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A symmetry for vanishing cosmological constant: Another realization

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Abstract

A more conventional realization of a symmetry which had been proposed towards the solution of cosmological constant problem is considered. In this study the multiplication of the coordinates by the imaginary number i in the literature is replaced by the multiplication of the metric tensor by minus one. This realization of the symmetry as well forbids a bulk cosmological constant and selects out $2(2n+1)$-dimensional spaces. On contrary to its previous realization the symmetry, without any need for its extension, also forbids a possible cosmological constant term which may arise from the extra-dimensional curvature scalar provided that the space is taken as the union of two $2(2n+1)$-dimensional spaces where the usual 4-dimensional space lies at the intersection of these spaces. It is shown that this symmetry may be realized through space–time reflections that change the sign of the volume element. A possible relation of this symmetry to the E-parity symmetry of Linde is also pointed out.

Recently a symmetry [1–3] which may give insight to the origin of the extremely small value [4] of the cosmological constant compared to its theoretical value [2,5] was proposed. As in the usual symmetry arguments the symmetry forces the cosmological constant vanish and the small value of the cosmological constant is attributed to the breaking of the symmetry by a small amount. In [1] the symmetry is realized by imposing the invariance of action functional under a transformation where all coordinates are multiplied by the imaginary number i. It was found that this symmetry select out the dimensions D obeying $D = 2(2n+1)$ $n = 0, 1, \dots$, that is, $D = 2, 6, 10, \dots$ and it gives some constraints on the form of the possible Lagrangian terms as well. Moreover that symmetry has more chance to survive in quantum field theory when compared to the usual scaling symmetry because the n-point functions are invariant under this symmetry. In this Letter we study a symmetry transformation where the coordinates remain the same while the metric tensor is multiplied by minus one. We show that this symmetry is equivalent to the one given in [1]. Although its results are mainly the same as [1] it is more conventional in its form, in the sense that the space–time coordinates remain real. On contrary to [1] we use the same symmetry to forbid 4-dimensional cosmological constant as well as to forbid a bulk cosmological constant. Moreover we show that the multiplication of the metric tensor by minus one may be related to a parity-like symmetry in the extra dimensions. We also discuss the relation of this symmetry to the anti-podal symmetry of Linde [6–8], whose relation to the previous realization of the present symmetry is discussed also in [3] for the 4-dimensional case.

The symmetry principle given in [1] may be summarized as follows: the transformation

$$x_A \to i x_A, \tag{1}$$

implies

$$R \to -R, \qquad \sqrt{g}\, d^D x \to (i)^D \sqrt{g}\, d^D x, \tag{2}$$

$$ds^2 = g_{AB}\, dx^A\, dx^B \to -ds^2, \tag{3}$$

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where $A, B = 0, 1, 2, \dots, D-1$ and $D = 1, 2, \dots$ is the dimension of the space–time. The requirement of the invariance of the gravitational action functional

$$S_R = \frac{1}{16\pi G} \int \sqrt{g}\, R\, d^D x \tag{4}$$

under (1) selects out the dimensions

$$D = 2(2n+1), \quad n = 0, 1, 2, 3, \dots \tag{5}$$

and forbids a bulk cosmological constant Λ in the action

$$S_C = \frac{1}{16\pi G} \int \sqrt{g}\, \Lambda\, d^D x. \tag{6}$$

Extension of this symmetry to the full action requires that the Lagrangian should transform in the same way as the curvature scalar, that is,

$$\mathcal{L} \to -\mathcal{L} \quad \text{as } x_A \to i x_A, \quad \text{for } D = 2(2n+1),\ A = 0, 1, \dots, D-1 \tag{7}$$

(i.e. for the dimensions given by Eq. (5)). The kinetic terms of the scalar and vector fields automatically satisfy Eq. (7) while the potential terms (e.g. ϕ^4 term) are, in general, allowed in the lower-dimensional sub-branes. The fermionic part of the Lagrangian does not satisfy (7) in general so fermionic fields may live only on a lower-dimensional subspace (brane). For example the free fermion Lagrangian is allowed on a $(4m+1)$-dimensional subspace of the $2(2n+1)$-dimensional space, where $m \leqslant n$ $n, m = 0, 1, 2, \dots$. Although the transformation rules for the fields are similar to the ones for scale transformations (where the scale parameter is replaced by the imaginary number i) this symmetry has a better chance of surviving after quantization because the two point functions (e.g. $\langle 0|T\phi(x)\phi(y)|0\rangle$ for scalars and $\langle 0|T\psi(x)\bar{\psi}(y)|0\rangle$ for fermions), which are the basic building blocks for connected Feynman diagrams, are invariant under this symmetry transformation.

