# Enabling Cooperation, Resource Allocation and Receiver Selection Across Cells: Complementary Fractional Frequency Reuse 

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#### Abstract

For a multi-cell multiple access channel, we develop a comprehensive cooperative communication framework: we propose a novel complementary fractional frequency reuse (FFR) strategy tailored specifically for pairwise user cooperation, also taking into account cell sectoring. This strategy allows the cell edge users not only to pool their resources and cooperate across cells, but also to choose the best receiver. We divide the users into cooperating inner and outer user pairs, and assign each pair orthogonal resources using OFDMA. We employ pairwise bidirectional cooperation based on block Markov superposition encoding among user pairs. We derive the achievable rates, while taking into account the geometry dependent interference at the users and the receiver. We find the jointly optimal power allocation, partner selection and receiver selection strategies that maximize the sum rate of the system. We then propose a heuristic matching algorithm, which operates based only on user and receiver locations. We compare the performance of our proposed strategies with several non-cooperative models, and demonstrate that the sum rate can nearly be doubled, while using the same resources.


## I. Introduction

The widespread use of wireless communication technologies in densely populated environments, brings along the need to revise the traditional frequency reuse and orthogonal multiple access techniques, and to build new models that accommodate more advanced opportunistic approaches such as user cooperation. In this paper, we develop such a system model by bringing together many communication and information theoretical concepts, such as user cooperation, cell planning, frequency reuse, power control, receiver selection and partner selection; and we optimize the total rate of the users in the system.
The term "user cooperation" is best suited for systems with mutually cooperating encoders, where all cooperating parties have their own messages to be transmitted. Such a system was studied in [1], where the authors identified the fading cooperative multiple access channel with Gaussian noise as a special case of a multiple access channel with generalized feedback, and characterized the achievable rates using block Markov superposition encoding and backwards decoding. In [2], the achievable rate region introduced in [1] was extended to include channel adaptive power allocation, and the optimum power control strategy was derived. However, these works mostly deal with toy information theoretic models, which involve only two transmitters and one receiver. While some extensions to simultaneous cooperation among more than two

[^0]users also exist, the encoding strategies, and the resulting rate expressions become too complicated even with only three users. Therefore, in order to implement multiuser cooperation in a large multi-cell network, it seems preferable to break the system into small orthogonal cooperative networks consisting of two users each, by means of orthogonal multiple access techniques such as OFDMA. In order to break the system into cooperating pairs, partnering decisions also need to be made before allocating the resources. Several approaches to partner selection in cooperative networks can be found in [3]-[5].

Resource allocation in OFDMA systems was studied extensively in the literature, see for example [6], [7], [8]. There are also works on resource allocation for cooperative OFDMA, such as [9], however, the cooperative models used, often involve only dedicated relays as opposed to mutual cooperation. Recently, for a full duplex two user mutually cooperative OFDMA channel, optimal power control strategies were found in [10]. The partnering strategies, which are jointly optimal with the power control strategy of [10] were obtained in [11]. However, the jointly optimal power allocation and partnering strategy of [11] is limited in two aspects. First, it is limited to a single cell setup, hence it does not take into account inter-cell interference, and the possibility of inter-cell partnering. Second, it operates by pairing strongest users by weakest cell edge users, thereby maximizing the system's sum rate at the expense of cell edge user performance, which is not desirable in an interference limited multi-cell setup.
In this paper, we deal with a multi-cell cooperative OFDMA model. To avoid inter-cell interference, traditional systems employ the concept of frequency reuse, so that, especially the cell-edge users do not suffer from adjacent cell interference. However, this approach is against the spirit of user cooperation as the cell edge users can overhear each other, even accross cells, and can be allowed to cooperate. Therefore, we introduce an unorthodox frequency reuse pattern, also factoring in cell sectoring, to allow cell edge users to share the same frequency bands in adjacent cells. In doing so, we develop a technique called complementary fractional frequency reuse, which separates the users in each cell into groups of inner and outer users just as in traditional FFR [13], but assigns frequency bands to outer users so as to allow cooperation. Moreover, by allowing cooperation across cells, we benefit from receive diversity, as the cell-edge users can now select the base station to communicate with. Our contributions are: (i) the introduction of the complementary FFR model with sectoring, which is unique to cooperative OFDMA, and which favors the cell-edge users; (ii) the calculation of interference at both inner and cell-
edge users, as well as at the receiver, based on the geometry; (iii) the characterization of achievable rates by cooperation; (iv) the solution of jointly optimal partnering, power control and receiver selection problem, using convex optimization techniques in conjunction with maximum weighted matching (MWM) and (v) the development of practical algorithms for partner selection as in the single cell setup.

