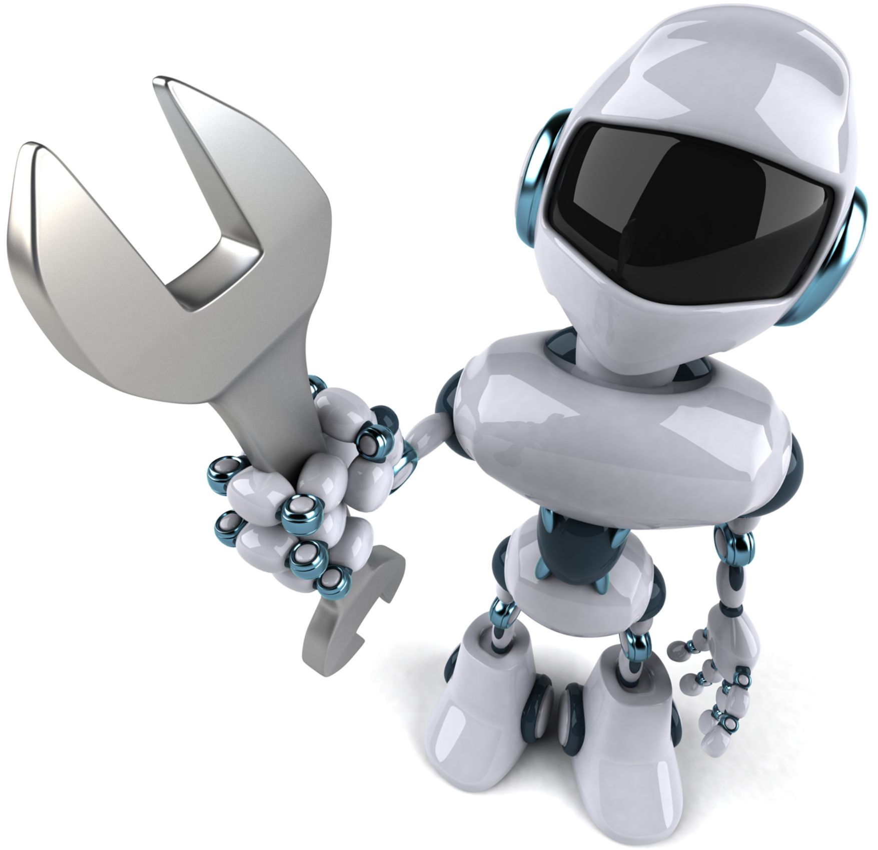

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Trajectory Tracking Control Strategy using Co-Reference for Rear-Steered Vehicle

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***Abstract*— A trajectory tracking control strategy using a co-reference for a rear-steered non-holonomic vehicle is presented. Given a reference moving on a path, the vehicle has to track the reference and reduce the vehicle-to-reference distance to the minimum possible distance. In addition, the vehicle has to maintain the change rate of the vehicle-to-reference distance non-positive at the neighborhood of the reference from any initial configuration. A class of barrier Lyapunov Function (BLF) is utilized to analyze the stability of the proposed control strategy. The co-reference is used to guide the vehicle to enter the neighborhood of the reference from the rear side of the reference. The performance of the proposed control strategy is evaluated by simulations and the results are presented.**

Keywords: Trajectory tracking control, automated guided vehicle, distance reduction.

I. INTRODUCTION

Research on tracking of moving reference for nonholonomic car-like vehicles has been almost-mature in the latest years. The ultimate mission of such the problem is to drive the vehicles moving towards to their respective references. Some successful works were reported to have Cartesian coordinate system-based control strategies [1]-[5]. However, it is obvious that the control strategies cannot include vehicle-to-reference distance as state space variable. Therefore, it cannot accommodate the requirement of reducing the distance. Meanwhile, in some particular cases of multiple vehicle systems, this issue is important. The ability of a vehicle to reduce its distance to the reference leads to the ability to avoid collision to other vehicles under preplanned trajectories, especially when the two trajectories have some interception points.

