EXPECTATION HETEROGENEITY AND WEALTH INEQUALITY

by
FURKAN SARIKAYA

Submitted to the Graduate School of Social Sciences
in partial fulfilment of
the requirements for the degree of Master of Arts

Sabancı University
July 2019

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Approved by:

Asst. Prof. Remzi Kaygusuz ....
(Thesis Supervisor)

Assoc. Prof. İnci Gümüş ....

Assoc. Prof. Türkmen Göksel ....

Date of Approval: July 8, 2019

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ABSTRACT

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FURKAN SARIKAYA

Economics M.A. Thesis, July 2019

Thesis Supervisor: Asst. Prof. Remzi Kaygusuz

Keywords: Wealth Inequality, Expectation Heterogeneity, Optimism, Pessimism, Incomplete markets

This study examines the effect of expectation heterogeneity on wealth inequality assuming that people with higher income are more optimistic about future returns of their savings, through a modified version of Krusell - Smith heterogeneous agent's model with uninsured idiosyncratic risk and aggregate uncertainty. Our main finding is that wealth distribution is jointly determined by general equilibrium effect, individual policy functions and income mobility under heterogeneous expectations assumption. As a result, an inverse U-shape relation between wealth inequality and level of expectation heterogeneity is observed.

ÖZET

BEKLENTİLERİN HETEROJENLİĞİ VE SERVET EŞİTSİZLİĞİ

FURKAN SARIKAYA

Ekonmomi Yüksek Lisans Tezi, Temmuz 2019

Tez Danışmanı: Dr. Öğr. Görevlisi Remzi Kaygusuz

Anahtar Kelimeler: Heterojen Beklentiler, Optimizm, Pesimizm, Servet Eşitsizliği, Eksik Piyasalar

Bu çalışma insanların gelir seviyesi arttıkça yaptıkları birikimlerden bekledikleri getirinin de arttığı varsayımına dayanarak heterojen beklentilerin servet eşitsizliği üzerindeki etkisini incelemektedir. Bu amaç doğrultusunda Krusell - Smith modelinin sigorta edilmemiş bireysel risk ve makro belirsizlik mekanizmalarının birlikte çalışabileceği bir versiyonu geliştirilip, yukarıdaki varsayım altında model simule edilmiştir. Çalışmanın temel bulgusu bahsettiğimiz mekanizma altında servet eşitsizliğinin iki etki tarafından eş zamanlı olarak belirlendiğidir: Genel denge etkisi ve bireysel karar fonksiyonu etkisi. Sonuç olarak, beklenti heterojenliğinin seviyesi ve gelir eşitsizliği arasında ters U şeklinde bir ilişki gözlenmiştir.

ACKNOWLEDGEMENTS

I would like to express my gratitude and thanks to my advisor Dr. Remzi Kaygusuz for the useful comments and remarks. His guidance helped me in research and writing of this thesis and in solving the questions about my career path.

Besides my advisor, I would like to thank the rest of my thesis committee: Dr. Türkmen Göksel, and Dr. İnci Gümüş, for their encouragement, insightful comments, and hard questions. My sincere thanks also goes to Dr. Hakkı Yazıcı, and Prof. Abdurrahman Aydemir, for helping me to improve a better understanding in Economics during my M.A degree.

Finally, I thank my friends, Oğuz Cicak, Gökay Aytekin, Adem Yaşlı, and Yusuf Süha Kulu for their precious friendship. I would like to thank my family: my parents Fatma and Metin Sarıkaya, my sister Nisanur Sarıkaya for supporting me spiritually. Without you, I would not be the person I am today. My wife Özgen Sarıkaya deserves special thanks for her improving comments and valuable love, but most of all, for being my best friend.

Last but not least, I would like to thank my beautiful country and its lovely people for offering me the opportunity to acquire my educational background and for shaping my personality with its tremendous cultural heritage.

To Sarıkaya family

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1. INTRODUCTION

There is a growing body of empirical literature that documents the expectation heterogeneity among agents (see Branch (2004) and Hommes (2015)). Most of the researches in this strand of literature focus on the question of where the expectation comes from and which model produces more consistent heterogeneous expectations with the ones observed in the data (Evans and Honkapohja (2001), Ball, Gregory Mankiw, and Reis (2005), Waters (2009)). In the theoretical literature, the discussion on the sources of heterogeneity concentrates on three main approaches: People have different expectations because (i) they are using different models, (ii) they have different information sets, and (iii) the capacity to process information is not the same for everyone.(Pfajfar (2013))

Leaving aside the vigorous debate on the source of expectation heterogeneity among agents, it is generally conceded that people's expectations about future inflation, exchange rate, interest rates, and unemployment rate exhibit substantial variation, leading significant macroeconomic implications. More recently, Das, Kuhnen, and Nagel (2017) explore a further characteristic of this expectation heterogeneity, revealing that individuals' macroeconomic expectations are affected by their socioeconomic status. To be more specific, their study infers that people with higher income are more optimistic about the future return on their savings.

Our study is motivated by this empirical finding of Das, Kuhnen, and Nagel (2017) and it examines the effect of future interest rate expectation heterogeneity on the wealth distribution by assuming a positive relationship between individuals' income and their optimism about anticipated future interest rate on their assets. Due to the absence of quantitative findings for the functional form or magnitude of the expectation heterogeneity conditioned on the socioeconomic status individuals. [1], a general model of heterogeneous expectation is developed, which are built upon incomplete markets model with uninsured idiosyncratic risk and aggregate uncer-

[1] Das, Kuhnen, and Nagel (2017) use the Michigan Surveys of Consumers(MSC), which individual are asked to express their beliefs about future values of several macroeconomic variables. The survey questions are far from being quantitative. Rather, they contain categorical answers like economics will be good or bad.

tainty of Krusell and Smith (1998). This model is well fit for our purpose because of its ability to create heterogeneity in the expected future return of capital through anticipated future aggregate capital. In other words, manipulating agents' aggregate forecasting rules of the economy according to their current income level generates a variation in their anticipation of future return of capital. It also allows us to investigate general equilibrium implications of the expectation heterogeneity.

In the calibration procedure, we closely follow the Acedański (2017). To solve for the individual policy rule, we use an Euler-equation iteration on a grid of pre-specified points (Maliar, Maliar, and Valli (2010)). To solve aggregate law of motion of capital, we modify the standard Krusell-Smith algorithm (Krusell and Smith (1998)).

Our main finding is that compared to benchmark model in which all agents have rational expectations, expectation differences between agents produce a substantial level of inequality in the society, measured by Gini coefficient. As the expectation differences rise, the level of inequality increases firstly then starts to decrease. This result is driven by two competing forces in our model: the effect of expectation differences on individual policy functions and the general equilibrium level of interest rate. On the one side, as the level of heterogeneity among agent rises, the spread in incentive to save between individual's expands. Agents at the top of the income distribution, who expect high return on their savings in the future save more and people at the bottom of income distribution loss their incentive to save. Hence, wealth inequality is aggravated by level of expectation heterogeneity. On the other side, as the optimism level of high - income people increase, they save more whereas pessimistic low-income people can not dissave enough because of the borrowing constraint of the incomplete market. This mechanism implies higher aggregate capital at the general equilibrium, and thus lower interest rate. Hence, even if rich people become more optimistic relative to the general equilibrium level of interest rate, their interest rate expectation will decrease because of the deterioration in the general equilibrium level of interest rate. As a result, an inverse U-shape relationship between the Gini coefficient and the level of expectation heterogeneity arises.

