

# Adaptive Output Tracking Control of a Surface Vessel

DongBin Lee, Enver Tatlicioğlu, Timothy C. Burg\*, Darren M. Dawson

**Abstract**—In this paper, the tracking control of a three degree-of-freedom marine vessel is examined. The novelty of this work is the transformation of the asymmetric inertia matrix into a symmetric, positive definite matrix. The asymmetry arises from the added mass common to practical surface vessels and creates a significant challenge for control design. The control design is further complicated by the parametric uncertainties in the dynamic model of the vessel. Two adaptive control schemes with a projection-based adaptation law are proposed: a full-state feedback controller and an output feedback controller. Both controllers are known to yield a uniformly ultimately bounded tracking result in the presence of parametric uncertainty. Numerical simulation results are shown to demonstrate the validity of the proposed controllers.

## I. Introduction

From a control perspective, the properties of the dynamic model of a marine vessel are of great importance. Specifically, the symmetry and the positive definiteness of the inertia matrix are crucial for control design and it is usually assumed to be constant (or slowly varying), strictly positive definite, and usually symmetric. However, these are satisfied only under the ideal conditions such as moving at low speed or interacting with an ideal fluid by assuming close to zero relative velocity. Non-ideal conditions such as imbalance of the ship, hydrodynamic forces and moments due to the ship and ocean current velocities, disturbances, parametric uncertainties or limited availability of signals can be encountered in many situations.

The inertia matrix of a vessel is commonly defined to be equal to the sum of the rigid-body inertia matrix and the added mass terms. The added mass terms result from the hydrodynamic forces and moments due to the motion of the vessel body and the interaction with the ocean fluids. The rigid-body inertia matrix is a strictly symmetric matrix, but the added mass matrix in surface vessel dynamic models can easily become asymmetric, especially in non-ideal conditions. This asymmetry in the added mass terms will result in an asymmetric inertia matrix which may cause system instability or a failure in meeting the control objectives when ignored.

This work has been supported in part by a DOE Contract, a Honda Corporation Grant.

D. Lee, T. C. Burg, D. M. Dawson are with the Department of Electrical and Computer Engineering, Clemson University, Clemson, SC 29634, USA. email: [dlee@clemson.edu](mailto:dlee@clemson.edu), [tburg@clemson.edu](mailto:tburg@clemson.edu) (\*corresponding author: 864-656-1368), [darren.dawson@ces.clemson.edu](mailto:darren.dawson@ces.clemson.edu). E. Tatlicioğlu is with the Department of Electrical and Electronics Engineering, Izmir Institute of Technology, Urla, Izmir 35430, Turkey. email: [enver@envertatlicioğlu.com](mailto:enver@envertatlicioğlu.com)

The research into control of marine surface vessels can be grouped by topic into maneuvering [1,2], dynamic positioning [3,4], tracking (including path following or way-point tracking) [7,9], and formation control [5]. In [6], Skjetne et al. identified the model of a three degree-of-freedom (DOF) marine vessel, *Cybership II*, and designed an adaptive maneuvering controller when the inertia matrix was symmetric and positive-definite. In [7], Do and Pan presented a global tracking controller where the system matrices are positive definite with nonzero off-diagonal entries. In [9], Behal et al. designed a tracking controller for underactuated surface vessels with nonintegrable dynamic models where the inertia matrix was diagonal. In [8], Do proposed robust and adaptive output feedback controllers for positioning of a surface vessel subject to parametric uncertainties and disturbances, and assumed the system matrices to be positive definite for low speed. Braganza et al. [12] investigated the positioning of a disabled vessel by controlling six tugboats where the uncertain inertia matrix for the ship was assumed to be symmetric and positive definite. Much of the relevant past research assumed a symmetric inertia matrix. However, some researchers have addressed the asymmetry of the inertia matrix. For example, Skjetne et al. in [2] developed the dynamic model with Coriolis-centripetal matrix model of the ship derived from the asymmetric added mass matrix. Recently, Lee et al. [13] proposed robust controllers for a surface vessel having an asymmetric inertia matrix.

In this paper, the dynamic model of a surface vessel is assumed to have asymmetric added mass terms which results in an asymmetric inertia matrix. To address this problem, this research has focused on transforming the inertia matrix into a symmetric form. The novelty of this transformation is the multiplication of the dynamic model with an upper triangular matrix to obtain a dynamic model with a symmetric inertia matrix. After this transformation, the adaptive full-state feedback (FSFB) and the adaptive output feedback (OFB) control strategies in [10]<sup>1</sup> are tailored for the control of surface vessels. Projection-based, on-line parameter estimation laws are then designed to estimate the unknown dynamic parameters.

The paper is organized as follows. Section II presents the dynamic and the kinematic models of a 3 DOF

<sup>1</sup>The adaptive control development in [10] differs from the development in [11] in a matrix decomposition, which does not affect our development. Thus, although we refer to [10] throughout the paper, the control development in [11] could also be utilized.

surface vessel. The error system development and the feedback control strategies including adaptation law are provided in Section III. A FSFB controller is proposed in Section III-A and an OFB controller is proposed in Section III-B. The numerical simulation results are shown in Section IV and the concluding remarks are presented in Section V.

