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RANDOM VIBRATION OF A ROAD VEHICLE

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by
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İZMİR

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ABSTRACT

RANDOM VIBRATION OF A ROAD VEHICLE

In this study, considering four different car models, random vibration characteristics of a road vehicle are investigated by using Mathematica. A road vehicle is modeled as quarter car, half car, and bicycle car. Natural frequencies of each model are found in first step. Then, responses to harmonic and random base excitation for each models are determined. The accuracy of the developed programs for different car models in Mathematica is evaluated by comparing the natural frequencies available in the literature. The effects of modeling approach on natural frequencies and response to harmonic and random base excitation are obtained. The results are presented in graphical and functional forms.

ÖZET

BİR YOL TAŞITININ RASTGELE TİTREŞİMİ

Bu çalışmada, bir yol taşıtının dört değişik modelini gözönüne alarak, rastgele titreşim karekteristikleri Mathematica kullanılarak araştırılmıştır. Bir yol taşıtı çeyrek araba, yarım araba ve bisiklet modelleri ile temsil edilmişlerdir. İlk adımda her model için doğal frekanslar bulunmuştur. Daha sonra, harmonik ve rastgele mesnet zorlamalarına karşı modellerin cevapları belirlenmiştir. Mathematica'da gelişritilen programlar, kaynaklarda mevcut olan doğal frekans ve titreşim biçimi sonuçları kullanılarak doğrulanmıştır. Modellemedeki yaklaşımların, doğal frekanslara, harmonik ve rastgele mesnet zorlamalarına karşı oluşan cevaplara etkileri elde edilmiştir. Sonuçlar grafikler ve fonksiyonlar halinde sunulmuştur.

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LIST OF SYMBOLS

a_1	*distance from mass center to front axle*
a_2	*distance from mass center to rear axle*
b_1	*distance from mass center to left wheel*
b_2	*distance from mass center to right wheel*
c	*damping coeffiient*
$[c]$	*damping matrix*
$f(t)$, $F(t)$	*force*
k	*stiffness coeffiient*
k_R	*antiroll bar torsional stiffness*
$[k]$	*stiffness matrix*
I	*mass moment of inertia*
L	*wave length*
m	*mass*
m_s	*sprung mass*
m_u	*unsprung mass*
$[m]$	*mass matrix*
R_{xy}	*crosscorrelation*
S_x	*probability density function of x*
T	*period*
$[u]$	*modal matrix*
x	*displacement*
$\ddot{x}$	*accelarion*
$\dot{x}$	*velocity*
α, β	*proportional damping coefficients*
ξ	*damping ratio*
θ	*body pitch motion coordinate*
ω	*natural frequency*
$H(\omega)$	*requency response function*

CHAPTER 1

GENERAL INTRODUCTION

In a road vehicle, mainly there are two types of vibrations. The first one is deterministic vibration which is caused by rotating parts of the vehicle. Vibration characteristic of this type can be predicted by analytical or numerical methods. At least approximate solutions can be found. The second type is random vibration which is caused by unpredicted loads such as road roughness and wind. Because of the unpredictable loads, future behavior of the system can not be precisely predicted. Responses of randomly excited systems are usually treated using statistical or probabilistic approaches.

Studying random vibrations is particularly important, because practically all real physical systems are subjected to random dynamic envoriments. Random vibration analysis in some systems such as airplanes, road vehicles, and spacecraft is very critical. Machine elements may have several damage due to the unpredictable loads which are random. While fatigue is the most critical concept and vital in aeroplane design, the passengers comfort are very important and determine the design parameter of the suspension systems in road vehicles.

In spite of the importance of the subject, there are few publications about mechanical random vibrations and fewer about random vibration of vehicle models.

Selected past studies can be summarized as follows: Stochastic modeling of vehicles for calculation of ground vibration was presented and four degree of freedom half car model established. Response spectrum of ground vibration was found and experimental work has done (Hunt 1989). Random road surface roughness is described by its power spectrum, and the dynamic response of vehicles to the road roughness is calculated from their frequency-response functions (Hunt 1990). Usage of finite element method in random vibration analysis was presented for more complicated geometries. The results are compared by analytical results in literature. Frequencies and spectral density of response was obtained.(Elishakoff and Zhu 1993) The root mean square acceleration response of a vehicle dynamic system subjected to actual random road excitations is obtained so as to account for the effect of the actual power spectral density of road excitation and the frequent changes in vehicle velocity (Tamboli 1999). Quarter car model is used to study the response of the vehicle to profile imposed excitation with randomly varying traverse velocity and variable vehicle forward velocity. Root mean square response of the vehicle to white and colored noise velocity road inputs is analyzed (Türkay and Akçay 2005). The history of random vibrations have been reviewed (Paez 2006).

Computer programs are developed in Mathematica to calculate the natural frequencies, modal transformations for decoupling the equations of motions of multi degrees of freedom car models, and plotting the response graphs. The solutions is made for different car models considering different stiffness coefficients and base excitation frequencies.

The accuracy and numerical precision of the developed deterministic models are compared by using analytical results in given in literature for all car models.

Chapter 1 is general introduction, basic information about what have done in this thesis in which methods. Also literature survey has been told in this chapter.

Chapter 2 is theoretical background about deterministic and random vibration analysis of different kinds of car models. In this chapter initial and base excitation of quarter, half and bicycle car model has been introduced. Then, random vibration analysis of single and multi degree of freedom car model has been presented.

Chapter 3 contains numerical examples of harmonically and randomly base excited car models. In this chapter natural frequencies, response of car models are found. Random vibration analysis is done depending on damping parameter β and white noise parameter S_0 which are design parameters.

CHAPTER 2

THEORETICAL BACKGROUND

2.1. Introduction

In this section, theoritical background for response to harmonic and random base excitation of different car models are given. The single degree of freedom quarter car model, two degree of freedom quarter car model, bicycle car model and half car model are introduced. The equations of motion for the car models are expressed. The matrix form of the equations and their responses are written for all car models considered. Brief theoritical information about random vibration is given.

2.2. Equations of Motion

In this section, general equations of motion will be given for different car models in both scalar and matrix form. Equation of motion for single degree of freedom system is written as,

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{2.1}$$

where

$$f(t) = ky + c\dot{y} \tag{2.2}$$

For multi degrees of freedom system, the equations of motions is expressed in matrix form as,

$$[M]\{\ddot{x}\} + [C]\{\ddot{x}\} + [K]\{x\} = \{F(t)\} \tag{2.3}$$

2.2.1. Quarter Car Model : Single Degree of Freedom

In this section, vibration analysis of a single-degree of freedom system is presented to model the vehicle as quarter car shown in Figure 2.1. In this model, only one-forth of the vehicle's mass and suspension are considered.

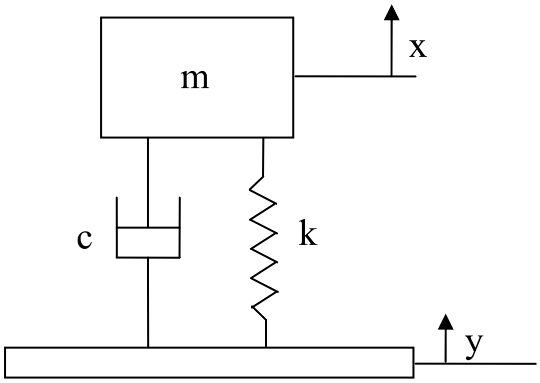

Figure 2.1. A quarter car model having single degree of freedom.