An alternative manner to solve the problem starts from the application of polar coordinate system-based navigation, where vehicle-to-reference distance is one of the axis [6]-[16]. However, a problem appears in using such the coordinate system, for instance, when the vehicle enters the neighborhood of the reference from significantly large orientation angle errors. This situation is not advantageous, since the vehicle-to-reference distance need be increased for a short time as a compensation of orientation adjustment in small distance. To attack such the problems, a new technique of applying guidance point which takes a role as a "co-reference" is introduced. The co-reference will guide the vehicle such that it will get closer to the reference from the rear of the actual reference. In this paper, a control algorithm utilizing barrier Lyapunov function (BLF) is designed to drive the vehicle to its respective reference focusing on reducing vehicle-to-reference distance.

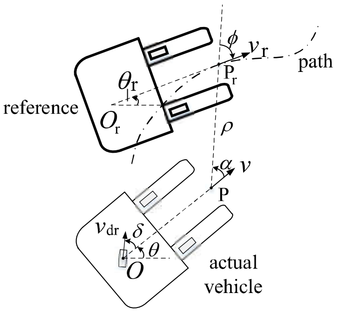

Figure 1. Trajectory tracking scenario. The actual vehicle intends to get closer to the related reference.

The contribution of this study is the design of trajectory tracking control algorithm to reduce the vehicle -to-reference distance under significantly distinct initial orientation.

II. VEHICLE'S KINEMATICS MODEL

Let a rear-steered vehicle and its respective preplanned moving reference are represented by their configurations (position in X-Y plane and orientation with respect to X-axis), i.e., $P(x,y,\theta)$ and $P_{\mathrm{r}}(x_{\mathrm{r}},y_{\mathrm{r}},\theta_{\mathrm{r}})$, respectively. The reference moves along a predefined path with linear and angular velocities v_{r} and ω_{r}, respectively. In addition, the direction of the reference is symbolized as θ_{r}.

For navigation purpose, we use navigation variables, as shown in Fig. 1, as $\mathbf{q}(t)=[\rho \quad \alpha \quad \phi]^{\mathrm{T}}$, where $\rho\in\Re$, $\alpha\in(-\pi,\pi]$, and $\phi\in(-\pi,\pi]$ are the vehicle-to-reference distance, the inclination angle of the vehicle-to-reference line PP_r with respect to the vehicle's orientation, and the inclination of the line PP_r with respect to θ_{r}, respectively. The formulations of the navigation variables are described as $\alpha=\arctan 2(\tilde{y},\tilde{x})-\theta$, $\phi=\theta_{\mathrm{r}}-\arctan 2(\tilde{y},\tilde{x})$, and $\rho=(\tilde{x}^2+\tilde{y}^2)^{0.5}$ if $(x,y)\in\Omega_{\mathrm{r}}$ and $\rho=-(\tilde{x}^2+\tilde{y}^2)^{0.5}$ if $(x,y)\in\Omega_{\mathrm{f}}$, where $\tilde{x}=x_r-x$ and $\tilde{y}=y_r-y$; Ω_{r} and Ω_{f} represents "the vehicle is in the rear of its reference" and "the vehicle is in the front of the reference", respectively, and are described as $\Omega_{\mathrm{r}}=\{(x,y)|\ \alpha\in[-\pi/2,\pi/2]\}$ and

$\Omega_{\mathrm{f}} = \{(x,y)| \quad \alpha \in (-\pi,-\pi/2) \vee (\pi/2,\pi)\}$. The time derivative of the navigation variables is expressed as

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} v_{\mathrm{r}} \cos\phi - v_{\mathrm{dr}} \cos\alpha \cos\delta \\ -v_{\mathrm{r}}\rho^{-1} \sin\phi + v_{\mathrm{dr}}\rho^{-1} \sin\alpha \cos\delta + v_{\mathrm{dr}} l^{-1} \sin\delta \\ v_{\mathrm{r}}\rho^{-1} \sin\phi - v_{\mathrm{dr}}\rho^{-1} \sin\alpha \cos\delta + \omega_{\mathrm{r}} \end{bmatrix}, \quad (1)$$

where v_{dr} and δ are the driving velocity and steering angle of the vehicle, respectively.

