $K_P$ are coefficients of wavelength sensitivity to strain, temperature and pressure. They are given by (Moyo et al., 2004):
\[ K_e = [1 - 0.5\eta_{eff} (\rho_{12} - \nu(\rho_{11} - \rho_{12}))]\lambda \]  (4-3)

\[ K_T = [1 + \xi]\lambda \]  (4-4)

\[ K_p = [- (1 - 2\nu)/E + \eta^2 (1 - 2\nu)(2\rho_{12} + \rho_{11}) / 2E]\lambda \]  (4-5)

where \( \rho_{11} \) and \( \rho_{12} \) are the components of the fiber optics strain tensor and \( \nu \) is Poisson’s ratio, \( \xi \) is the thermo-optics coefficient, \( E \) is Young’s modulus, \( \eta \) is the refractive index.

### 4.3 FBG SENSING SYSTEM

One of the distinct advantages of FBG sensors is their multiplexing capability. The FBG can be easily multiplexed to measure in different locations, which is a kind of typical quasi-distributed sensor as shown in Figure 4-2. Within the same fiber, many gratings in different Bragg wavelengths can be written at different locations, which can be measured simultaneously. As mentioned before, Bragg gratings are cross sensitive to strain, pressure and temperature. The sensing system is able to measure different parameters. The characteristic of multiplexing capacity is described in details in Section 4.9.1.

![Figure 4-2](image)

**Figure 4-2** Quasi-distributed sensor system using FBG elements

### 4.4 FABRICATION METHODS

The formation of the first permanent grating in fibers was first reported in 1978 (Hill et al.). The basic principle of formation of fiber grating is to expose a
photosensitive fiber to intense UV light. In general, the fibers used for manufacturing FBGs are germanium-doped silica fibers. Their cores are photosensitive. When they are exposed to a UV light, the core absorbs the light, and therefore causes a permanent change of the refractive index. The amount of change depends on the intensity of the UV light, duration of the exposure and the photosensitivity of the fiber. In practice, UV light source is commonly generated by a Krypton fluoride (KrF) excimer laser at 248 nm or Argon fluoride (ArF) excimer laser at 193 nm (Hill & Meltz, 1997).

There are two main methods of writing FBG: interferometric method and phase mask method. Interferometric method was a method first widely used for the fabrication of FBG using interference of two UV beams. The interferometric method splits an incident UV beam into two beams using a splitter and emerges to produce an interference pattern in the fiber core (Meltz et al., 1988). This method is highly sensitive to the optical alignment of the system; therefore it requires high mechanical stability and isolation from ambient vibration (Singh et al., 2005). Furthermore, since the FBG writing process would last a few minutes using interferometric methods, a UV source with a spatial coherence is required. Phase mask method is now commonly used for writing FBG (Hill et al., 1993; Singh et al., 2005) as described in Figure 4-3. When light interacts from a broadband source to the fiber, the grating wavelength is reflected, known as Bragg wavelength.
The phase mask is made by fused silica etched with a periodic structure on its surface. When the UV light passes through the phase mask, the UV light would be diffracted to split into $\pm 1$ diffractive order. The photosensitive fiber is placed close to the phase mask. The diffracted beams are exposed to the fiber to generate a periodic pattern in order to write a grating into the fiber core. Depending on the space of the periodic pattern of the phase mask, the grating period $\lambda$ is half of the period length of the phase mask. Fabricating various grating lengths of FBGs can use a reflection prism with an adjustable linear guide to adjust the distance of the UV beam passing through the optical fiber. This method highly simplifies the fabrication of the FBG and requires highly coherent UV light. Therefore an inexpensive excimer laser can be used for writing FBG. Phase masks have become less expensive in recent years. For these reasons, the phase mask method is suitable for the mass production environment.
4.5 STRIPPING METHODS

Silica glass optical fibers are commonly coated with coating (also known as jacket) such as acrylate during the drawing process, to provide mechanical strength to fibers and protect them from damage. In recent times, polyimide is also available used for coating, as it provides a higher protection and the thickness is less than acrylate coating. But these coatings can only provide protection for handling purposes. Extra protections such as using patch cord or armed cable for practical applications are available on the market.

Stripping the coating from an optical fiber is a practically important and necessary process when fabricating FBGs or splicing fibers. Bad stripping may break the fiber and also create a crack. This raises a concern for reliability of FBGs and their signal characteristics in applications (Kang et al., 2007). Mechanical strength and peak split are concerned for the reliability of FBGs. The peak split problem would be caused by birefringence or strain gradients.

There are two main stripping methods to remove the coating, mechanical and chemical. Mechanical stripping methods use manual or automatic strippers, as described as Figure 4-4, which are commonly available in the markets. Manual strippers are similar wire stripping tools. They cut into the coating with a precise blade along the fiber to peel the coating from the surface of fiber. The cutting diameter is just enough for cutting into the coating. But, using manual strippers requires an experienced person to prevent physical contact to the fiber that would cause scratch and strength degradation of the fibers. Automatic strippers are widely available in the markets. They require no skill to handle. The duration of the stripping is only few seconds. As the process is automatic, it can eliminate irregular force acting on the fiber during stripping process. Mechanical stripping can only work for acrylate coating. That means this method does not work for polyimide coating that has a higher strength.
The physical contact between the stripper and the fiber may still occur due to an excess of stripping force applied and irregular of the fiber; these would cause minor scratches on the surface of the fiber. Besides, the residues left on the surface of the fiber must be removed by solvents, typically isopropyl alcohol. This process also causes a physical contact. All of these may result in degradation of the mechanical strength of the fiber.

The chemical stripping method entails using sulfuric acid at a high temperature (200°C) to dissolve the coating. Unlike mechanical stripping, there is no mechanical force acting on the fiber. This method can remove nearly all coating material including polyimide. However, using chemical stripping is not practical for field applications, because the chemical is toxic and corrosive. Neutralization or dilution is required to dispose of the used chemical safely. This method would be employed in a laboratory or factory environment, especially when extremely high strength is required. This method does not degrade the strength when sufficient care is taken during the stripping and handling (Matthewson et al., 1997).

### 4.6 ADHESIVE AND EPOXY

For fabrications of FBG sensors, adhesives and epoxies are often used for alignment and mounting optical fibers to a component of a sensor. Adhesives harden rapidly with exposure to moisture in the air. Although they commonly set in a time period of from a few seconds to a few minutes, the curing continues for at least 24 hours. The curing time depends on the relative humidity and the material of the substrate as well as the bonding gap. Higher relative humidity results in a shorter
curing time but the ultimate strength may be weakened. The optimum relative humidity is 40% to 60% at room temperature. The shear strength is varied by the material of the substrate and the roughness of the substrate surface.

Epoxies are typically mixed by two parts as resin and hardener. Different compounds of epoxies have various curing methods and curing times. Depending on the compound of the epoxy, there are different curing methods including exposure of the moisture in air, at a high temperature or under UV light. In general, epoxies provide higher shear strength, temperature resistance and chemical resistance. As epoxies are suitable for filling voids, they can be used for embedding the optical fiber into a component of a sensor.

4.7 HYDROGEN LOADING

FBGs are fabricated by exposing the photosensitivity core of optical fiber under a UV laser beam. The first grating was formed using germanium-doped silica fiber (Hill et al., 1978).

In recent times, hydrogen loading has been commonly used to allow FBG fabrication in standard telecommunication optical fibers. This method is to load hydrogen into fiber with high pressure (140atm) and under high temperature (70°C) in a period (five days) (Guan et al., 2000) in order to enhance the photosensitivity of the standard telecommunication fiber core. As the photosensitivity is increased, it can reduce the duration of UV exposure to the fiber and increase the Bragg reflectivity (Atkins & Espindola, 1997). Therefore, hydrogen loading is commonly used to allow FBG fabrication in standard telecommunication optical fibers. However, the mechanical strength of the hydrogen loaded FBGs are degraded about 50% compared with hydrogen unloaded FBGs using a same fabrication procedure (Wei et al., 2002).

4.8 INFLUENCE OF EXPOSURE PARAMETERS ON THE MECHANICAL STRENGTH OF FBGS

The mechanical strength of FBGs during the fabrication would be degraded such as in the stripping method and hydrogen loading, as mentioned in Sections 4.5 and 4.6. Besides, exposure parameters during fabrication of FBGs such as wavelength of the UV laser and UV laser intensity would also influence the
mechanical strength of the FBGs. Comparing the tensile strengths of unirradiation and irradiation fiber, it is degraded about 25%. Using a shorter wavelength of UV laser to fabricate FBGs has a greater mechanical strength than when using a longer wavelength. Higher pulse energy used for writing FBGs also degrades the mechanical strength of FBGs (Wei et al., 2002).

4.9 CHARACTERISTICS

Most of the conventional sensors used in health monitoring applications are based on the transmission of electric signals. However, there are certain limitations of these sensors. These sensors are usually not small or durable enough to be embedded in a structure to measure interior properties. They are local (or point) sensors, which are restricted to measure parameters at one location only and cannot be easily multiplexed. The long lead lines also create problems for large civil structures, which often span several tens of kilometres. In some cases, the signals cannot be discriminated from noise due to electrical or magnetic interference (EMI). In addition, various demodulation techniques are required for different sensors. They all result in increasing the inconveniences of conventional sensors in SHM. (Li et al., 2004)

4.9.1 Multiplexing Capabilities

FBG sensors, which can be easily multiplexed to measure different measurands at many locations, are a kind of typical quasi-distributed sensor. Many gratings can be written in the same fiber at different locations and tuned to reflect at different wavelengths. This allows the measurement of strain at different places along a fiber using a single cable. Typically, a single fiber in a single channel of a sensing system could allow measurements of many gratings. The amount of FBGs used in a single channel depends on the wavelength shift range of each FBG and the wavelength range of the interrogator. A typical interrogator with wavelength range of 80 nm can measure up to 20 FBGs in a single channel assuming the wavelength range of ±2nm per FBG. Typical commercial interrogators have four channels and can expand to 16 channels using channel multiplexers. Hence, many sensors can be installed on a structure with minimum wiring. As the measurands are wavelengths instead of the
sensing parameters of the sensors, different types of sensors such as strain, temperature and inclination sensor can be coupled in the same channel. There are three FBG array systems: serial system, parallel system and branching system, as described as Figure 4-5. The sensor wiring systems are very flexible. The cost of wiring and the capital cost of the interrogator can be reduced. For the proposed vertical displacement measurement methods, a large amount of sensors are required and they are installed along the bridge’s span. This outstanding characteristic can increase the usability of these methods for bridges.

Figure 4-5  Multiplexing FBG arrays. (a) Serial system, (b) Parallel system, (c) Branching system (Kersey et al., 1997)

4.9.2 High Sampling Rate

Typically, the method of interrogation and multiplexing is based on a Fabry–Perot scanning filter. The scanning time of the filter configures the acquisition speed of the interrogator. In general, the sampling rate of a commercial FBG interrogator is from 1Hz to 1 kHz. For research purposes, it can be achieved up to 500 kHz (http://www.micronoptics.com). The measurement can be implemented in real time while the bridges are in service.
4.9.3 Small Size

Diameters of fibers with acrylate coating and polyimide coating are usually 250μm and 150μm, respectively, and the length of FBGs is usually around 10 mm. Due to compactness of size, FBGs can be used as a replacement for conventional strain gauges. Moreover, this is promising for developing different types of sensors.

4.9.4 Electro-Magnetic Interference (EMI) Immunity

As fibers are made of silica, FBG sensors are uniquely capable of multi-point EMI-resistant measurement. The measurement signals are not affected by high voltage and electro-magnetic interference. This is particularly suitable for outdoor application. This is a distinct advantage when compared to the conventional electrical based sensors.

4.9.5 Absolute Parameter of Wavelength

As the sensing parameter is wavelength instead of intensity of signal, which is absolute, the measurement does not depend on the intensity of signal or source power as well as loss in the connecting fibers and couplers. Therefore, the length of fibers could be achieved in term of kilometres, which is suitable for bridges by remote control.

4.9.6 High Resolution and Precision

Typically, resolution and accuracy of the interrogating system are 1 pm and 5pm, respectively. Converted to strain, they have a resolution of about 0.83με and an accuracy of 4.2με in 1550nm of Bragg wavelength.

4.9.7 Strain and Temperature Discrimination

As FBGs are cross-sensitivity of strain and temperature, changes in both induce the shift of Bragg wavelength. Each variable should be separated for accurate measurements. The common way is to use reference FBG, isolated from one parameter, which is placed near the FBG.

4.10 TYPES OF FBG SENSORS

FBG technology is applied to various measurements such as strain, curvature, displacement, temperature, acceleration and inclination. A calibration test is required
to convert the shift of Bragg wavelength into strain measurement because it is affected by different parameters such as epoxy resin, temperature and structure’s material. This section describes applications of FBG technology in sensor developments.