A quarter car model having single degree of freedom is shown in Figure 2.1. Equation of motion of the system given by Equation 2.1 and 2.2 are written in terms of ξ and ω_n as

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = \omega_n^2 f(t) \tag{2.4}$$

where

$$f(t) = \frac{2\xi}{\omega_n}\dot{y}(t) + y(t) \tag{2.5}$$

2.2.2. Quarter Car Model : Two Degree of Freedom

Tires and suspensions are considered in two degrees of freedom quarter car model as shown in Figure 2.2. This model is more realistic than a quarter car model with single degree of freedom. Equations of motions are given as, (Jazar 2009).

$$m_s\,\ddot{x}_s = -k_s(x_s - x_u) - c_s(\dot{x}_s - \dot{x}_u) \tag{2.6}$$

$$m_u\ddot{x}_u = k_s(x_s - x_u) + c_s(\dot{x}_s - \dot{x}_u) - k_u(x_u - y) - c_u(\dot{x}_u - \dot{y}) \qquad (2.7)$$

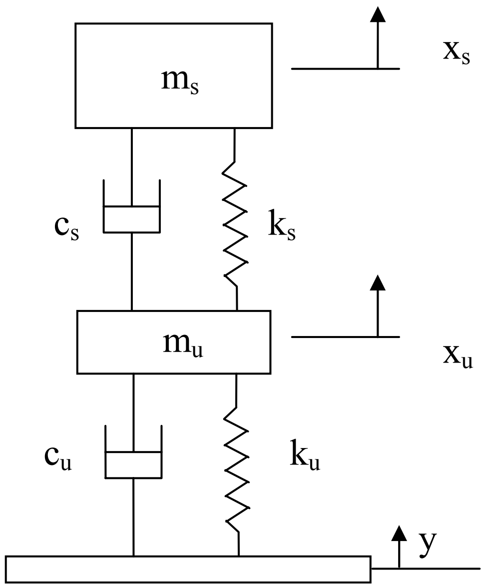

Figure 2.2. A quarter car model having two degrees of freedom.

If the Equations 2.6 and 2.7 are written in the form of Equation 2.3 depending on the displacement vector,

$$\{x(t)\} = \begin{Bmatrix} x_s(t) \\ x_u(t) \end{Bmatrix} \qquad (2.8)$$

then mass, stiffness, damping matrices, and force vector are written as,

$$[M] = \begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix} \qquad (2.9)$$

$$[K] = \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_u \end{bmatrix} \qquad (2.10)$$

$$[C] = \begin{bmatrix} c_s & -c_s \\ -c_s & c_s + c_u \end{bmatrix} \qquad (2.11)$$

$$\{F(t)\} = \begin{bmatrix} 0 \\ k_u y + c_u \dot{y} \end{bmatrix} \tag{2.12}$$

2.2.3. Bicycle Car Model

Quarter car model is excellent to examine and optimize the body bounce mode of vibrations. However, vibration model of vehicle must be expanded for including pitch and other modes of vibrations. Bicycle model includes body bounce and body pitch which are shown in Figure 2.3.

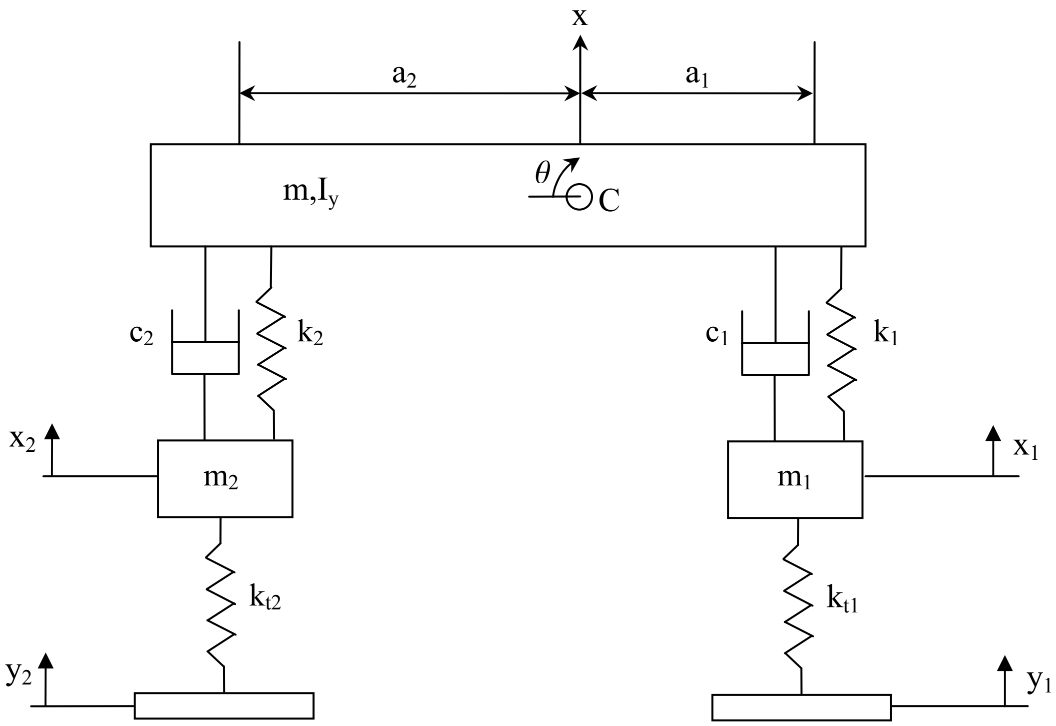

Figure 2.3. A bicycle car model having four degree of freedom.

Equations of motion for the bicycle vibrating model of a vehicle are given as follows, (Jazar 2009).

$$\begin{aligned} & m\ddot{x} + c_1\left(\dot{x} - \dot{x}_1 - a_1\,\dot{\theta}\right) + c_2\left(\dot{x} - \dot{x}_2 - a_2\,\dot{\theta}\right) \\ & + k_1\left(x - x_1 - a_1\,\theta\right) + k_2\left(x - x_2 - a_2\,\theta\right) = 0 \end{aligned} \tag{2.13}$$

$$
\begin{aligned}
&I_z\,\ddot{\theta} - a_1\,c_1\left(\dot{x} - \dot{x}_1 - a_1\,\dot{\theta}\right) + a_2\,c_2\left(\dot{x} - \dot{x}_2 - a_2\,\dot{\theta}\right) \\
&- a_1 k_1\left(x - x_1 - a_1\,\theta\right) + a_2 k_2\left(x - x_2 - a_2\,\theta\right) = 0
\end{aligned} \tag{2.14}
$$

$$
m_1 \ddot{x}_1 - c_1\left(\dot{x} - \dot{x}_1 - a_1\,\dot{\theta}\right) + k_{t1}\left(x_1 - y_1\right) - k_1\left(x - x_1 - a_1\,\theta\right) = 0 \tag{2.15}
$$

$$
m_2 \ddot{x}_2 - c_2\left(\dot{x} - \dot{x}_2 - a_2\,\dot{\theta}\right) + k_{t2}\left(x_2 - y_2\right) - k_2\left(x - x_2 - a_2\,\theta\right) = 0 \tag{2.16}
$$

If the Equations 2.13-16 are written in the form of Equation 2.3 depending on the displacement vector,

$$
\{x(t)\} = \begin{Bmatrix} x(t) \\ \theta(t) \\ x_1(t) \\ x_2(t) \end{Bmatrix} \tag{2.17}
$$

then mass, stiffness, damping matrices, and force vector are written as

$$
[M] = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_z & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix} \tag{2.18}
$$

$$
[C] = \begin{bmatrix} c_1 + c_2 & a_2 c_2 - a_1 c_1 & -c_1 & -c_2 \\ a_2 c_2 - a_1 c_1 & c_1 a_1^2 + c_2 a_2^2 & a_1 c_1 & -a_2 c_2 \\ -c_1 & a_1 c_1 & c_1 & 0 \\ -c_2 & -a_2 c_2 & 0 & c_2 \end{bmatrix} \tag{2.19}
$$

$$
[K] = \begin{bmatrix} k_1 + k_2 & a_2 k_2 - a_1 k_1 & -k_1 & -k_2 \\ a_2 k_2 - a_1 k_1 & k_1 a_1^2 + k_2 a_2^2 & a_1 k_1 & -a_2 k_2 \\ -k_1 & a_1 k_1 & k_1 + k_{t1} & 0 \\ -k_2 & -a_2 k_2 & 0 & k_2 + k_{t2} \end{bmatrix} \tag{2.20}
$$

$$\{F(t)\} = \begin{bmatrix} 0 \\ 0 \\ y_1 k_{t1} \\ y_2 k_{t2} \end{bmatrix} \tag{2.21}$$

2.2.4. Half Car Model

To examine and optimize the roll vibration of a vehicle, half car vibrating model must be used. This model includes the body bounce and body roll. The half car model may be different for the front and rear half due to different suspension and mass distrubition. Furthermore, different antiroll bars with different torsional stiffness may be used in the front and rear halves (Jazar 2009). Half car model can be seen in Figure 2.4.

