SIMULATION OF LIGHTWAVE PROPAGATION IN PHOTONIC DEVICES

by

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Loss analysis is based on full vectorial H-field computation using finite element with perfectly match layer boundary condition. In the grating waveguide, (a) TM and (b) TE mode losses at waveguide width of 6.5 $\mu$m are 1.85 dB/mm and 1.93 dB/mm respectively. Mode loss in resonator at slab width of 1.212 $\mu$m for TM mode is 1.2 dB/mm.

The splitting ratio for TM (SR$_{TM}$) and TE (SR$_{TE}$) modes computed in the fabrication tolerance of the (a) resonator width, (b) resonator’s gap, (c) grating waveguide slab width, and (d) grating waveguide gap spacing. The SR value of 13 dB is the minimum acceptable SR for a practical polarization diverse scheme.
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Figure 7.9 The solid lines represent the normalized transmitted output power computed versus the length of 1-D PhC MMI section, for (a) W8, (b) W10 and (c) W14 multimode PhC waveguide couplers respectively. The smoothly varying dashed lines are the output responses as predicted by multimode interference without the effect of multiple reflection and mode coupling at the input/output junctions.

Figure 7.10 The field intensity profile of (a) W8, (b) W10 and (c) W14 waveguides. The length of the 1-D PhC MMI section shown for all the waveguides is 40 $\mu$m. The surface plots of the field intensities follow the same color legend as shown in Figure 7.8.

Figure 8.1 The WDM based on hybrid PhC directional coupler with one row of dielectric rod in between the coupled line defect channel waveguide region. Sections (a) and (b) are the 1-D PhC and 2-D PhC interaction regions respectively.

Figure 8.2 The Bloch mode PBG diagram for the 2-D PhC input/output waveguides. The defect mode dispersion curve is shown as dashed-dot line. The normalized frequencies of 0.277 and 0.328 correspond to the wavelengths 1.55 $\mu$m and 1.31 $\mu$m.

Figure 8.3 1-D PhC slab mode photonic bandgap diagram computed using EME. The bandgap is between the upper and lower bands of modes. The dashed arrows show two defect supermodes in the coupled line defect 1-D PhC at $f$= 0.328 ($\lambda$=1.31 $\mu$m) and 0.277 ($\lambda$=1.55 $\mu$m) respectively.

Figure 8.4 The odd and even parity modes in the coupled 1-D PhC waveguide at wavelength of 1.55 $\mu$m.

Figure 8.5 The coupling length of the 1-D PhC dashed line) and 2-D PhC (solid line).

Figure 8.6 The optimization curves of the 1-D PhC coupling length. The dashed line is the coupling length at wavelength 1.31 $\mu$m and the solid line shown is 2 $\times$ coupling length at wavelength 1.55 $\mu$m, computed at various waveguide widths.
Figure 8.7 The sections $l_1$, $l_3$, $l_4$ and $l_5$ were optimized for maximum TM modes transmission at 1.31 $\mu$m and 1.55 $\mu$m exiting port 3 and 2 respectively.

Figure 8.8. The transmission spectrum of a 1.31 $\mu$m/1.55 $\mu$m WDM with interaction length $l_1 = 12.33$ $\mu$m and $l_2 = 2.15$ $\mu$m calculated by (a) EME and (b) FEM. In both the figures, the dashed and dashed-dot lines are outputs for ports 2 and 3 respectively, as calculated by CMT. The solid and dotted lines are outputs for ports 2 and 3 respectively, as calculated by (a) EME and (b) FEM.

Figure 8.9 (a) The 2-D PhC WDM design. (b) The transmission spectrum of a purely 2-D WDM with interaction length $l = 35a$. In the figure, the dashed and dashed-dot lines are outputs for ports 2 and 3 respectively, as calculated by CMT. And the solid and dotted lines are outputs for ports 2 and 3 respectively, as calculated by EME.

Figure 8.10 (a) The normalized transmitted power of 1-D and 2-D PhC WDM with interaction length 14 $\mu$m and 15.23 $\mu$m respectively, as shown in Figures 8.10(b) and (c). The dashed and dotted lines represent the transmitted output signals through ports 2 and 3 for the 1-D PhC WDM, respectively. Similarly the solid and dashed-dot lines are the transmitted signals through ports 2 and 3 in the 2-D PhC WDM, respectively. The maximum transmission through ports 2 and 3, are 1.55 $\mu$m and 1.31 $\mu$m wavelengths respectively.

Figure 8.11 (a) The hybrid PhC directional coupler model used in the simulation of the channel interleaver. (b) The output signals transmission spectrum of the channel interleaver with interaction length $l_1 = 2800$ $\mu$m and $l_2 = 2.15$ $\mu$m. In figure 8.14(b) the dotted and solid lines are transmitted signal for output ports 2 and 3 respectively, calculated by EME.

Figure 8.12 The field intensity pattern observed in the hybrid WDM with the input pulse excited in port 1 for wavelengths (a) 1.55 $\mu$m and (b) 1.31 $\mu$m respectively, as computed by FEM. The color legend is shown below the figures.

Figure A1-1 Bloch waves (a), (b), (c) and (d) observed at the band edges of the (e) PBG diagram, were computed at normalized frequency of 0.43 (wavelength at 1 $\mu$m) for five array of periodic dielectric slabs. At normalized frequency of 0.277 (wavelength at 1.55 $\mu$m) only the lower band of modes are present.
Figure A1-2 The 1-D PhC PBG diagram and the Bloch waves were computed using CAMFR [61]. The first and second bands of modes were obtained by applying PMC and PEC boundary condition at the symmetry of the outer most dielectric layers respectively.

