

## PROXIMAL VORONOÏ REGIONS, CONVEX POLYGONS, & LEADER UNIFORM TOPOLOGY

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*Dedicated to the Memory of Som Naimpally*

**ABSTRACT.** This article introduces proximal Voronoï regions. A main result in this paper is the proof that proximal Voronoï regions are convex polygons. In addition, it is proved that every collection of proximal Voronoï regions has a Leader uniform topology.

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