Solving Large-Scale Transmission Network Problems

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Büyük Ölçekli Elektrik Dağıtım Ağları Modellemesi

Deniz Beşik

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Anahtar Kelimeler: doğrusal programlama; sütun türetme; parçalı doğrusal yakınsama

Özet

Bir elektrik ağında, elektrik üreticileri iletim hatlarını kullanarak sistemdeki talebi karşılarlar. Literatürde, sistemdeki talebi karşılayan, elektrik ağının fiziksel kısıtlarına uyan ve elektrik üreticileri için en düşük maliyeti öneren matematiksel modeller bulunmaktadır. Fakat, elektrik dağıtımı birçok dış nedenden dolayı aksamaya uğrayabilir. Bu aksamalar hava koşullarına, terörist saldırılarına, insandan ve insan dışı gerçekleşen teknik hatalara veya voltaj düşüşü yüzünden gerçekleşen kayıplara bağlı olabilir. Bu dış nedenler sistemdeki talebin karşılanmasında bir risk oluşturmaktadır. Ayrıca, elektrik üreticisi ve talep noktası arasındaki uzaklık arttığında bu risk daha da büyümektedir. Bu tezde sunulan elektrik ağları için eniyileme modelinin amacı uzun mesafeli elektrik iletiminden kaynaklanan riskin önemini vurgulamaktır. Bir elektrik ağı düşünüldüğünde, elektrik üreticileri yakın çevrelerindeki talebi karşıladıkları zaman kayıp riskini azaltabilirler. Bu bağlamda, önerdiğimiz modelde değişken olarak üretici ve talep noktası arasında bulunan yol üzerinden geçen yükü kullanılırken,

amaç fonksiyonu bu yolun uzunluğuna ve yolun üzerinden geçen yüke bağlı olan bir risk fonksiyonunu enküçükleyecek şekilde sunulmaktadır. Risk fonksiyonu, elektrik üretcisinin dışbükey ve kareli ortalama maliyet fonksiyonu ile üretici ve talep noktası arasındaki yolun uzunluğuna bağlı olan bir risk katsayısı ile birleştirilerek elde edilmektedir. Bu çalışmanın literatürdeki diğer çalışmalardan farkı, üretici ve talep noktası arasındaki uzaklığı bir risk etkeni olarak sunulması ve bu riskin modele katılmasıdır. Sunduğumuz matematiksel eniyileme modelini çözmek için sütun türetme yöntemi kullanılmaktadır. Fakat, sütun türetme yönetimi, dışbükey ve kareli ortalama amaç fonksiyonuna sahip olan eniyileme modelinde kullanılamamaktadır. Bu nedenle öncelikli olarak amaç fonksiyonu parçalı doğrusal fonksiyonlar ile yakınsanmıştır. Fakat, ortaya çıkan amaç fonksiyonunu doğrusal olarak modellemek, satır sayısında artışa neden olmaktadr. Bu artış, önerilen çözüm yönteminin değiştirilmesine sebep olacaktr. Bu sebeple, amaç fonksiyonu literatürdeki bir yöntem ile satır sayısını arttırmayacak şekilde doğrusal olarak modellenmiştir. Elde edilen doğrusal programlama modeli sütun türetme yöntemiyle çözülmüş ve bu yaklaşım örnek problemler üzerinde sınanmıştır.

Large-Scale Transmission Network Problems

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Keywords: linear programming; column generation; piece-wise linear approximation

Abstract

Electricity is supplied by generators to meet the demand of the customers through the transmission lines. The flow-based optimization models in the literature seek for optimal generation cost while satisfying the demand and the physical constraints of the network. However, electricity transmission can be disrupted by exogenous factors such as weather conditions, terrorist attacks, human and operational errors or voltage drop due to line losses. These factors can generate a risk in the system leading to unmet demand of customers. Furthermore, this risk increases when the distances between the generators and the demand points becomes larger. In this thesis, we propose an electric network optimization model which emphasizes the risk arising from the long distance electricity transmission. In an electric network, if generators satisfy the demand in their vicinity, the arising risk from long distance electricity transmission can be reduced. In this regard, we use a path-based electric network optimization model where the objective is to minimize a risk function based on the path lengths and the flows. This risk function is obtained by incorporating a path length dependent risk coefficient into the convex quadratic generator cost function. Our work differs from the works in the literature as we consider such at risk function. To solve the resulting model, we employ column generation. However, column generation is not applicable when the objective function is convex quadratic. Therefore first, the convex quadratic function is approximated by a piece-wise linear convex function. However, the linear programming equivalent of this model causes a row-wise increase. This increase would cause to change the given solution approach. Thus second, an equivalent linear programming model without a row-wise increase is presented. The resulted model is solved with standard column generation and the numerical results are obtained for example networks.

