The Case for Comparability

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Abstract

We argue that all gradable expressions in natural language obey a principle that we call Comparability: if *x* and *y* are both *F* to some degree, then either *x* is at least as *F* as *y* or *y* is at least as *F* as *x*. This principle has been widely rejected among philosophers, especially by ethicists, and its falsity has been claimed to have important normative implications. We argue that Comparability is needed to explain the goodness of several patterns of inference that seem manifestly valid. We reply to some influential arguments against Comparability, raise and reject some new arguments, and draw out some surprising implications of Comparability for debates concerning preference and credence.

1   The thesis of comparability

Our topic is the logic of *comparative constructions*: paradigmatically, the comparative forms of adjectives ('*F*-er' or 'more *F*') and the equative form ('[at least] as *F* as').

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To help make the case that there is a subject matter here to be investigated, we list some relatively uncontroversial examples of valid schemas involving these constructions.

Restricted Reflexivity If x is at least as F as y, then x is at least as F as x and y is at least as F as y.

Comparative/Equative x is F-er than y if and only if x is at least as F as y and y is not at least as F as x.

Equative/Comparative x is at least as F as y if and only if either x is F-er than y or x and y are equally F.

Transitivity If x is at least as F as y and y is at least as F as z, x is at least as F as z.

Monotonicity If x is F and y is at least as F as x, then y is F.

We take all these schemas to be valid in the sense that their instances always express necessary truths, so long as any ambiguity or context-sensitivity they may harbor is resolved uniformly.[1] We are not committing ourselves to their having any such disputed status as analyticity. And while we will be taking their validity for granted here, on the grounds that anyone who denied it would be unlikely to agree with the more controversial claim that we will be focusing on, we regard this as a matter of legitimate and genuine philosophical debate. Transitivity in particular has been vocally disputed by Rachels (1998) and Temkin (2012), who have given much-discussed arguments that there are cases where one thing is better than a second and the second is better than some third thing without the first thing being better than the third thing.[2] Nevertheless, our attitude is

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[1]For this notion of validity, see Dorr (2014).

[2]In the literature on "comparative probability", it is—surprisingly—occasionally *taken for granted* that Transitivity (or rather, a minor grammatical generalisation of it) fails for 'at least as confident' when the domain includes irrational people. For example, Hawthorne (2009, p. 53) writes down a formal principle that he glosses in English as "if α is at least as confident that A as that B, and α is at least as confident that B as that C, then α is at least as confident that A as that C", and remarks that "Read this way, α is clearly supposed to be a logically ideal agent". We speculate that this is because of the tendency in this literature to equate 'α is at least as confident that A

that the instances of Transitivity are on an altogether firmer footing than the premises that philosophers like Rachels and Temkin appeal to in arguing against them, or indeed than all but the blandest and least controversial claims about the relative goodness of any specific things. We think it would be methodologically appropriate for philosophers to treat these general schemas, like other familiar laws of logic, as firm constraints on their theorising about more specific matters.[3]

The case for the validity of the schemas is based on certain ingrained patterns of usage. For example, one consequence of the validity of Comparative/Equative is that sentences of the form '*x* is *F*-er than *y* and *y* is *F*-er than *x*' are like sentences of the form '*x* is *F* and *x* is not *F*' in being necessarily false on any uniform interpretation. And indeed, our treatment of such sentences is analogous in revealing ways—just as we would react to 'The soup is good and also not good' by looking for plausible *non*-uniform interpretations where the two 'good's mean different things, we would likewise react to 'The soup is better than the dessert, but the dessert is also better than the soup' by looking for non-uniform interpretations of the two 'better than's.

Although our schemas involve just a few specific syntactic constructions involving gradable adjectives and present-tense verbs, they admit several natural syntactic generalisations, which we can harmlessly treat as if they were instances of the schemas, since they are valid if the original schemas are. First: some comparative adjectives take an "internal argument". For example, 'close', 'fond', and 'confident' normally appear in the constructions 'close to *x*', 'fond of *x*', and 'confident that *P*'. Any case for the validity of, say, Transitivity will surely carry over to to its natural generalisations for adjectives with internal arguments, such as:

as that *B*' with something like '*α* judges that it is at least as probable that *A* as that *B*', for which transitivity is *prima facie* implausible. We will have more to say about this in the final paragraph of section 7.

[3]Against the view that Transitivity is part of the logic of gradable adjectives, Temkin (2012, p. 495) insists that views that violate Transitivity "represent *substantive* normative positions" (emphasis his). We agree; we just think they are incorrect (see, e.g., Nebel 2018, Pummer 2017). We emphatically reject the assumption that logical disputes must be non-substantive.

(1) If x is at least as fond of x' as y is of y', and y is at least as fond of y' as z is of z', then x is at least as fond of x' as z is of z'.

Second: while our schemas are in the present tense and indicative mood, they have natural analogues for other patterns of tense and moods, including those in which the verb in the comparative clause differs in tense or mood from the main verb:

(2) If y is at least as hungry now as x was at sunrise and z will be at least as hungry at sunset as y is now, then z will be at least as hungry at sunset as x was at sunrise.

Third, the schemas extend naturally to comparative constructions not based on adjectives. For example, Transitivity has natural analogues for adverbs, verbs, and nouns (Fleischhauer 2016, Nicolas 2010):

(3) If x is campaigning at least as effectively as y and y is campaigning at least effectively as z, then x is campaigning at least as effectively as z,

(4) If x likes x' at least as much as y likes y' and y likes y' at least as much as z likes z', then x likes x' at least as much as z likes z',

(5) If John had at least as much fun as Mary and Mary had at least as much fun as Rover, then John had at least as much fun as Rover.

Subsuming these different forms under unified explanatory generalizations would require a fully general semantic analysis of comparative constructions, which we will not be trying to provide (see Wellwood 2019). Nevertheless, in what follows we will take it for granted that if any of the schemas to be considered is valid, its generalizations along the above lines are valid as well.

With these preliminaries out of the way, we are ready to state our thesis, which is that the following schema is valid:

Comparability If x is at least as F as x and y is at least as F as y, then either x is at least as F as y or y is at least as F as x.[4]

Note that given Restricted Reflexivity, the antecedent of Comparability is equivalent to the more cumbersome, but perhaps less odd-sounding, 'either x is at least as F as something or something is at least as F as x, and either y is at least as F as something or something is at least as F as y'.

As with the schemas considered earlier, we intend this claim of validity to apply not just to sentences that are strictly speaking instances of the schema, but to various natural syntactic generalisations, e.g.

(6)   If x is at least as confident that P as x is that P, and y is at least as confident that Q as y is that Q, then either x is at least as confident that P as y is that Q, or y is at least as confident that Q as x is that P.

(7)   If x was at least as hungry at sunset as x was at sunset and y will be at least as hungry at sunrise as y will be at sunrise, then either x was at least as hungry at sunset as y will be at sunrise or y will be at least as hungry at sunrise as x was at sunset.

(8)   If x likes x' at least as much as x likes x' and y likes y' at least as much as y likes y', then either x likes x' at least as much as y likes y' or y likes y' at least as much as x likes x'.[5]

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[4]Relations with the property that, whenever x and y are in the field of R, either xRy or yRx, are sometimes called "complete"; other names for this property include "totality", "connectedness", and "connexitity". To avoid multiplying terminology unnecessarily, we will characterize such relations as "satisfying Comparability," even though strictly speaking Comparability is a natural language schema and not a property of relations.

[5]A complication: some sentences that might seem like natural extensions of Comparability involve an implicit generic element, which might arguably make for invalidity. For example, suppose that when Amanda and Bob go on hikes, Amanda spends half the time walking slower than Bob and the other half of the time walking faster than Bob. One might argue that 'Amanda walks at least as fast as Bob' and 'Bob walks at least as fast as Amanda' are both false in this situation, despite the obvious truth of 'Amanda walks at least as fast as Amanda' and 'Bob walks at least as

We can harmlessly treat sentences like these as honorary instances of Comparability, since it would be completely implausible to suppose that they are invalid if it is valid.[6]

We state Comparability in conditional form rather than just endorsing its consequent because we do not want to be committed either way as regards the truth of odd disjunctions like (9) and (10):

(9)   Either the square root of two is at least as tall as the Eiffel Tower, or the Eiffel Tower is at least as tall as the square root of two.

(10) Either my left arm is at least as expensive as this laptop, or this laptop is at least as expensive as my left arm.

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fast as Bob'. The same issue can arise even for instances of Comparability proper, when the present-tense 'is' is given a habitual reading: understood in this way, 'Amanda is at least as happy as Bob' and 'Bob is at least as happy as Amanda' are both arguably false if Amanda is happier half the time and less happy the other half of the time. Any reasonable account of habituals should be able to explain such exceptions in a way that is compatible with the validity of Comparability (and its extensions) on non-habitual readings: presumably, the habitual interpretation involves something analogous to an 'at most times' quantifier taking scope over each disjunct. Since opposition to Comparability in the literature is not driven by habitual readings, we will set them aside in what follows. Thanks to REFERENCE DELETED FOR BLIND REVIEW for discussion of this issue.

[6]One possible generalization that we are inclined to resist is the analogue of Comparability for comparisons involving distinct adjectives:

> **Cross-Adjectival Comparability**   If x is F to some degree and y is G to some degree, then either x is at least as F as y is G, or y is at least as G as x is F.

Some instances of this schema, involving pairs of adjectives which "correspond to the same scale"—e.g., 'tall' and 'wide'—sound fine. But, for typical arbitrary pairs of adjectives, cross-adjectival comparisons are hard to interpret. For example, 'He is as old as she is heavy' sounds bizarre. And analogues for comparative constructions not involving adjectives—e.g., 'I had as much fun as you had coffee'—sound even worse. This seems like prima facie reason not to count Cross-Adjectival Comparability as valid. True, we are often not that bad at coming up with meanings for cross-adjectival comparisons—e.g., 'You are at least as beautiful as I am clever'. But following most of those who have discussed this phenomenon (Cresswell 1976, p. 282; Kennedy 1999, §3.2) we are inclined to think that sensible interpretations of such sentences require invoking some additional, non-automatic pragmatic mechanism. (Bale (2008), by contrast, gives a semantics in which ordinary comparisons using single adjectives (at least those like 'beautiful' that are not associated with any "system of measurement") are treated in exactly the same way as 'You are more beautiful than I am clever'. Given this view, it seems that Cross-Adjectival Comparability would have to be classified as valid if Comparability is. This surprising result would be tenable only if the oddity of many of its instances could be explained away, and it is not clear that Bale has the resources for such an explanation.)

Given Comparability, we are free to say that (9) and (10) are false, so long as we say that (11) and (12) are also false:

(11) The square root of two is at least as tall as itself.

(12) My left arm is as expensive as itself.

It is not *obvious* that we should want to say that any of these sentences are false. They are certainly quite odd, but this oddity can arguably be adequately explained by appeal to the fact that they have false *presuppositions* (as witness the fact that they remain odd when embedded under negation or turned into questions), and on some views of presupposition, true sentences can have false presuppositions. So, the claim that the consequent of Comparability is already valid is not a complete non-starter. But from our point of view, all that matters is that the central reasons why one might want to deny the truth of (9) and (10) also apply to (11) and (12), so whatever we decide to say about such sentences, they will not threaten Comparability.

To avoid awkwardness, we will stipulatively use 'x is F to some degree' to mean that x is at least as F as itself.[7] This would not look very plausible if it were intended as an account of the ordinary English meaning of 'to some degree'. (In the ordinary sense, 'Having one's foot broken is pleasant to some degree' seems false, but 'Having one's foot massaged is more pleasant than having it broken' seems true, which given Comparative/Equative and Restricted Reflexivity means that 'Having one's foot broken is at least as pleasant as itself' must also be true.) Two further stipulations will also be convenient: we will use 'x and y are comparable with respect to being F' to mean 'Either x is at least as F as y or y is at least as F as x', and 'x and y are incomparable with respect to being F' to mean 'x and y are both F to some degree but are not comparable with respect to being F'. Comparability can be restated using these definitions as 'No two things are incomparable with respect to being F'. Again, these stipulations are not supposed to reflect the ordinary meanings

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[7]This restricts our attention to what axiologists might call 'bearers' of F-ness, in a way that (unlike the jargon of 'bearers') naturally extends to the syntactic generalizations of Comparability.

of 'comparable' and 'incomparable'. (In the ordinary sense, 'x and y are incomparable' seems compatible with, and indeed tends to suggest, 'either x is incomparably better than y or y is incomparably better than x', which in turn evidently entails that 'x and y are comparable' is true in our technical sense. In the opposite direction, Chang (2002) argues that there are pairs of items that are incomparable in our sense, but that all of these pairs are comparable in the ordinary sense.)

To help bring out what would be involved in accepting or rejecting Comparability, it may be helpful to consider some closely related schemas which it entails given the less controversial validities listed earlier. (Some of the arguments for Comparability we will be giving below can also be adapted to provide direct support for these schemas.) First, given Comparative/Equative, Comparability is equivalent to the more natural-sounding

> **Exclusive Comparability** If x and y are both F to some degree, then either x is at least as F as y or y is F-er than x.