Now I introduce a symmetry transformation which is essentially equivalent to (1) while formulated in a more conventional form, that is,

$$g_{AB} \to -g_{AB}. \tag{8}$$

Eq. (8) induces

$$R \to -R, \qquad \sqrt{g}\, d^D x \to (\pm i)^D \sqrt{g}\, d^D x, \tag{9}$$

$$ds^2 = g_{AB}\, dx^A\, dx^B \to ds^2. \tag{10}$$

The requirement of the invariance of the gravitational action (16) under the transformation (8) selects out the dimensions given by

$$D = 2(2n+1), \quad n = 0, 1, 2, 3, \dots \tag{11}$$

as in Eq. (5), and for $D = 2(2n+1)$, $n = 0, 1, 2, 3, \dots$. Eqs. (9), (10) become identical with Eqs. (2), (3) [9,10]. Moreover one notices that the requirement of the invariance of the action functional under (8) forbids a bulk cosmological constant term (given by (6)) in $2(2n+1)$ dimensions. In other words the requirements of the invariance of the action functional under (8) and non-vanishing of its gravitational piece (5) implies $D = 2(2n+1)$ and the vanishing of the bulk cosmological constant as in [1]. Although the implications of this symmetry for Lagrangian are similar to those of [1] there are some differences. We find it more suitable to consider this point after we consider the realization of this symmetry through reflections in extra dimensions in the paragraph after the next paragraph.

We have shown that the invariance of the gravitational action under Eq. (8) requires the vanishing of the bulk cosmological constant. The next step is to show that Eq. (8) results in the vanishing of the possible contributions due to extra-dimensional curvature scalar as well so that the 4-dimensional cosmological constant vanishes altogether. On contrary to [1] we use the same symmetry ((i.e. (8)) that we have used to forbid the bulk cosmological constant) to forbid a possible 4-dimensional cosmological constant induced by extra-dimensional curvature scalar as well. To this end we take the 4-dimensional space–time be the intersection of two $2(2n+1)$-dimensional spaces; one with the dimension $2(2n+1)$ (e.g. (6)) and the other with the dimension $2(2m+1)$ (e.g. (6)) so that the total dimension of the space being $2(2m+1) + 2(2n+1) - 4 = 4(n+m)$ (e.g. (8)). Then Eq. (8) takes the following form

$$g_{AB} \to -g_{AB}, \quad A, B = 0, 1, 2, 3, 4', \dots, D'-1, \quad D' = 2(2n+1), \tag{12}$$

$$g_{CD} \to -g_{CD}, \quad C, D = 0, 1, 2, 3, 4'', \dots, D''-1, \quad D'' = 2(2m+1), \tag{13}$$

which transforms the metric and the curvature scalar as

$$\begin{aligned} &ds^2 = g_{MN}\, dx^M\, dx^N = g_{\mu\nu}\, dx^\mu\, dx^\nu + g_{ab}\, dx^a\, dx^b \to g_{\mu\nu}\, dx^\mu\, dx^\nu - g_{ab}\, dx^a\, dx^b, \\ &R_4 \to R_4, \qquad R_e \to -R_e, \qquad \sqrt{g}\, d^D x \to \sqrt{g}\, d^D x, \end{aligned} \tag{14}$$

where $R_4 = g^{\mu\nu} R_{\mu\nu}$ stands for the 4-dimensional part of the curvature scalar and $R_e = g^{ab} R_{ab}$ stands for the extra-dimensional part of the curvature scalar and

$$\mu\nu = 0, 1, 2, 3, \qquad a, b = 4', 5', \ldots, D' - 1, 4'', 5'', \ldots, D'' - 1.$$

It is evident that the extra-dimensional part of the gravitational action, that is,

$$S_{R_e} = \frac{1}{16\pi G} \int \sqrt{g}\, d^D x\, R_e \tag{15}$$

is forbidden by (14). So only

$$S_{R_4} = \frac{1}{16\pi G} \int \sqrt{g}\, R_4\, d^D x \tag{16}$$

may survive. In other words the requirement of the invariance of the action under (12) and (13) separately insures the vanishing of the bulk cosmological constant while the requirement of the invariance of the action under the simultaneous applications of (12) and (13) insures the vanishing extra-dimensional curvature scalar.