III. PROBLEM DESCRIPTION

In this study, we focus on a trajectory-tracking problem where the motion of the reference (linear and angular velocities) is independent to the position of the vehicle. The reference, whose linear and angular velocities are known, moves along a predefined path. Let $\Omega_1 = \{(\rho,\alpha,\phi)| \rho \le \rho_{\min}\}$, where $\rho_{\min}$ is sufficiently small. Suppose that the vehicle starts from an initial configuration $(x_0, y_0, \theta_0) \in \Omega_1$ such that the initial navigation variables $(\rho_0, \alpha_0, \phi_0) \in \Omega_1$. Also, suppose that $v_{\mathrm{dr}} > 0$ and $\delta \in [-\pi/2, \pi/2]$ are applied such that $\dot{\rho} < 0$. It can be investigated that α and ϕ will abruptly leap. Assume that there exists a Lyapunov candidate function such the control law makes the equilibrium point of the system asymptotically stable. It can be concluded that $\dot{\alpha}$ and $\dot{\phi}$ will swing away from zero and the vehicle will rotate rapidly towards Ω_1 . Therefore, if $\rho \approx 0$ and $\sin\phi >> \rho$ then $|\dot{\alpha}|$ grows larger and make α unstable.

Let us focus on the more specific condition. Suppose that the initial configuration of the vehicle is in $\Omega_2 = \{(\rho,\alpha,\phi) \mid \rho \le \rho_{\min}, \alpha \approx 0\}$. Therefore, the change rate of $\dot{\alpha}$ and $\dot{\phi}$ becomes $\dot{\alpha} \approx -v_{\mathrm{r}}\rho^{-1}\sin\phi + v_{\mathrm{dr}} l^{-1} \sin\delta$, $\dot{\rho} \approx v_{\mathrm{r}} \cos\phi - v_{\mathrm{dr}} \cos\delta$, and $\dot{\phi} \approx v_{\mathrm{r}}\rho^{-1}\sin\phi + \omega_{\mathrm{r}}$. It can be concluded that $\dot{\alpha}$ is influenced by uncontrolled variables, i.e., v_{r} and ϕ , besides the control law $(v_{\mathrm{dr}}, \delta)$. In this situation, it is difficult to drive α to zero. Meanwhile, $\dot{\phi}$ is fully uncontrollable. In other case, if the initial configuration is in $\Omega_3 = \{(\rho,\alpha,\phi)| \quad \rho \le \rho_{\min}, |\phi| \approx 0\}$, then that the influence of v_{r} and ω_{r} to $\dot{\alpha}$ can be eliminated.

However, the term ρ^{-1} in the equations of $\dot{\alpha}$ is still appeared and might cause scattering on α , especially under the condition of $\rho \approx 0$ and $\sin\alpha >> \rho$. Therefore, we need to drive α to zero under a proper value of ρ such that $\dot{\alpha}$ is bounded. It can be concluded that by achieving $|\phi| \approx 0$ first following by zeroing α and reducing ρ until $\rho \le \rho_{\min}$, we can guarantee that the condition of $\dot{\rho} < 0$ never occur for all $t \ge 0$. Another problem exists when we intend to guarantee that at small vehicle-to-reference distance ρ , the changing rate of ρ is always non-positive or ρ does not exceed $\rho_{\min}$. From (1), we can conclude that there are some initial configurations that makes ρ increase in some time interval, i.e., under the condition of $v_{\mathrm{dr}} \cos\alpha \cos\delta < v_{\mathrm{r}} \cos\phi$.

The objective of the study is to design a control law $(v_{\mathrm{dr}}, \delta)$ for the vehicle such that the vehicle can reach $\rho \le \rho_{\min}$, $|\alpha| = 0$, and $|\phi| = 0$ at $t \ge T$, where T is finite time for any initial configuration.

IV. CONTROL DESIGN

A. Transformed Barrier Lyapunov Functions (TLBF)

Consider the following system,

$$\dot{z} = f(z,u), \; z(0) = z_0, \quad (2)$$

where $z \in \Re$ and $u \in \Re$ are the state and control input of the system, respectively; $z_0 \in \Re$ is the initial state; $f \in \Re$ is a piecewise continuous and locally Lipschitz function. Also, define $z_{\mathrm{a}} \in \mathbf{Z}$ and $z_{\mathrm{b}} \in \mathbf{Z}$, such that $z_{\mathrm{a}} < z_{\mathrm{b}}$ and $z_0 \in (z_{\mathrm{a}}, z_{\mathrm{b}})$. For the design of u , we introduce a scalar, continuous function called "transformed barrier Lyapunov function (TBLF)" for each z_i , symbolized as $V_{z,i}$, formulated as follows.