Figure A1-3 The individual slab waveguides are shown with its line of symmetry and the relevant boundary conditions that preserve its field profiles. The modal field profiles are shown for (a) first and (b) second modes in the slab waveguide.
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARROW</td>
<td>Anti-reflection resonant optical waveguide</td>
</tr>
<tr>
<td>AWG</td>
<td>Array waveguide gratings</td>
</tr>
<tr>
<td>BC</td>
<td>Boundary condition</td>
</tr>
<tr>
<td>BPM</td>
<td>Beam propagation method</td>
</tr>
<tr>
<td>CAMFR</td>
<td>Cavity Modelling Framework</td>
</tr>
<tr>
<td>CF</td>
<td>Confinement factor</td>
</tr>
<tr>
<td>CMT</td>
<td>Coupled mode theory</td>
</tr>
<tr>
<td>DWDM</td>
<td>Dense wavelength division multiplexing</td>
</tr>
<tr>
<td>EDFA</td>
<td>Erbium doped fiber amplifier</td>
</tr>
<tr>
<td>EIM</td>
<td>Effective index method</td>
</tr>
<tr>
<td>EME</td>
<td>Eigen-mode expansion</td>
</tr>
<tr>
<td>ER</td>
<td>Extinction ratio</td>
</tr>
<tr>
<td>FB</td>
<td>Floquet-Bloch</td>
</tr>
<tr>
<td>FDM</td>
<td>Finite difference method</td>
</tr>
<tr>
<td>FDTD</td>
<td>Finite difference time domain</td>
</tr>
<tr>
<td>FE</td>
<td>Finite element</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite element method</td>
</tr>
<tr>
<td>FMM</td>
<td>Film mode matching</td>
</tr>
<tr>
<td>MMI</td>
<td>Multimode interference</td>
</tr>
<tr>
<td>OTDM</td>
<td>Optical time-domain multiplexing</td>
</tr>
<tr>
<td>PBC</td>
<td>Periodic boundary condition</td>
</tr>
<tr>
<td>PBG</td>
<td>Photonic bandgap</td>
</tr>
<tr>
<td>PBI</td>
<td>Polarization bit interleaving</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>PEC</td>
<td>Perfect electric conductor</td>
</tr>
<tr>
<td>PhC</td>
<td>Photonic crystal</td>
</tr>
<tr>
<td>PIC</td>
<td>Photonic integrated circuits</td>
</tr>
<tr>
<td>PMC</td>
<td>Perfect magnetic conductor</td>
</tr>
<tr>
<td>PML</td>
<td>Perfectly matched layer</td>
</tr>
<tr>
<td>PR</td>
<td>Polarization rotator</td>
</tr>
<tr>
<td>PS</td>
<td>Polarization splitter</td>
</tr>
<tr>
<td>SOI</td>
<td>Silicon-on-insulator</td>
</tr>
<tr>
<td>SR</td>
<td>Splitting ratio</td>
</tr>
<tr>
<td>TBC</td>
<td>Transparent boundary condition</td>
</tr>
<tr>
<td>TE</td>
<td>Transverse electric</td>
</tr>
<tr>
<td>TM</td>
<td>Transverse magnetic</td>
</tr>
<tr>
<td>WDM</td>
<td>Wavelength division multiplexers</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>-------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>AlGaAs</td>
<td>Aluminium gallium arsenide</td>
</tr>
<tr>
<td>$A$</td>
<td>Amplitude</td>
</tr>
<tr>
<td>$A_r$</td>
<td>Amplitude coefficient for partly reflected wave</td>
</tr>
<tr>
<td>$B_{m,1}^{**}$</td>
<td>Amplitude coefficient matrix for partly transmitted wave</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
</tr>
<tr>
<td>$a_m^-$, $b_m^-$</td>
<td>Backward amplitudes of the traveling waves</td>
</tr>
<tr>
<td>$[A], [B]$</td>
<td>Band matrix, sparse and symmetric</td>
</tr>
<tr>
<td>$L_{B,i}$</td>
<td>Beat length, and the subscript $i$ is either 1 or 2 representing 1-D or 2-D PhC coupling sections</td>
</tr>
<tr>
<td>$E_k$</td>
<td>Bloch wave electric field</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Conductivity of material</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Correction factor in Soref et. al. [22] single mode equation</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Coupling coefficient, and $i = 1$ and 2, denote the 1-D and 2-D PhC coupling sections respectively.</td>
</tr>
<tr>
<td>$l'_c1$, $l'_c2$</td>
<td>Coupling length at 1.31 $\mu$m wavelength, subscript 1 and 2 representing 1-D or 2-D PhC coupling sections respectively.</td>
</tr>
<tr>
<td>$l_c1$, $l_c2$</td>
<td>Coupling length at 1.55 $\mu$m wavelength, subscript 1 and 2 representing 1-D or 2-D PhC coupling sections respectively.</td>
</tr>
<tr>
<td>$L_{c,\lambda}$</td>
<td>Coupling length of the coupled section at the respective wavelengths</td>
</tr>
<tr>
<td>$\Gamma, M, X$</td>
<td>Critical points comprising irreducible Brillouin zone</td>
</tr>
<tr>
<td>dB</td>
<td>Decibel</td>
</tr>
<tr>
<td>W2, W8, W10, W14</td>
<td>Defect channel waveguide. Denote by Wn where n is the number of defect (or missing) rows</td>
</tr>
</tbody>
</table>
\( s, s_1 \) Defect or gap width

\( \nabla \) Del (nabla) operator

\( \varepsilon \) Dielectric constant of material

\( \Delta \beta \) Difference in propagation constant

\( r \) Direction of propagation

\( n_{\text{eff}} \) Effective index

\( E_x, E_y, E_z \) Electric field components for each rectangular coordinate

\( E \) Electric field vector

\( \text{Er} \) Erbium

\( \text{ER}_{1.55 \mu m}, \text{ER}_{1.31 \mu m} \) Extinction ratios (ER) are defined as the ratio of the residue power in the other port as compared to the output power in the desired port. \( \text{ER}_{1.55 \mu m} \) and \( \text{ER}_{1.31 \mu m} \) – extinction ratio at 1.55 \( \mu m \) and 1.31 \( \mu m \) wavelengths.

\( a_{i}, a_{r}, a_{ij} \) Field amplitude coefficient

\( \psi \) Field function

\( a_m^+, b_m^+ \) Forward amplitudes of the traveling waves

\( \text{TE}_{\text{frac}} \) Fraction of the real Poynting vector flux in TE polarization to the total real Poynting vector flux.

\( \text{TM}_{\text{frac}} \) Fraction of the real Poynting vector flux in TM polarization to the total real Poynting vector flux.

\( k_0 \) Free space wave number

\( \text{GaAs} \) Gallium arsenide

\( \text{GaN} \) Gallium nitride

\( \text{Ge} \) Germanium

\( \text{GeO}_2 \) Germanium dioxide

\( L_s \) Half-beat length or the coupling length

\( h_0, h_x, h_y \) Hybridness of the first fundamental mode, quasi-TM and quasi-TE modes respectively.
\( i \) Imaginary constant

\( \text{InP} \) Indium phosphide

\( a \) Lattice constant

\( l_1, l_2, l_3, l_4 \text{ and } l_5 \) Lengths of various sections in photonic crystal directional coupler

\( \text{LiNbO}_3 \) Lithium niobate

\( H_x, H_y, H_z \) Magnetic field components for each rectangular coordinate

\( H \) Magnetic field vector

\( \mu_m \) Micrometer or micron

\( \psi_0^{(1)}(x) \) Modal wave-functions in \( W_a \) and \( W_b \) waveguide section of MMI coupler respectively

\( \psi_n^{(2)}(x) \)

\( A_0^-, B_0^-, C_0^- \) Mode coefficient amplitudes in backward direction

\( A_0^+, B_0^+, C_0^+ \) Mode coefficient amplitudes in forward direction

\( D_n \) Normal displacement field component

\( B_n \) Normal induction field component

\( n \) Normal vector

\( f \) Normalized frequency, \( a/\lambda \)

\( P_2, P_3 \) Normalized power output from ports 2 (1.55 \( \mu m \) output) and 3 (1.31 \( \mu m \) output)

\( u_k \) Periodic function

\( \mu_0 \) Permeability of free space

\( \varepsilon_0 \) Permittivity of free space

\( \phi \) Phase constant

\( P \) Phosphorus

\( d_n \) Primitive lattice vector

\( b \) Primitive reciprocal lattice vector
\( \beta, \beta_m \)  
Propagation constant

\( \beta_{\text{even},i} \) and \( \beta_{\text{odd},i} \)  
Propagation constant for the even and odd modes, and the subscript \( i \) is either 1 or 2 representing 1-D or 2-D PhC coupling sections.