Now I take the discrete symmetry in (8) (or (12) and (13)) be a realization of a reflection symmetry in extra dimensions and study its implications. The simplest setup is to realize (12) and (13) by two reflections in two extra dimensions. To be more specific consider the following metric (where 4-dimensional Poincaré invariance is taken into account [11])

$$ds^2 = \Omega_1(y)\Omega_2(z) g_{\mu\nu}(x)\, dx^\mu\, dx^\nu + \Omega_1(y) g_{AB}(w)\, dx^A\, dx^B + \Omega_2(z) g_{CD}(w)\, dx^C\, dx^D, \tag{17}$$

where

$$\begin{aligned} & x = x^\mu, \qquad y = x^A, \qquad z = x^C, \qquad w = y, z, \\ & \mu, \nu = 0, 1, 2, 3; \qquad A, B = 4', 5', \ldots, D' - 1; \qquad C, D = 4'', 5'', \ldots, D'' - 1, \end{aligned} \tag{18}$$

$$D' = 2(2n + 1), \qquad D'' = 2(2m + 1), \quad n, m = 1, 2, 3, \ldots \tag{19}$$

and $\Omega_1(y)$, $\Omega_2(z)$ are odd functions of y, z; respectively, under some reflection; and $\tilde{g}_{AB}$, $\tilde{g}_{CD}$, are even functions of y, z. For simplicity we assume that Ω_1 and Ω_2, each depends only on one dimension, that is,

$$\Omega_1(y) = \Omega_1(y_1) \quad \text{and} \quad \Omega_2(z) = \Omega_2(z_1), \tag{20}$$

where y_1 is one of x^A and y_1 is one of x^C. For definiteness one may assume that $y_1 = x^A = x^{4'}$ and $z_1 = x^C = x^{4''}$. In other words $y_1 = x^A = x^{4'}$ and $z_1 = x^C = x^{4''}$ are taken as the directions where $\sqrt{g}\, d^D x$ changes sign under (a set of) space–time reflections in that direction. The volume element and the curvature scalar corresponding to (17) are

$$\sqrt{g}\, d^D x = \Omega_1^{2n+1}(y_1)\Omega_2^{2m+1}(z_1)\sqrt{\tilde{g}}\, d^D x, \tag{21}$$

$$\begin{aligned} R = (\Omega_1\Omega_2)^{-1}\Bigg[& R_4 + \tilde{R}_e - (D-1)\left(\tilde{g}^{4'4'} \frac{d^2 \ln(\Omega_1)}{dy_1^2} + \tilde{g}^{4''4''} \frac{d^2 \ln(\Omega_2)}{dz_1^2}\right) \\ & - \frac{(D-1)(D-2)}{4}\left(\tilde{g}^{4'4'}\left(\frac{d\ln(\Omega_1)}{dy_1}\right)^2 + \tilde{g}^{4''4''}\left(\frac{d\ln(\Omega_2)}{dz_1}\right)^2\right)\Bigg], \end{aligned} \tag{22}$$

where

$$\begin{aligned} & D = D' + D'' - 4 = 2(2n+1) + 2(2m+1) - 4 = 4(n+m), \\ & \tilde{g}^{MN} = \Omega_1(y_1)\Omega_2(z_1) g^{MN}, \qquad \tilde{g}^{4'4'} = \Omega_2(z_1) g^{4'4'}, \qquad \tilde{g}^{4''4''} = \Omega_1(y_1) g^{4''4''}. \end{aligned} \tag{23}$$

