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DESIGN OF A 2-DOF 8R LINKAGE FOR TRANSFORMABLE HYPAR STRUCTURE#

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Double curved geometries including hyperbolicparaboloids (hypars) have become a trend in contemporary architecture. However, most of the constructed architectural examples of the hypars are static and cannot offer any form variability. In this paper, a 2-DOF 8R linkage mechanism is introduced to buildtransformable hypar structures. It is inspired from the basic design principles of Bennett linkage. By its distinctive connection details and additional links, this novel mechanism can change its form fromplanar geometries to various hypars. The paper begins with the brief summary of the applications of hypar structures in architecture. Secondly, main principles and deficiencies of the Bennett linkage are presented. According to these deficiencies, structural synthesis of the novel mechanism is considered. Finally, advantages and potential uses of the proposed novel mechanism are explained.

Keywords: Bennett linkage; Deployable; Hypar; Kinetic architecture; Transformation.

INTRODUCTION

Parallel to the development of construction technologies and materials, nonorthogonal forms have become the demand of contemporary architecture. Architects design double-curved geometries as different building components, mostly roof or facade elements. As one of these double-curved geometries, a hyperbolic paraboloid (hypar) is obtained by sweeping a straight line over a straight path at one end and another nonparallel straight path (Moore, 2006). Hypar is a ruled surface which has convex one way along the surface and concave along the other. When the potential use of such surfaces in architecture is investigated, it is seen that many static examples have been constructed as shell structures. As a spatial lightweight ruled surface, these shell structures are capable of carrying their own weights and additional loads due to their curvature and twist. Constructed from masonry, concrete, or a grid of steel or timber members, they are assembled to large-span buildings in which a large space is covered without using columns. A concrete shell example of hypar is presented in Fig. 1.

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Figure 1 Hypar concrete shell.

Although the hypars offer nontraditional architectural forms, these structures do not offer any form flexibility. They are not adaptable. However, from the demand for today's dynamic, flexible and constantly changing activities, a need for the adaptive buildings and building components has emerged in architecture. Today, the most impressive examples which meet this need are the single degree-of-freedom (DOF) deployable structures that are capable of large shape changes. When present solutions of such structures are investigated, it is seen that many concepts have been proposed by varying in size, geometry, retraction methods and structural systems in terms of their morphological and kinematic characteristics. There is a vast literature on deployable planar and spatialstructures (Agarwal et al., 2002; Akgün et al., 2011; Dai and Jones, 1999; Escrig and Valcarcel, 1993; Gantes et al., 1993; Jiten and Ananthasuresh, 2007; Kiper et al., 2008; Kokawa, 2000; Korkmaz, 2005).

Deployable structures do not offer real flexible alternatives because they have only two shape configurations; changing from a closed compact configuration to a predetermined, expanded form (Fig. 2). Designing a single DOF deployable structure brings some advantages as well. These structures can be driven by a single actuator, which reduces the cost and simplifies the control.

The motivation of this paper is to add flexibility to the deployable hypar structures, because in recent years architecture requires structures with various configurations. These are transformable structures with at least two DOF. Architects seek for thekinetic buildings that can adapt to functional, spatial, environmental, socialandcultural changes. They try to obtain unique and

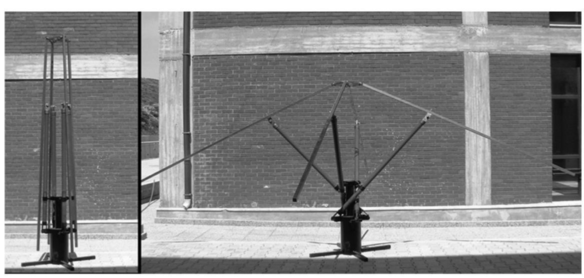

Figure 2 Deployable architectural umbrella (Korkmaz, 2005).