4.10.1 FBG Strain Sensors

When a bare FBG is directly attached onto a surface of a specimen, it can only measure the surface strain changes. The strain difference may be influenced by thermal expansion of a specimen or creep and shrinkage of a concrete specimen. It cannot directly reflect the curvature of a specimen. Direct attachment onto a surface can be only suitable when used for a smooth surface of specimen and in a steady temperature environment.

As a result, an FBG strain sensor is designed, which a pre-tensioned bare FBG that is attached on a carrier or package. It can be directly mounted on the specimen by fastener, spot weld or epoxy. Then, the curvature of the specimen can be determined by the measured strain. Besides, it can make fiber handling easier and the sensor installation faster and more repeatable. As there is cross-sensitivity of strain and temperature, some designs include a free FBG for temperature discrimination.

4.10.2 FBG Inclination Sensors

Inclination sensors (also known as inclinometer, tilt sensors or clinometers) measure the inclination of objects with respect to gravity. Conventional inclination sensors are commonly electrical based. Most of them are based on measuring acceleration using Micro-electro-mechanical systems (MEMS). Each sensor requires an independent cable for output signal and a power supply of 5V to 12V. It is sometime impractical to be used for slope measurement for bridges because the distance between the sensor and demodulator are limited. Besides, they are not insulated from the EMI.

Conventional inclination sensors offer a measurement range of 360° and resolution of 0.003°. A high-end inclination sensor can achieve a resolution of 0.001° ("US Digital ", 2012). Although they have high resolutions, they are sensitive to vibration. To measure an accurate inclination, they need to take some time to wait for
the results to become steady. A damper such as magnetic damping may be provided to minimise the overshoot and oscillations during the sensor tilts. Using a digital filter can filter out the noise due to motion that can increase the accuracy and reduce the respond time. The settling times still require a few seconds. For a high end inclination sensor with a higher sampling rate, the settling time can be reduced to less than a second, but the cost of the sensor is higher.

As FBGs have widely been used for measurement in recent years, several inclination sensors equipped with FBGs have been reported in the literature to measure the inclination from horizontal in one-axis (Aneesh et al., 2011; Chen et al., 2008) and two-axis (Au et al., 2011; Bao et al., 2010a; Bao et al., 2010b; Bao et al., 2010c; Dong et al., 2005; Guan et al., 2004; He et al., 2010; Ni et al., 2010). These sensors are all pendulum based. When the sensors tilt, the masses inside the sensors induce a movement due to the gravity. Their configuration and performances, including sensitivity, accuracy, resolution and measurement range, are summarised in Table 4-1.

Guan et al. (2004) reported an FBG inclination sensor with a slope accuracy and resolution of better than 0.1° and 0.007 °. The basic concept is based on four FBGs incorporated in a vertical pendulum. The sensor is capable of detecting the magnitude as well as the direction of the inclination from the horizontal direction. As the sensor is independent to temperature and achieves all the outstanding advantages of FBG sensors, the sensor would largely enhance the application for vertical displacement measurements of bridges.

Dong et al. (2005) reported an FBG inclination sensor by measuring the reflected optical power from the chirped FBGs. The sensor uses a pre-deflection of steel flakes with a complicated pendulum suspension mechanism. When the sensor is tilted, a non-uniform strain is induced on the surface of the steel flakes that results in a chirp in the period of the originally-uniform FBG. As the measurement is based on the reflected optical power, the intensity loss of reflected power causes an uncertainty of accuracy when using the sensor over a long distance.
### Table 4-1 Summary of the FBG inclination sensors

<table>
<thead>
<tr>
<th>Author</th>
<th>Sensors</th>
<th>Sensing system</th>
<th>Sensitivity pm/deg</th>
<th>Accuracy</th>
<th>Resolution</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Guan et al., 2004)</td>
<td>4 FBGs for 2axis</td>
<td>Interrogator</td>
<td>75</td>
<td>0.1°</td>
<td>0.007°</td>
<td>-3° to +9°</td>
</tr>
<tr>
<td>(Dong et al., 2005)</td>
<td>3 chirped FBG for 2 axis</td>
<td>OSA^2 and OPM^3</td>
<td>-</td>
<td>0.13°</td>
<td>0.02°</td>
<td>±5°</td>
</tr>
<tr>
<td>(Chen et al., 2008)</td>
<td>1 FBG for 1 axis</td>
<td>OSA</td>
<td>60</td>
<td>0.167°</td>
<td>0.0067°</td>
<td>±15°</td>
</tr>
<tr>
<td>(Ni et al., 2010)</td>
<td>4 FBGs for 2 axis</td>
<td>Interrogator</td>
<td>192</td>
<td>0.1°</td>
<td>0.005°</td>
<td>±12°</td>
</tr>
<tr>
<td>(He et al., 2010)</td>
<td>3 FBGs for 2 axis</td>
<td>Interrogator</td>
<td>53.7</td>
<td>15pm</td>
<td>0.009°</td>
<td>±10°</td>
</tr>
<tr>
<td>(Bao et al., 2010a)</td>
<td>2 FBGs for 2 axis</td>
<td>OSA and OPM</td>
<td>97.1 &amp; 89.9</td>
<td>0.125°</td>
<td>0.027°</td>
<td>±4°</td>
</tr>
<tr>
<td>(Bao et al., 2010b)</td>
<td>2 FBGs for 2 axis</td>
<td>OSA</td>
<td>54 or 46</td>
<td>0.27°</td>
<td>0.19°</td>
<td>0° to 2°</td>
</tr>
<tr>
<td>(Bao et al., 2010c)</td>
<td>4 FBGs for 2 axis</td>
<td>Interrogator</td>
<td>96, reduce to 80</td>
<td>0.2°</td>
<td>0.013°</td>
<td>±40°</td>
</tr>
<tr>
<td>(Au et al., 2011)</td>
<td>4 FBGs for 2 axis</td>
<td>Interrogator</td>
<td>39.5</td>
<td>0.051°</td>
<td>0.013°</td>
<td>±30°</td>
</tr>
<tr>
<td>(Aneesh et al., 2011)</td>
<td>2 FBGs for 1axis</td>
<td>Interrogator</td>
<td>62.6</td>
<td>0.36°</td>
<td>0.008°</td>
<td>±10°</td>
</tr>
</tbody>
</table>

^2 Optical spectrum analyser

^3 Optical power meter
Chen et al. (2008) reported a single axis FBG inclination sensor that is based on gravity force change while the sensor is tilted. The friction between the mass and the case causes instability of accuracy and repeatability.

He et al. (2010) and Ni et al. (2010) reported inclination sensors based on three and four FBGs in series on a single fiber, making an inverted pyramidal structure with three and four fiber arms and a mass suspended from the vertex, respectively. They have high sensitivities. However, as the mass hangs directly on the fibers, any unwanted add-on oscillation is expected to lead to a random vibration/oscillation of the fiber-mass pendulum system of both the sensor designs. This is bound to give measurement errors and makes these sensors highly unstable. Also, there is a possibility of slacking of one fiber arm in the plane of inclination during the tilt.

Three similar designs of FBG inclination sensors are reported (Bao et al., 2010a; Bao et al., 2010b; Bao et al., 2010c) that are based on a cantilever based pendulum suspension mechanism. The FBGs are directly attached on the cantilevers that have a mass hanging at the free end. The FBGs are not easy fragile. They all have a high measurement range but the sensitivities are limited.

Au et al. (Au et al., 2011) reported another inclination sensor based on four FBGs in the horizontal plane. The sensor has a high measurement range for over \( \pm 30^\circ \). However, the mass is directly hanged at the FBGs that are highly sensitive to impact force that may damage the sensor.

Most of the reported FBG inclination sensors are based on pendulum suspension mechanics. The sensor amplifies the small force induced by the inclination of the pendulum to a greater axial force into the FBGs. A high sensitivity results in a high resolution. The sensitivity can be increased by increasing and reducing the lever arm of the mass and FBG, respectively, from the joint. However, the high sensitivity limits the measurement range because the induced force of the FBG exceeds the ultimate strength of the FBG.

Many FBG inclination sensors have been reported in the literature but their performances for field applications such as vibration response, oscillation effect and resonance have not been reported. These sensors are limited for static measurements.
Their performance cannot compete against conventional sensors, which have much higher resolutions and accuracies. These increase the uncertainty of the application of the FBG inclination sensors for bridges. So the FBG inclination sensors are not commonly used in the field.

However, FBG offers a lot of outstanding advantages that are suitable to use for measurements in bridges. This prompts the need to develop a novel FBG inclination sensor, which can compete against conventional sensors, and also helps in applying the approaches proposed in Chapter 3 for the vertical displacement measurements of bridges. The requirements of a desired FBG inclination sensor for bridges could be described as follows:

- High accuracy and resolution
- High sensitivity
- Measurement range within ±2° for serviceability limit state
- Fast response
- Low frequency vibration (static or quasi-static)

According to the Bridge Design of Australian Standard Part 2 design loads <AS 5100.2-2004>, the deflection for the serviceability limit state of a road bridge shall be not greater than 1/600 of the span. For a simply supported bridge, the allowable slope shall be less than 0.4°. Therefore measurement range within ±2° can meet the allowable slope change for the serviceability limit state of road bridges. The objective of the slope measurement in this thesis is to measure the slope of bridges due to displacement. This slope change is usually in static or quasi-static and the amplitude is measurable using inclination sensors. The change of slope due to ambient vibration is in high frequency and the amplitude is very small. Accelerometers are commonly used to determine the slope using acceleration measurements, but they can only be applied for high frequency conditions.

### 4.11 CURVATURE MEASUREMENT USING FBG SENSORS

The Bragg wavelength shifts of FBGs are cross-sensitivity of strain change $\Delta \varepsilon$ and temperature change $\Delta T$ (Meltz et al., 1988), given by
\[
\Delta \lambda / \lambda = (1 - p_e) \Delta \varepsilon + (\alpha_f + \xi) \Delta T
\]  

(4-6)

where \( \Delta \lambda \) is the Bragg wavelength shift, \( \lambda \) is the strain-free Bragg wavelength, \( p_e \) is the photoelastic coefficient of fiber, \( \alpha_f \) is the coefficient of thermal expansion (CTE) of fiber and \( \xi \) is the thermo-optic coefficient. If the FBG is bonded onto a structure, the thermal expansion of the structure material causes a change in the grating period. Assuming the CTE of structure is much larger than the CTE of fiber, the value of \( \alpha_f \) should be replaced with \((1 - p_e)\alpha_s \) where \( \alpha_s \) is the CTE of the structure. Then, Eq. 4-6 becomes (Magne et al., 1997)

\[
\Delta \lambda / \lambda = (1 - p_e) \Delta \varepsilon + [(1 - p_e)\alpha_s + \xi] \Delta T
\]  

(4-7)

Hence, the relationship of the total Bragg wavelength, strain and temperature is given by

\[
\Delta \varepsilon = [\Delta \lambda - \lambda K_T \Delta T]/[\lambda (1 - p_e)]
\]  

(4-8)

where \( K_T = (1 - p_e)\alpha_s + \xi \). Substituting it into Eq. 3-8, the curvature is given by

\[
\kappa_l = \frac{((\Delta \lambda - \lambda K_T \Delta T)/[\lambda (1 - p_e)])^b - ((\Delta \lambda - \lambda K_T \Delta T)/[\lambda (1 - p_e)])^t}{h}
\]  

(4-9)

Assuming the temperature variations of the FBGs are of the same amount and \( \lambda^b \approx \lambda^t \), the equation is simplified as

\[
\kappa_l = \frac{(\Delta \lambda/\lambda (1 - p_e))^b - (\Delta \lambda/\lambda (1 - p_e))^t}{h}
\]  

(4-10)

As \( \Delta \lambda \ll \lambda \) and \( \lambda^b \approx \lambda^t \), the curvature can be described in wavelength separation \( \Delta (\lambda^b - \lambda^t) \) as

\[
\kappa_l = \frac{\Delta (\lambda^b - \lambda^t)}{h(1 - p_e)}
\]  

(4-11)

A pair of FBG strain sensors at the corresponding longitudinal location is proposed for curvature measurement, which eliminates the problem of interference amongst strain and temperature.
Although an FBG is sensitive to strain, it is difficult to directly apply a bare FBG sensor in infrastructure without any protection due to fragility of fibers. Commercial FBG strain sensors with carriers are designed for easy handling and fast installation.

Their measurements on the surface of structure include the strain induced by deformation and surface crack. For vertical displacement measurement of bridges, only the strain induced by deformation is concerned for determining the curvature of bridges. Hence, an FBG curvature sensor would be designed for vertical displacement measurement. It is noted that the strain due to surface crack should be eliminated. Additionally, the temperature effect could be eliminated by using additional free FBG sensors for temperature compensation.

The curvature sensor’s design is based on FBG sensors embedded on a beam structure. It can either be embedded in or attached to the bridges. The local strains of the FBG sensors are converted to curvature of the bridge.