## II. Proposed Cooperation and Frequency Reuse Model

We consider a cellular multiple access setup, consisting of several fading Gaussian multiple access channels operating in parallel. Multiple access towards each base station is facilitated using OFDMA, and frequency reuse is employed to increase the user capacity of the system. Yet, in our model, the users are further assumed to cooperate in pairs based on overheard information. Note that if we allow receiver selection, two cell-edge users belonging to two distinct neighboring cells could be ideal candidates for a cooperating pair. Therefore, traditional multiple access and frequency reuse techniques, which target orthogonal transmissions and especially try to avoid interference from neighboring cells are not suitable in our cooperative setup. Hence, we first develop a novel frequency reuse and multiple access model, which is directly tailored for pairwise cooperation. In [11], it was shown that in power controlled single cell cooperative OFDMA channels, optimal partner selection results in users close to base station being paired with cell edge users. As a result, cell center users abuse, rather than help, the cell edge users by taking over their subchannels while cooperating minimally, yielding a sum rate optimal but unfair resource allocation and partnering strategy. In a typical multi-cell environment, the cell edge users are more prone to interference and also suffer more from path loss; therefore fairer strategies compared to the partnering in [11] should be developed. Keeping this in mind, we propose to use a frequency reuse scheme which forces inner and outer users to cooperate in separate groups. This idea coincidentally leads to a fractional frequency reuse setup, an example of which is shown in Figure 1(a). In Figure 1(a) we assume three-cell clusters, which use four orthogonal frequency bands, $F_{1}, F_{2}, F_{3}$ and $F_{4}$, each denoted by different shades of grey. The main goal in FFR, is to increase the user capacity by allowing reuse of frequencies near the cell center, while still protecting cell edge users by assigning them orthogonal bands. Note however that orthogonalizing cell edge users in adjacent cells is completely against the spirit of user cooperation, as it rules out the possibility of cooperating across cells. Therefore, we propose the use of a rather unorthodox FFR scheme, called complementary fractional frequency reuse, which purposely assigns the same frequency sub-bands to neighboring cell sectors facing each other. This scheme is shown in Figure 1(b), where again distinct orthogonal frequency bands, $F_{1}, F_{2}, F_{3}$ and $F_{4}$ are used. Note that, the model in Figure 1(b) creates a translated frequency reuse pattern, with pseudo-cells that are composed of one sector from each cell being assigned a common frequency sub-band which is reused throughout. This
not only enables cooperation across cells, but it also allows cooperating users to select an optimal receiver, as each pseudocell is now served by any one of the three base stations in the cluster. In our model, we divide the cells in the system into 3cell clusters, and repeat the frequency reuse pattern over each cluster, as shown in Figure 1(b). We assume that there are $K=12 N$ users in a given cluster, where $N$ is an integer, and that these users are uniformly distributed over the cluster surface, yielding $4 N$ users per cell. Assuming hexagonal cells with radius $r$, each cell is divided into two concentric regions: the users inside a circle of radius $r_{i n}=r / 2$ surrounding the base station of each cell are called the inner users, and the remaining users are called outer users. Since the number of users is proportional to the area they are distributed on, there are on average $N$ inner and $3 N$ outer users in each cell. This also amounts to an average of $3 N$ users per each pseudo-cell sharing the same frequency resource.

Due to symmetry, it is sufficient to focus on a single cluster, which is highlighted by the bold boundary in Figure 1(b), and shown separately in Figure 1(c). The light gray region at the center of the cluster, consisting of one sector from each cell, will be our pseudo-cell of interest. A sample user distribution is also given in Figure 1(c), showing only the set of outer users, $U_{\text {out }}$, belonging to the pseudo cell of interest, and the inner users, $U_{i n, b}$ in each cell, where $b=\{1,2,3\}$ is the receiver, or equivalently, cell index. Other outer users may be communicating with receivers from a different cluster, and hence are not shown on Figure 1(c).