\( k_x \)  
Wavevector component in x-direction

\( k_y \)  
Wavevector component in y-direction

\( P_z \)  
Real Poynting vector flux

\( k \)  
Reciprocal lattice vector

\( x \)  
Rectangular coordinate (x-axis)

\( y \)  
Rectangular coordinate (y-axis)

\( z \)  
Rectangular coordinate (z-axis)

\( R_{\text{ee}} \)  
Reflection coefficient

\( R \)  
Reflection matrices

\[ \left[r^{-}(l)_{\text{multi-refl.}} \right]_{m,n} \]  
Reflection matrices in backwards direction including multiple reflections and mode coupling between the excited modes

\[ \left[r^{+}(l)_{\text{multi-refl.}} \right]_{m,n} \]  
Reflection matrices in forward direction including multiple reflections and mode coupling between the excited modes

\[ \left[R(l)_{\text{multi-refl.}} \right]_{m,n} \]  
Reflection matrix including multiple reflections and mode coupling

\( n \)  
Refractive index of material

\( \varepsilon_r \)  
Relative dielectric constant

\( g \)  
Resonator's air gap

\( \Delta x \)  
Sampling step of FDTD in x-direction

\( \Delta y \)  
Sampling step of FDTD in y-direction

\( \Delta z \)  
Sampling step of FDTD in z-direction

\( \Delta t \)  
Sampling time interval of FDTD

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\[ \phi, F, G \quad \text{Scalar field variable} \]
\[ S \quad \text{Scattering matrix} \]
\[ \text{SiO}_2 \quad \text{Silica} \]
\[ \text{Si} \quad \text{Silicon} \]
\[ \text{SiO}_2\text{N}_2 \quad \text{Silicon oxinitride} \]
\[ r \quad \text{Slab height over rib height ratio} \]
\[ h \quad \text{Slab height or waveguide height} \]
\[ d \quad \text{Slab waveguide thickness} \]
\[ c_0 \quad \text{Speed of light} \]
\[ \rho \quad \text{Surface charge density} \]
\[ j \quad \text{Surface current density} \]
\[ t \quad \text{Time variable} \]
\[ [B^- (l)_{\text{multi-refl}}]_{m,1} \quad \text{Total backward propagation modal coefficients matrices including multiple reflections} \]
\[ \psi_{\text{Total}} (x, z) \quad \text{Total fields} \]
\[ \psi^-_{\text{Total}} (x, z, l) \quad \text{Total fields in the backward directions} \]
\[ \psi^+_{\text{Total}} (x, z, l) \quad \text{Total fields in the forward directions} \]
\[ [B^+ (l)_{\text{multi-refl}}]_{m,1} \quad \text{Total forward propagation modal coefficients matrices including multiple reflections} \]
\[ R_{\text{multi-refl}} \quad \text{Total reflection matrices including multiple reflections} \]
\[ T_{\text{multi-refl}} \quad \text{Total transmission matrices including multiple reflections} \]
\[ [T']_{m,1} \quad \text{Transmission coefficient including mode coupling} \]
\[ T \quad \text{Transmission matrices} \]
\[ [T(l)_{\text{multi-refl}}]_{1,m} \quad \text{Transmission matrix including multiple reflections and mode coupling} \]
\[ \nabla_t \quad \text{Transversal nabla operator} \]
\( t \) Transverse component
\( E_t \) Transverse electric field component
\( H_t \) Transverse magnetic field component
\( \hat{x}, \hat{y}, \hat{z} \) Unit vector in x, y and z-axes
\( H \) Waveguide/Rib height
\( W_a, W_b \) and \( W_c \) Waveguide sections in the photonic crystal MMI coupler
\( w, W_a, W_b, W \) Waveguide width
\( \lambda \) Wavelength
\( \text{YIG} \) Yittrium iron garnet
SIMULASI RAMBATAN GELOMBANG CAHAYA DI DALAM PERANTI-FOTONIK

ABSTRAK

Disertasi ini adalah tertumpu kepada simulasi rambatan gelombang cahaya dalam peranti fotonik leper, ianya termasuk penyiasatan keadaan satu mod dalam pandu gelombang rusuk, reka bentuk pemutar dan pembelah polarisasi, penyiasatan pantulan berganda dalam pengganding interferens-berganda hablur fotonik dan reka bentuk pembahagi panjang-gelombang hablur fotonik hibrid 1-D dan 2-D dan antara-silih saluran.


Secara keseluruhan, kesemua objektif yang ditetapkan dalam tesis ini adalah tercapai. Dalam penyiasatan pandu-gelombang rusuk satu mod dari gallium arsenid/aluminium gallium arsenid (GaAs/AlGaAs), formulasi satu mod bagi pandu gelombang rusuk silikon atas silica didapati adalah juga terpakai kepada pandu-gelombang rusuk GaAs/AlGaAs. Satu kaedah langsung bagi menetapkan had satu

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Akhir sekali dalam tesis ini, peranti-peranti optik yang berasaskan hablur fotonik hibrid 1-D dan 2-D telah dicadangkan dan disiasat. Dalam penyiasatan pengganding interferens berganda, kesan dari isyarat pantulan berganda dilihat sebagai sebab utama mengapa imej berkala dalam pengganding tersebut menyimpang dari prinsip pengimejan-diri. Dalam reka-bentuk pembahagi berganda panjang gelombang, satu reka-bentuk yang efisien dan padat telah dicadangkan bagi panjang gelombang 1.31/1.55 μm, dengan panjang keseluruhan peranti tersebut adalah 12.33 μm. Kecekapan pemindahan kuasa peranti tersebut adalah 91% dan nisbah kepupusan adalah -23.7 dB dan -20.8 dB bagi panjang gelombang 1.31 μm dan 1.55 μm masing-masing. Bagi peranti antara-silih saluran, dengan menggunakan panjang interaksi sepanjang 2800 μm, ianya menghasilkan renggangan saluran sebesar 0.8 nm dengan kecekapan keluaran sebanyak 90%
This dissertation focuses on the simulation of lightwave propagation in planar devices, which include investigation of single mode conditions in rib waveguide, designs of compact silica polarization rotator and splitters, investigation of multiple reflections in photonic crystal (PhC) multimode interference couplers and design of hybrid 1-D and 2-D PhC wavelength division multiplexers and channels interleavers.