### 4.12 DEVELOPMENT OF FBG INCLINATION SENSOR

The degree of inclination of bridges is correlated to the displacements of the bridges. Many of the inclination sensors have been developed; they are commonly used in construction and flight controls. They generate an artificial horizon and measure slope with respect to the horizon. Their common mechanisms include accelerometers and pendulum based. The accelerometer based inclination sensors are commonly used micro-electromechanical systems (MEMS). They determine the slopes by measuring components of the gravitational acceleration of the three axes.

Certain high sensitivity MEMS inclination sensors can achieve an accuracy of 0.001° but the sensors are limited to the temperature drift, sensitivity, repeatability and electromagnetic interference. They also require temperature compensation, digital filtering for noise and damping effect, and power supply. Hence, they are not outdoor friendly. As FBG technology can overcome the EMI and the power supply problem, an FBG inclination sensor is desired to measure slope of bridges. In practical applications for bridges, the requirements of the FBG inclination sensors
are not only accuracy and resolution, but are also stability and repeatability for the long term applications.

Although several FBG inclination sensors have been published (Aneesh et al., 2011; Au et al., 2011; Bao et al., 2010a; Bao et al., 2010b; Bao et al., 2010c; Chen et al., 2008; Dong et al., 2005; Guan et al., 2004; He et al., 2010; Ni et al., 2010) and commercialized in the market, they are only applied for static measurements. Their performance under vibration have not been studied. As there is a lack of FBG inclination sensors that fulfil the requirements for the slope measurements of bridges, five FBG inclination sensors are self-developed for the vertical displacement measurement experiments, which will be described in Chapter 6. For a high sensitivity sensor applying to slope measurement in bridges, the accuracy of the sensor depends very much on the friction of the bearing joint. Low friction needle bearing universal joints are used in these five sensors in order to reduce errors caused by the friction of the joints. They are tested for their performance in static measurement.

4.12.1 Theory

As the allowable deflections of bridges for the serviceability limit state are much smaller towards the spans of bridges, the inclination variation of bridges is very small. Hence, a high sensitivity, resolution and precision inclination sensor is required for vertical displacement measurement of bridges. For the purpose of vertical displacement measurements along the span in a beam-like bridge structure such as slab-on-girder and box-girder bridges, the sensors are considered for the axis along the span slope measurement. A new FBG inclination sensor is proposed, as described in Figure 4-6.

The design of the sensor is pendulum based. The joint is a key component of the sensor. Since the friction of the joint used is significant to the sensor performance, especially the accuracy, a high precision, low friction and low backlash joint is required. In the markets, joints or bearings are designed to use in motion. A minor friction is not a design concern for the general applications, because it does not affect their performance much while in motion. However, the required joint for this sensor
is almost in stationary because the FBGs keep the equilibrium of the mass. Several bearings and joints have been investigated. Finally, a universal joint with a needle roller bearing is proposed to be used for the sensor. The joint is fitted with pre-lubricated needle bearings, and uses small cylindrical rollers to reduce friction of a rotating surface.

One end of the joint is embedded at the top of a steel frame. The other end is bonded into a steel rod with two arms. The other end of the rod is bonded with a vertical pendulum. Two FBGs that are pre-strained equally are glued to the two arms at one end and the frame at another end.

![Figure 4-6  Proposed FBG inclination sensor](image)

The sensor amplifies the small force induced by the inclination of the pendulum to a greater axial force into the FBGs. As applying an impact force to the sensor may break the FBGs, a movement stopper is assigned inside the steel frame to prevent the free movement of the pendulum while transporting and handling the sensor. In addition, it can control the vertical pendulum movement within the ultimate inclination range while an over-inclination is accidentally applied to the sensor.
When the sensor is inclined about the x-axis, the axial strain is induced to the FBGs to keep the vertical pendulum in equilibrium.

Taking moment at the joint,

\[ AE(\varepsilon_0 + \Delta\varepsilon_1)a - AE(\varepsilon_0 + \Delta\varepsilon_2)a = mgl \sin \theta \]  \hspace{1cm} (4-1)

where \( A \) and \( E \) are the cross section area and elastic modulus of the fibers, \( \varepsilon_0 \) is the pre-tension strain, \( m \) is the mass of the vertical pendulum, \( g \) is gravitational acceleration, \( a \) is the length of the arms, \( l \) is the length of the vertical pendulum, respectively. The equation is then be simplified to

\[ \Delta\varepsilon_1 - \Delta\varepsilon_2 = \frac{mgl}{AEa} \sin \theta \]  \hspace{1cm} (4-2)

Substituting Eq. 4-8 into Eq. 4-2,

\[ \frac{\Delta\lambda_1 - \lambda_1 K_T \Delta T}{\lambda_1} - \frac{\Delta\lambda_2 - \lambda_2 K_T \Delta T}{\lambda_2} = \frac{mgl}{AEa} (1 - p_e) \sin \theta \]  \hspace{1cm} (4-3)

Assuming the temperature variations of the FBGs are of the same amount, \( \Delta \lambda_i \ll \lambda_i \) and \( \lambda_1 \approx \lambda_2 \), the equation of the wavelength separation is given by

\[ \Delta(\lambda_1 - \lambda_2) = \frac{mgl}{AEa} (1 - p_e) \lambda_1 \sin \theta \]  \hspace{1cm} (4-4)

Due to the small variation of inclinations, \( \sin \theta \approx \theta \). The slope is linear proportional to the wavelength separation and can be expressed as

\[ \theta = \Delta(\lambda_1 - \lambda_2) / \frac{mgl (1 - p_e) \lambda_1}{AEa} \]  \hspace{1cm} (4-5)

where \( \frac{mgl (1 - p_e) \lambda_1}{AEa} \) is the sensitivity of the sensor. Since the slope is encoded by the wavelength separation between two FBGs, the problem of cross-sensitivity of temperature and strain is eliminated because the wavelength shifts induced by temperature of the FBGs are of the same amount due to the same temperature sensitivity. Hence, the wavelength is separated between two FBGs that are temperature independent.
4.12.2 Fabrication of the FBG Inclination Sensors

Five FBG inclination sensors, IS1 to IS5, were constructed and tested for the purpose of slope measurements of the beam tests in Chapter 6. Due to the available resources, the sensors are fabricated with different dimensions. The frames of the sensors were made using steel grade C45 with hardening and anti-corrosive treatment or stainless steel. The mass was made using brass or stainless steel.

The fibers used for writing FBGs are standard single mode telecommunication fiber \( E = 7.2E10 \text{ N/m}^2, A = 1.23 \times 10^{-8} \text{ m}^2, \rho_e \approx 0.22 \) with high bend resistance. Before writing the FBGs, the fibers were hydrogen-loaded (Figure 4-7) at 80°C for three days at 100 bar in order to increase the photosensitivity of the fiber cores. The coating of each fiber for FBG writing was stripped along 10mm using an automatic mechanical stripper. The FBGs were written by exposing the UV laser, using the phase mask method. The grating lengths are 6mm. The FBGs were annealed at 100 °C for 24 hours in order to remove the un-reacted hydrogen on the fiber. The initial wavelengths of the sensors are listed in Table 4-2.

Before mounting the FBGs to the sensors, the coatings at the gluing location of the fiber were stripped, in order to increase the bonding strength between the sensor and the FBGs. Firstly, one side of the FBGs were bonded to the steel frame with epoxy resin. After setting, the FBGs were tensioned using interrogator to monitor the actual strain induced (Figure 4-8) that is equal to about 2000µε (corresponds to 2.4 nm of wavelength shift from the strain-free state). Then, the other side of the FBGs were boned to the pendulum arms with epoxy resin.

<table>
<thead>
<tr>
<th>Table 4-2 Initial wavelength of the FBG inclination sensors</th>
</tr>
</thead>
<tbody>
<tr>
<td>FBG inclination sensors</td>
</tr>
<tr>
<td>IS1</td>
</tr>
<tr>
<td>Initial wavelengths (nm) ( \lambda_1 )</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
</tr>
</tbody>
</table>
Figure 4-7   Hydrogen loading process at 100 bar and 80°C
4.12.3 Testing and Calibration

The sensor was calibrated on a tilt platform that inclined along the x-axis with interval of 0.5° from 0° to +2°, then decreasing to -2° and returning to 0°. The slope was monitored by a digital inclination sensor with resolution of 0.003°. The wavelength shifts of the FBGs were measured with an interrogator. The objective of the tests is to determine the sensitivity, accuracy and resolution of the sensors. Each sensor was tested three times. The sensitivities of the sensors are determined by averaging the sensitivity found in each test.

The calibration test results for the FBG inclination sensor, IS1, are plotted in Figure 4-10. It is noted that when the direction of the platform changes, an approximate 0.3° backlash occurs due to the gear of the tilt platform. However, it does not affect the calibration test, as the slope of the platform is measured by the reference inclination sensor.
Figure 4-9  Program controlled tilt stage for testing the inclination sensors

Figure 4-10  Measured wavelength separation $\Delta(\lambda_1 - \lambda_2)$ of IS1 versus slope along x-axis
From Figure 4-10, it can be seen that the sensitivities in three tests are about 386pm/°. The coefficient of determination $R^2$ is 0.9999, so the repeatability of the sensor is very high. The discrepancy of the measured slope encoded from the wavelength separation is less than ±0.006°, as shown in Figure 4-11. The resolution of slope measurement depends on the wavelength resolution of the FBG interrogator. For an interrogator with a resolution of 0.5pm, the inclination sensor resolution is 0.001°.

The results of the other FBG inclination sensors, IS2 to IS5 are plotted in Figure 4-12 to Figure 4-15. The sensitivities, accuracies and resolutions of the FBG inclination sensors are summarised in Table 4-3.
Figure 4-12  Results of the FBG inclination sensors, IS2

Figure 4-13  Results of the FBG inclination sensors, IS3
Figure 4-14  Results of the FBG inclination sensors, IS4

Figure 4-15  Results of the FBG inclination sensors, IS5
### Table 4-3 Results of the FBG inclination sensors

<table>
<thead>
<tr>
<th>FBG Inclination Sensors</th>
<th>IS1</th>
<th>IS2</th>
<th>IS3</th>
<th>IS4</th>
<th>IS5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity (pm/deg)</td>
<td>386.2</td>
<td>463.2</td>
<td>1152.4</td>
<td>609.2</td>
<td>164.1</td>
</tr>
<tr>
<td>Resolution (deg)</td>
<td>0.00129</td>
<td>0.00108</td>
<td>0.00043</td>
<td>0.00082</td>
<td>0.00305</td>
</tr>
<tr>
<td>Ultimate Measurement Range (deg)</td>
<td>±6</td>
<td>±5</td>
<td>±2</td>
<td>±4</td>
<td>±15</td>
</tr>
<tr>
<td>Accuracy (deg)</td>
<td>0.006</td>
<td>0.045</td>
<td>0.06</td>
<td>0.143</td>
<td>0.03</td>
</tr>
<tr>
<td>Accuracy (% F.S)</td>
<td>0.09</td>
<td>0.83</td>
<td>2.77</td>
<td>3.48</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Comparing the sensors reported in Section 4.10.2, the results show that these five sensors have much better performances including sensitivity, accuracy and resolution. The performances can also compete against conventional inclination sensors.

The resolutions of the sensors depend on the sensitivity. From the result of the sensor IS3, it is observed that high resolution does not cause high accuracy, because the accuracy does not only depend on the resolution but also the mechanics’ design of the sensors, such as the machining tolerance of the components, alignment of the grating and friction of the joint.
4.13 DEVELOPMENT OF A NOVEL FRICTIONLESS FBG INCLINATION SENSOR WITH EXTREMELY HIGH SENSITIVITY AND RESOLUTION

The accuracy of an inclination sensor depends on its resolution, the friction of its joint and its repeatability. The resolution of the sensor is controlled by the resolution of interrogator and sensitivity. When a high resolution of interrogator and high sensitivity of sensor are used, the resolution of the sensor is increased. To increase the accuracy and resolution of the sensor, it is important to have a high sensitivity that depends on the mechanics of the sensor structure. For a pendulum based sensor, increasing the length of the arm and the weight of the mass as well as reducing the FBG lever arm can increase the sensitivity of the sensor as the tensile stresses of the FBGs are increased. However, it is limited by the strength of the optical fiber of the FBGs, because the fibers are under a tensile stress to keep the equilibrium of the mass. It also reduces the measurement range. For slope measurement of bridges, the measurement range is relatively small.

For a high sensitivity sensor applying for slope measurement in bridges, the accuracy of the sensor depends very much on the friction of the joint. The smaller the friction, the more accurate it will be. Although a low friction needle bearing universal joint is proposed to be used in the previous sensors described in Section 4.12, the friction of any mechanical joint cannot be completely eliminated. In order to eliminate the friction in the sensor, a foil steel is proposed to replace the bearing joint. A novel, frictionless FBG inclination sensor with extremely high sensitivity and resolution is proposed. To enhance the sensitivity of the sensor and maintain a suitable measurement range of \( \pm 2^\circ \), a high tensile strength coated fiber, such as polyimide coated fiber, is suggested for use. The schematic representation of the inclination sensor is shown in Figure 4-16.