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Chapter 1

Introduction

Electricity power is one of the most crucial elements for the financial, industrial and social developments of any country. Since electricity cannot be stocked, the market regulation depends on the hourly supply and demand balance. Electricity is supplied by generators to meet the demand of the customers through the transmission lines. However, the electricity transmission can be disrupted by exogenous factors such as weather conditions, terrorist attacks, human and operational errors or voltage drop due to line losses (Simonoff et al., 2007). These factors create a risk of not satisfying the customer demand in the system. In addition, this risk may become more crucial when the distances between suppliers and demand points become larger. Moreover, a disruption on a single line can cause unmet demand at multiple demand points. In this thesis, we propose an electric network optimization model which emphasizes the risk arising from the long distance electricity transmission. In an electric network, if the generators satisfy the demand in their vicinity, this risk can be reduced. In this regard, we use a path-based electric network optimization model where the objective function minimizes the risk arising from an exogenous factor in long distance electricity transmission. The risk is defined as a function, which depends on the amount of flow between a generator and demand point as well as the length of the path. We use one example of the exogenous factor, which is the incurred voltage due to line losses. The path-based formulation may have excessive number of paths even for moderate size networks so that column generation is a viable approach to solve the resulting problem. However, column generation approach requires a linear programming model but the risk function in the objective function is nonlinear, in particular, convex quadratic. To overcome this difficulty, we first approximate the convex quadratic function by a piece-wise linear convex function. However, the linear programming equivalent of this model gives a row-wise increase. For such a model, we need to change the solution approach. Instead of changing the solution approach, we give an equivalent linear programming model that does not grow row-wise. Finally, the resulting linear programming model is solved by column generation.

There are other network optimization models in the literature. Generally, these models have the objective of minimizing the generation cost while conforming the operational constraints of the electric network. These constraints ensure the power flow between nodes under physical restrictions that govern the network. These models can be referred to as flow-based models. The path-based formulation of this model that reaches the same capacities as the flow-based model can be written using theoretical results. However, we also incorporate a risk function into the objective which results in a more condensed electricity distribution. This result cannot be obtained in the flow-based model due to the structure of the risk function. The structure contains a risk component which depends on the path length.

In this chapter, the problem definition is given in Section 1.1. Then, the motivation behind our study is explained in Section 1.2. The contributions of the thesis are given in Section 1.3. Lastly, Section 1.4 describes the flow of the thesis.

1.1 Problem Definition

Finding the optimal generation quantity in a transmission network dates back to the beginning of the 20^{th} century. In 1960s, an electric network optimization model, called optimal power flow (OPF) model is introduced. Basically, this model seeks to minimize the generator cost subject to the operational constraints of the given electric network. This problem is originally nonlinear and nonconvex due to the physical laws governing the network. Lavaei and Low (2010) shows that OPF problem is NP-hard. Through linearization, the problem can be simplified. The linear formulations of the OPF are frequently used by the energy industry due to their simplicity. However, none of these formulations consider the possibility of incurred risk due to long distance electricity transmission. We present a path-based model with a convex quadratic objective function and linear constraints. Also, we propose a risk function that is defined for every path between generators and demand points with respect to the path length and the power flow on it. Since the risk function requires a path-based formulation, we alter the linear flow based formulation of Villumsen and Philpott (2011) into a path-based model. Then, we incorporate the path-dependent risk function into the objective function of the path-based model in the form of convex quadratic function. This form arises as a result of a risk coefficient, which alters the original convex quadratic generator cost function with respect to the path length.

1.2 Motivation

Electricity transmission carries a risk of encountering a voltage drop due to line losses, terrorist attacks or unexpected changes in weather conditions. As a result of these exogenous factors, the customer demands may not be satisfied or the costs of generators may increase. The risk here becomes a more crucial issue when the long distance transmission is considered. With this motivation, we determine a model with a risk function, which depends on the path length and the path flow. To clarify our motivation, we give an example in Figure 1.1. In this figure, both generators g_1 and g_2 supply electricity power to demand point i_3. The lengths of transmission lines are given on each link. First, suppose that the generation cost of g_1 is slightly lower than g_2 and neglect the path lengths between $g_1 - i_3$ and $g_2 - i_3$. In this case, the generator with the lowest generation cost will supply electricity to the demand point assuming that the operational constraints are satisfied. Now consider the path length between generators and the demand point. As mentioned earlier, the exogenous factors increase the risk of having an unsatisfied demand in long distance electricity transmission. Considering this risk may result in favoring the generators that are closer to demand points. In Figure 1.1, the path length of $g_1 - i_3$ is significantly longer than $g_2 - i_3$. If any one of the risk factors is realized through the path between $g_1 - i_3$ the demand of i_3 may not be satisfied. As a result, supplying electricity from generator g_2 is less risky as it is much closer to the demand point i_3.

In this thesis, we aim to reduce the risk of experiencing a demand loss while minimizing the generation costs. To satisfy this goal, a risk function depending on the path length and the flow is presented. Then, we incorporate the risk function into the

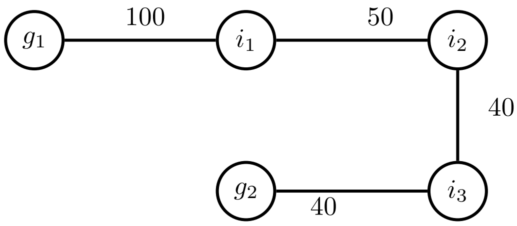

Figure 1.1: An example network to illustrate the motivation of the thesis

objective function of a path-based power flow model. The risk function is obtained by multiplying two components. The first component is the generator cost function where the independent variable is the flow on the path. This cost function is assumed to have a convex quadratic form. Another factor that has detrimental effect is the voltage drop on the path due to loss. In this regard, we use a predictive loss function for the second component. The output of this function returns a positive risk coefficient. This function depends on the path length. When the path length becomes larger, the value of this coefficient increases. With this risk function, we propose a model that considers the risk of long distance electricity transmission.