## II. System Model

In this section, the system model and relevant properties are presented. The coordinate frame of the surface vessel is depicted in Figure 1 where  $B$  is the body-fixed reference frame of the vessel and the fixed inertial frame, approximated by the earth-fixed frame (North-East-Down convention), is denoted by  $I$ . The dynamic and kinematic models of a 3 DOF surface vessel are given as [1], [2]

$$M\dot{\nu} + C\nu + D\nu = \tau \quad (1)$$

$$\dot{x} = R\nu \quad (2)$$

where  $M(\theta_1, \theta_2)$ ,  $C(\nu, \nu_r, \theta_1, \theta_3)$ , and  $D(\nu, \nu_r, \theta_4, \theta_5) \in \mathbb{R}^{3 \times 3}$  represent the inertia matrix, Coriolis-centripetal matrix, and hydrodynamic damping terms, respectively, which are assumed to be uncertain and continuously differentiable up to their second time derivatives. In (1), the vectors  $\nu(t) = [u, v, \dot{\psi}]^T \in \mathbb{R}^3$  and  $\dot{\nu}(t)$  denote the linear and angular velocities and accelerations of the ship, respectively where  $u(t)$  describes the forward velocity of the vessel,  $v(t)$  is the velocity perpendicular to  $u(t)$ , and  $\dot{\psi}(t)$  is the angular velocity about the  $Z$ -axis.  $\tau(t) = [\tau_1, \tau_2, \tau_3]^T \in \mathbb{R}^3$  represents the control input vector in which  $\tau_1(t)$  and  $\tau_2(t)$  are the translational forces along the  $X$ - and  $Y$ -directions and  $\tau_3(t)$  is the moment about the  $Z$ -axis. The vector  $v_r(t) \in \mathbb{R}^3$  denotes the relative velocity between the vessel and the current and  $\theta_i$  ( $i = 1, \dots, 5$ ) are the unknown constant parameters in the system matrices. In (2),  $x(t) = [x_p, y_p, \psi]^T \in \mathbb{R}^3$  denotes the linear position ( $x_p, y_p$ ) along the  $X$ - and the  $Y$ -axes and the yaw angle,  $\psi(t)$ , which is the angle measured from the  $X$ -axis in the  $I$ -frame to the positive  $u(t)$  direction in  $B$ -frame in a clock-wise direction, and  $\dot{x}(t) \in \mathbb{R}^3$  is the position and orientation rate expressed in the  $I$ -frame. The matrix,  $R(\psi) \in SO(3)$ , denotes the rotation matrix from the origin of  $B$  to the origin of  $I$  and requires only the yaw angle. All the body-fixed states are measured from the center point (CP) of the ship frame in  $B$  rather than the center of gravity (CG) point which causes some extra forces and moments in the system matrices and  $x_g$  denotes the distance between the CP and CG of the ship. The inertia matrix,  $M(\theta_1, \theta_2)$ , of the ship is defined as [2]

$$M \triangleq M_{RB}(\theta_1) + M_A(\theta_2) \quad (3)$$

where  $M(\theta_1, \theta_2)$  is a real matrix with non-zero leading principals,  $M_{RB}(\cdot) \in \mathbb{R}^{3 \times 3}$  represents strictly symmetric rigid-body inertia matrix, and  $M_A(\cdot) \in \mathbb{R}^{3 \times 3}$  accounts for the asymmetric added mass which causes  $M(\cdot)$  to be an

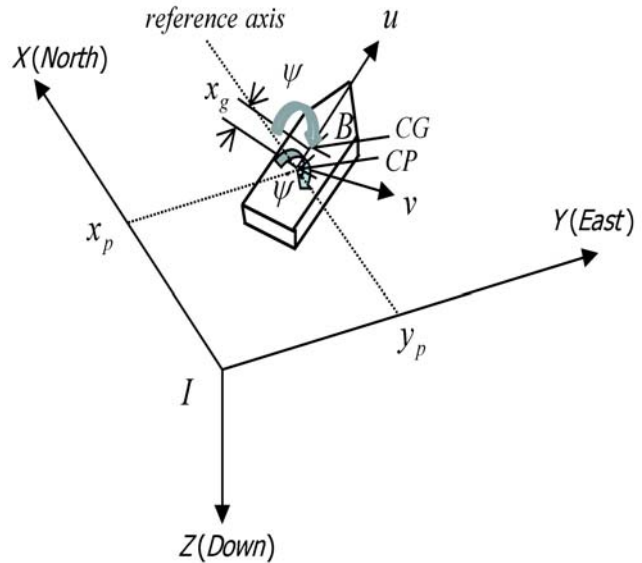


Fig. 1. Surface Vessel Coordinate Frames

asymmetric matrix. To facilitate the subsequent control development, the dynamic model in (1) will now be modified to obtain a symmetric inertia matrix. An upper triangular matrix,  $T(\cdot) \in \mathbb{R}^{3 \times 3}$ , is proposed such that the multiplication of  $T(\cdot)$  and  $M(\cdot)$  results in a symmetric, positive definite matrix, denoted by  $M_s(\cdot) \in \mathbb{R}^{3 \times 3}$ . After multiplying (1) with  $T(\cdot)$  the following expression is obtained

