FUNCTION GENERATION SYNTHESIS OF PLANAR 5R MECHANISM

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Abstract: *This paper deals with the function generation problem for a planar five-bar mechanism. The inputs to the mechanism are selected as one of the fixed joints and the mid-joint, whereas the remaining fixed joint represents the output. Synthesis problem of the five-bar mechanism is analytically formulated and an objective function is expressed in polynomial form. Function generation synthesis is performed with equal spacing and Chebyshev approximation method. The four unknown construction parameters and the error are evaluated by means of five design points and the coefficients of the objective function are determined by numerical iteration using four stationary and one moving design point. Stationary points are placed at the boundaries of the motion and the moving point is re-selected at each iteration as the point corresponding to the extremum error. Iterations are repeated until the values are stabilized. The stabilization usually occurs at the third iteration. By this method, the maximum error values are approximately equated, hence the total error is bounded at certain limits. Finally the construction parameters of the mechanism are determined.*

Keywords: kinematic synthesis, function generation, five bar mechanism.

1. INTRODUCTION

Underlying idea of kinematic synthesis can be expressed as the determination of kinematic dimensions for a certain type of mechanism to perform a specific task such as function generation, path generation or rigid body guidance with acceptable errors. In function generation synthesis, input(s) and output(s) of the mechanism are related with the input(s) and output(s) of the desired function.

In literature, various methods are used for function generation synthesis problems. Levitskii presented function generation synthesis problem for the planar four-bar mechanism with interpolation, least-square and Chebyshev approximation methods [1]. Alizade et.al used analytical methods for synthesis of function generating spherical 4R mechanisms for five precision points, [2]. Various analytical and geometrical function synthesis methods exist for single degree-of-freedom (dof) mechanisms [3], but methods of synthesis of multi-dof mechanisms are usually done by optimization (See ex. [4]). Also Davitashvili et.al worked on path generation of some 2-dof mechanisms [5].

This study investigates kinematic synthesis of 2-dof planar mechanisms by an example. The example selected is a five-bar mechanism with a fixed and moving input joint and fixed output joint.

2. DERIVATION OF OBJECTIVE FUNCTION

The construction parameters are presented on the kinematic diagram of the five-bar mechanism shown in Fig. 1. Scaling the mechanism does not change the input/output

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relationship, so without loss of generality assume the fixed length l5 = 1. θ and α are the input angles and ψ is the output angle. The function to be synthesized is given by $\psi = f(\theta, \alpha)$.

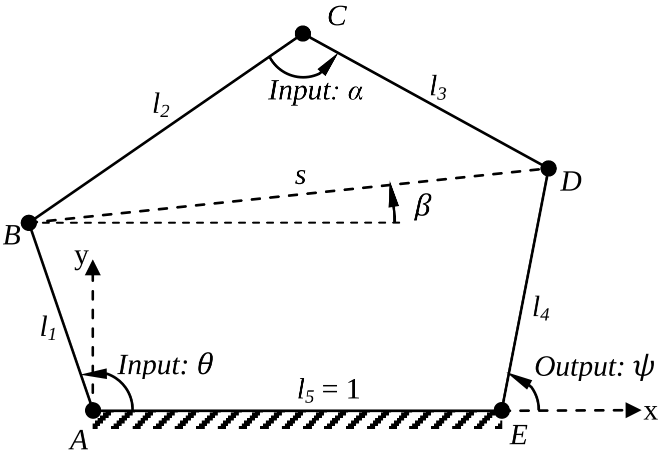

Fig.1 Five-Bar Mechanism

Writing coordinates of B and D:

$$x_B = l_1 cos\theta \; ; \; y_B = l_1 sin\theta \; ; \; x_D = 1 + l_4 cos\psi \; ; \; y_D = l_4 sin\psi \tag{1}$$

The square of the magnitude of $\overrightarrow{BD}$:

$$s^2 = \left|\overrightarrow{BD}\right|^2 = (x_D - x_B)^2 + (y_D - y_B)^2 = (1 + l_4 cos\psi - l_1 cos\theta)^2 + (l_4 sin\psi - l_1 sin\theta)^2$$
$$= 1 + {l_1}^2 + {l_4}^2 - 2l_1 cos\theta + 2l_4 cos\psi - 2l_1 l_4 cos(\psi - \theta) \tag{2}$$

writing cosine theorem for triangle BCD:

$$s^2 = {l_2}^2 + {l_3}^2 - 2l_2 l_3 \cos\alpha \tag{3}$$

Equating, right hand sides of Eqs. (2) and (3) we obtain the implicit input/output relationship for the mechanism:

$$1 + {l_1}^2 - {l_2}^2 - {l_3}^2 + {l_4}^2 + 2l_2 l_3 \cos\alpha + 2l_4 \cos\psi - 2l_1 \cos\theta - 2l_1 l_4 \cos(\psi - \theta) = 0 \tag{4}$$