The remaining of the paper is organized as follows. Section 2 examines the related literature briefly. Section 3 presents the theoretical model that we studied. Section 4 presents the information regarding calibration exercise. In section 5, the findings of the study are presented. Section 6 concludes. The computational procedure of the study is presented in the appendix section in detail.

2. LITERATURE REVIEW

This study is attached to two areas of the literature. First, it contributes the area of research that investigates the source of heterogeneity in wealth[1]. Almost all general equilibrium models in this literature are based on Bewley Model in which the agents are *ex ante* identical but *ex post* heterogeneous due to different realization of shocks. Aiyagari (1994) and Huggett (1996) initialized the discussion by working quantitatively on the models with incomplete market and uninsured idiosyncratic shocks. Although Huggets's calibration achieves to match US wealth gini considerably, many moments of the wealth distribution especially wealth concentration at the top of distribution and extreme poverty remains unexplained.

Many solutions to this problem have been developed. Carroll (1998) objectifies "capitalist spirit", which is a sociological concept proposed by Max Weber, within a quantitative general equilibrium model. He believes that the wealth enters the utility function as a luxury good and this in assumption accompanied by an explanation for why rich people have higher saving rates in the actual data. Krusell and Smith (1998) formulate a model with heterogeneous time preference which allows the more patient agents to save more. They reveals that heterogeneous discount factor generates more realistic variance of the cross-sectional distribution of wealth.

Although many moments of the wealth distribution is well explained by these studies, they were still not good enough to capture wealth concentration among the richest households. Another strand of the literature proposes that incorporating entrepreneurship in the incomplete market framework is crucial to understand this mysterious observation in actual wealth distribution data (see Quadrini (2009), M. and Glenn (2004), and Cagetti and Nardi (2005)). The main motivation of this argument is the pattern that entrepreneurs have higher saving rates among rich people, which is documented by Quadrini (1999) and Buera (2009). Quadrini (1999) points out that the key element driving this pattern is costly financial intermediaries which make interest rate on borrowing more than return of savings and capital imperfec-

[1] A detailed literature review is presented by Cagetti and De Nardi (2008) and Nardi (2015)

tions which motivate agents who have an entrepreneurial idea to save more. His model produces better results to match with the upper tail of wealth distribution observed in actual data compared to previous ones.

Furthermore, bequest motive in the form of both accidental and voluntary is considered as a source of heterogeneity in wealth. (see Huggett (1996)) and Yang (2005)). While accidental bequest is conceptualized as a possible result of life span risk and thus, it does not alter the saving behavior of the agents, voluntary bequest motive impacts saving decisions directly as insurance against labor shock and life span risk do. Voluntary bequest motive also produces more consistent lifetime saving profiles with the ones observed in the data. In addition, Nardi (2004) takes into account inter-generational transition of human capital and del Rio (2015) introduces the market skills of agents and heterogeneity in labor disutility to capture substantial heterogeneity in the wealth distribution.

The second area of literature related with our study is on the heterogeneity in expectations [2]. A bulk of this strand of literature has focused on the source of heterogeneity in expectations about macroeconomic variables, which is documented by several studies (see Mankiw, Reis, and Wolfers (2003), Vissing-Jorgensen (2004), Dreger and Stadtmann (2008) and, and Gnan, Langthaler, and Valderrama (2011)). Ball, Gregory Mankiw, and Reis (2005) articulate the systematic errors of individuals' expectations as a result of infrequently update of information by agents and posits it as a main source of expectation heterogeneity. On the other hand, Evans and Honkapohja (2001) consider agents as econometricians who do know the model but not the parameters of the model. Although there are many studies on the source of expectation heterogeneity, the research in the potential macroeconomic outcomes of this heterogeneity remains limited. As far as we know, the only research which has investigated the role of expectation heterogeneity on the wealth inequality is written by Acedański (2017). His study focuses on the role of randomly assigned heterogeneous expectations related to interest rate and the exogenous frequency of switch between different expectations on the wealth inequality. This study improves this question in the sense of the empirical findings reported by Das, Kuhnen, and Nagel (2017), a positive relation between agents' macroeconomic optimism and their income level. What if the expectation heterogeneity distributed systematically across the population rather than being random is arguably an important question to be addressed.

[2] A detailed literature review is presented by Pfajfar (2013).

3. MODEL

In this section, we describe our model economy which is a modified version of Krusell - Smith heterogeneous agents model with uninsured idiosyncratic risk and aggregate uncertainty. The main sources of heterogeneity are aggregate capital forecasting rule of agents, which is conditioned on the individual's current income level, and idiosyncratic income shock which implies variation in individuals' employment histories.

3.1 Environment

The economy is populated by a continuum of infinitely-lived many agents. We assume that the agents have one unit of time endowment, which gives rise to $\epsilon_t \bar{l}$ unit of labor input, where $\bar{l}$ is agent's labor supply and ϵ_t represents the employment status of the agent in period t and it evolves according to two-stage Markov chain. If he/she is unemployed in period t, $\epsilon_t = 0$, otherwise $\epsilon_t = 1$. In the case that the agent is employed, he/she earns the net wage $(1 - \tau_t)\bar{l}w_t$, where $0 \leq \tau_t \leq 1$ is tax rate and w_t is the wage. Unemployed agents receive unemployment benefit, μw_t, where $0 \leq \mu \leq 1$ is the unemployment benefit rate. In addition to labor income, agents receive interest on their capital holding k_t, which is equal to $(r_t - \delta)k_t$, where r_t and δ represent interest rate and depreciation rate respectively. Note that subscript t denotes time during this study.

3.2 Agents' Problem

In this environment, the agents decide how to divide their wealth which is a summation of labor income and capital income between consumption and saving in each period. The agent's problem can be characterized as follows:

$$\max_{c_t,k_{t+1}} E_t \sum_{h=0}^{\infty} \beta^h u(c_{t+h}) \qquad \text{s.t.} \tag{3.1}$$

$$c_{t+h} + k_{t+h+1} = (1-\delta - r_{t+h})k_{t+h} + \Big[(1-\tau_{t+h})\bar{l}\epsilon_{t+h} + \mu(1-\epsilon_{t+h})\Big] w_{t+h} \tag{3.2}$$

$$k_{t+h} \geq \underline{k}, \quad h = 1,2\ldots \tag{3.3}$$

$$c_{t+h} \geq 0, \quad h = 1,2\ldots \tag{3.4}$$

where c is consumption, $0 \leq \beta \leq 1$ represents discount factor of the agents, and $\underline{k}$ denotes the borrowing constraint. We assume that the utility function of the agents is constant relative risk aversion (CRRA) with γ representing relative risk aversion coefficient:

$$u(c) = \frac{c^{1-\gamma}-1}{1-\gamma} \tag{3.5}$$

3.3 Production Sector and Government

Production sector consists of a representative firm which has a technology for producing single consumption good according to Cobb-Douglas production function.

$$Y_t = z_t K_t^{\alpha} (\bar{l} L_t)^{1-\alpha} \tag{3.6}$$

where K and L represent total amount of capital and labor supplied by agents respectively, z is aggregate productivity shock following two stage Markov chain, and $0 < \alpha < 1$ denotes capital share.

