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CHAPTER 9
CO-LEARNING IN THE COLLABORATIVE MATHEMATICS CLASSROOM

Abstract. Co-learning agreements between researchers and practitioners have been suggested as a way of generating research findings that are useful to improving schools and informing classroom practice. In this chapter we describe how a co-learning agreement was developed as a basis for researching aspects of the learning of mathematics in the classroom of a teacher whose pedagogy emphasised collaborative problem-solving as a major vehicle for learning mathematics. A study of co-operative group work in mathematics found that the perceptions of high achieving students (in terms of the purpose and benefits of group work) were more in line with those of their teacher than those of low achieving students. In contrast, the research reported in this chapter of a collaborative mathematics classroom revealed no such differences between students. In using a co-learning approach both authors felt that they learnt more about the processes of collaborating in researching and learning mathematics than they would have done with either a more traditional 'data gathering' approach led by the researcher or as a piece of action research by the teacher. The chapter suggests that co-learning can be a useful way of collaborating and describes the benefits such an approach can offer when researching classroom processes.

INTRODUCTION

The benefits to learning of working in groups has been known for some time. In 1981 an influential meta-analysis by Johnson *et al* of more than 120 research studies indicated that group work in learning situations was considerably more effective than competitive or individualistic goal structures. Yet group work is not a panacea that can be applied without thought to the classroom. Research has suggested that the composition of the groups and the form of tasks the groups tackle are important factors in determining the quality of learning achieved through such group work (Barnes & Todd, 1974; Cohen, 1994). It is also vital to properly distinguish between different *forms* of group work. An especially crucial distinction is between *co-operative* group work and *collaborative* group work (see, for example, Damon and Phelps, 1989). It is usual to refer to *co-operative* group work as distinct principles and practices such as specific role assignments in a group, and goal-related accountability of both individuals and the group. Co-operative group work is primarily task-orientated with roles and goals assigned at the outset, often by someone outside the group such as the teacher. In contrast, *collaborative* group work involves students working jointly on the same problem at all times. Within a collaborative group, decisions are shared and the negotiation of roles and relationships constantly evolves. Such a collaborative approach to learning is linked to Vygotskian ideas such as situated cognition, scaffolding, and the zone of proximal development (Forman & Cazden, 1985).

A good deal is known about *co-operative* small group learning in mathematics (Good, Mulryan & McCaslin, 1992). Some of this research has indicated that *co-*

operative group work can *increase* the separation between high and low achieving students. This can happen because, for instance, certain individuals dominate the group by taking on a leadership role leaving other group members passive and perhaps only involved in low-level tasks (Good et al., 1992, 172-173 and 176-177). Much less is known about collaborative small group work in mathematics classrooms where decisions are shared and roles and relationships are shared (Lyle, 1996). One recent study of students' perceptions of *co-operative* small group work in mathematics (Mulryan, 1994) found that with *co-operative* group work, the perceptions of high achieving students (in terms of the purpose and benefits of group work) were more in line with those of their teacher than those of low achieving students. This is a likely contributor to *increasing* the gap between high and low achieving students. This chapter reports the findings of one component of an ethnographic study of the classroom practice of an experienced teacher of secondary mathematics whose pedagogy emphasises *collaborative* problem-solving as a major vehicle for learning mathematics (Edwards and Jones, 1999a). One aim of the research was to examine the perspectives of both high and low achieving students who had experienced *collaborative* group work in secondary mathematics for a considerable period of time to see whether there was a difference in their perceptions of working in such a way (Edwards and Jones, 1999b).

In what follows we describe how we developed a 'co-learning agreement' (Wagner 1997; Jaworski, 1999) between the secondary school classroom teacher and the university researcher as a way of approaching the research. What we found from analysing data from representative samples of secondary school students who had been taught using a collaborative small-group approach for varying lengths of time (from two to four years) was *no difference* between the perceptions of high attaining students and those of low attaining students. This is in direct contrast to Mulryan's findings with co-operative group work. This chapter concludes with our reflections on these findings and on using this co-learning approach to researching the collaborative classroom.