1.3 Contributions

Originally, the OPF formulations do not consider the risk factors in the transmission of electricity from a generator to a demand point. However, electricity transmission may contain a disruption risk, which may result in unmet demand. When the distance between a generator and a demand point becomes larger, the risk is expected to become higher. In this thesis, we considered this risk through a function, which depends on the path length and the flow. We give a path-based formulation for OPF problem with a risk function in the objective. This path-based model obtains the optimal generator capacities that satisfy the demand and the operational constraints while minimizing the generator costs along with the incurred risk in the network. As far as we know, a similar model does not exist in the literature.

1.4 Outline

We give a literature review of optimal power flow models in Chapter 2. The formulations that are reviewed in this chapter are flow-based formulations. In Chapter 3, the flow-based and the path-based models are explained. Introduction and integration of the risk function into the path-based model is also given in the same chapter. We select the column generation approach as a solution method. The approximation of the risk function by piece-wise linear functions is presented at the end of Chapter 3. In Chapter 4, an equivalent linear programming model is given for the model with a piece-wise linear separable objective function to employ the column generation method. The column generation approach with the selected sub-pricing problem is explained at the last section of Chapter 4. The computational results are presented for IEEE 14 Bus and 118 Bus networks in Chapter 5. Finally, we conclude the thesis in Chapter 6.

Chapter 2

Literature Review

An electric network or a transmission network is formed by the connection of the electricity suppliers and the customers through the transmission lines. This power system can be mathematically formulated as a network optimization problem. Representation of the electrical state of the network in an optimization model is given by the system variables such as generation power, transmission line flow, voltage and phase angle (Frank et al., 2012). The major optimization model in the literature for the electric network optimization problem is the Optimal Power Flow (OPF) model.

The objective of the OPF problem is minimizing the generation cost or the system losses (Zhang, 2010). The operational constraints ensure that the physical characteristics of the transmission network is satisfied with the given capacity on the system variables. There are equality and inequality constraints in standard form of the optimal power flow model (Kundur et al., 1994). Generally, the equality constraints are given for the power flow equations. These equations correspond to the conservation of power flow and Kirchhoff's Voltage Law constraints. In most of the formulations, power flow equations are given for both real and reactive power. On the other hand, the inequality constraints are given for capacity of the system variables.

OPF model, which is formulated at the beginning of 1960s by Carpentier (1962), is a nonlinear programming problem. The nonlinearity in this problem arises from the Kirchhoff's Voltage Law equation (Bukhsh et al., 2011) as it reflects the nonlinear relationship between the voltage and phase angles. The resulting problem is classified as an NP-hard problem (Lavaei and Low, 2010). Since, OPF problem is NP-hard, several approximations are introduced. Deterministic, stochastic and hybrid formulations (combination of deterministic methods) for OPF are presented in the literature (Suthat and Vyas, 2013).

Later in 1960s, gradient based solution methods are used for solving the model proposed by Carpentier (1962). One of the early examples in the literature is given by Dommel and Tinney (1968). They insert a penalty factor into the objective function for the bound constraints of basic variables and solve the model by reduced gradient method. They did not use the Newton's method due to being computationally expensive at that time. However in 1970s, Sasson et al. (1973) present a solution for OPF problem with Newton's method. Later, Sun et al. (1984) also use Newton's method to solve the OPF problem. They approximate the Lagrangian function as a quadratic one and use the sparsity characteristic of the Hessian matrix to reduce the computational time. Burchett et al. (1984) present a newly sparse implementation of an optimization method where the exact second derivative can be computed. Their method is applicable to large scale networks having 350 to 2000 nodes.

An efficient solution of OPF with linear constraints is presented by Carpentier (1968, 1972) through application of generalized reduced gradient method in 1972. In the same year, Peschon et al. (1972) also describe the application of generalized reduced gradient to solve the OPF problem. They also present sensitivity and efficiency analyses. Yu et al. (1986) propose a new nonlinear programming formulation for OPF, where the model includes network performance measures such as scheduled bus voltages and topological constraints. There are also quadratic programming based approximations to OPF. Contaxis et al. (1986) solve OPF problem with a quadratic programming based approach. Grudinin (1998) present a reactive power optimization with quadratic programming formulation in which the solution is given by Newton's method. Fletcher (1971)'s method is used by Nanda et al. (1989) to solve OPF problem for minimum generation cost and minimum losses. Jabr (2008) gives a conic quadratic representation of OPF and solves the problem by primal-dual interior point method. In this thesis, we use an OPF model where the objective function is convex quadratic and the constraints are linear.