$$M_s \dot{\nu} = -T(C + D)\nu + T\tau \quad (4)$$

From (4), the following expression can be obtained

$$M_s R^T \ddot{x} = [\dot{\psi} M_s S_3 - T(C + D)] R^T \dot{x} + T\tau \quad (5)$$

where (2) and its time derivative were used along with the time derivative of the rotation matrix and a skew-symmetric matrix  $S_3 \in \mathbb{R}^{3 \times 3}$  according to

$$\dot{R}^T(\psi) = -\dot{\psi} S_3 R^T, \quad S_3 \triangleq \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (6)$$

After premultiplying (5) with  $R(\psi)$ , the following model is obtained

$$\bar{M} \ddot{x} = \bar{C} \dot{x} + R T \tau \quad (7)$$

where  $\bar{M}(\psi, \theta_1, \theta_2)$  and  $\bar{C}(x, \dot{x}, \nu, \nu_r, \theta_3, \theta_4, \theta_5) \in \mathbb{R}^{3 \times 3}$  are defined as

$$\bar{M} \triangleq R M_s R^T, \quad (8)$$

$$\bar{C} \triangleq R [\dot{\psi} M_s S_3 - T(C + D)] R^T. \quad (9)$$

Property1: The matrix  $\bar{M}(\cdot)$  is positive definite, symmetric, and satisfies the following inequalities

$$\lambda_1 \|\xi\|^2 \leq \xi^T \bar{M} \xi \leq \lambda_2 \|\xi\|^2, \quad \forall \xi \in \mathbb{R}^3 \quad (10)$$

where  $\lambda_1, \lambda_2 \in \mathbb{R}$  are positive bounding constants.

### III. Control Development

#### A. Full-State Feedback Control

The controller development in this section is based on the assumption that all the states of the vessel are measurable.

1) Error Development: The tracking error, denoted by  $e_1(t) \in \mathbb{R}^3$ , is defined as

$$e_1 \triangleq x_d - x \quad (11)$$

where  $x_d(t) \in \mathbb{R}^3$  is the desired trajectory. For the subsequent analysis, the desired trajectory and its first and second time derivatives are assumed to be bounded (i.e.,  $x_d(t)$ ,  $\dot{x}_d(t)$ , and  $\ddot{x}_d(t) \in \mathcal{L}_\infty$ ). To facilitate the subsequent error development, a filtered error, denoted by  $e_2(t) \in \mathbb{R}^3$ , is defined as

$$e_2 \triangleq \dot{e}_1 + e_1. \quad (12)$$

In order to simplify the error signals and to facilitate the stability analysis, another filtered tracking error,  $r(t) \in \mathbb{R}^3$ , is introduced as

$$r \triangleq e_1 + e_2. \quad (13)$$

The time derivative of  $r(t)$  can be obtained as follows

$$\dot{r} = \ddot{x}_d - \ddot{x} + 2\dot{e}_1 \quad (14)$$

where the time derivative of (12) was utilized along with the second time derivative of (11). After premultiplying (14) with  $\bar{M}(\cdot)$ , the following expression is obtained

$$\bar{M}\dot{r} = \bar{M}\ddot{x}_d - \bar{C}\dot{x} - R\tau + 2\bar{M}\dot{e}_1 \quad (15)$$

where (7) was utilized. The expression in (15) can be rearranged after adding and subtracting the terms  $\frac{1}{2}\dot{\bar{M}}r(t)$ ,  $e_1(t)$ , and  $R\tau(t)$  to the right-hand side as

$$\bar{M}\dot{r} = Y\theta - \frac{1}{2}\dot{\bar{M}}r - e_1 - R\tau \quad (16)$$

where  $Y(\cdot) \in \mathbb{R}^{3 \times p}$  is a measurable regression matrix,  $\theta \in \mathbb{R}^p$  is the unknown parameter vector, and  $Y(\cdot)\theta$  is a linear parameterization defined as

$$Y\theta \triangleq \bar{M}(\ddot{x}_d + 2e_1) - \bar{C}\dot{x} + \frac{1}{2}\dot{\bar{M}}r + e_1 - R(T - I_3)\tau. \quad (17)$$

Paralleling the approach in [10], an auxiliary error vector denoted by  $z(t) \in \mathbb{R}^6$  is introduced as

$$z = [e_1^\top, r^\top]^\top, \quad (18)$$

and taking its time derivative yields

$$\dot{z} = Az + B\dot{r} \quad (19)$$

where (12) and (13) were utilized and  $A(\cdot) \in \mathbb{R}^{6 \times 6}$ , and  $B(\cdot) \in \mathbb{R}^6$  are defined as

$$A = \begin{bmatrix} -2I_3 & I_3 \\ O_3 & O_3 \end{bmatrix}, \quad B = \begin{bmatrix} O_3 \\ I_3 \end{bmatrix} \quad (20)$$

where  $I_3 \in \mathbb{R}^{3 \times 3}$  is an identity matrix and  $O_3 \in \mathbb{R}^{3 \times 3}$  is a matrix of zeros.