Since the firm operates in a competitive environment, the production function implies that wage and interest rate are the followings:

$$w_t(z_t, K_t, L_t) = (1-\alpha)z_t \left(\frac{K_t}{\bar{l}L_t}\right)^{\alpha} \tag{3.7}$$

$$r_t(z_t, K_t, L_t) = (\alpha)z_t \left(\frac{K_t}{\bar{l}L_t}\right)^{(1-\alpha)} \tag{3.8}$$

The government's role in our model follows the one which is specified by Haan, Judd, and Juillard (2010) and Acedański (2017). Government balances its budget every period by collecting taxes, τ_t on labor in order to finance unemployment benefits, μ. Thus, the government's budget constraint can be defined as follows:

$$\tau_t w_t \bar{l} L_t = \mu u_t w_t \tag{3.9}$$

where u_t represents unemployment rate in period t. This constraint implies that the government set the tax rate τ in period t, according to following equation:

$$\tau_t = \frac{\mu u_t}{\bar{l}L_t} \tag{3.10}$$

3.4 Expectation Heterogeneity

Krusell and Smith (1998) describe the law of motion of capital perceived by agents with a stochastic process for a finite-dimensional vector of moments of the wealth distribution. Specifically, they reveal that the behavior of macroeconomic aggregates at the stationary stochastic equilibrium can be almost perfectly characterized by the first moment of wealth distribution. As a result, they proposed that the following log-linear functional form with proper set of parameter $P^b = \{a_0, a_1, b_0, b_1\}$ is good enough to make agents almost fully rational:

$$\log K_{t+1} = a_0 + a_1 \log K_t \qquad if \quad z_t = z_g \tag{3.11}$$

$$\log K_{t+1} = b_0 + b_1 \log K_t \qquad if \quad z_t = z_b \tag{3.12}$$

where z_g and z_b represent good(boom) or bad(recession) states of the economy. Reducing all moments of wealth distribution to the first moment of wealth distribution leads consumers facing manageable prediction problem. More specifically Krusell and Smith (1998) shows that an agent perceiving above simple laws of motion for aggregate capital makes extremely small mistakes. Briefly, following the mean of the wealth distribution under above functional form is almost enough to make agents fully rational.

In order to impose expectation heterogeneity into the model, we assume the same law of motion of aggregate capital with only one difference - assigning the parameter set $P^b = \{a_0, b_0, a_1, b_1\}$ conditional on agent's income ranking. Since we assume that people with higher income are more optimistic about the future return of their savings, we manipulate the anticipated future interest rate, r' with a function $\Phi(i) : I \to [-m, m]$, where $i \in I$ represents the income ranking of the agent, $m \in \mathbb{R}_{++}$ and $\Phi'(i) \geq 0$. We characterize interest rate expectation of an agent who is at the income ranking i, r'_i as the following:

$$r'_i = (1 + \Phi(i)) r'_b \tag{3.13}$$

where r'_b is the anticipated future interest rate in the case that every agent has the rational expectation. To be more precise, we set the interest rate produced by a model without any heterogeneity of expectation as an anchor, and define the heterogeneous expectation of future interest rate relative to this anchor.

Implied expected aggregate capital stock is derived from equation 3.11 and 3.12. It is as the followings:

$$log(K_b) = \frac{a_0}{1 - a_1} \quad if \quad z = z_g \tag{3.14}$$

$$log(K_b) = \frac{b_0}{1 - b_1} \quad if \quad z = z_b \tag{3.15}$$

The following formula is derived by using equation 3.8 and equation 3.13

$$K_i = K_b/(1+\Phi(i))^{\frac{1}{\alpha-1}} \qquad \forall i \in I \tag{3.16}$$

Given a_0, a_1, b_0, b_1 and $\Phi(i)$ implies different aggregate capital K_i value for each income group i, and similar to 3.14 and 3.15.

$$log(K_i) = \frac{a_{0i}}{1-a_{1i}} \quad if \quad z = z_g \tag{3.17}$$

$$log(K_i) = \frac{b_{0i}}{1-b_{1i}} \quad if \quad z = z_b \tag{3.18}$$

Thereby, the set of parameter $P = \{a_{0i}, b_{0i}, a_{1i}, b_{1i}, \forall i \in I\}$ is set in order to satisfy 3.17 and 3.18. As a result, the forecasting rule of aggregate capital turns out to be following equations whose the parameters are conditional of income ranking of the agent:

$$\log K_{i,t+1} = a_{0i} + a_{1i} \log K_{i,t} \qquad if \quad z_t = z_g \tag{3.19}$$

$$\log K_{i,t+1} = b_{0i} + b_i 1 \log K_{i,t} \qquad if \quad z_t = z_b \tag{3.20}$$

3.5 Recursive Formulation

In the following notation, the time subscripts are omitted and the next period's variables are denoted with primes. The recursive decision problem of the agents can be characterized as follows:

$$v(k,K,\epsilon,z)=\max_{c,k'}\{u(c)+\beta Ev(k',K',\epsilon',z')\} \qquad \text{s.t} \tag{3.21}$$

$$k'+c=(1-\delta-r)k+[(1-\tau)\bar{l}\epsilon+\mu(1-\epsilon)]w \tag{3.22}$$

$$\log K'=a_{0i}+a_{1i}\log K \qquad if \quad z=z_g \tag{3.23}$$

$$\log K'=b_{0i}+b_i 1\log K \qquad if \quad z=z_b \tag{3.24}$$

$$k',k\geq \underline{k} \tag{3.25}$$

where v denotes value function, and subscript i denotes the current income group of the agent.

Note that expectation heterogeneity is imposed to model by conditioning parameters of law of motion of aggregate capital on agent's current income and it is neither state variable nor a stochastic process[1]. This method has some implications which are worth to be discussed. Firstly, agents are not aware that their optimism and pessimism about market prices are related to their current income level. Thus, a high - income optimist agent may turns out to be a pessimist agent in the next period if her income is adequately deteriorated by a negative employment shock. At the first glance, this radical set up may be seen as unrealistic. However, Das et al. (2017) employ a panel sub-sample of their data set and show that the relation between income and optimism remains to be statistically significant, implying that optimism may change even within an agent's life depending on her income level. In addition, considering expectation heterogeneity as a state variable of the problem may lead complications in our framework since it is dependent to income which is partly determined by agents via saving decisions. In this case, expectation heterogeneity would be a tool which can be manipulated by agents in order to increase individual welfare rather than being a behavioral anomaly that we want to investigate.