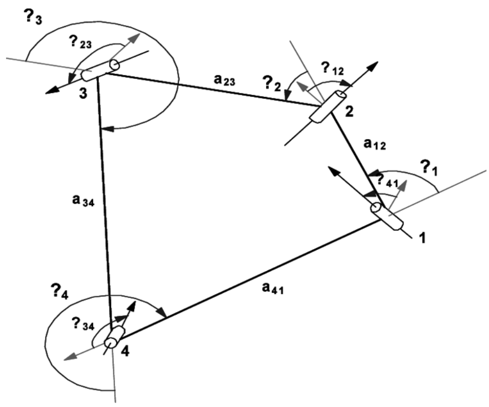

Figure 3 Abstraction of Bennett linkage.

remarkable structures. To reach these aims, they search for alternative ways of producing architecture. At this point, movement is a principle concept that equips their unique works.

A single DOF deployable hypar surface can be constructedby using Bennett linkage which is a 4R spatial linkage. 4R closed chains can be classified into two types: the axes of rotation are all parallel to one another or they intersect at a point, leading to planar 4R or spherical 4R linkages. Any disposition of the axes from these two special arrangements is known as a rigid chain and so furnishes no mechanism at all. But there is an exception, which is called as the Bennett linkage.

Bennett linkage is one type of spatial overconstrained mechanisms consisting of four straight rods. Discovered by Bennett in 1903, Bennett linkage is a spatial 4R closed chain whose four rigid links are connected by four revolute joints (Fig. 3). It has the simplest geometric form and is the only 4R spatial linkage among all overconstrained linkages. The axes of revolute joints are neither parallel nor concurrent while they are either parallel or concurrent in common 4R linkages. This special characteristic of Bennett linkage has attracted the attention of many researchers and engineers. Most research has focused on kinematic characteristics, mathematical descriptions, geometric configurations and combination methods of Bennett linkage to generate new mechanisms for further applications.

In his “skew isogram mechanism,” Bennett (1914) proved that all hinge axes of Bennett linkage can be regarded as the same regulus on a certain hyperboloid at any configuration of the linkage. Yu (1981) found that Bennett linkage is directly related to the hyperboloid defined by its joint axes. He set up the plucker coordinates of its four hinge axes and derived the equation of hyperboloid. Later, Yu (1987) and Baker (1988) investigated quadric surfaces associated with the Bennett linkage. Although Yu studied sphere which pass through the vertices of the Bennett linkage and generated a spherical 4R linkage, Baker scrutinized J-hyperboloid defined by the joint axes of the Bennett linkage and L-hyperboloid defined by the links of the Bennett linkage. In his later works, Baker (2000, 2001) focused on the relative motion and the axodes of the motion of the Bennett linkage. He found that relative motion could be neither purely rotational nor purely translational at any time. Moreover, he demonstrated that the fixed axode of the linkage is ruled surface.

On the other hand, Huang (1997) explored finite kinematic geometry of Bennett linkage based on the intersection of two basis screws associated with two RR dyads. Because the combination of two screws forms a cylindroid that is one type of ruled surface, the axis of cylindroid coincides with the line symmetry of the Bennett linkage. Further, Perez and McCarthy (2003) established that the cylindroids can be generated directly from the three position synthesis of a spatial RR chain.

In addition, Chen (2003) introduced and verified simple methods to build the linkage in its alternative form. By using the Bennett linkage as the basic element, Chen and You developed large mobile assemblies for aerospace applications. They offered many alternative forms of the linkage to obtain the most compact folding and maximum expansion for large structures (Chen and You, 2005, 200, 2007, 2008).

Furthermore, Yu (2007) proposed a deployable membrane structure based on the alternative form of the Bennett linkage. Besides, many other structural mechanisms associated with Bennett linkage have been developed by combining two or three linkages in such a way that common link is removed and adjacent links are attached to each other, but this paper does not cover the studies about 5R and 6R linkages based on Bennett linkage.

To understand the main principles and superiority of the proposed 8R linkage mechanism, its ancestor Bennett linkage should be thoroughly investigated.

GEOMETRIC AND KINEMATIC PRINCIPLES OF BENNETT LINKAGE

It is known that the mobility M of a spatial linkage can be determined by Grübler mobility criterion.

$$M = 6(n-1) - \sum_{i=1}^{5}(6-i)p_i \tag{1}$$

where n is number of links and $\{p_i\}_1^5$ is number of joints having i DOF of joints. According to the above equation, mobility of Bennett linkage can be calculated as follows.

$$M = 6(4-1) - 5 \times 4 = -2. \tag{2}$$

It must be a rigid chain. Even though Bennett linkage does not meet the mobility criterion, it has full-range mobility. Bennett is an exception like all other overconstrained mechanisms because of their special geometry conditions among the links and joint axes.