Proposition 1: Define constants $k_{\mathrm{a},1} > 0$, $k_{\mathrm{a},2} > 0$, $k_{\mathrm{b},1} > 0$, $k_{\mathrm{b},2} > 0$, $\beta_z > 0$. Consider two open region $D_1 = \{z \in \Re : -k_{\mathrm{a},1} < z < k_{\mathrm{b},1}\}$ and $D_2 = \{y \in \Re : -k_{\mathrm{a},2} < y < k_{\mathrm{b},2}\}$. Let f maps D_1 to D_2 as $e_z = f(z) = \beta_z z - \psi_z$, where $\psi_z \in \Re$ is a translation along z -axis. For any $z \in D_1$, there exists a $e_z \in D_2$ if and only if ψ_z satisfies $\beta_z z - k_{\mathrm{b},2} < \psi_z < \beta_z z + k_{\mathrm{a},2}$.

Proof: Let M be a non-negative constant. Applying $\psi = \beta_z z - k_{\mathrm{b},2} + M$ to the equation of e_z yields $e_z = k_{\mathrm{b},2} - M$. Since M is nonnegative, the maximum e_z is $e_z = k_{\mathrm{b},2}$. In other words, if $M > 0$ then e_z belongs to D_1 . On the other hand, applying $\psi_z = \beta_z z + k_{\mathrm{a},2} - M$ to the equation of e_z yields $e_z = -k_{\mathrm{a},2} + M$. Since M is nonnegative, the minimum e_z is $e_z = -k_{\mathrm{a},2}$, or in other words, if $M > 0$, e_z belongs to D_1 , as well. ■

Definition 1: Let us consider a region and $D_2 = \{e_z \in \Re : -k_{\mathrm{a},2} < e_z < k_{\mathrm{b},2}\}$, where $e_z = \beta_z z - \psi_z$, z belongs to $D_1 = \{z \in \Re : -k_{\mathrm{a},1} < z < k_{\mathrm{b},1}\}$, and $\psi_z \in \Re$. A transformed barrier Lyapunov function (TBLF) is a continuous and positive definite function $V(e_z)$ that is defined with respect to a system (2) and has continuous first-order partial derivatives on every point in D_2 and has a property $V(e_z) \to \infty$ as e_z approaches the boundaries of D_2, i.e., $e_z = -k_{\mathrm{a},2}$ and $e_z = k_{\mathrm{b},2}$, and satisfies $V(e_z) \le b$ for all $t \ge 0$ along the solution of $\dot{z} = f(z,u)$ for $z \in D_2$ and some positive constant b.

Let a barrier Lyapunov function (BLF) is defined as $V(z) = 0.5(q(z)k_{z,1} \ln k_{\mathrm{a}}^2 (k_{\mathrm{a}}^2 - z^2)^{-1} + (1-q(z))k_{z,1} \ln k_{\mathrm{b}}^2 (k_{\mathrm{b}}^2 - z^2)^{-1})$, where $q(z)$ is 1 if $z \le 0$ and 0 otherwise. The function lies on an open set $D_1 = \{z \in \Re : -k_{\mathrm{a}} < z < k_{\mathrm{b}}\}$. We introduce a transformed barrier Lyapunov function (TBLF) defined on the domain $D_2 = \{e_z \in \Re : -k_{\mathrm{a}} < e_z < k_{\mathrm{b}}\}$, where $e_z = f(z)$ and $\psi_z \in \Re$, as follows: $V(e_z) = 0.5(q(e_z)k_{z,1} \ln k_{\mathrm{a}}^2 (k_{\mathrm{a}}^2 - e_z^2) + (1-q(e_z))k_{z,1} \ln(k_{\mathrm{b}}^2 (k_{\mathrm{b}}^2 - e_z^2)))$. It can be investigated that $\min V = 0$ at $y = \arg\min_y V = 0$. One can rewrite V as the function of z, i.e., $V(e_z) = 0.5(q(e_z)k_{z,1} \ln k_{\mathrm{a}}^2 (k_{\mathrm{a}}^2 - e_z^2)^{-1} + (1-q(e_z))k_{z,1} \times \ln k_{\mathrm{b}}^2 (k_{\mathrm{b}}^2 - e_z^2)^{-1})$. Consequently, $\min V = 0$ at $\arg\min_z V = \psi_z \beta_z^{-1}$. The time-derivative of $V(e_z)$ is $\dot{V} = (k_{z,1} q(y) e_z \beta_z (k_{\mathrm{a}}^2 - e_z^2)^{-1} + k_{z,1}(1-q(y)) e_z \beta_z (k_{\mathrm{a}}^2 - e_z^2)^{-1}) \times \dot{e}_z$.

