Assessing offensive/defensive strategies in a football match using DEA

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Abstract

Football teams play every match following two basic strategies: defensive and offensive. The length of time in which the team plays under each these two modes will depend basically on two factors: who the rival team is and the technical play developed by the teams during the match. The coach is responsible for devising a proper offensive/defensive strategy aiming to maximize the goals scored and minimize the goals against. Once the match is over, an analysis of the team performance is necessary to determine which aspects of the offensive/defensive strategies used during the match failed and which succeeded. Although there are some studies in the literature that assess the offensive and defensive efficiencies of football teams in a season, none of them deals with the performance in a single match. In this paper a multiple modes of functioning Data Envelopment Analysis (DEA) model is proposed. In addition to computing efficiency scores and goals targets, this methodology is able to determine the percentage of time in which the team should have played in defensive and offensive modes, in order to maximize those efficiencies. The procedure has been applied to the matches played in the Spanish First Division league during 2014/2015 season.

Keywords: performance assessment; offensive/defensive strategies; DEA; multiple modes of functioning

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1. Introduction

Data Envelopment Analysis (DEA) is a well-known non-parametric methodology aimed at benchmarking similar operating units. These units are assumed to consume inputs and produce outputs. DEA allows determining whether the observed level of outputs could have been achieved with a lower level of inputs or, alternatively, whether a larger amount of output could have been produced with the observed input consumption. DEA has been widely used in multiple applications related to sports (Li et al., 2015; Wu et al. 2010; Lozano et al. 2003; Moreno & Lozano, 2015; etc.) including football (e.g. Villa and Lozano 2016).

In this paper, however, we are not interested in assessing the performance of teams along a season as most DEA studies do. We focus our attention on a specific match taking into account the team data as well as the data from the opposing team. Analyzing the way a team plays is important to understand its success or its failure in a match. Basically, two strategies can be deployed during a match: the offensive game (when the team has the possession of the ball), and the defensive game (when it does not). Thus, we aim at assessing the efficiency of the offensive/defensive strategies of each team. Moreover, we propose a novel DEA approach recently proposed in Lozano and Villa (2016) and Lozano et al. (2017) for systems with multiples modes of functioning (MMF). The working of that type of systems results from the interplay of different subprocesses. The key feature of MMF DEA, one that distinguishes it from network DEA, for example, is the fact that the subprocesses do not run parallel- Instead, the operation of the system alternates between the different subprocesses, which thus run on a time-sharing basis.

In this paper, the offensive and defensive strategies of the game played by a team are considered its two distinct modes of functioning (MF). In each of them, the team uses certain inputs to produce outputs. Thus, for example, goals scored is the output of the offensive strategy while goals against (a variable that the team is interested in reducing s much as possible) is the input of the defensive MF. An important variable in MMF DEA is the time allocation between the different MF. That corresponds, in this application, to the fraction of time the team is in possession of the ball. Not only the proposed approach can compute efficient targets for each strategy (and corresponding efficiency scores) but also it can determine the optimal time allocation. Thus, this analysis provides a more detailed and rich analysis that the conventional single-process, black box DEA approach with which it is also compared in this paper.

The structure of the paper is the following. Section 2 carries out a literature review of DEA applied to football teams. In Section 3, we formulate and explain the proposed MMF DEA models. In Section 4, the proposed approach is illustrated on the different games played by the 20 football teams of the Spanish First Division in the season 2014/15. Finally, Section 5 summarizes and concludes.

2. Literature review

Many studies point to a direct relation between the success of a team and the strategic mix used during a certain number of games played. Some empirical studies focused on the success and failure of football teams are: Hughes et al. (1988) examine patterns of play for successful (semifinals) and unsuccessful (eliminated) teams in the 1986 World Cup finals, concluding that successful teams played significantly more touches of the ballper possession than unsuccessful teams. Hughes and Frank (2005) study the relationship between number of passes and goals in the 1990 and 1994 World Cup finals. Tenga et al. (2010) deduced that counter-attacks were more effective than elaborate attacks when playing against an imbalanced defence, using a sample of 163 matches of Norwegian professional football league. Lago and Martín (2007) used a sample of 170 matches in the First Division of Spanish League to study the relationship between winning, drawing or losing and the percentage of the possession of the ball. Taylor et al. (2008) focused their study on 40 games of a professional English team concluding that at winning it performed more interception, clearance and aerial challenge and fewer crosses, passes and dribbles, while atlosingit made more crosses, dribbles and passes andfewer clearances and interceptions.

As regards the DEA methodology used in this paper, a number of papers have studied the performance of football teams in order to assess their efficiency. Thus, Haas et al. (2003a), was the first study in measuring the efficiency of football teams using DEA. Since then, many other studies have addressed this issue in different perspectives. Espitia-Escuer and García-Cebrián (2004; 2006; 2008 and 2010) studied the Spanish football teams using different approaches (CCR-I, CCR-O, BCC-I, BCC-O models). Guzmán and Morrow (2007) used the directors' remuneration of English Premier League teams in order to explain the sportive results measured through points won and turnover. Sala et al. (2009) used Windows Analysis to measure the efficiencies evaluated from 2000/01 to 2007/08 seasons. Recently, Zambom-Ferraresi et al. (2017) analyze the technical, pure technical and scale efficiencies of the 32 Europe's top international football competition at club level through 10 sports seasons (2003/04 to 2013/14). They use as economical output the coefficients applied by the UEFA from UCL revenue distribution.

To group all these DEA studies we have followed the approach in García-Sánchez (2007) which considers three categories depending on how the analysis has been carried out empirically: considering economic variables, using the available precise statistical data, and taking into account the emotional aspects. Table 1 shows the variables used by the different DEA references divided into these three categories. Thus, 'operating cost', 'team payroll', 'coach salary' and 'season total revenues' are examples of economic variables most often used by the authors; 'number of spectators per match' is the most used in the emotional variables category; finally, the empirical category includes variables such as 'number of points', 'goals scored', 'shots at goal', 'possession', etc. By this classification the approach proposed in this paper mainly uses empirical variables plus the team and opposing team budgets as additional economic-type variables.

======================== Table 1 (about here) ====================

Note that García-Sánchez (2007) was the first who measure explicitly the offensive and defensive efficiencies in order to determine the social effectiveness of a team during a season. Later, Boscá et al. (2009) analyzed the offensive and defensive efficiencies in the Italian and Spanish leagues from season 2000/2001 to 2002/2003. To do so, they used the average values of goals scored and conceded, shots on/at goal, attacking plays, centre plays/in area, attacks in area and possession of the ball collected from each season.

In addition, most previous DEA applications study the performance of teams in a season. However, we believe that once a givenmatch is over, an analysis of the game played is essential in order to identify the strengths of one's own team, and its weaknesses, with the aim of improving for the next match. To the best of our knowledge only Villa and Lozano (2016) have analyzed the efficiency of football teams in a single match. Specifically, they used a network DEA model with two parallel processes: the home and the away team. However, they did not study the offensive/defensive strategies used by each team.