The dimension of active motion of rigid body for planar mechanisms is RPP, for spherical mechanism is RRR, and for Bennett is RRP (Bennett, 1914). So the dimension of subspace is $\lambda = 3$ and all mechanisms moving in this subspace are called overconstrained mechanisms. All the kinematic joints will have constraint $d = 6 - \lambda$, or $d = 3$. The Grübler formula (1) for subspace $\lambda = 3$ gives:

$$M = 3(n-1) - 2p_1 - p_2 = 3(4-1) - 2 \times 4 = 1. \tag{3}$$

The movement of Bennett is allowed due to its special configurations. The conditions defined by Bennett (1914) are as following:

(a) The twists that are the skewed angles between the axes of two revolute at the ends of links and the lengths of the bars on opposite sides are equal.

$$\begin{aligned} a_{12} &= a_{34} = a \\ a_{23} &= a_{41} = b \\ \alpha_{12} &= \alpha_{34} = \alpha \\ \alpha_{23} &= \alpha_{41} = \beta \end{aligned} \tag{4}$$

(b) The twists and the lengths must satisfy the condition of

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \tag{5}$$

(c) Because the skewed angles α and β are restricted to the range $(0, \pi)$, the displacement closure equations based on the four variable joint angles θ_i can be written as

$$\theta_1 + \theta_3 = 2\pi = \theta_2 + \theta_4 \tag{6}$$

$$\tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2} = \frac{\sin\frac{1}{2}(\beta+\alpha)}{\sin\frac{1}{2}(\beta-\alpha)}. \tag{7}$$

These closure equations are only in use when one of the revolute variables θ_i is independent. In addition to above conditions, there is a special case in which a different type of Bennett linkage is generated (Bennett, 1914). When $\alpha + \beta = \pi$ and $a = b$, an equilateral linkage is obtained (Fig. 4). Therefore, Eq. (7) becomes as;

$$\tan\frac{\theta_1}{2} \times \tan\frac{\theta_2}{2} = \frac{1}{\cos\alpha} \tag{8}$$

DEFICIENCIES OF BENNETT LINKAGE

The revolute joints used in the Bennett linkage are mechanically simple. However, their placement along each link and angle between the pin axes make the manufacturing process complicated. Therefore, there is a need to determine the link cross-sections or design a new joint.

Geometry of the Bennett linkage defines a hypar surface but it is deployable with single DOF. From an architect's perspective, sometimes a single DOF structure is not enough to create the adaptable space for rapidly changing activities of modern life.

Another disadvantage of the Bennett is the distance betweenthe mid-points of the two opposite links a_{12}, a_{34} and a_{23}, a_{41} during the deployment. The connection

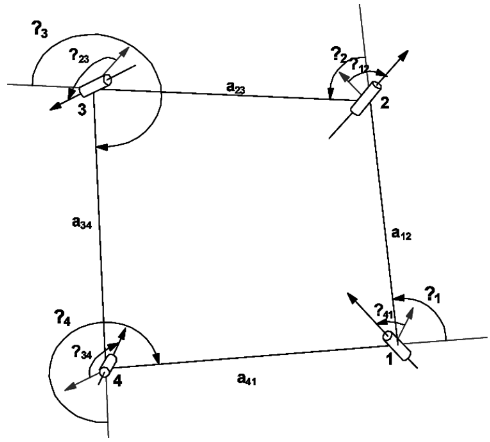

Figure 4 Equilateral Bennett linkage.

points of the intermediate links on opposite sides of the linkage come closer during the deployment process. Therefore, it is impossible to attach straight rods whose lengths are identical to the linkage. Each intermediate link should be designed in order toextend or shorten (Fig. 15). This argument can be proved by Denavit–Hartenberg (D–H) convention. The transformation matrix of D–H is

$$T_{i-1}^{i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

where θ_i is joint angle, α_i is link twist, a_i is link length, and d_i is link the offset which is zero for Bennett mechanism (Fig. 5).