Proposition 2: Consider the TBLF $V(e_z)$ and the time derivative of V, i.e., $\dot{V}$. The equilibrium of $z = \psi_z \beta_z^{-1}$ will be asymptotically stable if for $z \le \psi_z \beta_z^{-1}$, $\dot{z} < (k_{\mathrm{a}}^2 - (\beta_z z - \psi_z)^2)$, and for $z > \psi_z \beta_z^{-1}$, $\dot{z} > -(k_{\mathrm{b}}^2 - (\beta_z z - \psi_z)^2)$.

Proof: It is obvious that $V(e_z)$ is positive definite. We need to determine the necessary condition such that $\dot{V}$ is negative if $z \ne \psi_z \beta_z^{-1}$ and zero otherwise. For $z \le \psi_z \beta_z^{-1}$, the term $\beta_z z - \psi_z$ is negative. Choosing the upper limit of $\dot{z}$ is $\sup z = (k_{\mathrm{a}}^2 - (\beta_z z - \psi_z)^2)$ and applying $\dot{z} < \sup z$ yields $\dot{V} = k_{z,1}(\beta_z z - \psi_z)\beta_z < 0$. For $z > \psi_z \beta_z^{-1}$, the term $\beta_z z - \psi_z$ is positive. Therefore, we choose the lower limit of $\dot{z}$ as $\dot{z}^* = -(k_{\mathrm{b}}^2 - (\beta_z z - \psi_z)^2)$. Selecting $\dot{z} > \dot{z}^*$ yields $\dot{V} = -k_{z,1}(\beta_z z - \psi_z)\beta_z < 0$.

B. Trajectory Tracking Control Design

In this study, we introduce a co-reference directing the vehicle to the right way such that the vehicle could approach its reference from the right direction. The main idea is that the co-reference will take a proper place such that the vehicle must get closer to the guide prior to approaching the reference (see Fig. 2). Suppose that the guide is located at point $E(x_{\mathrm{e}}, y_{\mathrm{e}}, \theta_{\mathrm{e}})$, where $(x_{\mathrm{e}}, y_{\mathrm{e}})$ and θ_{e} are the position and orientation of the, as shown in Fig. fff5. Let us define the following parameters. Roughly speaking, there must be a rule such that the co-reference is always in the rear side of the reference. In other words, the position of the co-reference must satisfy $\theta_{\mathrm{r}} - \arctan 2(y_{\mathrm{r}} - y_{\mathrm{e}}, x_{\mathrm{r}} - x_{\mathrm{e}}) \in (-\pi, \pi)$.

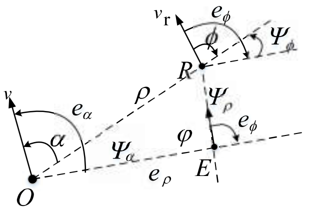

Figure 2. The vehicle O will move towards guidance point *E* while *E* moves towards the reference *R*.

As shown in Fig. 2, the co-reference E performs new navigation variables, i.e., $(\psi_\rho, \psi_\alpha, \psi_\phi)$, where ψ_ρ is the distance between the vehicle to the co-reference, ψ_α is the inclination angle of the vehicle's direction with respect to the vehicle-to-co-reference line, and ψ_ϕ is the direction of the co-reference with respect to the vehicle-to-co-reference line.

As the consequence of the co-reference existence, we formulate new navigation variables e_ρ, e_α, and e_ϕ as follows: $e_\rho = \sqrt{(y_e - y)^2 + (x_e - x)^2}$, $e_\alpha = \arctan 2(y_e - y, x_e - x) - \theta$, and $e_\phi = \theta_{\mathrm{g}} - \arctan 2(y_e - y, x_e - x)$. We construct the relationship between the new navigation variables and the original ones as $e_\rho = \beta_\rho \rho - \psi_\rho$, $e_\alpha = \hbar(\beta_\alpha \alpha - \psi_\alpha)$, and $e_\phi = \hbar(\beta_\phi \phi - \psi_\phi)$, where $\hbar(z) = z$ if $z \in (-\pi, \pi]$ and $\hbar(z) = z - 2\mathrm{sign}(x)\pi$, otherwise.