The receiver of each cell in the cluster is located at the center of the cell. The inner users in cell $b$, labeled $U_{i n, b}$, are to be grouped in cooperating pairs, exclusively within that cell; i.e., there is no inter-cell cooperation for inner users. Each pair of users $\{i, j\} \in U_{i n, b} \times U_{i n, b}$ is assigned a distinct set of subchannels $S_{i j} \subset F_{1}$, and both users of the pair simultaneously utilize these sub-channels. The outer users $U_{\text {out }}$ in the pseudocell shared by receivers $b=\{1,2,3\}$ are also to be grouped in cooperating pairs. If a cooperating pair has users from two different cells, an intended receiver is also to be selected optimally. Each pair $\{i, j\} \in U_{\text {out }} \times U_{\text {out }}$ is assigned a distinct set of sub-channels $S_{i j} \in F_{2}$, and both users of the pair simultaneously utilize these subchannels. It is easy to check that, assuming $n$ subchannels are assigned to each pair, there needs to be a total of $n N / 2$ sub-channels in $F_{1}$, and $3 n N / 2$ subchannels in $F_{2}$. This subchannel assignment is assumed to be made once, and is fixed throughout the transmission.

Regardless of the cooperating pair being an inner or outer pair, the signals received by the users $i, j$ and the receiver $b$, over each subchannel $s \in S_{i j}$, are respectively given by,

$$
\begin{align*}
Y_{i} & =\sqrt{h_{j i}^{(s)} d_{i j}^{-\alpha}} X_{j}^{(s)}+I_{i}^{(s)}+N_{i}^{(s)}  \tag{1}\\
Y_{j} & =\sqrt{h_{i j}^{(s)} d_{i j}^{-\alpha}} X_{i}^{(s)}+I_{j}^{(s)}+N_{j}^{(s)}  \tag{2}\\
Y_{b} & =\sqrt{h_{i b}^{(s)} d_{i b}^{-\alpha}} X_{i}^{(s)}+\sqrt{h_{j b}^{(s)} d_{j b}^{-\alpha}} X_{j}^{(s)}+I_{b}^{(s)}+N_{b}^{(s)} \tag{3}
\end{align*}
$$

where, for each subchannel $s, N_{i}^{(s)}, N_{j}^{(s)}$ and $N_{b}^{(s)}$ denote


Fig. 1. Illustration of complementary FFR scheme, compared to strict FFR.
independent, zero mean white Gaussian noise components, having variances $\sigma_{i}^{(s)^{2}}, \sigma_{j}^{(s)^{2}}, \sigma_{b}^{(s)^{2}} ; I_{i}^{(s)}, I_{j}^{(s)}$ and $I_{b}^{(s)}$ denote intercell interference at users $i, j$ and receiver $b ; X_{i}^{(s)}$ and $X_{j}^{(s)}$ denote the codewords transmitted by users $i$ and $j ; h_{i j}^{(s)}, h_{j i}^{(s)}$, $h_{i b}^{(s)}$ and $h_{j b}^{(s)}$ are i.i.d. exponential power fading coefficients. The variables $d_{i j}, d_{i b}$ and $d_{j b}$ denote the user $i$ to user $j$, user $i$ to receiver and user $j$ to receiver distances respectively; and $\alpha$ denotes the path loss exponent. We assume that pairwise channel state information $\mathbf{h}=\left\{h_{i j}^{(s)}, h_{j i}^{(s)}, h_{i b}^{(s)}, h_{j b}^{(s)}, \forall s \in S_{i j}\right\}$, is only available at the corresponding cooperating pair and the receiver, and pairing is done at the receiver, based only on the channel statistics. The calculation of intercell interference terms, $I_{i}, I_{j}$ and $I_{b}$ require special attention, and will be discussed in the following section.