APPROACHES TO RESEARCHING THE COLLABORATIVE CLASSROOM: A CO-LEARNING AGREEMENT

Education research is, at times, subject to adverse comment and criticism. For example, Hargreaves has argued that education research "is poor value for money in terms of improving the quality of education in schools" (Hargreaves, 1996, 1). In the UK, two recent, government-sponsored reports, both came to somewhat similar adverse conclusions (see Tooley, 1998; Hillage, 1998), while in the US, effort has been put into determining directions for education research that might more directly improve the learning taking place in schools (NERPPB, 1999a/b). Education research is generally criticised for two main reasons. First, it is said that without the active participation of teachers, education research cannot generate findings, and perhaps even research questions, that are useful to improving schools. Secondly, traditional forms of education research reflect asymmetries of power and knowledge

with teachers as the objects of research rather than an integral part of the research process. Of course, all education research in schools does involve co-operation of one form or another between researchers and practitioners if only at the level of teachers allowing data to be gathered in their classrooms. Various types of teacher research can also involve forms of co-operation, with perhaps a researcher offering advice to the practitioner.

Yet both these approaches to researching educational issues have flaws. As Wagner (1997) describes, while 'data-extraction agreements' led by researchers are one of the most widely used approaches in education research, a significant problem is the possibility of bias and partisanship (Delgado-Gaitan, 1993; Tooley, 1998). Various forms of teacher-led research exist, including action research (which focuses primarily on the impact of adjustments in the actions of teachers) and practitioner research (which generally has a wider social concern than action research) are not free from drawbacks either. Such teacher-led research, with its exclusive focus on problems of immediate practice, generally has less interest in contributing to a wider understanding of educational issues and so, as such, does not look for evidence of general impact on classroom learning or generalisable findings that can inform policy in education.

Co-learning agreements have been suggested by Wagner (1997) as a way of integrating practitioners into the research process so that rigorous and systematic joint analysis of educational issues can be tackled as a collaborative venture between researchers and teachers. Such co-learning is collaborative in that it involves the development of a model of joint planning, joint implementation, and joint evaluation between classroom teacher(s) and university researcher(s). Wagner argues that both participants in a co-learning agreement are involved in the processes of action and reflection throughout the research. In this chapter we argue that, beyond active participation, it is the 'voice' of the teacher which, in this case, provides a crucial contribution to findings that can increase knowledge about particular education environments.

The research described in this chapter was developed as a co-learning agreement between a secondary school classroom teacher, the first author of this chapter, and a university researcher, the second author. The development of the co-learning agreement that formed the basis for the research study described in this chapter was assisted by a number of factors. Firstly, there had been previous co-operation between the first and second author when the classroom of the teacher (the first author) was used as the site for a research study of the learning of certain aspects of geometry led by the second author. Secondly, the class teacher had also completed a Master's degree in Mathematics Education (Edwards, 1997) at the second author's institution and, at the time the study reported in the chapter was being planned and carried out, was undertaking a Master of Philosophy in Research Methodology (Edwards, 1999) also at the second author's institution (neither qualification was supervised by the second author, neither is the research reported in this chapter from either dissertation). Such advanced study involved the first author in numerous conversations with members of the research group at the university about issues in

mathematics education, especially those concerned with mathematics pedagogy and its impact on mathematics learning. It was the class teacher, the first author of this chapter, who drew the attention of the research team to the findings of Mulryan that the perceptions of small group work in mathematics of high achieving students might be more in line with those of their teacher than those of low achieving students and thus likely to increase the gap between high and low achieving students. Such a finding did not concur with the impression of the class teacher about her classroom and, if the findings were reproducible, appeared to her to directly undermine her pedagogical intentions of ensuring the full achievement of all the pupils she taught.