In order to build transformed Lyapunov functions (TBLFs), we define $\rho_{\mathrm{a}} \in \Re^+$ and $\rho_{\mathrm{b}} \in \Re^+$, $\alpha_{\mathrm{a}} \in (-\pi, 0)$ and $\alpha_{\mathrm{b}} \in [0, \pi]$, and $\phi_{\mathrm{a}} \in (-\pi, 0)$ and $\phi_{\mathrm{b}} \in [0, \pi]$ as the lower and upper limits of ρ, α, and ϕ, respectively; and ψ_ρ, ψ_α, and ψ_ϕ are the equilibrium points of ρ, α, and ϕ, respectively, and are assumed as constant at a time instance. Therefore, the changing rate of e_ρ, e_α, and e_ϕ with respect to time can be expressed as follows: $\dot{e}_\rho = \beta_\rho (v_{\mathrm{r}} \cos\phi - v_{\mathrm{dr}} \cos\alpha \cos\delta)$, $\dot{e}_\alpha = \beta_\alpha \left(-v_{\mathrm{r}} \rho^{-1} \mathrm{s\ i}\,\phi + v_{\mathrm{dr}} \rho^{-1} \sin\alpha \cos\delta + v_{\mathrm{dr}} l^{-1} \sin\delta\right)$, and $\dot{e}_\phi = \beta_\phi (v_{\mathrm{r}} \rho^{-1} \times \sin\phi - v_{\mathrm{dr}} \rho^{-1} \sin\alpha \cos\delta + \omega_{\mathrm{r}})$.

To solve the problem of "curse of small ρ", two types of motions are defined. The first is "ϕ-adjusting motion", i.e., the vehicle attempts to reduce ϕ to zero while keeping the distance ρ sufficiently far to avoid the large altering of α. The second one is "ρ-regulating motion". The motion is executed under almost-zero ϕ and α. Problem of scattering appears in this motion, especially when ρ is closer to zero.

Note that ψ_ρ, ψ_α, and ψ_ϕ are designated as planned ρ, α, and ϕ at which the minimum values of V_ρ, V_α, and V_ϕ are located. In other words, ψ_ρ, ψ_α, and ψ_ϕ are designated to be temporary goals for each associated navigation variables.

Let the following parameters to represent a switching mechanism, i.e., c_ρ, c_α, and c_ϕ, be designed as follows: $c_\rho = (q(e_\rho)(\rho_\mathrm{a}^2 - e_\rho^2)^{-1}(\rho_\mathrm{b}^2 - e_\rho^2)^{-1})k_\rho e_\rho$, $c_\alpha = 0$ if $e_\alpha \approx 0$ and $c_\alpha = (q(e_\alpha)\left(\alpha_\mathrm{a}^2 - e_\alpha^2\right)^{-1} + (1-q(e_\alpha)) \times(\alpha_\mathrm{b}^2 - e_\alpha^2)^{-1})k_\alpha e_\alpha$ elsewhere, and $c_\phi = 0$ if $e_\phi \approx 0$ and $c_\phi = (q(e_\phi)(\phi_\mathrm{a}^2 - e_\phi^2)^{-1} + (1-q(e_\phi))(\phi_\mathrm{b}^2 - e_\phi^2)^{-1})k_\phi e_\phi$ elsewhere. Furthermore, let us define the following parameters: $h_1 = v_\mathrm{r} e_\rho^{-1}\sin e_\phi + 1$, $h_2 = v_\mathrm{r} e_\rho^{-1}\sin e_\phi + \omega_\mathrm{r} + c_\phi$, and $h_3 = v_\mathrm{r}\cos e_\phi$. Then, we construct the following proposition.