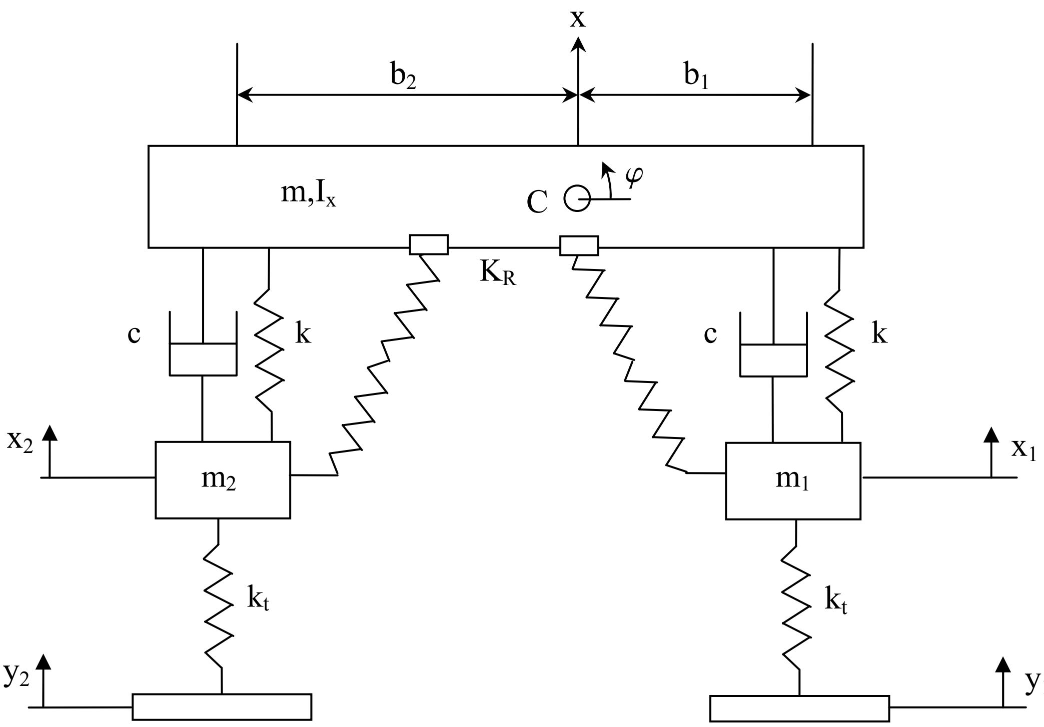

Figure 2.4. A half car model having four degree of freedom.

Equations of motion for the half car vibrating model of a vehicle are given as follows (Jazar 2009).

$$m\ddot{x}+c\left(\dot{x}-\dot{x}_1-b_1\,\dot{\varphi}\right)+c\left(\dot{x}-\dot{x}_2-b_2\dot{\varphi}\right)$$
$$+k\left(x-x_1-b_1\,\varphi\right)+k\left(x-x_2-b_2\,\varphi\right)=0 \quad (2.22)$$

$$I_x\,\ddot{\varphi}+b_1\,c\left(\dot{x}-\dot{x}_1+b_1\,\dot{\varphi}\right)-b_2\,c\,\left(\dot{x}-\dot{x}_2-b_2\,\dot{\varphi}\right)$$
$$+b_1k\left(x-x_1-b_1\,\varphi\right)-b_2k\left(x-x_2-b_2\,\varphi\right)+K_R\varphi=0 \quad (2.23)$$

$$m_1\ddot{x}_1-c\left(\dot{x}-\dot{x}_1+b_1\,\dot{\varphi}\right)+k_t\left(x_1-y_1\right)-k\left(x-x_1+b_1\,\varphi\right)=0 \quad (2.24)$$

$$m_2\ddot{x}_2-c\left(\dot{x}-\dot{x}_2+b_2\,\dot{\varphi}\right)+k_t\left(x_2-y_2\right)-k\left(x-x_2+b_2\,\varphi\right)=0 \quad (2.25)$$

If the Equations 2.22-25 are written in the form of Equation 2.3 depending on the displacement vector,

$$\{x(t)\}=\begin{bmatrix} x(t) \\ \varphi(t) \\ x_1(t) \\ x_2(t) \end{bmatrix} \quad (2.26)$$

then mass, stiffness, damping matrices, and force vector are written as

$$[M]=\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & I_X & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix} \quad (2.27)$$

$$[C]=\begin{bmatrix} 2c & cb_1-cb_2 & -c & -c \\ cb_1-cb_2 & cb_1^2+cb_2^2 & -cb_1 & cb_2 \\ -c & -cb_1 & c & 0 \\ -c & cb_2 & 0 & c \end{bmatrix} \quad (2.28)$$

$$[K]=\begin{bmatrix} 2k & kb_1-kb_2 & -k & -k \\ kb_1-kb_2 & kb_1^2+kb_2^2+k_R & -kb_1 & kb_2 \\ -k & -kb_1 & k+k_t & 0 \\ -k & kb_2 & 0 & k+k_t \end{bmatrix} \tag{2.29}$$

$$\{F(t)\}=\begin{bmatrix} 0 \\ 0 \\ y_1k_t \\ y_2kt_2 \end{bmatrix} \tag{2.30}$$

2.3. Natural Frequincies of Car Models

The natural frequency equations of different car models are introduced in this section. Natural frequencies of vehicle models can be found by using the undamped and free vibration of equations of motions. The normal mode solution of the Equation 2.3 is given by the following equation,

$$\{x(t)\}=\{X\}\,e^{i\omega t} \tag{2.31}$$

Substituting Equation 2.31 into Equation 2.3 without damping and force terms, the following generalized eigenvalue problem for the natural frequencies ω and their corresponding mode shape vectors $\{X\}$ can be written,

$$([K]-\omega^2[M])\{X\}=0 \tag{2.32}$$

The natural frequencies are obtained from the following equation,

$$\det([K]-\omega^2[M])=0 \tag{2.33}$$

Then, the corresponding eigenvectors, mode shapes, $\{X_i\}$ for natural frequencies ω_i are found by Equation 2.32. The natural frequencies for the car models given in Section 2.2 will be presented in the following subsections.

2.3.1. Quarter Car Model: Single Degree of Freedom System

Natural frequency of a single degree of freedom quarter car model is found by using the following equation,

$$\omega = \sqrt{k/m} \tag{2.34}$$

2.3.2. Quarter Car Model: Two Degree of Freedom System

From Equations 2.9, 2.10, and 2.33, following equation can be written

$$\det\begin{bmatrix} k_s - m_s\omega^2 & -k_s \\ -k_s & k_s + k_u - m_u\omega^2 \end{bmatrix} = 0 \tag{2.35}$$

2.3.3. Bicycle Car Model

From Equations 2.18, 2.20, and 2,33 following equation can be written

$$\det\begin{bmatrix} \mathrm{k}_1 + k_2 - m\omega^2 & -a_1k_1 + a_2k_2 & -k_1 & -k_2 \\ -a_1k_1 + a_2k_2 & a_1^2k_1 + a_2^2k_2 - I_z\omega^2 & a_1k_1 & -a_2k_2 \\ -k_1 & a_1k_1 & k_1 + k_2 - m_1\omega^2 & 0 \\ -k_2 & -a_2k_2 & 0 & k_2 + k_{t2} - m_2\omega^2 \end{bmatrix} = 0 \tag{2.36}$$

2.3.4. Half Car Model

From equations 2.27, 2.29, and 2.33 following equation can be written

$$\det\begin{bmatrix} 2k - m\omega^2 & b_1k - b_2k & -k & -k \\ b_1k - b_2k & b_1^2k + b_2^2k + kR - I_x\omega^2 & -b_1k & b_2k \\ -k & -b_1k & k + k_t - m_1\omega^2 & 0 \\ -k & b_2k & 0 & k + k_t - m_1\omega^2 \end{bmatrix} = 0 \tag{2.37}$$

