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Violation of the holographic principle in the loop quantum gravity

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Abstract – In this paper, we analyze the holographic principle using loop quantum gravity (LQG). This will be done by using polymeric quantization for analysing Yurtsever's holographic bound on the entropy, which is obtained from local quantum field theories. As the polymeric quantization is the characteristic feature of loop quantum gravity, we will argue that this calculation will indicate the effect of loop quantum gravity on the holographic principle. Thus, we will be able to explicitly demonstrate the violation of the holographic principle in the loop quantum gravity.

Introduction. – Loop quantum gravity (LQG) is a background independent, nonperturbative approach to quantize Einstein's General Theory of Relativity [1,2]. It is one of the main approaches to the quantum theory of gravity, and so LQG necessitates a quantized structure for space. The microscopic degrees of freedom in LQG are discrete, and it is important to relate these discrete degrees of freedom to the macroscopic geometry which is described by a continuous differential manifold. Thus, the LQG is constrained to produce a continuous differential manifold in the infrared limit. This has been done by using polymeric quantization, which is characterized by a polymeric length scale [3,4]. At scales much larger than the polymeric length scale, the geometry of spacetime can be approximated by a differential manifold. However, at scales comparable to the polymeric length scale, the discrete behavior becomes more manifest. This has also led to the development of polymer quantum mechanics. So, polymeric quantum mechanics can be used to understand the behavior of LQG, as it is the main technique used in loop quantum gravity. It may be noted that quantum field theory consistent with polymeric quantization has been studied [5]. This has been done by analysing the propagator of the scalar field theory using a method which took the polymeric effects into consideration. Thus, an expression for the propagator, consistent with polymeric quantization, was obtained both in the infrared and the ultraviolet regimes. The semi-classical dynamics of a free massive scalar field in a homogeneous and isotropic cosmological spacetime has been used for analyzing polymeric inflation [6,7].

In this paper, we will analyse the effects of the polymeric quantization on the holographic principle. The holographic principle states that the degrees of freedom in a region in space are equal to the degrees of freedom on the boundary surrounding that region of space [8,9]. 't Hooft's holographic principle has been motivated by the Bekenstein entropy-area relation, as according to this relation the entropy of the black hole scales with its area, $S = A/4$ [10–13]. The Bekenstein entropy-area relation is expected to get modified at very small scales where quantum gravitational effects become significant [14–24]. So, it is expected that the holographic principle will get violated near the Planck scale [25,26].

It may be noted that a different kind of holographic bound on entropy called Yurtsever's holographic bound occurs in local quantum field theories [27–30]. Unlike 't Hooft's holographic bound where the entropy scales as A [9], in Yurtsever's holographic bound the entropy scales as $A^{3/4}$ [27–30]. Yurtsever's holographic bound is obtained by imposing an upper bound on the total energy of the corresponding Fock states which ensures that the system is in a stable configuration against gravitational collapse, and imposing a cutoff on the maximum energy of the field modes of the order of the Planck energy. It is this bound that will be violated by polymeric quantization because the polymeric quantization effectively modifies the measure of the phase space. It may be noted that it has been demonstrated that this bound also gets violated due to the existence of a minimum measurable length scale because such a minimum measurable length scale modifies the usual uncertainty principle to a generalized uncertainty principle, and this generalized uncertainty principle in turn modifies the measure of the phase space [31]. As the generalized uncertainty principle is related to the polymeric quantization [32], it was expected that a similar result will also hold for polymeric quantization. However, this was never explicitly demonstrated, and this is what we do in this paper. Furthermore, as the loop quantum gravity depends on polymeric quantization, our calculations indicate a violation of the holographic principle in loop quantum gravity.