Proposition 3: Under the free workspace, applying the following control law

$$\begin{bmatrix} v_\mathrm{dr} \\ \delta \end{bmatrix} = \begin{bmatrix} (k_{v,1}e_\rho + \exp(-k_{v,2}e_\rho))v_\mathrm{r} \\ \arctan\left(\dfrac{-(c_\alpha + h_1 - h_2)l}{(((c_\rho h_3 + 1)h_3)^2 + h_2^2)^{0.5}}\right) \end{bmatrix} \tag{3}$$

to the co-reference point *E* with the following guidance rules: $\psi_\rho(t) = \rho^* - \lambda t$, and $\psi_\alpha = -\arccos((\rho^2 + e_\rho^2 - \psi_\rho^2)/2\rho e_\rho)$, and $\psi_\phi = -\psi_\alpha$ leads the original navigation function to $\rho < \rho_\mathrm{min}$, $\alpha = 0$, $\phi = 0$.

Proof: Let us introduce a Lyapunov candidate function formulated as $V = V_\rho + V_\alpha + V_\phi$, where V_ρ, V_α, and V_ϕ are candidate TLBFs and are described as $V(e_\rho) = 0.5(q(e_\rho)k_\rho \times \ln\rho_\mathrm{a}^2(\rho_\mathrm{a}^2 - e_\rho^2)^{-1} + (1-q(e_\rho))k_\rho \ln\rho_\mathrm{b}^2(\rho_\mathrm{a}^2 - e_\rho^2)^{-1})$; $V(e_\alpha) = 0.5(q(e_\alpha)k_\alpha \ln\alpha_\mathrm{a}^2(\alpha_\mathrm{a}^2 - e_\alpha^2)^{-1} + \left(1-q(e_\alpha)\right)k_\alpha + (1-q(e_\alpha))k_\alpha \ln\alpha_\mathrm{b}^2(\alpha_\mathrm{b}^2 - e_\alpha^2)^{-1})$; $V(e_\phi) = 0.5(q(e_\phi)k_\phi \times\ln\phi_\mathrm{a}^2(\phi_\mathrm{a}^2 - e_\phi^2)^{-1}$, and $V(e_\phi) = 0.5(q(e_\phi)k_\phi \ln(\phi_\mathrm{a}^2(\phi_\mathrm{a}^2 - e_\phi^2)^{-1}) + (1-q(e_\phi))k_\phi \ln\phi_\mathrm{b}^2(\phi_\mathrm{b}^2 - e_\phi^2)^{-1})$, where $q(e_m) = 1$ if $e_m \le 0$ and 0, otherwise. Furthermore, we have $\dot{V}_\rho = c_\rho\beta_\rho^2(v_\mathrm{r}\cos e_\phi - v_\mathrm{dr}\times\cos e_\alpha\cos\delta)$, $\dot{V}_\alpha = c_\alpha\beta_\alpha^2 \times(-(v_\mathrm{r}/e_\rho)\sin e_\phi + (v_\mathrm{dr}/e_\rho)\sin e_\alpha\cos\delta + (v_\mathrm{dr}/l)\sin\delta)$, $\dot{V}_\phi = c_\phi\beta_\phi^2((v_\mathrm{r}/e_\rho)\sin e_\phi - (v_\mathrm{dr}/e_\rho)\sin e_\alpha\cos\delta + \omega_\mathrm{r})$.

Assume that V_ρ, V_α, and V_ϕ are positive definite and the time-derivatives are negative semi-definite. Consequently, if $e_\alpha \approx 0$ then $\dot{V}_\alpha = 0$ and the Lyapunov candidate function V depends only on V_ρ and V_ϕ. It means that the controller's objective is to decrease e_ρ and e_ϕ. The situation is similar for $e_\phi \approx 0$.

