European Scientific Journal May 2014 edition vol.10, No.15 ISSN: 1857 – 7881 (Print) e - ISSN 1857- 7431

INTUITIONISTIC FUZZY SET AND ITS APPLICATION IN CAREER DETERMINATION VIA NORMALIZED EUCLIDEAN DISTANCE METHOD

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Abstract

Intuitionistic fuzzy set (IFS) is very useful in providing a flexible model to elaborate uncertainty and vagueness involved in decision making. In this paper, we reviewed the concept of IFS and proposed its application in career determination using normalized Euclidean distance method to measure the distance between each student and each career respectively. Solution is obtained by looking for the smallest distance between each student and each career.

Keywords: Fuzzy sets, intuitionistic fuzzy sets, career choice, career determination

INTRODUCTION

Fuzzy sets (FS) introduced by (Zadeh, 1965) has showed meaningful applications in many fields of study. The idea of fuzzy set is welcome because it handles uncertainty and vagueness which Cantorian set could not address. In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. However in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some hesitation degree. Therefore, a generalization of fuzzy sets was proposed by (Atanassov, 1983, 1986) as intuitionistic fuzzy sets (IFS) which incorporate the degree of hesitation called hesitation margin (and is defined as 1 minus the sum of membership and non-membership degrees respectively). The notion of defining intuitionistic fuzzy set as generalized fuzzy set is quite interesting and useful in many application areas. The knowledge and semantic representation of intuitionistic fuzzy set become more meaningful, resourceful and applicable since it includes the degree of belongingness, degree of non-belongingness and the hesitation margin (Atanassov, 1994, 1999). Szmidt and Kacprzyk (2001) showed that intuitionistic fuzzy sets are pretty useful in situations when description of a problem by a linguistic variable given in terms of a membership function only seems too rough. Due to the flexibility of IFS in handling uncertainty, they are tool for a more human consistent reasoning under imperfectly defined facts and imprecise knowledge (Szmidt and Kacprzyk, 2004).

De *et al* (2001) gave an intuitionistic fuzzy sets approach in medical diagnosis using three steps such as; determination of symptoms, formulation of medical knowledge based on intuitionistic fuzzy relations, and determination of diagnosis on the basis of composition of intuitionistic fuzzy relations. Intuitionistic fuzzy set is a tool in modelling real life problems like sale analysis, new product marketing, financial services, negotiation process, psychological investigations etc. since there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object (Szmidt and Kacprzyk, 1997, 2001). Atanassov (1999, 2012) carried out rigorous research based on the theory and applications of intuitionistic fuzzy sets. Many applications of IFS are carried out using distance measures approach. Distance measure between intuitionistic fuzzy sets is an important concept in fuzzy mathematics because of its wide applications in real world, such as pattern recognition, machine learning, decision making and market prediction. Many distance measures between intuitionistic fuzzy sets have been proposed and researched in recent years (Szmidt and Kacprzyk, 1997, 2000 and Wang and Xin, 2005) and used by (Szmidt and Kacprzyk, 2001, 2004) in medical diagnosis.

We show a novel application of intuitionistic fuzzy set in a more challenging area of decision making (i.e. career choice). An example of career determination will be presented, assuming there is a database (i.e. a description of a set of subjects S, and a set of careers C). We will describe the state of students knowing the results of their performance. The problem description uses the concept of IFS that makes it possible to render two important facts. First, values of each subject performance changes for each student. Second, in a career determination database describing career for different students, it should be taken into account that for different students aiming for the same career, values of the same subject performance can be different. We use the normalized Euclidean distance method given in (Szmidt and Kacprzyk, 1997, 2000, 2014) to measure the distance between each student and each career. The smallest obtained value, points out a proper career determination based on academic performance.

CONCEPT OF INTUITIONISTIC FUZZY SETS

Definition 1 (Zadeh, 1965): Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{\langle x, \mu_A(x)\rangle : x \in X\}$, where $\mu_A(x)$: $X \longrightarrow [0,1]$ is the membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership i.e. having degrees of membership.