It may be noted that the area of the black hole increases by a Planck area when one bit of information crosses the horizon [11]. So, even using the usual black-hole thermodynamics it can be demonstrated that 't Hooft's holographic entropy bound is an integer multiple of the Planck area. This restricts the number of microstates for the black hole in any theory of quantum gravity. In fact, it has been explicitly demonstrated that the entropy of a black hole is an integer multiple of the Planck area using the space-time foam picture, which is based on the usual Wheeler-DeWitt approach [33,34]. So, discreteness of space cannot increase 't Hooft's holographic bound on the entropy, as this discreteness cannot modify the number of microstates for a black hole. It can only explain the nature of such microstates. However, the discretization of space in polymeric quantization effectively changes the measure used in the phase space. This can increase Yurtsever's holographic bound on the entropy in local quantum field theories, and this is what we will explicitly demonstrate in this paper.

Polymer quantization. – In this section, we will review the polymer quantization [3,4,35]. Polymer quantization is a singular representation of the Weyl-Heisenberg algebra. It is unitarity inequivalent to the familiar Schrödinger representation. In order to implement this quantization scheme, a representation for the algebra of this theory has to be constructed. So, we have to obtain a suitable inner product to obtain the kinematic Hilbert space. After defining a suitable inner product, the dynamics of this theory can be analysed. This can be done by first noting that the polymer Hilbert space is a non-separable space which is defined as $H_{poly} = L^2(\mathcal{R}_d, d_{\mu_d})$. Thus, the polymer Hilbert space is the Cauchy completion of the space of complex-valued functions which are cylindrical with respect to a graph on the real line. Now if $|x_i\rangle$ are the normalizable eigenvectors of the position operator which span H_{poly}, then we can write an inner product as

$$\langle x_i|x_j\rangle = \delta_{i,j}. \tag{1}$$

This inner product can be used to obtain the Cauchy completion of the cylindrical function space. The position and momentum operators cannot be defined simultaneously. This is because the states in H_{poly} have support on graphs which have countably many points in them. Thus, we cannot define the momentum operator through differentiation. However, the Hamiltonian of a system contains the kinetic terms, and so we need to have a representation for the momentum operator. Thus, the momentum operator is defined by regularizing the Hamiltonian. This is done by introducing a lattice structure on the position space, and then using it to define a translation operator. This translation operator replaces the momentum operator in polymeric quantization. Now we can write the operators which are commonly used in polymer quantization,

$$\hat{x}|x_j\rangle = x_j|x_j\rangle, \tag{2}$$

$$\hat{V}(\mu)|x_j\rangle = |x_j - \mu\rangle, \tag{3}$$

where μ is the regulation measure. This regulation measure cannot be removed from the results of the theory. It may be noted that this regulation measure shows an ambiguity related to its exact value. Now the momentum operator can be written as

$$\hat{p} = \frac{\hat{V}(\mu) - \hat{V}^{\dagger}(\mu)}{2i\mu}. \tag{4}$$

In the limit $\mu \to 0$, this operator reduces to $\hat{p} = -i\frac{\partial}{\partial_x}$, which is the usual momentum operator. Even though this polymerization measure μ depends on the position, in the semi-classical approximation, a constant μ is used. Furthermore, the translation operator is replaced by $\hat{V}(\mu) \to e^{i\mu\hat{p}}$. Thus, the momentum operator can be written as $\hat{p}_\mu = \frac{\sin(\mu p)}{\mu}$.

Now let us consider the effect of polymer quantization on a one-dimensional system, with the Hamiltonian

$$H = \frac{p_\mu^2}{2m} + V(x), \tag{5}$$

where we have used the polymer modified momentum $p_\mu = \frac{\sin(\mu p)}{\mu}$ instead of the canonical one. The modified equations of motion [36] now take the form

$$\dot{x} = \{x, H\} = \{x, p_\mu\}\frac{\partial H}{\partial p_\mu} = \cos(\mu p)\frac{p_\mu}{m} = \sqrt{1-\mu^2 p_\mu^2}\,\frac{p_\mu}{m}, \tag{6}$$

$$\dot{p}_\mu = \{p_\mu, H\} = -\{x, p_\mu\}\frac{\partial H}{\partial x} = \cos(\mu p)\left(-\frac{\partial V}{\partial x}\right) = \sqrt{1-\mu^2 p_\mu^2}\left(-\frac{\partial V}{\partial x}\right). \tag{7}$$

Using (6) and (7), we can write the acceleration as

$$\ddot{x} = \frac{1}{m}\left(-\frac{\partial V}{\partial x}\right)\left[1 - 2\mu^2 p_\mu^2\right]. \tag{8}$$

In this section, we reviewed some basic results related to polymer quantization. In the next section, we will apply the polymer quantization to the Liouville theorem.