And given Equative/Comparative, Exclusive Comparability is in turn equivalent to:

> **Trichotomy** If x and y are both F to some degree, then either x is F-er than y, y is F-er than x, or x and y are equally F.

Finally, given the combination of Transitivity and Comparative/Equative, Comparability implies:

> **Negative Transitivity** If x is not F-er than y, and y is not F-er than z, and y is F to some degree, then x is not F-er than z.[8]

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[8] *Proof:* Suppose the antecedent of Negative Transitivity is true but the consequent is false: x is F-er than z. Then x and z, as well as y, are F to some degree. So by Exclusive Comparability, y is at least as F as x and z is at least as F as y. So by transitivity, z is at least as F as x, which by Comparative/Equative is incompatible with our assumption x being F-er than z.

One might wonder whether there are any plausible conditions under which Negative Transitivity is *equivalent* to Comparability. The following would do the trick:

> If either x is F-er than everything y is F-er than, or everything F-er than x is F-er than y, and x and y are both F to some degree, then x is at least as F as y.

Recall that given Restricted Reflexivity, 'is F to some degree' can be replaced in any of these principles with either 'is at least as F as something' or 'is such that something is at least as F as it'.

One final caveat: some philosophers reject (or at least decline to accept) the validity of instances of the Law of the Excluded Middle, 'Either P or it is not the case that P'. Particularly germane in the present context are views according to which the *vagueness* of P can make instances of this schema unacceptable. These philosophers have reason to be suspicious of Comparability and many of its consequences. For example, they should not accept the validity of Exclusive Comparability. For the combination of Exclusive Comparability with the seemingly valid 'If y is F-er than x, x is not F-er than y' seems to imply 'If x and y are both F to some degree, then either x is F-er than y or x is not F-er than y.' But it would be strange for anyone who did not accept Excluded Middle to regard this scheme as valid, since the comparative forms of adjectives are often vague. And since Exclusive Comparability follows rather unproblematically from Comparative/Equative and Comparability, any doubts about Excluded Middle should carry over to one or both of these schemas as well. Since a defence of Excluded Middle would take us too far afield from our topic, we will here simply assume its validity. For a variety of views about vagueness that are compatible with the validity of Excluded Middle, see Lewis 1969, Fine 1975, Williamson 1994, Keefe 2000, Graff 2000, Dorr 2003, and Bacon 2018.[9]

However, this would be false if it could happen that there are at least two things that are F to some degree and any two things which are both F to some degree are incomparable. And there is no obvious reason for opponents of Comparability not to allow for such possibilities.

[9]Comparability has a variant not involving disjunction, which is not obviously affected by doubts related to Excluded Middle:

> **Conditional Comparability** If x and y are both F to some degree, and x is not at least as F as y, then y is at least as F as y.

In a non-classical setting like that of Field (2003), where Conditional Comparability does not imply Comparability, Conditional Comparability may retain much of the philosophical significance of Comparability, and be supported by variants of some of the arguments which we will be presenting below. We will not however attempt to track which of our arguments could be adapted to any particular non-classical background logic.

Our arguments for Comparability will be presented in §3. First, in §2, we will discuss some arguments against Comparability that have been influential especially in the ethics literature (where there has, unsurprisingly, been a special focus on the adjective 'good'). We will reply to these arguments in §4, and to some other arguments against Comparability in §5. §6 and §7 will combine Comparability with some further premises to argue for certain unpopular claims about *preference* and *credence*. §8 concludes.

2   The debate about comparability

Comparability seems to be taken for granted in most contemporary work by semanticists working on gradable adjectives.[10] However, we are not aware of explicit arguments for Comparability in the linguistics literature, and it is not obvious how indispensable the assumptions that lead to its validity are in the linguists' accounts of the phenomena they are primarily concerned to explain.

Many philosophers, by contrast, either argue explicitly against certain instances of Comparability, or rely on such failures in accounts of other phenomena, in a way that could easily be retooled as an abductive argument against Comparability.[11] Some of these

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[10]See, e.g., Kennedy (1999), Schwarzschild (2008), and Wellwood (2019). Klein (1980) develops a trivalent semantics which posits a distinctive category of 'nonlinear' adjectives for which it can happen that both '*a* is at least as *F* as *b*' and '*b* is at least as *F* as *b*' are neither true nor false; but he uses supervaluationist machinery in evaluating complex sentences in such a way that instances of Comparability will always come out true. The earlier literature is less committed: Wheeler (1972) presents a kind of analysis for comparatives which is neutral not only about Comparability but about Transitivity, and sees the desire for a semantic theory that settles such questions as "strategic and theoretical mistake" that "turns semantics into physics" (319). (He does go on to suggest that "the grammatical phenomenon of the '-er' construction... serves as a notification that" the relation in question is transitive, but makes no parallel suggestion about completeness.) Likewise, Cresswell (1976) is explicitly neutral about comparability, and even suggests that "perhaps we should even be liberal enough not to insist on transitivity and antisymmetry" for '*F*-er than' (p. 266).

[11]Perhaps the most influential examples are Chang (2002) and Raz (1985). Examples related to rational choice and preference include Handfield (2014), Hsieh (2005), Rabinowicz (2008), Rabinowicz (2012), Raz (1999), and Schoenfield (2014). Examples related to moral decision-making include Hare (2016), Richardson (1994), and Sinnott-Armstrong (1985). Examples from population ethics include Chang (2016), Frick (2017), Gustafsson (2019), Parfit (2016), Qizilbash (2007), Qizilbash (2018), and Rabinowicz (2009a). Examples from formal epistemology include R. Bradley

arguments are highly specific to specific expressions of interest to some particular fields of philosophy, and cannot be addressed in this paper. But it will be useful to consider some particularly influential arguments against Comparability which, if successful, would generalize to a wide class of comparative expressions. These arguments turn on the so-called "multidimensionality" exhibited by many comparative expressions. For example, in the case of 'good', it often seems deeply tendentious to characterise either of two things as at least as good as the other, because each is better than the other in different respects, and there seems to be no non-arbitrary basis for aggregating the different respects into a single overall judgment. For example, according to Parfit (2016, p. 113),

> When two painful ordeals differ greatly in both their length and their intensity, there are no precise truths about whether, and by how much, one of these pains would be worse. There is no scale on which we could weigh the relative importance of intensity and length.

Sen (1997b, p. 5) makes a similar point about 'more unequal than':

> Most statistical measures of the inequality level assume a high degree of measurement. . . . It is, however, possible to argue that the implicit notion of inequality that we carry in our mind is, in fact, much less precise and may correspond to an incomplete quasi-ordering. We may not indeed be able to decide whether one distribution x is more or less unequal than another, but we may be able to compare some other pairs perfectly well. The notion of inequality has many aspects, and a coincidence of them may permit a clear ranking, but when these different aspects conflict an incomplete ranking may emerge.

Earlier still, Keynes (1921, p. 31) gives a similar justification for putative counterexamples to Comparability with respect to 'probable':

(2017), Hawthorne (2009), Kaplan (1996), Konek (2019), Schoenfield (2012), and Smith (2014). Amartya Sen in particular has utilized incomparability across a wide variety of domains, including the theory of inequality, welfare aggregation, justice, and social and rational choice (Sen 1970, Sen 1980, Sen 1997a, Sen 1997b, Sen 2009).

Consider three sets of experiments, each directed towards establishing a generalisation. The first set is more numerous; in the second set the irrelevant conditions have been more carefully varied; in the third case the generalisation in view is wider in scope than in the others. Which of these generalisations is on such evidence the most probable? There is, surely, no answer; there is neither equality nor inequality between them. We cannot always weigh the analogy against the induction, or the scope of the generalisation against the bulk of the evidence in support of it.

We take it that Parfit, Sen, and Keynes are reasoning as follows. Consider a case in which x and y are both F, but in quite different ways. If Comparability is true, then either one is F-er than the other or they are equally F. Whichever it is, the comparison would have to be explained by some particular way of weighing between the relevant dimensions of F. But there is no reason to privilege any particular way of weighing between those dimensions. We should therefore deny that one of the items must be at least as F as the other.

There are doubtless subtle differences between the ways in which our various authors are thinking about this mode of argument, but we will lump them all together under as "tradeoff arguments". While not all comparative adjectives are subject to the kind of multidimensionality that drives these arguments, a great many of them are: for example one could easily run parallel arguments for 'clever', 'funny', 'useful', 'hairy', and 'tidy'.[12]

Another kind of argument prominent in the axiological literature is the "argument from small improvements".[13] Here, instead of citing the difficulties posed by tradeoffs among dimensions in the same way to support all three of 'x is not F-er than y', 'y is not F-er than x' and 'x and y are not equally F', we start with the first two of these claims (perhaps motivated by thoughts about the difficulty of tradeoffs), and these claims are then used

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[12]See Sassoon (2013) for some proposed diagnostics for multidimensional adjectives.

[13]The small improvement argument was first made, by De Sousa (1974), as an objection to the completeness axiom of expected utility theory. The axiological version is pressed, most influentially, Raz (1985).

as premises in a more complicated argument for the third claim (of non-equality). Here is an example from Chang (1998, pp. 23–24), based on Raz (1985):

> Suppose we rationally judge that a particular career as a clarinetist is neither better nor worse than a particular career as a lawyer, say, with respect to goodness of careers. . . . We can improve the clarinetist career a little with respect to goodness of careers, perhaps by increasing the salary by ten dollars. Are we thereby compelled to judge that the improved music career is better than the legal one? It seems rational to resist this conclusion. If it is rational, then the original careers cannot be equally good, since if they were, a small improvement in one must make it better than the other.

The general form of the argument can be reconstructed as follows:

(P1) x is not F-er than y and y is not F-er than x.

(P2) $x+$ is F-er than x.

(P3) $x+$ is not F-er than y.

(Conclusion) x is not at least as F as y and y is not at least as F as x.

This argument is valid given Transitivity and Comparative/Equative.[14] So the challenge is to find particular instances of x, y, and $x+$ for which all three premises are plausible. As we saw in the quote from Chang, the strategy is to choose an x and y for which P1 is plausible not because of some very extensive similarities or some perfectly balanced competing considerations, but because x and y differ along multiple relevant dimensions in such a way that it would seem deeply tendentious to characterise either as F-er than

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[14]Suppose for contradiction that either x is at least as F as y or y is at least as F as x. By P1, each of x and y must at least as F as the other. But P2 implies that $x+$ is at least as F as x, so, by Transitivity, $x+$ is at least as F as y. So the only way P3 could be true would be if y were also at least as F as $x+$. But if this were the case, then by Transitivity, x would have to be at least as F as $x+$, which is ruled out by P2.

the other. We then choose an $x+$ that is similar to x but slightly improved along some dimensions. This makes P2 plausible, while keeping the pattern of relations between $x+$ and y similar enough to the pattern of relations between x and y that insofar as that pattern supports P1, it will provide a similar level of support to P3.

The small improvement argument is arguably an improvement over the tradeoff argument, since in the presence of the relevant kind of multidimensionality, the claim that the relevant items are equally F seems less immediately repugnant than the claims to the effect that one is F-er than the other.[15]

Like the tradeoff argument, the small improvement argument can be run for a very broad range of adjectives. For example, Schoenfield (2012) gives a small improvement argument for the claim that there are not only *possible* counterexamples to Comparability for 'confident', but that there are people who *should* be (such that they are) counterexamples. Having described a case where you have masses of disparate evidence pointing to each of two suspects in a murder, she first elicits the judgment that you could rationally be neither more nor less confident that Smith is the murderer than that Jones is; she then argues that if you were rational and were in this state, you should continue to be in the state even if you acquired a tiny bit more evidence that Smith was the murderer which made you slightly more confident that Smith was the murderer and slightly less confident that Jones was the murderer.

We will come back to the tradeoff and small improvement arguments in §4, after we have laid out our positive case for Comparability; we will suggest that there are good independent reasons for being suspicious of both arguments, so that overall the considerations in favour of Comparability should be regarded as more compelling.

Despite the broad influence of tradeoff and small improvement arguments, philosophers have certainly not been unanimously opposed to Comparability. Some have defended the validity of Comparability for specific adjectives on specific interpretations:

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[15]We will discuss why this should be in §4.

for example, Regan (1997) appeals to requirements of practical reason to argue that Comparability holds for the specific sense of 'good' used by G. E. Moore. But there seems to be only one widely discussed argument that might have some chance of supporting Comparability in full generality, namely that of Broome (1997).