Definition 2 (Atanassov, 1999): Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x), \nu_A(x)$: $X \longrightarrow [0,1]$define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1$. Furthermore, we have $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x \in X$ to the IFS A and $\pi_A(x) \in [0,1]$ i.e.,$\pi_A(x)$: $X \longrightarrow [0,1]$ and $0 \leq \pi_A \leq 1$ for every $x \in X$. $\pi_A(x)$ expresses the lack of knowledge of whether x belongs to IFS A or not.

For example, let A be an intuitionistic fuzzy set with $\mu_A(x) = 0.5$ and $\nu_A(x) = 0.3 \Rightarrow \pi_A(x) = 1 - (0.5 + 0.3) = 0.2$. It can be interpreted as "the degree that the object x belongs to IFS A is 0.5, the degree that the object x does not belong to IFS A is 0.3 and the degree of hesitancy is 0.2".

BASIC RELATIONS AND OPERATIONS ON INTUITIONISTIC FUZZY SETS

1. [inclusion] $A \subseteq B \leftrightarrow \mu_A(x) \leq \mu_B(x)\ and\ \nu_A(x) \geq \nu_B(x)\ \forall x \in X$
2. [equality] $A = B \leftrightarrow \mu_A(x) = \mu_B(x)\ and\ \nu_A(x) = \nu_B(x)\ \forall x \in X$
3. [complement] $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$
4. [union]
   $A \cup B = \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle : x \in X\}$
5. [intersection]
   $A \cap B = \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle : x \in X\}$
6. [addition] $A \oplus B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x)\rangle$: $x \in X\}$
7. [multiplication] $A \otimes B = \{\langle x, \mu_A(x)\mu_B(x),$
   $\nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)\rangle$: $x \in X\}$

8. [difference] $A - B = \{x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle : x \in X\}$
9. [symmetric difference] $A\Delta B = \{\langle x, max[\min(\mu_A, \nu_B), \min(\mu_B, \nu_A)], min[\max(\nu_A, \mu_B), \max(\nu_B, \mu_A)]\rangle : x \in X\}$
10. [Cartesian product] $A \times B = \{\langle \mu_A(x)\mu_B(x), \nu_A(x)\nu_B(x)\rangle : x \in X\}$.

Theorem 1: Let A and B be two IFS in a nonempty set X, then; (i) $A - B = A \cap B^c$
(ii) $A - B = B - A$ iff $A = B$ (iii) $A - B = B^c - A^c$.

Proof: (i) Let $A = \{\langle \mu_A(x), \nu_A(x)\rangle x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle x \in X\}$ for $A, B \in X$, then $A - B = \{x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x)), \mu_B(x))\rangle x \in X\}$. But $B^c = \{\langle x, \nu_B(x), \mu_B(x)\rangle x \in X\}, \Rightarrow$
$A \cap B^c = \{x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle x \in X\}$ since $A \cap B = \{x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x))\rangle x \in X\}$. The result follows.

(ii) $A - B = \{x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle x \in X\}$. If $A = B \Rightarrow \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)\ \forall x \in X$. From this, it is certain that $B - A = A - B$ and the result follows.

(iii) $A - B = \{x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x))\rangle x \in X\}$. Given that, $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle x \in X\}$ and $B^c = \{\langle x, \nu_B(x), \mu_B(x)\rangle x \in X\}$, it implies that, $B^c - A^c = \{x, \min(\nu_B(x), \mu_A(x)), \max(\mu_B(x), \nu_A(x))\rangle x \in X\}$ and the result is straightforward.

Corollary 1: Whenever $A = B$, $A\,\Delta\,B = B\,\Delta\,A\ \forall A, B \in X$.

Proof is straightforward from the proof of theorem 1 (ii).

Theorem 2: Let A and B be two IFS in a nonempty set X, then;
(i)$A - A = \Phi$ (ii) $A - \Phi = A$ (iii) $A - B \subseteq A$ (iv) $A - B = \Phi$ iff $A = B$ (v) $A - B = A$ iff $B = \Phi$ (vi) $A - B = A$ iff $A \cap B = \Phi$

It is easy to prove the above results.

Theorem 3: For IFS A, B, C in X and$A \subseteq B \subseteq C$, then we have; (i) $B - A \subseteq C - A$ (ii)$B\,\Delta A \subseteq C\,\Delta A$.