The findings of Mulryan were also interesting to the university researchers for theoretical reasons, concerned with the under-researched aspects of group learning (Dillenbourg, 1999), and because of recent developments in education policy at national level in the UK which appeared to be promoting 'whole-class teaching' at the expense of group work (see e.g. Ofsted, 1998a and 1999). These two diverse research perspectives on the findings from the Mulryan study provided the basis for developing an approach to a research design which would achieve the objectives of both perspectives. Through discussion, the research question became a joint problem with both parties to the co-learning agreement wishing to learn about the pupil perspective on collaborative learning in mathematics. In practice, the co-learning agreement was a resolution, through constant discussion and negotiation, of inevitable tensions raised by the differing perspectives. These are described below.

Issues relating to data collection created the first tensions which needed to be resolved between the partners. Discussion about the forms of data to be collected and how such data should be gathered resulted in agreement that the maintenance of the emancipatory classroom environment and the consequent relationships between teacher and students necessitated that the teacher undertake the interviews of students. Such a choice, however, raised issues of reliability from the theoretical perspective. These were addressed by use of Mulryan's framework of questions. The co-learning agreement established itself in the form of an iterative process of raising particular issues from one perspective or the other and finding solutions to address these issues which satisfied both the practitioner and academic aspects of the research. This working relationship evolved more quickly as the confidence between the partners developed and tensions were resolved more expediently. At the stage of interpretation of data, the co-learning relationship was such that there was almost immediate mutual and justified agreement that the interpretative insights of the teacher who knew and worked with the students should form the initial basis for interpretation of data. This analysis was then reviewed by the university researcher with a critical perspective and using sampled original data to monitor the analysis – a role often termed the 'critical other' (see Stenhouse, 1975).

In the research study reported in this chapter, the co-learning agreement was not a formal document laying down responsibilities and roles. It was a way of working which we each found to be productive and capable of generating insights into the detail of classroom approaches. Though our roles and responsibilities, each as

teacher researcher and university researcher, were inevitably defined in practice, the co-learning agreement enabled us to work towards common research goals. The agreement is based on a mutual respect for each other's strengths, sensitivity towards each other's experiences, a willingness to be challenged about particular views, a willingness to justify viewpoints, and an acceptance of re-negotiated stances. It is a time-consuming process initially, during which trust is established and an equitable way of working is developed. It also relies on actively seeking to understand the views and perspectives of the other (Kapuscinski, 1997, 9). Common research goals mean that challenges to perspectives and re-negotiation is viewed as a positive commitment towards these goals. We provide some reflections on this co-learning approach in the final section of this chapter. In the next section we begin describing the research study we carried out.

COOPERATING AND COLLABORATING IN THE MATHEMATICS CLASSROOM

In a comprehensive review, Good, Mulryan and McCaslin (1992, 167) describe "clear and compelling evidence that small group work can facilitate student achievement as well as more favourable attitudes towards peers and subject matter". Reviews such as those by Good et al. (1992) and by Cohen (1994) suggest that research has established such *co-operative* learning as an effective means of teaching and learning. As Good and is colleagues. warn, however, small group work is not a panacea. These reviews describe inappropriate ways of using small group work as well as appropriate ways. Factors such as the nature of the task assigned to a group, the role of the teacher, the composition of the group, the status and roles of group participants, helping behaviours, mutual goal interdependence, reward inter-dependence and the effects on cognition have been the foci for research on small group activity.

Much less is known about collaborative small group work than co-operative small group work (Lyle, 1996). As a result, little has been reported on a range of issues concerning collaborative small group work such as how the composition and dynamics of collaborative groups affect their ability to function effectively (for a recent report, see Barnes, 1998), or whether the students themselves find it an effective way of working. Lyle's study was longitudinal (over a 10 year period) and undertaken in a constructivist environment with self-selecting groups of classroom friends. Her focus was the importance in collaborative small group work of careful planning related both to the task and organisational strategies. Her study confirms earlier research that the greater the complexity required in the problem(s) tackled by the group, the higher the level of achievement. For example, Phelps and Damon (1989) found that peer collaboration was effective for mathematical tasks which require reasoning but not for tasks which require rote learning.