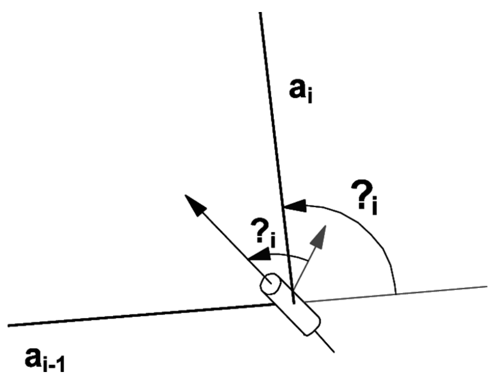

Figure 5 Parameters used in Denavit–Hartenberg convention.

By using Eq. (9) for equilateral Bennett linkage in Fig. 4:

$$T_1^2 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1\cos\alpha_{12} & \sin\theta_1\sin\alpha_{12} & a_{12}\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1\cos\alpha_{12} & -\cos\theta_1\sin\alpha_{12} & a_{12}\sin\theta_1 \\ 0 & \sin\alpha_{12} & \cos\alpha_{12} & d_{12} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

$$T_2^3 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2\cos\alpha_{23} & \sin\theta_2\sin\alpha_{23} & a_{23}\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2\cos\alpha_{23} & -\cos\theta_2\sin\alpha_{23} & a_{23}\sin\theta_2 \\ 0 & \sin\alpha_{23} & \cos\alpha_{23} & d_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{11}$$

$$T_1^3 = T_1^2 \times T_2^3 \tag{12}$$

Because $a_{12} = a_{23} = a_{34} = a_{41}$ and $\alpha_{12} = \alpha_{23} = \alpha_{34} = \alpha_{41}$, θ_i is variable. By using Eq. (8), θ_2 can be found as:

$$\theta_2 = 2\tan^{-1}\left(\frac{1}{\cos\alpha \times \tan(\theta_1/2)}\right) \tag{13}$$

In Fig. 6 the distance between two points in space (x_1, y_1, z_1) and (x_2, y_2, z_2) can be calculated as follows;

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}. \tag{14}$$

By using Eq. (10)–(14), a numerical example can be solved to find the distance between the mid-points of a_{23} and a_{41} in Fig. 6. Let's take $a_{12} = a_{23} = a_{34} = a_{41} = 100\,\text{cm}$, $\alpha_{12} = \alpha_{23} = \alpha_{34} = \alpha_{41} = 45°$ and $\theta_1 = 90°$.

First of all, θ_2 is found by using Eq. (13).

$$\theta_2 = 2\tan^{-1}\left(\frac{1}{\cos 45 \times \tan 45}\right) = 109.4712° \tag{15}$$

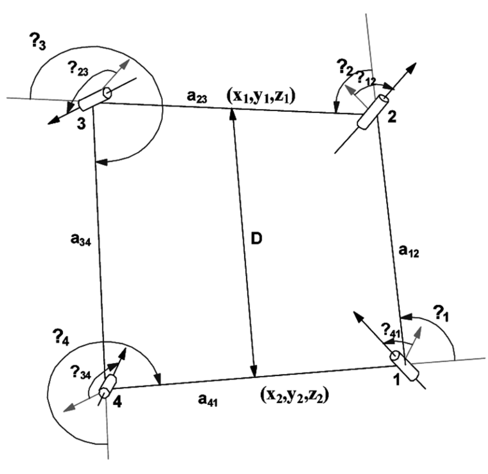

Figure 6 Mid-points of the two opposite links.

T_1^2 is found by using Eq. (10). The equation gives the coordinates of joint 2 as (0, 100, 0) in Fig. 6.

$$T_1^2 = \begin{bmatrix} 0 & -0.7071 & 0.7071 & 0 \\ 1 & 0 & 0 & 100 \\ 0 & 0.7071 & 0.7071 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

T_2^3 is found by using Eq. (11),

$$T_2^3 = \begin{bmatrix} -0.3333 & 0.6667 & 0.6667 & -33.3333 \\ 0.9428 & 0.2357 & 0.2357 & 94.2809 \\ 0 & 0.7071 & -0.7071 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{17}$$

T_1^3 is found by using Eq. (12). The equation gives the coordinates of joint 3 as (−66, 66, 66) in Fig. 6.