$R_4(x) = g^{\mu\nu} R_{\mu\nu}$ and $\tilde{R}_e$ are the curvature scalars of the metrics; $g_{\mu\nu}(x)\, dx^\mu\, dx^\nu$ and $\tilde{g}_{AB}(y,z)\, dx^A\, dx^B + \tilde{g}_{CD}(y,z)\, dx^C dx^D = \Omega_2^{-1}(z_1) g_{AB}(y,z)\, dx^A\, dx^B + \Omega_1^{-1}(y_1) g_{CD}(y,z)\, dx^C\, dx^D$; respectively. The action corresponding to (21) and (22) is

$$S_R = \frac{1}{16\pi G} \int \Omega_1^{2n}(y_1)\Omega_2^{2m}(z_1)\sqrt{\tilde{\tilde{g}}}\, d^D x\, \tilde{R}, \tag{24}$$

where

$$\begin{aligned} \tilde{R} = \Bigg[& R_4 + \tilde{R}_e - (D-1)\left(\tilde{g}^{4'4'} \frac{d^2 \ln(\Omega_1)}{dy_1^2} + \tilde{g}^{4''4''} \frac{d^2 \ln(\Omega_2)}{dz_1^2}\right) \\ & - \frac{(D-1)(D-2)}{4}\left(\tilde{g}^{4'4'}\left(\frac{d\ln(\Omega_1)}{dy_1}\right)^2 + \tilde{g}^{4''4''}\left(\frac{d\ln(\Omega_2)}{dz_1}\right)^2\right)\Bigg] \quad \text{and} \\ \tilde{\tilde{g}} = {} & \det(g_{\mu\nu}) \det(g_{AB}) \det(g_{CD}). \end{aligned} \tag{25}$$

One notices that all terms in $\tilde{R}$ in Eq. (24) except R_4 are odd either in y_1 or in z_1 provided that Ω_1 is odd (about some point) in y_1 and Ω_2 is odd (about some point) in z_1 and all other terms in (24) are even. So all terms in (24) except the R_4 contribution of $\tilde{R}$ vanish after integration. In other words the symmetry imposed (which makes $\Omega_{1(2)}$ odd in $y_{1(z_1)}$) guarantees the absence of cosmological constant. For example consider

$$\Omega_1 = \cos k_1 x_{5'}, \qquad \Omega_2 = \cos k_2 x_{6''}. \tag{26}$$

Because Ω_1, Ω_2 in (26) are odd under the parity operator about the point, $k_{1(2)} x_{5'(6'')} = \frac{\pi}{2}$ defined by

$$k_{1(2)} x_{5'(6'')} \to \pi - k_{1(2)} x_{5'(6'')} \tag{27}$$

and $\frac{d^2 \ln(\Omega_{1(2)})}{dy_1(z_1)^2}$ and $(\frac{d \ln(\Omega_{1(2)})}{dy_1(z_1)})^2$ are even hence the $\Omega_{1(2)}$-dependent terms in (25) are odd. By the same reason $\tilde{R}_e$ is odd as well. So the only even term in $\tilde{R}$ is R_4. So there is no contribution to the cosmological from the bulk cosmological constant or from the extra-dimensional part of the curvature scalar. One may consider other types of spaces as well; for example one may take the parity operator be defined by $x_{D-1} \to -x_{D-1}$ about the point $x_{D-1} = 0$ and either of $\Omega_{1(2)}$ or both of them change sign under the parity operator (for example, as $\Omega = \sin kx_{D-1}$). In fact one may consider a more restricted parity transformation which effectively corresponds to the interchange of two branes in the x_{D-1}-direction. For example one may take some dimensions, say the x_{D-1}th dimension, be identified by the closed line interval described by S^1/Z_2 so that $\Omega = \cos|kx_{D-1}|$, and there are two branes located at $x_{D-1} = 0$ and $kx_{D-1} = \pi$. Then the transformation in (8) is effectively induced by the interchange of the two branes.

The transformation rule for the Lagrangian under the requirement of the invariance of the action functional (where the metric (17) is considered for simplicity)

$$S_L = \int \sqrt{g}\, d^D x\, \mathcal{L} = \int \Omega_1^{2n+1}(y_1) \Omega_2^{2m+1}(z_1) \sqrt{\tilde{\tilde{g}}}\, d^D x\, \mathcal{L}, \tag{28}$$

where

$$\tilde{\tilde{g}} = \det(g_{\mu\nu}) \det(g_{AB}) \det(g_{CD})$$

under Eq. (8) (or under (12) and (13)) results in

$$\mathcal{L} \to -\mathcal{L} \quad \text{as } g_{MN} \to -g_{MN},\ M, N = 0, 1, 2, 3, \ldots, D-1, \tag{29}$$

which is similar to the condition obtained in [1]. To be more specific we consider the metrics of the form of Eq. (17). Then (29) becomes

$$\mathcal{L} \to -\mathcal{L} \quad \text{as } \Omega_1 \to -\Omega_1 \text{ and/or } \Omega_2 \to -\Omega_2. \tag{30}$$