In this paper, we will consider that a football team can be defined as a system with two modes of functioning (MF): offence and defence. When the offence mode is used, the team cannot operate in the defence mode, and vice versa. The possession of the ball determines the percentage of the total time that the team functions in the offence mode during the match. Also, the inputs and outputs involved when the team attacks are different from the inputs and outputs to be considered when the team defends. To deal with this situation, we propose using the multiple modes of functioning DEA approach proposed by Lozano and Villa (2016). This allows measuring the offensive and defensive efficiencies of a team in a given match and computing targets for the number of goals scored and against. Furthermore, the proposed approach also determines the optimal time allocation between the two strategies.

3. Proposed DEA approach

Recently Lozano and Villa (2016) have proposed a DEA approach to model situations in which the system can operate in one of multiple modes of functioning (MMF). In this section we present the application of the MMF DEA approach to the case of a football team playing a given match. Figure 1 shows a football team considered as a black box (BB) perspective as well as distinguishing its two MFs. Note that the inputs and outputs of the BB approach are the sum of those of the offence and defence modes.

========================= Figure 1 (about here) ====================

Inputs in the offence mode are non-discretionary since the amount of each one is not only determined by the team, but by the rival. Similarly,the outputs of the defence mode have to be considered as non-discretionary for the same reasons.In the offence MF the team tries to convert as many shots at goals, corners, penalties, etc. into goals scored while, in the defence mode, it tries to prevent the shots at goals against, corners against, penalties against, etc. to be converted into goals against. Note that the inputs and outputs of the whole system are the sum of the inputs and outputs of its two MFs and that, except for the budget variables, the set of inputs and outputs of the two MFs are disjoint.The budget of the team has been considered as a non-discretionary input in both MFs, since it contributes positively to their performance but it is considered fixedduring a season. Similarly, the budget of the rival is considered as a non-discretionary output in both MFs. These budget variables have been considered because there seems to be a correlation between the budget of a team and the average number of goals scored and average number of goals against during the season, with wealthier teams scoring more and receiving fewer goals against than more modest teams. Thus, Figure 2 shows those variables for the 20 teams of the Spanish First Division for the season 2014/2015.

========================= Figure 2 (about here) ====================

Before formulating the BB and MMF DEA models let us introduce the required notation:

I^{off} set of non-budget offensive inputs: shots at goal, corners, penalties and steals.

O^{def} set of non-budget defensive outputs: shots at goal against, corners against, penalties against and turnovers.

x_{ij}^{off} amount of input $i \in I^{off}$ consumed in offence mode by DMU j.

y_{kj}^{def} amount of output $k \in O^{def}$ produced in defence mode by DMU j.

pos_j ball possession (i.e. fraction of time in offence mode) for DMU j

gs_j number of goals scored by DMU j

ga_j number of goals against for DMU j

b_j budget of DMU j.

br_j budget of the rival team of DMU j.

3.1. BB DEA model

Conventional DEA(e.g. Cooper et al. 2000) does not distinguish modes of functioning. Hence it cannot take into account that the team can only play in either offence or defence mode. Thus, the BB DEA model represents the traditional perspective of assessing the efficiency considering the game played by a team in a football match as a black box. The key feature of the BB DEA model is that, as it is customary in conventional DEA, a single set of intensity variables is used to project the DMU. Thus

Let:

$\left(\lambda_1, \lambda_2, ..., \lambda_n\right)$ intensity variables

sgs slack for goals scored by DMU 0

sga slack for goals against DMU 0

<u>BB</u>

$$\text{Max} \quad sgs + sga \tag{1}$$

s.t.

$$\sum_j \lambda_j x_{ij}^{off} \le x_{i0}^{off} \quad \forall i \in I^{off} \tag{2}$$

$$\sum_j \lambda_j b_j \le b_0 \tag{3}$$

$$\sum_j \lambda_j ga_j = ga_0 - sga \tag{4}$$

$$\sum_j \lambda_j gs_j = gs_0 + sgs \tag{5}$$

$$\sum_j \lambda_j y_{kj}^{def} \ge y_{k0}^{def} \quad \forall k \in O^{def} \tag{6}$$

$$\sum_j \lambda_j br_j \ge br_0 \tag{7}$$

$$\sum_j \lambda_j = 1 \tag{8}$$

$$\lambda_j \ge 0 \quad \forall j \ sga, sgs \ \text{ integer} \tag{9}$$

The objective function of the above model corresponds to a non-oriented, Variable Returns to Scale (VRS), additive DEA model. Note that the objective function tries to increase goals scored and reduce goals against at the same time. Improving on any of these dimensions contribute to the overall objective of winning the match. Note that the non-discretionary inputs and outputs do not use slack variables and, hence, those inputs and outputs are not included in the objective function. Finally, since the goals scored and goals against targets should be integer variables, the corresponding slacks are integers (Lozano and Villa, 2006; Kuosmanen and Kazemi-Matin, 2009; Kazemi-Matin and Kuosmanen, 2009).

3.2. MMF1DEA model

As we said before, the proposed MMF DEA approach distinguishes two alternative MF when a team is playing: the offensive and the defensive game. Each MF has its own set of intensity variables, which allows computing itstarget operation point within its own mode-specific technology.Such MF target points are computed as linear combinations of the corresponding MF of the observed DMUs. This is done first computing the input consumption rates and output production rates of each DMU and then using the corresponding intensity variables to combine them. The input consumption rate represents the input consumption per unit time. Since the time unit considered is the duration of the whole game, the input consumption rate of a DMU corresponds to the input consumption if the given MF has been played by that DMU all the time. Similarly, the output production rate of a DMU corresponds to the amount of output that the DMU had obtained if it had played that MF all the time.For each MF, the intensity variables represent the fraction of time that the DMU 0 being assessed should replicate the corresponding MF operation point of the observed DMUs. Same as it happens in conventional DEA, in the optimal solution only efficient DMUs will have non-zero intensity variables. In other words, the MF targets can only result from linearly combining DMUs that are efficient and thus define the facet of the MF efficient frontier on which the MF target lies.

Let:

$\left(\lambda_1^{off},\lambda_2^{off},...,\lambda_n^{off}\right)$ intensity variables for offence mode

$\left(\lambda_1^{def},\lambda_2^{def},...,\lambda_n^{def}\right)$ intensity variables for defence mode

<u>MMF1</u>

$$\text{Max} \quad sgs + sga \tag{10}$$

s.t.

$$\sum_j \lambda_j^{off} \frac{x_{ij}^{off}}{pos_j} \leq x_{i0}^{off} \quad \forall i \in I^{off} \tag{11}$$

$$\sum_j \lambda_j^{off} \frac{b_j}{pos_j} \leq b_0 \tag{12}$$

$$\sum_j \lambda_j^{def} \frac{ga_j}{(1-pos_j)} = ga_0 - sga \tag{13}$$

$$\sum_j \lambda_j^{def} \frac{b_j}{(1-pos_j)} \leq b_0 \tag{14}$$

$$\sum_j \lambda_j^{off} \frac{gs_j}{pos_j} = gs_0 + sgs \tag{15}$$

$$\sum_j \lambda_j^{off} \frac{br_j}{pos_j} \geq br_0 \tag{16}$$

$$\sum_j \lambda_j^{def} \frac{y_{kj}^{def}}{(1-pos_j)} \geq y_{k0}^{def} \quad \forall k \in O^{def} \tag{17}$$

$$\sum_j \lambda_j^{def} \frac{br_j}{(1-pos_j)} \geq br_0 \tag{18}$$

$$\sum_j \lambda_j^{off} = pos_0 \tag{19}$$

$$\sum_j \lambda_j^{def} = 1 - pos_0 \tag{20}$$

$$\lambda_j^{off}, \lambda_j^{def} \geq 0 \quad \forall j \; sga, sgs \quad \text{integer} \tag{21}$$