Broome's strategy is to argue that that incomparability in any gradable adjective is incompatible with vagueness. In doing so, he relies on the following "collapsing principle" about vague comparatives: if y is not F-er than x and it is not determinately false that x is F-er than y, then x is F-er than y.[16] Unfortunately, if we assume with Broome that the operator 'it is not determinately false that...' is not logically redundant, this principle is subject to counterexample. For example, suppose that we are in the process of enlarging a statue that was originally made of a certain quantity of bronze, b; we have moulded a new piece of bronze into the shape of a crown, which we are just in the process of soldering on to the top of b. Let $b+$ be the larger quantity of bronze comprising b together with our new bronze. Let's suppose we are at a point in the process where it is neither determinately true nor determinately false that the new bronze is part of the statue. (This assumption should be acceptable to anyone who takes 'it is not determinately false that...' to be non-redundant.) Hence it is neither determinately true nor determinately false that the statue is heavier than b, and also neither determinately true nor determinately false that $b+$ is heavier than the statue. But clearly, b is not heavier than the statue, and the statue is not heavier than $b+$. Given two applications of Broome's principle, we can derive from these premises that the statue is heavier than b, and $b+$ is heavier than the statue. But we surely should not be committed to this conclusion, given that we are committed to its being neither determinately true nor determinately false. Indeed, it seems quite implausible: it's certainly not the case that only *some* of the new bronze is part of the statue.

There is a sizeable literature on Broome's collapsing principle and how it might be

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[16]Broome's wording lacks the 'determinately'; we find it hard not to read 'not false that' as logically redundant.

revised to avoid such counterexamples.[17] Suffice it to say that principles in the vicinity are highly controversial and, we believe, less independently plausible than Comparability itself. We will not rely on them in what follows.

We know of no other explicit arguments in favour of the validity of Comparability. In the next section we will supply several arguments, each of which appeals to some pattern of inference which seems manifestly valid, but which would seem to be invalid if Comparability is. None of our arguments are very intricate; some are so simple that opponents of Comparability may feel we are begging the question against them. Nevertheless, we think there is value in simply laying out the appearances that favour Comparability, since its opponents have not often acknowledged that there is even a *prima facie* case for its validity. Even if our arguments are not decisive, we believe that it will not be an easy task to provide an alternative, incomparability-friendly explanation of our data.

3   Arguing for Comparability

(i) 'Not as F as'.   Our first argument turns on the apparent validity of the following inference:

(13)   Max's room isn't as tidy as Josh's. So Josh's room is tidier than Max's.

The pattern is perfectly general: for any gradable adjective F, Not As F As seems intuitively valid:

Not As F As x is not as F as y. So, y is F-er than x.

Indeed, the converse of Not As F As also seems valid: 'x is not as F as y' is intuitively *equivalent* to 'y is Fer than x'. The appearances here are just as strong for the paradigm "multidimensional" adjectives where Comparability has been thought to fail as for other

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[17]See Andersson (2014), Andersson and Herlitz (2018), Carlson (2004), Carlson (2013), Constantinescu (2012), Elson (2014), Gustafsson (2013), Gustafsson (2018), Qizilbash (2007), and Rabinowicz (2009b).

adjectives: for example, 'Your favourite composer isn't as good as mine' seems interchangeable with 'My favourite composer is better than yours'.

How should these appearances be explained? Here is what we propose. 'As F as' is truth-conditionally equivalent to 'at least as F as'. As a result, given the validity of Exclusive Comparability, Not As F As can be turned into a valid argument-schema by adding an extra premise, 'x and y are both at least as as F as themselves'. Now, in many instances of Not As F As, this extra premise is an obvious necessary truth. For example, it is presumably necessary that all rooms are as tidy as themselves, and that all composers are at least as good as themselves. So in these cases, the appearance of validity can be accepted at face value. However, this observation doesn't take us far enough: Not As F As seems like a valid *form*; and its instances seem valid even when there are no specific grounds for assuming that the relevant objects are F to some degree. (For example, 'The thing he is thinking about isn't as expensive as the thing she is thinking about' seems to entail 'The thing she is thinking about is more expensive than the thing he is thinking about'.) One *could* consider explaining this by endorsing the strengthened version of Comparability that drops the 'to some degree' proviso. But this is not required: instead, the sense of validity can be explained by positing that comparative sentences like 'x is as F as y' carry a *presupposition* to the effect that x and y are both F to some degree (in our stipulative sense of being at least as F as themselves). Such a presupposition would not be surprising: in uttering 'x is as F as y' we are raising the question 'How F are x and y?', a question that intuitively presupposes that x and y are F to some degree. It is characteristic of presuppositions to "project through negation": a sentence and its negation presuppose the same things. So 'x is not as F as y' has a false presupposition in the case where one or other of x and y is not F to any degree. Given Comparability, it follows that Not As F As has the following status: whenever its premise neither entails nor presupposes anything false, its conclusion neither entails nor presupposes anything false. Whether or not we want to apply the technical term 'valid' to arguments with this status, it seems sufficient

to account for the intuitive feeling of validity that arguments like (13) inspire: compare 'Max doesn't know that Sue is a spy; so not every spy is known by Max to be a spy', or 'Every animal I own is well-trained; so at least one animal I own is well-trained'.

Our explanation of the apparent validity of Not As F As relies on the thesis that 'as F as' is equivalent to 'at least as F as'. An apparent problem for this claim comes from the fact that 'x is as F as y' often looks interchangeable with 'x and y are equally F', whereas 'x is at least as F as y' never does. But as Schwarzschild 2008 points out, the stronger meaning associated with 'x is as F as y' looks to be an instance of the phenomenon of *scalar implicature*, the same pragmatic process whereby 'I ate most of the cake' can be used to mean 'I ate most but not all of the cake', or 'You are allowed to type your essay' can be used to mean 'You are allowed but not required to type your essay'. One hallmark of scalar implicatures is that the "strengthened" meanings normally disappear in negative environments: 'I didn't eat most of the cake' won't normally seem true if I ate all of it; 'No students are allowed to type their essays' won't normally seem true if all students are required to type their essays. As we have seen, 'as F as' behaves similarly. 'x is not as F as y' wouldn't normally seem true if x and y are not equally F because x is F-er than y. Likewise, 'No graduate student's room is as tidy as Josh's' wouldn't normally seem true if no graduate student's room is such that it and Josh's are equally tidy but some of them are tidier than Josh's. The way 'at least' functions to block the strengthened meaning is also seen in other cases of scalar implicature: compare 'I at least ate *most* of the cake'; 'You are at least *allowed* to type your essay.'[18]

Could the apparent validity of (13) be explained without appealing to Comparability?

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[18]If one rejected the appeal to scalar implicature, the obvious alternative would involve positing a straightforward semantic ambiguity, whereby 'x is as F as y' has one meaning equivalent to 'x and y are equally F' and another equivalent to 'x as at least as F as y'. This less parsimonious view could also be used, together with Comparability, to explain why arguments like (13) strike us as valid, so long as we have some account of why we gravitate to the disambiguation that makes them valid. Part of this could be a general charitable tendency to favour disambiguations that make arguments presented as valid actually be valid, although there is surely more to it than that, since the 'equally F' reading seems in general hard to access in negative (downward-entailing) environments.

One idea is that 'x is as F as y' is equivalent, at least on one disambiguation, to 'y is not F-er than x', or to 'y is not F-er than x and x and y are both F to some degree'. But if Comparability is rejected, these proposals conflict with the very plausible thesis that 'as F as' is transitive on all of its disambiguations: 'a is as F as b, and b is as F as c, but a is not as F as c' seems flatly inconsistent.

A more promising strategy for opponents of Comparability would be to appeal to some pragmatic effect by which 'Max's room is not at tidy as Josh's' gets associated with a meaning that strengthens its standard truth-condition by entailing that the rooms are comparable (and hence that Josh's room is tidier than Max's). But such a strengthened meaning could not be generated by any *general* pragmatic mechanism, since no comparable strengthening is observed for other binary relations which uncontroversially do not obey the analogue of Comparability. Consider parthood. There are some things x and y such that x is not part of y and y is not part of x. But 'not part of' does not get pragmatically strengthened in anything like the way we are considering for 'not as tidy as'. There is no temptation to hear the following argument as valid:

(14) My carburettor is not part of your computer. So your computer is part of my carburettor.

So, it looks like the strategy we are considering would have to posit something distinctive about the conventional meanings of comparatives to generate the needed strengthening. The most obvious idea is to posit that 'x is [at least] as F as y' *presupposes* 'either x is at least as F as y or x is at least as F as y'. Given that presuppositions project through negation, this posit will secure for Not As F As the very same status of "presuppositional validity" that we have claimed for it.[19]

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[19]Magidor (2013, p. 145, n. 46) considers (without endorsing) the similar idea that "'x is smaller than y' triggers the presupposition that x and y are comparable in size", as one possible account of the oddity of 'My table is[/is not] smaller than the number three'. In §5, we will be endorsing an alternative account of the oddity of such sentences (one which Magidor also considers), in terms of polysemy or context-sensitivity in comparatives like 'smaller'.

Positing such a presupposition as a brute matter of lexical semantics seems worryingly ad hoc: in general, the ordinary truth-conditional meanings of expressions seem to place quite tight constraints on their presuppositions, even though there is no consensus about what form these constraints take. We can also argue directly against the presupposition-theoretic proposal, by pointing to various utterances which should, if it were true, strike us as infelicitous or at least risky, on account of their strong presuppositions. Consider polar questions, which have the same presuppositions as the corresponding declarative sentences:

(15) Is your Ferrari parked in the garage?

—Why are you assuming I have a Ferrari?

We can use this as a test for views that posit nontrivial presuppositions. Our proposal that 'x is as F as y' presupposes 'x and y are both F to some degree' (i.e. 'x is as F as x and y is as F as y') seems to pass the test:

(16) Was the performance by the children's choir as expensive as the wedding banquet?

—Why do you assume they were paid?[20]

But there is no hint that questions of the form 'Is x as F as y?' carry any non-trivial presupposition about the *relation* between x and y that goes beyond the presupposition that they are individually F to some degree:

(17) Is Max's room as tidy as Josh's room?

—? Why do you assume that one of their rooms was at least as tidy as the other?

Plausibly, every room is tidy to some degree. But if Comparability is false, one would expect it to be pretty easy for a pair of rooms to be such that neither is at least as tidy as

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[20]Examples like this suggest that 'expensive' carries *some* kind of presupposition, to the effect that the relevant item was or could be bought. Whatever exactly this comes to, it presumably entails (and might be argued to be equivalent to) being as expensive as oneself. For the difficulty of pinning down precise presuppositions in such cases, see Magidor (2013, §4.2).

the other. So unless we take the questioner in (17) to have some independent evidence about what each of the rooms is like, it should seem like they are unjustifiably ruling out a live scenario. But we have no sense that this is the case: (17) seems a perfectly innocent question.

The point is especially forceful when we turn to sentences involving quantifiers, which should (if 'as *F* as' carries a comparability presupposition but Comparability is false) have strong presuppositions that we can often know to be false. Consider:

(18) Most composers aren't as good as Beethoven.

(19) Are many contemporary cellists as talented as Pablo Casals was?

(20) I will buy an ice cream for everyone whose room is as tidy as Josh's.

We should expect such sentences to carry universal presuppositions, just as 'Most senior managers drive their Ferraris to work' carries the presupposition that every senior manager has a Ferrari, and 'Have many programmers stopped using Emacs?' carries the presupposition that all programmers used to use Emacs. So on the view that equatives carry comparability presuppositions, (18)–(20) should presuppose, respectively, that every composer is comparable in goodness to Beethoven; that every contemporary cellist is comparable in talentedness to Casals; and that everyone's room is comparable in tidiness to Josh's. And if Comparability fails, these presuppositions should seem tendentious or even clearly false. But in fact these sentences are all perfectly felicitous for speakers with no special evidence. So, there is an abundance of evidence against the thesis that that '*x* is as *F* as *y*' carries the comparability of *x* and *y* as a presupposition which is not entailed by the presupposition that *x* and *y* are both *F* to some degree.

(ii) 'No F-er'. Our second argument is based on the apparent validity of arguments like the following:

(21) Max's room is no tidier than Josh's. So, Josh's room is at least as tidy as Max's.

This certainly seems a good piece of reasoning. Since the appearances seem entirely the same for any other comparative adjectives (including 'good' and other paradigms of "multidimensionality"), it sees to us that its goodness is best explained by the view that No F-er Than is valid:

No F-er Than   If x and y are both F to some degree, and x is no F-er than y, then y is at least as F as x.

The validity of No F-er Than can in turn be explained by appeal to Comparability and Comparative/Equative, given the plausible further assumption that 'no F-er than' is truth-conditionally equivalent to 'not F-er than'.

Analogous to what we already saw in the case of Not As F As, the fact that (21) seems valid despite not having 'Max's room and Josh's room are both tidy to some degree' in the antecedent can plausibly be taken care of by invoking presuppositions. No F-er Than is also analogous to Not As F As in that opponents of Comparability could attempt to explain away the evidence for its validity by positing a comparability presupposition in addition to the 'F to some degree' presupposition; but as we have discussed, standard tests suggest that no such further presupposition exists.