The objective function can be expressed in polynomial form as.

$$P_1 {f_1}^i + P_2 {f_2}^i + P_3 {f_3}^i + P_4 {f_4}^i - F^i = \pm L \quad i = 1,2,3,4,5 \tag{5}$$

where

$$P_1 = \left(1 + {l_1}^2 - {l_2}^2 - {l_3}^2 + {l_4}^2\right)/(2l_1 l_4), \; P_2 = l_2 l_3/(l_1 l_4), \; P_3 = 1/l_1, \; P_4 = -1/l_4,$$
$${f_1}^i = 1, \; {f_2}^i = \cos\alpha_i, \; {f_3}^i = \cos\psi_i, \; {f_4}^i = \cos\theta_i, \; {f_5}^i = \pm 1, \; F^i = \cos(\psi_i - \theta_i)$$

P_1, P_2, P_3, P_4 are the unknown coefficients defined by l_1, l_2, l_3, l_4. ${f_1}^i, {f_2}^i, {f_3}^i, {f_4}^i, F^i$ are functions of the design points θ_i, α_i and ψ_i. ±L is the error, magnitude of which is to be equated at the design points.

Objective function of five bar mechanism with four unknown parameters "P_i" and the error "L" requires five design points over the topological surface of the error space, where i = 1,...,5. For the five design points Eq. 5 can be written in matrix form as

$$\begin{bmatrix} {f_1}^1 & {f_2}^1 & {f_3}^1 & {f_4}^1 & {f_5}^1 \\ {f_1}^2 & {f_2}^2 & {f_3}^2 & {f_4}^2 & {f_5}^2 \\ {f_1}^3 & {f_2}^3 & {f_3}^3 & {f_4}^3 & {f_5}^3 \\ {f_1}^4 & {f_2}^4 & {f_3}^4 & {f_4}^4 & {f_5}^4 \\ {f_1}^5 & {f_2}^5 & {f_3}^5 & {f_4}^5 & {f_5}^5 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ L \end{bmatrix} = \begin{bmatrix} F^1 \\ F^2 \\ F^3 \\ F^4 \\ F^5 \end{bmatrix} \tag{6}$$

Thus, unknown parameters "P_i" and the error "L" can be solved uniquely for the function generation synthesis problem with Chebyshev approximation.

3. CASE STUDY WITH EQUAL SPACING

As a case study four of the design points are placed at the boundaries of two dimensional domains and the fifth design point is initially located at the center of the domain. The design points at the boundary are selected at the corners of the rectangular boundary and are equidistant to the center. Such a selection is somewhat similar to equal spacing.

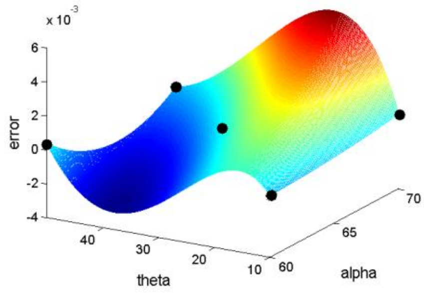

Fig.2 First Iteration
($10^{\circ} < \theta < 50^{\circ}, 60^{\circ} < \alpha < 70^{\circ}$)

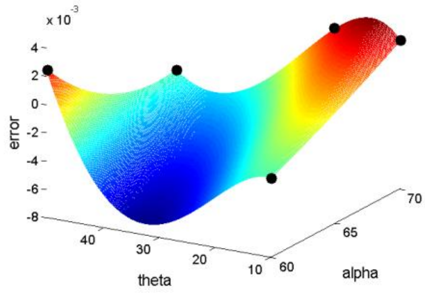

Fig.3 Second Iteration
($10^{\circ} < \theta < 50^{\circ}, 60^{\circ} < \alpha < 70^{\circ}$)

The input intervals for the task is chosen as $10^{\circ} < \theta < 50^{\circ}$, $60^{\circ} < \alpha < 70^{\circ}$. The desired generated function was selected as $\psi = \theta^{0,6}\alpha^{0,8}$. At the first iteration the error of the objective function is found as given in Fig.2. The maximum and minimum error values are calculated as $5{,}75 \times 10^{-3}$ and $-2{,}98 \times 10^{-3}$, respectively. The maximum absolute error is attained for $\theta = 21{,}72^{\circ}$ and $\alpha = 70^{\circ}$ and the design point at the center is replaced with this point. In the second iteration the maximum error value is found as $3{,}64 \times 10^{-3}$ and the minimum error value is equal to $-7{,}20 \times 10^{-3}$. Fig. 3 shows the error variation surface and selected design points. For the third iteration maximum absolute error is obtained for $\theta = 32{,}35^{\circ}$ and $\alpha = 60^{\circ}$.