2) Control Input: Based on assumption that all states are available, the control input  $\tau(t)$  is designed as [10]

$$\tau \triangleq R^\top \left[ Y\hat{\theta} + Kr + k_1 \|Y\|^2 r \right] \quad (21)$$

where  $\hat{\theta}(t) \in \mathbb{R}^p$  is the estimate of  $\theta$ ,  $K$  and  $k_1 \in \mathbb{R}^{3 \times 3}$  are constant diagonal positive-definite control gain matrices, and  $\|Y\|^2 r$  is a feedforward term. To avoid potential singularities in the control design, the unknown parameters are assumed to satisfy the following inequalities

$$\underline{\theta}_j \leq \theta_j \leq \bar{\theta}_j, \quad (j = 1, \dots, p) \quad (22)$$

where  $\theta_j$  is the  $j$ th parameter of  $\theta$  in  $Y\theta$ , and  $\underline{\theta}_j, \bar{\theta}_j \in \mathbb{R}$  are the lower and upper bounds of  $\theta_j$ , respectively. The adaptation law is designed using the projection-based update algorithm in [15] as

$$\dot{\hat{\theta}} = \text{Proj}\{\Gamma Y^\top r, \hat{\theta}\} \quad (23)$$

where  $\Gamma \in \mathbb{R}^{p \times p}$  is a constant diagonal matrix and  $\text{Proj}\{\cdot\}$  is the parameter projection operator [12]. In (23), the use of projection algorithm guarantees the parameter estimates  $\hat{\theta}(t)$  stay within a known compact set. After substituting (21) into (16), the following closed-loop error signals can be obtained

$$\bar{M}\dot{r} = Y\tilde{\theta} - \frac{1}{2}\dot{\bar{M}}r - e_1 - \left( K + k_1 \|Y\|^2 \right) r \quad (24)$$

where  $\tilde{\theta}(t) = \theta - \hat{\theta}$  denotes the parameter estimation error.

Remark 1: The development in this section is an application of the previously proposed general methodology in [10] where the stability analysis yields an uniformly, ultimately bounded (UUB) tracking result (the reader is referred to [10] for a detailed stability analysis).

#### B. Output Feedback Control

The controller development in this section is based on the assumption that the position and the orientation of the ship are the only states available to the controller.

1) Observer Design: To facilitate the subsequent control development, the following expression can be obtained for the dynamics of  $z(t)$

$$\dot{z} = \begin{bmatrix} -2e_1 + r \\ \dot{r} \end{bmatrix} \quad (25)$$

where (11) was utilized. An estimation form of  $z(t)$  is introduced as follows

$$\hat{z} \triangleq [\hat{e}_1^\top, \hat{r}^\top]^\top \quad (26)$$

where  $\hat{e}_1(t) \in \mathbb{R}^3$  and  $\hat{r}(t) \in \mathbb{R}^3$  are high-gain observers that are introduced to estimate the error signals  $e_1(t)$  and  $r(t)$ , respectively. The time derivative of (26) is defined as follows [10]

$$\dot{\hat{z}} \triangleq \begin{bmatrix} \dot{\hat{e}}_1 \\ \dot{\hat{r}} \end{bmatrix} = \begin{bmatrix} \hat{r} - 2\hat{e}_1 + \frac{\alpha_1}{\varepsilon}(e_1 - \hat{e}_1) \\ \frac{\alpha_2}{\varepsilon^2}(e_1 - \hat{e}_1) \end{bmatrix}. \quad (27)$$

where  $\varepsilon$ ,  $\alpha_1$ , and  $\alpha_2 \in \mathbb{R}$  are positive observer gains. To develop the subsequent analysis, observer errors, denoted by  $\eta_1(t)$  and  $\eta_2(t) \in \mathbb{R}^3$ , are introduced as

$$\eta_1 = \frac{1}{\varepsilon}(e_1 - \hat{e}_1) \quad (28)$$

$$\eta_2 = r - \hat{r}. \quad (29)$$

The dynamics for the observer errors can be obtained as follows

$$\dot{\eta}_1 = \frac{1}{\varepsilon}(-\alpha_1 \eta_1 + \eta_2 - 2\varepsilon \eta_1) \quad (30)$$

$$\dot{\eta}_2 = -\frac{\alpha_2}{\varepsilon} \eta_1 + \dot{r} \quad (31)$$

where (27), (28), and (29) were utilized. After combining (30) and (31), the following simplified expression can be obtained