Note that, although MMF1 has the same objective function as BB, the input and output constraints (11)-(18) are different to those of the BB model in two respects. One is that two different sets of intensity variables are used, one for each MF. The other is that, since the system can only operate in one of the two MF at any time and the fraction of time each MF is used is given by the observed possession percentage, the MF operation points result from operating a given fraction of time (given by the intensity variables) as the different observed DMUs. The MF target inputs and outputs are computed aggregating the inputs consumed and outputs produced during those fractions of time. Note that the overall sum of all the intensity variables is unity, meaning that the sum of the length of time that the team plays in offence and in defence modes is equal to the total match duration.

As indicated above, a key feature of theMMF1 DEA model is that the time allocation, i.e. the time that each MF is used, is fixed to the observed valueas per equations (19) and (20). This constraint will be relaxed in the alternative MMF DEA model presented below.

3.3. MMF2DEA model

In the MMF2DEA model,as in the previous model, the intensity variables determine, for each MF, the fraction of time that the team should play as each of the observed DMUs. The difference with respect to MMF1 is that in MMF2 the model is free to compute the most efficient time allocation, i.e. the fraction of time for both MFs that leads to the largest potential improvement in terms of increase in goals scored and decrease in goals against.

Let

α^{off} fraction of time that DMU 0 should be in offence mode

α^{def} fraction of time that DMU 0 should be in defence mode

TheMMF2 DEA model coincides with theMMF1 model just replacing constraints (19) and (20) with the following:

$$\sum_{j} \lambda_j^{\text{off}} = \alpha^{\text{off}} \tag{22}$$

$$\sum_j \lambda_j^{def} = \alpha^{def} \tag{23}$$

$$\alpha^{off} + \alpha^{def} = 1 \tag{24}$$

$$\alpha^{off}, \alpha^{def} \geq 0 \tag{25}$$

Note that since any feasible solution in MMF1 is also feasible in MMF2 (but not the opposite) the optimal solution of MMF2 involves a larger (or at least equal) value of the objective function, which is the sum of the increase in goals scored and reduction in goals against.

4. Illustration of proposed approach

In this section the application of the proposed approach to the performances of the teams of the Spanish First Division in each of the matches they played during the 2014/2015 season is presented. Tables 1 and 2 show the average value of the inputs and outputs of each MF observed for each team in the different matches played during that season.

========================= Table 2 (about here) ====================

========================= Table3 (about here) ====================

The three DEA models presented in Section 2 were solved for the 760 DMUs (corresponding to each of the 20 teams playing 38 matchesduring the season). Figure 3shows the absolute frequency counts for the pairwise comparisons of the optimal value of the objective function computed by the three models. For points above the diagonal line, the optimal objective function of the approach corresponding to the Y-axis islarger than the optimal objective function value of the approach corresponding to the X-axis. The opposite occurs for points below the diagonal. Note that a larger optimal objective function value means more inefficiency identified by the corresponding model. As expected, the optimal objective function values for MMF1 are always less than (or equal to) those of MMF2. As regards BB, its optimal objective function value is larger than that of MMF1 a few times, and larger than that of MMF2 in just two cases. It follows that MMF2 has more discriminant power than MMF1 and the latter more discriminant power than BB.

========================= Figure 3 (about here) ====================

The objective function of the proposed models provides scores that measure inefficiency and are not normalized. If normalized efficiency scores are desired, these can be computed, for any of the three approaches (BB, MMF1 and MMF2) as

$$\theta_0 = \frac{1}{1 + sgs^* + sga^*} \tag{26}$$

$$\theta_{0,off} = \frac{0.5}{0.5 + sgs^*} \tag{27}$$

$$\theta_{0,def} = \frac{0.5}{0.5 + sga^*} \tag{28}$$

It is easy to check that the overall system efficiency θ_0 is the harmonic mean of offensive ($\theta_{0,off}$) and defensive ($\theta_{0,def}$) efficiencies, i.e.

$$\frac{1}{\theta_{0,off}} + \frac{1}{\theta_{0,def}} = \frac{1}{0.5}\left(\frac{1 + sgs^* + sga^*}{1}\right) = \frac{2}{\theta_0} \quad \Leftrightarrow \quad \theta_0 = \frac{2}{\dfrac{1}{\theta_{0,off}} + \dfrac{1}{\theta_{0,def}}} \tag{29}$$

Table 3 shows,for each team, the average value of the overall, offensive and efficiencies (27)-(29), labelled as $\bar{\theta}$, $\bar{\theta}_{off}$ and $\bar{\theta}_{def}$ respectively, and the average value of the targets of goals scored ($gs_0 + sgs^*$) and goals against ($ga_0 - sga^*$) computed by each of thethree approachesconsidered. It can be seen that, generally, $\bar{\theta}^{BB} > \bar{\theta}^{MMF1} > \bar{\theta}^{MMF2}$, $\bar{\theta}_{off}^{BB} > \bar{\theta}_{off}^{MMF1} > \bar{\theta}_{off}^{MMF2}$ and $\bar{\theta}_{def}^{BB} > \bar{\theta}_{def}^{MMF1} > \bar{\theta}_{def}^{MMF2}$ which suggests, as mentioned above, that MMF2 is the approach with the greatest discriminant power. Note also that, for most teams, its defensive efficiency is higher than its offensive efficiency. This is more frequent for MMF2, in which it occurs for 17 out of the 20 teams.

========================= Table 4 (about here) ====================

With respect to the average target values, all teams present a higher value of goals scored than goals against for the three approaches. The range of values of the average target values of goals scored is larger (e.g. between 1.63 and 5.26 for MMF2) than the corresponding range for the target goals against (e.g. between 0.0 and 0.63 for MMF2). Note also how the targets of the top teams (such as Barcelona and R. Madrid) are more demanding, especially for goals scored. Finally, through its ability to optimize the time allocation, MMF2 may sometimes be rather optimistic, especially as regards the target goals against, which is often close to zero.

Following the suggestion of one of the reviewers we have tested whether the obtained results were robust with respect to deleting the matches played by the two top teams, Real Madrid and Barcelona. Tables 5 and 6 show, respectively, the average and the Pearson correlation of the efficiency scores of the different DEA models for the complete dataset and for the dataset without Real Madrid and Barcelona. As it can be seen from those tables, the results obtained in both cases are very similar. This must be due to the fact that the Budget variable, which is the one that sets these two teams apart from the rest, is considered non-discretionary and, therefore, the proposed approach benchmarks the small teams against similar small teams.