The polymer quantization and Liouville theorem. – It is possible to analyse the effect of the polymeric quantization on the Liouville theorem. The number of states inside a volume of phase space does not change with time, the time variations of position and momentum are $x_i' = x_i + \delta x_i$ and $p'_{\mu\,i} = p_{\mu\,i} + \delta p_{\mu\,i}$. Using (6) and (7) the infinitesimal changes in position and momentum become

$$\delta x_i = \{x_i, p_{\mu\,j}\}\frac{\partial H}{\partial p_{\mu\,j}}\delta t,$$
$$\delta p_{\mu\,i} = -\{x_j, p_{\mu\,i}\}\frac{\partial H}{\partial x_j}\delta t. \tag{9}$$

The infinitesimal phase space volume after this infinitesimal time evolution can now be written as

$$\mathrm{d}^D x' \mathrm{d}^D p'_\mu = \left|\frac{\partial(x'_1, \ldots, x'_D, p'_{\mu\,1}, \ldots, p'_{\mu\,D})}{\partial(x_1, \ldots, x_D, p_{\mu\,1}, \ldots, p_{\mu\,D})}\right| \mathrm{d}^D x\, \mathrm{d}^D p_\mu, \tag{10}$$

where

$$\left|\frac{\partial(x'_1, \ldots, x'_D, p'_{\mu\,1}, \ldots, p'_{\mu\,D})}{\partial(x_1, \ldots, x_D, p_{\mu\,1}, \ldots, p_{\mu\,D})}\right| = 1 + \left(\frac{\partial \delta x_i}{\partial x_i} + \frac{\partial \delta p_{\mu\,i}}{\partial p_{\mu\,i}}\right) + \cdots,$$
$$\left(\frac{\partial \delta x_i}{\partial x_i} + \frac{\partial \delta p_{\mu\,i}}{\partial p_{\mu\,i}}\right)\frac{1}{\delta t} = -\left[\frac{\partial}{\partial p_{\mu\,i}}\{x_j, p_{\mu\,i}\}\right]\frac{\partial H}{\partial x_j}. \tag{11}$$

We know that

$$\{x_i, p_{\mu\,j}\} = \delta_{ij}\sqrt{1 - \mu^2 p^2_{\mu\,j}},$$
$$-\left[\frac{\partial}{\partial p_{\mu\,i}}\{x_j, p_{\mu\,i}\}\right] = \delta_{ij}\mu^2 p_{\mu\,i}. \tag{12}$$

Now we can write (10) as

$$\mathrm{d}^D x' \mathrm{d}^D p'_\mu = \left(1 + \mu^2 p_{\mu\,i}\frac{\partial H}{\partial x_i}\delta t\right)\mathrm{d}^D x\, \mathrm{d}^D p_\mu. \tag{13}$$

We have to get rid of the term in the parenthesis in order to cast (13) into an invariant form. To this end, let us consider the infinitesimal evolution of $(1 - \mu^2 {p'_\mu}^2)$, up to the first order in μ^2 and δt. It is given by

$$\left(1 - \mu^2 {p'_\mu}^2\right) = 1 - \mu^2\left(p_{\mu\,i} - \sqrt{1 - \mu^2 p^2_{\mu\,i}}\frac{\partial H}{\partial x_i}\delta t\right)^2, \tag{14}$$

where we have used the fact that

$$p'_{\mu\,i} = p_{\mu\,i} + \delta p_{\mu\,i} = p_{\mu\,i} - \sqrt{1 - \mu^2 p^2_{\mu\,i}}\left(\frac{\partial H}{\partial x_i}\right)\delta t. \tag{15}$$