Both the reviews by Good et al. and by Cohen advocate a future focus for research on the socially situated learning which occurs in small groups. They argue that research on small groups has gone beyond a need to justify its overall benefit

through improved learning outcomes. They emphasise the need for work on the factors which affect *discourse processes* as well as factors which affect *achievement outcomes*. Mulryan's (1994) work on interviewing students about their experiences of doing mathematics in *co-operative* small groups attempts to address the issue of the process of *co-operative* small group work. For her study she interviewed 48 students from six classes (four high attaining and four low attaining students from each class) with equal numbers of boys and girls in each category. The teachers of these six classes were also interviewed. The purpose of the study was to examine the differences between high and low attaining students in mathematics in their experiences of small group work and in their perceptions of expectations in relation to each other. Classroom observations directed towards identifying these differences were also undertaken. Mulryan found that high attaining students remained on task for longer than low attaining students, they were more active with peers and there was a significant difference in the types of interaction involving question and answer situations for these different attaining students. Interviews were conducted with students and teachers about the following perceptions:

- purpose and benefits of co-operative small group work in mathematics,
- appropriate student behaviour during co-operative small group work,
- characteristics of co-operative small groups that are important for successful co-operative activity in small groups,
- determination of group composition.

Her results indicated that high attaining students understood the complexity of small group work better than low attaining students. In addition, the understanding of high attaining students of the purposes of small group work was closer to that of their teachers than was the understanding of low attaining students. The implication is that the narrower understanding of the purposes of small group work shown by low attaining students is a result of their less active participation in class and the greater time off task.

One of the factors implicit in effective small groups is the time involved in working together (Laborde, 1994). This is not accounted for in most studies because it is a social factor and, as such, a difficult variable to control in studies adopting an experimental approach. Mulryan's (1994) study used classes in which "students worked in co-operative small groups for mathematics at least once a week" (p. 281). It is unclear whether the findings from Mulryan's study are applicable to students who work *daily* in mathematics lessons in *collaborative* groups *over a period of years*.

WAYS OF WORKING IN THE COLLABORATIVE MATHEMATICS CLASSROOM

This overview of differing outcomes from research on co-operative and collaborative work may suggest that the role of the teacher and the epistemological basis within which these groups are examined may have a significant influence on

the outcomes visible in the classroom. In the case of the class teacher in the study reported in this chapter, the classroom ethos is established through both emancipatory and socio-constructivist theoretical frameworks. Emancipatory (or socially-just or inclusive) pedagogy is being developed from work in feminist and other emancipatory endeavours. Within such a pedagogical approach, the teacher is intent on recognising and valuing a plurality of forms of knowledge and ways of knowing (Becker, 1995; Povey, 1996; Solar, 1995). This is borne out of critical reflection and action. Emancipatory practice also challenges traditional classroom power structures and discourses.

Aligned with an emancipatory movement which uses socio-constructivist mathematics learning, is the view that socio-constructivist learning recreates a model of how mathematicians actually do mathematics. This is a second reason why this approach to mathematics learning is used in the classroom in this study. Humanly constructed knowledge implies a social, cultural and gendered dimension to this construction as individuals construct knowledge based on their personal experiences. The emergence of social constructivism in mathematics (see Ernest, 1991, 1998) reflected a response to this socio-cultural dimension to learning.

As a result of the combination of these two learning epistemologies, there are four main pedagogical elements which emerge for the class teacher in this study:

- refutation of mathematics as a set of 'truths' and recognition of mathematical knowledge as socially constructed,
- open-ended and problem-solving opportunities which allow process rather than knowledge to prevail,
- social and environmental contexts for learning,
- collaborative learning situations.

These four elements form the basis of collaborative learning in which social structures provide for egalitarian contributions to the processes of learning, challenging the status difficulties which emerge in co-operative groups. Students engage in open-ended problems in the genuine sense of a mathematician contributing jointly to a body of knowledge.