Based on the experience gained from the previous sensor fabrication, the fiber can easily be broken at the splicing location, while assembling the FBGs into the sensor due to handling. Extreme care is required to prevent the breaking of the FBGs. To reduce the chance of their breaking, the two FBGs were fabricated in a single
fiber without splicing. The positions of FBGs in the sensors are changed into linearly horizon in order to keep the pre-strain force identical.

The FBGs of the sensor were written by acrylate coated fibers. The sensor, TS1, was tested three times, similar to the above testing procedures described in Section 4.12.3. The initial wavelengths of the FBGs are 1565.4 nm and 1568.5 nm. The sensor was placed on the tilt platform that inclined within ±1°. The slope was monitored by a digital inclination sensor with resolution of 0.003°. The wavelength shifts of the FBGs were measured with an interrogator. The test results are shown in Figure 4-17 and Figure 4-18. The average sensitivity is 3271pm/° and the linearity is very high where $R^2$ is 0.9999. The resolution is 0.00015° for an interrogator with a resolution of 0.5pm. The sensitivity and resolution of the developed sensor are much better than the sensors reviewed previously (Section 4.10.2).
4.13.1 Vibration Response

The inclination sensor is a pendulum spring system that is sensitive to vibration. When the inclination sensor is subject to a slope change, the net restoring force is set up in the body, which tends to bring the pendulum back to equilibrium position.
However, during the course of the pendulum moving backward, momentum is acquired that keeps them moving beyond the original position. Then, it establishes a new restoring force that causes an oscillation.

To implement the real time slope measurement, the vibration response of the high sensitivity inclination sensors is studied. The inclination sensor is described as a pendulum spring system, as shown in Figure 4-19.

![Diagram of the inclination sensor](image)

Figure 4-19   Mathematics model of the inclination sensor

When the inclination sensor is tilted, the pendulum is subjected to an inertia force $F_i$ and a restoring force $F_R$ due to gravity $G$ that will accelerate it back towards to the equilibrium position and is given by

$$F_R = -G \sin \theta = -mg \sin \theta$$  \hspace{1cm} (4-6)

$$F_i = ma = m\dot{\theta}$$ \hspace{1cm} (4-7)
where $m$ is the mass of the pendulum, $g$ is the acceleration due to gravity, $\theta$ is the slope from vertical and $\dot{\theta}$ is angular acceleration.

For a pendulum supported by a massless, inextensible hanging arm, the pendulum can move only on a circle with radius $L$. Take the moment at joint O, where the Newtonian equation of the motion that describes the model is given by

$$mL^2\ddot{\theta} + c\theta(L_2 \cos \theta)^2 + kL_1 \sin \theta(L_1 \cos \theta) + mgL \sin \theta = 0$$

(4-8)

where $L$ is the distance from the joint to the center of mass, $c$ is the viscous damping constant, $k$ is stiffness of the sensor system, $L_1$ is distance from the joint to the FBGs, $L_2$ is the distance from the joint to the insertion of the damping force and $mL^2\ddot{\theta}$ is the inertial moment of the system $c\theta(L_2 \cos \theta)^2$ is the moment due to damping, $kL_1 \sin \theta(L_1 \cos \theta)$ is the restoring torque due to the elastic properties of the FBGs and $mgL \sin \theta$ is the restoring torque due to gravity.

Since the amplitude of oscillation is small, $\sin \theta \approx \theta, \cos \theta \approx 1$. The equation can be simplified through a linear approximation as

$$mL^2\ddot{\theta} + cL_2^2 \dot{\theta} + kL_1^2 \theta + mgL \theta = 0$$

(4-9)

The natural frequency of oscillation $\omega_0$ of this system is determined by its inertial and stiffness characteristics. Dividing by $mL^2$, the equation is given by

$$\ddot{\theta} + \frac{cL_2^2}{mL^2} \dot{\theta} + \frac{kL_1^2 + mgl}{mL^2} \theta = 0$$

(4-10)

where the natural angular frequency of oscillation $\omega_0$ and the damping coefficient $\gamma$ are

$$\omega_0 = \sqrt{\frac{kL_1^2 + mgl}{mL^2}}$$

(4-11)

$$\gamma = \frac{cL_2^2}{mL^2}$$

(4-12)

The equation can then be simplified as a second order differential equation as
\[ \dot{\theta} + \gamma \dot{\theta} + \omega_0^2 \theta = 0 \]  \hspace{1cm} (4-13)

Critical damping occurs when the damping coefficient \( \gamma \) is equal to \( 2 \omega_0 \). The damping ratio \( \zeta \) is defined as the ratio of damping coefficient \( \gamma \) to the critical damping constant as

\[ \zeta = \frac{\gamma}{2\omega_0} \]  \hspace{1cm} (4-14)

The equation then has the form

\[ \ddot{\theta} + 2\zeta \omega_0 \dot{\theta} + \omega_0^2 \theta = 0 \]  \hspace{1cm} (4-15)

The general solution of Eq. (4-15) is

\[ \theta(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t} \]  \hspace{1cm} (4-16)

where \( A \) and \( B \) are constants, and \( \lambda_1 \) and \( \lambda_2 \) are the two roots of the quadratic equation created by substitution of a trial solution \( e^{\lambda t} \) into Eq. (4-15).

\[ \lambda^2 + 2\zeta \omega_0 \lambda + \omega_0^2 = 0 \]  \hspace{1cm} (4-17)

\[ \lambda = -\zeta \omega_0 \pm \sqrt{\left(\zeta \omega_0\right)^2 - \omega_0^2} \]  \hspace{1cm} (4-18)

or

\[ \lambda = \omega_0 (-\zeta \pm \zeta^2 - 1) \]  \hspace{1cm} (4-19)

The solutions have three distinct cases as described as Figure 4-20.
1. Overdamped case: When $\zeta > 1$, the roots are real and distinct. The general solution is given by

$$\theta = Ae^{\lambda_1 t} + Be^{\lambda_2 t} \quad (4-20)$$

2. Critical damped: When $\zeta = 1$, the roots are real and equal. The general solution is given by

$$\theta = (A + Bt)e^{\zeta \omega_0 t} \quad (4-21)$$

3. Underdamped: When $\zeta < 1$, the roots are complex numbers given by

$$\lambda = -\zeta \omega_0 \pm i \omega_d \quad (4-22)$$

where the damped angular frequency, $\omega_d$ is

$$\omega_d = \omega_0 \sqrt{1 - \zeta^2} \quad (4-23)$$

The general solution is given by

$$\theta(t) = a_1 e^{\lambda_1 t} + a_2 e^{\lambda_2 t} \quad (4-24)$$
where $a_1$ and $a_2$ are complex numbers. Substitution Eq. (4-23) into Eq. (4-24),

$$\theta(t) = e^{-\zeta \omega_0 t} (a_1 e^{i\omega t} + a_2 e^{-i\omega_0 t})$$  \hspace{1cm} (4-25)

Using the Euler’s formula $e^{ix} = \cos x + i \sin x$ and $e^{-ix} = \cos x - i \sin x$,

$$\theta(t) = e^{-\zeta \omega_0 t} [(a_1 + a_2) \cos(\omega_0 t) + (a_1 - a_2) j \sin(\omega_0 t)]$$ \hspace{1cm} (4-26)

Let the real numbers $A_2 = (a_1 + a_2)$ and $A_1 = (a_1 - a_2)j$, the solution becomes

$$\theta(t) = e^{-\zeta \omega_0 t} [A_1 \cos(\omega_0 t) + A_2 \sin (\omega_0 t)]$$ \hspace{1cm} (4-27)

Then, defining the constant $A = \sqrt{A_1^2 + A_2^2}$ and the angle $\phi = \tan^{-1}(A_2/A_1)$, thus $A_1 = A \cos \phi$ and $A_2 = A \sin \phi$, the solution becomes

$$\theta(t) = e^{-\zeta \omega_0 t} [A \cos \phi \cos(\omega_0 t) + A \sin \phi \sin (\omega_0 t)]$$ \hspace{1cm} (4-28)

Recalling the trigonometric relation $\cos \alpha \cos \beta + \sin \alpha \sin \beta = \cos(\alpha - \beta)$, the solution becomes

$$\theta(t) = A e^{-\zeta \omega_0 t} \cos (\omega_0 t - \phi)$$ \hspace{1cm} (4-29)

where $A$ and $\phi$ are the constants of integration to be determined from initial conditions.

### 4.13.2 Measurement

The mathematics model includes the mass, stiffness and damping ratio of the system. The mass and stiffness is determined by simple static measurements, while the damping ratio requires a dynamic measurement. The displacement response of an underdamped system is used to determine the damping ratio. The exponentially
The decaying amplitude of the oscillation is called envelope as shown in Figure 4-21. The equation is

\[ \theta(t) = Ae^{-\xi \omega_d t} = Ae^{(-\gamma/2)t} \tag{4-30} \]

Figure 4-21  Envelope of a underdamped oscillation

To understand the response of the system, a simple measurement can be implemented to find \( \zeta \) and \( \omega_d \). By the relationship of damping ratio and damped angular frequency in Eq. 4-14 and Eq. 4-23, the damping ratio is expressed as

\[ \zeta = \frac{\gamma/2}{\sqrt{\omega_d^2 + (\gamma/2)^2}} \tag{4-31} \]

where \( \omega_d = \frac{2\pi}{T_d} \). \( T_d \) is time of a oscillation period. The \( \gamma/2 \) can be solved by curve fitting using an exponential model.
4.13.3 Oscillation Effect

As mentioned before, the oscillation occurs when the inclination sensor is tilted (Figure 4-22). As the joint has a much small friction, the settling time was 125 seconds from 1 degree to 0.001 degree.

![Oscillation of the sensor](image)

Figure 4-22 Oscillation of the sensor

To reduce the oscillation effect, a viscous damper was applied to the inclination sensor. A viscosity of 3500cst shock oil was added to the sensor at a different level from the bottom. To measure the settling time, a quick tilt angle change was applied that causes a force excitation and a subsequent of free vibration oscillation. Using the free vibration oscillation data, the damping ratio is then determined. The oscillations are shown in Figure 4-23 and the results are summarised in Table 4-4.
The settling time is decreased when the damping oil was filled up higher. For the angle of sensor changing from 0.1° and settling at 0.001°, the settling time is 2.9 seconds when the damping oil is filled to 10mm from the bottom of the mass. It is reduced about 42 times compared with the settling time of the undamped case. The response time is suitable for static measurements. To reduce the settling time, it can increase the damping ratio by increasing the oil level in the sensor and replacing a
higher viscosity of damping oil. To reach the critical damping, the oil level is estimated to fill up to 100mm from the bottom of the mass.

It is noted that the settling time also depends on the interval of tilt angle change. A larger tilt angle change causes a greater momentum force to the mass that results in a longer oscillation time to settle. For the damping oil added 10mm from the bottom of the mass, the settling time is 2.9 seconds when the angle of sensor changes from 0.1° and settling at 0.001°. However, the settling time is 0.29 seconds when the angle of sensor changes from 0.001° to 0.002°. To further reduce the settling time, a digital filter is suggested as shown in Section 4.13.5.

4.13.4 Resonance

Ambient vibration from the environment cannot be avoided in a real application. Although its small amplitude should not affect the slope change of bridges, resonance will occur when an ambient vibration occurs to the sensor at the same natural frequency of the sensor. If resonance occurs, the response amplitude is enlarged, causing error to the sensor data. Thus, the natural frequency of the sensor should be kept away from the ambient vibration. The measurement frequency range under vibration will be limited to prevent resonance in order to maintain the accuracy of the sensor. Besides, the enlarged force may damage the fiber due to an unexpected increase of the amplitude of oscillation. To test the performance of the sensor under vibration, a vibration test was conducted where specific frequencies of tilt angle changes were applied to the sensor. A tilt angle stage was set up with a shaker as shown in Figure 4-24 and Figure 4-25. The left end of the stage is a pin support and the right end is a roller that allows the stage with free motion vertically. When the shaker shakes vertically, the stage tilts. The tilt angle of the stage was measured by a laser displacement sensor with an accuracy of 0.0001mm.
The procedures of the test are as follows:

1. The laser displacement sensor is fully calibrated by a static displacement test for reference. The measured displacements from the laser sensor are converted to tilt angle of the tilt stage.
2. Apply an oscillation of 0.1 Hz to the stage. As the peak-to-peak of tilt angle is measured, the magnitude of the oscillation is not required to be exact. Typically, the magnitude of the oscillation is close to the measurement range limit of the laser sensor. Also, monitoring the responses of FBGs are required to prevent the force exceeding the ultimate strength of the FBGs.