After one considers the kinetic part of the Lagrangian for the scalar fields

$$2\mathcal{L}_k = g_{MN} (\partial_M \phi)^\dagger \partial_N \phi = \Omega_1 \Omega_2 g_{\mu\nu} (\partial_\mu \phi)^\dagger \partial_\nu \phi + \Omega_1 g_{AB} (\partial_A \phi)^\dagger \partial_B \phi + \Omega_2 g_{CD} (\partial_C \phi)^\dagger \partial_D \phi \tag{31}$$

one notices that only the 4-dimensional part of (31) transforms as in the required form, (29) under both of $\Omega_{1(2)} \to -\Omega_{1(2)}$. So the extra-dimensional piece of the kinetic Lagrangian for scalar fields is forbidden by this symmetry. In other words the extra-dimensional part of the kinetic Lagrangian vanishes after integration. The scalar field is allowed to transform as

$$\phi \to \pm\phi. \tag{32}$$

If we adopt the plus sign in (32) then no potential term is allowed in the bulk (if we impose the symmetry) while the terms localized on branes may be allowed. However introducing potential terms in the bulk is not problematic once the symmetry is identified by reflections in extra dimensions because these terms cancel out after integration over the directions where the volume element is odd under these reflections. Therefore such terms are not dangerous and no restriction is put on them in this set-up while some restrictions were obtained for such terms in the case of [1]. Hence the only term which may survive after integration over the extra dimensions is the 4-dimensional piece of the kinetic term and it does not contribute to the cosmological constant since it depends on 4-dimensional coordinates non-trivially. So the realization of the symmetry introduced here gives more freedom for model building than its previous realization which introduced some constrains on the form of the potential terms and the dimensions where they may live. Similar conclusions are valid for the vector fields as well. The case of fermion fields is more involved. The potential term of the fermionic Lagrangian is not allowed in the bulk (i.e. in each of the $2(2n+1)$-dimensional spaces) by Eqs. (12), (13). However if Eqs. (12), (13) are identified as the results of reflections in extra dimensions as given in (27) then the potential terms cancel out after integration because they are even under (27). So potential terms do not pose a problem. Kinetic term of the fermionic Lagrangian is neither odd nor even under the separate applications of Eqs. (12), (13) so it does not seem to cancel out after integration. To see this better consider the specific case where the metric is in the form of (17) and $\Omega_{1(2)}$ are given as in (26) with

$$g_{MN}=\Omega\eta_{MN},\quad \text{where}\quad \Omega=\begin{array}{ll}\Omega_1(u_1)\Omega_2(u_2) & \text{for } M,N=\mu,\nu,\\ \Omega_1(u_1) & \text{for } M,N=A,B,\\ \Omega_2(u_2) & \text{for } M,N=C,D,\end{array} \tag{33}$$

where μ, ν, A, B, C, D stand for the coordinate indices defined in Eq. (18); $u_{1(2)}$ stands for $k_{1(2)}x_{5'(6'')}$, and η_{MN} is the D-dimensional flat metric containing the usual 4-dimensional Minkowski metric. The corresponding Lagrangian and action functionals are

$$\mathcal{L}_{fk}=i\bar{\psi}\Gamma^{\mu}\partial_{\mu}\psi+i\bar{\psi}\Gamma^{a1}\partial_{a1}\psi+i\bar{\psi}\Gamma^{a2}\partial_{a2}\psi, \tag{34}$$

where

$$\Gamma^{\mu}=\left[\left(\cos\frac{u_1}{2}\tau_3+i\sin\frac{u_1}{2}\tau_1\right)\left(\cos\frac{u_2}{2}\tau_3+i\sin\frac{u_2}{2}\tau_1\right)\right]^{-1}\otimes\gamma^{\tilde{\mu}},$$

$$\Gamma^{a1(2)}=\left(\cos\frac{u_{1(2)}}{2}\tau_3+i\sin\frac{u_{1(2)}}{2}\tau_1\right)^{-1}\otimes\gamma^{a\tilde{1(2)}},$$