In some formulations of OPF model, discrete variables representing the transformer tap ratios or switched capacitor banks are also used. These variables are especially used for network design problems. Lima et al. (2003) give a mixed integer linear programming model for finding the optimal locations of phase shifter transformers. However according to Suthat and Vyas (2013), mixed integer nonlinear programming approach is more accurate for representing the system behavior of discrete variables. A recent example of such a model is introduced by Kumar and Gao (2010). In this work, the optimal location and the number of power generators are determined in a hybrid electricity market.

Nonlinear programming formulations can reflect the transmission network behavior better than linear programming formulations. However, the nonlinear programming formulations are hard to solve (Almeida and Galiana, 1996). Therefore, linear programming approximations are frequently used. Solving the linear programming approximation of OPF, which is also called as the Direct Current(DC) formulation, is very fast(Rau, 003b). In this thesis, we use a DC approximation model where the model is adopted from the model of Villumsen and Philpott (2011). In their model, the objective is to acquire the optimal generator capacity with the minimum cost subject to linear operational constraints. The main advantage of this model is to use a linear Kirschhoff's Voltage Law constraint. We use this model to integrate a risk function where it changes with the path length s between the generators and the demand points. Due to this path dependent structure, a path-based OPF model is proposed.

While the electric network is operating, some lines may not be used to decrease the cost and the efficiency. In addition, the cyclic structure of the electric networks requires to take off those lines which this process of taking the lines off and using them again is called switching. Villumsen and Philpott (2011) used DC approximation of OPF model to find minimum cost dispatch and commitment of power generation units in a transmission network with active switching. We assume in this work that the switching constraint is not considered. Another work that uses DC approximation is given by Fisher et al. (2008). In their work, they solve the linear OPF problem with optimal transmission switching. Their work is similar to Villumsen and Philpott (2011) where there are some differences in the modeling phase. Also, Fisher et al. (2008) integrate the N-1 security constraint to the OPF model with active switching. N-1 security constraint signifies the deprivation of any element in the system such as generator or transmission line.

Heuristic methods are also used for solving the OPF problem. Yang et al. (1996) introduce an evolutionary algorithm for economical dispatch problem with nonsmooth cost function. This algorithm finds near optimal solutions. Sayah and Zehar (2008) propose modified differential evolution algorithm for the OPF problem with nonsmooth and nonconvex generator fuel cost functions. Lee et al. (1998) proposes a method based on neural networks to solve an OPF problem with piece-wise quadratic cost function. OPF is also solved with an enhanced Genetic Algorithm by Bakirtzis et al. (2002) where the model includes both discrete and continuous variables.

In this thesis, we use the DC approximated model where the objective function is convex quadratic. This objective function also involves the risk associated with transmission on long lines. That is, the risk function that we propose depends on path length and flow on that path. To the best of our knowledge, a model similar to ours has not been studied in the literature before.

Chapter 3

Mathematical Programming Models

The OPF model minimizes the generator costs in an electric network subject to power flow conservation and Kirchhoff's Voltage Law constraints. Consideration of the distances between the generators and the demand points is crucial regarding the effect of the exogenous factors. Because, there may be a risk of having a possible terrorist attack, line voltage drop due to loss or an unexpected weather condition. In such a case, customer demand may not be satisfied. In this thesis, we introduce a risk function that depends on the path length and the flow.

The OPF problem is a flow-based problem which can have a linear or convex quadratic objective function. This model can be equivalently formulated as a path-based model through the flow decomposition theorem (Ahuja et al., 1993). We replace the objective function of the path-based model with a risk function that is based on path length and path flow. Our motivation of using this model comes from the emerging risk of the long distance electricity transmission. The proposed risk function has two components. The first component is the original convex quadratic generator cost function which depends on the flow on the path. For the second component we consider a path length dependent risk factor. As mentioned before, disruption on electricity transmission occurs as a result of the outside factors. One example of these factors is the voltage drop due to the incurred loss. Since loss is a function of path length, the second component of the risk function considers the loss and the path length. We call this component as the risk coefficient which is determined through a predictive loss function from the literature. As a result of multiplying these two components, the risk function becomes convex quadratic and defined for each path separately.

This chapter consists of five sections. First, flow-based model is explained in Section 3.1.

3.1 Flow-Based Model

As mentioned in Chapter 2, the linear programming model is used to approximate the nonlinear optimal power flow model. This formulation is called DC approximation. Next we use the optimal power flow formulation of Villumsen and Philpott (2011). In the rest of the thesis, this model will be referred to as the flow-based model. The formulation is given as follows:

$$\begin{aligned}
\text{minimize} \quad & \sum_{g \in \mathcal{G}} c_g s_g, && && (3.1)\\
\text{subject to} \quad & \sum_{g \in \mathcal{G}(i)} s_g + \sum_{e \in \mathcal{I}(i)} f_e - \sum_{e \in \mathcal{O}(i)} f_e = d_i, && i \in \mathcal{N}, && (3.2)\\
& r_e f_e = \theta_j - \theta_i, && (i,j) = e \in \mathcal{E}, && (3.3)\\
& s_g^{\min} \leq s_g \leq s_g^{\max}, && g \in \mathcal{G}, && (3.4)\\
& f_e^{\min} \leq f_e \leq f_e^{\max}, && e \in \mathcal{E}, && (3.5)\\
& \theta_i^{\min} \leq \theta_i \leq \theta_i^{\max}, && i \in \mathcal{N}, && (3.6)
\end{aligned}$$