3. Record the peak-peak data of the laser displacement sensor and the FBG inclination sensor.

4. Repeat steps 3 and 4, applying oscillation of the other frequencies from 0.2 to 4 with interval of 0.1 Hz incrementally, then 5 Hz, 10 Hz, 20 Hz and 30 Hz.

5. Convert the laser displacement data to slope of the tilt stage and the FBG inclination sensor data to slope.

6. The transmissibility is the calculated by

   \[
   \text{Transmissibility} = \frac{\text{peak to peak slope response of the FBG inclination sensor}}{\text{peak to peak slope of the tilt stage}}
   \]

The result of the undamped case is shown in Figure 4-26. From the graph, it can be seen that the resonance occurs at 2.8 Hz. To reduce the settling time and the oscillation effect, viscous damping oil was added to the sensor. The damping oil was added in different levels of 0, 3, 5 and 10 mm from the bottom of the mass. The results are shown in Figure 4-27, which shows the transmissibility is linear within 1 Hz. So the available measurement range in dynamic is 1 Hz for the sensor with damping oil filled to 10 mm. To increase the measurement range in dynamic, the resonance response should be reduced and the natural frequency of the sensor should be increased. To reduce resonance response, it can easily increase the damping ratio by increasing the oil level in the sensor and replacing a higher viscosity of damping oil. It can reduce the mass to reduce resonance response, but the sensitivity would be reduced. To reach the critical damping, the oil level is estimated to be 100 mm from the bottom of the mass, as reported in the test results of Section 4.13.3.
Figure 4-26  Transmissibility of the undamped FBG inclination sensor

Figure 4-27  Transmissibility of the undamped and damped FBG inclination sensor
4.13.5 Finite Impulse Response (FIR) Filter

If an impact or a high frequency force is applied to the sensor, the force will transmit as a white noise to the sensor. The noise includes various frequencies. The response and their fast Fourier transform (FFT) are shown in Figure 4-28 and Figure 4-29.

As mentioned before, the available measurement range in dynamic is within 1Hz while the damping oil is filled up to 10mm. Hence, if the response is with frequency higher than 1Hz, it can be considered as a noise that can be filtered out by applying a digital filter.

![Figure 4-28 Response of the sensor while applying three impact to the sensor](image_url)
Finite Impulse Response (FIR) filter is used because the signal settles to zero in finite time. In order to filter out a frequency higher than 1Hz, a low pass filter with a cutoff frequency of 1 Hz is chosen. In this filter, the magnitude is set as 1 when frequency is less than 1Hz and the magnitude is set as 0 when frequency is higher than 1Hz. The response and the FFT of sensor when applying FIR filter are shown in Figure 4-30. The signals with frequency higher than 1 Hz, including the resonance frequency and noise, are filtered out. Hence, the noise from an impact can be eliminated.
As mentioned before, the settling time can be reduced by applying a damper in order to increase the damping factor. That can reduce the oscillation magnitude resulting lesser settling time. Besides, the oscillation can also be filtered out by the FIR filter. When the sensor is subjected to a quick tilt angle change, a force excitation and a free oscillation are occurred. By applying the FIR filter, the frequency of the free oscillation is detected and filtered out. For example, a quick tilt angle of $0.68\,^\circ$ change is applied to the sensor using an automated tilt stage. The results are shown in Figure 4-31. The response time of original data and filtered data are 2.8 seconds and 1.3 seconds. It is reduced by more than 50%. It has been demonstrated that an FIR filter can improve the performance of the sensor including reducing settling time and eliminating the noise from ambient vibration and impact.
4.14 CONCLUDING REMARKS

As conventional sensors could not meet the requirements for the proposed approaches, FBG sensors should be used to implement the two approaches proposed in Chapter 3 for the vertical displacement measurements of bridges. This chapter first describes the FBG sensing technology, including the principle, sensing system, fabrication methods, stripping methods, stripping methods, adhesive, hydrogen loading and influence of exposure parameters on the mechanical strength of FBGs. The characteristics of the FBG sensing and types of FBG sensors that could be used for applying the two proposed approaches are described. The curvature measurements of bridges using FBG sensors are illustrated.

For the proposed vertical displacement measurements methods for bridges, FBG inclination sensors are required to fulfill the requirements for the slope measurements including EMI immunity, outdoor user friendly, as well as high accuracy and resolution. As there is a lack of available FBG inclination sensors in the market for the vertical displacement measurement of bridges, five FBG inclination sensors were fabricated. They were tested in static tests. The performances of the sensors were reported including sensitivity, accuracies and resolutions.
For a high sensitivity sensor, the accuracy of the sensor depends very much on the friction of the joint for slope measurement in bridges. Although low friction needle bearing universal joints were used in these five sensors the friction of any mechanical joint cannot be completely eliminated. A foil steel was then proposed to replace the bearing joint. A novel frictionless FBG inclination sensor with extremely high sensitivity and resolution was developed and tested for static measurements. The static test results showed the sensor to have very high sensitivity, resolution, accuracy and linearity. As the sensor is sensitive to vibration, oscillation effect occurs when the settling time is increased. The resonance may cause error to the measurement data and damage fibers in the sensor. A viscous damper using a viscosity of 3500cst damping oil was employed in the sensor to reduce the oscillation due to ambient vibration. The settling time and resonance of the sensor with damping oil were determined by the dynamic test. To eliminate the error due to an ambient frequency, a finite impulse response (FIR) is a digital filter that was proposed to filter the unwanted frequency of response. The filter can also filter out the frequency of free oscillation where the inclination is settling from a quick tilt angle change. The results showed that the noise from ambient vibration and impact can be eliminated and the settling time can be reduced by 50%.
Chapter 5: Numerical Simulation Tests of Vertical Displacement Measurement

5.1 INTRODUCTION

This chapter describes a series of numerical simulation tests to study the implementation of the curvature and inclination approaches. Section 5.2 describes the details of a full scale bridge for the simulation tests, while Section 5.3 describes implementation of the approaches for bridges with various support conditions. Section 5.4 describes implementation of the self-compensation capacity. Section 5.5 describes implementation for the bridges with various flexural rigidities along the span. Section 5.6 describes implementation for the bridges under non-uniformly distributed loads. Section 5.7 describes a further study where a noise of 10% was added into the data. Concluding remarks are given in Section 5.8.

5.2 MODEL SET UP

A full-scale bridge model with a span of 27.4 m is set up as shown in Figure 5-1. A finite element model for structural analysis is established by MATLAB. The model is divided into 100 elements. Each element size is 274mm.

![Figure 5-1 Description of the bridge model](image)

A self-developed MATLAB code as a mass program is written for the tests. The mass program can modify the parameters such as support condition, loading,
noise ratio, sensor location and number of sensors used for simulating the various cases. The data for vertical displacement, slope and curvature are extracted from the sub-program for matrix structural analysis, for which the source code was written by Rahami (2010) and provided in Appendix A. The mass program also implements the proposed methods to determine the vertical displacements that are then compared to the simulated displacement. The flow of the program is illustrated in Figure 5-2.

In order to implement the approaches, data from nine curvature and nine inclination sensors at 2.74 m centre-to-centre along the span were generated by the finite element method. The deflection shape functions were then determined by these approaches respectively. The curvature and slope functions were selected as second and third order polynomials, respectively. The assumption of zero displacement at
both the supports was applied as a boundary condition. The deflection shape function is given in Eq. 3-21.

The bridge model is simply supported with nine curvature and nine inclination sensors along the span. The results are given in Figure 5-3, where k0, s0 and d0 are the curvature, slope and vertical displacement, respectively, determined for reference. As well, k3 and s2 are the simulated curvature and slope data that are used for implementing the curvature and inclination approaches, respectively. The curvature (K3) and slope (S2) curves that are determined by the curvature and slope approaches respectively are identical to the references (k0 and s0). The curves are exactly retrieved. The assumptions of the second order polynomial for the curvature function and the third order polynomial for the slope function have been verified. The vertical displacement (D3 and D2) curves are then determined and they are identical to the references (d0). It is then proven that both approaches could successfully retrieve the vertical displacement functions.

![Curvature, slope and vertical displacement](image)

Figure 5-3 Curvature, slope and vertical displacement without noise

The curvature and slope measurements in the field usually contain measurement noise. Therefore, a noise with 5% noise-to-signal-ratio was added to the generated input data using the ‘awgn’ function in MATLAB ("MATLAB," 2009) to simulate measurement noise interferences experienced during experimental testing.

In the field, bridges have different conditions such as supports, loading and flexural rigidities (EI). Twelve cases are simulated for these different conditions of bridges, as described in Table 5-1. The bridge models of cases 1 to 3 are illustrated.
with various support conditions, in order to verify the feasibility of the approaches for bridges with different support conditions. In cases 4 to 8, various numbers of sensors are selectively deactivated to simulate faulty sensors in order to verify the self-compensation capacity of the approaches. In cases 9 and 10, the flexural rigidities \((EI)\) of the bridge models vary along the spans, to verify the feasibility of the approaches for various cross sections along the spans. In cases 11 and 12, non-uniformly distributed loads are applied, with the aim of verifying the feasibility of the approaches under different loadings.

The discrepancies in any of the vertical displacements, \(\Delta v\) are given by

\[
\Delta v = |v - d0|
\]

(5-1)

where \(v\) is the vertical displacement determined by one of the proposed methods. The percentage discrepancy, \(\Delta v\%\) is given by

\[
\Delta v\% = \left| \frac{v - d0}{v} \right| \times 100\%
\]

(5-2)

The maximum discrepancies of the vertical displacements and their corresponding percentage discrepancies are summarised in Table 5-2, for all the different cases considered.
### Table 5-1  Simulation cases

<table>
<thead>
<tr>
<th>Cases</th>
<th>Sensors</th>
<th>Support conditions</th>
<th>Flexural rigidities (EI)</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>Simply supported</td>
<td>Constant</td>
<td>UDL</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>Fixed (left); roller (right)</td>
<td>Constant</td>
<td>UDL</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>Both Fixed</td>
<td>Constant</td>
<td>UDL</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>Simply supported</td>
<td>Constant</td>
<td>UDL</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>Simply supported</td>
<td>Constant</td>
<td>UDL</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>Simply supported</td>
<td>Constant</td>
<td>UDL</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>Simply supported</td>
<td>Constant</td>
<td>UDL</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>Simply supported</td>
<td>Constant</td>
<td>UDL</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cases</th>
<th>Sensors</th>
<th>Support conditions</th>
<th>Flexural rigidities (EI)</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>9</td>
<td>Simply supported</td>
<td>Varying</td>
<td>UDL</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>Simply supported</td>
<td>Varying</td>
<td>UDL</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cases</th>
<th>Sensors</th>
<th>Support conditions</th>
<th>Flexural rigidities (EI)</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>9</td>
<td>Simply supported</td>
<td>Constant tapered</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>Simply supported</td>
<td>Constant partially</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5-2  Maximum discrepancies of the vertical displacements between both of the approaches and the finite element model

<table>
<thead>
<tr>
<th>Case</th>
<th>Curvature Approach</th>
<th>Inclination approach</th>
<th>Curvature Approach</th>
<th>Inclination approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5% error mm (%)</td>
<td>10% error mm (%)</td>
<td>5% error mm (%)</td>
<td>10% error mm (%)</td>
</tr>
<tr>
<td>1</td>
<td>0.43 (2.9)</td>
<td>0.72 (4.4)</td>
<td>0.37 (2.3)</td>
<td>1.45 (10.8)</td>
</tr>
<tr>
<td>2</td>
<td>0.23 (3.5)</td>
<td>0.31 (4.6)</td>
<td>0.20 (3.4)</td>
<td>0.63 (10.1)</td>
</tr>
<tr>
<td>3</td>
<td>0.14 (5.0)</td>
<td>0.16 (4.9)</td>
<td>0.12 (3.7)</td>
<td>0.32 (13.8)</td>
</tr>
<tr>
<td>4</td>
<td>0.29 (2.4)</td>
<td>0.72 (5.1)</td>
<td>0.35 (2.6)</td>
<td>0.37 (5.9)</td>
</tr>
<tr>
<td>5</td>
<td>0.32 (3.0)</td>
<td>0.35 (2.1)</td>
<td>0.34 (2.4)</td>
<td>0.72 (4.4)</td>
</tr>
<tr>
<td>6</td>
<td>0.11 (1.0)</td>
<td>0.43 (4.2)</td>
<td>0.24 (1.9)</td>
<td>1.36 (8.2)</td>
</tr>
<tr>
<td>7</td>
<td>0.36 (2.3)</td>
<td>0.40 (2.5)</td>
<td>0.23 (2.1)</td>
<td>1.75 (14.9)</td>
</tr>
<tr>
<td>8</td>
<td>0.25 (1.5)</td>
<td>0.55 (3.3)</td>
<td>0.33 (2.0)</td>
<td>0.52 (5.5)</td>
</tr>
<tr>
<td>9</td>
<td>0.59 (3.4)</td>
<td>0.75 (4.3)</td>
<td>0.25 (1.5)</td>
<td>1.54 (11.5)</td>
</tr>
<tr>
<td>10</td>
<td>0.34 (2.6)</td>
<td>0.63 (4.1)</td>
<td>0.42 (2.6)</td>
<td>1.35 (10.5)</td>
</tr>
<tr>
<td>11</td>
<td>0.40 (2.6)</td>
<td>0.72 (4.4)</td>
<td>0.36 (2.2)</td>
<td>1.45 (10.9)</td>
</tr>
<tr>
<td>12</td>
<td>0.34 (5.1)</td>
<td>0.39 (5.1)</td>
<td>0.31 (4.0)</td>
<td>0.75 (10.7)</td>
</tr>
</tbody>
</table>
5.3 VARIOUS SUPPORT CONDITIONS