$$\varepsilon \dot{\bar{\eta}} = A_o \bar{\eta} + \varepsilon g \quad (32)$$

where  $\bar{\eta}(t) \triangleq [\eta_1^\top, \eta_2^\top]^\top \in \mathbb{R}^6$  and the signals  $g(t) \in \mathbb{R}^6$  and  $A_o \in \mathbb{R}^{6 \times 6}$  are defined as follows

$$A_o = \begin{bmatrix} -\alpha_1 I_3 & I_3 \\ -\alpha_2 I_3 & O_3 \end{bmatrix}, \quad g = - \begin{bmatrix} 2\eta_1 \\ -\dot{r} \end{bmatrix}. \quad (33)$$

2) Control Design: The controller is designed following the general approach given in [10] as

$$\tau \triangleq R^\top \left[ \hat{Y} \hat{\theta} + K \text{sat} \{ \hat{r} \} + k_1 \left\| \hat{Y} \right\|^2 \{ \hat{r} \} \right] \quad (34)$$

where  $\text{sat}\{\cdot\}$  represents the vector saturation function and  $\hat{Y}(\cdot) \in \mathbb{R}^{3 \times p}$  is the estimate of the regression matrix. Similar to (23), the adaptation law under the parameter condition (22) is updated using  $\hat{r}(t)$  and  $\hat{Y}(\cdot)$  as follows

$$\dot{\hat{\theta}} = \text{Proj} \{ \Gamma \hat{Y}^\top (\cdot) \hat{r}, \hat{\theta} \}. \quad (35)$$

Substituting (34) into (16) yields the following closed-loop error signals

$$\bar{M} \dot{r} = Y \theta - \frac{1}{2} \bar{M} \dot{r} - e_1 + \hat{Y} \hat{\theta} - K \text{sat} \{ \hat{r} \} - k_1 \left\| \hat{Y} \right\|^2 \text{sat} \{ \hat{r} \}. \quad (36)$$

Remark 2: Since the output feedback controller in this section is a special case of the development in [10], the uniformly, ultimately bounded (UUB) tracking result will apply to this control.

#### IV. Numerical Simulation Results

Two numerical simulations were performed to show the validity of the proposed controllers. The inertia matrix of the vessel is obtained by combining the rigid-body inertia matrix and the added mass terms as

$$M(\theta_1, \theta_2) = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & n_a & n_b \\ 0 & n_c & n_d \end{bmatrix} \quad (37)$$

where  $n_a, n_b, n_c$ , and  $n_d$  are auxiliary terms that are defined as follows

$$\begin{aligned} n_a &= m - Y_{\dot{v}}, & n_b &= mx_g - Y_{\dot{r}}, \\ n_c &= mx_g - N_{\dot{v}}, & n_d &= I_z - N_{\dot{r}}. \end{aligned} \quad (38)$$

If the values of  $Y_{\dot{r}}$  and  $N_{\dot{v}}$  are different ( $Y_{\dot{r}} \neq N_{\dot{v}}$ ), then the resulting matrix is asymmetric. For the simulation, the following values were chosen for these parameters

$$Y_{\dot{r}} = -0.5, \quad N_{\dot{v}} = -1.0, \quad (39)$$

which results in an asymmetric inertia matrix. Based on the inertia matrix in (37), the following matrix (similar to [10, 11]) is defined to transform the system dynamic model

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{n_c^2}{d} - \frac{n_a n_d}{d} & \frac{n_a n_b}{d} - \frac{n_a n_c}{d} \\ 0 & 0 & \frac{n_b n_c}{d} - \frac{n_a n_d}{d} \end{bmatrix} \quad (40)$$

where  $d \in \mathbb{R}$  is a non-zero term defined as follows

$$\begin{aligned} d &= -mI_z + mN_{\dot{r}} + Y_{\dot{v}}I_z - Y_{\dot{v}}N_{\dot{r}} + (mx_g)^2 \\ &\quad - mx_g N_{\dot{v}} - mx_g Y_{\dot{r}} + Y_{\dot{r}} N_{\dot{v}}. \end{aligned}$$

The Coriolis-centripetal term  $C(\cdot)$  in (1) is defined by combining the rigid-body matrix  $C_{RB}(\nu, \theta_1) \in \mathbb{R}^{3 \times 3}$  and corresponding added mass  $C_A(\nu, \nu_r, \theta_3) \in \mathbb{R}^{3 \times 3}$  as

$$C(\nu, \nu_r, \theta_1, \theta_3) = \begin{bmatrix} 0 & 0 & c_2 \\ 0 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{bmatrix} \quad (41)$$

where  $c_1(\nu_r) = mu + (-X_{\dot{u}} u_r)$  and  $c_2(\nu, \nu_r) = -m(x_g \dot{\psi} + v) + (Y_{\dot{v}} v_r + .5(Y_{\dot{r}} + N_{\dot{v}}) \dot{\psi})$ .