where the problem variables s_g, f_e and θ_i stand for the generated electricity at the generator g, the flow on an edge e and the voltage angle at node i, respectively. Here $\mathcal{N}$ is the set of nodes and $\mathcal{E}$ is the set of edges. There are two sets associated with the generators. The first one, $\mathcal{G}(i)$ is the set of generators at node i and it is a subset of the entire set of generators, $\mathcal{G}$. The set $\mathcal{I}(i)$ denotes all those edges entering node i. Similarly, $\mathcal{O}(i)$ is the set of all edges leaving node i. We assume that the following problem parameters are given: cost of generating one unit supply, c_g; demand at each node, d_i (if a node is not a demand point then simply $d_i = 0$); resistance factor, r_e; upper and lower bounds on supply, flow and voltage angle given by the pairs $(s_g^{\min}, s_g^{\max})$, $(f_e^{\min}, f_e^{\max})$ and $(\theta_i^{\min}, \theta_i^{\max})$, respectively. The objective (3.1) is to minimize the total cost of the supply at the generators. The constraints (3.2) correspond to the conservation of flow at each node. The Kirschoff rule on each edge $e = (i, j)$ is represented by constraint (3.3). The remaining constraints

(3.4)-(3.6) denote the bounds on the problem variables.

Solving flow-based problem is relatively simple as the objective function and constraints are linear. However, most of the nonlinear or quadratic programming approximations of flow-based model contain a nonlinear generator cost function. Some example approximations are given in the form of convex quadratic (Dieu and Schegner, 2013; Sayah and Zehar, 2008; Lee and Yang, 1998; Mahdad et al., 2010), third degree polynomial (Shoults and Mead, 1984) or as a discrete function (Wang et al., 2007). Also, in some cases the electricity suppliers prefer to use the cost function with a single linear segment or with multiple linear segments (Wood and Wollenberg, 2012). We assume a convex quadratic generator cost function (Park et al., 1993). This function is given by:

$$\sum_{g \in G} c_g(s_g) = \sum_{g \in G} as_g^2 + bs_g + d. \tag{3.7}$$

Note that this function is simply the sum of uni-variate functions.

In this section, flow-based model with linear and convex quadratic objective function is presented. In the following section, the flow-based model is converted to a path-based one thorough the flow decomposition theorem in Ahuja et al. (1993). The generation quantity s_g is written according to the summation of the flow on paths between generator and demand points. This alteration forms a base for the integration of the path dependent risk function in Section 3.3.

3.2 Path-Based Model

The goal of this thesis is to present a model for reducing the risk of long distance electricity transmission. We assume that the risk depends on the path length and the flow. Due to this structure, we decompose the flow-based formulation into a path-based one.

Before presenting the path-based formulation, we introduce some new notation using various collections of paths. Let $\mathcal{P}$ denote the set of all paths in the network. Then

$$\mathcal{P}_g^i = \{p \in \mathcal{P} : p \text{ is a path between generator } g \text{ and node } i\}.$$

This allows us to define for $g \in \mathcal{G}$, the set

$$\mathcal{P}(g) = \{p \in \mathcal{P} : p \text{ is a path starting from generator } g\} = \cup_{i \in \mathcal{N}} \mathcal{P}_g^i$$

and for $i \in \mathcal{N}$, the set

$$\mathcal{P}(i) = \{p \in \mathcal{P} : p \text{ is a path terminating at node } i\} = \cup_{g \in \mathcal{G}} \mathcal{P}_g^i.$$

The last set is associated with those paths traversing a given edge and it is given by

$$\mathcal{P}(e) = \{p \in \mathcal{P} : p \text{ includes edge } e\}.$$

We next give the path-based formulation:

$$\text{minimize} \quad \sum_{g \in \mathcal{G}} c_g \sum_{p \in \mathcal{P}(g)} f_p, \tag{3.8}$$

$$\text{subject to} \quad \sum_{p \in \mathcal{P}(i)} f_p = d_i, \qquad i \in \mathcal{N}, \tag{3.9}$$

$$r_e \sum_{p \in \mathcal{P}(e)} f_p = \theta_j - \theta_i, \qquad (i,j) = e \in \mathcal{E}, \tag{3.10}$$

$$s_g^{\min} \leq \sum_{p \in \mathcal{P}(g)} f_p \leq s_g^{\max}, \qquad g \in \mathcal{G}, \tag{3.11}$$

$$f_e^{\min} \leq \sum_{p \in \mathcal{P}(e)} f_p \leq f_e^{\max}, \qquad e \in \mathcal{E}, \tag{3.12}$$

$$\theta_i^{\min} \leq \theta_i \leq \theta_i^{\max}, \qquad i \in \mathcal{N}. \tag{3.13}$$

Here f_p denotes the flow on a path and the remaining variables as well as the parameters are as before. The objective function of this model has a linear structure. Recall that the convex quadratic function objective function of the flow-based model in (3.7). Same quadratic function can be given for the path-based problem

$$\sum_{g \in G} c_g (\sum_{p \in P(g)} f_p) = \sum_{g \in G} a (\sum_{p \in P(g)} f_p)^2 + b \sum_{p \in P(g)} f_p + d. \tag{3.14}$$

Note that the solutions of the flow-based and the path-based problems are interchangeable since $s_g = \sum_{p \in P(g)} f_p$.