The teacher in this study plans a unit of work for a period of weeks based around an open-ended activity which may be investigative or practical in nature. Tasks to support areas of mathematical content which are likely to arise during students' engagement with the activity are derived from a number of sources and used to 'scaffold' the activity (Wood, Bruner & Ross, 1976; Roehler & Cantlon, 1997). The open-ended activities are introduced as a whole-class discussion with students and teachers making possible suggestions for routes for exploration. The students work on the activity in collaborative groups of two to six students, though the class is sometimes drawn together at various points to enable a student or a group to explain a discovery or for the teacher to make a teaching point from a groups' work. The role of the teacher is that of a facilitator to the various mathematical directions that groups may take. The emphasis in this role is on questioning and challenging the students in their work on the activity. Advice is given and suggestions made as to

the areas of mathematical content which might be a fruitful direction to explore. Students are expected to present an outcome for their work which is usually in written form but may be a presentation to the class. This method of teaching "delegates [mathematical] authority to groups while holding them accountable for performance" (Cohen, Lotan & Holthuis, 1995, 157-8).

METHODOLOGY

This section describes the methodology, developed as part of the co-learning agreement, for a study of the perspectives of students who learn mathematics collaboratively. The overall approach is described and details are given of the sample and the interview protocol. An ethnographic case study using semi-structured interviews was most suitable for this research study for two reasons. Firstly, it allowed the students to say what they wished about their experiences of collaborative group work within the framework of the interview schedule (Hammersley and Atkinson, 1995, 25). Secondly, semi-structured interviews are known to be suitable for gathering information and opinions and exploring people's thinking and motivations (Drever, 1995). Strict procedures were adopted for the interviews in order to minimise any potential bias introduced by the interviewer.

The Sample

A random sample of seven students were chosen for the study, selected from the classes of a teacher who taught in a UK inner-city comprehensive secondary school whose mathematics results in national testing were approximately in line with the national average (note that the mathematics classes were set by attainment, a very common practice in the UK). This sample was approximately 10% of the students taught be the teacher. The classes from which the students were chosen were:

- a low attaining class of students aged 15-16 (known in the UK as Year 11 students) who had experienced small group collaborative work in mathematics for the previous four years,
- a high attaining class of students aged 14-15 (Year 10 students) who had experienced small group collaborative work for the previous three years,
- a middle attaining class of students aged 12-13 (Year 8) who had experienced two years of small group collaborative work.

In all cases, attainment was defined by the school in terms of performance of the pupils on standardised non-verbal reasoning tests. Grouping students for teaching in terms of their prior attainment (usually measured by some form of test scores) in subjects like mathematics is very common in the UK (the practice is known as 'setting'), especially in secondary schools, and increasingly so in primary schools (Ofsted, 1998b). The seven students were randomly selected from the following classes: two from the low attaining Year 11 class, three from the high attaining Year

10 class, and two from the middle attaining Year 8 class. All the students had been taught by the same mathematics teacher throughout their experience of collaborative group work in mathematics.

The Interview

An interview schedule based on the headings used by Mulryan (1994) was utilised as a set of general prompts. Questions were based around the following pupil perceptions:

- perceptions of the purpose and benefits of collaborative small group work in mathematics,
- perceptions of teacher expectations for appropriate student behaviour during small group work,
- perceptions of the characteristics of small groups that are important for successful groups,
- perceptions of the extent to which individual and group accountability exist in small groups,
- perceptions in relation to the stability of membership of small groups.

The opportunity was also offered to the students for more open comment on their experiences of collaborative small group work.

ANALYSIS OF DATA

Following transcription of the audio tapes, each response was systematically coded for a particular category or categories. These categories were developed in an on-going way as new student respondents contributed different categories until there was a stable set of categories. This process of grounded theorising was necessary as the sample size was too small to use the particular categories devised by Mulryan, who, even with a sample of 48 students, had no more than 5 responses in any one category. As part of this analytical process, some categories were grouped to reflect similar themes.

The grouped categories given below are those identified from analysing the interview transcripts. In each case, representative extracts are given from the stratified sample of students.

- Benefits of working together/ collaborating/ working as a team/ working as a group.

  This theme was evident in all seven respondents' descriptions of their experiences of collaborative group work. For example:

  > R (low attaining Year 11) said "I think its really good, because we're able to work ... as a team ... you just understand more about maths than you do just by writing down on pen and paper".

S (high attaining Year 10) said "you might only look at a problem one way, but ...if you give lots of different people a problem, and they look at it in .. different ways".