## III. Encoding, Decoding and Achievable Rates

Let us assume that users $i$ and $j$ are paired, and assigned a set of subchannels $S_{i j}$ and a base station $b$. The cooperation then proceeds according to the power controlled inter-subchannel cooperative OFDMA model of [10]. Namely, the users employ block Markov superposition encoding, and decode each other's message at the end of each block, and the receiver decodes the user messages using backwards decoding after receiving all blocks of information. The transmitted codeword, consisting of direct transmission, common message generation and common message transmission components,

$$
X_{i b}^{(s)}, X_{i j}^{(s)} \text { and } U^{(s)} \text { respectively is }
$$

$$
\begin{equation*}
X_{i}^{(s)}=\sqrt{p_{i b}^{(s)}(\mathbf{h})} X_{i b}^{(s)}+\sqrt{p_{i j}^{(s)}(\mathbf{h})} X_{i j}^{(s)}+\sqrt{p_{U_{i}}^{(s)}(\mathbf{h})} U^{(s)} \tag{4}
\end{equation*}
$$

where the powers, assigned to the codewords selected from zero mean Gaussian distributions, should satisfy the long term average constraint
$\sum_{s \in S_{i j}} E\left[p_{i b}^{(s)}(\mathbf{h})+p_{i j}^{(s)}(\mathbf{h})+p_{U_{i}}^{(s)}(\mathbf{h})\right] \triangleq \sum_{s \in S_{i j}} E\left[p_{i}^{(s)}(\mathbf{h})\right] \leq \bar{P}_{i}$,
For each pair $\{i, j\}$, and receiver $b$, the resulting achievable sum rate can be obtained by extending the rate regions in [10] and [11], to include the inter-cell interference parameters $I_{i}$, $I_{j}$ and $I_{b}$, which are modelled as Gaussian, resulting in (5) at the bottom of this page. Note however that, while the general form of the sum rate expression does not depend on whether we are dealing with an inner or outer pair, the interference terms do. Due to the cooperative nature of our model, and the geometry of complementary FFR, four different interference terms arise: inner user to base station, $I_{b, i n}$; inner user to inner user $j, I_{i n, j}$; outer user to base station, $I_{b, o u t}$; and outer user to outer user $j, I_{o u t, j}$.

The geometries used in the calculation of each interference term are shown in Figures 2(a)-2(d). While computing interference, we only consider first tier interferers, and assume worst case scenarios for the positions of the interferers. An important observation is, since we already use cell sectoring as a part of our complementary FFR setup, we can further exploit the sectorized structure to limit the interference at the base stations

$$
\begin{align*}
\left(R_{i}+R_{j}\right)_{b} \leq \min \{ & \sum_{s \in S_{i j}} E\left[\log \left(1+\frac{h_{i b}^{(s)} d_{i b}^{-\alpha} p_{i}^{(s)}(\mathbf{h})+h_{j b}^{(s)} d_{j b}^{-\alpha} p_{j}^{(s)}(\mathbf{h})+2 \sqrt{h_{i b}^{(s)} d_{i b}^{-\alpha} h_{j b}^{(s)} d_{j b}^{-\alpha} p_{u_{i}}^{(s)}(\mathbf{h}) p_{u_{j}}^{(s)}(\mathbf{h})}}{\sigma_{b}^{(s)^{2}}+I_{b}}\right)\right], \\
& \sum_{s \in S_{i j}} E\left[\log \left(1+\frac{h_{i j}^{(s)} d_{i j}^{-\alpha} p_{i j}^{(s)}(\mathbf{h})}{h_{i j}^{(s)} d_{i j}^{-\alpha} p_{i b}^{(s)}(\mathbf{h})+\sigma_{j}^{(s)^{2}}+I_{j}}\right)+\log \left(1+\frac{h_{j i}^{(s)} d_{j i}^{-\alpha} p_{j i}^{(s)}(\mathbf{h})}{h_{j i}^{(s)} d_{j i}^{-\alpha} p_{j b}^{(s)}(\mathbf{h})+\sigma_{i}^{(s)^{2}}+I_{i}}\right)\right] \\
& \left.+\sum_{s \in S_{i j}} E\left[\log \left(1+\frac{h_{i b}^{(s)} d_{i b}^{-\alpha} p_{i b}^{(s)}(\mathbf{h})+h_{j b}^{(s)} d_{j b}^{-\alpha} p_{j b}^{(s)}(\mathbf{h})}{\sigma_{b}^{(s)^{2}}+I_{b}}\right)\right]\right\} \tag{5}
\end{align*}
$$



Fig. 2. Interferer locations for inner and outer users. Only the interfering pseudo-cells are shown for outer users (Figures 2(c) and 2(d)). Cell sectors, shown by dashed blue lines, help reduce the interference at the receivers, but the interference at the users is affected by all first tier interferers.
by adjusting the receive antenna beams. As a result, while computing $I_{b, i n}$, we have only two first tier interferers, and for $I_{b, \text { out }}$, we have only three first tier interferers, see Figures 2(a) and 2(c). This way, the increased interference for inner users, which is typical for FFR, is significantly reduced, as a byproduct of our cooperative model. Since the users cannot do receive beamforming, we need to consider six interferers, while computing $I_{i n, j}$ and $I_{o u t, j}$.