Proof: (i) Given that $A, B, C \in X$ and $A \subseteq B \subseteq C \Rightarrow$ "$\subseteq$" is transitive i.e. $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$ i.e. $\mu_A(x) \leq \mu_B(x) \leq \mu_C(x)$ and $\nu_A(x) \geq \nu_B(x) \geq \nu_C(x)\quad \forall x \in X$. Since A is the smallest of B and C, subtracting A from both side of $B \subseteq C$ means nothing, i.e. $\Rightarrow B - A \subseteq C - A$. The result follows.

Since " Δ " is the extension of "$-$", the result of (ii) follows.

Corollary 2: From the basic operations, we deduced the following relations:

1 $A \times B = B \times A$

2 $(A \times B) \times C = A \times (B \times C)$

3 $A \times (B \cup C) = (A \times B) \cup (A \times C)$

4 $A \times (B \cap C) = (A \times B) \cap (A \times C)$

5 $A \times (B \oplus C) = (A \times B) \oplus (A \times C)$

6 $A \times (B \otimes C) = (A \times B) \otimes (A \times C)$

ALGEBRA LAWS IN INTUITIONISTIC FUZZY SETS

Let A, B and C be IFS in X, then the following algebra follow:

1 Complementary Law: $(A^c)^c = A$

2 Idempotent Laws: $(i) A \cup A = A$ $(ii) A \cap A = A$

3 Commutative Laws: (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$ $(iii) A \oplus B = B \oplus A$ (iv) $A \otimes B = B \otimes A$

4 Associative Laws:

$(i) (A \cup B) \cup C = A \cup (B \cup C)$ (ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (iii) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$ (iv) $A \otimes (B \otimes C) = (A \otimes B) \otimes C$

5 Distributive Laws:

$(i) A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $(ii) A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iii) $A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$ (iv) $A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$

(v) $A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C)$ (vi) $A \otimes (B \cap C) = (A \otimes B) \cap (A \otimes C)$

6 De Morgan's laws:

(i) $(A \cup B)^c = A^c \cap B^c$ (ii) $(A \cap B)^c = A^c \cup B^c$ (iii) $(A \oplus B)^c = A^c \otimes B^c$ (iv) $(A \otimes B)^c = A^c \oplus B^c$

7 Absorption Laws: (i) $A \cap (A \cup B) = A$ (ii) $A \cup (A \cap B) = A$

8 (i) $\Phi^c = X$ (ii) $X^c = \Phi$ (iii) $A \cup \Phi = A$ (iv) $A \cap \Phi = \Phi$ (v) $A \cap A^c = \Phi$

9 (i) $A \cup X = X$ (ii) $A \cup A^c = X$ (iii) $A \cap X = A$

Note: Distributive Laws hold for both right and left hands. The proofs follow from the basic operations.

Theorem 4: Let A, B, C be IFS in X and $B \subseteq C$, then we have; (i) $A \oplus B \subseteq A \oplus C$ (ii) $A \otimes B \subseteq A \otimes C$ (iii) $A \cup B \subseteq A \cup C$ (iv) $A \cap B \subseteq A \cap C$.

Proof: **(i)** Given that $A, B, C \in X$ and $B \subseteq C$, it means $\mu_B(x) \leq \mu_C(x)$ and $\nu_B(x) \geq \nu_C(x)$ for every $x \in X$. If another IFS $A \in X$ is added to $B \subseteq C$, it is certain that, $A \oplus B \subseteq A \oplus C$ and the result follows.

Results of (ii) – (iv) follow from the proof of (i)

Theorem 5: Let A and B be IFS in X, then (i) $A \cap B = A$ or $A \cap B = B$, (ii) $A \cup B = A$ or $A \cup B = B$ iff $A = B$.

Proof: (i) For $A, B, C \in X$, it implies that $A \cap B \in X$. If $A = B \Rightarrow \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x) \quad \forall x \in X$. Since $A = B$, from idempotent laws, $A \cap B = A$ or $A \cap B = B$. The result follows.

Result of (ii) follows from (i).