$$\{\Gamma^M,\Gamma^N\}=2g^{MN},\qquad \{\gamma^{\tilde{M}},\gamma^{\tilde{N}}\}=2\eta^{\tilde{M}\tilde{N}},\quad M,N=\mu,a1,a2, \tag{35}$$

$$S_{Lfk}=\int\sqrt{g}\,d^Dx\,\mathcal{L}_{fk}=\int\Omega_1^{2n}(y_1)\Omega_2^{2m}(z_1)\sqrt{\tilde{\tilde{g}}}\,d^Dx\,\mathcal{L}'_{fk}, \tag{36}$$

where

$$\mathcal{L}'_{fk}=\mathcal{L}'_{fk}=i\bar{\psi}\Gamma'^{\mu}\partial_{\mu}\psi+i\bar{\psi}\Gamma'^{a1}\partial_{a1}\psi+i\bar{\psi}\Gamma'^{a2}\partial_{a2}\psi, \tag{37}$$

$$\Gamma'^{\mu}=\Omega_1\Omega_2\Gamma^{\mu}=\cos u_1\cos u_2\left[\left(\cos\frac{u_1}{2}\tau_3+i\sin\frac{u_1}{2}\tau_1\right)\left(\cos\frac{u_2}{2}\tau_3+i\sin\frac{u_2}{2}\tau_1\right)\right]^{-1}\otimes\gamma^{\tilde{\mu}},$$

$$\Gamma'^{a1(2)}=\Omega_{1(2)}\Gamma^{a1(2)}=\cos u_{1(2)}\left(\cos\frac{u_{1(2)}}{2}\tau_3+i\sin\frac{u_{1(2)}}{2}\tau_1\right)^{-1}\otimes\gamma^{a\tilde{1(2)}}, \tag{38}$$

where $\gamma^{\tilde{A}}$ are the usual gamma matrices corresponding to η_{MN} in (33); τ_1, τ_3 are the Pauli sigma matrices; $\otimes$ denotes tensor product. Notice that the number of spinor components for the fermions and hence the size of gamma matrices are doubled by the introduction of the Pauli sigma matrices in (35) and (38). This choice is more advantageous than the gamma matrices containing the standard vielbeins involving $\sqrt{g_{MN}}\propto\sqrt{\Omega}=\sqrt{\cos u_{1(2)}}$ since $\sqrt{\cos u_{1(2)}}$ is ill defined under (30) while the gamma matrices introduced above do not pose such a problem. One notices that (37) is multiplied by $\pm i$ under

$$u_{1(2)}\to\pi-u_{1(2)} \tag{39}$$

so the argument of (37) is neither odd nor even under (39) on contrary to the scalar case, (31). Hence at first sight it seems that the method employed here to make possible extra-dimensional contribution from the fermionic kinetic term does not work. However one notices that $\cos u_{1(2)}(\cos\frac{u_{1(2)}}{2}\tau_3+i\sin\frac{u_{1(2)}}{2}\tau_1)^{-1}$ is odd under a parity operation about the point $u_{1(2)}=2\pi$ defined by

$$u_{1(2)}\to 2\pi-u_{1(2)} \tag{40}$$

and the other terms in (36) are even under (40) so that S_{Lfk} vanishes after integration. Therefore if a fermion lives in the whole bulk then its contribution to the vacuum energy (and hence to the cosmological constant) is zero if adopt the spaces (of the form of (17) and (26)) whose volume elements are odd under space–time reflections (of some of the extra dimensions). If one wants to avoid this result then the fermions must be confined into a subspace where (37) is invariant under (12) and (13), that is, the fermions must be localized in the directions (e.g. y_1 and/or z_1 in (21)) where the volume element is odd under their reflections so that the fermions live in a $4(n+m)-1$ or $4(n+m)-2$-dimensional subspace of the bulk.