Since these approaches are based on the geometry of the span of beam type bridges, relationships between vertical displacement, slope and curvature, support conditions do not contribute to the methods except for settling up boundary conditions. The deflected shapes with various support conditions are directly reflected into the curvature and slope measurements. In cases 1 to 3, these tests aim to verify the feasibility of these approaches for bridges with various support conditions. These bridge models have pinned-roller, fixed-roller and fixed-fixed supports, respectively, as shown in Figure 5-4. The curvature, slope and vertical displacement curves of these cases are plotted in Figure 5-5. The discrepancies of the curvature and inclination approaches are less than 0.43mm (2.9%) and 0.72mm (4.4%), respectively. It is demonstrated that both the approaches can be implemented to determine vertical displacements of bridges with various support conditions of a single span. As the approach can work for these supports, it can also be implemented for bridges with continuous supports.
Figure 5-4  Model description of cases 1 to 3 (a) simply supported; (b) fixed-roller supported and (c) fixed-fixed supported, respectively
Figure 5-5 Curvature, slope and vertical displacement of cases 1 to 3

5.4 SELF-COMPENSATION CAPACITY

For these bridge models, both of the approaches require only three sensors to retrieve the curvature and slope functions. Hence, if some sensors are faulty, the redundant sensor data can compensate the data loss. In cases 4 to 8, some curvature and inclination sensors are neglected in the simulation to depict faulty sensors in order to verify the self-compensation capacity.
Chapter 5: Numerical Simulation Tests of Vertical Displacement Measurement

(a) Case 4

(b) Case 5

(c) Case 6
Their curvature, slope and vertical displacement curves, with 5% noise, are shown in Figure 5-6. The discrepancies of the vertical displacements using the curvature and inclination approaches are less than 0.36mm (2.3%) and 0.72mm (5.1%), respectively.

Although only three curvature and inclination sensors would theoretically retrieve the curvature and slope functions, it is suggested that five or more sensors distributed along the span are used in order to provide enough information to exactly retrieve the curvature and slope functions.
5.5 DIFFERENT FLEXURAL RIGIDITIES ALONG THE SPAN

The tests in Cases 9 and 10 are to study the implementation of the approaches for bridges with varying flexural rigidities along the spans.

\[ EI' = \alpha \times EI \]

where \( \alpha \) is the reduction coefficient of the flexural rigidity. The vertical displacement curves are shown in Figure 5-8. The discrepancies of the curvature and inclination approaches are less than 0.59mm (3.4%) and 0.75mm (4.3%), respectively. It is
demonstrated that both the approaches are feasible to be implemented for bridges with varying flexural rigidities along the spans.

![Curvature, slope and vertical displacement of cases 9 and 10 with 5% noise](image)

(a) Case 9

(b) Case 10

Figure 5-8 Curvature, slope and vertical displacement of cases 9 and 10 with 5% noise

### 5.6 NON-UNIFORMLY DISTRIBUTED LOADS

In cases 11 and 12, the tests study the implementation of the approaches for bridges under a tapered distributed loading and a uniform load partially distributed, as described in Figure 5-9. Their curvature, slope and vertical displacement curves are shown in Figure 5-10. The discrepancies in the curvature and inclination approaches are less than 0.40mm (2.6%) and 0.72mm (4.4%), respectively. These
tests therefore have demonstrated that both the approaches are feasible to be implemented for bridges under various loading.

Figure 5-9  Loading descriptions of (a) case 11 and (b) case 12
Further studies were conducted for a noise of 10% in the measurements. The results of all cases are plotted in Figure 5-11. The discrepancies of the curvature and inclination approaches are less than 0.42mm and 1.75mm, respectively. For the curvature approach, although a higher noise was applied to the simulated data, the discrepancies do not significantly increase compared with those with a noise of 5%. It is because some parts of noise are filtered out when the curvature curve is retrieved. For the inclination approach, the maximum discrepancies are higher than those with a noise of 5%. It is concluded that the performance of the curvature approach is better than the inclination approach, even if a noise of 10% occurred in the measurements.

Figure 5-10  Curvature, slope and vertical displacement of cases 11 and 12 where noise of 5%

5.7 MEASUREMENT NOISE OF 10%

Further studies were conducted for a noise of 10% in the measurements. The results of all cases are plotted in Figure 5-11. The discrepancies of the curvature and inclination approaches are less than 0.42mm and 1.75mm, respectively. For the curvature approach, although a higher noise was applied to the simulated data, the discrepancies do not significantly increase compared with those with a noise of 5%. It is because some parts of noise are filtered out when the curvature curve is retrieved. For the inclination approach, the maximum discrepancies are higher than those with a noise of 5%. It is concluded that the performance of the curvature approach is better than the inclination approach, even if a noise of 10% occurred in the measurements.
Chapter 5: Numerical Simulation Tests of Vertical Displacement Measurement

(a) Case 1

(b) Case 2

(c) Case 3

(d) Case 4
(e) Case 5

(f) Case 6

(g) Case 7

(h) Case 8
Figure 5-11  Curvature, slope and vertical displacement of cases 1 to 12 with 10% noise
5.8 CONCLUDING REMARKS

In this study, the curvature and inclination approaches for vertical displacement measurements of bridges have been demonstrated for various cases using numerical simulation tests.

The bridge models of cases 1 to 3 have different support conditions, to verify the feasibility of the approaches for bridges with various support conditions. In most situations, the boundary conditions can be considered zero displacement at both the end supports. It has been demonstrated that the theory of the approaches is independent to the support conditions.

In cases 4 to 8, various numbers of sensors are selectively deactivated to simulate faulty sensors to verify the self-compensation capacity of the approaches. These approaches can theoretically use only three curvature and inclination sensors to determine the vertical displacements. If more than three sensors are used, these approaches have a self-compensation capacity that has been demonstrated in the simulation tests. If one or more sensors are faulty, other sensors can compensate the data loss. It is recommended that five or more sensors be used along the spans in order to provide enough data to retrieve the curvature and slope functions.

In cases 9 and 10, the flexural rigidities ($EI$) of the bridge models vary along the spans, in order to verify the feasibility of the approaches for various cross sections along the spans. Since the approaches are derived from the geometric relationships amongst vertical displacement, slope and curvature; applied loads and bridge structural properties such as size and elastic modulus are not required in the measurements. In other words, the approaches do not require any prior knowledge of loading and material properties. It is demonstrated that the approaches can be implemented for bridges that have varying cross sections along their spans.

In cases 11 and 12, the tests study the implementation of the approaches for bridges under a tapered distributed loading and a uniform load partially distributed. It is demonstrated that both the approaches are feasible to be implemented for bridges under these types of loading.
The results of the simulated tests show that the errors using the curvature and inclination approaches are below 0.59mm (3.4%) and 0.75mm (4.3%), respectively, with a noise of 5% added. When a noise of 10% is added, the results of curvature and inclination approaches are 0.42mm (2.6%) and 1.75mm (14.9%), respectively. The curvature approach has a good performance even if a higher noise has been applied. Hence, if both of the approaches are implemented for the bridge, it is suggested to use the results of the curvature approach for further applications.

The two proposed approaches can be implemented individually to determine vertical displacements. The choice of the particular approaches should be considered based on the budget, access of bridges and installation methods.

For the installation method of the sensors, the inclination sensors are only simply placed on the bridge directly or mounted on the surface of the bridge which is much easier than the installation of curvature sensors and this enables a reduction in installation time. It is because the curvature sensors are required to be glued or mounted on the surface of the bridge.

The inclination is an absolute parameter with respect to the ground or earth. When an inclination sensor is faulty, the replacement of the sensor is very easy. On the other hand, the curvature measurement requires at least two sensors at the corresponding longitudinal location to measure the mean curvature to eliminate the influence of neutral axis shift. If the sensors are installed on the surface of structures, the strain concentration may affect the results due to local surface crack. Also, the uneven temperature expansion may cause abnormal strain measurement.

As FBG curvature sensors have been commercialised, the cost of the curvature sensors would be less than the cost of FBG inclination sensors. However, the cost of sensors depends on the number of sensors required, which in turn depends on the type of bridges.

Implementing the curvature approach for a box-girder bridge, several curvature sensors may be required to be installed in the section of bridges at each longitudinal location in order to ensure the accuracy of curvature measurement. It is because the section of box-girder bridges is usually large. A pair of sensors may be not enough to
provide an accurate curvature measurement. For implementing the inclination approach, only one inclination sensor is required to be placed at each longitudinal location. If the inside box of a box-girder bridge can be accessed, the sensors can be placed inside the box.

For a slab-on-girder bridge, each girder would be considered as a beam and hence the approaches are required to be implemented individually for each girder. For implementing the curvature approach, only a pair of curvature sensors is required at each longitudinal location. Comparing the cost of a pair of FBG curvature sensors and an FBG inclination sensor, the curvature approach should be adopted for slab-on-girder bridges.
Chapter 6: Experimental Validation

6.1 INTRODUCTION

The proposed vertical displacement measurement methods, using the curvature and inclination approaches, have been demonstrated by the simulation tests in Chapter 5. In the simulation tests, noise of 5 and 10 % were added to the data in order to simulate a field situation. In order to further study the performance of the proposed methods when using FBG strain and inclination sensors in a real life situation, a beam model was set up for validation of the proposed vertical displacement measurement methods, as described in Section 6.2. Two loading tests were conducted for vertical displacement measurement using FBG strain and inclination sensors. Regression analyses for retrieving the curvature and slope curves using the measurements were conducted in order to demonstrate the approaches that can retrieve the optimum order of polynomial without prior knowledge of the loading condition. A beam test with increasing loads at the mid-span is described in Section 6.3. A beam test with different loading locations along the span is described in Section 6.4. Concluding remarks are given in Section 6.5.

6.2 EXPERIMENTAL SET UP

A 3.9 metre long aluminium beam model was set up as shown in Figure 6-1. The beam is a 100mm wide and 50mm deep rectangular hollow section and is simply supported at the ends. Two rigid supports are used as the beam supports to eliminate the influence of floor vibration. A yoke with two rollers was designed and fabricated in order to move the loading with ease (Figure 6-2). Nine FBG strain sensors in an array were directly attached on top of the surface and another array with nine FBG strain sensors attached underneath in the same longitudinal locations. Due to the budget, only five FBG inclination sensors were assembled for the test. They were located on the top along the span.
Figure 6-1  Beam set up
The fibers used for writing FBGs are standard single mode telecommunication fibers with high bending resistance. Before writing the FBGs, the fibers were hydrogen-loaded at 80°C for three days at 100bar in order to increase the photosensitivity of the fiber cores. The coating of each fiber for FBG writing was stripped along 10mm. The FBGs were written by exposing the UV laser using the phase mask method (Figure 6-3). The grating lengths are 6mm. The FBGs were annealed at 100 °C for 24 hours in order to remove the un-reacted hydrogen on the fibers.
Nine pairs of FBGs for strain measurement were written using the same phase mask. Their initial Bragg wavelengths are between 1519.3 and 1566.5 nm, as listed as Table 6-1. The nine pairs of FBGs are spliced into two arrays that were then glued on the top and underneath surface of the beam (Figure 6-4).