The quantitative solution method of our model is described in the Appendix in detail. In brief, given state variables and expectation parameters we discussed above, each agent solves its own problem by deciding individual asset holding level. All agents' decisions generate a specific level of aggregate capital. We iterate expectation parameters until the model reaches its stationary equilibrium in terms of aggregate capital.

[1]In the next section, we will elaborate how the expectation heterogeneity parameter is constructed.

4. CALIBRATION PROCEDURE

In this section, we explain how we choose parameter values of the model. In addition, we elaborate the method of specifying the function $\Phi(i)$ which decides the level of expectation heterogeneity among agents.

The parameters are set using a model period of one quarter. The calibration procedure closely follows the work of Krusell and Smith (1998). The discount factor β equals to 0.99. Utility parameter γ converges to 1, which lead CRRA utility function collapses the logarithmic utility, i.e, $u(c) = log(c)$. We use capital share α of 0.36 and depreciation rate δ of 0.025. The time endowment $\underline{l}$ is 0.31. We set the transition matrix of the Markov chain for idiosyncratic and aggregate shocks following the estimate of Krusell and Smith (1998). Transition probabilities are presented in Table 4.1. Aggregate shock values, z_g and z_b is set to 1.01 and 0.99, respectively in order to match aggregate fluctuations in the US economy. The calibration parameters are summarized in Table 4.2.

Table 4.1 Transition Probabilities

	R - U	R - E	B - U	B - E
R - U	0.525	0.35	0.03125	0.09375
R - E	0.03889	0.83611	0.00208	0.12292
B - U	0.9375	0.03125	0.29167	0.58333
B - E	0.00912	0.11589	0.02431	0.85069

R and B represent recession and boom states of the economy, respectively. U and E denote being unemployed and employed, respectively. The table presents transition probabilities from period t to period t+1.

We split the population into four groups in each period according to their current income, which implies $i \in \{1,2,3,4\}$. If $i = 1$ the agents at the bottom 25% of the current income distribution. $i = 2$ and $i = 3$ implies that the agent's income is between 26 - 50 and 51 - 75 percentiles, respectively. Finally, If $i = 4$, the agents at top 25% of the income distribution. We constraint the number of income group as four because of the trade of between the explanatory power of the model and

Table 4.2 Calibration Parameters

Parameter	Description	Value
α	capital share	0.36
β	discount parameter	0.99
δ	depreciation rate	0.025
$\bar{l}$	time endowment	0.31
z_g	aggregate shock in a boom	1.01
z_b	aggregate shock in a recession	0.99
μ	unemployment replacement rate	0.15
$\underline{k}$	debt limit	-4
k_{min}	lower bound of individual capital holding	-4
k_{max}	lower bound of individual capital holding	200
K_{min}	lower bound of aggregate capital level	8
K_{max}	lower bound of aggregate capital level	16
n_k	number of grid points on k	200
n_K	number of grid points on K	17
η_1, η_2	tolerance parameters	10^{-8}, 10^{-3}
π	update parameter	0.5
a_0^b, a_1^b	benchmark aggregate law of motion coefficients	0.0786, 0.9660
b_0^b, b_1^b		0.0898, 0.9636

computational work. On the one side, a smaller number of groups prevents us from evaluating general equilibrium, income mobility, and individual policy rule effects simultaneously since the transition of agents between income groups is altered considerably as the number of groups decreases. On the other side, a higher number of groups requires much more computational power. However, we control the model results for higher number of groups, as well. It is observed that although the numerical results change substantially, the general patterns that we interpreted in Section 3 remained unchanged.

Following function depending on expectation heterogeneity function that we describe in previous section is defined for the sake of simplicity:

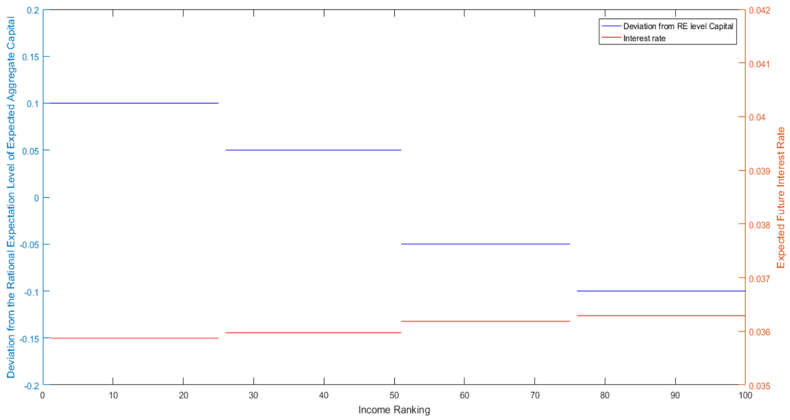
$$\Psi(i) = \frac{1}{(1+\Phi(i))^{\frac{1}{\alpha-1}}} \tag{4.1}$$

The following form is assigned to the function $\Psi(i)$[1]:

[1]see Figure 4.1 in order to understand what the functional form in equation 4.2 implies in terms of future interest rate expectations of the heterogeneous groups.

$$\Psi(i) = \begin{cases} (1+\Delta) & i=1 \\ (1+\Delta/2) & i=2 \\ (1-\Delta/2) & i=3 \\ (1-\Delta) & i=4 \end{cases} \tag{4.2}$$

Figure 4.1 Design of Heterogeneity



This figure illustrates the method of imposing heterogeneity to expected interest rate for $\Delta = 0.1$ and $K = 13$. The blue line shows expectation bias imposed by dispersion parameter where zero at y-axes means there is no bias. The red line represents anticipated future interest rates by income ranking, which is produced by related biases shown with blue line.

Various values of Δ are considered to evaluate the model[2]. It is important to restate that the model is far from making quantitative implication because of the lack of information on the functional form of the expectation heterogeneity conditional on income. Rather, a general parametrization is used in order to investigate theoretically how expectation heterogeneity depending on socioeconomic status and its magnitude affects wealth inequality. Note that the higher Δ, the more optimistic people at the top of income distribution and the more pessimistic people at the bottom of income distribution. At the rest of the paper, we may use poorest, poor, rich and richest groups to refer the agents whose income rankings are equal to $i = 1$, $i = 2$, $i = 3$, and $i = 4$ respectively for the sake of fluency.

[2]Note that $\Delta = 0$ means the model is the same with the benchmark model which there is no expectation heterogeneity. The other values of Δ as the followings: 0.005, 0.01, 0.02, 0.035, 0.055, 0.1, 0.2, 0.3, 0.4, 0.5, 0.8. Among them, 0.02 is close to the standard deviation of the time series of aggregate capital produced by benchmark model with no heterogeneous expectations and 0.055 is equal to the standard deviation of the time series of aggregate capital estimated from data (Acedański 2017).