3.3 Risk Function

Long distance electricity transmission may result in unsatisfied demand due to possible terrorist attacks, voltage drop along the line due to loss or an unexpected weather condition. In this regard, we present a risk function that considers the path length and the flow on the path. Instead of considering all of the possible risks, we exemplify this risk function according to the possibility of voltage drop on the path due to loss.

Power movement in an electrical device, such as a conductor or a regulator, acquires a certain amount of loss because of the resistance to the flow of electricity on the device (Willis, 2010). Considering the transmission line loss in an electrical network is crucial for determining the quantity of power generation. Power loss could effect the quantity of the transmitted power when transmission line length is several hundred kilometers (Gustafson and Baylor, 1988). Total generation quantity equals to the summation of demand and the line losses (Wood and Wollenberg, 2012). The optimal power flow models that consider line loss use this equation as the flow of conservation constraint. In these models, loss is taken as a decision variable and the objective function either minimizes the loss or the generation cost. Sharif et al. (1996) propose a mathematical model where the objective is to minimize the total loss in the network while maintaining the acceptable voltage limits. Sinsuphun et al. (2011) also minimize the total loss in the system. They use a method based on swarm intelligence for minimizing the nonlinear loss function. Smita and Vaidya (2012) also use particle swarm optimization. Baldwin and Makram (1989) presented the optimal generation cost through a quadratic loss function in the constraint. Furthermore, Baran and Wu (1989) propose a method in network configuration for loss reduction and load balancing.

Bamigbola et al. (2014) define loss through a predictive loss function. In this work, the loss is divided into two components as ohmic loss and corona effect. Ohmic loss is defined as the flow resistance in the transmission lines where the resistance results in the form of heat (Smed et al., 1991). On the other hand, corona effect occurs when the applied voltage exceeds a critical level (Sakhavati et al., 2012). Summation of these two types of losses leads to an exponential loss function with the parameters line length and power flow. This relatively simple definition of the loss inspired us to present the risk function that we propose in here. The resulting path-based risk function is given by:

$$r_p(f_p, l_p) = \beta(l_p)c_g(f_p),\ p \in \mathcal{P}_g^i, \tag{3.15}$$

where

$$\beta(l_p) = t - e^{-l_p}. \tag{3.16}$$

The risk function in (3.15) is obtained by multiplying of two components $\beta(l_p)$ and $c_g(f_p)$. The latter component is the original convex quadratic cost function in (3.14) where f_p is the power that is sent on the path p. The former component is presented through the predictive loss function where l_p represents the length of the path. Since the path length is known, the result of the exponential function returns a positive coefficient. We call this positive number as the risk coefficient. The cost of the path increases with respect to the risk coefficient. The justification for the usage of exponential function can be formed through considering the boundary conditions. When the generator supplies electricity through a path with the length of infinity, the risk coefficient gives the maximum value possible which is t. In the computational study section a sensitivity analysis is given for different values of t. On the other hand, if the path length is zero, the risk coefficient becomes one.

Notice that, the risk function shifts the cost function up with respect to the path length and the path flow. To clarify this issue, consider the example network in Figure 3.1.

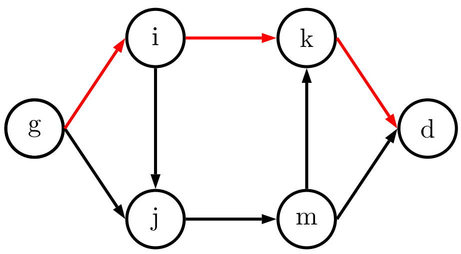

Figure 3.1: Example network to illustrate the structure of the risk function

Now, consider the paths $g - i - k - d$ and $g - j - m - d$ where the path lengths are assumed to be 50 and 300 km respectively. We call these paths as p_1 and p_2.

Suppose that the flow on the paths are the same and the path length is ignored, then the risk function becomes:

$$r_p(f_p, 0) = af_p^2 + bf_p + c. \tag{3.17}$$

If we now consider the path lengths, then we obtain:

$$r_{p_1}(f_{p_1}, 50) = (2 - e^{-50})(af_{p_1}^2 + bf_{p_1} + c), \tag{3.18}$$

and

$$r_{p_2}(f_{p_2}, 300) = (2 - e^{-300})(af_{p_2}^2 + bf_{p_2} + c). \tag{3.19}$$

Notice that, the risk coefficient shifts the functions with respect to the path length. Illustration of this shift can be seen in Figure 3.2.