2.4. Response To Harmonic Base Excitation

In this section, response to harmonic base excitation of the damped car models are given for a single and multi degrees of freedom systems. The base excitation frequency ω_b depending on the car velocity v and wave length of the road L can be written as

$$\omega_b = \frac{2\pi v}{L} \tag{2.38}$$

2.4.1. Single Degree of Freedom System

The equation of motion for a single degree of freedom system is given by Equation 2.4. The base excitation function $y(t)$ is generally given as

$$y(t) = \mathrm{Re}\,(Y \exp(i\omega_b t) \tag{2.39}$$

The displacement response $x(t)$ of the car model with single degree of freedom under the base excitaion function given in Equation 2.39 is found as

$$x(t) = X\,Cos\left(\omega_b\, t - \phi_1\right) \tag{2.40}$$

where

$$X = Y\left[1 + \left(\frac{2\,\xi\,\omega_b}{\omega_n}\right)^2\right]^{1/2} \left|H(\omega)\right| \tag{2.41}$$

and

$$\phi_1 = \tan^{-1}\left(\frac{2\,\xi\left(\omega_b/\omega_n\right)^3}{1-\left(\omega_b/\omega_n\right)^2 + \left(2\xi\,\omega_b/\omega_n\right)^2}\right) \tag{2.42}$$

in which

$$\left|H(\omega)\right| = \frac{1}{\{[1-\left(\omega_b/\omega_n\right)^2]^2 + [2\,\xi\,\omega_b/\omega_n]^2\}^{1/2}} \tag{2.43}$$

2.4.2. Multi Degree of Freedom System

The equations of motion of multi degrees of freedom models given by Equation 2.3 generally have n coupled differential equations. To solve these n coupled equations for harmonic $\{F(t)\}$, they must be uncoupled by using modal transformation. The uncoupled equations of motions in modal coordinates are expressed as

$$\ddot{q}_r(t) + 2\,\xi_r\,\omega_r\,\dot{q}_r(t) + \omega_r^2\,q_r(t) = N_r(t) \qquad r=1, 2, \ldots n \tag{2.44}$$

where $N_r(t)$ are associated generalized forces which are expressed in vector form as

$$\{N(t)\} = [u]^T\{F(t)\} \qquad r=1, 2, \ldots n \tag{2.45}$$

in which $[u]$ is modal matrix of the system. To accomplish the decoupling of the Equation 2.3, damping matrix $[C]$ is assumed as

$$[C] = [2\,\xi\,\omega] = \alpha\,[M] + \beta\,[K] \tag{2.46}$$

Equation 2.46 represents the proportional damping.

2.5. Fundementals of Random Vibratioıns

The mean square value of $x(t)$ is given as

$$\psi_x^2 = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt \tag{2.47}$$

There are two types of correlations which are named as autocorrelation and cross-correlation. Cross-correlation is a measure of similarity of two samples and defined by

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, y(t+\tau)\, dt \qquad (2.48)$$

Autocorrelation is the cross-correlation of a sample with itself and provides information concerning properties of a random varibale in the time domain. It is expressed as

$$R_{x}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, x(t+\tau)\, dt \qquad (2.49)$$

The maximum value of the auto-correlation function is obtained at τ=0 which can be recognized by considering the Equation 2.47 as

$$R_x(0) = \psi_x^2 \qquad (2.50)$$

The power spectral density is the fourier transform of the autocorrelation function and provides information concerning properties of a random varibale in the frequency domain. It is expressed as

$$S_x(\omega) = \int_{-\infty}^{\infty} R_x(\tau) e^{-i\omega\tau} d\omega \qquad (2.51)$$

Equation 2.51 implies that the auto-correlation function can be obtained in the form of the inverse Fourier transform

$$R_x(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_x(\omega) e^{i\omega\tau} d\omega \qquad (2.52)$$

The most common excitation assumptions in random vibration analysis is ideal white noise which includes all frequencies.

2.6. Response To Random Base Excitation

In this section. the methods of random vibration analysis will be presented for single and multi degrees of freedom systems. The analysis assumptions are ideal white noise excitation, ergodic and stationary process.

2.6.1. Single Degree of Freedom Systems

If the equation of motion of the single degree of freedom system given in Equations 2.4 and 2.5 are re-written here for convenience

$$\ddot{x}(t) + 2\xi\omega_n\dot{x}(t) + \omega_n^2 x(t) = \omega_n^2 f(t) \qquad (2.53)$$

where

$$f(t) = \frac{2\xi}{\omega_n}\dot{y}(t) + y(t) \qquad (2.54)$$

The power spectral density of the response $x(t)$ is given as (Meirovitch 1975)

$$S_x(\omega) = |H(\omega)|^2 S_f(\omega) \qquad (2.55)$$

where $S_f(\omega)$ is the power spectral density of the forcing function $f(t)$. In this study, $S_f(\omega)$ is assumed as ideal white noise which can be written as

$$S_f(\omega) = S_0 \qquad (2.56)$$

Substituting Equation 2.43 and 2.56 into Equation 2.55, the following equation is obtained:

$$S_x(\omega) = \frac{S_0}{[1-(\omega/\omega_n)^2]^2 + [2\xi\omega/\omega_n]^2} \qquad (2.57)$$

The mean square value of the response x(t) is obtained by substituting Equation 2.57 into Equation 2.52, letting τ=0 and integrating as

$$R_x(0) = \frac{S_0 \omega_n}{4\xi} \quad (2.58)$$

2.6.2. Multi Degree of Freedom Systems

Excitation spectral matrix associated with the generalized forces is given as (Meirovitch, 1975).

$$[S_f(\omega)] = [\omega^2]^{-1} [u]^T [S_F(\omega)][u][\omega^2]^{-1} \quad (2.59)$$

where

$$[\omega^2] = [u]^T [K][u] \quad (2.60)$$

and

$$[S_F(\omega)] == \int_{-\infty}^{\infty} R_F(\omega) e^{-i\omega\tau} d\tau \quad (2.61)$$

The response correlation matrix is written as

$$[R_x(\tau)] = \frac{1}{2\pi}[u] \int_{-\infty}^{\infty} [H^*(\omega)][S_f(\omega)][H(\omega)] e^{i\omega\tau} d\omega \, [u]^T \quad (2.62)$$

where $[H(\omega)]$ is the diagonal matrix of the frequency response function $H_r(\omega)$ which is given as

$$H_r(\omega) = \frac{1}{1 - (\omega/\omega_r)^2 + i\,\xi_r \omega/\omega_r} \qquad r=1, 2, \ldots n \quad (2.63)$$

and $[H^*(\omega)]$ represents the complex conjugate of $[H(\omega)]$. ξ_r in Equation 2.63 can be found from the following equation:

$$[2\,\xi\,\omega] = [u]^T [C]\,[u] \quad (2.64)$$

The autocorrelation function associated with the response random process $x_i(t)$ can be written denoting by $[u]_i$ for the i-th row matrix of the modal matrix $[u]$ as

$$R_{xi}(\tau) = \frac{1}{2\pi}[u]_i \int_{-\infty}^{\infty} [H^*(\omega)] [S_f(\omega)] [H(\omega)] e^{i\omega\tau} d\omega [u]_i^T \qquad (i=1, 2, \ldots n) \quad (2.65)$$

The mean square value of the response $x_i(t)$ is obtained from Equation 2.65 by letting τ=0, nameley,

$$R_{xi}(0) = \frac{1}{2\pi}[u]_i \int_{-\infty}^{\infty} [H^*(\omega)][S_f(\omega)][H(\omega)] d\omega [u]_i^T \qquad (i=1, 2, \ldots n) \quad (2.66)$$

CHAPTER 3

RESULTS OF NUMERICAL EXAMPLES AND DISCUSSION

3.1. Introduction

In this chapter, the car models under harmonic and random base excitation will be analized using the numerical car parameters available in the literature. The calculations was carried out by using computer programs developed in Mathematica. The codes have been tested by natural frequencies and mode shapes available in the literature.

3.2. Response to Harmonic Base Excitation for Different Car Models

In this section, three different car models presented in Chapter 2 will be examined. The single degree of freedom sytem summarized in Chapter 2 is not discussed here because of the simplicity. Proportional damping coefficients: for mass matrix α=0, for stiffness matrix β=0.08 are taken in all numerical examples in this section. Raod profiles are assumed that Y=0.01 m and different L which will be given in next subsections. Car velocity v is selected 50 km/h.