The other obvious option is to accept the validity of No F-er Than, but reject the thesis that 'no F-er than' is equivalent to 'not F-er than'. But note that for the argument to go through, it is enough that 'x is no F-er than y' follows from 'x and y are both F to some degree, and x is not F-er than y'. And it is very hard to see what more could reasonably be required, given that 'no F-er' results from combining 'F-er' with the mass-quantifier 'no', which also occurs in other contexts like 'There is no butter in the fridge'. Given the validity of 'If is not the case that there is butter in the fridge, there is no butter in the fridge', it is hard to see how any systemic semantic account of 'no' could avoid likewise validating 'If it is not the case that x is F-er than y, x is no F-er than y'.[21]

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[21]If you are inclined to resist here, it might help to use 'x is not at all F-er than y' or 'x isn't any F-er than y' as a bridge.

Admittedly, we do seem happier to accept 'not *F*-er than' than 'no *F*-er than' in some circumstances, even when there is no doubt that the relevant items are *F* to some degree. 'Beethoven is no better than Bach' perhaps suggests more confidence in one's musical judgment than 'Beethoven isn't better than Bach'. But it does not seem promising to explain this contrast by assigning 'no *F*-er' a special new kind of semantics that one could not predict by thinking about the general behaviour of 'no' and '*F*-er'. We find it much more likely that the contrast is a pragmatic one, related to whatever makes "wide scope" negations often feel weaker than narrow-scope negations: e.g 'He is not bald' feel weaker than 'He is non-bald', and 'It's not an issue' feel weaker than 'It's a non-issue'. For example, we might want to say that wide-scope negations can, but narrow-scope negations cannot, be pragmatically reinterpreted as "metalinguistic negation" (Horn 1985), rejecting some other contemplated speech act rather than making an ordinary assertion.

(iii) 'F things are F-er than non-F things'. Our third argument is based on the apparent validity of the following piece of reasoning:

(22) Kara is healthy, and Sam is not healthy. So Kara is healthier than Sam.

Again, (22) seems like a good piece of reasoning for any comparative construction. Its apparent validity is, we submit, best explained by the validity of the following schema (which is a strengthening of Monotonicity, stated on page 2):

> **Strong Monotonicity** If x is *F* and y is not *F* but y is *F* to some degree, then x is *F*-er than y.[22]

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[22]For similar principles, see Chisholm and Sosa (1966, p. 248) and Van Benthem (1982, p. 198). Nebel (2018) states instances of Strong Monotonicity for 'good' and 'bad' and observes that they pose problems for incomparability in specific contexts, but fails to notice its more general implications. Flanigan and Halstead (2018) propose a similar "dyadic–monadic" principle to defend the more limited claim that, given epistemicism about vagueness, options must be comparable.

Gustafsson (2019) rejects Strong Monotonicity for 'good' on the grounds that (i) something can be incomparable to a "neutral" thing (he calls such things "undistinguished") and (ii) anything better than a neutral thing must be good. If x is neutral and y is incomparable to x (and thus "undistinguished"), then y should also be incomparable to some $x+$ that is slightly better than

Strong Monotonicity does not *quite* certify (22) as a valid argument, since it requires the extra premise 'Sam is healthy to some degree'. But it is obviously necessary that every living creature is healthy to some degree (in our stipulative sense of being at least as healthy as itself), and in considering (22) we naturally assume that 'Sam' is the name of a living creatures (as opposed to, say, a number); so Strong Monotonicity is adequate to explain why (22) seems cogent. Moreover, it is plausible that '*F* to some degree' is a presupposition of an unmodified adjective *F*, and hence also of its negation, so that arguments like (22) will have the good status of "presuppositional validity" discussed earlier.

One might reject Strong Monotonicity on the grounds that 'This mouse is big, and that elephant is not big (but is big to some degree)' does not entail 'This mouse is bigger than that elephant'. But we can account for this apparent counterexample without giving up the validity of Strong Monotonicity by claiming that the natural interpretation of the premise involves resolving the context-sensitivity in the two occurrences of 'is big' in different ways. For example, on some accounts, context supplies a "comparison class",

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x. (ii) implies that *x*+ must be good, and Strong Monotonicity would then require *x*+ to be better than *y*, when they should be incomparable. Regarding (i), obviously we are unmoved by an argument against Strong Monotonicity that presupposes the negation of our conclusion, Comparability, though our own argument from Strong Monotonicity may be similarly dialectically ineffective against staunch proponents of Gustafsson's axiology; we offer other principles below that, together with Strong Monotonicity, form a package they might find harder to resist. But even if we accepted (i), we would reject (ii). Analogous principles don't seem plausible for other comparatives: one room might be at a neutral temperature while being warmer than another room at a neutral temperature. An alternative picture would involve a range of neutral things, some of which are better than others (as in Rabinowicz 2009a). The idea that anything better than a neutral thing must be good seems to rely on a very specific interpretation of 'neutral' (so specific, in fact, that on Gustafsson's own view, no possible lives are neutral in value), which we take to be less pre-theoretically compelling than Strong Monotonicity itself. If Gustafsson's view were true, and if incomparability were possible, we would expect the notion of undistinguishedness not to seem as unfamiliar as it does. And we see no evidence that it is a more general phenomenon that extends beyond Gustafsson's focus of population axiology. So, despite the interest of Gustafsson's axiology, more detailed discussion of his arguments would be out of step with this paper's non-domain-specific focus.

which is plausibly different for the two uses.[23, 24]

Strong Monotonicity is a special case of a more general pattern. The following inference also seems valid:

(23) Kara is very healthy, and Sam is not very healthy, so Kara is healthier than Sam.

The impression of validity remains if we replace 'very' by other "positive degree modifiers": 'extremely', 'somewhat', 'pretty', 'quite', etc. The case for the validity of Strong Monotonicity thus extends to the following more general schema, where 'V' is to be replaced any positive degree modifier:

Degree-Modified Strong Monotonicity   If x is V F and y is not V F, but y is F to some degree, then x is F-er than y.

Strong Monotonicity can be seen as a special case of Degree-Modified Strong Monotonicity if (following the standard treatment in semantics going back to Bartsch and Vennemann 1972), we analyse sentences like 'Kara is healthy' as containing a phonologically null degree modifier 'POS' that plays the same role that 'very' would play if it were present.

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[23]Even if we do not treat 'is big' as containing an unpronounced comparison-class-denoting variable, we might still treat it is containing an unpronounced "degree expression" POS (see below), in which case we might plausibly attribute the relevant contextual variation to this element rather than to the root word 'big'.

[24]'Is big' also seems to admit a reading that can be paraphrased as 'is big for one's kind' (where the relevant kind might be one's biological species or genus, or some more "folk" category): consider for example 'Due to pervasive overfeeding, almost every animal in that zoo is big'. That reading of 'big' would allow for 'This mouse is big and that elephant is not big' to be true without any equivocation. So as proponents of Strong Monotonicity, we are committed to the claim that 'This mouse is bigger than that elephant' is true when 'big' is interpreted in that way. This seems defensible. That sentence *does* have a possible, though highly dispreferred, reading on which it is true when the demonstrated mouse weighs much more than most mice while the demonstrated elephant weighs less than most elephants. (Similarly, 'more expensive' has a reading where 'That is an expensive sandwich, but this cup of coffee is more expensive' could be true if the sandwich cost $15 and the cup of coffee cost $8.) It seems reasonable to suppose that this unusual reading of 'bigger than' is the one that arises when we give 'big' the "big for its kind" reading, for which one's position on the relevant scale turns on the relation between one's size and the size-statistics characteristic of one's kind.

For those who are not satisfied by this response, we note that the role of Strong Monotonicity in our argument could equally well be played by the schema 'If x is F for a K and y is not F for a K, but y is a K and is F to some degree, then x is F-er than y'.

Again, arguments like (23) are not quite certified as valid by Degree-Modified Strong Monotonicity, without the additional assumption that both things are at least as *F* as themselves. But this assumption seems even more clearly to be a presupposition of a degree-modified adjective '*V F*'. And, as in (23), this presupposition projects through negation (in addition to being obvious). For example, 'The square root of two is not very tall' sounds bizarre because it presupposes that the square root of two has some degree of height.

The validity of Degree-Modified Strong Monotonicity is hard to deny. But it makes it very hard to see how there could be any significant incomparability. For, given Degree-Modified Strong Monotonicity, items that are divided by a degree-modified, positive form of a gradable adjective—as in (23)—must be comparable. And any gradable adjective can be degree-modified in many different ways. For example, imagine a possibly partial order of people from the not at all funny to the supremely funny: in between there are (in no particular order) people who are barely funny, somewhat funny, pretty funny, moderately funny, slightly more than moderately funny, extremely funny, and so on. All of these degree modifiers would have to impose bottlenecks in the ordering, across which there can be no incomparability. This is implausible. If there is incomparability in *F*ness, it is best explained by tradeoffs between dimensions of *F*: for example, two people who are each funny in different respects, where our use doesn't privilege any particular weighting of these respects in such a way that one of them gets to count as funnier overall. One would expect to be able to find such tradeoffs anywhere in the ordering, not just in between the bottlenecks imposed by degree modifiers.

Put differently, Degree-Modified Strong Monotonicity implies that, if some *x* is *V F*, then every *y* to which *x* is incomparable is also *V F*. It follows that positive, degree modified adjectives spread through chains of incomparability. But, if "multidimensional" adjectives give rise to incomparability in the way that has been supposed, one would expect such chains to be ubiquitous. For example, one would expect the following kind of

case to be possible. Consider two sequences of careers $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$, where the subscript represents one's annual salary in dollars. For example, the x's might be careers as a philosopher and the y's might be careers as an artist. For some value of n, x_n and y_n might both be very good careers. If there is incomparability, we would expect it to be possible for each x_i to be neither better nor worse than, nor just as good as, y_i. And we would expect a dollar not to break the incomparability: if making i dollars per year as a philosopher is neither better nor worse than, nor just as good as, making i dollars per year as an artist, then making $i - 1$ dollars per year as a philosopher cannot plausibly be worse than making i dollars per year as an artist. That is the kind of intuition that motivates judgments of incomparability rather than equality in the first place. So suppose that x_n—a very good career—is not better than y_n, which is not better than x_{n-1}, which is not better than $y_{n-1}, \ldots$, which is not better than x_1, which is not better than y_1. Because they reject Negative Transitivity, fans of incomparability can say that x_n is better than y_1. But, given Degree-Modified Strong Monotonicity, they must say that y_1 is still a very good career. And that is absurd: no career in which one makes only a dollar a year is very good!

(iv) 'The F-est things'. Strong Monotonicity has an analogue for superlatives that seems similarly compelling:

> **Superlatives** If x is one of the *F*-est *K*s, and y is a *K* and is *F* to some degree but is not one of the *F*-est *K*s, x is *F*-er than y.[25]

Here *K* is schematic for a noun phrase, perhaps complex. The claim that Superlatives is valid can be supported by appeal to the intuitive validity of various specific arguments: for example 'Parmesan is one of the most beloved cheeses, and Grana Padano is not one of the most beloved cheeses; so, Parmesan is more beloved than Grana Padano'.[26]

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[25]Note that the converse principle 'If x is one of the *F*-est *K*s, and x is *F*-er than y, y is not one of the *F*-est *K*s' is not very plausible: 'Anne and Bert are the tallest children in the class' is consistent with 'Anne is taller than Bert'.

[26]As usual, we can explain why we don't feel the need for the extra premise 'Grana Padano is at least as beloved as itself' by appeal to presupposition. This also applies to 'Grana Padano is a

The upshot of Superlatives is that the only pluralities eligible to be denoted by 'the *F*-est *K*s' are those whose boundary marks a "bottleneck" for the relation expressed by 'is an *F*-er *K* than', in the same sense in which Degree-Modified Strong Monotonicity requires the extension of '*V F*' to be a set whose boundary marks a bottleneck for the 'is *F*-er than' relation. But as we have seen, the standard motivations for positing incomparability suggest that for many gradable adjectives *F*, when the extension of *K* is big enough, we will be able to reach any *K* from any other *K* by a series of steps in which each item is a *K* and is incomparable with its predecessor, so that there aren't any such bottlenecks. In such a case, then given Superlatives, the only candidate plurality to be denoted by 'the *F*-est *K*s' will be the plurality of *all K*s. This seems absurd: claims like 'A poorly-remunerated career in manure-shovelling is not one of the best careers' are unproblematic in a way that they should not be if incomparabilities were common and Superlatives were true.

Indeed, it is rather tempting to think that whenever *x* is *F*-er than *y* and both are *K*s, we can find *some* candidate denotation for 'the *F*-est *K*s' that includes *x* but not *y*. If so, then given Superlatives, it would follow that any such *x* and *y* are separated by a bottleneck, so that Negative Transitivity is true restricted to the *K*s. We could sharpen this hunch by turning to plural definite descriptions of the form 'the *n F*-est *K*s', which do not seem to be subject to the peculiar context-sensitivity of 'the *F*-est *K*s' (although they of course inherit any context sensitivity there might be in '*F*' and '*K*'). The following principle concerning such cardinality-specific plurals seems rather natural:

> **Cardinal Superlatives** If *x* is *F*-er than *y*, *x* and *y* are both *K*s, and there are at most *n K*s, then either *x* is the *F*st *K*, or *x* is one of the two *F*-est *K*s and *y* is not, or *x* is one of the three *F*-est *K*s and *y* is not, . . . , or *x* is one of the *n* − 1 *F*-est *K*s and *y* is not.