So, after a little bit of algebra, we can write

$$\left(1 - \mu^2 {p'_\mu}^2\right) = \left(1 - \mu^2 p_\mu{}^2\right)\left[1 + 2\mu^2\, p_{\mu\,i}\frac{\partial H}{\partial x_i}\delta t\right]. \tag{16}$$

Thus, we obtain

$$\left(1 - \mu^2 {p'_\mu}^2\right)^{-\frac{1}{2}} = \left(1 - \mu^2 p_\mu{}^2\right)^{-\frac{1}{2}}\left[1 - \mu^2\, p_{\mu\,i}\frac{\partial H}{\partial x_i}\delta t\right]. \tag{17}$$

Now, if we multiply both sides of (13) by (17) we obtain

$$\frac{\mathrm{d}^D x' \mathrm{d}^D p'_\mu}{\sqrt{\left(1 - \mu^2 {p'_\mu}^2\right)}} = \frac{\mathrm{d}^D x\, \mathrm{d}^D p_\mu}{\sqrt{(1 - \mu^2 p_\mu{}^2)}}\left[1 + \mu^2 p_{\mu\,i}\frac{\partial H}{\partial x_i}\delta t\right] \times \left[1 - \mu^2 p_{\mu\,i}\frac{\partial H}{\partial x_i}\delta t\right], \tag{18}$$

where

$$\left[1 + \mu^2 p_{\mu\,i}\frac{\partial H}{\partial x_i}\delta t\right]\left[1 - \mu^2 p_{\mu\,i}\frac{\partial H}{\partial x_i}\delta t\right] \approx 1. \tag{19}$$

Therefore, the invariant volume element of the phase space takes the following form:

$$\frac{\mathrm{d}^D x' \mathrm{d}^D p'_\mu}{\sqrt{\left(1 - \mu^2 {p'_\mu}^2\right)}} = \frac{\mathrm{d}^D x\, \mathrm{d}^D p_\mu}{\sqrt{(1 - \mu^2 p_\mu{}^2)}}. \tag{20}$$

After integrating over the coordinates, the infinitesimal volume element of the phase space can be written as $V\,\mathrm{d}^D p_\mu / \sqrt{(1 - \mu^2 p_\mu{}^2)}$. The number of states per momentum space volume is assumed to be $V\mathrm{d}^D p_\mu / (2\Pi\hbar)^D \sqrt{(1 - \mu^2 p_\mu{}^2)}$. The modification of the number of states per momentum will modify the entropy of a local quantum field theory. This will in turn violate the holographic bound.

Yurtsever's holographic bound. – The holographic principle states that the degrees of freedom in a region of space are equal to the degrees of freedom on the boundary of that region of space. The holographic principle can be used to analyse a closed space-like surface containing quantum bosonic fields [27–31]. Now at a temperature T, the energy of the most probable state for this field theory can be written as $E = a_1 Z T^4 V$ where Z is the number of different fundamental particles with mass less than T, and a_1 is a suitably chosen constant. In natural units it can be taken to be of order one. We can also write the entropy of this system as $S = a_2 Z V T^3$, where a_2 is another constant which can again be chosen to be of order one. We obtain the holographic limit $2E < \frac{V}{\frac{4}{3}\pi}$, and we also obtain $T < a_3 Z^{-\frac{1}{4}} V^{-\frac{1}{6}}$. So, the entropy bound is given by

$$S < a_4 Z^{\frac{1}{4}} V^{\frac{1}{2}} = a_4 Z^{\frac{1}{4}} A^{\frac{3}{4}}, \tag{21}$$

where A is the area of the boundary. At low temperatures, Yurtsever's holographic bound is small compared to 't Hooft's holographic bound, $S = A/4$ [8,9]. It may be noted that here the bulk degrees of freedom for any closed surface are equal to the boundary degrees of freedom. So, the physics inside a closed surface can be be represented by surface degrees of freedom.