The damping matrix  $D(\cdot)$  in (1) is defined by combining the linear matrix term  $D_L(\nu, \theta_4) \in \mathbb{R}^{3 \times 3}$  and nonlinear matrix  $D_{NL}(\nu, \nu_r, \theta_5) \in \mathbb{R}^{3 \times 3}$  as

$$D(\nu, \nu_r, \theta_4, \theta_5) = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix} \quad (42)$$

where  $d_{11}(\nu_r) = -X_u + (-X_{|u|u} |u_r| - X_{uuu} u_r^2)$ ,  $d_{22}(\nu, \nu_r) = -Y_v + (-Y_{|v|v} |u_r| - Y_{rv} |\dot{\psi}|)$ ,  $d_{33}(\nu, \nu_r) = -N_r + (-Y_{|v|v} |u_r| - Y_{|r|v} |\dot{\psi}|)$ ,  $d_{23}(\nu, \nu_r) = -Y_r + (-Y_{|v|v} |u_r| - Y_{|r|v} |\dot{\psi}|)$ , and  $d_{32}(\nu, \nu_r) = -N_v + (-N_{|v|v} |v_r| - N_{rv} |\dot{\psi}|)$ . The relative velocity  $\nu_r(t)$  between the body-fixed vessel and the ocean current velocity  $\nu_c(t) \in \mathbb{R}^3$ , acting as a disturbance, is defined as [2,7]

$$\nu_r(t) = \nu - \nu_c = \nu - R(\psi)^\top \dot{x}_c \quad (43)$$

where  $\dot{x}_c(t) = [3, 3, 0]^\top$  [m/s] is expressed in  $I$ . The upper and lower bounds of the vector saturation function used in each control input were set to  $\pm 200$ . The upper and lower bounds for the unknown parameters were set to  $\pm 30\%$  of their real values.

The parameter values describe the Cybership II, refer to [2], which has the dimensions LxB=1.255 m  $\times$  0.29 m and mass 23.8 kg. The desired position and orientation are specified as

$$x_d(t) = \begin{bmatrix} 10 \sin(0.1t) \text{ (m)} \\ 10 \cos(0.1t) \text{ (m)} \\ -0.1t \text{ (rad)} \end{bmatrix} \quad (44)$$

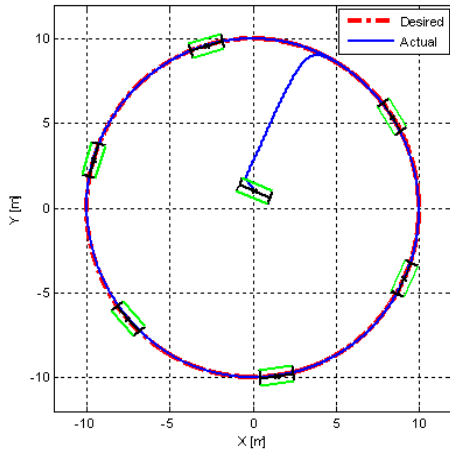


Fig. 2. Tracking Demonstration in the XY-Plane (OFB)

and the vessel was considered to be initially at rest in the following configuration

$$x(0) = [0.1, 1, -\frac{\pi}{8}]^T$$

#### A. Full-State Feedback Control

For the FSFB controller, the constant control parameters were chosen as

$$K = \text{diag} \{ 500 \ 500 \ 200 \},$$

$$k_1 = \text{diag} \{ 1 \ 1 \ 1 \}, \quad \gamma = 200.$$

where  $\gamma$  is the diagonal gain value of  $\Gamma(\cdot)$ .

(the simulation results can be seen in [16])

#### B. Output Feedback (OFB) Control

For the OFB controller simulation, the control gains were chosen as

$$K = \text{diag} \{ 600 \ 600 \ 200 \} \text{ and } k_1 = \text{diag}(1, 1, 1),$$

$$\gamma = 200, \text{ and } \alpha_1 = 100, \alpha_2 = 800, \varepsilon = 0.05.$$

The upper and lower bounds of the vector saturation function for the estimated error  $\hat{r}(t)$  in (34) were set to  $\pm 100$ . In addition, the estimated velocity,  $\hat{v}(t)$ , was used in  $C(\cdot)$  and  $D(\cdot)$  matrices instead of  $v(t)$ . In Figure 2, the actual position and the desired trajectory of the surface vessel in the  $XY$ -plane is demonstrated. Figure 3 displays the tracking error signals at each axis where small, bounded tracking errors are seen. From Figure 4 it is clear that the control objectives were met. In Figure 4, the control input  $\tau(t)$  is presented. The high-gain observer outputs defined in (27) are shown in Figure 5. Figure 6 shows the relative velocity as an added disturbance. Some parameter estimates are shown in Figures 7 (more simulation results in OFB can be seen in [16]).