In the design approach, we have adopted semi-analytical technique which allows us to incorporate fast optimization process. Global and local simplex search techniques were extensively used. In the investigation of single mode conditions in rib waveguide and the design of silica polarization rotator, full vectorial film mode matching and finite element methods were used. Propagation analysis in the silica polarization rotator was conducted using finite element eigen-mode expansion method. The quasi 2-D effective index method with eigen mode expansion method was used in the design of compact silica grating waveguide polarization splitter, investigation of multiple reflections in hybrid PhC multimode interference coupler and design of hybrid PhC wavelength division multiplexers and channel interleavers.

Basically all the objectives set in the thesis were achieved. In the investigation of single-mode gallium arsenide/aluminium gallium arsenide (GaAs/AlGaAs) rib waveguide, single-mode formulation for silicon on silica is shown to be equally applicable to GaAs/AlGaAs rib waveguide. A direct method of identifying vertical single-mode cut-off limit is also established by using power confinement factor in the waveguide. In the design and optimization of polarization controlling optical components, compact silica polarization rotator and polarization splitter were investigated. A compact silica polarization rotator of length 790 μm
with slanted sidewall angle of 46° was designed. It has an overall conversion efficiency of 99%, with very low crosstalk value of -38 dB and a power transfer efficiency of about 81%. As for the silica polarization splitter it consisted of a symmetric three channel directional coupler, with intermediate conventional waveguide at the center and two grating waveguides on the outside. A compact design was obtained with an overall length of 340 μm and splitting ratio for the TE and TM polarized signals obtained at 36 dB and 15 dB respectively.

Finally in this thesis, optical components based on hybrid 1-D and 2-D photonic crystal structure were proposed and investigated. In the investigation of hybrid photonic crystal multimode interference coupler, the effect of multiple reflections is observed to be one of the main reasons why image periodicity in the couple departs from self-imaging principle. And in the proposed wavelength division multiplexer, an efficient design was proposed for 1.31/1.55 μm wavelength, with a total device length of 12.33 μm. Its power transfer efficiency is 91% and extinction ratios of -23.7 dB and -20.8 dB at wavelength 1.31 μm and 1.55 μm respectively. For the channel interleaver, an interaction length of 2800 μm give a channel spacing of 0.8 nm wavelength, with output efficiency of 90%.
CHAPTER 1
INTRODUCTION

1.0 Introduction

Photonics is the science and technology of generating and controlling photons, particularly in the visible and near infra-red light spectrum. Photonics as a science is closely related to quantum optics and optoelectronics. The emergence of photonic in communications and other applications are largely due to the advent of semiconductor laser (1960’s) [1] and low loss optical fibers (1970’s) [2]. Especially in telecommunication it is helped further by the development of erbium doped fiber amplifier (EDFA) [3,4] in 1987 and dense wavelength division multiplexing (DWDM) [5] in the 1990’s.

1.1 Photonic Devices and Integrated Circuits

Photonic devices lie at the heart of the optical communications revolution. Major photonic devices include optical fibers/waveguides, couplers, electro-optic devices, magneto-optic devices, acousto-optic devices, nonlinear optical devices, optical amplifiers, lasers, light-emitting diodes, and photodetectors. Development and progress in these devices have been achieved not on single material platform but on many different materials, shown in Tables 1 and 2 [6].

Optical components are built using materials including indium phosphide (InP), gallium arsenide (GaAs), lithium niobate (LiNbO3), silicon (Si), silica-on-silicon (SOI) and organic polymer. Lithium niobate offers little practical promise as a material platform for integration since it cannot be used to practically implement active opto-electronic components like lasers and detectors.
Table 1.1: Functions achieved to date at 1550 nm wavelength in key integrated optical material systems [6].

<table>
<thead>
<tr>
<th>Material System</th>
<th>Lasers</th>
<th>Amplifiers</th>
<th>Detectors</th>
<th>Modulators</th>
<th>Polarization Controllers</th>
<th>Couplers</th>
<th>Filters</th>
<th>Switches</th>
<th>Attenuators</th>
<th>Isolators</th>
<th>Circulators</th>
<th>Wavelength Converters</th>
<th>Chromatic Dispersion Compensators</th>
<th>PMD Compensators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silica [SiO₂]</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
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<tr>
<td>Silicon [Si]</td>
<td>×</td>
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<tr>
<td>Silicon Oxynitride [SiO₂N]</td>
<td>×</td>
<td>×</td>
<td>×</td>
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<td>Polymers</td>
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<tr>
<td>Lithium Niobate [LiNbO₃]</td>
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<td>×</td>
<td>×</td>
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<td>Indium Phosphide [InP]</td>
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<td>Gallium Arsenide [GaAs]</td>
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<td>Garnets (e.g. YIG)</td>
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</table>

Table 1.2: Elemental optical functions and the corresponding ideal integrated optical material system(s) for operation at 1550 nm wavelength [6].

<table>
<thead>
<tr>
<th>Elemental Optical Function</th>
<th>High Performance Material System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lasers</td>
<td>InP/GaAs</td>
</tr>
<tr>
<td>Amplifiers</td>
<td>Er-doped Silica, InP/GaAs</td>
</tr>
<tr>
<td>Detectors</td>
<td>InP/GaAs</td>
</tr>
<tr>
<td>Modulators</td>
<td>LiNbO₃, InP</td>
</tr>
<tr>
<td>Polarization Controllers</td>
<td>LiNbO₃</td>
</tr>
<tr>
<td>Couplers/Splitters/Combiners/Taps</td>
<td>Silica, Polymers</td>
</tr>
<tr>
<td>Multiplexer/Demultiplexer</td>
<td>Silica, Silicon Oxynitride</td>
</tr>
<tr>
<td>Tunable Notch Filters</td>
<td>Polymers</td>
</tr>
<tr>
<td>Switches</td>
<td>Polymers</td>
</tr>
<tr>
<td>Attenuators</td>
<td>Polymers</td>
</tr>
<tr>
<td>Isolators</td>
<td>Garnets</td>
</tr>
<tr>
<td>Circulators</td>
<td>Garnets</td>
</tr>
<tr>
<td>Wavelength Converters</td>
<td>LiNbO₃, InP/GaAs</td>
</tr>
<tr>
<td>Chromatic Dispersion Compensators</td>
<td>Silica, Silicon, Polymers</td>
</tr>
<tr>
<td>PMD Compensators</td>
<td>Polymers, LiNbO₃</td>
</tr>
</tbody>
</table>

Although active opto-electronic devices can be implemented on GaAs, the intrinsic band-gap of GaAs generally only allows operation in the 850 nm telecommunication wavelength window, limiting its usefulness to local area network applications. Indium phosphide [7] and recently silicon [8], has demonstrated the ability to integration of both active and passive optical devices operating in the 1310 nm or 1550 nm telecommunication wavelength windows, since it supports all the key high-value opto-electronic functions required in an optical transport to be integrated on a single substrate.
In silica-on-silicon technology, waveguide is formed in a silica layer by doping it with phosphorus or germanium atoms. Low-loss integrated silica waveguides on silicon allow for low loss coupling with optical fibers and are used to form a variety of planar lightwave circuits for optical branching, switching and filtering. Silica waveguides can also be used as platforms (motherboards) for hybrid optoelectronic integration [9]. Increases in integration density with this technology are restricted however, by the large minimal bending radius of silica waveguides that is of the order of a few centimeters. A significant denser integration has been demonstrated with higher index contrast silicon oxynitride (SiON) technology [10]. An index contrast in the order of 3.3% between the core of the SiON waveguide and silica cladding, allows the minimum bending radius to be reduced to below 1 mm. Further scaling is possible with silicon-on-insulator technology, where the waveguide is formed in a thin silicon layer. Extremely high refractive index contrast between the silicon core \((n = 3.5)\) and silica cladding \((n = 1.444)\) allows the waveguide core to be shrunk down to a submicron cross-section, while still maintaining single mode propagation at telecommunications wavelengths. Such extreme light confinement allows the minimal bending radius to be reduced to the micron range, opening an avenue to realize ultra-dense photonic integrated circuits on a single silicon chip [11]. However it also results in significantly enhanced propagation losses due to increased interaction of the waveguiding mode with the sidewall surface roughness [12].