========================= Tables 5 and 6 (about here) ====================

Figure 4 shows the value of the optimal time allocation α^{off} (i.e. the fraction of the time in a match that the team should have played in an offensive mode) computed by MMF2 versus the actual, observed possession of the ball for each of the 760 DMUs. The points with $\alpha^{off} > pos_0$ (above the diagonal line) correspond to cases in which teams should play in the offensive mode longer, while for points below the diagonal line (which occurs more often) the opposite happens (i.e. the team should play longer more in a defensive mode). The symbol distinguishes whether the DMU won, drew or lost the corresponding match. Note that the points above and farthest from the diagonal tend to be wins. Those points correspond to the common situation in which a winning team plays defensive mode (trying to maintain its advantage and letting the initiative to the rival) contrary to the more offensive game suggested by the MMF2 model.Analogously, some points below and farthest from the diagonal line correspond to teams that lost or drew the match. Those points corresponds to situations in which a team is losing or drawing and, trying to improve that score, the MMF2 model suggests a more offensive game than the one actually played.

======================== Figure 4 (about here) ====================

Figure 4 also shows that, according to the adage “the best defence is a good offence”, it may seem counterproductive to increase the share of the defence MF. In defensive mode you cannot make goals (and so you cannot increase your output), while you are at risk of receiving goals (increasing your inputs). Recall that the MMF DEA objective function involves two slacks, one for goals scored and the other for goals against. Increasing the possession of the ball aims at increasing the goals scored, which would impact positively on the objective function. However, by playing an efficient defence game, a team may reduce its goals against, especially if the observed DMU had many goals against, perhaps because it played too much offense, disregarding its defence. In that case, more defence can be beneficial for the objective function.

5. Summary and conclusions

In this paper, a new MMF DEA approach is used to assess the offensive/defensive strategies of every single match played by a team. To do so, the team has been modelled as a system that works under two modes of functioning: the offence and the defence mode. Depending on whether the time allocated to each MF is respected or optimized, two different models (labelled MMF1 and MMF2, respectively) are proposed. The proposed approach can compute MF efficiency scores as well as MF targets and optimal MF time allocation. For comparison, the conventional BB DEA model has also been considered. It is important to note that the main contribution of this article to the existing football DEA literature is this new perspective that seeks to determine the optimal fractions of time for both strategies of the game (defence and offense) as well as target improvements in goals scored and reductions in the goals against which, respectively, measure the inefficiencies in the effectiveness of these two strategies (MFs).

The proposed approach has been applied to the teams in the Spanish First Division for the matches they played during the 2014/2015 season.The results obtained by the proposed MMF DEA models have been compared with the BBDEA model it has been confirmed that the proposed approach has more discriminant power than conventional DEA and makes better use of the available information on how the real system works. Overall and mode-specific efficiency scores have been computed for each team in each match together with target goals scored and goals against. The optimal offensive/defensive time allocation has also been computed and compared with the observed possession of the team.

As regards limitations if the study, note that the model proposed in this article may not be applicable in certain situations in which the best strategy to be played may be influenced not just by the variables considered but also by other external factors (e.g. the score of matches played by third teams, or by the score of the previous match in knock-out rounds).

As regards possible topics for further research, it would be very interesting to consider additional variables such as the ranking of the DMUs at the time the match is played, to analyze if the ranked below in the table team tends to play in a defense mode most of the time (and the opposite for the team ranked above in the table). Another interesting research line would be using a real-time data collection system. Thus, it would be possible to incorporate these models to a useful tool capable of monitoring the game and gathering data from the match (such as the remaining duration game and the current score) so as to decide to change the initial strategy if necessary in order to improve the team performance. On the other hand, this procedure can be applied to other sports whose game is based on defensive and offensive tactics such as, for example, basketball.

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List of table and figure captions

	Variable	References
ECONOMIC	operating cost	Barros and García-del-Barrio (2011);Guzmán and Morrow (2007); Sala et al. (2009)
	team value	Villa and Lozano (2016)
	total assets	Barros and García-del-Barrio (2011)
	team payroll	Barros and García-del-Barrio (2011);Barros and Leach (2006); Haas (2003b); Haas et al. (2004);
	attendance receipts	Barros and García-del-Barrio (2011)
	other receipts	Barros and García-del-Barrio (2011)
	net assets	Barros and Leach (2006)
	expenditures	Barros and Leach (2006)
	staff costs	Guzmán and Morrow (2007)
	directors’ remuneration	Guzmán and Morrow (2007)
	total wages and salaries (excl. coach)	Haas (2003a)
	UEFA’s coefficient revenue distribution	Zambom-Ferraresi et al. (2017)
	coach salary	Haas (2003a); Haas (2003b); Haas et al. (2004)
	season total revenues	Haas (2003a); Haas (2003b); Haas et al. (2004)
EMOTIONAL	stadium facilities	Barros and Leach (2006)
	stadium capacity	García-Sánchez (2007)
	population of the teams’ home town	García-Sánchez (2007); Haas (2003a)
	number of spectators per match	Barros and Leach (2006);García-Sánchez (2007); González-Gómez and Picazo-Tadeo (2010); Haas (2003b); Haas et al. (2004); Picazo-Tadeo and González-Gómez (2009); Roboredo, et al. (2015)
EMPIRICAL	number of players	Barros and Leach (2006); Espitia-Escuer and García-Cebrián (2004); Espitia-Escuer and García-Cebrián (2006); Espitia-Escuer and García-Cebrián (2008); Espitia-Escuer and García-Cebrián (2010); González-Gómez and Picazo-Tadeo (2010); Picazo-Tadeo and González-Gómez (2009);
	points	Barros and Leach (2006); Espitia-Escuer and García-Cebrián (2004); Espitia-Escuer and García-Cebrián (2006); Espitia-Escuer and García-Cebrián (2008); González-Gómez and Picazo-Tadeo (2010); Guzmán and Morrow (2007); Haas (2003a); Haas (2003b); Haas et al. (2004); Picazo-Tadeo and González-Gómez (2009); Roboredo, et al. (2015)
	goals scored	Boscá et al. (2009); Espitia-Escuer and García-Cebrián (2004); Espitia-Escuer and García-Cebrián (2008); García-Sánchez (2007); Villa and Lozano (2016)