$$T_1^3 = T_1^2 \times T_2^3 = \begin{bmatrix} -0.6667 & 0.3333 & -0.6667 & -66.6667 \\ -0.3333 & 0.6667 & 0.6667 & 66.6667 \\ 0.6667 & 0.6667 & -0.3333 & 66.6667 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

According to aforementioned solution, the coordinates of midpoint a_{23} are found as $(-33.3, 83.3, 33.3)$ while the coordinates of midpoint of a_{41} is $(-50, 0, 0)$. By using Eq. (14) the distance between two mid-points is found 91.25 cm. When $\theta_1 = 45°$, the coordinates of mid-points a_{23} is (27.5, 54.4, 19.1) and the midpoint of a_{41} is $(-50, 0, 0)$. The distance between a_{23} and a_{41} is found 96.59 cm. The results show that the distance between mid-points of a_{23} and a_{41} will change when the linkage deploy. Therefore, the intermediate links must consist of two links connected withthe prismatic joints.

Due to the restrictions of the linkage, researchers have sought for alternative forms of Bennett linkage. For instance, Chen (2003) explores the possibilities of constructing largestructural mechanisms by using Bennett linkage as the basic element. For this purpose, first of all Chen identifies basic element from Bennett linkage with skew square cross-section bars to obtain compact folding and maximum expansion (Fig. 7). With this particular example Chen shows that construction of a Bennett linkage with compact folding and maximum expansion is not only mathematically feasible, but also practically possible. Secondly she develops a way by which the basic element can be connected to form a large deployable structure while retaining the single DOF.

Basic element based on the alternative forms of Bennett linkage provide the most efficient compact folding with a maximum expansion, but it is still limited to single DOF deployable mechanism. Basic element changes its form from a closed compact configuration to a predetermined, expanded form. To obtain real flexible solutions for transformable structures, there is a need to develop novel 2-DOF linkage mechanism.

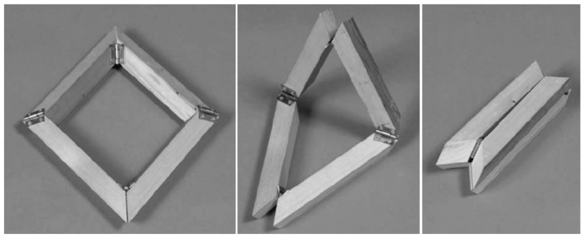

Figure 7 Bennett linkage with skew square cross-section bars (Chen, 2003).

PROPOSED 2-DOF 8R LINKAGE MECHANISM

Figure 8 shows three equilateral Bennett linkages with same joint angle θ_1. Even though these three linkages have same lengths and same joint angles their configurations are different. This is because of the different skewed angles α_{12} and α_{23} for each linkage. Consequently, skewed angles are different but constant for each linkage, because of this reason any linkage cannot transform to another one.

The main idea is to develop a novel equilateral mechanism withvariable skewed angles α_{12}, α_{23}, α_{34}, α_{41} so that it can take form of any equilateral Bennett linkage. The proposed mechanism is an 8R closed chain system with eight cylindrical link cross-sections (Fig. 9). Revolute joints between links 1 and 2, 3 and 4, 5 and 6, 7, and 8 let rotations about the links' axes. This rotation makes skewed angles α_{12}, α_{23}, α_{34}, α_{41} variables. Kinematic diagram of this novel mechanism is represented in Fig. 10. With its additional links and joints this novel mechanism has a superior transformation capability overcommon Bennett linkage.

According to Grübler's criterion, the mobility of this novel mechanism is:

$$M = 6(n-1) - 5p_1 = 6(8-1) - 5 \times 8 = 2 \tag{19}$$

As one of the most important superiority of this novel mechanism over classical Bennett linkage, the mechanism can physically move with overlapped links. This property increases the transformation capability and the novel mechanism can stow and deploy completely. Furthermore, both side of the mechanism can

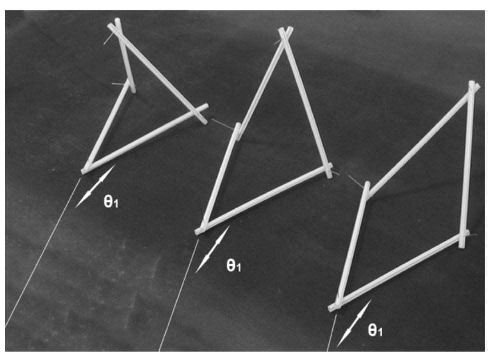

Figure 8 Equilateral Bennett linkages with same joint angle θ_1.