In recent years, organic photonic polymeric materials have undergone rapid development and have exhibited large improvements in performance. Organic polymer waveguide do not have the limitation due to intrinsic bandgap as in group III-IV semiconductor material such as GaAs and InP, making it suitable for wide range of wavelength including blue light generated by gallium nitride (GaN) laser.
Complex switching and routing circuits with state-of-the-art performance have been demonstrated on a polymer waveguide platform. Organic photosensitive polymeric materials offer low cost, flexibility of design and fabrication, and performance suitable for photonic devices [13].

Extensive research is also currently being done on photonic crystal (PhC) integrated circuits [14]. Light propagating in one or two dimensional, periodically coupled, waveguiding structures shows effects that are not observed in bulk media. The new degrees of freedom that arise due to the micro structuring of optical sample allow for almost arbitrary tailoring of the dispersion relation, according to specific needs of device. For higher power of the optical excitation, nonlinear effects broaden the diversity of observable phenomena that arise due to the interaction of the nonlinearity and the unique features of the dispersion relation. Infiltration of these periodic structures with polymers or chemical substance [15,16] added a further ability to manipulate the dispersion properties using electro or acoustic optical effect.

1.2 Motivations and Objectives in the Thesis

The motivation to explore planar optical waveguide technology started with the national top-down project in the year 2002 [17]. And the basic structure that is vital to the design and fabrication of photonic devices is the photonic waveguide. Therefore the goal and scope of this thesis is to conduct theoretical investigation on the propagation of lightwave in photonic waveguides and devices. The basic waveguide structures identified for investigations are the gallium arsenide/aluminium gallium arsenide (GaAs/AlGaAs) rib waveguide and silica channel waveguide. In this thesis the single mode condition was studied in GaAs/AlGaAs rib waveguide by investigating the lateral and vertical higher order modes, the
polarization birefringence of the fundamental polarized modes and compared some of these results with the silicon on insulator (SOI) rib waveguide properties available in literature. Similarly for the silica waveguide, basic properties such as modal birefringence and hybridness due to structural deformation were investigated. Based on these results, polarization controlling devices such as polarization rotator and polarization splitter in silica waveguide were designed and characterized. Further enhancement in the polarization birefringence property was obtained by using the slotted waveguide structure [18,19]. The silica polarization splitter was designed and characterization based on slotted silica waveguide structure.

As a natural progression, the slotted structures were investigated further. The slotted array structures are periodic in nature, however a full 3-D propagation analysis of these structures are very memory intensive and in order to reduce the computer resources required, only a 2-D effective index analysis was conducted on the periodic structure. Models for the multimode interference coupler, wavelength division multiplexer and channel interleavers based on 1-D and 2-D photonic crystal structures consisting of gallium arsenide (GaAs) dielectric rods in air were investigated. The waveguiding in these structures are in the air defect channel waveguide due to bandgap mirror confinement [20]. In real practical devices however it would be based on finite height columns of rods with omni-directional reflectors above and below the structure [21].

As an overview to this thesis, following the introduction given in chapter 1, chapter 2 gives some literature background about electromagnetic wave theory and chapter 3 gives the computational aspect used in this thesis. In the proceeding chapters, the optical waveguide and devices investigated are the single mode condition in large GaAs/AlGaAs rib waveguide, design and optimization of silica
based polarization controlling devices such as polarization rotators and splitters, the
effect of multiple reflections in multimode photonic crystal waveguide coupler and
finally the design of wavelength division multiplexer and channels interleaver for
dense wavelength division multiplexers based on photonic crystal directional
coupler. The following sections below summarized the motivations, objectives and
results of these research topics and full report are found in chapters 4, 5, 6, 7 and 8
respectively. Finally a conclusion on the research work presented in this thesis and
future extension of these works are given in chapter 9.

1.2.1 Single Mode Condition in GaAs/AlGaAs Rib Waveguide.

In chapter 4, single mode condition in gallium arsenide/aluminium gallium
arsenide (GaAs/AlGaAs) rib waveguide was investigated. Rib waveguide is
important optical device due to its ability to propagate light in single mode at large
size comparable to single mode fibre. This would allow better power transfer
between rib waveguide and single mode fiber optic, since the spot size of the
lightwave are comparable [22,23].

In the single-mode condition in rib waveguide, various formulations such as
one proposed by Soref et. al. [22] and Pogossian et. al. [23] were analysed. Even
though these formulations are for silicon on insulator (SOI) rib waveguides, its
validity to GaAs/AlGaAs rib waveguide was investigated. Therefore one of the
objectives in the study of GaAs/AlGaAs rib waveguide was to investigate the single-
mode cut off conditions, i.e. the maximum allowable width and slab height that
maintain only single-mode propagation in the waveguide. Comparison were made
between the effective indices of the modes in the waveguide, specifically the quasi-
TE ($H_{11}^L$, $H_{21}^L$, $H_{31}^L$ and $H_{12}^L$) modes to the slab mode effective index. The second
The objective was to analyze the power confinement factors of the higher order ($H_{31}^p$, $H_{31}^q$, and $H_{12}^q$) modes.

The result obtained shows that the formulation given by Soref et al. [22] is valid for GaAs/AlGaAs rib waveguides. It is observed that at single-mode GaAs/AlGaAs rib waveguide dimensions, only the fundamental mode effective index is larger than the slab waveguide mode effective index. And in the analysis of the power confinement factor, as the higher order modes evolve from leaky modes into guided modes, the power confined in the rib waveguide due to these modes would increase drastically, thus the results lead to a simple identification of the vertical (slab height) single mode cutoff. Investigation was also conducted to identify polarization independent rib waveguide structures. In a polarization independent rib waveguide, the fundamental quasi-TE and quasi-TM modes effective indices would be the same. Two optical waveguide's mode solving methods were employed in the investigation, the semi analytical film mode matching (FMM) method [24,25] and a full numerical finite element method (FEM) [26].