V (middle attaining Year 8) said "it's lot easier to work in a group because you can help each other and you can find out the answers and make sure yours are right".

- Putting ideas together/ contributing/ using different skills (described as a process).

As for the theme above, this was widespread throughout the transcripts. For example:

R (middle attaining Year 8) said "you put all your ideas together, and by putting everyone's ideas together, you come up with good ideas and just get good knowledge".

R (low attaining Year 11) describes a similar experience, "and even if one person did say ... this is the right answer, we wouldn't just write it down, you'd, you know, make it more deeper and everybody'd put more to extend the answer".

J (high attaining Year 10) related that "K came up with an idea once, and then we .. started working on that, and then other people ... put in other ideas on top of it, so we were always building up".

- Listening to/ respecting others in the group/ sharing knowledge.

This theme is distinct from merely recognising the skills offered by others. It is described by pupils in the following ways:

R (middle attaining Year 8 class) "We can all listen to people's ideas, which I think is good and ... we all bring up our own ideas,"

S (high attaining Year 10 class) "people come up with different ideas ... and you get to explore other people's ideas which helps".

Z (low attaining Year 11) said "someone would say [something].. and then we all would .. put our different words in and talk about it".

- Confidence building/ feeling successful/ being motivated

Some pupils, including the higher attaining students, described collaborative group work as a vehicle for increasing their mathematical confidence. For example:

L (high attaining Year 10) said "I think in my case, ... if I know someone else thinks the same thing, I'm more confident about what I think".

There were several instances of pupils describing the experience of group work as making them feel more successful.

J (also high attaining Year 10) explained "I just think its better than working by yourself, really. I think you learn a lot more".

Pupils also seemed to find the group dynamics a more motivating learning environment.

Z (low attaining Year 11) affirmed "we just didn't want to leave it ... we used to stay behind lessons ... we wanted to get the work done ... I prefer doing maths ... with group work ".

- Friendship/ knowledge of collaborators/ stability of groups.

Questions about group structure revealed that all the pupils believed that their performance in a group was positively affected by working with others who

were well known to them. Friendship seemed to provide successful working relationships in the view of all those interviewed.

> V (middle attaining Year 8) explained "If you're not friends with somebody, ... you might not get along with them, and they might start getting into a bit of an argument about the answers".
>
> R (low attaining Year 11) said "no others could be as good as working with some friends".
>
> S (high attaining Year 10) said "well, obviously, you've all got to get on quite well, you've got to know ... I think it's easiest if you know each other first".

- Speed/ volume of learning.

  Students across the age and achievement range thought that collaborative learning in small groups allowed learning to happen more quickly and that they could learn more.

> J (high attaining Year 10) summed this up: "I think you learn a lot more, ... I think if people ... work together you can get a lot more done and you ... understand a lot more ... I think its probably quicker, because if you're working by yourself, it's you that does all the work,".
>
> R (middle attaining Year 8) said "it's easier if you do group work because you can get through it quicker and .. get to know a lot more".
>
> R (low attaining Year 11) offered a more reflective comment "I don't think it's quick or slow, it's in the middle, but because it's like that, you get a deeper meaning, you know what you're doing, you don't just skim it over the top".

The students also commented on: helping one another, thinking hard, enjoyment, autonomy and independence, and awareness of the possibilities of distraction. All the students were also aware of the expectations of the teacher in terms of what was appropriate for successful collaborative group work in mathematics.

DISCUSSION

This analysis of the interview transcripts for the categories described above allows some comment to be made, both on emerging global patterns in the student responses and on local patterns within groups. Examples of such local patterns relate to the age of the students and the length of their experience of collaborative small group work.

Overall, the full range of students in this study seemed to recognise the benefits of collaborative group work in mathematics. They realised the necessity of listening to one another, felt collaborative working made them confident and successful, and judged that they learnt more mathematics more rapidly by working in that way. There also appears to be clear indications that working with friends, that is working with those with whom you get on well, is important. It may be that this helps with the sharing and respecting of each others ideas and that, in the end, this helps with learning. These benefits of working with friends are noted by Zajac and Hartup (1997) in their review. Whicker, Boll and Nunnery (1997), in their study of co-operative groups in secondary school mathematics, found that their students "disliked having groups pre-assigned and permanent, and suggested alternating

group membership". A feature of co-operative group work tends to be the dominant role of the teacher in determining both the constitution of group and the roles and responsibilities of group members.