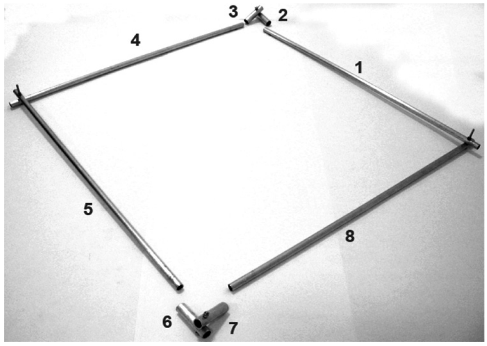

Figure 9 Proposed novel 2-DOF8R linkage mechanism.

move individually, and the system can transform from planar geometries to various hypars. Figure 11 shows shape alternatives of the proposed 8R linkage.

This mechanism can be used as atransformable building component (such as a roof, façade system, or furniture, etc.) which can change its shape according to the expectations of the users. In order to use this system as a roof structure, some intermediate links (additional struts) are necessary to plug the cover material (Fig. 15). During the experimental studies with prototypes and SolidWorks 2010, it is realized that there is also rotation besides translation between the two intermediate links. Because of this reason, cylindrical joints have been used instead of prismatic joints (Fig. 14). Kinematic diagram of the proposed mechanism with additional struts is seen in Fig. 12.

By linking the intermediate links tothe proposed mechanism, a transformableroof can be constructed and remains 2-DOF. It remains 2-DOF because the intermediate links are structural groups which have zero mobility. Each structural group consists of two links and three p_2 joints as shown in Fig. 13.

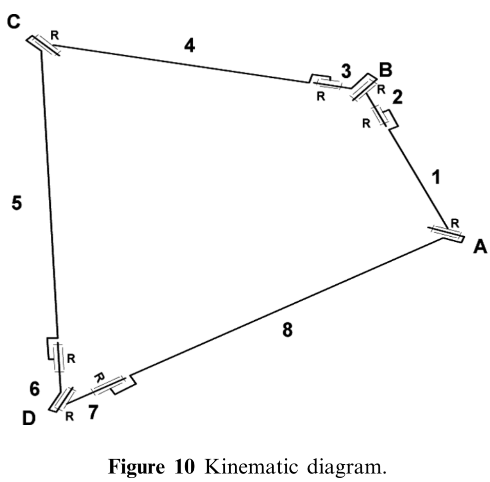

Figure 10 Kinematic diagram.

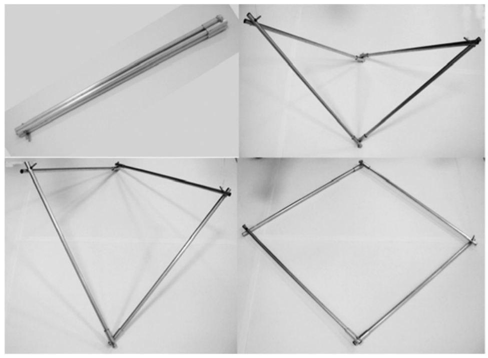

Figure 11 Transformation capability of the proposed 2-DOF 8R Linkage.

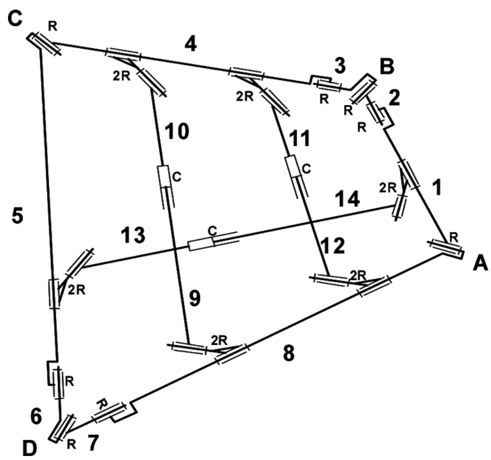

Figure 12 Kinematic diagram of the proposed linkage with intermediate links.