But given the obvious extension of Superlatives to cardinality-specific definites, Cardinal Superlatives entails that Negative Transitivity holds restricted to the *K*s whenever there

cheese': 'is [not] one of the *F*st *K*s' seems generally to presuppose 'is a *K*'.

are finitely many Ks. It follows that Negative Transitivity holds in full generality: just take K to be 'thing identical to x or y or z' for any given trio. And as we saw in section 1, there is no interesting way of accepting Negative Transitivity while rejecting Comparability.

(v) 'Much Fer than'. Our final argument is in the spirit of Degree-Modified Strong Monotonicity but does not require any appeal to the positive or superlative forms of gradable expressions. Consider the following inferences:

(24) Ann is much more creative than Bob. Cat is more creative, but not much more creative, than Bob. So Ann is more creative than Cat.

(25) Io had a lot more fun than Jim. Kyle had more fun, but not a lot more fun, than Jim. So Io had more fun than Kyle.

(26) Xan swam way faster than Yair. Zev swam faster, but not way faster, than Yair. So Xan swam faster than Yair.

These seemingly impeccable inferences suggest the following generalization (where V now stands for positive degree modifiers of the same sort as 'much', 'a lot', and 'way'):

> **Much F-er Than**   If x is V F-er than z and y is F-er than z but not V F-er than z, then x is F-er than y.[27]

Much F-er Than poses a problem for incomparability, because it entails that chains of incomparability collapse the distinction between modified comparatives ('much F-er') and their unmodified versions ('F-er'). To see this, return to our two sequences of careers $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$. Plausibly, for some n, x_n is a much better career than x_1. Suppose again, though, that these sequences form a chain of incomparability: x_n is not better than y_n, which is not better than $x_{n-1}, \ldots$, which is not better than x_1. Then, by Much F-er Than, if y_n is better than x_1, then it must be much better than x_1; otherwise, x_n would be better

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[27]We would also accept the stronger version of this principle that replaces 'y is F-er than z' with 'y is at least as F as itself'. But we focus here on the weaker version for dialectical reasons.

than y_n. By the same reasoning, x_{n-1} can be better than x_1 only if it's much better than x_1; otherwise, y_n would be better than x_{n-1}. More generally, for any k, x_k is better than x_1 only if it's much better. But, intuitively, though x_2 is better than x_1, it's not much better: they only differ by a dollar per year!

One could do something similar by appealing to a schema involving explicit comparisons of the amounts by which two objects improve on some third object:

> **Not As Much** *F*-**er As**   If x and y are both *F*-er than z, but y is not as much *F*-er than z as x is, then x is *F*-er than y.

The validity of this schema explains the badness of speeches like the following:

(27)   The tuna and the sable were both tastier than the whitefish. But the sable wasn't as much tastier as the tuna. However, the tuna was not tastier than the sable.

Like Much *F*-er Than, Not As Much *F*-er As collapses the distinction between modified comparatives and their unmodified versions along chains of incomparability. For example, applied to our sequences of careers, Not As Much *F*-er As implies that any career better than x_1 must be as much better than x_1 as x_n is. But this is absurd. Clearly x_2, for example, isn't better than x_1 by as much as x_n.

This concludes our battery of arguments for Comparability. Some opponents may try to dismiss our arguments as naively relying on deference to untutored folk intuitions (or linguistic practices) which need not survive reflection on philosophical arguments against Comparability, such as those discussed in §2. We have two responses. First, we need not rely on any intuitions or judgments other than our own, and judgments we rely on need not be *unreflective* (except to the extent that all argumentation on any subject whatsoever must begin from a starting point prior to reflection). We have endeavoured to subject the judgments of validity that provide the basis for our arguments to at least as much reflective scrutiny as opponents of Comparability have brought to bear on the judgments that drive their arguments. Second, although the folk are prone to many errors, there

should be a strong presumption that deeply ingrained dispositions to treat certain general argument-forms as if they were valid should not be dismissed as mistakes, unless there is a compelling explanation for why we should systemically go wrong in these specific ways. After all, use (and dispositions thereof) determines meaning, and it is plausible that these sorts of 'logical' dispositions play an especially central role in giving our words the meanings they have. So, opponents of Comparability are saddled with a heavy explanatory burden: namely, to explain how comparative constructions get to express relations that violate Comparability (and, more generally, how related constructions such as the positive, superlative, equative, and degree-modified forms get to mean what they do) contrary to the entrenched facts of usage. Our burden is less great: it is not at all remarkable for philosophers to make errors, and the fact that many philosophers are disposed to make certain kinds of inferences generates very little pressure towards a semantics on which those inferences are truth-preserving. Still, in view of the wide influence of the tradeoff and small improvement arguments, it is incumbent on us to say something about them. We will do this in the next section.

4   Tradeoff and small improvement arguments revisited

We take the data considered in the previous section to provide a strong *prima facie* case that Comparability is valid for every comparative construction in English. Let us now revisit the influential arguments against Comparability that we surveyed in §2.

First: we admit that when one considers the multiplicity of dimensions and the difficulty of making tradeoffs, speeches of the following kind can sound fine:

(28)   Neither George Carlin nor Richard Pryor was funnier than the other. But they were not equally funny.

The tradeoff argument works by eliciting such judgments, and using them to draw the inference that there are cases of incomparability. We think they should be treated along the same lines as

(29) George Costanza is not bald, but he is also not *not* bald.

We are tempted to say things like (29) when it is vague whether or not someone is bald. But the acceptability of such utterances is not a compelling argument against the thesis that sentences of the form 'P but not P' are logically inconsistent in English. We don't need to have a theory about the nature of vagueness to see that (29) should not be taken literally. Since "multidimensional" comparative adjectives are certainly vague, we should similarly not take utterances like (28) to express literal truths. Their role is, rather, to convey the speaker's awareness that some (at least two) of the sentences '*x* is *F*-er than *y*', '*y* is *F*-er than *x*', '*x* and *y* are equally *F*' are extensionally vague (i.e. not "definitely true" or "definitely false").

This diagnosis of the motivations that led philosophers to find utterances like (28) obviously correct is strongly confirmed in many cases by an examination of the justifications offered for the utterances. For example, we have seen some say that 'there are no precise truths' (Parfit), that we 'may not . . . be able to decide' (Sen), or that there is 'no answer' (Keynes) to whether one thing is *F*-er than another. In one of the first statements of the small improvement argument (quoted approvingly by Raz) Mackie (1977) suggests that there may be no 'objectively right and determinable answer' to the question of whether one thing is *F*-er than another. In his own discussion of the argument, Raz imagines a 'small but definite improvement' to one of two options, which suggests a contrast to a possibly 'indefinite' comparison. Most tellingly, perhaps, is that proponents of incomparability rarely make outright assertions like (28). Parfit (1984, p. 431) says we 'might claim' that neither of two things is *F*-er than the other, not that neither is *F*-er. Chang (1998, p. 23) supposes that we 'rationally judge' that neither of two things is *F*-er than the other. Carlson (2013, p. 449) gives a case in which neither of two things can 'be said to be' *F*-er. Such hedged assertions and assessments of assertibility and reasonableness are precisely what one would expect in the presence of vagueness. There is a marked contrast between the sentences typically used to express the purported counterexamples to Comparability

and, say, the sentences used to express purported counterexamples to Transitivity—e.g., "B is worse than A, C is worse than B, D is worse than C, ... and Z is worse than Y, yet Z is better than A" (Rachels 1998).

We are far from the first to appeal to vagueness in explaining away the appearance of incomparability (see especially Broome 1997). Proponents of incomparability have taken great pains to reject such appeals. Their arguments have been discussed at length elsewhere.[28] Our general impression is that the attempts to dismiss vagueness-based diagnoses rely on highly tendentious characterizations of vagueness. For example, Chang (2002) argues that borderline cases have a distinctive phenomenology in which, "insofar as we are willing to judge that the predicate applies, we are also willing to judge that it does not apply." She suggests that, in putative cases of (what we're calling) incomparability, we are instead inclined to judge that one thing is not F-er than another, but not similarly inclined to judge that it is. Chang admits that there might, in certain cases, be "perplexity" about the comparative judgments. But Chang claims that the perplexity is distinct from the kind that characterizes borderline cases, in which "it is perfectly permissible to resolve the indeterminacy in favor of application or not" (683). In putative cases of incomparability, by contrast, Chang suggests that "given a list of admissible ways in which the perplexity might be resolved, there is still a further question as to how the perplexity is to be resolved, for that resolution is not simply given by arbitrarily opting for one admissible resolution over another" (685). For example, people who offer conflicting comparative judgments in putative cases of incomparability are involved in a "genuine substantive disagreement" (685), but people who offer conflicting resolutions of vague predicates are not.

Here is a simple way to see that these arguments are not compelling. Recall our two sequences of careers $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$, where, for every i, the proponent of incomparability is inclined to judge that x_i is neither better nor worse than y_i, which is neither better nor worse than x_{i-1}. Anyone with that inclination should be no less

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[28]See, e.g., Elson (2017), Gustafsson (2013), and Williams (2016).

inclined, for every $i > 1$, to judge that it is not the case that x_i is a good career and y_i is not, and not the case that y_i is a good career and x_{i-1} is not. But these negated conjunctions lead to the absurd conclusion that it's not the case that x_n is good and and x_1 is not.[29] This pattern is characteristic of vagueness if anything is. But it is not plausible that there is a distinctive phenomenology to the negated conjunctions that is absent in the comparative judgments, or that one can permissibly settle on an arbitrary value of i at which one rejects the negated conjunctions while remaining at a loss as to how to resolve the perplexity raised by the corresponding comparisons, or that disagreement about the comparative judgments is more genuinely substantive than conflicting resolutions of the predicate 'good'.[30] Proponents of the tradeoff argument may continue to find it incredible that there should be a single correct way to weigh between the different dimensions of a multidimensional concept. But this seems no more incredible than familiar analogous propositions about vague predicates—e.g., that a single cent can make the difference between a career that is good and a career that is not.

The above diagnosis of the tradeoff argument could also be extended to the argument from small improvements. This argument begins with a judgment concerning two rather disparate items x and y, to the effect that neither is F-er than the other. This could be rejected in the same way that we rejected (28), as a mistaken reaction to the recognition that, due to vagueness in 'F-er', neither 'x is F-er than y' nor 'y is F-er than x' is *definitely* true.

It is worth noting, however, that small improvement arguments look like suspiciously powerful tools for arguing against claims of the form 'x and y are equally F' even when

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[29]At least, they do so in classical logic, and in prominent nonclassical logics like intuitionism and the paraconsistent logic of Field (2003). They do not do so in the very weak logic of Fine (2017), in which vagueness is held to invalidate not only Excluded Middle but the rule of Conjunctive Syllogism ('P; not (P and Q); therefore not-Q').

[30]Note also that the monadic predicate isn't the only one that is clearly susceptible to vagueness. We are also inclined, for every $i > 1$, to judge that it's not the case that y_i is better than x_1 but y_{i-1} is not. But these judgments, which are suspiciously similar to the "small improvement" intuition, lead to the absurd conclusion that it's not the case that y_n is better than x_1 and y_1 is not.

we *haven't* already ruled out '*x* is *F*-er than *y*' and '*y* is *F*-er than *x*'. Claims of the form '*x* and *y* are equally *F*' are commonplace in everyday life. For example, a hiring committee chair might utter

(30) These two candidates are equally well qualified on paper, so we will have to make a case based on our opinions of the writing samples.

A small-improvement-monger might object:

> Really? So, you are committed to the view that if Candidate *A* had TAed for just one additional class, they would have been slightly better qualified than Candidate *B* (since clearly in that case they would have been slightly better qualified than they actually are)? Isn't that an implausibly strong thing to be committed to?

In the context of an actual conversation in non-philosopher-speak, the above objection sounds silly and pedantic. Raz himself says, of a person choosing between a legal career and a musical career, "He is equally suited for both, and he stands an equal chance of success in both" (126).