	Variable	References
EMPRIRICAL (cont.)	shots at goal	Boscá et al. (2009);Espitia-Escuer and García-Cebrián (2006); Espitia-Escuer and García-Cebrián (2008); García-Sánchez (2007); Villa and Lozano (2016)
	attacking moves	Espitia-Escuer and García-Cebrián (2004); Espitia-Escuer and García-Cebrián (2006); Espitia-Escuer and García-Cebrián (2008); Espitia-Escuer and García-Cebrián (2010); García-Sánchez (2007)
	centre plays/in area	Boscá et al. (2009)
	possession	Boscá et al. (2009); Espitia-Escuer and García-Cebrián (2004); Espitia-Escuer and García-Cebrián (2006); Espitia-Escuer and García-Cebrián (2008); Espitia-Escuer and García-Cebrián (2010); Villa and Lozano (2016); Zambom-Ferraresi et al. (2017)
	shots and headers	Espitia-Escuer and García-Cebrián (2004)
	goals attempts	Espitia-Escuer and García-Cebrián (2008); Espitia-Escuer and García-Cebrián (2010); Zambom-Ferraresi et al. (2017)
	passes	Zambom-Ferraresi et al. (2017)
	passes to the penalty area	García-Sánchez (2007)
	ball recovery	García-Sánchez (2007);Villa and Lozano (2016);Zambom-Ferraresi et al. (2017)
	goalkeeper's actions	García-Sánchez (2007); Villa and Lozano (2016)
	number of seasons played in the First Division	González-Gómez and Picazo-Tadeo (2010); Picazo-Tadeo and González-Gómez (2009); Roboredo, et al. (2015)
	trophies won in competitions	González-Gómez and Picazo-Tadeo (2010)
	number of matches played in European competitions	Espitia-Escuer and García-Cebrián (2010); González-Gómez and Picazo-Tadeo (2010); Picazo-Tadeo and González-Gómez (2009);
	number of matches played in the King's Cup	González-Gómez and Picazo-Tadeo (2010); Picazo-Tadeo and González-Gómez (2009)
	attacks/in area	Boscá et al. (2009)
	centre plays/in area	Boscá et al. (2009)
	turnover	Barros and Leach (2006); Guzmán and Morrow (2007); Villa and Lozano (2016)
	corners and penalties	Villa and Lozano (2016)
	inverse of goals received	Boscá et al. (2009); García-Sánchez (2007)
	position in the final league table	García-Sánchez (2007); Roboredo, et al. (2015)

Table 1. A review of the variables used in the literature on performance assessment of football teams using DEA

OFFENSE MODE	INPUTS					OUTPUTS		
TEAM	# SHOTS AT GOAL	# CORNERS	# PENALTIES	# STEALS	BUDGET (10^6€)	GOALS SCORED	BUDGET OF RIVAL TEAM (10^6€)	% POSSESSION
ALMERIA	3.18	5.24	0.16	40.68	18.3	0.92	50.29	46.04
ATHLETIC	3.89	5.21	0.13	47.24	73.532	1.11	48.84	52.09
ATLETICO	4.97	4.89	0.16	47.08	171.7	1.76	46.25	49.06
BARCELONA	7.24	6.16	0.18	50.82	509.6	2.89	37.36	68.84
CELTA	4.37	6.24	0.16	51.34	30.3	1.24	49.98	57.11
CORDOBA	3.24	4.47	0.08	41.87	23.071	0.58	50.17	36.86
DEPORTIVO	3.50	4.61	0.18	42.79	30.08	0.92	49.98	46.52
EIBAR	3.29	4.24	0.05	43.18	15.8	0.89	50.36	40.92
ELCHE	3.39	4.37	0.18	42.00	26.4	0.92	50.08	32.17
ESPANYOL	3.63	4.53	0.08	45.50	48.2	1.24	49.50	49.66
GETAFE	3.84	4.79	0.05	45.55	36	0.87	49.83	49.06
GRANADA	3.34	5.29	0.16	38.13	28	0.76	50.04	50.12
LEVANTE	2.68	4.68	0.05	42.16	26.253	0.89	50.08	49.91
MALAGA	4.42	5.16	0.16	46.18	38	1.11	49.77	50.78
R MADRID	7.47	6.24	0.32	48.84	529.5	3.11	36.84	50.45
R SOCIEDAD	3.55	5.32	0.08	46.34	51.872	1.16	49.41	50.49
RAYO	4.53	5.16	0.08	48.37	21.967	1.21	50.19	50.87
SEVILLA	4.95	5.21	0.24	45.89	99.7	1.87	48.15	50.00
VALENCIA	4.32	4.58	0.18	46.84	89	1.84	48.43	50.39
VILLARREAL	4.68	5.68	0.05	45.74	62.099	1.26	49.14	49.57

Table 2. Average of the inputs and outputs of football teams operating in offence mode

DEFENSE MODE	INPUTS		OUTPUTS					
TEAM	# GOALS AGAINST	BUDGET (10^6€)	# SHOTS AT GOAL AGAINST	# CORNERS AGAINST	# PENALTIES AGAINST	# TURNOVERS	BUDGET OF RIVAL TEAM (10^6€)	1-POSSESSION (%)
ALMERIA	1.68	18.3	5.71	6.82	0.05	66.00	50.29	53.96
ATHLETIC	1.08	73.532	3.76	4.84	0.08	74.74	48.84	47.91
ATLETICO	0.76	171.7	2.58	4.26	0.08	64.66	46.25	50.94
BARCELONA	0.55	509.6	2.45	3.63	0.11	65.76	37.36	31.16
CELTA	1.16	30.3	3.92	3.79	0.16	72.95	49.98	42.89
CORDOBA	1.79	23.071	5.13	5.68	0.18	67.82	50.17	63.14
DEPORTIVO	1.58	30.08	4.84	4.82	0.16	72.92	49.98	53.48
EIBAR	1.45	15.8	5.16	6.00	0.13	69.08	50.36	59.08
ELCHE	1.63	26.4	4.71	5.71	0.24	62.74	50.08	67.83
ESPANYOL	1.34	48.2	4.39	5.03	0.03	72.11	49.50	50.34
GETAFE	1.68	36	4.53	5.21	0.21	70.50	49.83	50.94
GRANADA	1.68	28	4.24	4.50	0.11	63.71	50.04	49.88
LEVANTE	1.76	26.253	4.89	5.32	0.21	63.34	50.08	50.09
MALAGA	1.26	38	4.03	4.58	0.11	70.21	49.77	49.22
R MADRID	1.00	529.5	3.58	4.92	0.13	68.82	36.84	49.55
R SOCIEDAD	1.34	51.872	4.55	6.26	0.16	64.32	49.41	49.51
RAYO	1.79	21.967	4.84	4.97	0.13	73.76	50.19	49.13
SEVILLA	1.18	99.7	3.95	4.66	0.21	66.26	48.15	50.00
VALENCIA	0.84	89	3.37	5.61	0.21	66.32	48.43	49.61
VILLARREAL	0.97	62.099	3.87	5.45	0.05	67.97	49.14	50.43

Table 3. Average of the inputs and outputs of football teams operating in defence mode