It is possible to derive Yurtsever's holographic bound on the entropy using local quantum field theories in flat spacetime, where the total energy of Fock states is in a stable configuration [27–30]. This bound is obtained by imposing a cutoff on the maximum energy that modes of the quantum field can attain. This cutoff is usually of the order of the Planck energy. So, the total number of the quantized modes for a massless bosonic field confined to a cubic box of size L can be written as

$$N = \sum_{\vec{k}} 1 \to \frac{L^3}{(2\pi)^3}\int \mathrm{d}^3\vec{p} = \frac{L^3}{2\pi^2}\int_0^\Lambda p^2 \mathrm{d}p = \frac{\Lambda^3 L^3}{6\pi^2}, \tag{22}$$

where Λ is the ultraviolent cutoff in the energy of these modes. This cutoff makes N finite, and now the Fock states can be written in terms of the occupying number n_i,

$$\mid \Psi\rangle =\mid n(\vec{k}_1), n(\vec{k}_2), \cdots, n(\vec{k}_N)\rangle \rightarrow\mid n_1, n_2, \cdots, n_N\rangle, \quad (23)$$

The dimension of the Hilbert space can be calculated from the number of occupancies $\{n_i\}$. If no gravitational collapse occurs, we obtain

$$E = \sum_{i=1}^{N} n_i \omega_i \leq E_{BH} = L. \quad (24)$$

Now the bound on the energy can be obtained as follows:

$$E \rightarrow \frac{L^3}{2\pi^2}\int_0^{\Lambda} p^3 \mathrm{d}p = \frac{\Lambda^4 L^3}{8\pi^2} \leq E_{BH}. \quad (25)$$

So, we can write $\Lambda^2 \leq \frac{1}{L}$. Now we can write the maximum entropy as

$$S_{\max} = -\sum_{j=1}^{W} \frac{1}{W}\ln\frac{1}{W} = \ln W, \quad (26)$$

where W is given by

$$W = \dim\mathcal{H} < \sum_{m=0}^{N}\frac{z^m}{(m!)^2} \leq \sum_{m=0}^{\infty}\frac{z^m}{(m!)^2} = I_0(2\sqrt{z}) \sim \frac{e^{2\sqrt{z}}}{\sqrt{4\pi\sqrt{z}}}. \quad (27)$$

Here I_0 is the zeroth-order Bessel function of the second kind. As z is given by

$$z = \sum_{i=1}^{N} L_i \rightarrow \frac{L^3}{2\pi^2}\int_0^{\Lambda}\left[\frac{E_{BH}}{p}\right] p^2 \mathrm{d}p = \frac{\Lambda^2 L^4}{4\pi^2}, \quad (28)$$

so we obtain, $z \leq L^3$. We also have $A \sim L^2$, so we can write Yurtsever's holographic bound on the maximum entropy as [27–30]

$$S_{\max} = \ln W \leq A^{3/4}. \quad (29)$$

This is Yurtsever's holographic bound on the entropy which is obtained from local quantum field theories.

Violation of the holographic bound. – In this section, we will analyse the violation of the holographic principle from LQG. We again consider a massless bosonic field confined to a cubic box of size L. However, now we will also consider the modifications to this analysis which stem from polymeric quantization. As the polymeric quantization is a characteristic feature of LQG, we will use this analysis to demonstrate the violation of the holographic principle in LQG.

Here, from this point on, we will drop subindex μ from all the momentum operators for notational clarity. Now using (20) and integrating over the position coordinates the total number of the quantized modes gets modified in the polymeric framework as

$$N \rightarrow \frac{L^3}{2\Pi^2}\int_0^{\lambda}\frac{p^2 \mathrm{d}p}{\sqrt{1-\mu^2 p^2}}. \quad (30)$$

Now using the condition $\frac{1}{\mu} \geq \lambda$, we obtain the following result:

$$N \approx \frac{L^3}{2\Pi^2}\left(\int_0^{\lambda} p^2\left(1+\frac{1}{2}\mu^2 p^2\right)\mathrm{d}p\right) = \frac{L^3}{2\Pi^2}\left(\frac{\lambda^3}{3}+\frac{\mu^2}{10}\lambda^5\right). \quad (31)$$