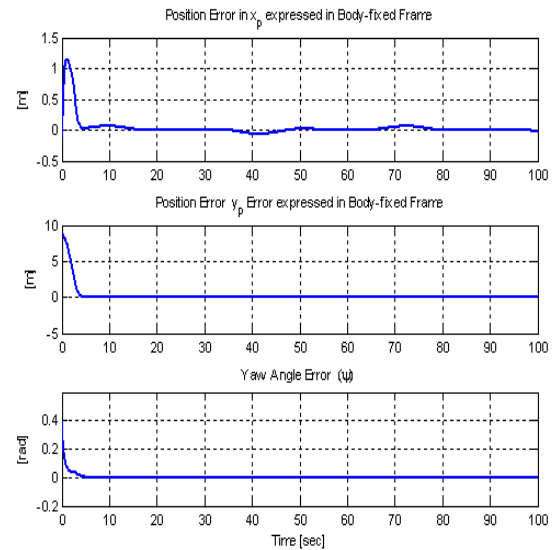
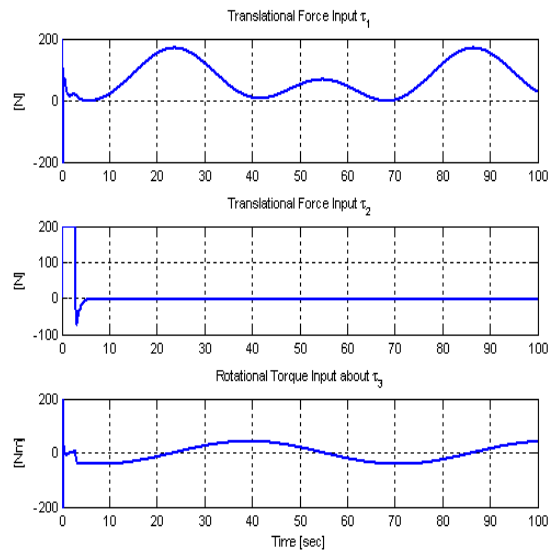
Fig. 3. Tracking Errors in Position ( $x_p$ ,  $y_p$ ) and Yaw Angle ( $\psi$ ) (OFB)

Fig. 4. Forces and Torque Input (OFB)

## V. Conclusion

The control problem of surface vessels having asymmetric inertia matrices and parametric uncertainties was addressed. An important contribution of this work was the modification of the inertia matrix by pre-multiplying an upper triangular matrix to obtain a symmetric form. Then, a full-state feedback and an output feedback controller were designed. The estimated parameters were updated by the adaptation laws utilizing the projection algorithm. Numerical simulation results were shown to demonstrate the proposed control strategies.

## References

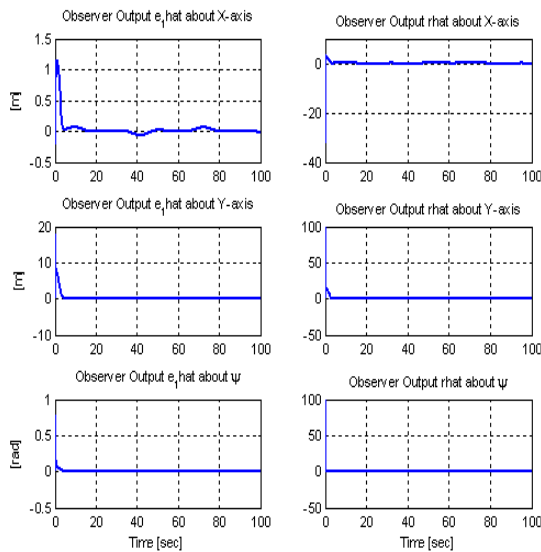
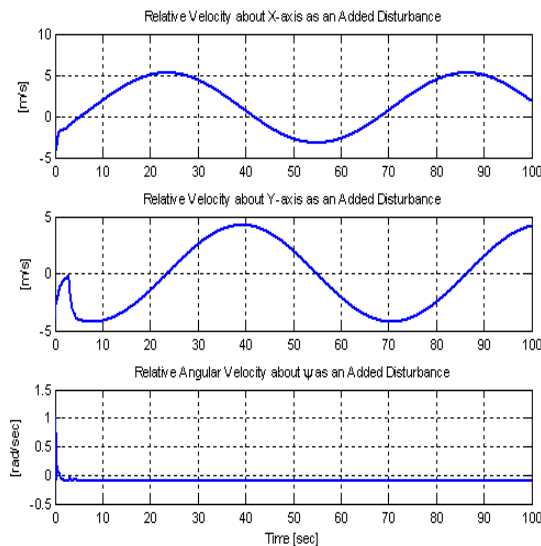
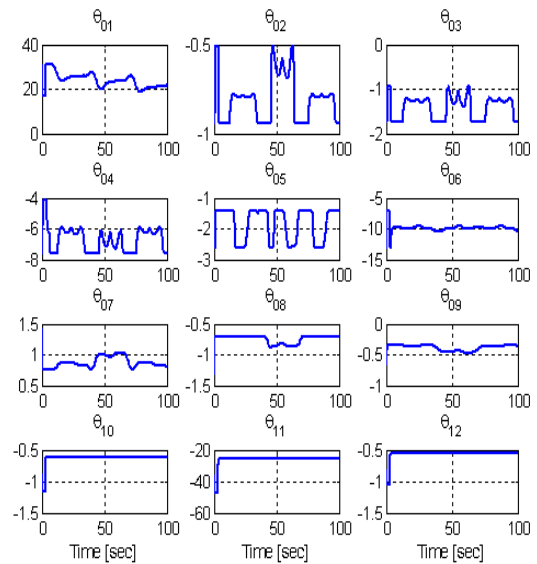
Fig. 5. Observer Output,  $\hat{e}_1(t)$  (left) and  $\hat{r}(t)$  (right), (OFB)