One possible account of what is going on appeals to context-sensitivity. According to this account, comparative expressions can in some contexts take on "coarse-grained" meanings, where the domain is "chunked" into some finite number of discrete sections. The relations expressed in a coarse-grained context by 'equally *F*' and 'at least as *F* as' will hold universally within each chunk, while the relation expressed by '*F*-er than' holds only between things in different chunks. When we start paying attention to small differences in the relevant underlying factors, that generates pressure to change the context to a more fine-grained one, which means the extension of '*F*-er than' will expand to include certain pairs of items that were previously in the same chunk.[31] On this account, the

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[31]Schwarzschild and Wilkinson (2002) propose a "chunking" account for 'as *F* as', 'at least as *F* as' and 'exactly as *F* as'. However they do not extend the account to '*F*-er than', so their theory

small improvement argument can be diagnosed as turning on a shift in the context. When we initially consider the disparate items x and y, the most natural resolution of context-sensitivity is a rather coarse-grained one that puts them in the same chunk, since, because of vagueness, there is no non-arbitrary grounds for choosing a resolution of context sensitivity that makes one but not the other of 'x is F-er than y' and 'y is F-er than x' true. The first premise, 'x is not F-er than y and y is not F-er than x', is thus true in the context in which it is uttered; 'x and y are equally F' is also true in this context. But when we are invited to pay attention to the small differences between x and x^+, we are pushed into a new, more fine-grained context, in which 'x^+ is F-er than x' is true.

An alternative diagnosis appeals to pragmatics rather than semantic context-sensitivity. On this account, when we say 'These things are equally F', we are very often speaking *loosely* (non-literally). Analogously, some philosophers argue that when we say 'The cities are 853 miles apart' or 'They arrived at 3pm' we are almost never speaking literally, since the literal truth conditions are ultra-demandingly exact (Hoek 2018). But if that's the diagnosis, it is plausible that the same kind of looseness is in play when, at the beginning of the small improvement argument, we are inclined to accept 'Neither of x and y is F-er than the other'. This seems to be a different kind of nonliteral speech from the kind exemplified by 'He is not bald, but he is also not not bald', since it doesn't have anything special to do with negation. However, the two sources of nonliteralness could sometimes both be in play.

We need not commit ourselves to any particular diagnosis of the apparent assertibility of sentences like (30). It seems likely that, whatever the diagnosis, it can help to explain away the apparent truth of the key premises of the small improvement argument—which, as we have said, are rarely asserted outright in the first place.

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surprisingly predicts the consistency of 'a is F-er than b and b as at least as F as a'. While they could give a pragmatic account of the oddity of this conjunction by positing that the first conjunct tends to push us into a more "fine-grained" context where the chunks are small enough to make the second conjunct false, it seems better to us to preserve the validity of Comparative/Equative by applying the "chunking" to comparatives as well as equatives.

5   Other objections to Comparability

Even after we set the tradeoff and small improvement arguments aside, there are some putative counterexamples to Comparability which might convince someone that it cannot be a valid schema. One class of counterexamples is typified by (31):

(31)   Either this cup of coffee is at least as good as the latest *Star Wars* movie, or the latest *Star Wars* movie is at least as good as this cup of coffee.

There is an air of absurdity to (31). It's not that there are some respects in which the coffee is better than the movie which we have trouble weighing against respects in which the movie is better than the coffee: it is hard to even imagine a case where either of the disjuncts of (31) is straightforwardly true. But, given Comparability, (31) is a logical consequence of 'This cup of coffee is good to some degree and the latest Star Wars movie is good to some degree'. And given Restricted Reflexivity, this is in turn a logical consequence of

(32)   This cup of coffee is at least as good as the last one I ordered in Starbucks, and the latest Star Wars movie is at least as good as *The Phantom Menace*

which seems perfectly fine.

It seems to us that whatever is going on in (31) is also going on in (33), which is perhaps more suggestive of what the diagnosis should be:

(33)   Either this seminar is at least as long as Fifth Avenue, or Fifth Avenue is at least as long as this seminar.

This is at least as absurd as (31). But given Comparability and Restricted Reflexivity it should be a logical consequence of perfectly fine conjunctions like

(34)   This seminar is longer than the one last week, and Broadway is longer than Fifth Avenue.

The right way to block the inference from (34) to (33), we claim, is to appeal to context-sensitivity (or polysemy, if that's not just a kind of context-sensitivity) in 'long'. 'Long' has one reading where it has to do with time, and another where it has to do with space. On the former, 'x is as long as y' is true only when x and y are events; on the latter, it is true only when x and y are (appropriately shaped and oriented) physical objects. The natural reading of (34) is a non-uniform one where the first occurrence of 'longer than' has the temporal meaning and the second one has the spatial meaning; but this does not entail (33) on any of its possible readings (whether uniform or mixed).

A parallel diagnosis is plausible for (31), given the widely recognised context-sensitivity (or polysemy) of 'good'. This also generates a multiplicity of possible interpretations for 'better than' and 'at least as good as': for example, if A is a professional cellist and entry level safecracker and B is a professional safecracker and beginning cellist, 'A is better than B' could be uttered truly in a conversation among bank robbers while 'B is at least as good as A' was uttered truly in a conversation between the organisers of a musical *soirée*. Among the salient possible interpretations of 'x at least as good as y' are relations that hold only among cups of coffee; relations that hold only among movies; relations that hold only among servings of food or drink; relations that hold only among cultural products (movies, plays, novels, songs. . .); and so on. Neither of the disjuncts of (31) is true on any of these interpretations. But (32) isn't true either if we interpret it *uniformly* using any of these interpretations. The reason (32) seems so much better than (31) is that it is true on its most natural interpretations, which are non-uniform ones where the first 'at least as good as' corresponds to some relation that movies can't bear to anything (even themselves), and the second 'at least as good as' corresponds to some relation that cups of coffee can't bear to anything (even themselves).[32]

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[32](31) cannot be saved by interpreting its two occurrences of 'good' non-uniformly, since no salient interpretation makes either disjunct true. It is interesting to consider what happens if we expand the sentence so that there are two occurrences of 'good' in each disjunct:

(31*) Either this cup of coffee is at least as good as the latest *Star Wars* movie is good, or the latest *Star Wars* movie is at least as good as this cup of coffee is good.

It wouldn't be plausible to hold that 'at least good as' and 'better than' can *never* express relations whose relata include both cups of coffee and movies. For example, in the right context 'good' could mean 'good as a conversation topic on your next date'. But in such a context, (31) seems fine. The fact that (31) is odd out of the blue can be explained by saying that interpretations of this sort are sufficiently unsalient, and sufficiently disparate, that we would expect speakers to provide some contextual pointers before saying something that relies on one of them.

Another quite different family of putative counterexamples to Comparability turns on terms of art like "is stronger than," meaning *entails.* If *p* does not entail *q*, and *q* does not entail *p*, neither is stronger than the other, but they are not equally strong, in this technical sense.

We grant that speakers can go against the linguistic grain and stipulate meanings for comparative adjectives that do not conform to the general rules. For example, one can, with diligent effort, impose a use on 'westerly' such that 'Tokyo is more westerly than New York', 'New York is more westerly than Istanbul', 'Istanbul is more westerly than Tokyo', and 'Nothing is more westerly than itself' are all true. Such stipulative uses should not be taken to refute the thesis that transitivity is guaranteed by the meaning of comparative morphology in spoken dialects of non-artificial languages.[33] If the stipulation only applies

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One might expect that if our diagnosis of the oddity of (31) is correct, (31*) should be much better than (31) because of the possibility of interpreting the two 'good's in each disjunct differently. ((31) would standardly be treated as derived from (31*) by ellipsis; however since ellipsis normally seems to rule out non-uniform interpretation, this is compatible with one sentence being much better than the other.) We are not clear on whether there is a relevant contrast between (31) and (31*): it's hard to get past the fact that the latter just sounds strange and stilted. But if it's true that (31*) is odd in the same way as (31), one possible explanation of this would be to say that the two salient interpretations of 'good' are not only distinct, but not "associated with the same scale", so that the cross-adjectival comparison is analogous to 'You are more beautiful than I am clever' rather than 'The mountain is taller than the lake is deep' (see n. 6). If (unlike Bale 2008) takes the former to require some special, non-default semantic or pragmatic interpretation, one could appeal to the lack of salience of the extra ingredients required for this to explain why (31*) is not markedly better than (31).

[33]Even if one does take such stipulative uses to refute Transitivity, that position is not helpful to opponents of Comparability who are motivated by the small improvement argument, which (as we saw on page 13) requires Transitivity for its validity.

to the two-word expression 'stronger than', it involves some noncompositional use of that expression where it only orthographically appears to embed the gradable adjective 'strong,' in which case it is not really a counter-example to Comparability. Moreover, when one goes against the linguistic grain in this way, ordinary usage will tend to reassert itself, as people use other syntactic forms not covered by the original stipulation, relying on the ordinary meanings of those forms in ways that are apt to lead to all kinds of confusions. For example, philosophers will sometimes say that one theory is "much stronger than" or "only a little stronger than" another, and will be tempted to assume unreflectively that if T_3 is much stronger than T_1 and T_2 is only a little stronger, then T_3 is stronger than T_2, even though this is not guaranteed given the stipulated meaning for 'stronger than'. Such confusions are to be expected on our view that there is a general logic of gradable adjectives, since this logic would make it easy to slip into modes of reasoning that it supports. And we conjecture that, as speakers start to treat these originally stipulative expressions like gradeable adjectives in other ways, they will also start to use them in ways that conform to Comparability and the more specific patterns of inference that we have highlighted in its support.

6 Preference

This completes our case for Comparability. In this section and the next, we argue that it has two philosophically interesting consequences: the completeness of preferences, and the ubiquity of credences.

The small improvement argument made its debut not as an argument against any instance of Comparability, but as an argument against a thesis about *preference* and *indifference*:

> **Rational Preference Completeness**   Necessarily, if one is rational, then if one has some preferences about *A* and some preferences about *B*, one either prefers *A* to *B*, prefers *B* to *A*, or is indifferent between *A* and *B*.

This thesis plays a central role in the canonical axiomatic developments of decision theory. But from the beginning there has been a widespread skepticism. According to von Neumann and Morgenstern (2007, p. 630), 'It is very dubious, whether the idealization of reality which treats this postulate ['the completeness of the individual's system of preferences'] as a valid one, is appropriate or even convenient.' Aumann (1962, p. 446) agrees: 'Of all the axioms of utility theory, the completeness axiom is perhaps the most questionable. Like others of the axioms, it is inaccurate as a description of real life; but unlike them, we find it hard to accept even from the normative viewpoint.' Even authors who accept Comparability seem willing to reject Rational Preference Completeness (e.g., Broome 1991, pp. 92–93).

Though these these authors and many others have their doubts about Rational Preference Completeness, a thesis about *rational* preferences, they take for granted that the preferences of actual (possibly irrational) human beings are incomplete, i.e. that Descriptive Preference Completeness is false:

> **Descriptive Preference Completeness**   Necessarily, if one has any preferences about x and y, either one prefers x to y, prefers y to x, or is indifferent between x and y.

For example, Bales, Cohen, and Handfield (2013) assert that "incomplete preferences are ubiquitous in ordinary life." And many recent contributions to descriptive decision theory either argue that some agents have incomplete preferences, or simply take for granted that they do and go on to offer models of such agents (Bleichrodt 2007, Eliaz and Ok 2006, Galaabaatar and Karni 2013, Mandler 2004, Mandler 2005, Ok 2002). We suspect that almost everyone would reject Descriptive Preference Completeness.[34] Everyone, that is, but us.

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[34]Steele and Stefánsson (2016) raise doubts about a 'Completeness' principle they write as 'For any $A, B \in S$, either $A \preccurlyeq B$ or $B \preccurlyeq A$', where S is a set of options, and '$\preccurlyeq$ represents a weak preference relation, i.e., the relation "... is not preferred to ... "'. So, if we take them at their word, and assume the validity of de Morgan's laws, their Completeness principle actually says that it never happens that someone both prefers A to B and prefers B to A. Even opponents of Descriptive

As unpopular as Descriptive Preference Completeness may be, Comparability can be leveraged into a powerful argument in its defense. Of course, 'prefer to' is not the comparative form of any gradable expression, so our arguments for Comparability do not immediately bear on Descriptive Preference Completeness. But they can be brought to bear indirectly by appealing to certain plausible equivalences between 'prefer' and the sort of gradable items to which Comparability applies. Consider:

Comparative Preference a prefers x to y if and only if a likes x better than y.

Comparative Indifference a is indifferent between x and y if and only if a likes x and y equally well.

Both of these equivalences seem valid to us. But if they are, the validity of Descriptive Preference Completeness will follow from that of Comparability.[35]

In support of the validity of the equivalences, we can point to the fact that speeches like 'She likes vanilla more than chocolate, but she doesn't prefer vanilla to chocolate', or 'He prefers scuba diving to hang gliding although he likes hang gliding more than scuba diving' seem flatly incoherent. Likewise, 'She likes these two restaurants equally well but she's not indifferent between them' sounds bizarre: what more, we might think, does the speaker think is required for the agent to be indifferent between options? (In fact Comparative Indifference isn't really crucial to our argument. For opponents of Descriptive Preference Completeness will certainly think that the analogue of Negative Transitivity fails for preference: one can prefer x^+ to x while failing to prefer x^+ to y or y to x, and still preferring y to something or other. But Comparative Preference and Negative Transitivity jointly imply that this never happens.)