Table 6-1  Initial wavelengths of the FBG strain sensors attached on the beam surface

<table>
<thead>
<tr>
<th>Location (mm)</th>
<th>350</th>
<th>750</th>
<th>1150</th>
<th>1550</th>
<th>1950</th>
<th>2350</th>
<th>2750</th>
<th>3150</th>
<th>3550</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor number</td>
<td>T1</td>
<td>T2</td>
<td>T3</td>
<td>T4</td>
<td>T5</td>
<td>T6</td>
<td>T7</td>
<td>T8</td>
<td>T9</td>
</tr>
<tr>
<td>Wavelength of the top sensors (nm)</td>
<td>1565.8</td>
<td>1554.1</td>
<td>1549.5</td>
<td>1528.8</td>
<td>1523.8</td>
<td>1543.1</td>
<td>1533.8</td>
<td>1537.8</td>
<td>1519.3</td>
</tr>
<tr>
<td>Sensor number</td>
<td>B1</td>
<td>B2</td>
<td>B3</td>
<td>B4</td>
<td>B5</td>
<td>B6</td>
<td>B7</td>
<td>B8</td>
<td>B9</td>
</tr>
<tr>
<td>Wavelength of the bottom sensors (nm)</td>
<td>1566.5</td>
<td>1553.9</td>
<td>1548.8</td>
<td>1529.1</td>
<td>1523.9</td>
<td>1543.9</td>
<td>1534.1</td>
<td>1539.0</td>
<td>1518.7</td>
</tr>
</tbody>
</table>

Figure 6-4  FBG strain sensor on the beam surface (a) before glue and (b) glued
The Bragg wavelengths of the sensors were measured by a 1 kHz interrogator with 1000 sample averaging. The top (T1 to T9) and bottom (B1 to B9) arrays of FBG strain sensors used channel one and two of the interrogator. The FBG inclination sensors IS1 to IS6 were coupled by patch cords in series, using channels 3 and 4 of the interrogator. The initial wavelengths and sensitivities of the FBG inclination sensors are listed as Table 6-2. Four dial gauges with a resolution of 0.01mm were placed on the beam for comparison of results. The procedures used in capturing the measurements are given in Figure 6-5.

Table 6-2  Sensitivities of the FBG inclination sensors

<table>
<thead>
<tr>
<th>Location (mm)</th>
<th>IS1</th>
<th>IS2</th>
<th>IS3</th>
<th>IS4</th>
<th>IS5</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>1536.5</td>
<td>1562.3</td>
<td>1568.1</td>
<td>1525.8</td>
<td>1543.7</td>
</tr>
<tr>
<td>550</td>
<td>1548.7</td>
<td>1564.6</td>
<td>1570.5</td>
<td>1530.8</td>
<td>1558.0</td>
</tr>
<tr>
<td>1750</td>
<td>386.4</td>
<td>463.4</td>
<td>1133.8</td>
<td>608.2</td>
<td>164.2</td>
</tr>
<tr>
<td>3350</td>
<td>3350</td>
<td>3350</td>
<td>3350</td>
<td>3350</td>
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</table>

Table 6-2  Sensitivities of the FBG inclination sensors

<table>
<thead>
<tr>
<th>FBG inclination sensors</th>
<th>IS1</th>
<th>IS2</th>
<th>IS3</th>
<th>IS4</th>
<th>IS5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location (mm)</td>
<td>150</td>
<td>550</td>
<td>1750</td>
<td>3350</td>
<td>3750</td>
</tr>
<tr>
<td>Initial wavelengths (nm)</td>
<td>1536.5</td>
<td>1562.3</td>
<td>1568.1</td>
<td>1525.8</td>
<td>1543.7</td>
</tr>
<tr>
<td>Sensitivity (pm/deg)</td>
<td>386.4</td>
<td>463.4</td>
<td>1133.8</td>
<td>608.2</td>
<td>164.2</td>
</tr>
</tbody>
</table>
6.3 INCREASING LOAD APPLIED AT THE MID-SPAN

According to the Australian Bridge design code, Part 2 design loads <AS 5100.2-2004>, the deflection for serviceability limit state of a road bridge shall be not greater than 1/600 of the span. For this test set up, the limit of deflection for serviceability limit state is 6.5mm. In order to investigate the performances of the approaches when different amounts of vertical displacements are induced below and above the deflection limit, the induced vertical displacement is designed to reach double the deflection limit. The corresponding loading to be added at mid span causing the limit deflection is estimated as 18kg. The loadings were increasingly applied at the mid-span, shown in Figure 6-6. The amounts of loading added on the beam in each step are listed in Table 6-3.
The curvature and slope of the beam were measured by encoding the wavelength shift of the FBGs. The curvature and inclination approaches are implemented to determine the vertical displacements respectively. The Bragg wavelength shift of the FBG strain sensors at the top and bottom surface of the beam under the increasing loading are plotted in Figure 6-7. The data are then converted to curvature using Eq. 4-11 as plotted in Figure 6-8.
Figure 6-7  Bragg wavelength shift of the FBG strain sensors under the increasing loading, (a) top and (b) bottom.
According to the curvature profile shown in Figure 6-8, the loading is obviously concentrated at the mid span and therefore the curvature curves should be linear. A regression analysis was conducted in order to demonstrate whether it could find the optimum polynomial for retrieving the curvature curve. First to fourth degree polynomials are used to fit the curvature data and their coefficients of determination, $R^2$ are listed in Table 6-4. The $R^2$ of the first degree polynomials in all loading steps are highest. This agrees with the observation from the curvature data. The retrieved curvature curves compared with the measurement and simulation are plotted in Figure 6-9.

### Table 6-4  Coefficients of determination of the curvature data fitting

<table>
<thead>
<tr>
<th>Degree of polynomial</th>
<th>1 (Left segment)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Left segment)</td>
<td>0.99953</td>
<td>0.99945</td>
<td>0.99930</td>
<td>0.99925</td>
<td>0.99910</td>
<td>0.99906</td>
<td></td>
</tr>
<tr>
<td>1 (Right segment)</td>
<td>0.99905</td>
<td>0.99913</td>
<td>0.99912</td>
<td>0.99920</td>
<td>0.99924</td>
<td>0.99926</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.93826</td>
<td>0.93733</td>
<td>0.93767</td>
<td>0.93672</td>
<td>0.93641</td>
<td>0.93618</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.93829</td>
<td>0.93736</td>
<td>0.93770</td>
<td>0.93675</td>
<td>0.93645</td>
<td>0.93621</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.98356</td>
<td>0.98288</td>
<td>0.98253</td>
<td>0.98193</td>
<td>0.98143</td>
<td>0.98133</td>
<td></td>
</tr>
</tbody>
</table>
Figure 6-9  Fitted curvature curves comparing with the measurement and theoretical results

The measured slope using the FBG inclination sensors are plotted in Figure 6-10. From the graph, it is not possible to determine the order of polynomials for retrieving the slope curve. A regression analysis was also conducted to find the optimum order of polynomials of the slope curves in all steps. First to fourth order polynomials are used to fit the slope data and the $R^2$ are listed in Table 6-5. The $R^2$ of the fourth order polynomials in all loading steps are highest. The fourth order polynomials are adopted to determine the vertical displacements. The retrieved slope curves compared with the measured slope data are plotted in Figure 6-11.
Figure 6-10  Measured slope along the beam

Table 6-5  Coefficients of determination of the slope data fitting

<table>
<thead>
<tr>
<th>Order of polynomial</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.99224</td>
<td>0.99104</td>
<td>0.99086</td>
<td>0.99168</td>
<td>0.99099</td>
<td>0.99155</td>
</tr>
<tr>
<td>2</td>
<td>0.99322</td>
<td>0.99166</td>
<td>0.99127</td>
<td>0.99201</td>
<td>0.99177</td>
<td>0.99218</td>
</tr>
<tr>
<td>3</td>
<td>0.99982</td>
<td>0.99990</td>
<td>0.99979</td>
<td>0.99983</td>
<td>0.99984</td>
<td>0.99986</td>
</tr>
<tr>
<td>4</td>
<td>0.99985</td>
<td>0.99994</td>
<td>0.99995</td>
<td>0.99993</td>
<td>0.99999</td>
<td>0.99999</td>
</tr>
</tbody>
</table>
The vertical displacements are then determined by the approaches respectively. The results are plotted in Figure 6-12, where the solid curves are the determined vertical displacements, the markers are the displacement reading of dial gauges and dash curves are the theoretical vertical displacements.

The maximum differences and the corresponding percentage of difference of the vertical displacements determined by the curvature measurements compared to dial gauge readings and theoretical results are 0.14mm (1.13%) and 0.05mm (0.37%), as shown in Figure 6-13 and Figure 6-14. Those by the slope measurements are 0.41mm (3.35%) and 0.32mm (2.64%). The results showed that both of the approaches have been successfully implemented to determine the vertical displacements with high accuracies.

In the literature, Kim et al. (2004) and Chung et al. (2008) conducted a similar test using curvature measurements. Their accuracies are 7.35% and 6%. The current study has been validated to measure vertical displacements by compared with the literature.
Figure 6-12  Measured vertical displacements using (a) curvature and (b) slope measurement compared with the dial gauges reading and theory
Figure 6-13  Differences between the measured vertical displacement, dial gauge readings and theoretical results; (a) by curvature measurement; (b) by slope measurement
Figure 6-14  Percentage differences between the measured vertical displacement, dial gauge readings and theoretical results; (a) by curvature measurement; (b) by slope measurement
6.4 LOADING AT DIFFERENT LOCATIONS ALONG THE SPAN

An 18 kg concentrated load was applied to the beam at 17 different locations along the span to investigate the performance of the approaches when a loading was applied at different locations along the span. The first loading was applied at 0.35m from the left support and moved to 3.55m in stages of 0.2m, as described in Figure 6-15. The configuration of the sensors and dial gauges are the same as those used in Section 6.3.

![Figure 6-15 Applying loading at different locations along the span](image)

The measured curvature and slope are plotted in Figure 6-16 and Figure 6-17, respectively. The vertical displacements are then determined by the approaches discussed earlier. The results are plotted in Figure 6-18. The differences and percentage differences of measured vertical displacement using the approach compared with the dial gauge readings are listed in Table 6-6. The maximum differences of the measured vertical displacements using the curvature and inclination approaches are 0.38mm and 0.74mm, respectively. Their corresponding percentage differences are 5.5% and 10.56%, respectively. When the loading was applied at mid-span, the maximum differences are 0.12mm (1.46%) and 0.16mm (1.87%), respectively. They are less than those where the load was applied close to the supports. This is because the response of the structure is higher when loaded at mid-span and consequently the differences between the measured values and those predicted by the approaches are smaller. The tests have demonstrated that both approaches can be implemented to determine the vertical displacements while the beam is under concentrated loads with different magnitudes.
Figure 6-16  Measured curvature along the span

Figure 6-17  Measured slope along the span
Figure 6-18  Vertical displacements determined by (a) curvature measurements and (b) slope measurements with load was applied at different locations along the span.
Table 6-6  Differences of the measured vertical displacement comparing with the dial gauge readings

<table>
<thead>
<tr>
<th>Location (mm)</th>
<th>950</th>
<th>1450</th>
<th>1950</th>
<th>2950</th>
<th>950</th>
<th>1450</th>
<th>1950</th>
<th>2950</th>
<th>950</th>
<th>1450</th>
<th>1950</th>
<th>2950</th>
<th>950</th>
<th>1450</th>
<th>1950</th>
<th>2950</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading @ (mm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>P@350</td>
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<td>0.26</td>
<td>0.25</td>
<td>0.03</td>
<td>0.01</td>
<td>-0.02</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-7.65</td>
<td>-7.70</td>
<td>-7.34</td>
<td>-1.36</td>
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<td>0.64</td>
<td>0.92</td>
<td>1.59</td>
</tr>
<tr>
<td>P@550</td>
<td>0.15</td>
<td>0.05</td>
<td>-0.04</td>
<td>-0.02</td>
<td>0.13</td>
<td>0.04</td>
<td>-0.05</td>
<td>-0.06</td>
<td>-3.28</td>
<td>-0.92</td>
<td>0.74</td>
<td>0.72</td>
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<td>-0.72</td>
<td>1.05</td>
<td>1.98</td>
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<tr>
<td>P@750</td>
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<td>0.15</td>
<td>0.03</td>
<td>-0.14</td>
<td>-0.17</td>
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<td>2.64</td>
<td>4.84</td>
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<td>-0.40</td>
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<td>3.99</td>
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<td>P@950</td>
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<td>-0.33</td>
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<td>-0.01</td>
<td>0.23</td>
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<td>0.01</td>
<td>-0.15</td>
<td>0.07</td>
<td>0.19</td>
<td>0.02</td>
<td>-0.22</td>
<td>0.12</td>
<td>-1.17</td>
<td>-0.08</td>
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<td>-1.67</td>
<td>-0.16</td>
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<td>-0.23</td>
<td>-0.21</td>
<td>0.02</td>
<td>0.10</td>
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<td>4.05</td>
<td>2.29</td>
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<td>2.01</td>
<td>-0.21</td>
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<td>-0.19</td>
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<td>0.27</td>
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<td>3.74</td>
<td>-3.17</td>
<td>-2.29</td>
<td>-3.09</td>
<td>-4.51</td>
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</tbody>
</table>

Max  
0.43  0.73  0.74  0.16  0.26  0.30  0.16  0.38  8.95  11.80  10.56  3.74  3.83  2.95  3.09  5.49
As the vertical displacement curves have been determined when the loading is applied at different locations along the span, deflection influence lines can be constructed using those vertical displacement curves. In this test, four deflection influence lines at dial gauge locations are constructed as shown in Figure 6-19, and the difference between them and the dial gauges is given in Figure 6-20. It is demonstrated that both the approaches can be implemented to determine the vertical displacements when a loading is applied along the span at different locations.
Figure 6-19  Deflection influence lines determined by (a) curvature measurements and (b) slope measurements
Figure 6-20 Differences in the deflection influence line between the (a) curvature measurements and (b) slope measurements and dial gauge readings.
Figure 6-21  Percentages of differences in the deflection influence line obtained from (a) curvature measurements & (b) slope measurements and the dial gauge readings
6.5 CONCLUDING REMARKS

A 3.9 metre long, simply supported aluminium beam model was set up. Nine pairs of FBG strain sensors were installed on top of the surface and underneath the beam for curvature measurements and five self-developed FBG inclination sensors were located on the beam for slope measurements. A static loading test with increasing loads and a loading beam test with different loading locations were conducted. In the static test, the maximum differences of the vertical displacements determined by curvature and slope measurements compared to the dial gauges are 0.14mm and 0.41mm, respectively. The corresponding percentages of the differences are 1.13 and 3.35 %, respectively. The test has demonstrated that both approaches can be implemented to determine the vertical displacement while the beam is under different amounts of concentrated loading.