Nevertheless, the responses of all the groups were not identical. In particular, the responses of the younger students, those aged 12-13 (from the Year 8 class), who had only experienced collaborative small group work in mathematics for two years, were different in several respects. These students found it more difficult to articulate their perceptions of collaborative group work. Overall, their responses during the semi-structured interviews were much shorter, less reflective, and demonstrated less understanding of the pedagogic process, than the older students. In addition, the younger students seemed more orientated towards outcome, rather than process or understanding. For example, student V, middle attaining Year 8, said that working in a group means "you can find out the answers and make sure yours are right", and, later in the interview, that it was more enjoyable to work in a group because "you can get more accurate answers from it".

Such responses from the younger students, and the contrasting answers from the older students, may indicate that, in addition to maturation, it takes quite some time for the teacher to establish fully collaborative groups. The research on co-operative groups has already established that simply placing the students in groups does not mean that group work will take place. Indeed a frequent complaint about common practice in UK primary schools is that the pupils are arranged in groups in the classroom yet they do essentially individual work. Training in co-operative working was found necessary for successful co-operative group work, and research on collaborative learning suggests that for collaborative group work some form of teaching of relevant skills is required (Gillies & Ashman, 1996). A range of other factors is likely to influence the successful development of collaborative group work, including, in secondary schools, the experience of the students in other curriculum subjects.

Finally, unlike Mulryan (1994), we found no difference between the perceptions of high attaining students and those of low attaining students. All the students in our sample felt that collaborative group work had a positive effect on their rate of learning and depth of understanding. The reason for this difference, however, may not lie solely with the grouping structure. In our study it is likely that the philosophical and epistemological stance of the teacher, in developing a strongly inclusive pedagogy, is the influencing factor.

Evidence from other studies in which small group work is peripheral to the ethos of the "normal" working of the classroom may provide the key to differences between studies of co-operative and collaborative classroom situations. The theoretical bases around which the teacher in this study structured her classroom pedagogy may well be the most significant factor in influencing the views of the students involved in this pedagogy over a period of years. The differences in perceptions of the students in this study from those in Mulryan's study may, in part, be due to the frequency and duration of their experience of working in such a way.

REFLECTIONS

The Collaborative Mathematics Classroom

Mulryan's finding that, with *co-operative* group work, the perceptions of high achieving students were more in line with those of their teacher than those of low achieving students might suggest that *co-operative* group work could *increase* the separation between high and low achieving students, a possibility implied in other studies of co-operative learning (Good et al., 1992, 172-173 and 176-177). A generalisation from Mulryan's findings is that the less active participation and the greater time off task displayed by low attaining students is a result of their narrower understanding of the purposes of small group work. Such results contrast directly with the findings of the study described in this chapter where active participation was the norm for all students and there was, in general, little off-task activity. Accounting for these differences in findings between co-operative and collaborative group work in mathematics entails considering a range of issues.

Firstly, and in contrast to co-operative learning where there appear to be differential effects, collaborative learning in an emancipatory classroom strives to reduce these effects based on status through the creation of a learning ethos in which all contributions are equally important. In this way, differential perspectives are likely to be less pronounced than in a situation where differential contributions are the norm. Secondly, the research studies cited by Good *et al* (including those mentioned earlier in this chapter) were, in general, not undertaken from an inclusive socio-constructivist perspective, as was the study reported in this chapter. Without such a perspective on learning and on peer interaction, any differential effects may be exacerbated. Finally there are issues of detail that inevitably have an impact on the outcome. In Mulyran's 1994 study, for example, she reports that in the classrooms she studied, "the students worked in co-operative small groups for mathematics at least once a week" (p. 281). This may also be a factor in the different findings for high and low attaining students between Mulryan's study and the study that we describe in this chapter. The students in our study worked daily in mathematics lessons in collaborative groups over a period of years.