	$\bar{q}$			$\bar{q}_{off}$			$\bar{q}_{def}$			gs+sgs*			ga-sga*		
TEAM	**EM**	**MMF1**	**MMF2**	**EM**	**MMF1**	**MMF2**	**EM**	**MMF1**	**MMF2**	**EM**	**MMF1**	**MMF2**	**EM**	**MMF1**	**MMF2**
ALMERIA	0.644	0.451	0.398	0.793	0.595	0.517	0.685	0.539	0.485	1.32	2.42	1.97	0.92	0.21	0.45
ATHLETIC	0.507	0.434	0.391	0.510	0.449	0.414	0.698	0.629	0.560	2.39	2.82	2.55	0.5	0.55	0.11
ATLETICO	0.501	0.404	0.378	0.476	0.396	0.348	0.760	0.653	0.653	3.13	2.39	3.61	0.32	0.45	0.03
BARCELONA	0.559	0.418	0.361	0.500	0.338	0.295	0.846	0.832	0.753	4.37	2.95	5.26	0.24	0.55	0.05
CELTA	0.532	0.411	0.344	0.554	0.423	0.357	0.806	0.611	0.507	2.42	2.53	2.79	0.74	0.29	0.18
CORDOBA	0.536	0.413	0.336	0.593	0.522	0.378	0.675	0.483	0.437	1.45	2.66	2.13	0.97	0.45	0.32
DEPORTIVO	0.555	0.416	0.344	0.695	0.543	0.421	0.611	0.525	0.472	1.55	2.82	2.32	0.74	0.21	0.16
EIBAR	0.800	0.546	0.498	0.869	0.697	0.573	0.844	0.632	0.613	1.16	2.47	1.63	1.11	0.18	0.63
ELCHE	0.644	0.408	0.382	0.748	0.558	0.507	0.715	0.481	0.458	1.47	2.61	2.16	0.92	0.37	0.24
ESPANYOL	0.527	0.375	0.350	0.590	0.475	0.460	0.648	0.503	0.493	2.13	2.58	2.55	0.58	0.26	0
GETAFE	0.492	0.292	0.269	0.524	0.364	0.335	0.632	0.398	0.354	1.97	2.97	2.53	0.89	0.53	0.18
GRANADA	0.576	0.375	0.327	0.745	0.525	0.425	0.641	0.480	0.454	1.34	2.79	2.03	0.97	0.37	0.16
LEVANTE	0.573	0.400	0.379	0.656	0.544	0.506	0.692	0.457	0.415	1.47	2.76	1.92	0.97	0.37	0.29