5. RESULTS

In Table 5.1, several moments of wealth distribution at the equilibrium are presented for different dispersion parameter, Δ values. The wealth inequality measured by Gini coefficient, at the second row of the table, is determined by two opposing forces: The amount of expectation bias and general equilibrium effect. The amount of expectation bias is characterized by Δ as we discussed in Section 2.4. As Δ increases, level of heterogeneity towards the expectation on future interest rate rises. This exacerbates the inequality resulting from different employment histories since agents who have more favorable employment history are more likely to be optimistic while agents who have got negative shocks are more likely to be pessimistic. Hence, higher value of Δ or the more distance between agents' anticipated future return of their savings implies higher level of wealth inequality since agents' saving decisions vary with anticipated future return of their savings.

The second mechanism which drives the level of wealth inequality can be called general equilibrium effect. As we discussed in section 2.4, the expectation heterogeneity on aggregate capital is formed around general equilibrium level of aggregate capital. As an example, in the case that $\Delta = 0.005$, richest people in the economy expect 0.005 less capital than general equilibrium level of aggregate capital while poorest people in the economy expect 0.005 more capital than general equilibrium level of aggregate capital. The first column of the Table 5.1 shows that the average aggregate capital level at the general equilibrium rises with dispersion parameter, implying that as the expectation difference among income groups increases, the equilibrium level of expected interest rate decreases. This relation arises as a result of incomplete market structure of our model economy. To be more specific, general equilibrium interest rate decreases with Δ because extra savings of rich people stemming from their over optimism on future returns cannot be absorbed by poor people who are willing to borrow since total amount of debt stock of each agent is not allowed to exceed borrowing constraint, $\underline{k}$. Thus, higher values of Δ implies less incentive to save for all agents compared to lower values of Δ. In other words, increase in average aggregate capital exerts downward pressure on wealth inequality by providing a

Table 5.1 Results (Main Model)

Δ:	0	0.005	0.01	0.02	0.035	0.055	0.1	0.2	0.3	0.4	0.5	0.8
Av. K	10.97	11.00	11.04	11.22	11.19	11.31	11.58	12.16	12.68	13.17	13.72	15.42
Gini	0.545	0.728	0.779	0.806	0.812	0.816	0.820	0.821	0.815	0.798	0.779	0.684
Top 1	8.9	5.28	5.45	5.11	5.32	5.24	5.15	5.03	4.85	4.52	4.13	3.23
Top 10	35.9	41.8	45.6	46.8	48.3	48.6	48.8	48.5	47.2	44.15	40.3	31.56
Neg.	6.5	34.6	47.5	55.6	61.2	65.6	71.1	73.8	67.0	51.2	49.2	50

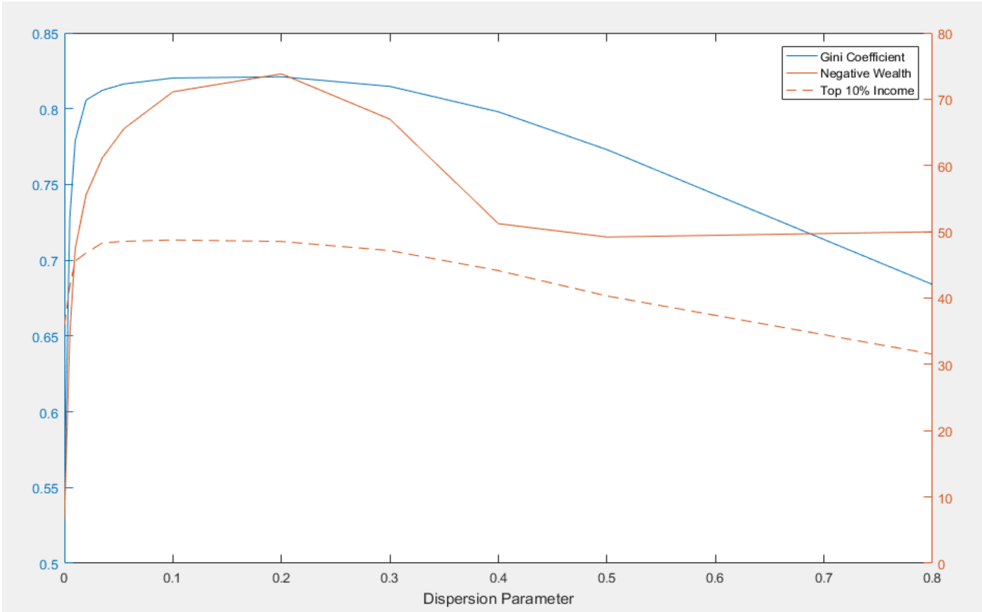
The results are obtained from a simulation with 10000 agents and 35000 periods. The first 5000 periods are treated as a burn in the sample. 'Av. K' means average aggregate capital in the economy. 'Gini' means gini coefficient of the wealth distribution. 'Top 1' and 'Top 10' represents the total share of wealth held by wealthiest 1 % and 10 % of the population. 'Neg.' is the fraction of population with negative wealth.

disincentive for the top earners. The forth row of the Table 5.1 presents the average share of total wealth held by the wealthiest 10 % of the populations. It reveals that higher optimism does not imply higher share of wealth for top earners.

Under these two competing effects of level of expectation heterogeneity on income inequality, the Gini coefficient peaks its maximum value when $\Delta = 0.2$ and then starts to decrease. A sufficiently large value of Δ allows the general equilibrium effects to dominate the effects of individual decision rules, wherein the Gini coefficient stops increasing. As a result, an inverse U-shape relation between inequality and level of heterogeneity parameter arises (see Figure 5.1).

Figure 5.2 illustrates the wealth distribution in the model under various Δ values. A concentration around borrowing constraint is observed even for very small expectation differences. As the expectation difference rise, wealth holding level of heterogeneous groups diverge more from each other. This deteriorates the income mobility between heterogeneous groups, which exacerbates the divergence implied by different future interest rate expectations. To be more precise, low-income level agents have lower saving rate because of the lower level of interest rate expectation while high-income agents have a tendency to save more due to more optimist expectations toward future interest rates. Thus, they have realized different wealth holdings. However, the total income, which determines the expectation of agents have a second component, labor income. Idiosyncratic unemployment shocks produced by a stochastic process generates labor income for each agent. Therefore, if a negative shock hits optimistic agent and positive shock hits pessimistic agent, they can change their socioeconomic status determined by the summation of labor income and capital income in the case that their capital holdings are not so much different.