1.2.2 Germanium Dioxide Doped Silica Polarization Rotator.

One of the most important polarization controlling device in optical integrated circuit is the polarization rotator [27]. It is being used for example to improve isolation between signals in adjacent communication channels in optical transmission system, and by implementing polarization mode interleaving [28] in long distance transmission network. These applications require efficient on-chip polarization rotator that can be integrated with other optical components. Numerous design have been proposed [27,29-32], but in this thesis the design and optimization
of silica polarization rotator based on slanted sidewall waveguide [27] was adopted due to its high conversion efficiency compared to other designs.

Therefore the objectives in chapter 5 were to investigate the effect of structural deformations (variation in width, height and slant angle) on the polarization rotation properties of a germanium doped silica waveguide, and to design and optimize a compact silica waveguide polarization rotator. The polarization rotator properties studied were the hybridness and polarization birefringence. As a result from this study, a polarization rotator design that is highly efficient and tolerant to fabrication errors, was obtained with a slanted structure with sidewall angle of 46°, height and width of 8.4 μm and 5.8 μm respectively. In the fabrication tolerance analysis, the critical parameter observed are the variation in the slant angle of the waveguide sidewall, where as the polarization rotator response is observed to be relatively stable with respect to the variation in the operating wavelengths. The optimum polarization rotator design obtained for operating wavelength of 1.55 μm consisted of a rotator waveguide section, with an intermediate tapered slanted section to facilitate a better power transfer between the input/output sections and the rotator section. The overall polarization conversion and power transfer efficiencies are about 99% and 80% respectively.

1.2.3 Low Refractive Index Grating Waveguide Polarization Splitter based on Resonant Tunneling.

In optical communication system, components based on polarization diverse scheme [33] would have the incoming optical signal split into TE and TM polarized state. One of the polarized states is then rotated and recombined using a power combiner. Splitting of optical signal into its respective polarization components is performed by a polarization splitter. Polarization splitters are based on different
principles [34-37] and these devices mainly utilize material with high refractive indices thus realizing a short and compact device due to its large polarization birefringence. Silica waveguide due to its low refractive index contrast would have low polarization birefringence thus the design of polarization splitters is normally rather large. In this work by the incorporation of slotted or grating structure [38], it enhances the difference in effective indices of the two polarization states therefore a short and compact silica waveguide polarization splitter may be achieved.

The objectives in chapter 6 were to implement the design of a short and compact silica waveguide polarization splitter based on resonant tunneling [37] and to investigate its fabrication tolerance and loss analysis. In the design process, quasi 2-D effective index method with global and local search method is employed to obtain the optimum design. In the polarization splitter design obtained the overall transmission efficiencies for the splitting of TM and TE optical components at a wavelength of 1.55 μm are 88% and 83% respectively.

1.2.4 Analysis of Multiple Reflections in Hybrid Photonic Crystal Multimode Interference Coupler.

In this topic, multiple reflections in hybrid 1-D and 2-D gallium arsenide photonic crystal [39-41] multimode waveguides was investigated. In conventional multimode waveguide, self-imaging [42] is one of the most important phenomena observed. It is being exploited to design and fabricate numerous optical devices [42-44]. However in photonic crystal multimode waveguide where the wave confinement is due to bandgap confinement [20], the reflectivity of all the modes in the waveguide is very large at the end facet of the waveguide. Therefore the objectives in chapter 7 were to investigate the lightwave propagation in multimode photonic crystal waveguide, and the effect of multiple reflections on self-imaging phenomena. And it
is observed that multiple reflections are one of the main reasons why image periodicity in photonic crystal multimode interference coupler departs from self-imaging principle.

1.2.5 Hybrid Photonic Crystal Wavelength Division Multiplexer and Channel Interleaver.

Wavelength selective components in optical communication system are used for wavelength multiplexing/demultiplexing of signals. In conventional optical waveguide structure, wavelength division multiplexer and channels interleaver may be realized in a form such as an array waveguide grating [45] and cascaded Mach-Zehnder interferometer [46] respectively. In photonic crystal structure, numerous design of highly compact wavelength selective components has been reported. They are based on superprism [47,48], couple cavity waveguide [49], mini stopband effect [50,51], and directional couplers [21,52].

For directional couplers based on 2-D photonic crystal, if the total length are large they are not suitable for channel interleaving application since a 2-D photonic crystal waveguide device exhibits relatively high propagation loss [53]. However 1-D photonic crystal waveguide exhibit lower loss [54,55]. Therefore in chapter 8, the objectives were to design and optimize a hybrid 1-D and 2-D gallium arsenide photonic crystal air defect channels wavelength division multiplexer for 1.31/1.55 μm and channels interleaver suitable for dense wavelength multiplexer. In this design, a coupled 1-D photonic crystal air defect channels was used in the interaction region sandwiched between input/output 2-D photonic crystal waveguides. The 2-D photonic crystal section was incorporated due to its superior wave bending efficiency [56,57]. An efficient design with power transfer efficiency of 91% and its extinction ratios of -23.7 dB and -20.8 dB at wavelength 1.31 μm and 1.55 μm respectively,
were obtained. And for the design of dense wavelength division multiplexer's channels interleaver with channel spacing of 0.8 nm wavelength, the power output efficiency obtained is about 90%.

1.3 Major New Contributions

New contributions in this thesis are:

1. Investigation of vertical mode cut-off in large GaAs/AlGaAs rib waveguide using the analysis of power confined in the rib section of the waveguide.

2. Optimum and efficient silica polarization rotator design based on conventional slanted waveguides are presented.

3. Design and characterization of a compact silica polarization splitter based on resonant tunneling effect and grating waveguide.

4. Computation of photonic band diagram for 1-D photonic crystal using quasi 2-D effective index -eigenmode expansion method is introduced.

5. Investigation of lightwave propagation and multiple reflections in photonic crystal multimode interference couplers.

6. Design of a hybrid 1-D and 2-D GaAs photonic crystal wavelength division multiplexer for 1.31/1.55 μm channel splitting and channels interleaver suitable for dense wavelength division multiplexing application with 0.8 nm wavelength spacing.

1.4 Methodology and Computation Tools used in this Thesis

This dissertation blends two types of lightwave simulations i.e. theoretical investigation of physical phenomena and design of photonic waveguide devices. The investigations of physical phenomena include investigation of single mode conditions
in rib waveguides and multiple reflections in photonic crystal multimode interference couplers. Photonic waveguide devices proposed include the compact silica waveguide polarization rotators, silica grating waveguide polarization splitters and the modeling of hybrid 1-D and 2-D photonic crystal wavelength division multiplexer and channels interleaver suitable for dense wavelength division multiplexer. In the design and investigation of the photonic waveguide devices, advanced optimization technique using global and local simplex algorithm were employed [58-60]. A combination of semi-analytical methods [61-63] to design and optimize, and numerical technique [64] in the verification of the designs were used. The general methodology adopted in the design and optimization process is given by the flow chart shown in Figure 1.1.