For the beam test with loads at different locations, the results were represented as four influence lines for deflection at dial gauge locations and compared with dial gauge readings. The maximum differences of the vertical displacements determined by curvature and slope measurements compared with the dial gauge readings are 0.38mm and 0.74mm, respectively. The corresponding percentage differences are 5.5 and 10.6 %, respectively. When the loading was applied at mid-span, the maximum differences are 0.12mm (1.46%) and 0.16mm (1.87%), respectively. This test has demonstrated that both the approaches can be implemented to determine the vertical displacements when a loading is applied at different locations along the span.
Chapter 7: Conclusions

Vertical displacements are one of the most relevant parameters for structural health monitoring of bridges in both the short and long term. They can be used to evaluate performance via loading tests, and rate the bridge capacity. However, it is difficult to obtain vertical displacements precisely and practically. In response to the demand for providing feasible and effective solutions for vertical displacement measurements, this research program addresses the following topics:

- This thesis reviews the vertical displacement measurement methods including conventional, image based and indirect methods in the literature as well as the damage evaluation for bridges.

- It proposes methods that use curvature and inclination measurements for determining vertical displacements. The feasibility for applying these methods under different conditions such as loading, supports, varying flexural rigidities of structure and number of sensors has been verified. Thus, these methods can be used to measure vertical displacements for most of the beam type bridge structures, such as the slab-on-girder and box-girder bridges. Also, the theory is based on span geometry, and hence, support settlement and suspension bearing at the support do not affect the measurement.

- This study proposes to use FBG sensors for the measurements due to their distinct advantages such as multiplexing capability and immunity of electro-magnetic interference (EMI). This will overcome the problems of electrical based sensors used in the field, such as influenced by EMI, large amounts of wiring and acquirement processes. The FBG sensing technology, especially in terms of fabrication and practical application, has been investigated.
• Using FBG strain sensors for curvature measurement is studied. Using wavelength separation of a pair of FBGs is proposed for curvature measurement to eliminate the effect due to neutral axis shift.

• Five FBG inclination sensors were constructed and their static performances are reported. Based on the experience gained from the fabrication of these sensors, a novel frictionless FBG inclination sensor has been developed for bridges with extremely high sensitivity and resolution. Its performances under static conditions and under vibration are reported.

• The numerical verifications prove that these approaches can measure vertical displacements under various conditions, such as different support conditions, varying flexural rigidities along the spans, different loading, without any prior knowledge of the loading and structural parameters. The self-compensation capacity is also verified. When one or more sensors are faulty, the data from the other sensors can compensate the data loss.

• Experimental beam tests have been conducted to verify the approaches that can be implemented using FBG sensors.

The curvature and inclination approaches of vertical displacement measurement methods have shown that both can successfully be used to measure the vertical displacements of bridges. Typically, the curvature approach has a better performance compared to the inclination approach in term of accuracy. It is suggested that the results from the curvature approach be used when both approaches are available for the bridge. However, inclination sensors can be easily installed on bridges, e.g. placing on the deck of bridges enables a reduction in installation time. The inclination is an absolute parameter with respect to the ground or earth. When an inclination sensor is faulty, the replacement of the sensor is very easy. On the other hand, the curvature measurement requires at least two sensors at the corresponding longitudinal location to measure the mean curvature to eliminate the influence of neutral axis shift. If the sensors are installed on the surface of structures, the strain concentration may affect the results due to local surface crack. Also, the uneven
temperature expansion may cause abnormal strain measurement. For box-girder bridges, more curvature sensors may be required to obtain an accurate measurement.

Considering the cost, installation methods and the type of bridges, the inclination approach should be used for box-girder bridges and the curvature approach should be used for slab-on-girder bridges.

7.1 RECOMMENDATIONS FOR FURTHER RESEARCH

The proposed methods have been proved to be successful in measuring vertical displacements of bridges. A novel, frictionless FBG inclination has been developed and its performance has been established under both static conditions and under vibration. In order to apply the vertical displacement methods to bridges, additional research may be extended based on the achievements in this research. Recommendations for future work include the following:

- The proposed vertical displacement measurement methods are simple and practical to measure vertical displacements of bridges. As the FBG sensors are inexpensive and suitable for outdoor long-term usage, the sensors can be permanently installed on bridges. These methods can be used for developing a fully automatic, real time vertical displacement monitoring system for bridges. As the signals of sensors are optical, the raw data can be directly transferred to the office using the optical fiber cable without amplifier. The raw data can be analysed remotely and the results can be monitored in real time. If excessive deflection occurs while the bridge is in service, the bridge manager can take necessary action to prevent the bridge from failure causing casualties and economic loss.

- The methods proposed herein also measure the curvature and slope. They are all sensitive to damage. The change of slope and curvature between the undamaged and damaged structures are reliable indicators for locating damage. Studying how these parameters indicate the damage in real life applications can be an interesting topic. A displacement based damage detection (DBDD) technique can be further developed.
As these methods are based on the span of bridges, the support conditions would not affect the measurement. The geometry and support conditions are mainly designed in single span. Further study can be done in other structural forms, such as suspension bridges, to evaluate the feasibility of these methods for a more complex structure.

These methods can also be applied for identifying moving load. Identification of moving loads on bridges is an important inverse problem (Yu & Chan, 2007). It can be used to understand the interaction between the bridge and vehicles. Using vertical displacement measurements for identification of moving loads could be further investigated.

The vertical displacements can also be applied to determine the actual capacities of bridges via a loading test (Section 7.1.1). It is suggested that a standard procedure of loading tests be established for determining the actual capacities of bridges in the future.

7.1.1 Load Testing for Rating Bridges

The performance and structural capacity of a bridge may be reduced during the designed service life due to damage, corrosion, loss of section and material deterioration. Field measurements of a bridge involve assessments of section properties such as the actual member size, deterioration and the uncertainties of the position of internal components. These can be used for rating bridges.

The objective of a non-destructive loading test is to determine the load capacity for the load rating of a bridge. The performance and structural capacity of a bridge can be quantified by the loading test. When a controlled and predetermined load is applied to a bridge, the actual structural response can be evaluated. Results should take into account the rating of the bridges, which can be determined by comparison of the previous test results. The principle of the loading test is to compare the field measurement, such as load versus deflection or load versus strain, of a bridge, with its theoretical performance. When the actual deflection of a bridge subjected with a controlled load is measured, the capacity of the bridge can be inversely evaluated by
comparing the theoretical result. This will enable to estimate the capacity of the bridge to carry live loads.

Load testing can be classified as static load tests and dynamic load tests. AASHTO (2011) defines that a static load test is conducted using stationary loads to avoid bridge vibrations. The intensity and position of the load may be changed during the test. A dynamic load test is conducted with time-varying loads or moving loads that excite vibrations in the bridge. Dynamic tests may be performed to measure modes of vibration, frequencies, dynamic load allowance, and to obtain load history and stress ranges for fatigue evaluation.

There are mainly two types of non-destructive static load tests which are the proof load testing and performance load testing ("Standards Australia AS 5100.7," 2004). Both tests are based on the measurement of the bridge response to the vehicle loading.

Proof load testing involves loading the bridge incrementally, using a load close to the ultimate limit state where the bridge behaviour is within the linear-elastic range. The aim of the test is to proof the ability of the carrying capacity of the bridge. A satisfactory proof load test usually provides higher confidence in the load capacity than a calculated capacity (AASHTO, 2011). However, proof load tests are often considered as a high risk approach and may lead to a rapid deterioration and collapse (Mehrkar-Asl & Brookes, 1997). Proof load testing should not be used unless the bridge is fully instrumented to measure strains in steel and concrete, and rotation and deflection at critical positions (Mehrkar-Asl & Brookes, 1997).

A static performance loading test that is a serviceability limit state test, involves monitoring a structure using normal road or railway traffic loads, or specific vehicles loaded to pre-determined weights to determine specific responses, such as vertical and horizontal forces, deflection and strains, to assist in assessing load distribution, to identify weak or failed components and to understand the structural performance. Load versus deflection ratio can be used as an indicator to access the current performance of the bridge. The test may also be employed regularly to monitor the degradation of structural performance and assist in detecting defective
components. Comparing the load versus deflection ratio of the original design and
the current performance, the capacity of the bridge can be evaluated ("Standards
Australia AS 5100.7," 2004).

The proposed vertical displacement measurement approaches are applicable for
static load testing of a bridge. The measurements for load testing are usually strain,
vertical displacement and slope (2011). In the proposed methods, strain/curvature
and slope will be measured and the vertical displacement is determined by the
implementation of the approaches. An extra instrument for the vertical displacement
measurements is not required when the proposed methods are implemented. The cost
of the loading tests can be reduced.

Although load testing can determine the actual load carrying capacity of the
bridge, the tests are not suitable to be conducted when the cost of testing reaches or
exceeds the cost of bridge strengthening, and the test may be impractical because of
access difficulties or site traffic conditions. As mentioned in Chapter 2, existing
vertical displacement measurement methods are sometimes impractical because a
specialised operator is required or measurements can be affected by weather. For
these instances, the proposed methods are useful, as they are inexpensive and a
stationary reference is not required.

The proposed methods can measure the required parameters from the
beginning of the test to continuously monitor the responses of the bridge under
incremental loading in order to keep it under linear-elastic behaviour and limit
distress due to cracking or other physical damage.
Bibliography


Appendix

Appendix A
Matrix Structural Analysis

The MATLAB code of the stiffness method for structural analysis of two and three dimensional frame structure is given as follows,

4 Copyright (c) 2010, Hossein Rahami All rights reserved.
function [Q,V,R]=MSA(D)

m=D.m;n=D.n;N= zeros (12,12,m);S=zeros(6*n);PF=S(:,1);Q=zeros(12,m);Qf=Q;Ei=Q;

for i=1:m

H=D.Con(:,i);C=D.Coord(:,H(2));D.Coord(:,H(1));e=[6*H(1)-5:6*H(1),6*H(2)-5:6*H(2)];c=D.be(i);
[a,b,L]=cart2sph(C(1),C(3),C(2));ca=cos(a);sa=sin(a);cb=cos(b);sb=sin(b);cc=cos(c);sc=sin(c);

r=[1 0 0;0 0 1;0 1 0];T=kron(eye(4),r);
cos=2*t*[(6/L)^2+1];x=D.A(i,1)*L^2+y*D.(i,1)*l*i=p+D.Q(i)+D.X(i)+L^2*D.E(i);

K1=diag([(x(1,1))]);K2=[0 0 0 0 0 0 0 0];K3=diag([g,0,0,0,0,0,0,0]);K4=diag([g,0,0,0,0,0,0,0]);

K=D.E(i)/L^3*[K1 K2 -K1 K2 K3 -K3 K4 -K4 K1 K1 -K2 K2 -K2 K4 -K4 K3];

w=D.w(:,i);Qf=[w^2/2*[6*w/w 0 -w(3) w(1) w(2) 0 0 w(3) -w(2)]]*Qf*K*T*D.ST(e);

A=diag([0 -0.5 -0.5]);B(2,3)=1.5/L;B(3,2)=-1.5/L;W=diag([1,0,0]);Z=zeros(3);M=eye(12);p=4.6;q=10.12;

switch n^2+H(4)

case 0;B=2*B(3);M(:,[p,q])=[-B -B;W Z;B B Z W];

end

K=M*K;N(:,i)=K*T;S(:,e)=S(:,e)+T*N(:,j);Qf(:,j)=M*Qf;Pf(:,e)=Pf(:,e)+T*M(1,1);Qf(:,j)=Ei(:,i)=e;

end

V=1-D.Re[D.St];f= fnd(V);Vf=S(f,f)(D.Load(f)-Pf(f));R=reshape(S*Vf(:,1)+Pf,6,6);R(i)=0;V=V+D.St;

for i=1:m

Q(:,j)=N(:,j)*V(Ei(:,j))+Qf(:,j);

end