3.2.1. Quarter Car Model

The quarter car model shown in Figure 2.2. is considered. The numerical values of the parameters used in this model is selected as follows (Jazar 2008)

m_s= 1085/4 kg

m_u=40 kg

k_s=10000 N/m

k_u=150000 N/m

Equations 2.8-2.12 are subsituted into Equations 2.3 and solved for harmonic base excitation. The natural frequencies are found to be $\omega_1 = 5.87$ rad/s and $\omega_2 = 63.26$ rad/s. The input and response functions are plotted in Figures 3.1-3 for different *L* values which are given in graphs.

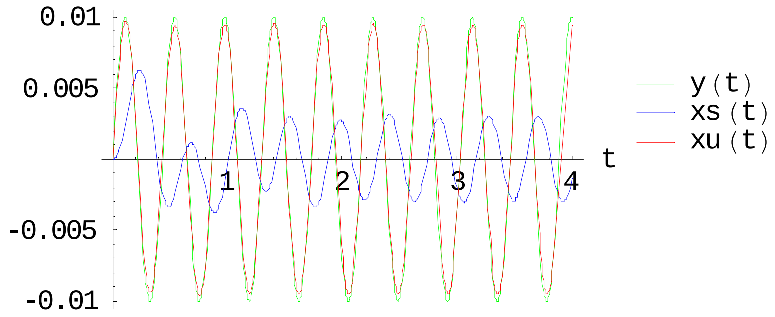

Figure 3.1. Input and response functions for *L*=6 m

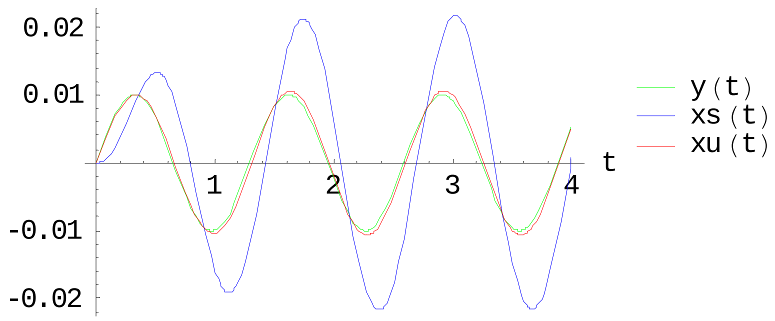

Figure 3.2. Input and response functions for *L*=18 m

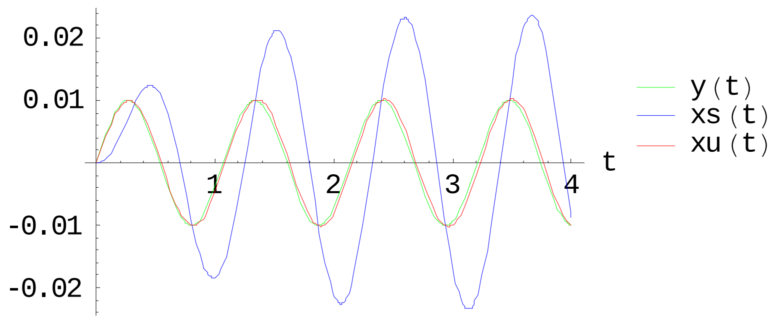

Figure 3.3. Input and response functions for *L*=14.85 m

3.2.2. Bicycle Car Model

The bicycle car model shown in Figure 2.3. is considered. The numerical values of the parameters used in this model is selected as follows (Jazar 2008)

m= 1085/2 kg, m_1=40 kg, m_2=40 kg,
I_y=1100 kg m^2,
a_1=1.4 m, a_2=1.47 m,
k_1=10000 N/m, k_{t1}= k_{t2} =150000 N/m.

Equations 2.17-2.21 are subsituted into Equations 2.3 and solved for harmonic base excitation. The natural frequencies are found to be $\omega_1 = 5.376$ rad/s, ω_2 = 5.829 rad/s, ω_3 = 62.861 rad/s, and ω_4 = 63.264 rad/s for k_2=8000 N/m and $\omega_1 = 5.825$ rad/s, ω_2 = 5.976 rad/s, ω_3 = 63.264 rad/s, and ω_4 = 63.265 rad/s for k_2=10000 N/m. The input and response functions are plotted in Figures 3.4-9 for L=6 m and Figures 3.10-15 for L=15 m choosing different k_2 values which are given in graphs.