The energy bound of the local field is modified to

$$E \rightarrow \frac{L^3}{2\Pi^2}\int_0^{\lambda}\frac{p^3 \mathrm{d}p}{\sqrt{1-\mu^2 p^2}} \approx \frac{L^3}{2\Pi^2}\left(\frac{\lambda^4}{4}+\frac{\mu^2}{12}\lambda^6\right) \leq E_{BH}, \quad (32)$$

where in the last step we have used the fact that the black-hole energy is the upper bound to the energy of the local field theory. Using $E_{BH} = L$ and $\lambda^2 \leq \frac{1}{L}$, up to first order in μ^2, we obtain the UV cutoff as

$$\frac{L^3}{8\Pi^2}\left(\lambda^4 + \frac{\mu^2}{48}\lambda^6\right) \leq L, \qquad \frac{1}{L}\left(1-\frac{\mu^2}{96L}\right) \geq \lambda^2. \quad (33)$$

The modified entropy can now be written as $S_{\max} = \ln W$ with $W \sim e^{2\sqrt{z}}$, where

$$z \rightarrow \frac{L^3}{2\Pi^2}\int_0^{\lambda}\left(\frac{E_{BH}}{p}\right)\frac{p^2 \mathrm{d}p}{\sqrt{1-\mu^2 p^2}} \approx \frac{L^4}{2\Pi^2}\left(\frac{\lambda^2}{2}+\frac{\mu^2}{8}\lambda^4\right). \quad (34)$$

From (33) and (34) the bound on z is

$$z \leq L^3\left(1+\frac{5\mu^2}{96L}\right). \quad (35)$$

Thus, we can write the maximum entropy $S_{\max} = \ln W = \ln e^{\sqrt{z}} = \sqrt{z}$ as follows:

$$S_{\max} \leq L^{\frac{3}{2}}\left(1+\frac{5\mu^2}{96L}\right)^{\frac{1}{2}} = A^{\frac{3}{4}} + \frac{5\mu^2}{192}A^{\frac{1}{4}}. \quad (36)$$

Thus, the polymer quantization violates Yurtsever's holographic bound on entropy by a term which is of the order of $A^{\frac{1}{4}}$. As the polymer quantization is the main characteristic technique used in loop quantum gravity, we expect that the loop quantum gravity will also violate the holographic principle.

Conclusion. – In this paper, we have analysed the effect of loop quantum gravity on the holographic principle. We have demonstrated that Yurtsever's holographic bound on entropy in the local quantum field theories is violated due to the effects coming from polymeric quantization. As the polymeric quantization is the basis of loop quantum gravity, our calculations indicate a violation of the holographic principle in loop quantum gravity. It would be interesting to analyse the violation of the holographic principle in a more general scene, since such a violation can have interesting physical consequences. This is because various interesting physical systems have been analysed using the general form of the holographic principle. The holographic principle has become the basis of the holographic cosmology [37–40]. The holographic cosmology is based on the idea that the difference between the degrees of freedom in a region and the degrees of freedom on the boundary surrounding that region drives the expansion of the universe. This model is analysed using the Jacobson formalism [41]. In this formalism, the Einstein equations are viewed as the Clausius relation. This is done by requiring the entropy to be proportional to the area of the cosmological horizon. Thus, the Friedmann equations are obtained from the Clausius relation using this formalism [42]. It will be interesting to analyse the effect of the violation of the holographic principle on the holographic cosmology. It is expected that in this case, the speed of acceleration of the universe will get affected by the violation of the holographic principle. The holographic principle has led to the development of the AdS/CFT correspondence, and the AdS/CFT correspondence has in turn had many important applications [43,44]. Apart from the conventional applications in the string theory [45], and M-theory [46], the AdS/CFT correspondence has also been used for understanding condensed-matter systems [47,48]. So, a violation of the holographic principle can have interesting applications for all these systems.

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