Now we want to point out the relation between this scheme and the E-parity model of Linde [6]. In Linde's model the total universe consists of two universes; the usual one and ghost particles universe. The corresponding action functional is taken as

$$S=N\int d^4x\,d^4y\sqrt{g(x)}\sqrt{g(y)}\left[\frac{M_{\mathrm{Pl}}^2}{16\pi}R(x)+\mathcal{L}\big(\psi(x)\big)-\frac{M_{\mathrm{Pl}}^2}{16\pi}R(y)-\mathcal{L}\big(\hat{\psi}(y)\big)\right], \tag{41}$$

where ψ and $\hat{\psi}$ stand for the usual particles and ghost particles, $R(x)$, $R(y)$ are the scalar curvatures of the usual and the ghost parts of the universe with the metric tensors $g_{\mu\nu}$ and $\hat{g}_{\mu\nu}$; respectively. If one imposes the symmetry

$$P:\quad g_{\mu\nu}\leftrightarrow\hat{g}_{\mu\nu},\qquad P:\quad \psi\leftrightarrow\hat{\psi} \tag{42}$$

then any constant which may be induced through Lagrangians is canceled by the symmetry so that the vacuum energy hence the cosmological constant is zero. In this scenario two universes are assumed to be non-interacting (which is a rather strong condition). Other variants and refinements of this model are proposed in [7,8]. However the main idea of the scheme is preserved in these studies as well. So we do not consider them separately. I think the symmetry proposed by Linde has some relation with the symmetry studied in this Letter. The parity-odd part of the actional function in this Letter is forbidden or cancels out (depending on if you just impose the symmetry or identify it as reflection in extra dimension(s)). So as long as the vanishing part of the action functional is concerned, the action functional in this study transforms as in Eq. (41). In the present scheme

$$g_{\mu\nu} \to -g_{\mu\nu} \quad \text{implies} \quad \begin{array}{l} \text{if } R, \mathcal{L} \to -R, -\mathcal{L} \text{ and } S \to -S \text{ then } S = 0, \\ \text{if } R, \mathcal{L} \to -R, -\mathcal{L} \text{ and } S \to S \text{ then } S \neq 0, \\ \text{if } R, \mathcal{L} \to R, \mathcal{L} \text{ and } S \to -S \text{ then } S = 0, \\ \text{if } R, \mathcal{L} \to R, \mathcal{L} \text{ and } S \to S \text{ then } S \neq 0. \end{array} \tag{43}$$

In other words the vanishing cosmological constant is related to $S \to -S$ in the present study as well. As long as the cosmological constant is concerned the conclusion of both schemes are similar. The relation between two schemes can be seen better if one considers two branes in a space respecting this symmetry. Let us consider a space whose metric tensor transforms like (12), (13) and whose volume element is odd under reflections in the direction of the $x_{4'4''}$th dimension(s) and forms a closed line interval described by S^1/Z_2 with the metric of the form of (17) where $\Omega_{1(2)} = \cos|k_{1(2)x_{4'(4'')}}|$. Hence there are two branes (for each direction) located at $x_{4'(4'')} = 0$ and at $x_{4'(4'')} = \pi$. Then under the transformation given in Eqs. (12) and (13) two branes are interchanged and for the even terms in R and $\mathcal{L}$ (e.g. for cosmological constant), $S \to -S$ so that the contribution of the branes cancel each other after integration in a way similar to the Linde's model. Of course there are essential differences between the two models. The space–time in Linde's model is 4-dimensional and the volume element of the space was taken to be not effected by the symmetry while the space–time in the present model is higher-dimensional and its volume element is odd under the symmetry transformation. So in our model the parts of R, $\mathcal{L}$ which are even under the symmetry cancel out to maintain the cosmological constant zero while in Linde's model R and $\mathcal{L}$ (or at least $\mathcal{L}$) is odd under the symmetry to make the cosmological constant zero. In Linde's model symmetry is ad hoc while in the present study the symmetry arises from $g_{AB} \to -g_{AB}$, which can be identified by reflection symmetry in extra dimensions.

In this study we have studied the symmetry induced by reversal of the sign of the metric tensor. We have identified this symmetry by reflections in extra dimensions. In this way we may find some higher-dimensional spaces which satisfy the symmetry and forbid both bulk and 4-dimensional cosmological constants. We have also discussed the relation between this symmetry and the E-parity symmetry of Linde. Another point worth to mention is that throughout this study we take the gravity propagate in the whole extra dimensions while standard model particles are localized in a brane (or branes) in the bulk so that the contribution of the curvature scalar and the Lagrangian terms in the bulk which depend on only extra dimensions cancel out while the standard model effects survive.

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