Since our main goal is to optimize the powers, partnering strategies and receiver selection; and channel state information at the transmitters is limited, we take the powers of the interferers outside the cluster of interest to be equal to their average, say $\bar{P}$, while computing total interference at each channel state, which is a common assumption. This way, the convexity of the optimization problem is preserved. Also, we assume that the fading from the interferers is averaged out, and we only consider a simplified path loss model from the interferers. The resulting average interference powers are given by,

$$
\begin{align*}
I_{b, \text { in }} & =2 \times \bar{P} /\left(r \sqrt{3}-r_{i n}\right)^{\alpha}  \tag{6}\\
I_{b, \text { out }} & =2 \times \bar{P} /(r \sqrt{7})^{\alpha}+\bar{P} /(r \sqrt{10})^{\alpha}  \tag{7}\\
I_{i n, j} & =\sum_{m=1}^{6} \bar{P} / d_{j m, \text { in }}^{\alpha}  \tag{8}\\
I_{\text {out }, j} & =\sum_{m=1}^{6} \bar{P} / d_{j m, \text { out }}^{\alpha} \tag{9}
\end{align*}
$$

where $d_{j m, i n}$ (respectively, $d_{j m, o u t}$ ) is the distance of the $m$ th first tear inner (respectively, outer) interferer to inner (respectively, outer) user $j$, and depends on user coordinates. Finally, if $\{i, j\} \in U_{i n, b} \times U_{i n, b}$, we set $\left\{I_{b}, I_{i}, I_{j}\right\}=\left\{I_{b, i n}, I_{\text {in }, i}, I_{\text {in }, j}\right\} ;$ if $\{i, j\} \in U_{\text {out }} \times U_{\text {out }}$ we set $\left\{I_{b}, I_{i}, I_{j}\right\}=\left\{I_{b, \text { out }}, I_{\text {out }, i}, I_{\text {out }, j}\right\}$ in (5).

## IV. Jointly Optimum Power, Cooperating Partner and Receiver Selection

The sum rate of the system can be written as a sum of inner and outer user pair rates, and due to the orthogonality of the subchannels, the sum rate of inner users and outer users can be optimized separately. As far as inner user sum rate maximization is concerned, there is no issue of base station selection, and for each inner cell, the problem can be reduced
to the joint partnering and power control problem of [11], by adding the intercell interference powers computed in the previous section to noise variances. Hence, we will focus here on the outer user rate maximization, which is considerably more involved. Note that the sum rate maximization for each pseudo-cell can be solved separately, thanks to orthogonality supplied by OFDMA. The goal is then to solve,

$$
\begin{align*}
\max _{\substack{\Gamma_{l} \in \Gamma, b_{i j} \in\{1,2,3\}, \mathbf{p}(\mathbf{h})}} & \sum_{\{i, j\} \in \Gamma_{l}}\left(R_{i}+R_{j}\right)_{b_{i j}} \\
\text { s.t. } & \sum_{s \in S_{i j}} E\left[p_{i b_{i j}}^{(s)}(\mathbf{h})+p_{i j}^{(s)}(\mathbf{h})+p_{U_{i}}^{(s)}(\mathbf{h})\right] \leq \bar{P}_{i} \\
& \left(R_{i}+R_{j}\right)_{b_{i j}} \text { satisfies }(5), \quad \forall\{i, j\} \in \Gamma_{l}
\end{align*}
$$

where $\Gamma_{l}$ is a two user partition of the set $U_{\text {out }}$ of users in the pseudo-cell of interest, $\Gamma$ is the set of all such distinct partitions $\Gamma_{l}, b_{i j}$ is the receiver selected by $\{i, j\}$ and $\mathbf{p}(\mathbf{h})$ denotes the vector of all power variables at all channel states.