MALAGA	0.491	0.382	0.333	0.592	0.409	0.378	0.635	0.519	0.460	2.16	2.47	2.79	0.47	0.45	0.08
R. MADRID	0.481	0.301	0.290	0.435	0.301	0.296	0.754	0.586	0.576	4.68	2.47	5.26	0.47	0.26	0.05
R. SOCIEDAD	0.465	0.389	0.329	0.524	0.513	0.407	0.649	0.487	0.453	2.26	2.79	2.53	0.55	0.34	0.11
RAYO	0.626	0.529	0.329	0.742	0.549	0.390	0.710	0.716	0.507	1.68	2.63	2.63	0.95	0.26	0.34
SEVILLA	0.546	0.365	0.316	0.550	0.423	0.398	0.770	0.621	0.530	2.87	2.47	3.34	0.71	0.45	0.03
VALENCIA	0.566	0.477	0.450	0.591	0.496	0.465	0.725	0.648	0.630	2.74	2.55	3.18	0.34	0.26	0.16
VILLARREAL	0.412	0.309	0.292	0.389	0.354	0.324	0.728	0.612	0.590	2.82	2.58	3.19	0.34	0.34	0.03

Table 4. Average overall, offence and defence efficiency and goals-scored and goals-against targets computed by BB and MMF models

		EFF	EFF_OFF	EFF_DEF
BB DEA model	Data without RM and BAR	0.343	0.390	0.500
	Complete Data	0.343	0.389	0.504
MMF1 DEA model	Data without RM and BAR	0.414	0.489	0.564
	Complete Data	0.395	0.458	0.555
MMF2 DEA model	Data without RM and BAR	0.635	0.708	0.739
	Complete Data	0.601	0.668	0.721

Table 5. Average efficiency scores with and without Real Madrid (RM) and Barcelona (BAR)

	EFF	EFF_OFF	EFF_DEF
BB DEA model	0.929	0.870	0.828
MMF1 DEA model	0.946	0.928	0.979
MMF2 DEA model	0.981	0.983	0.985

Table 6. Pearson correlation coefficient of efficiency scores with and without Real Madrid and Barcelona

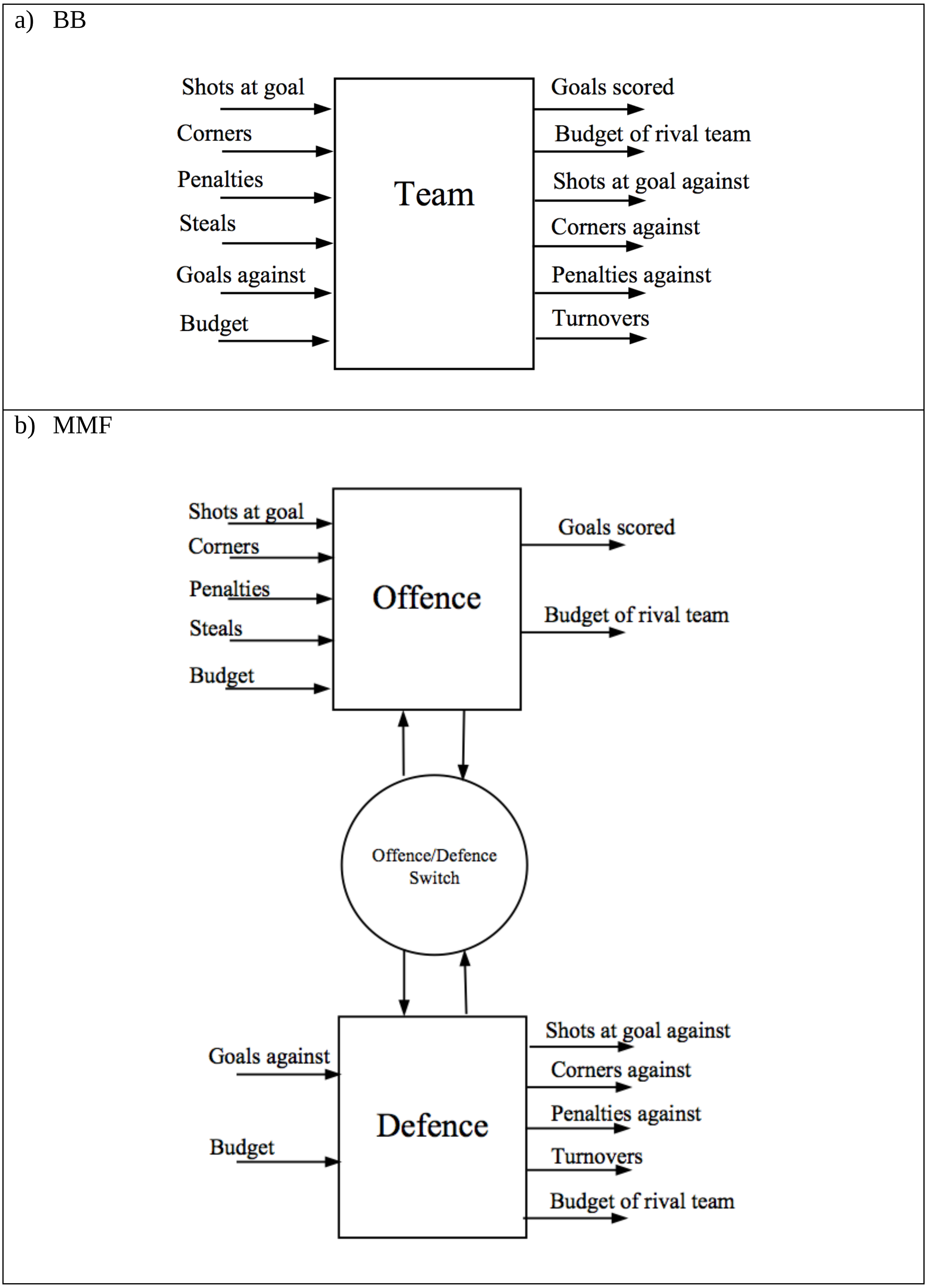
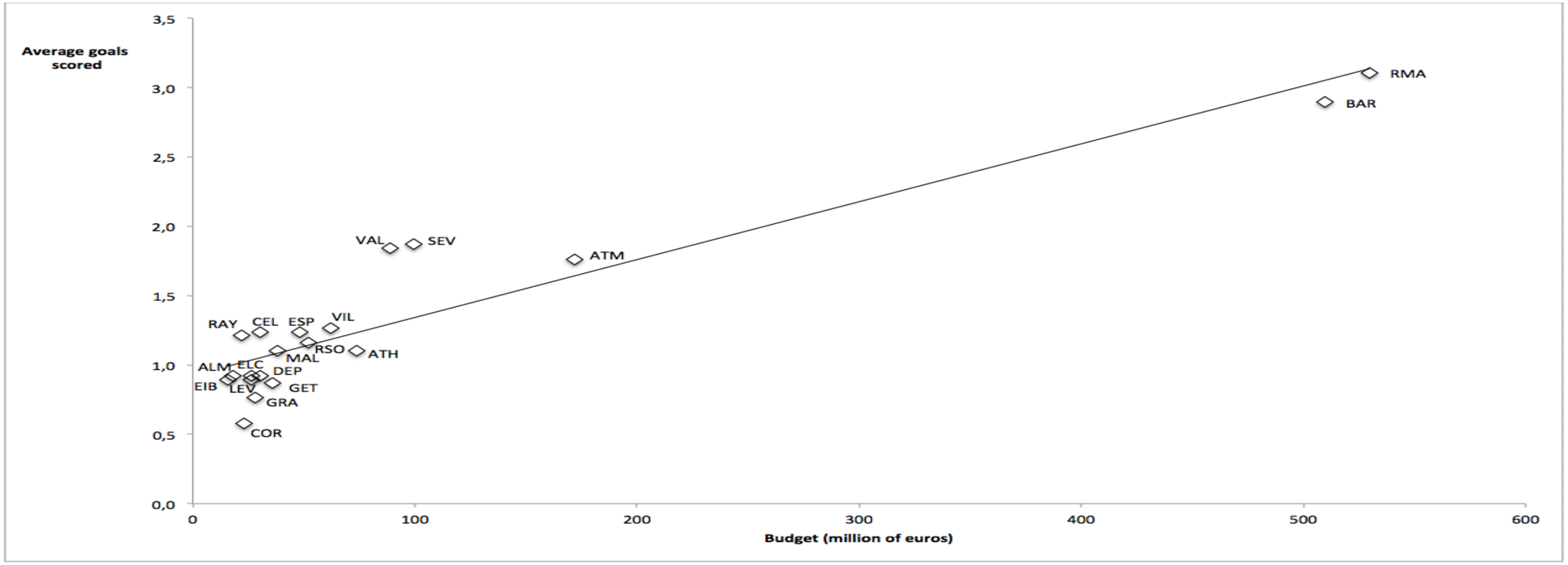