Figure 5.1 Moments of Wealth Distribution and Level of Expectation Heterogeneity



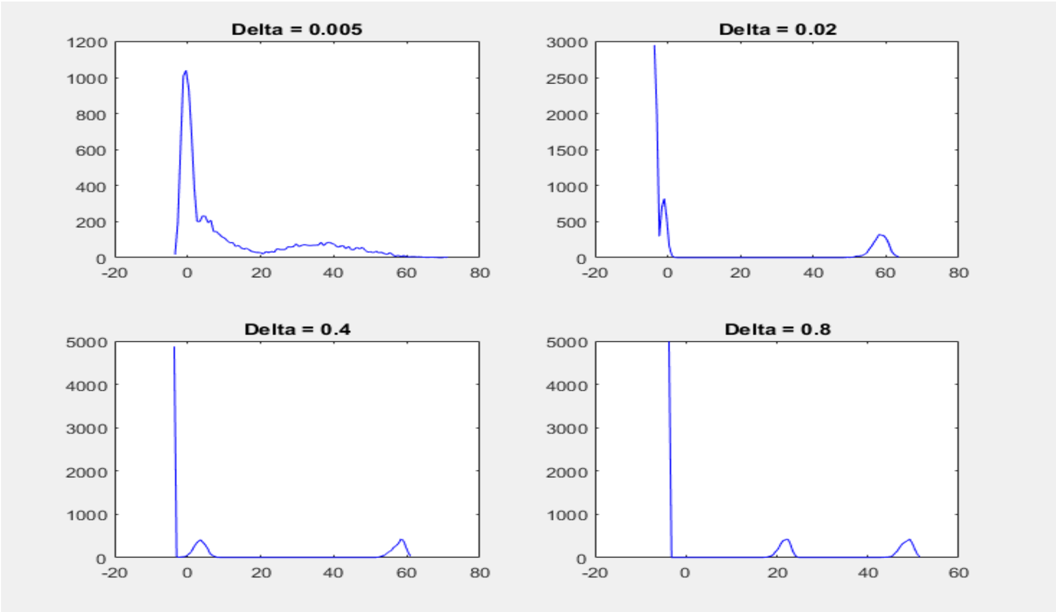
The relationship between several wealth distribution moments and level of heterogeneity represented by Δ on the x-axis. The blue line is gini coefficient with the values on the left y-axis. Red line and the dashed red line represent fraction of agents with negative wealth and share of total wealth held by richest 10 %, respectively.

Thus, continuous transitions between groups alleviate the wealth inequality stemming from different saving rate decisions. When the expectation heterogeneity is large enough, the wealth holdings of income groups implied by their expectations cluster far from each other as the panel B of figure 4.2 illustrates. Thus, Idiosyncratic shocks turn out to be inadequate to create income mobility between heterogeneous groups. The model with $\Delta = 0.005$ produces on average 26 transition between rich group to richest groups each period, which is approximately equal to 1 percent of the group population. However, almost zero income mobility between these groups is produced by the model with $\Delta = 0.02$.

5.1 Expectation on Future Wage

It is possible to notice that agents' expectation bias towards aggregate capital generates expectation heterogeneity on not only anticipated future interest rates but also future wages. Put differently, our model implies that level of optimism about

Figure 5.2 Wealth Distribution



Wealth distributions for $\Delta = 0.05$, $\Delta = 0.02$, $\Delta = 0.4$, and $\Delta = 0.8$, are presented in this figure. Number of agents and wealth holding are on the y-axis and x-axis, respectively.

future wages decreases with income level of agents. However, Das, Kuhnen, and Nagel (2017) also reveal that having a higher income rank is the significant predictor of the level of optimism in the expectation on employment rate. It is possible to consider that this pattern about labor market condition might be applicable for expected wage rates, even if it is not directly measured by Das, Kuhnen, and Nagel (2017). It would be contradictory with the implications of our model. Therefore, we simulate the model by excluding the heterogeneity on anticipated future wage. The results are presented in Table 5.2. Fortunately, many of the patterns that we discussed above are not altered considerably. Based on Figure 5.3, the inverse U-shape relation between inequality and level of heterogeneity parameter, Δ is preserved when we isolate our model from expectation heterogeneity on future wage rates. Excluding this heterogeneity generates a level effect on the relation between gini coefficient and Δ. To be more specific, the higher inequality in our main specification is a result of precautionary saving mechanism, in which the richer agents have a tendency to save more in order to insure themselves against lower anticipated future wage rates. Similarly, higher future wage rate anticipated by poorer agents implies lower saving rate for them. Excluding anticipated wage heterogeneity improves the wealth inequality by eliminating this mechanism.

Table 5.2 Results (Expectation on Future Wage)

Δ:	0	0.005	0.01	0.02	0.035	0.055	0.1	0.2	0.3	0.4	0.5	0.8
Av. K	10.97	10.99	10.99	11.02	11.04	11.07	11.15	11.31	11.46	11.60	11.76	12.09
Gini	0.545	0.621	0.699	0.762	0.792	0.805	0.814	0.814	0.796	0.766	0.718	0.645
Top 1%	8.9	4.7	5.2	5.5	5.6	5.56	5.4	5.1	4.6	4.1	3.6	2.9
Top 10%	35.9	36.2	40.1	44.6	47.2	48.4	49.0	47.8	43.8	39.6	34.6	27.8
Neg.	6.5	18.1	28.5	41.8	50.4	55.5	63.8	63.9	52.4	50.2	49.9	50

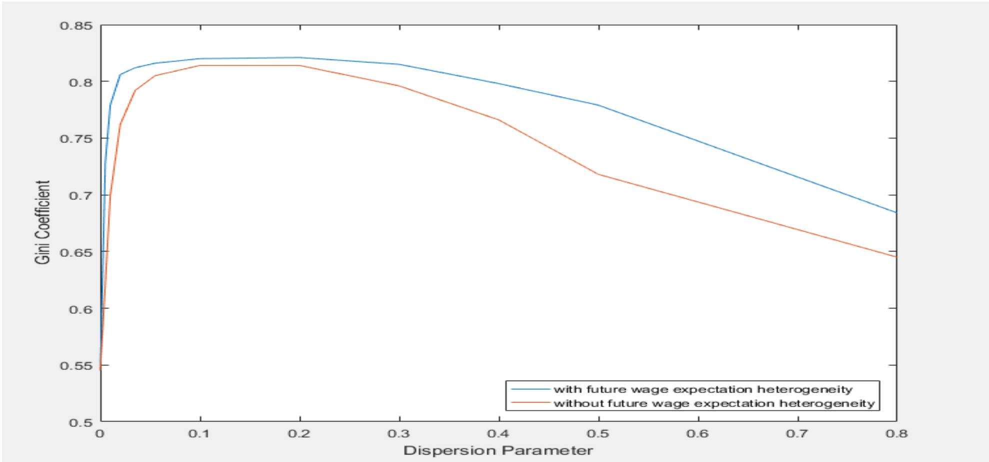
The results are obtained from a simulation with 10000 agents and 35000 periods. The first 5000 periods are treated as a burn in the sample. 'Av. K' means average aggregate capital in the economy. 'Gini' means gini coefficient of the wealth distribution. 'Top 1' and 'Top 10' represents the total share of wealth held by wealthiest 1 % and 10 % of the population. 'Neg.' is the fraction of population with negative wealth.