The joint maximization problem is rather difficult to solve, as the channel gains, distances, and hence the sum rates themselves depend on which users are paired, and which base station is selected. A brute force search clearly results in a combinatoric problem, and is not a viable option. The key to solving (10) is to realize that like its single-cell counterpart [11], it can be reduced to a maximum weighted matching problem on a graph, if the sum rate obtainable by each pair of users and the selected receiver, after power control, is viewed as the weights assigned to the edges of the graph. A simple four user, three receiver example is shown in Figure 3(a). The resulting weighted graph is shown in Figure 3(b). Each of the three parallel edges connecting each user pair corresponds to selecting a distinct receiver. Clearly, in the final solution, each pair should be assigned only one edge, so that it is served by only one base station. The trick is to realize that the edge selection for each potential pair may in fact be done before solving the matching problem: one can simply keep only the edge corresponding to the most powerful receiver for each pair, and delete the other two, without considering which partners or receivers are selected by the other users. This can be shown easily by contradiction. Let us assume we know that users $i$ and $j$ are paired in the optimal strategy,


Fig. 3. Determination of weights for edge cell users
and let them be served by base station $b$. Now, if there exists $b^{\prime} \neq b$, for which $\left(R_{i}+R_{j}\right)_{b}<\left(R_{i}+R_{j}\right)_{b^{\prime}}$ in the original graph, the edge between $i$ and $j$ corresponding to receiver $b$ can be removed, and selecting $b^{\prime}$ as the new receiver will result in a strictly better system sum rate, as the sum rate of the other users remain unchanged. This contradicts the optimality of $b$, and shows that any edge corresponding to such inferior $b$ can be removed initially, without compromising optimality. As a result, the model in Figure 3(c) is obtained, and the structure of the problem once again reduces to that of single cell partnering. The jointly optimal partnering, receiver selection and power allocation problem can therefore be stated as an equivalent three stage problem,

$$
\begin{align*}
\max _{\Gamma_{l} \in \Gamma,} & \sum_{\{i, j\} \in \Gamma_{l}} \max _{b} \max _{\mathbf{p}_{i}(\mathbf{h}), \mathbf{p}_{j}(\mathbf{h})}\left(R_{i}+R_{j}\right)_{b} \\
\text { s.t. } & \sum_{s \in S_{i j}} E\left[p_{i b}^{(s)}(\mathbf{h})+p_{i j}^{(s)}(\mathbf{h})+p_{U_{i}}^{(s)}(\mathbf{h})\right] \leq \bar{P}_{i} \\
& \left(R_{i}+R_{j}\right)_{b} \text { satisfies }(5), \quad \forall\{i, j\} \in \Gamma_{l} \tag{11}
\end{align*}
$$

which can further be converted to

$$
\begin{equation*}
\max _{\Gamma_{l} \in \Gamma,} \sum_{\{i, j\} \in \Gamma_{l}}\left(R_{i}+R_{j}\right)^{*} \tag{12}
\end{equation*}
$$

and being a maximum weighted matching problem on a complete graph, (12) can be solved in polynomial time using methods such as Edmonds algorithm [12]. Algorithm 1 below summarizes the stages of our three step optimization. Instead of calculating the optimum powers to obtain the graph weights
for each pair of users, it is also possible to resort to some heuristic distance based algorithms to perform the matching step. We now propose such an algorithm: the distances among each pair of outer users in each pseudo-cell are computed and sorted. The users closest to each other are matched, removed from the list of users, then the same procedure is applied to the remaining users. Once the matching is found, power allocation and receiver selection steps are performed. The distance based matching for the inner users is identical to single cell matching, and is performed using [11, Algorithm E]. The performance of the heuristic algorithm will be evaluated in the following section.

```
Algorithm 1 Algorithm for outer cell users
    for all \((i, j) \in U_{\text {out }}\) do
        for all receivers \(b \in\{1,2,3\}\) do
            Compute optimal powers using the algorithm from [10]
            Calculate \(\left(R_{i}+R_{j}\right)_{b}\) by equation (5)
        end for
        Select \(b^{*}=\arg \max \left(R_{i}+R_{j}\right)_{b}\),
        Use \(\left(R_{i}+R_{j}\right)^{*} \triangleq\left(R_{i}+R_{j}\right)_{b^{*}}\) as graph weights
    end for
    Run MWM algorithm on weighted graph for optimal pair-
    ing.
```