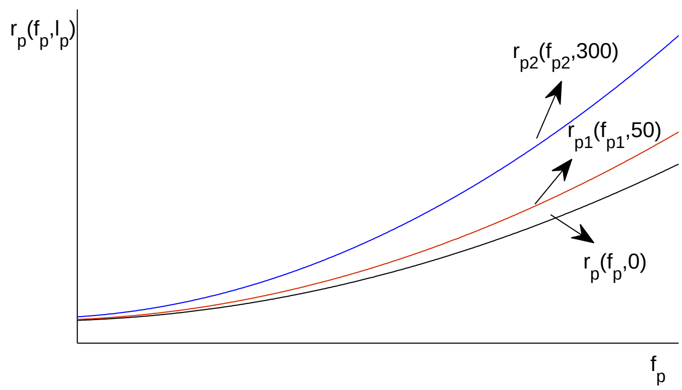

Figure 3.2: Shift in the risk functions with respect to path lengths

We replace the objective function of the path-based model given in (3.8) with the risk function in (3.15). This change signifies the usefulness of the path-based model over the flow-based model. It is important to notice that the risk function does not simply consider the summation of the incurred risks on the individual lines. In other words; it is not separable. Therefore a flow-based model cannot be directly used.

The convex quadratic path-based model can be solved by the standard methods.

However, the number of paths exponentially grows especially when there is considerable number of nodes in the network. As a result, employ column generation approach to solve the model. In the next section, the convex quadratic risk function is approximated by piece-wise linear convex function. Afterwards in Chapter 4 an equivalent linear programming model is given for the piece-wise model so that column generation approach can be applied.

3.4 Piece-wise Linear Approximation

In this section, quadratic convex objective function of path-based problem is linearized by piece-wise linear upper and lower approximation. There are piece-wise quadratic and piece-wise linear approximations in the literature for the flow-based model with a convex quadratic generator cost curve. Lin and Viviani (1984) introduces a method for solving the optimal power flow model by piece-wise quadratic cost functions. They use a hierarchical solution methodology that the decentralized computations can be possible. Furthermore, Dieu and Schegner (2013) also approximate the generator cost curve by a piece-wise quadratic function. Then, they are solving nonlinear flow-based model.

Every path between a generator and a demand point has its own quadratic convex risk function which depends on the path length and the path flow. However, since the path length only effects the value of the risk coefficient, the decision variable for the piece-wise linear approximation is the flow on the path. The piece-wise linear path-based model becomes

$$\begin{aligned}
\text{minimize} \quad & \sum_{p \in \mathcal{P}(g)} \phi_p(f_p), && && (3.20)\\
\text{subject to} \quad & \sum_{p \in \mathcal{P}(i)} f_p = d_i, && i \in \mathcal{N}, && (3.21)\\
& r_e \sum_{p \in \mathcal{P}(e)} f_p = \theta_j - \theta_i, && (i,j) = e \in \mathcal{E}, && (3.22)\\
& s_g^{\min} \leq \sum_{p \in \mathcal{P}(g)} f_p \leq s_g^{\max}, && g \in \mathcal{G}, && (3.23)
\end{aligned}$$

$$f_e^{\min} \leq \sum_{p \in \mathcal{P}(e)} f_p \leq f_e^{\max}, \qquad e \in \mathcal{E}, \tag{3.24}$$

$$\theta_i^{\min} \leq \theta_i \leq \theta_i^{\max}, \qquad i \in \mathcal{N}, \tag{3.25}$$

where $\phi_p(f_p)$ represents the set of approximated piece-wise linear convex functions for every path p in $\mathcal{P}(g)$. The function $\phi_p(f_p)$ is defined as

$$\phi_p(f_p) = \text{maximize} \quad \{\alpha_{pk} f_p + \delta_{pk}, k = 1, ..., m_p\} \tag{3.26}$$

where m_p denotes the number of linear pieces that is given for each path. The slopes and the intercepts are denoted by α_{p_k} and δ_{p_k}, respectively. Since a convex function is approximated, the slopes and the intercepts satisfy

$$\alpha_{p_1} \leq \alpha_{p_2} \leq ... \leq \alpha_{p_{m_p-1}} \leq \alpha_{p_{m_p}} \tag{3.27}$$

and

$$\delta_{p_1} \geq \delta_{p_2} \geq ... \geq \delta_{p_{m_p-1}} \geq \delta_{p_{m_p}}. \tag{3.28}$$

In the next chapter we will discuss how to obtain a linear programming model that can be solved by column generation.

Chapter 4

Solution Approach

The path-based model with piece-wise linear convex objective function can be solved by simplex method after a simple transformation. The drawback of this approach arises if the network includes considerable number of nodes because the increase in the number of nodes results in an exponential increase in the number of paths. This issue can be handled by column generation. However, column generation method needs a linear model with a fixed number of rows to obtain the reduced costs properly. If the piece-wise linear convex objective function is linearized by introducing rows, then column generation cannot be applied directly.