5.2 Role of Conditioning on Income

Acedański (2017) investigates the role of expectation heterogeneity on wealth inequality without conditioning the heterogeneity on income. His study focuses on the role of randomly assigned heterogeneous expectation and the exogenous frequency of switch between different expectations on the wealth inequality. However, our model produces endogenous transition between heterogeneous groups by assigning the expectation bias to each agent according to his/her income level in every period. In order to understand how this set up drives the wealth inequality among agents, we compare our main results reported in Table 5.1 and the results of a modified model which transition between heterogeneous groups is assigned randomly, as Acedański (2017) reports. Table 5.3 presents the comparison, which columns with 'M' and 'RT' show the results of the main model and results with random transition, respectively. The number of transition between heterogeneous groups and average aggregate capital generated by main model is imposed to the random transition model, as well. In other words, the only differences between main and random transition model is the functions which determine the expectation bias of each agent.

For each Δ value, our model generates at least 20 % higher gini coefficient than random transition model does. It exhibits that a substantial part of the inequality produced by our main model derives from the conditioning expectation heterogeneity towards anticipated future return of saving on income. Random transition structure

Figure 5.3 Gini Coefficients with and without Wage Expectation Heterogeneity



A comparison between gini coefficients of the models with and without anticipated future wage heterogeneity.

Table 5.3 Results (Role of Conditioning on Income)

Δ:	0	0.005 M	0.005 RT	0.02 M	0.02 RT	0.2 M	0.2 RT
Average Capital	10.97	11.00	11.00	11.22	11.22	12.16	12.16
Gini Coefficient	0.545	0.728	0.571	0.806	0.641	0.821	59.6
Top 1% Wealth	8.9	5.28	7.96	5.11	5.04	5.03	6.46
Top 10% Wealth	35.9	41.8	30.21	46.8	46.1	48.5	43.22
Negative Wealth	6.5	34.6	17.42	55.6	47.1	73.8	50

The comparison between results of the main model and results of the model with random transition. 'M' and 'RT' represents the main model and random transition model, respectively.

compensates a part of inequality resulting from idiosyncratic employment shocks by switching agents' groups randomly. In other words, an agent with bad employment history may be optimistic and, thereby he/she decides to allocate higher resource for saving in order to earn more marginal benefit from higher interest rate. Thus, random transition model also generates lower share of top 10% income and lower fraction of population with negative wealth than our model does.

6. CONCLUSION

This paper examines the effect of heterogeneity in expectation towards future interest rates on the wealth inequality. The aim of the research is investigating this effect theoretically rather than making quantitative statements which requires more structured micro foundations about the functional form and the magnitude of expectation heterogeneity. The main finding of this paper is an inverse U shape relation between the level of expectation heterogeneity and the wealth inequality measured by gini coefficient. This pattern arises as a result of individual decisions under biased expectations and general equilibrium effect. In addition, income mobility between heterogeneous groups is a significant determinant of inequality in our model set up. While most research has focused on the empirical validation or rationalization of the expectations' heterogeneity, less attention has been given to an examination of its consequences. This study intends to fill this gap by studying the role of heterogeneous expectations conditioned on income as Das, Kuhnen, and Nagel (2017) documents in shaping the distribution of wealth in the incomplete markets model with uninsured idiosyncratic risk and aggregate uncertainty of Krusell and Smith (1998) .

Many questions which may be arguably important to be addressed have remained unanswered such as how is the wealth distribution determined under other forms of expectation heterogeneity?, how does a risky asset motive accompanied by expectation heterogeneity towards return on saving alter the wealth inequality?, and what would be the consequences if agents are allowed to make entrepreneurial choices with their biased expectations? Advancements in the empirical studies which document the heterogeneity in expectation conditioned by income more accurately may help to answer these questions in a comprehensive way.

BIBLIOGRAPHY

Acedański, Jan. 2017. "Heterogeneous expectations and the distribution of wealth." *Journal of Macroeconomics* 53: 162 – 175.

Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *The Quarterly Journal of Economics* 109(3): 659–684.

Ball, Laurence, N. Gregory Mankiw, and Ricardo Reis. 2005. "Monetary policy for inattentive economies." *Journal of Monetary Economics* 52(May): 703–725.

Branch, William A. 2004. "The Theory of Rationally Heterogeneous Expectations: Evidence from Survey Data on Inflation Expectations*." *The Economic Journal* 114(497): 592–621.

Buera, Francisco. 2009. "A dynamic model of entrepreneurship with borrowing constraints: theory and evidence." *Annals of Finance* 5(June): 443–464.

Cagetti, Marco, and Mariacristina De Nardi. 2005. Entrepreneurship, frictions, and wealth. Technical report.

Cagetti, Marco, and Mariacristina De Nardi. 2008. "Wealth Inequality: Data And Models." *Macroeconomic Dynamics* 12(September): 285–313.

Carroll, Christopher D. 1998. "Why Do the Rich Save So Much?" (May).

Das, Sreyoshi, Camelia M Kuhnen, and Stefan Nagel. 2017. "Socioeconomic Status and Macroeconomic Expectations." (November).

del Rio, Jose Mustre. 2015. "Wealth and Labor Supply Heterogeneity." *Review of Economic Dynamics* 18(July): 619–634.

Dreger, Christian, and Georg Stadtmann. 2008. "What drives heterogeneity in foreign exchange rate expectations: insights from a new survey." *International Journal of Finance & Economics* 13(4): 360–367.

Evans, George, and Seppo Honkapohja. 2001. *Learning and expectations in macroeconomics.* Princeton University Press.

Gnan, Ernest, Johannes Langthaler, and Maria Teresa Valderrama. 2011. "Heterogeneity in Euro Area Consumers' Inflation Expectations: Some Stylized Facts and Implications." *Monetary Policy & the Economy* (2): 43–66.

Haan, Wouter J. Den, Kenneth L. Judd, and Michel Juillard. 2010. "Computational suite of models with heterogeneous agents: Incomplete markets and aggregate uncertainty." *Journal of Economic Dynamics and Control* 34(1): 1 – 3.

Hommes, Cars. 2015. *Behavioral Rationality and Heterogeneous Expectations in Complex Economic Systems.* Number 9781107564978 *in* "Cambridge Books" Cambridge University Press.

Huggett, Mark. 1996. “Wealth distribution in life-cycle economies.” *Journal of Monetary Economics* 38(December): 469–494.

Krusell, Per, and Anthony Smith. 1998. “Income and Wealth Heterogeneity in the Macroeconomy.” *Journal of Political Economy* 106(5): 867–896.

M., Gentry William, and Hubbard R. Glenn. 2004. “Entrepreneurship and Household Saving.” *The B.E. Journal of Economic Analysis & Policy* 4(August): 1–57.

Maliar, Lilia, Serguei Maliar, and Fernando Valli. 2010. “Solving the incomplete markets model with aggregate uncertainty using the Krusell–Smith algorithm.” *Journal of Economic Dynamics and Control* 34(1): 42 – 49.