For some time in the UK the effectiveness of group work of any sort has been called into question, particularly in the primary phase where it is far more common than in secondary schools. Since the publication in 1992 of an influential policy document which concluded that "in many [primary] schools the benefits of whole class teaching have been insufficiently exploited" (Alexander, Rose & Woodhead, 1992, 35), reports by the UK government office charged with inspecting the quality of school education have consistently pressed the case for more whole class teaching. For example, the report on 'Standards in the Primary Curriculum' (Ofsted, 1998a) states that "the quality of mathematics teaching is improving with the recognition that pupils benefit from direct teaching". Similarly an overview of primary education claims that improvements in primary schools in the period 1994-8 are "largely due to an increase in the amount of direct, whole-class teaching" (Ofsted, 1999, section 1.1). The focus of these publications has invariably been on

the problems and difficulties that can arise with group work. It seems to us that, based on the research reported in this chapter and elsewhere, to date the benefits of *collaborative* group work in mathematics might well have been insufficiently exploited.

The Co-learning Agreement

In using a co-learning approach to researching the collaborative mathematics classroom both authors feel that we learnt more about the processes of collaboration in learning mathematics than we would have done with either a more traditional 'data gathering' approach devised by the researcher or as a piece of action research implemented by the teacher. For the class teacher it meant:

- being more confident about maintaining objectivity, particularly in the analysis of data phase, by working with a critical 'other',
- being able to engage with research such as that by Mulryan rather than rejecting it as not being consistent with the lived classroom experience,
- gaining new research skills to add to those of a successful classroom action researcher.

For the university-based researcher it meant:

- being a partner in developing and working on a research problem that had clear practical, theoretical and policy level aspects,
- gaining a better and more insightful understanding of classroom approaches and interactions through the collaborative data collection and analysis,
- acquiring new insights into the research process and how it can support the development of pedagogical knowledge.

These benefits of a co-learning agreement for both participants were not acquired easily. They are the result of a set of constant inevitable tensions between the teacher and the university-based researcher (Johnston, 1997 p113) and the resolution of conflicts arising from these tensions. These tensions are borne out of differing experiences and the consequent differing perspectives. What a co-learning agreement enabled us to do was to utilise these tensions in positive argumentation towards common research goals. It meant that the 'voice' of the teacher was evident throughout the process of the research. Working within the co-learning agreement included the exposition of our personal positionings through a reflexive and critical research design, a process often used and advocated in feminist research methodologies (Lather, 1995, 301). Revealing our own perspectives was sometimes difficult as it involved the possibility of vulnerability for each of us. Much time and energy was committed to establishing a framework of trust and this was crucial to the success of the co-learning agreement.

CONCLUDING COMMENTS

The ease with which the co-learning agreement continues to function through time reflects our findings in terms of students in collaborative mathematics work. For example, as we found with the pupils we studied, the success of collaborating is likely to be related to the frequency of the interactions between those involved and length of time over which the co-learning agreement takes place. It seems to us that the factors involved in successful collaborative work in the classroom are also applicable to the collaboration we developed as co-learning partners. Adapting the words of Wood and O'Malley (1996, 8), we would say that the success of our collaboration is due to:

- each of us having a detailed knowledge of our own field of expertise and a desire to learn more about the other's,
- a choice of research topic that was sufficiently 'shareable' to achieve and maintain mutual understanding,
- the development of a way of working that was mutually contingent and seemed likely to be of mutual benefit,
- sources of help in the form of colleagues who appreciated the aims of the study.

As Wood and O'Malley observe, such conditions are not easy to achieve, but, like us, they remain optimistic that it is possible to realise the potential of collaborative learning in the classroom. We feel the same way about co-learning agreements.

Co-learning could be said to be what characterises collaborative learning in the classroom. Both involve learning about each other as well as about the subject matter at hand. Both benefit from recognising and valuing other 'voices'. From the experience of undertaking the research reported in this chapter, co-learning appears to offer significant benefits when researching classroom processes.

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