We choose the following equations consisting v_dr and δ, i.e., $v_\mathrm{dr}\cos e_\alpha\cos\delta = (c_\rho h_3 + 1)h_3$, $(e_\rho^{-1}\sin e_\alpha\cos\delta + l^{-1}\sin\delta)v_\mathrm{dr} = -c_\alpha + h_1$, $h_2 = v_\mathrm{dr}e_\rho^{-1}\sin e_\alpha \times\cos\delta$ and we obtain $\dot{V}_\alpha = -c_\alpha^2\beta_\alpha^2$, $\dot{V}_\phi = -c_\phi^2\beta_\phi^2$, $\dot{V}_\rho = -c_\rho^2\beta_\rho^2 v_\mathrm{r}^2\cos^2 e_\phi$, respectively. Therefore, the equilibrium $(e_\rho, e_\alpha, e_\phi) = (0,0,0)$ is stable. Eliminating the term $v_\mathrm{dr}e_\rho^{-1}\sin e_\alpha\cos\delta$ and assuming that δ is in $-\pi/2 \le \delta \le \pi/2$ gives us the formulation for the steering angle as shown in (3). Since $\dot{V}_\rho$, $\dot{V}_\alpha$, and $\dot{V}_\phi$ guarantee that the navigation variables are bounded near the equilibrium for any v_dr, then we can select any v_dr to guarantee that $\dot{e}_\rho < 0$ for $e_\rho > 0$. By applying the following necessary condition for $\dot{e}_\rho < 0$, we have $(k_{v,1}e_\rho + \exp(-k_{v,2}e_\rho))\cos\alpha\cos\delta > \cos\phi$. Suppose that $\alpha = 0$, $\phi = 0$. Then the values of $k_{v,1}$ and $k_{v,2}$ must satisfy $k_{v,1}e_\rho > 1 - \exp(-k_{v,2}e_\rho)$.

V. SIMULATION RESULTS

A simulation was conducted to evaluate the performance of the designed control strategy. In the simulation, a reference starts from $(30\,\mathrm{m}, 10\,\mathrm{m}, \pi\,\mathrm{rad})$ and move to the direction of $x-$ with linear velocity $v_\mathrm{r} = 2\,\mathrm{m/s}$. The vehicle starts from $(20\,\mathrm{m}, 20\,\mathrm{m}, \pi/2\,\mathrm{rad})$. In the simulation, the values of k_ρ, k_α, and k_ϕ are set such that $c_\rho = -\cos e_\phi / v_\mathrm{r}$, $c_\alpha = -\tan e_\alpha / l + c_\phi + \omega_\mathrm{r} + 1$, and $c_\phi = 1$, respectively, especially at $e_\rho \approx 0$, $e_\alpha \approx 0$, and $e_\phi \approx 0$. For other situation, their values are $k_\rho = 1$, $k_\alpha = 1$, and $k_\phi = 1$. The resulted trajectory, vehicle- and co-reference-to-revealed in Figs. 3, 4, and 5, respectively. As shown in Fig 4, reference distances, and the orientation navigation angles arethe co-reference's initial distance to the reference is $\rho^* = 20$ m and is constantly decreased linearly with respect to time with $\lambda = 4$. The important result here is that if $|\phi| > \pi/2$, instead of directly moving towards the reference, the vehicle moves to the co-reference. That is the reason that in Fig 4, the vehicle-to-reference reduces slowly (from t=3s to t=9s).

VI. CONCLUSIONS

A tracking control strategy for moving reference is presented. The main feature of the proposed method is the use of guidance points positioned behind the reference point. For controller design, a type of Barrier Lyapunov Function (BLF) is used. A simulation is performed and the

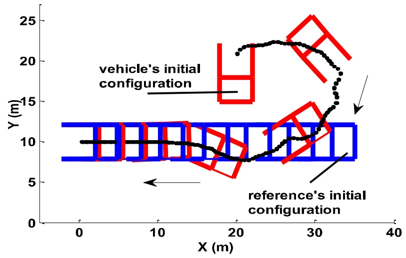

Figure 3. Resulted trajectory.

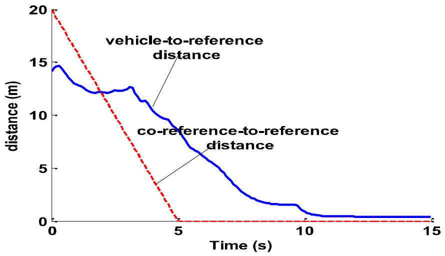

Figure 4. Vehicle-to-reference distance (solid) and co-reference-to-reference distance (dashed).

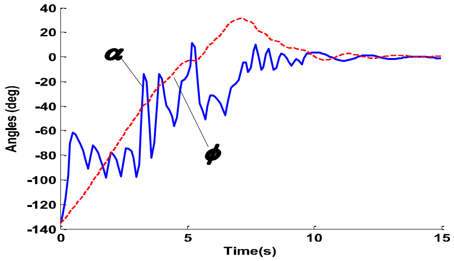

Figure 5. Resulted α and ϕ.

results show that the proposed control strategy is stable. The future works are planned to extend the control strategy to the case of adversarial workspaces.

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