Fig. 6. Added Disturbance: Relative Velocity (OFB)

Fig. 7. Parameter Estimates,  $\hat{\theta}_1(t) - \hat{\theta}_{12}(t)$ , (OFB)

- [1] T.I. Fossen, Marine Control Systems : Guidance, Navigation, and Control of Ships, Rigs and Underwater Vehicles, Trondheim: Marine Cybernetics AS, Norway, 1st ed. 2002.
- [2] R. Skjetne, O.N. Smogeli, and T.I. Fossen, "A Nonlinear Ship Manoeuvring Model: Identification and adaptive control with experiments for a model ship", Modeling, Identification and Control, Vol 25, No. 1, pp. 3-27, 2004.
- [3] T.D. Nguyen, A.J. Sørensen, and S.T. Quek, "Design of Hybrid Controller for Dynamic Positioning from Calm to Extreme Sea Conditions", Automatica, Vol. 43 , No. 5, pp. 768-785, May 2007.
- [4] E.A. Tannuri and H.M. Morishita, "Experimental and numerical evaluation of a typical dynamic positioning system", Applied Ocean Research Vol. 28, No. 2, pp. 133-146, April 2006.
- [5] F. Fahimi, "Nonlinear Model Predictive Formation Control for Groups of Autonomous Surface Vessels", International Journal of Control, Vol. 80, Issue 8, pp. 1248-1259, August 2007.
- [6] R. Skjetne, T.I. Fossen, and P.V. Kokotovic, "Adaptive maneuvering, with experiments, for a model ship in a marine control laboratory", Automatica, Vol. 41, No. 2, pp. 289 - 298, 2005.
- [7] K.D. Do and J. Pan, "Global tracking control of underactuated ships with nonzero off-diagonal terms in their system matrices", Automatica, Vol. 41, pp. 87 - 95, 2005.
- [8] K.D. Do, "Global Robust and Adaptive Output Feedback Dynamic Positioning of Surface Ships", IEEE International Conference on Robotics and Automation, Rome, Italy, pp. 4271 - 4276, April 2007.
- [9] A. Behal, D.M. Dawson, W.E. Dixon, and Y. Fang, "Tracking and Regulation Control of an Underactuated Surface Vessel with Nonintegrable Dynamics", IEEE Transactions on Automatic Control, Vol. 47, No. 3, pp.495 - 500, March 2002.
- [10] J. Chen, A. Behal, and D.M. Dawson, "Adaptive Output Feedback Control for a Class of MIMO Nonlinear Systems", Proc. of the American Control Conference, Minneapolis, Minnesota, pp.5300-5305, June 2006.
- [11] J. Chen, A. Behal, D.M. Dawson, and X. Zhang, "Robust and Adaptive Output Feedback Control Strategies for a Class of Uncertain MIMO Nonlinear Systems", Technical Report, CRB, Clemson University, April 2006, Available: <http://www.ece.clemson.edu/ece/crb/publicn/tr.htm>.
- [12] D. Braganza, M. Feemster, and D.M. Dawson, "Positioning of Large Surface Vessels using Multiple Tugboats", Proc. of the American Control Conference, New York, NY, pp. 912 - 917, July 2007.
- [13] D. Lee, E. Tatlicioglu, T.C. Burg, and D.M. Dawson, "Robust Output Tracking Control of a Surface Vessel," Proc. of American Control Conference, accepted to appear, Seattle, WA, June 2008.
- [14] A.N. Atassi and H.K. Khalil, "A Separation Principle for the Stabilization of a Class of Nonlinear Systems", IEEE Transactions on Automatic Control, Vol. 44, No. 9, pp.1672 - 1687, Sept. 1999.
- [15] J.B. Pomet and L. Praly, "Adaptive Nonlinear Regulation: Estimation from the Lyapunov Equation", IEEE Trans. on Automatic Control, Vol. 37, pp. 729-740, June 1992.
- [16] D. Lee, E. Tatlicioglu, T.C. Burg, and D.M. Dawson, "Adaptive Output Feedback Tracking Control of a Surface Vessel", Clemson University, ECE, CRB Technical Report, CU/CRB/3/03/08/#1, Available: <http://www.ece.clemson.edu/crb/publicn/tr.htm>.