## V. Simulation Results

We simulate our proposed frequency reuse, partner selection, base station selection and power allocation strategy for a system with $4 N=24$ users per cell, $r=2 r_{i n}=100 \mathrm{~m}$. We assume that the average power of each user is unity, and the fading is exponential with mean 1 . Each user is assigned an average of one subchannel, that is, in the cooperative scenario, the user pair is assigned two subchannels and share both of these subchannels. This amounts to a total of 60 subchannels reused in the system. Note that, if FFR, whether complementary or strict, is not used, each cell can support only 20 users, in which case the worst 4 users should be blocked. The rate maximization is carried out for only the outer users in the central pseudo-cell of the cluster, and the inner users; and per cell sum rate is found by averaging.

In Figure 4, we compare the sum rates of four strategies: our proposed jointly optimized strategy, our heuristic strategy, strict FFR with single user power control but no cooperation, and power control only (no FFR). Each index on the horizontal axis refer to a different user geometry. While the use of noncooperative strict FFR increases the user capacity, it yields less sum rate compared to no FFR, due to the added interference at the inner users. In fact, it was noted in [13] that when $r_{i n}=r / 2$, FFR and no FFR give nearly the same rate, as validated here. However, our proposed strategy, as well as the heuristic partnering approach nearly double the rates of both non-cooperative techniques, thanks to the gain from cooperation, reduction of interference due to the sectorized complementary FFR model, and flexibility in choosing partners and receivers.


Fig. 4. Sum rate comparison of proposed model with non-cooperative models.


Fig. 5. Sample optimal partnering strategy obtained by MWM.

| User Pair | No FFR <br> User Rates | Strict FFR <br> User Rates | Comp. FFR <br> Sum rate |
| :---: | :--- | :--- | :---: |
| $12-16$ | $1.20-1.14$ | $1.20-1.14$ | 4.92 |
| $6-17$ | $0.99-1.21$ | $0.99-1.21$ | 4.29 |
| $2-5$ | $0.98-0.84$ | $0.98-0.84$ | 4.24 |
| $3-10$ | $0.75-0.78$ | $0.75-0.78$ | 3.88 |
| $4-14$ | $0.71-0.75$ | $0.71-0.75$ | 3.76 |
| $8-11$ | $1.01-1.10$ | $1.01-1.10$ | 3.44 |
| $1-9$ | $0.69-\mathrm{x}$ | $0.69-0.65$ | 3.37 |
| $15-18$ | $\mathrm{x}-\mathrm{x}$ | $0.68-0.59$ | 3.13 |
| $7-13$ | $0.80-\mathrm{x}$ | $0.80-0.68$ | 3.07 |

TABLE I
COMPARISON OF USER RATES FOR COOPERATIVE VS. NONCOOPERATIVE PROTOCOLS.
In Figure 5, we give the optimal partnering strategy for a sample geometry. Dividing the cell into two has the effect of increasing the connectivity of the users, and encourages cooperation, compared to a single cell setup [11]. As a result, especially the cell edge users with comparable direct link gains tend to pair with close-by helpers, as opposed to the observations in the single cell scenario [11]. This leads to a fairer solution and higher rates for cell edge users. This is further illustrated in Table I, where we tabulate the rates of the outer users, falling into the pseudo-cell in Figure 5. In non-cooperative strict FFR and no FFR scenarios, each user is assigned a single subchannel and performs single user optimal power control, leading to the individual rates given in Table
I. Note that, without FFR, only $7 N / 3=14$ of the $3 N=18$ outer users can be supported, hence the worst four users are denied access to the channel. In the cooperative FFR scenario, each pair is assigned two subchannels, and their sum rate is shown. The pairs shown on Table I correspond to optimal partnering obtained by MWM. It can be observed from Table I that the worst case users benefit more from cooperation, as the sum rates of user pairs are more nearly equal compared to the non-cooperative setup.

## VI. Conclusion

We proposed a new fractional frequency reuse technique, to be used in conjunction with pairwise cooperation in cooperative multicell multiple access channels. This technique allows cell edge users, potentially from adjacent cells, to share the same subchannels, and select their receiver, which is also convenient for soft hand-off scenarios. We obtained the jointly optimal partner selection, power allocation and receiver selection policy, and demonstrated that this policy not only doubles the system sum rate compared to non-cooperative techniques, but also it provides a fairer rate distribution for cell edge users.

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