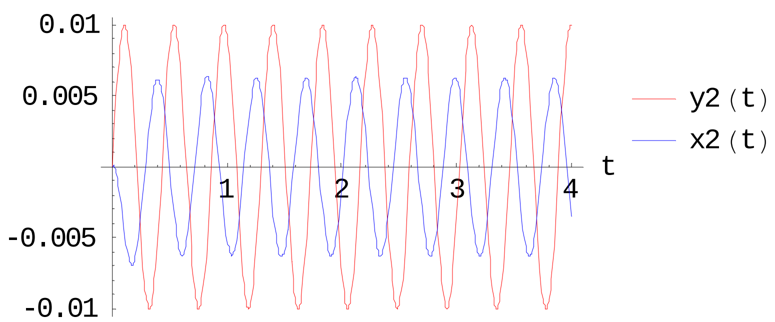

Figure 3.4. Input and response functions for k_2=8000 N/m

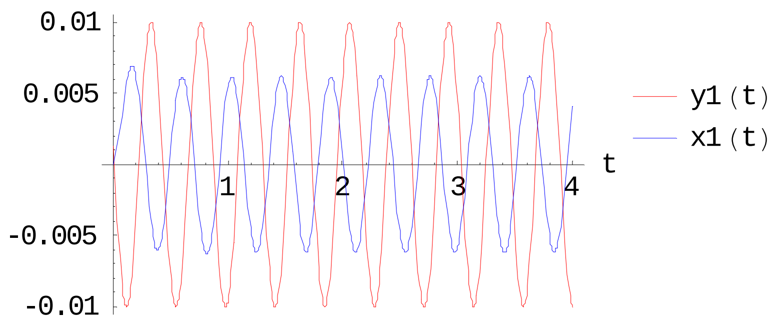

Figure 3.5. Input and response functions for k_2=8000 N/m

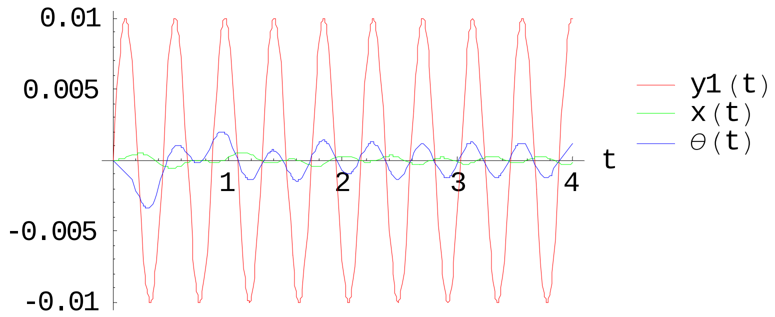

Figure 3.6. Input and response functions for k_2=8000 N/m

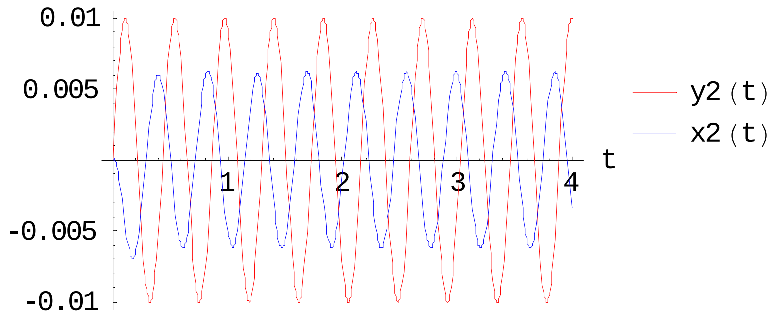

Figure 3.7. Input and response functions for k_2=10000 N/m

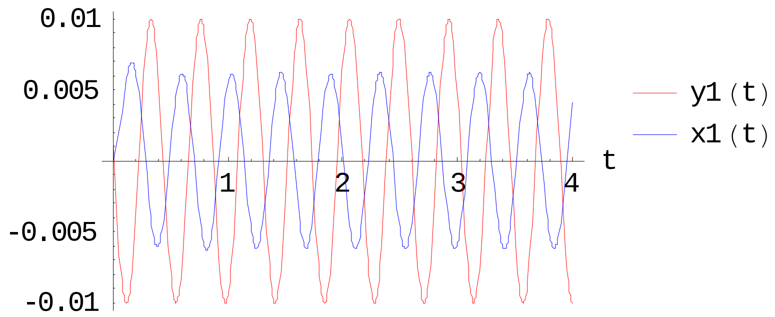

Figure 3.8. Input and response functions for k_2=10000 N/m

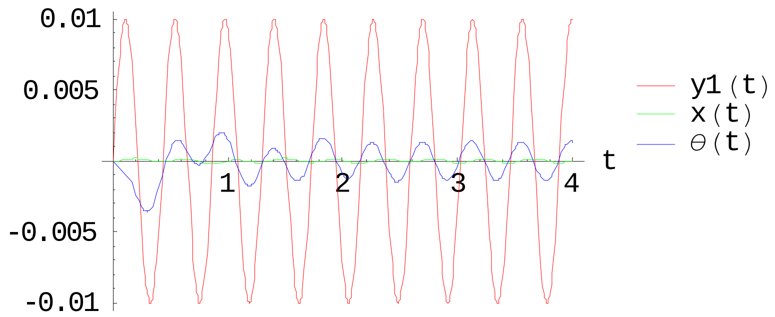

Figure 3.9. Input and response functions for k_2=10000 N/m

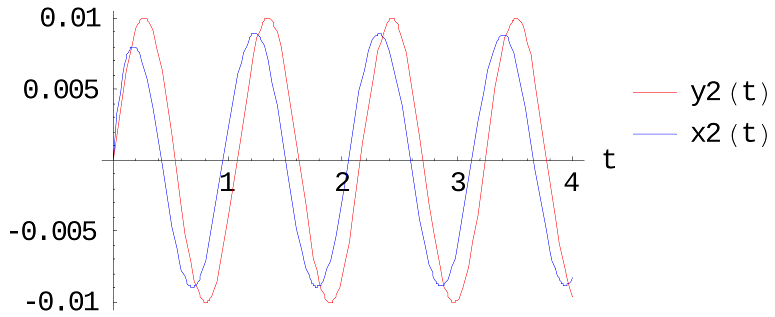

Figure 3.10. Input and response functions for k_2=8000 N/m

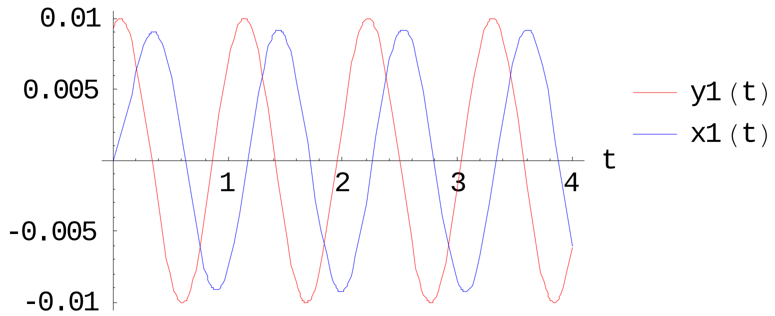

Figure 3.11. Input and response functions for k_2=8000 N/m

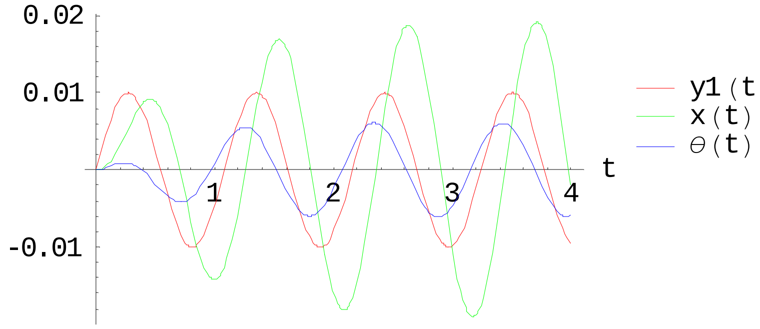

Figure 3.12. Input and response functions for k_2=8000 N/m

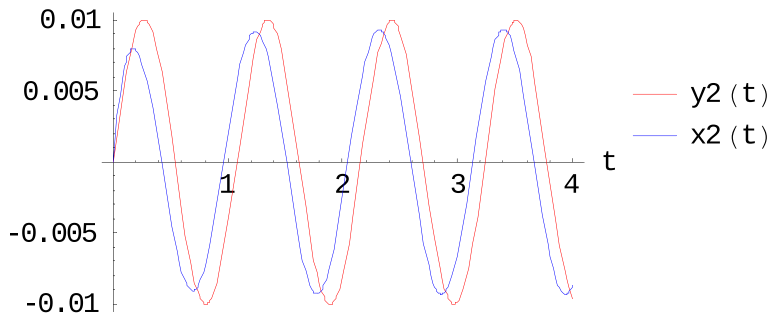

Figure 3.13. Input and response functions for k_2=10000 N/m

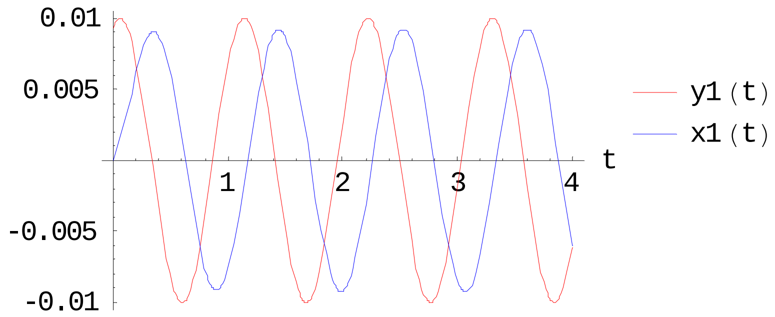

Figure 3.14. Input and response functions for k_2=10000 N/m

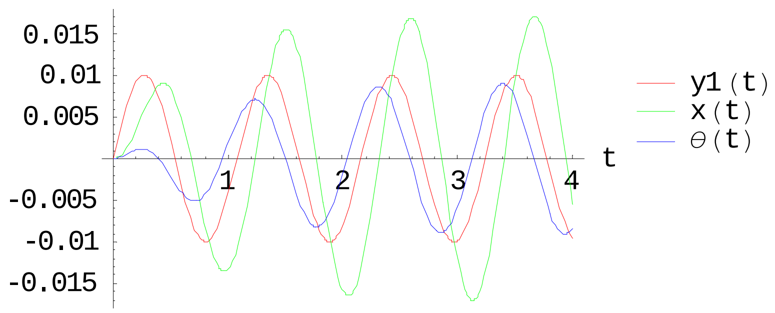

Figure 3.15. Input and response functions for k_2=10000 N/m

3.2.3. Half Car Model

The half car model shown in Figure 2.4. is considered. The numerical values of the parameters used in this model is selected as follows (Jazar 2008)

m= 1085/2 kg, m_1=40 kg, m_2=40 kg
I_x=820 kg m^2
b_1=0.7 m, b_2=0.75 m
k_1=10000 N/m, k_{t1}= k_{t2} =150000 N/m

Equations 2.23-2.28 are subsituted into Equations 2.3 and solved for harmonic base excitation. The natural frequencies are found to be $\omega_1 = 3.465$ rad/s, ω_2 = 5.879 rad/s, $\omega_3 = 63.252$ rad/s, and ω_4 = 63.264 rad/s for k_R=0 and $\omega_1 = 4.917$ rad/s, ω_2 = 5.881 rad/s, ω_3 = 63.252 rad/s, and ω_4 = 63.264 rad/s for k_R=100000 N/m. The input and response functions are plotted in Figures 3.16-21 for L=6 m and Figures 3.22-27 for L=15 m choosing different k_R values which are given in graphs.