![Figure 1.1: Flow chart - general methodology in the design and optimization process.](chart.png)
Photonic softwares used in this thesis, are FIMMwave, FIMMPROP [63] and KALLISTOS [60] from Photon Design, CAMFR [61] which is freely available in the internet, and also multiphysics simulation tool Comsol v. 3.4 [64] that is based on finite element analysis. The propagation analyses employed are the quasi 2-D effective index method with eigenmode expansion (EIM-EME) [63] and 3-D finite element method with eigenmode expansion (FE-EME) [63].
2.0 Introduction – Maxwell Equations

Guided wave optical devices form the basis of modern optical communication systems. The basic physics governing these devices are the Maxwell equations [65-67]. The four underlying Maxwell equations for electromagnetic fields in a source free, non-magnetic medium are

\[
\nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \tag{2.1}
\]

\[
\nabla \times H = \varepsilon_0 \varepsilon_r \frac{\partial E}{\partial t} \tag{2.2}
\]

\[
\nabla \cdot \varepsilon_0 \varepsilon_r E = 0 \tag{2.3}
\]

\[
\nabla \cdot \mu_0 H = 0 \tag{2.4}
\]

In Eqs. (2.1) – (2.4), \(E\) is the electric field intensity vector and \(H\) is the magnetic field intensity vector, and the operator \(\nabla\) is defined as

\[
\nabla = \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \tag{2.5}
\]

The constants in Maxwell equations are the dielectric constant of free space,

\[
\varepsilon_0 = \frac{1}{c^2 \mu_0} \approx \frac{1}{36\pi} \times 10^{-9} \ \text{F/m}
\]

The permeability of free space, \(\mu_0 = 4\pi \times 10^{-7} \ \text{H/m}\) and the quantity \(c\) is the velocity of light in vacuum.

Taking the curl \((\nabla \times)\) of Eq. (2.1) and substituting Eq. (2.2) yields an equation that depends only on the electric field intensity vector \(E\)
Using the identity
\[ \nabla \times \nabla \times E = -\mu_0 \varepsilon_0 c^2 \frac{\partial^2 E}{\partial t^2} \]  
(2.6)

and substituting in Eq. (2.6), yields the wave equation

\[ \nabla^2 E - \mu_0 \varepsilon_0 c^2 \frac{\partial^2 E}{\partial t^2} = 0 \]  
(2.7)

Considering only time harmonic fields with the time dependence represented by the real part of \( e^{i\omega t} \), results in the time harmonic wave equation

\[ \nabla^2 E + n^2 k_0^2 E = 0 \]  
(2.8)

where \( n = \sqrt{\varepsilon_r} = \sqrt{\frac{\varepsilon}{\varepsilon_0}} \) is the refractive index, and \( k_0 = \omega \sqrt{\mu_0 \varepsilon_0} \) the wavenumber of free space. The dielectric constant of the medium \( \varepsilon = \varepsilon_0 \varepsilon_r \), where \( \varepsilon_r \) is the relative dielectric constant.

\[ \varepsilon_2 > \varepsilon_1 \]

Figure 2.1. A typical symmetric dielectric slab waveguide with its transverse and normal components to the dielectric interface shown as \( t \) and \( n \) respectively.

At the boundary between the two media, shown in Figure 2.1, distinguish by the dielectric constants \( \varepsilon_1 \) and \( \varepsilon_2 \), in the absence of charge and current, the boundary conditions on the electric field component transverse (\( t \)) and electric displacement component normal (\( n \)) to the dielectric interface are

\[ E_{nt} = E_{2t} \]  
(2.10)
\[ D_{1n} = D_{2n} \]  
\[ (2.11) \]

Hence the electric field normal component becomes, \( \varepsilon_1 E_{1n} = \varepsilon_2 E_{2n} \).

Similarly, the boundary conditions on the magnetic field components transverse and normal to the interface are

\[ H_{1t} = H_{2t} \]  
\[ (2.12) \]
\[ B_{1n} = B_{2n} \]  
\[ (2.13) \]

For a non magnetic material, \( \mu_r = 1 \), the normal magnetic field component, \( H_{1n} = H_{2n} \)

2.1 Dielectric Waveguide

Waveguides come in a variety of different forms: slab waveguide, rectangular waveguide and circular waveguide in the form of optical fiber and holey fibers. The cores are assumed to have an average refractive index (dielectric constant, \( \varepsilon_1 \)) higher than the cladding's (surrounding's) refractive index (dielectric constant \( \varepsilon_2 \)). In the absence of conducting boundaries, electromagnetic fields exist both inside and outside the dielectric waveguide. The relative amount of energy propagating inside the core increases with an increase in the refractive index contrast between the core and the cladding. Considering the number of different waveguide structures [67,68], we present our discussion based on the simplest type waveguide: the dielectric slab waveguide.

Some of the typical dielectric waveguides are shown in Figures 2.2 (a)-(d): (a) slab dielectric waveguide [69], (b) rectangular dielectric waveguide [69], (c) circular dielectric waveguide (optical fiber) [69] and (d) holey circular dielectric waveguide (holey fiber) [70].
Figure 2.2. Type of dielectric waveguides. The dielectric constant of the surrounding is assumed to be $\varepsilon_2$, and the dielectric constant of the core, $\varepsilon_1$ is assumed to be larger than $\varepsilon_2$.

For a general waveguide structure that propagates light along the $z$-direction, the dielectric constant can be written as $\varepsilon(x,y)$, i.e. the dielectric function is invariant along the direction of propagation. The field solutions for Maxwell equations are called modes and may be written in the following forms:

$$E(x, y, z, t) = E(x, y) \exp(-i\beta z + i\omega t) \tag{2.14}$$

$$H(x, y, z, t) = H(x, y) \exp(-i\beta z + i\omega t) \tag{2.15}$$

These solutions or modes, would maintain their shape as the field propagates along the $z$-direction. Each mode is also characterized by its frequency $\omega$, and its wavevector along the $z$-direction $\beta$ (also known as propagation constant). The relation between the frequency and the propagation constant along the $z$-direction defines the dispersion relation.
For a structure with uniform dielectric constant \( \varepsilon \), Eq. (2.14) and (2.15) can also be written as

\[
E(x, y, z, t) = E \exp(-ik_x x - ik_y y) \exp(-i\beta z + i\omega t)
\]

(2.16)

\[
H(x, y, z, t) = H \exp(-ik_x x - ik_y y) \exp(-i\beta z + i\omega t)
\]

(2.17)

where

\[
\varepsilon \mu_0 \frac{\omega^2}{c^2} = k_x^2 + k_y^2 + \beta^2
\]

(2.18)

and \( k_x \) and \( k_y \) are the wavevector components along the x and y directions.