Preference Completeness might well accept this. While Steele and Stefánsson's definition of $\preceq$ is perhaps just a slip, we think it is a telling one since it shows how easy it is to slip into taking Descriptive Preference Completeness for granted even while attempting to raise objections to it.

[35]Indeed, we only need the right-to-left directions of these schemas, although the left-to-right directions seem equally plausible.

There is room to fuss about the details of the way we formulated Comparative Preference: for example, about the grammatical objects of 'prefer' and 'like' (the form we used, with 'to', can be used with noun phrases and gerunds; but we need to switch to 'than' if we want *x* and *y* to be infinitival clauses or 'that' clauses); the mood of the verbs ('would prefer' and 'would like' are more natural in some cases, and yield interpretations that are more anchored to a particular occasion as opposed to one's standing dispositions[36]); the verb and adverb on the right-hand side (instead of 'like better' we could have 'like more' or 'want more', 'desire more strongly', etc.) None of these variations are important for our argument, which requires only that there is some equivalence (or even a mere entailment) between 'prefers' and some expression in the class to which Comparability applies.

The validity of Comparative Preference can be further supported by the observation that in many languages, the natural translations of English sentences involving 'prefer' use some explicitly comparative adjective, adverb, or verb. For example, here's how you would say 'I prefer tea to coffee' in Irish:

Is fearr   liom   tae ná   caife.
is better with me tea than coffee

Mandarin:

Bǐ qǐ   kāfēi, wǒ gèng xǐhuān chá
compare coffee, I   more like   tea

Hindi:

Mujko coffee chai se   adhik pasand   hai
For me coffee tea   than more pleasing is

and Finnish:

Pidän teestä enemmän kuin kahvista
I like tea   more   than coffee

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[36]Note that despite appearances, 'would prefer' and 'would like' are not normally about the psychological state the relevant person *would* have been in under some counterfactual circumstances: cf. 'I would prefer my grave to be garlanded with peonies'.

Some of these languages do also have non-comparative words that could be used to provide less natural translations of sentences involving 'prefer'.[37] But Irish at least doesn't seem to have any word like that, while in Finnish it is a recent import. Of course, there is always the option of saying that the languages in question cannot, or cannot easily, be used to talk about preference. But this sits very oddly with the great explanatory importance ascribed to the notion of preference by the orthodox decision-theoretic tradition: it is hard to believe that some human languages have cut themselves off from something so psychologically fundamental.

One salient argument against Descriptive Preference Completeness proceeds from the identification of preferences with behavioral dispositions, along the lines of:

Preferences as Dispositions a prefers x to y if and only if a is disposed to choose x when presented with a choice between x and y.[38]

Since someone could presumably be disposed to choose x^+ when given the choice between x^+ and x without being disposed to choose x^+ when given the choice between x^+ and y or disposed to choose y when given the choice between y and x, Preferences as Dispositions is in clear tension with Descriptive Preference Completeness.[39] But even setting aside our argument for Descriptive Preference Completeness, it is quite implausible that there is any possible meaning for 'prefer' (or any other piece of psychological vocabulary in ordinary English) that is so tightly tied to behavioural dispositions. For good reasons, philosophers of mind since around 1960 have been pretty much universally agreed on the falsity of behaviorism. So it is somewhat odd that behaviorism continues to be treated as

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[37]In Mandarin you could also say 'Bǐ qǐ kāfēi, wǒ piān'ài chá'—likewise, while the natural German translation is 'Ich habe Tee lieber als Kaffee', one can also say 'Ich ziehe Tee dem Kaffee vor'.

[38]See, e.g., Savage 1954, p. 17. While few these days seem to regard Preferences as Dispositions as unequivocally true, it seems quite common to think that 'prefer' has multiple senses, one of which comes down to behaviour in something like this way (see, e.g., Hausman 2011, §1.1).

[39]It's a little hard to be fully specific here, since 'disposed' is itself a gradable adjective (Manley and Wasserman 2007), and it is unclear where one should set the threshold for its positive (non-comparative) form in evaluating claims like Preferences as Dispositions.

a live option in the literature on decision theory.[40]

Some moral philosophers (Broome 2006, Hausman 2011) and decision theorists (e.g. Joyce 1999, p. 40) understand preferences as a kind of *judgment*, endorsing something like:

> **Preferences as Judgments**   *a* prefers x to y if and only if *a* judges x to be better than y.

Given that it seems obvious that there are cases where someone (even someone rational) judges x^+ to be better than x without judging x^+ to be better than y or judging y to be better than x, Preferences as Judgments plausibly requires giving up Descriptive Preference Completeness (and perhaps also Rational Preference Completeness).[41] Of course, if one reasons in accordance with Comparability and judges x^+ to be better than x, one will judge that *either* x^+ is better than y or y is better than x; but in general, we would expect it to be possible to judge that P or Q without judging that P or judging that Q. Our case for Descriptive Preference Completeness thus counts against Preferences as Judgments and a wide array of other claims with a similar structure: for example, views that equate preferring x to y with judging x to be *more desirable* than y, *finding x to be better* than y, with x's *seeming* better to one than y, etc. But even setting aside our arguments for Descriptive Preference Completeness, the equation of preferences with judgments does not strike us as well-motivated.[42] 'I prefer vanilla to chocolate although I'm not sure it's better' certainly

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[40]The tradition of functionalism which superceded behaviourism in the philosophy of mind has provided a range of ways of making sense of the idea that there are "constitutive links" between mental states and behavioural dispositions that are more like presumptions that hold in normal cases than necessary and sufficient conditions. However, it is not obvious how one could go about arguing against Descriptive Preference Completeness from a weakening of Preferences as Dispositions along these lines.

[41]Indeed, since it is also independently plausible that one could *prefer x^+* to x without judging x^+ to be better than y or judging y to be better than x, the left-to-right direction of Preferences as Judgments is already enough to generate the tension. This direction would be accepted, for example, by R. Bradley (2017, ch. 4), who holds a "hybrid" theory on which preference is the conjunction of a judgment with a behavioural disposition.

[42]Some authors seem to think that if preferences are not behavioural dispositions they must be judgments. For example, Steele and Stefánsson (2016) equate the thesis that preferences are "mental attitudes" with the thesis that they are "considered judgments about whether an option

doesn't sound incoherent in the way that 'I like vanilla better than chocolate although I don't prefer it' does.[43] Moreover, as soon as we recognise the possibility of suspension of judgment as regards which of two options is better—as we must if the judgment theory is to motivate rejection of Descriptive Preference Completeness—we will be naturally led to think about differences in confidence about which of two options is better, and by how much; and such differences this will bring into view a whole new array of reasons to prefer one option to another *without* judging it to be better. For example, if one has middling confidence that x is much better than y, while being sure that y is at most slightly better than x, one could naturally end up preferring x to y. More generally, we find it strange to suppose that a binary cognitive attitude such as judgment should be foundational for the purposes of (normative or descriptive) decision theory, given that so much of decision theory has been concerned with the kinds of graded attitudes that become important in situations of uncertainty.

Some readers may find Descriptive Preference Completeness so implausible that they prefer to treat our argument in this section as a reductio of Comparability. But even this move would have some surprising implications. Many theorists who reject Descriptive Preference Completeness would also deny the analogous principle for transitivity of preference: while the claim that *rational* people always have transitive preferences is widely endorsed, discussions of that thesis typically treat it as obvious that at least irrational people can have intransitive preferences.[44] These theorists would insist that it is possible for an agent to prefer x to y and y to z without preferring x to z. But, as we said at the outset of this paper, Transitivity is far less controversial than Comparability. And, by

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is better or more desirable than another"; likewise, R. Bradley (2017, ch. 4) treats 'mental attitude' as interchangeable with 'judgment'. We find this conflation mysterious.

[43]We take 'I'm not sure it's better' to entail or implicate 'I don't judge that it is better'. Some may feel that 'I don't think it's better' makes the sentence sound worse, but this is plausibly because 'thinks' *neg-raises* so that 'I don't think it's better' pragmatically implies 'I think it's not better'.

[44]E.g., Resnik (1987, pp. 23–4) thinks that 'Experiments can easily determine that humans are not always able to have transitive preferences', although 'the transitivity conditions characterize the preferences of an ideally rational agent'. For the minority view that even rational people can have intransitive preferences, see Fishburn 1991.

Comparative Preference, we can similarly argue from Transitivity to the conclusion that, necessarily, everyone has transitive preferences. This suggests that readers who find Descriptive Preference Completeness too incredible to be believed would be better off trying to find a way of resisting the arguments for Comparative Preference that we gave above, rather than attempting to use it as a lever to argue against Comparability.

7 Confidence and credence

Bayesians and many others have found it fruitful to theorise about a relation between people, propositions, and real numbers: a given real number in the unit interval $[0, 1]$ can be a given person's *credence* in a given proposition, also known as their "degree of belief" in it, or its "subjective probability" for them. This technical terminology is often characterised in a way that links it to the familiar notion of *confidence*, from Savage (1954, p. 3) ('[P]robability measures the confidence that a particular individual has in the truth of a particular proposition') to Titelbaum (2019, p. 1) ('Credences are numerical degrees of confidence'). This naturally suggests two principles linking the technical use of 'credence' to the ordinary word 'confident':

> **Credence Existence**   If a is confident to some degree that P, then there exists a real number x in the unit interval that is a's credence that P.

> **Credence/Confidence**   If there exist real numbers x and y such that $x > y$ and x is a's credence that P and y is b's credence that Q, then a is more confident that P than b is that Q.[45]

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[45]Note that given Credence/Confidence, Credence Existence is equivalent to the following principle: if a is at least as confident that P as b is that Q, then there exist real numbers x and y such that $x \geq y$ and x is a's credence that P and y is b's credence that Q. The converse of this would be slightly stronger than Credence/Confidence, since it also implies that sameness of credence suffices for equal confidence. We confine our attention to the weaker version in order to allow for the possibility of fine-grained differences in confidence without differences in credence. For example, perhaps one could be less confident that a certain fair coin will not land heads every day throughout an infinite future than that it either will or will not do this, despite having credence 1 in both propositions. As Koopman (1940, p. 767) puts it: "Numerical probability gives but a

Other gradable adjectives we could use in place of 'confident' here include 'certain' and 'sure'.

While Credence/Confidence looks hard to deny, Credence Existence is quite controversial. In the large literature on "imprecise probability" (going back to Keynes 1921), it is a commonplace that, with a few isolated exceptions primarily involving logical truths and gambling setups, real human beings almost never have unique real numbers that are their credences in any propositions. For example:

> As many commentators have observed..., numerically sharp degrees of belief are psychologically unrealistic. It is rare, outside casinos, to find opinions that are anywhere near definite or univocal enough to admit of quantification. (Joyce 2010)

> But probability theory seems to impute much richer and more determinate attitudes than seems warranted. What should your rational degree of belief be that global mean surface temperature will have risen by more than four degrees by 2080? Perhaps it should be 0.75? Why not 0.75001? Why not 0.7497? ...It seems there are many events about which we can (or perhaps should) take less precise attitudes than orthodox probability requires. (S. Bradley 2019)

> [T]here are plenty of circumstances in which we manifest confidence without lending point-valued subjective probability. Indeed this looks to be more the norm than the exception in everyday life. (Sturgeon 2019, ch. 3)

Claims like these are generally treated as obvious starting points in the relevant literature, which focuses instead on the normative question whether it is ever rationally permissible (or even rationally compulsory) to have such "imprecise credence". (The dominant answer is yes on both counts.)[46]

blurred rendering of the ultimate logical relations between probability and certainty".

[46]The frequent occurrences of words like 'precise' and 'definite' in the literature (as in the above quotes from Joyce and Bradley) might suggest an interpretation on which their claims are not in

So, it is not uninteresting to observe that Comparability generates a lot of pressure to accept Credence Existence. The pressure arises from the undisputed *possibility* of each rational-valued credence:

> **Credence Possibility**   For every rational $x \in [0,1]$, it is possible that for some a and P, x is a's credence that P.

For example, it seems quite apt to characterise someone who is absolutely certain that a ball will be fairly chosen from an urn containing m red balls and n green balls, and takes this into account in the usual way, as having credence $m/(m+n)$ in the proposition that a red ball will be chosen. None of our authors would dispute this: their objection is to the extension of the same numerical measure to cases in which the available evidence is sparser or more disparate. But by appealing to Comparability together with the seemingly unproblematic Credence/Confidence, we can use these uncontroversial cases of credence as a yardstick to assign a real-valued credence to any *any* pair of a person and a proposition in which they are confident to some degree.