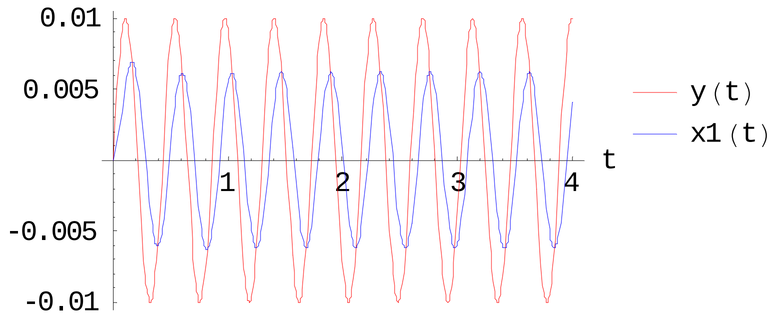

Figure 3.16. Input and response functions for k_R=0

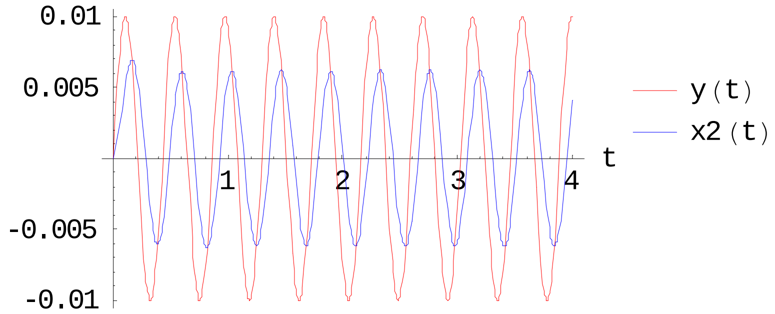

Figure 3.17. Input and response functions for k_R=0

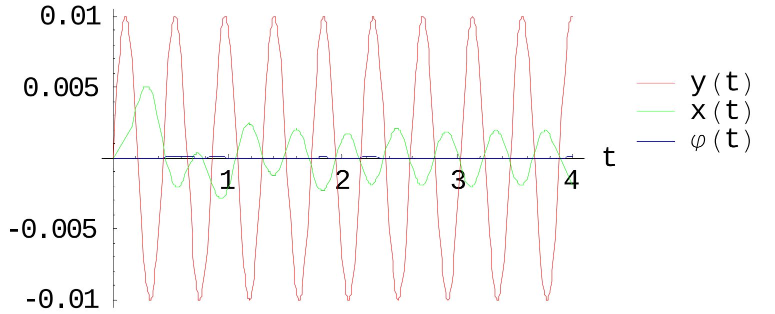

Figure 3.18. Input and response functions for k_R=0

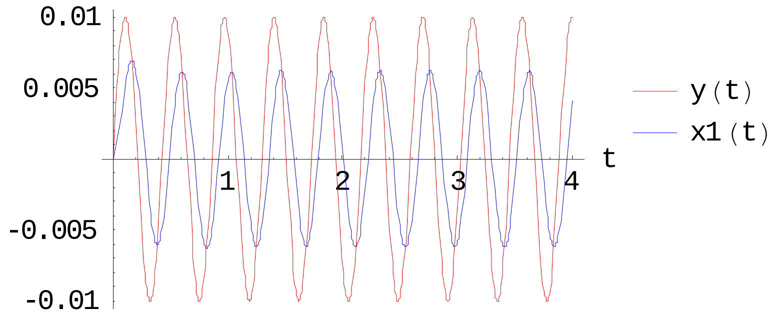

Figure 3.19. Input and response functions for k_R=10000 N/m

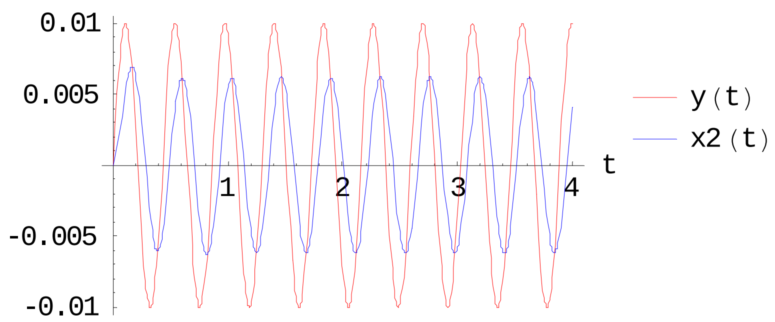

Figure 3.20. Input and response functions for k_R=10000 N/m

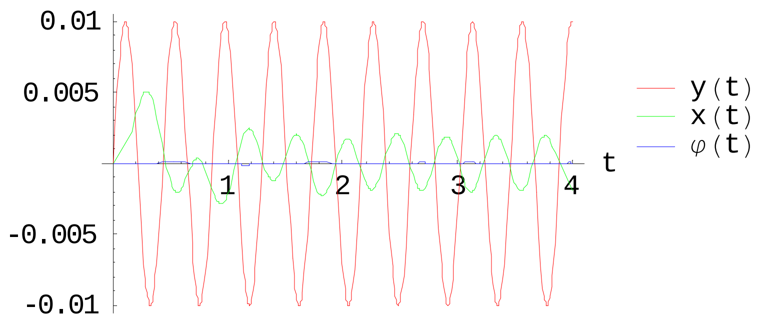

Figure 3.21. Input and response functions for k_R=10000 N/m

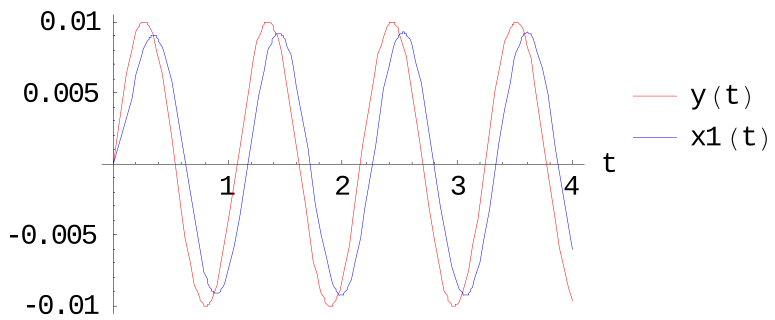

Figure 3.22. Input and response functions for k_R=0

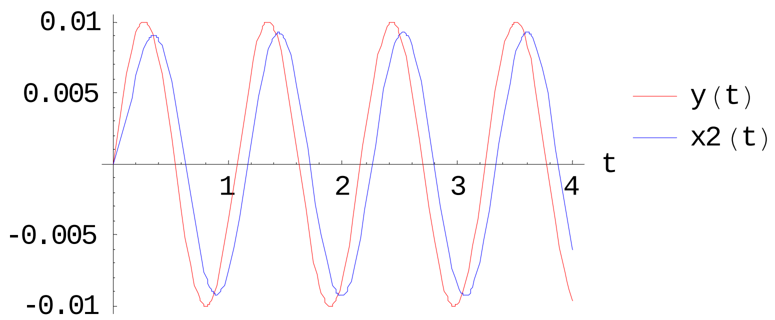

Figure 3.23. Input and response functions for k_R=0

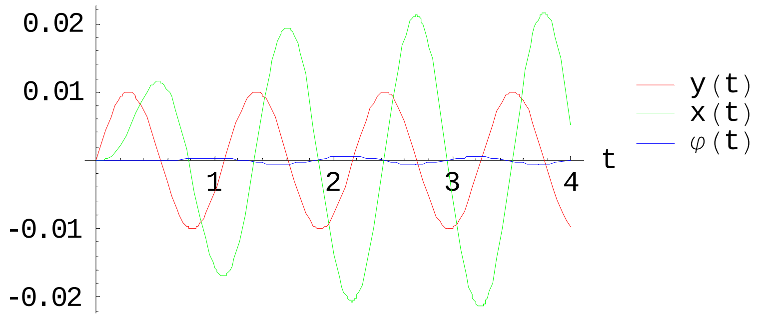

Figure 3.24. Input and response functions for k_R=0

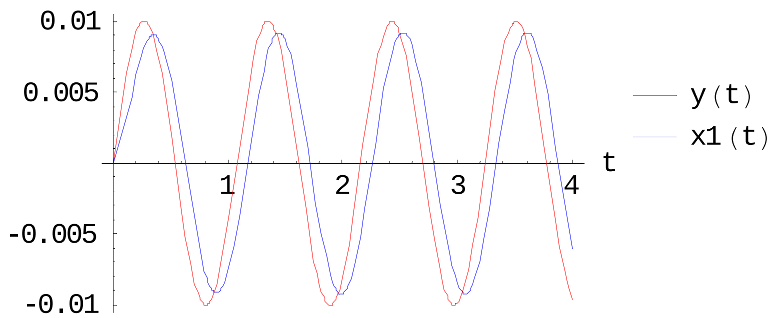

Figure 3.25. Input and response functions for k_R=10000 N/m

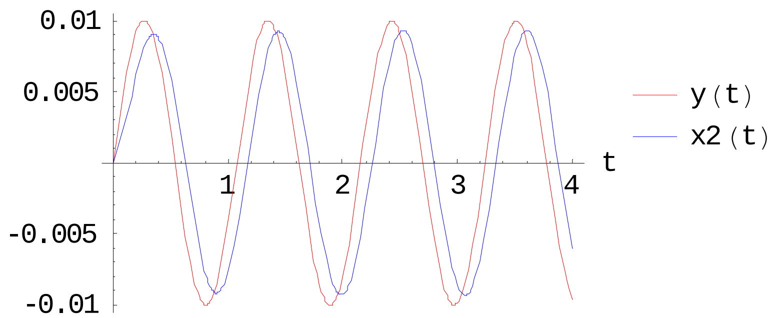

Figure 3.26. Input and response functions for k_R=10000 N/m

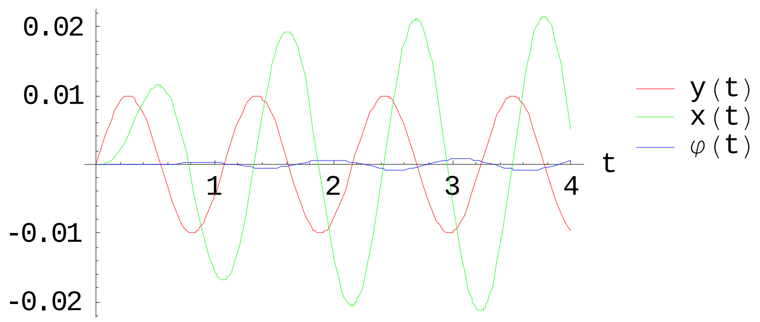

Figure 3.27. Input and response functions for k_R=10000 N/m

3.3. Response to Random Base Excitation for Different Car Models

3.3.1. Quarter Car Model

The quarter car model shown in Figure 2.2. is considered. The numerical values of the parameters used in this model is selected as in Section 3.2.1. Because of the white noise assumption for excitation, the excitation spectral matrix associtaed with the physical forces is expressed as

$$[S_F(\omega)] = \begin{bmatrix} S_0 & 0 \\ 0 & 0 \end{bmatrix}$$

Neglecting the off diagonal terms in the triple matrix multiplication appeared in Equation 2.65, the mean square value of the response in terms of damping coefficient and S_0 are found as

$$R_{xu}(S_0,\beta) = 2.26125\ \ 10^{-11}\ \frac{1}{\beta} S_0$$

$$R_{xs}(S_0,\beta) = 1.95756\ \ 10^{-11}\ \frac{1}{\beta} S_0$$

3.3.2. Bicycle Car Model

The bicycle car model shown in Figure 2.3. is considered. The numerical values of the parameters used in this model is selected as in Section 3.2.2. Because of the white noise assumption for excitation, the excitation spectral matrix associtaed with the physical forces is expressed as

$$[S_F(\omega)] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & S_0 & 0 \\ 0 & 0 & 0 & S_0 \end{bmatrix}$$

Neglecting the off diagonal terms in the triple matrix multiplication appeared in Equation 2.65, the mean square value of the response in terms of damping coefficient and S_0 are found for k_2=8000 N/m as

$$R_x(S_0,\beta) = 1.11877\ \ 10^{-11}\ \frac{1}{\beta} S_0$$

$$R_\theta(S_0,\beta) = 5.53626\ \ 10^{-12}\ \frac{1}{\beta} S_0$$

$$R_{x1}(S_0,\beta) = 1.95762\ \ 10^{-11}\ \frac{1}{\beta} S_0$$

$$R_{x2}(S_0,\beta) = 2.0056\ \ 10^{-11}\ \frac{1}{\beta} S_0$$

for k_2=10000 N/m as

$$R_x(S_0,\beta) = 1.13129\ \ 10^{-11}\ \frac{1}{\beta} S_0$$

$$R_\theta(S_0,\beta) = 5.49204\ \ 10^{-12}\ \frac{1}{\beta} S_0$$

$$R_{x1}(S_0,\beta) = 1.95763\ \ 10^{-11}\ \frac{1}{\beta} S_0$$

$$R_{x2}(S_0,\beta) = 1.95742\ \ 10^{-11}\ \frac{1}{\beta} S_0$$

3.3.3. Half Car Model

The half car model shown in Figure 2.4. is considered. The numerical values of the parameters used in this model is selected as in Section 3.2.3. Because of the white noise assumption for excitation, the excitation spectral matrix associtaed with the physical forces is expressed as