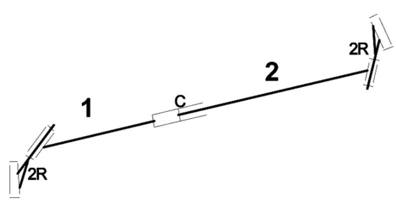

Figure 13 Structural group with zero DOF.

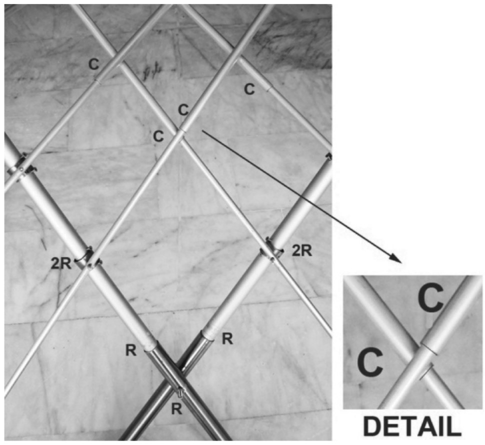

Figure 14 Joints on the proposed mechanism.

Mobility of the structural group is 0 as:

$$M = 6m - 4p_2 \tag{20}$$

$$M = 6 \times 2 - 4 \times 3 = 0 \tag{21}$$

where M is the mobility of the structural group, m is the number of moveable links. Equation proves that any number of intermediate links do not change the DOF of the whole system. According to two different mobility equations, mobility of the mechanism with intermediate links is 2 (Fig. 12).

By using Grübler equation:

$$\begin{aligned} M &= 6(n-1) - 5p_1 - 4p_2 \\ M &= 6(14-1) - 5 \times 8 - 4 \times 9 = 2 \end{aligned} \tag{22}$$

With additional struts, the proposed transformable structure is a multi-loop mechanism. The second equation is useful for the mechanisms which contain mixed independent loops with variable general constraint. Mobility of the multi-loop mechanism is found by Freudenstein–Alizade equationas (Freudenstein and Alizade, 1975);

$$M = \sum_{i=1}^{j} f_i - \sum_{K=1}^{L} \lambda_K = 26-(4 \times 6) = 2, \tag{23}$$

where f_i is DOF of I-kinematic pair, λ_K is number of space or subspace of k-closure loop, L is number of independent loops.

In each direction seven intermediate links are added to the proposed mechanism and its transformation capability is observed (Fig. 15). It is possible to watch the movie of the prototype from our website (Mecart).

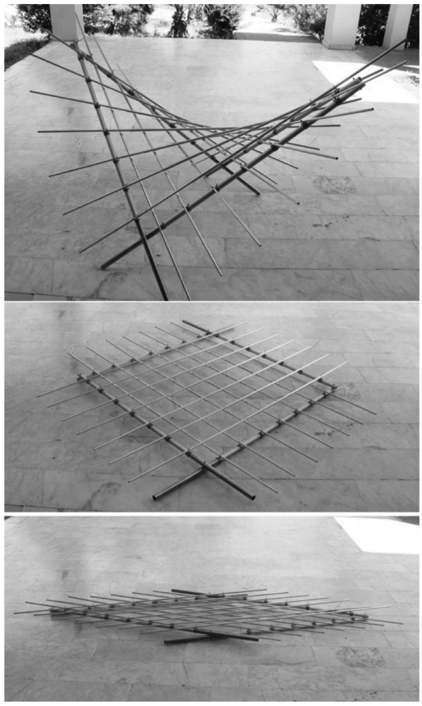

Figure 15 Transformation capability of the roof structure.

CONCLUSIONS

This paper has exposed a two DOF 8R linkage mechanism to build transformable hypar structure for architectural applications. Geometric and structural principles of the mechanism and the superiority overBennett linkage have been thoroughly explained. Finally, application of this linkage mechanism as an adaptive roof structure has been presented. Contrary to previous single DOF deployable structures, the proposed two DOF transformable structure has wider form flexibility. This increased flexibility is achieved by means of at two actuators.

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