Definition 3 (Szmidt and Kacprzyk, 2014): The normalized Euclidean distance d_{n-H} (A, B) between two IFS A and B is defined as

$$d_{n-H}(A, B) = \left(\frac{1}{2n} \sum_{i=1}^{n} \left[(\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2 \right]\right)^{\frac{1}{2}}$$

, $X = \{x_1, x_2, \ldots, x_n\}$ for $i = 1, 2, \ldots, n$.

APPLICATION OF INTUITIONISTIC FUZZY SETS IN CAREER DETERMINATION

The essence of providing adequate information to students for proper career choice cannot be overemphasized. This is paramount because the numerous problems of lack of proper career guide faced by students are of great consequence on their career choice and efficiency. Therefore, it is expedient that students be given sufficient information on career determination or choice to enhance adequate planning, preparation and proficiency. Among the career determining factors such as academic performance, interest, personality make-up etc.; the first mentioned seems to be overriding. We use intuitionistic fuzzy sets as tool since it incorporate the membership degree (i.e. the marks of the questions answered by the student), the non-membership degree (i.e. the marks of the questions the student failed) and the hesitation degree (which is the mark allocated to the questions the student do not attempt).

Let $S = \{s_1, s_2, s_3, s_4\}$ be the set of students, $C =$ {medicine, pharmacy, surgery, anatomy} be the set of careers and Su ={English Language, Mathematics, Biology, Physics, Chemistry} be the set of subjects related to the careers. We assume the above students sit for examinations (i.e. over 100 marks total) on the above mentioned subjects to determine their career placements and choices. The table below shows careers and related subjects requirements.

Table 1 Careers vs Subjects

	English Language	Mathematics	Biology	Physics	Chemistry
medicine	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.9,0.0,0.1)	(0.6,0.3,0.1)	(0.8,0.1,0.1)
pharmacy	(0.9,0.1,0.0)	(0.8,0.1,0.1)	(0.8,0.1,0.1)	(0.5,0.3,0.2)	(0.7,0.2,0.1)
surgery	(0.5,0.3,0.2)	(0.5,0.2,0.3)	(0.9,0.0,0.1)	(0.5,0.4,0.1)	(0.7,0.1,0.2)
anatomy	(0.7,0.2,0.1)	(0.5,0.4,0.1)	(0.9,0.1,0.0)	(0.6,0.3,01)	(0.8,0.0,0.2)

Each performance is described by three numbers i.e. membership μ, non-membership ν and hesitation margin π. After the various examinations, the students obtained the following marks as shown in the table below.

Table 2 Students vs Subjects

	English Language	Mathematics	Biology	Physics	Chemistry
S_1	(0.6,0.3,0.1)	(0.5,0.4,0.1)	(0.6,0.2,0.2)	(0.5,0.3,0.2)	(0.5,0.5,0.0)
S_2	(0.5,0.3,0.2)	(0.6,0.2,0.2)	(0.5,0.3,0.2)	(0.4,0.5,0.1)	(0.7,0.2,0.1)
S_3	(0.7,0.1,0.2)	(0.6,0.3,0.1)	(0.7,0.1,0.2)	(0.5,0.4,0.1)	(0.4,0.5,0.1)
S_4	(0.6,0.4,0.0)	(0.8,0.1,0.1)	(0.6,0.0,0.4)	(0.6,0.3,0.1)	(0.5,0.3,0.2)

Using Def. 3 above to calculate the distance between each student and each career with reference to the subjects, we get the table below.

Table 3 Students vs Careers

	medicine	pharmacy	surgery	Anatomy
S_1	0.1006	0.0897	0.0904	0.0805
S_2	0.0738	0.0856	0.0591	0.0954
S_3	0.0781	0.0805	0.0811	0.0857
S_4	0.0831	0.0749	0.0928	0.1014

From the above table, the shortest distance gives the proper career determination. S_1 is to read anatomy (anatomist), S_2 is to read surgery (surgeon), S_3 is to read medicine (doctor), and S_4 is to read pharmacy (pharmacist).

Conclusion

This novel application of intuitionistic fuzzy sets in career determination is of great significance because it provides accurate and proper career choice based on academic performance. Career choice is a delicate decision making problem since it has a reverberatory effect on efficiency and competency if not properly handled. In the proposed application, we used normalized Euclidean distance to calculate the distance of each student from each career in respect to the subjects, to obtained results.

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