To spell this out, let us introduce the following definitions:

> x is a [lower/upper] bound on a's credence that P := a is [more/less] confident that P than anyone who had credence x in some proposition could be in that proposition.

conflict with Credence Existence. Perhaps some authors in the tradition are merely saying that the word 'credence' (and 'degree of belief' and 'subjective probability') are vague, in such a way that there is typically no real number to which 'x's degree of belief in p' *definitely* applies, just as there is no natural number to which 'the least number of hairs one could have without being bald' definitely applies. This claim is consistent with Credence Existence, and should be uncontroversial: outside of logic, mathematics, and perhaps certain parts of physics, almost all words are rather vague, and it would be naïve to suppose that the technical term 'credence' was introduced in a way that avoided it. However, some authors in the tradition are quite explicit in rejecting Credence Existence, such as Sturgeon in the above quote. Moreover, many authors develop further theories about "imprecise probability" which suggest that they were not just making the banal point about vagueness. For example, they describe alternative abstract objects, such as *sets* of functions from propositions to real numbers ('representors': van Fraassen 1990), which are supposed to be able to represent certain states of opinion that are not aptly represented by functions from propositions to real numbers. But if the only problem with the latter was one of vagueness, it is unclear one would take this to be progress: it would be just as naïve to assume that 'representor' was introduced in such a way as to make predicates involving it, like 'the set of values to which the members of a's representor map proposition p', have *definite* application beyond a few special cases.

Using these definitions, we can state the following very plausible sufficient condition for having a credence:

> **Sufficiency** If every non-negative rational number less than x is a lower bound on a's credence that P, and every rational number greater than x and no greater than 1 is an upper bound on a's credence that P, then a's credence that P is x.

This is motivated by the idea that if a real number x satisfies the antecedent, it does everything we could sensibly demand of a "numerical measure" of a's degree of confidence that P.

But given Comparability, Credence Possibility, and Credence/Confidence, we can show that necessarily, whenever A is confident to some degree that P, there is some x which satisfies the antecedent of Sufficiency.[47] These three principles together with Sufficiency thus entail Credence Existence.

Given Comparability, this argument looks hard to resist. None of the worries about real-valued credence suggests any reason to worry about Credence Possibility or Credence/-

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[47]First, we show that $y < y'$ whenever y and y' are rational and respectively a lower and an upper bound on a's credence that P. Suppose they are. It can't be that $y = y'$, since by Credence Possibility there is a possible world w at which some person b has credence y in some Q, which would mean that b at w was both more and less confident that Q than a actually is that P: this is impossible by Comparative/Equative. Also, it can't be that $y > y'$. For by Credence Possibility there are w, w', b, b', Q, Q' such that at w, b has credence y that Q and at w', b' has credence y' that Q'. By Credence/Confidence, b' is more confident at w that Q' than b is at w' that Q. Since y is a lower bound on a's credence that P, a is actually more confident that P than b is at w that Q. And since y' is an upper bound, b' is, at w', more confident that Q' than a actually is that P. By Transitivity, it follows that a is more confident that P than a is that P, which is inconsistent given Comparative/Equative.

Second, we show that for any rational y and y' such that $0 \leq y < y' \leq 1$, either y is a lower bound on a's credence that P or y' is an upper bound. For this to be false, there would have to be some w, w', b, b', Q, and Q' such that b's credence at w that Q is y; b''s credence at w' that Q' is y'; a is not actually more confident that P than b is at w that Q; and a is not actually less confident that P than b' is at w' that Q'. Given Negative Transitivity and the fact that a is confident to some degree that P, it follows that b' is not more confident at w' that Q' than b is at w that Q, contradicting Credence/Confidence.

It follows from these two results that the least upper bound of the set of rational lower bounds of a's credence in P is identical to the greatest lower bound of the set of rational upper bounds of a's credence in P. This number thus satisfies the antecedent of Sufficiency.

Confidence.[48] And denying Sufficiency seems simply to change the subject from credence understood as a measure of confidence to something else. Proponents of this option urgently owe us some alternative positive conception of what they take having a credence in a proposition to require. And even if they answer this challenge, we can still just use the antecedent of Sufficiency to define some new technical term, say 'x is a's confidence-number that P', and continue to theorise using the analogues of Credence Existence and Credence/Confidence for this.

The idea that Credence Existence stands or falls with Comparability for adjectives like 'confident' is not at all new. Opposition to real-valued credence has often been inferred from, or used to motivate, failures of completeness for so-called "comparative probability" relations, which are typically explained using gradable adjectives. A central source for the tradition is Keynes (1921), whose explicit denial of comparability with regard to 'probable' we have already noted in §2. Koopman (1940), an early proponent of

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[48]Note that the argument would still work if we weakened Credence/Confidence by restricting it to propositions about balls in urns and agents who assign credences to those propositions in the canonical way by learning about the chances, so long as we strengthen Sufficiency accordingly (by inserting the same restriction in the definition of 'lower bound' and 'upper bound'); the strengthened version seems just as plausible as the original.

The one author we are aware of who has raised doubts about Credence/Confidence is Williamson (2019), who argues that "The normal use of the word 'confident' does not fit the ideology of credences". His idea is that even if, for example, I have moderately higher credence that horse 7 will win the race than that any other horse in the race will win, 'I have no confidence that horse 7 will win' might still be true. He takes this to entail that 'I am more confident that horse 7 will win than that horse 6 will win' is false (despite my higher credence). In response: this 'no confidence' seems analogous to other cases where we treat small quantities as if they were zero, e.g. 'I have no interest in your explanations', 'There is no prospect of his being elected', etc. On the 'chunking' account of this kind of phenomeon suggested in section 4, Williamson's claims are correct in some coarse-grained contexts, but this does not count against the truth of Credence/Confidence in fine-grained contexts (which are natural for comparatives: consider 'If a's height exceeds b's height by one micron, a is taller than b'). Williamson supports his thesis by pointing out that 'Which horse are you most confident will win?' feels to have a false presupposition in the imagined circumstances. But for many adjectives it can seem a bit odd to make comparisons between objects near the bottom of the scale, even when such comparisons have a salient true reading. 'Ordinary headaches are quite unpleasant, but at least they are more pleasant than migraines' has a salient true reading, even though 'Ordinary headaches are not at all pleasant' seems fine, and 'Which is more pleasant, ordinary headaches or migraines?' tends to suggest the the questioner is under a serious misapprehension.

the reduction of numerical probability to something comparative, also takes for granted that his chosen comparative relation gives rise to incomparability.[49] For explicit rejection of Comparability for 'confident' (or its consequences such as Trichotomy), see Hawthorne 2009, p. 59, Schoenfield 2012, and Konek 2019.[50] R. Bradley (2017) says similar things using 'credible' instead of 'confident'. Kaplan (1996) develops an interesting argument against Comparability for 'confident' (and thus against Credence Existence) from a premise about preference-incompleteness—specifically, that even the preferences of an ideally rational person who cared only for money could be incomplete.[51] These authors generally focus on the claim that the relevant failures of Comparability can arise even for *rational* people, while taking it for granted such failures arise for ordinary, imperfectly rational people.[52]

Credence Existence places a important constraint on theorising about credence, confidence, and probability, ruling out several generalisations that might otherwise seem attractive, and which perhaps bear some of the blame for the widespread rejection of Credence Existence. For example, the time-honoured idea that credences are correlated with betting dispositions does not fit comfortably with Credence Existence, since one can plausibly be confident to some degree that P while being firmly disposed to refuse all

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[49]Koopman glosses his primitive as "p_1 given p_2 is no more probable than p_3 given p_4", so that (assuming the validity of double-negation elimination) the "incomparability" he endorses entails, surprisingly, that it can be simultaneously true that p_1 given p_2 is more probable than p_3 given p_4 and that p_3 given p_4 is more probable than p_1 given p_2. While he would have done better to write 'at least as' rather than 'no more than', the slip illustrates how hard it is not to slip into assuming Comparability even when one explicitly rejects it.

[50]Sturgeon (2019, §3.2) is more cautious, suggesting that 'more confident than' admits multiple 'most-natural generalizations' to states of confidence that do not involve credences, and that counterexamples to Trichotomy arise on some, but perhaps not all, ways of resolving this indeterminacy.

[51]Our argument for Descriptive Preference Completeness will clearly undermine this premise, although even setting that argument aside, we don't find Kaplan's premise compelling enough to threaten Comparability.

[52]Hawthorne (2009) justifies his rejection of Comparability on the grounds that 'Real agents may well be unable to assess their comparative confidence in some pairs of statements'. This could be taken as an appeal to some kind of "luminosity" thesis, according to which any facts about comparative confidence must be within the capacity of the agent to "assess": see Williamson 2000 for general reasons to be suspicious of such theses.

offers to buy and sell bets on whether P.[53] Another view that is in tension with Credence Existence takes credences to be or entail *probability judgments*:

> **Credences as Judgments** a has credence x that P only if a judges that the probability that P is x.[54]

Whatever sense of 'probability' might be in question, Credences as Judgments is prima facie in tension with Credence Existence, since it seems rare for people to make such specific judgments about the value of any real-valued quantity whatsoever.[55] The tension remains when we replace 'judge' in these biconditionals with other attitude-type expressions like 'believe', 'take', 'regard', 'find'. And it applies whether we are thinking of the principles as elucidating credence in terms of some antecedently understood sense of 'probability', or as elucidating some sense of 'probability' in terms of an antecedently understood notion of credence.[56]

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[53]de Finetti (1937) defines credences in terms of betting dispositions. Walley (1991) endorses a view on which betting dispositions are necessary but not sufficient for credences, which would be sufficient to generate the tension with Credence Existence.

[54]Talk of "probability judgments" is pervasive in the literature on "imprecise probability": e.g. Levi (1974) says his "chief concern is to argue that rational men should sometimes avoid adopting numerically precise probability judgments". It is unclear to us whether authors like Levi are implicitly accepting Credences as Judgments, or are simply not talking about credence in our sense.

[55]This principle has an analogue for comparative confidence:

> **Comparative Confidence as Judgment** a is more confident that P than that Q if and only if a judges that it is more probable that P than that Q.

Assumptions structurally like this are frequent in the literature on comparative confidence. For example, Konek 2019 uses 'opinion' and 'plausible' instead of 'judge' and 'probable': 'Often I simply lack an opinion about which of two propositions is more plausible. I am not more confident that copper will be greater than £2/lb in 2025. . . than I am that nickel will be greater than £3/lb in 2025. . . . Neither am I less confident. . . , nor equally confident. I simply lack an opinion on the matter.' Cariani, Santorio, and Wellwood (MS) defend the surprising combination of Comparability with a variant of Comparative Confidence as Judgment using 'believe' instead of 'judge' and 'likely' instead of 'probable'.

[56]For the former attitude, see Holton 2014. The latter way of thinking about claims like Credences as Judgments, which is pervasive in the literature on credence, can be seen as a kind of *expressivism* about (the relevant sense of) 'probability', by analogy with the view that equates judging that lying is wrong with disapproving of lying, and takes this as an elucidation of 'wrong' rather than of 'disapprove'. For recent defences, and compositional semantic implementations, see Cariani,

8 Conclusion

Many questions of philosophical interest—questions about goodness, probability, confidence, preference, inequality, beauty, the strength of reasons, and so on—are posed using familiar, natural-language gradable adjectives and other comparative expressions. Our arguments, if they work, show that such expressions must satisfy Comparability. They do not, of course, show that relations that violate Comparability are uninteresting or theoretically unimportant. If you find our arguments convincing but were antecedently disposed to reject Comparability for some particular philosophically important term, you might be inclined to say "So much the worse for natural language". If you have in mind some specific definition of a technical use of the relevant term which could in principle be used to simply eliminate all uses of it, well and good: just be sure that your choice of this familiar-sounding way of expressing yourself doesn't mislead readers by encouraging them to conflate your technical sense with the ordinary sense. (You can stipulate a sense of 'knowledge' that means 'true belief' if you like, but doing so certainly risks confusion.) If you don't have such a definition up your sleeve, you might be tempted to continue theorizing anyway: after all, novel theoretical terms can often be introduced in science without the help of explicit definitions (Lewis 1970), and perhaps philosophy shouldn't be any different. But when it comes to particular gradable adjectives such as 'good', 'confident', 'probable', and other gradable expressions of interest to ethics, epistemology, and other areas of philosophy, the difficulty of this path should not be underestimated. Once the ordinary meanings of the expressions are set aside, it is far from clear that the theoretical role that remains will be anywhere near rich enough to pin down their meanings, and that

Santorio, and Wellwood MS, Moss 2016, Rothschild 2012, Swanson 2016, Yalcin 2012. Most focus on 'probably', but Swanson explicitly considers operators of the form 'there is an $x\%$ chance that'. Rothschild and Moss appeal to states like *being such that every member of one's representor maps P to some number greater than 0.5* rather than *having a credence in P that is greater than 0.5*; this avoids the worry that the account requires us to hold implausibly specific judgments about probability, at the cost of making it far less clear that we have any independent grip on the relevant mental states other than as patterns of judgment about probability.

we can rely on intuitive judgments about their application as freely as some philosophers do.

Be that as it may, the original meanings of the relevant expressions are *also* philosophically interesting. And there is a big disadvantage of hijacking the words and investing them with some technical sense: one will then be left without a natural way to discuss important and philosophically central questions about goodness, probability, confidence, preference, inequality, beauty, the strength of reasons, and other such topics. Those, at any rate, are the questions that most interest us, and Comparability tells us something important about them.

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