Figure 1. BB and MMF perspectives for performance analysis of football team in a match



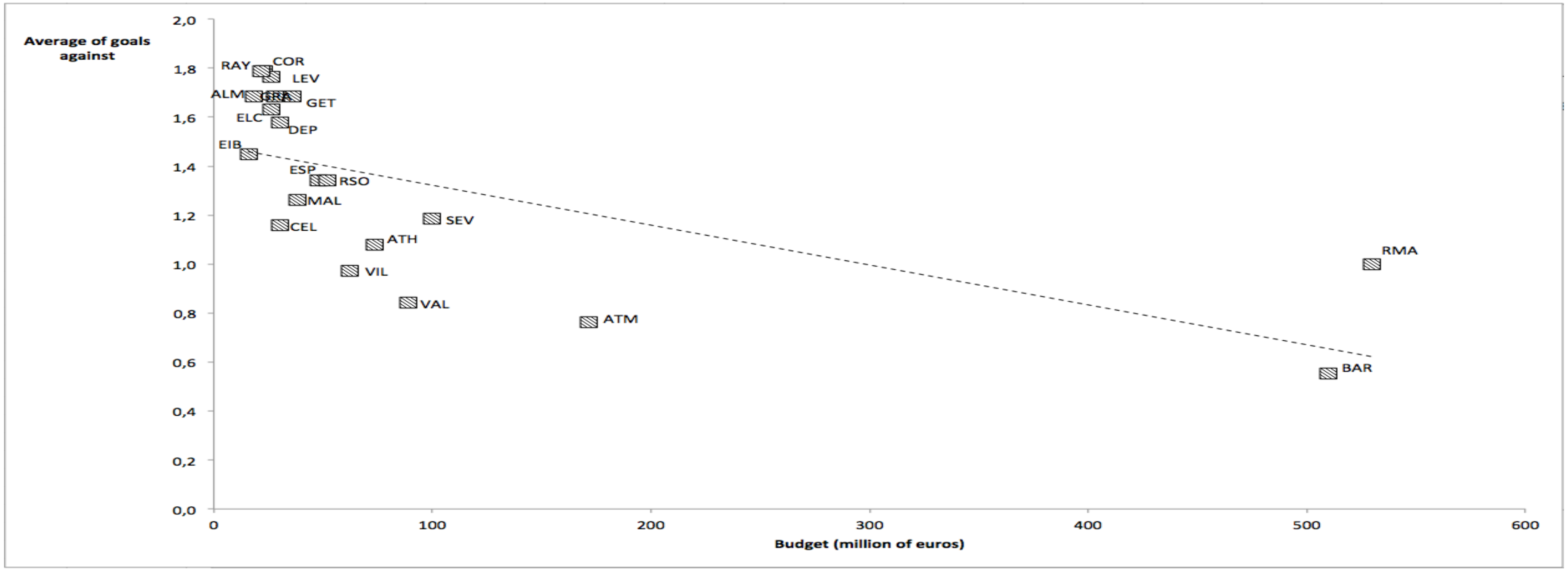


Figure 2. Average number of goals scored and of goals against versus budget

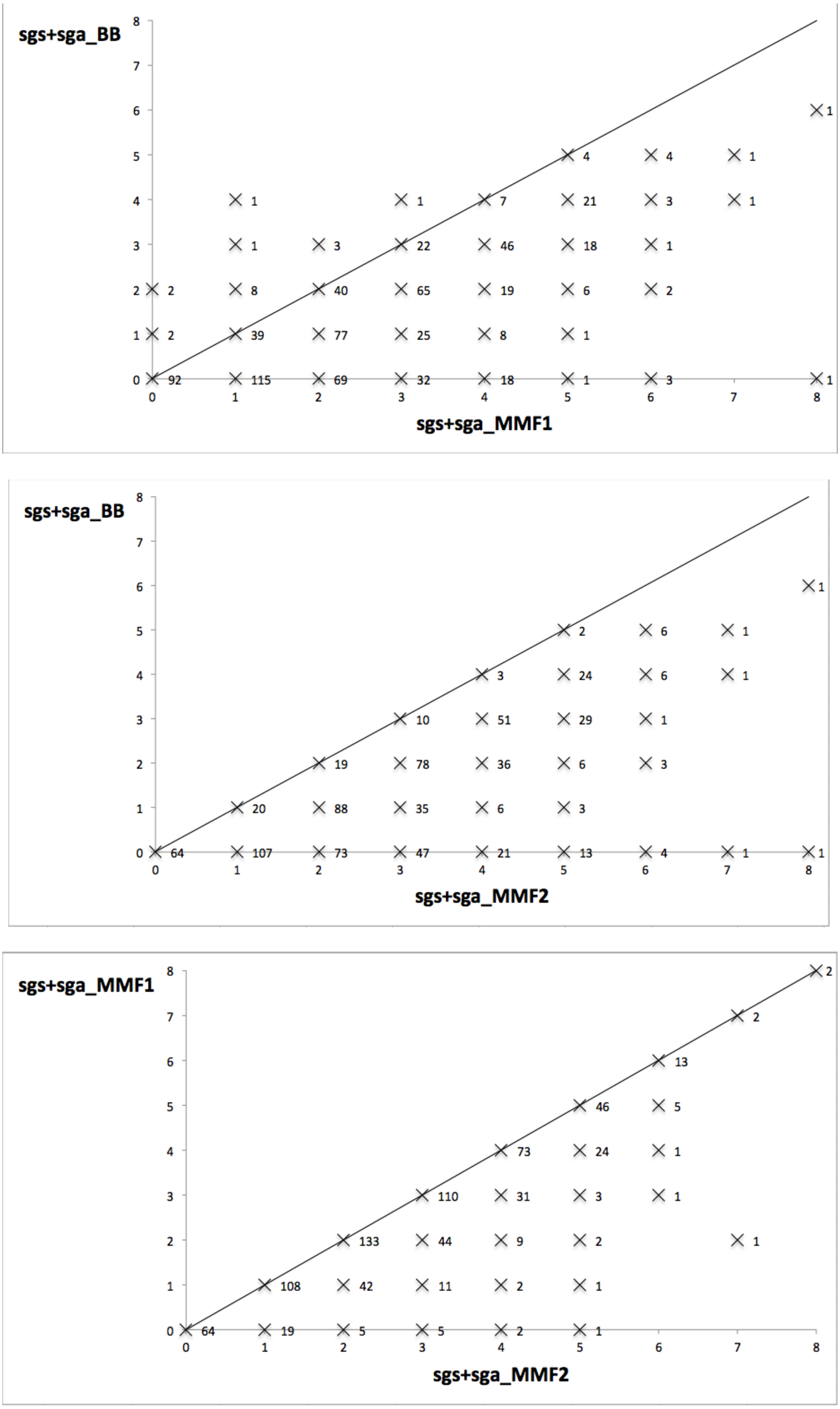


Figure 3. Sum of goals-scored and goals-against slacks computed by BB and MMF models

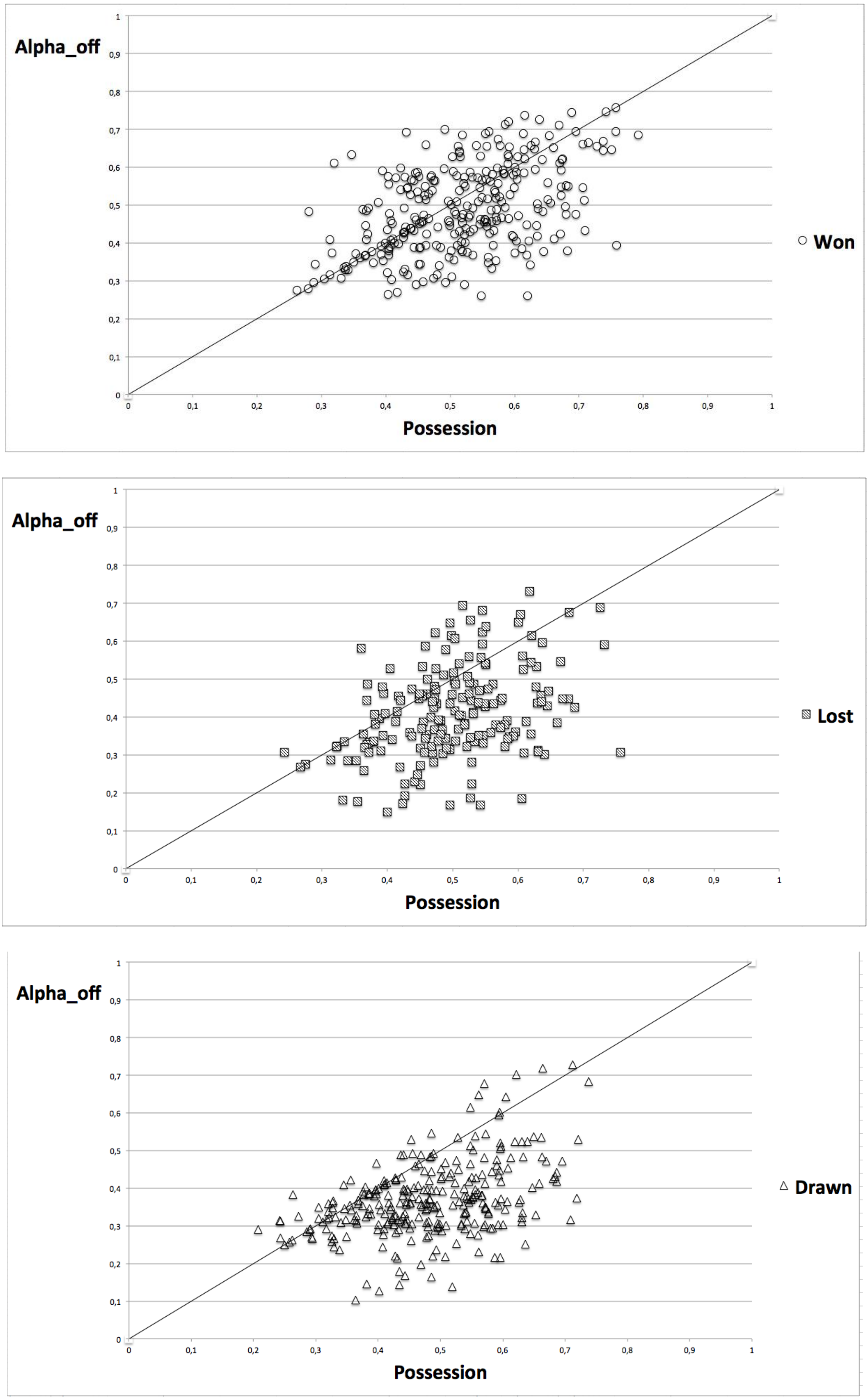


Figure 4. Observed possession versus optimal α^{off} for the 760 DMUs grouped by win, lose or draw