$$[S_F(\omega)] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & S_0 & 0 \\ 0 & 0 & 0 & S_0 \end{bmatrix}$$

Neglecting the off diagonal terms in the triple matrix multiplication appeared in Equation 2.65, the mean square value of the response in terms of damping coefficient and S_0 are found for k_R=0 as

$$R_x(S_0,\beta) = 1.13195\ 10^{-11}\ \frac{1}{\beta} S_0$$

$$R_\varphi(S_0,\beta) = 2.12666\ 10^{-11}\ \frac{1}{\beta} S_0$$

$$R_{x1}(S_0,\beta) = 1.959\ 10^{-11}\ \frac{1}{\beta} S_0$$

$$R_{x2}(S_0,\beta) = 1.9589\ 10^{-11}\ \frac{1}{\beta} S_0$$

for k_R=10000 N/m as

$$R_x(S_0,\beta) = 1.12579\ 10^{-11}\ \frac{1}{\beta} S_0$$

$$R_\varphi(S_0,\beta) = 5.30603\ 10^{-12}\ \frac{1}{\beta} S_0$$

$$R_{x1}(S_0,\beta) = 1.95551\ 10^{-11}\ \frac{1}{\beta} S_0$$

$$R_{x2}(S_0,\beta) = 1.95577\ 10^{-11}\ \frac{1}{\beta} S_0$$

3.4. Discussion of Results

It can be seen from Figures 3.1-3.3 that the vibration amplitude of sprung mass of the quarter car model is highly effected by the base excitation frequency which is found to be 14.544 rad/s, 4.848 rad/s, and 5.877 rad/s for the wave lengths 6m, 18 m, and 14.85 m, respectively. However, the vibration amplitude of the unsprung mass is lightly effected from the road profiles.

For bicycle car model, it is seen from Figures 3.4-9 that the translational vibration amplitude of the body is acceptable level since the base excitation frequency is far from the natural frequencies. On the other hand, when the car is excited near to natural frequency/frequencies, the translational vibration amplitude of the body is larger then the excitation amplitude. This circumstances can be seen from Figures 3.10-15. However, similar to quarter car model, the amplitude of unsprung mass is lightly effected from the road profiles.

For half car model, the similar tendencies to other car models can be seen from Figures 3.16-27.

The mean square value of the response of the sprung mass is less than the mean square value of the response of the unsprung mass for bicycle and half car models but not for quarter car model.

CHAPTER 4

CONCLUSIONS

Random vibration analysis of a road vehicle is investigated using different car models which are quarter car model, bicycle car model, and half car model. Computer programs in Mathematica are developed for all car models. To understand the base excitation response behaviors of the sprung mass in all car models, firstly deterministic vibration analysis are carried out and the results are presented by graphs. This graphs show the vibration amplitudes vs time under the excitation frequencies which are near to and far from the natural frequencies. To simplify the calculations, proportional damping is considered for damping properties in car models.

Car models having multi degrees of freedoms are formulated in matrix form and the developed computer programs are coded using matrix calculations. Random base excitations for all car models are assumed as white noise which gives mostly reasonable results. The mean square value of the response are found in terms of damping coefficient β and S_0. Damping coefficient β is primary suspension design parameter. On the other hand, the value S_0 can only be obtained from the actual road profiles. Because of the restrictions, the numerical values of this parameter is not given in this study. Therefore, for the sake of completeness, semi-numerical results for random vibration analysis of different car model's results are presented.

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