Mankiw, N. Gregory, Ricardo Reis, and Justin Wolfers. 2003. “Disagreement about Inflation Expectations.”.

Nardi, Mariacristina De. 2004. “Wealth Inequality and Intergenerational Links.” *Review of Economic Studies* 71(3): 743–768.

Nardi, Mariacristina De. 2015. “Quantitative Models of Wealth Inequality: A Survey.” (April).

Pfajfar, Damjan. 2013. “Formation of rationally heterogeneous expectations.” *Journal of Economic Dynamics and Control* 37(8): 1434 – 1452.

Quadrini, Vincenzo. 1999. “The Importance Of Entrepreneurship For Wealth Concentration And Mobility.” *Review of Income and Wealth* 45(March): 1–19.

Quadrini, Vincenzo. 2009. “Entrepreneurship in macroeconomics.” *Annals of Finance* 5(June): 295–311.

Vissing-Jorgensen, Annette. 2004. “Perspectives on Behavioral Finance: Does "Irrationality" Disappear with Wealth? Evidence from Expectations and Actions.” (Mayo): 139–208.

Waters, George A. 2009. “Chaos in the cobweb model with a new learning dynamic.” *Journal of Economic Dynamics and Control* 33(6): 1201 – 1216.

Yang, Fang. 2005. “Accounting for the heterogeneity in retirement wealth.”.

APPENDIX

Individual Decision Rule

This study closely follows the study of Maliar, Maliar, and Valli (2010) in order to solve individual policy function. As shown in the cited study, the Euler equation is the following:

$$
\begin{aligned}
k' = & \left(1 - \frac{\mu u}{\bar{l} L}\right) w \bar{l} \epsilon + \mu w (1-\epsilon) + (1-\delta+r)k \\
& - \left\{ \Lambda + \beta E \left[\frac{1-\delta+r'}{\left(\left(1 - \frac{\mu u'}{\bar{l} L'}\right) w' \bar{l} \epsilon' + \mu w'(1-\epsilon') + (1-\delta+r')k' - k''\right)^{\gamma}} \right] \right\}^{-1/\gamma}
\end{aligned}
\tag{6.1}
$$

where Λ is lagrange multiplier of the budget constraint and k'' refers two period ahead policy function.

Equation 6.1 is derived from euler equation of recursive problem of the agents (see section 3.5) by eliminating current and future consumption using budget constraint and Kuhn - Tucker conditions. Lagrange multiplier and policy function are defined as a function of current states of the economy, $k, \epsilon, K, z,$ i.e $\Lambda = \Lambda(k, \epsilon, K, z)$ and $k' = k'(k, \epsilon, K, z)$. We define the individual wealth holding k, and level of aggregate capital, K on the intervals $[k_{min}, k_{max}]$ and $[K_{min}, K_{max}]$, and dicretize these intervals with n_k and n_K number of points respectively to construct a grid of points for (k, ϵ, K, z). Euler equation is solved on this grid using the following iterative procedure:

Step I: Set the initial policy function at $k'_0(k, \epsilon, K, z) = 0.9k$ for all k, ϵ, K, z values.

Step II: Using the law of motion of capital compute the future prices, r' and w'. Set lagrange multiplier $\Lambda = 0$. Calculate $k''_i =$

$k_i'(k_i'(k,\epsilon,K,z),\epsilon,K,z)$. Since the values $k_i'(k,\epsilon,K,z)$ may not overlap with grid points on k, use spline interpolation in order to decide unmatched values.

Step III: Given values of future prices, lagrange multiplier, policy function and two period ahead policy function, calculate the r.h.s value of equation 6.1, k_{i+1}' by using the transition probabilities stated in Table **??**.

Step IV: Set each point for which $k_{i+1}'(k,\epsilon,K,z)$ does not belong $[k_{min},k_{max}]$ equal to the closest boundary values.

Step V: If the distance between k_i' and k_{i+1}' is less than a tolerance level, η_1 end up the process and consider k_{i+1}' as true policy function. Otherwise, update the $k_{i+1}' = \pi k_i' + (1-\pi)k_{i+1}'$ and return to step II.

Aggregate Law of Motion of Capital

This section describes the procedure which calculates aggregate law of motion of the economy. To be more precise, we calibrate the set of parameters $P = \{a_{0i}, b_{0i}, a_{1i}, b_{1i}, \forall i \in I\}$ (see Section **??**) by using a modified version of Krusell - Smith algorithm ((Krusell and Smith 1998)). The procedure can be summarized as follows:

Step I: Fix the initial values of coefficients to the benchmark values (see Table 4.2), i.e $a_0 = a_0^b$, $a_1 = a_1^b$, $b_0 = b_0^b$, and $b_1 = b_1^b$.

Step II: Generate T period length aggregate shock and for each of the N agents, generate T period of idiosyncratic shock. Fix the initial distribution of capital across N heterogeneous agents.

Step III: Define each income groups' coefficients for aggregate law of motion as the following[1]:

$$a_{0i} = a_0 + \Psi(i) \tag{6.2}$$

$$b_{0i} = b_0 + \Psi(i) \tag{6.3}$$

$$a_{1i} = a_1 \tag{6.4}$$

$$b_{1i} = b_0 \tag{6.5}$$

Step VI: Given set of parameters $P = \{a_{0i}, b_{0i}, a_{1i}, b_{1i}, \forall i \in I\}$, calculate anticipated future prices r_i' and w_i' for each i using equations (3.17), (3.18), (3.7), and (3.8).

Step V: Given anticipated future interest rate and future wage, find the individual decision rules for each group of people using the procedure described in previous section.

Step V: Simulate the economy T period forward by using individual policy rules of each agent. In each period, calculate aggregate level of capital, K_t.

Step VI: Regress K_{t+1} on K_t with the functional forms of equations (3.11) and (3.12). Derive the coefficients $\widetilde{a_0}, \widetilde{a_1}, \widetilde{b_0}, \widetilde{b_1}$.

Step VI: If $max\left\{|\frac{a_0}{1-a_1} - \frac{\widetilde{a_0}}{1-\widetilde{a_1}}|, |\frac{b_0}{1-b_1} - \frac{\widetilde{b_0}}{1-\widetilde{b_1}}|\right\} < \eta_2$, finish the procedure and take set of parameters P as true for the model, otherwise update

[1] Adjusting expectations only via intercept coefficients is not problematic ((Acedański 2017)). This part will be explained more detail.

the initial values a_0, a_1, b_0, b_1 as the following:

$$a_0 = \pi\widetilde{a_0} + (1-\pi)a_0 \tag{6.6}$$

$$a_1 = \pi\widetilde{a_1} + (1-\pi)a_1 \tag{6.7}$$

$$b_0 = \pi\widetilde{b_0} + (1-\pi)b_0 \tag{6.8}$$

$$b_1 = \pi\widetilde{b_1} + (1-\pi)b_1 \tag{6.9}$$

and return to step III.