Figure 2.3 shows that for a waveguide structure as shown in Figure 2.2(a), total internal reflection occurs when \( \varepsilon_1 \mu_0 \frac{\omega^2}{c^2} < \beta^2 \), in which case the field decays exponentially outside the guiding layer. Thus in the \( \beta - \omega \) plane, the guided modes can only exist in the phase space region above the light line of the cladding media. A further constraint imposed on allowed modes is the propagation constant is always less than the core light line, \( \varepsilon_1 \mu_0 \frac{\omega^2}{c^2} > \beta^2 \). Thus the allowed phase space region for
guided modes exist between two straight lines that go through the origin in the $\beta$-$\omega$ plane. The propagation constant is related to the effective index of the modes by the relation $n_{\text{eff}} = \frac{\beta}{k_0}$, and the effective index of the guided modes based on the above results must be between the range $n_1 > n_{\text{eff}} > n_2$, where $n = \sqrt{\varepsilon_r}$.

2.1.1 Dielectric Slab Waveguide Modes

The slab waveguide supports a finite number of guided modes, leaky modes and infinite continuum of radiation modes. The guided modes, leaky modes and the radiation modes have to satisfy the wave equation, and can be obtained as solutions of a boundary value problem. Figure 2.4 shows different type of dielectric slab waveguides, symmetrical slab waveguide, asymmetrical slab waveguide and anti-reflection resonant optical waveguide (ARROW) [20], which supports guided leaky modes. The refractive indices of the different layers are given by $n_1$, $n_2$, $n_3$ and $n_4$, and the thickness of the core (wave-guiding) layer shown with refractive index $n_1$ is $2d$.

![Different types of dielectric slab waveguides](image)

Figure 2.4. Types of dielectric slab waveguides, (a) Symmetrical waveguide, (b) Asymmetrical waveguide and (c) Anti-reflection resonant optical waveguide (ARROW)
The modes of a dielectric slab waveguide shown in Figures 2.4 are hybrid modes in general, but they can be decomposed into quasi-transverse electric (TE) and quasi-transverse magnetic (TM) modes.

A quasi-TE polarized light has one magnetic field component that is pointed out of the propagation plane and two electric field components which are within the plane. Quasi-TM polarized light has one electric field component pointing out of the plane and two magnetic field components which are within the plane. The wave equations (with x-z propagation plane) for quasi-TE mode are

\[
\frac{\partial^2 H_y}{\partial x^2} + \left( k_0^2 n^2 - \beta^2 \right) H_y = 0
\]

(2.19)

\[
E_x = \frac{\beta}{\omega \varepsilon} H_y
\]

(2.20)

\[
E_z = -i \frac{\omega \varepsilon}{\omega \mu} \frac{\partial H_y}{\partial x}
\]

(2.21)

and the wave equations for quasi-TM mode are

\[
\frac{\partial^2 E_y}{\partial x^2} + \left( k_0^2 n^2 - \beta^2 \right) E_y = 0
\]

(2.22)

\[
H_x = -\frac{\beta}{\omega \mu_o} E_y
\]

(2.23)

\[
H_z = i \frac{\omega \mu_o}{\omega \mu} \frac{\partial E_y}{\partial x}
\]

(2.24)

Typical field solutions for quasi-TM modes, assuming an asymmetrical slab structure shown in Figure 2.4(b), are shown in Figure 2.5. The dispersion diagram and the dependence of propagation constant on frequency is plotted in Figure 2.6. The modal propagation constant of the modes supported by the waveguide increases as the operating frequencies is increased. At lower frequencies (higher wavelengths)
the modes would couple into the radiation mode region. However in the forbidden region, no guided mode would be found as the fundamental mode effective index cannot be larger than the bulk refractive index of the core layer in the waveguide.

Figure 2.5. Typical field profiles of the first four quasi-TM modes supported in an asymmetric slab waveguide shown in Figure 2.4(b). Typical propagation constants of the modes would have its propagation constants, \( \beta_0 > \beta_1 > \beta_2 > \beta_3 \).

Figure 2.6. A typical dispersion diagram for quasi-TM modes in a dielectric slab waveguide: shown in terms of propagation constant and angular frequency.
2.1.2 Mode Expansion

Apart from the discrete set of guided modes, radiation modes and leaky modes are also solutions to Maxwell equations. Radiation modes form a continuum and having the property \( k_0^2 n_2^2 < \beta^2 < k_0^2 n_3^2 \) for substrate radiation modes and \( \beta^2 < k_0^2 n_2^2 \) for true radiation modes. A detailed discussion of radiation modes can be found in reference [69]. Leaky modes are also radiation modes but they are considered as a continuation of guided modes, since they will become part of guided modes as the frequency (wavelength) increases (decreases), seen as the guided modes dispersion curves extending into the radiation modes region, as shown in Figure 2.6. The guided modes together with the radiation modes form a complete orthogonal set.

2.2 Coupled Waveguide Array

In coupled waveguide array, the array mode is described as a collective excitation or a "supermode" of the individual waveguides modes, evanescently coupled to each other [71,72]. The supermode is described in terms of the individual modes complex amplitudes, with the details of the field between the waveguides contained in the coupling constant. A more general approach, in which waveguide arrays are regarded as an example of a general one-dimensional periodic optical structure, is the Floquet-Bloch (FB) analysis [73]. It predicts that the propagation-constant spectrum of the array's eigenmodes is divided into bands, separated by gaps in which propagating modes do not exist.

Considering the case of three parallel waveguides as illustrated in Figure 2.7, suppose that the central direction of propagation is in the direction of z-axis, i.e. 90° normal to the direction of periodicity, the periodic dielectric permittivity given as
\[ e(x) = e(x + a) \]  
(2.25)

with \( a \) being the periodic lattice constant.

Figure 2.7. Two-dimensional view of arrayed waveguides in parallel made of a layered structure. The direction of wave propagation is 90° normal to the direction of periodicity.

Figure 2.8. Mode patterns of orthogonal modes when three parallel waveguides are brought close for directional coupling. (a)–(c): The field profiles for the six lowest-order guided modes for the index structure of three coupled waveguides. (d) The relative magnitudes of \( k_x^{(b)}/k \) positioned with respect to \( n(x) \), the refractive-index profile of three coupled waveguides [72].

The amplitude patterns of guided modes created by bringing the three guides closely are schematically illustrated in Figures 2.8(a), (b) and (c) [72]. The three propagation constants diverge from the original value of an isolated waveguide as the
three guides are put closer. A typical dispersion diagram shown in Figure 2.9 is for three waveguide arrays clearly show the mode splitting as a result of coupling between adjacent waveguides. If we increase the number of parallel waveguides in the array, the diverging propagation constants form a near continuum which is called a band.

![Dispersion Diagram](image)

Figure 2.9. A typical dispersion diagram for TM modes in a coupled dielectric waveguide as in Figure 2.8. As the three coupled waveguides are put closer, the three propagation constants diverge from the original value of an isolated waveguide, depicted in Figure 2.6.

### 2.2.1 Photonic Band Diagram of 1-D Periodic Structure

The one-dimensional arrays of high and low refractive index material would exhibit eigenmodes divided into bands, separated by gaps in which propagating modes do not exist. Suppose the periodic dielectric array as shown in Figure 2.10 is consisted of a polymeric compound [74] of high refractive index \( n = 1.59 \) layer of width \( d \), and low refractive index \( n = 1.0 \) layer of width \( s \). The period for the layer is \( a = d + s \).