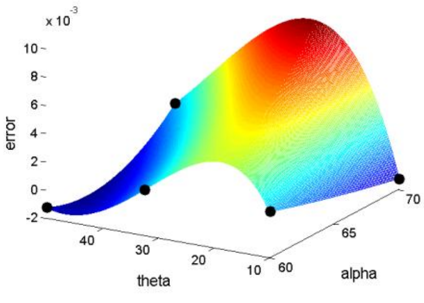

Fig.4 Third Iteration
($10^{\circ} < \theta < 50^{\circ}, 60^{\circ} < \alpha < 70^{\circ}$)

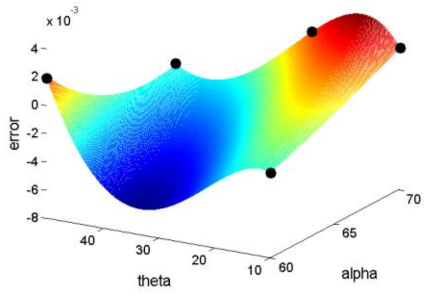

Fig.5 Fourth Iteration
($10^{\circ} < \theta < 50^{\circ}, 60^{\circ} < \alpha < 70^{\circ}$)

At the third iteration, the maximum error value is equal to $8{,}88 \times 10^{-3}$ and minimum error value is equal to $-1{,}96 \times 10^{-3}$ as shown in Fig. 4. The new design points for the next, iteration are found as $\theta = 25{,}73^{\circ}$ and $\alpha = 70^{\circ}$ corresponding to the maximum absolute error. The error of function generating mechanism for fourth iteration is shown in Fig. 5, which has a maximum and a minimum error values equal to $3{,}87 \times 10^{-3}$ and $-6{,}17 \times 10^{-3}$

respectively. The new design points for the next iteration are found as $\theta = 33{,}60^{\circ}$ and $\alpha = 60^{\circ}$ corresponding to the maximum absolute error.

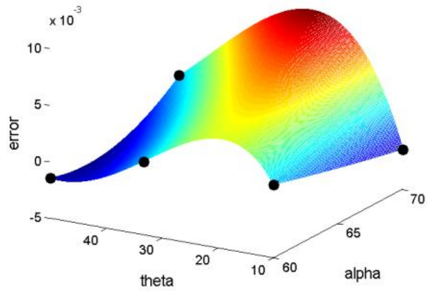

Fig.6 Fifth iteration
($10^{\circ} < \theta < 50^{\circ}, 60^{\circ} < \alpha < 70^{\circ}$)

Finally as shown in Fig.6 at the fifth iteration, the maximum error value is equal to $8{,}88 \times 10^{-3}$ and minimum error value is equal to $-1{,}96 \times 10^{-3}$ and after the fifth iteration the error values start to alternate between the values of the fourth iteration and fifth iteration, which means the error values are stabilized. The calculated construction parameters for fourth iteration are presented in Table 1.

Table 1. Numerical values of parameters for the first task

l_1	l_2	l_3	l_4	l_5	L
5,5323	0,4340	0,9676	4,1190	1	$1{,}97\times10^{-3}$

4. CONCLUSION AND DISCUSSION

In this paper, function generation problem for a planar five-bar mechanism is worked out semi-analytically for five design points. The objective function is derived and its values at the design points are expressed in matrix form. A case study is presented with four stationary design points and a moving design point. Starting with equal spacing, the calculations are iterated until the error stabilizes with Chebyshev approximation method. The errors are given as three dimensional graphical representations and finally the construction parameters are determined.

In Chebyshev approximation approach the main aim is to bound the extremum error values in a band of ±L. For single dof mechanisms it is easier to achieve this. However, as seen in the case study, the problem gets more complicated with a two dimensional error variation and it is not guaranteed that the extremum errors are bound in between ±L. Still, the designer has the option to change several initial constraints such as the bounds of inputs and seek for a feasible solution for the same function to be generated. Another option may be the initial selection of the design points at the input boundaries. These points may also be selected as midpoints of the boundaries. Also it is possible to iterate these design points using the extremum errors at the boundaries.

Another commonly encountered problem is about the link dimensions obtained at the end of the procedure. In some cases the ratio of maximum link length to minimum link length is quite large and this is usually not desirable in practice. Once again, the designer should perform repetitive trials by changing the input boundaries and/or change selection of design points at the boundaries in order to obtain a better design.

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СИНТЕЗ ПЕРЕДАТОЧНОГО ПЯТИЗВЕННОГО МЕХАНИЗМА

Чохан Кипер, Тунч Билгинджан, Мехмет Исмет Джан Деде.

Резюме: В работе рассматривается задача синтеза передаточного пятизвенного механизма с подвижными и неподвижными осями входных звеньев. Проблема син теза решена аналитически и представлена функция цели, которая реализована мето дом Чебышева. Задание четырех стационарных и одной подвижной точек Чебышева позволило определить минимальную точку исследуемой поверхности. Даны результаты синтеза и определены параметры механизма.