Fourer (1985, 1988, 1992) introduced a solution method for piece-wise linear convex models by introducing auxiliary variables. This approach leads to an increase in the number of constraints with respect to the number of piece-wise linear equations in the objective function. This increase in the number of rows also creates a problem for column generation as the rows depend on the generated columns. In the literature, there are also methods to solve problems with column dependent rows. One recent example is given by Muter et al. (2013).

In this thesis, we use a solution method called *Dantzig Reformulation*. This solution approach provides an equivalent linear programming model without any change in the number of constraints. However, application of this methodology causes an increase in the number of columns. This increase can again be handled by column generation.

4.1 Row-wise Expanding Linear Model

An equivalent linear programming formulation of (3.20)-(3.25) by a simple transformation using auxiliary variables. This variable defines the cost of every path between generator and demand point in the network. That is

$$z_p = \text{maximize} \quad \{\alpha_{pk} f_p + \beta_{pk}, k = 1, ..., m_p\}. \tag{4.1}$$

Then, we obtain

$$
\begin{aligned}
\text{minimize} \quad & \sum_{p \in \mathcal{P}(g)} z_p, && && (4.2)\\
\text{subject to} \quad & \sum_{p \in \mathcal{P}(i)} f_p = d_i, && i \in \mathcal{N}, && (4.3)\\
& r_e \sum_{p \in \mathcal{P}(e)} f_p = \theta_j - \theta_i, && (i,j) = e \in \mathcal{E}, && (4.4)\\
& s_g^{\min} \leq \sum_{p \in \mathcal{P}(g)} f_p \leq s_g^{\max}, && g \in \mathcal{G}, && (4.5)\\
& f_e^{\min} \leq \sum_{p \in \mathcal{P}(e)} f_p \leq f_e^{\max}, && e \in \mathcal{E}, && (4.6)\\
& \theta_i^{\min} \leq \theta_i \leq \theta_i^{\max}, && i \in \mathcal{N} && (4.7)\\
& z_p \geq \alpha_{pk} f_p + \beta_{pk}, && p \in \mathcal{P}(g),\ k = 1, ..., m_p. && (4.8)
\end{aligned}
$$

As it can be seen from the model, the constraints in (4.8) depend on p. Thus, the problem size increases row-wise as new paths are added. Even for small networks, this increase can be cumbersome. For example, suppose there are $1,200$ paths between generators and demand points in an electric network. Also, assume that the piece-wise linear approximation is done with 100 linear pieces. In this case, $1,200$ times 100 additional rows are included to the model. Especially in large scale problems, the numbers of rows and columns increase exponentially due to the number of paths in the network. Even though column generation can handle the increase in number of columns, the increase in the number of rows changes the solution approach. Therefore, we use *Dantzig Reformulation* instead of the standard reformulation, since *Dantzig Reformulation* does not add rows to the model.

4.2 Dantzig Reformulation

Dantzig (1956) reformulates the piece-wise model in a way that the increase in the number of constraints is avoided. This solution method is referred to as *Dantzig Reformulation* or *Delta Formulation.* In this reformulation, every linear piece that approximates the convex quadratic objective function is considered as a new variable. Then, the decision variable in the piece-wise objective function is described as the summation of these new variables.

Consider the piece-wise linear convex function $\phi_p(f_p)$. The connected linear pieces that generates this function have bounds with respect to the distance between the breakpoints. An illustration is given in Figure 4.1, where the breakpoints are denoted by γ_k^p.

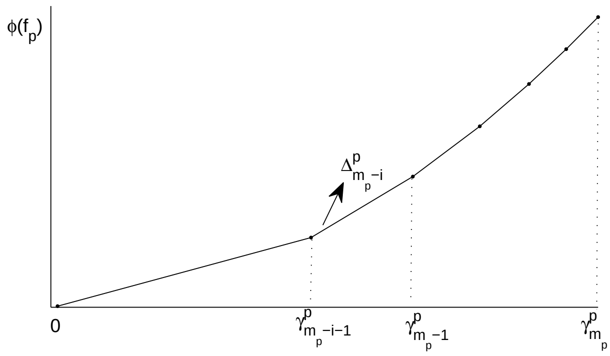

Figure 4.1: Illustration of Dantzig Reformulation

In *Dantzig Reformulation*, every linear piece is designated with a new decision variable. Summation of these variables gives the decision variable f_p.That is

$$f_p = \Delta_1^p + \Delta_2^p + ... + \Delta_{m_p}^p. \tag{4.9}$$

Then, the upper bound on Δ_k^p is simply the distance between the associated breakpoints,

$$0 \leq \Delta_k^p \leq \gamma_k^p - \gamma_{k-1}^p, \qquad k = 1...m_p. \tag{4.10}$$

The crucial point of this reformulation is that at the optimal solution Δ_k^p in (4.9) is nonzero if and only if Δ_{k-1}^p is equal to its upper bound. This situation can be interpreted through the cost perspective. The cost of these variables is represented in the objective function through the slope of the lines. Consider the slope of Δ_k^p and Δ_{k-1}^p as α_{p_k} and $\alpha_{p_{k-1}}$ respectively. Since the slopes occur in an increasing fashion, $\alpha_{p_k} \leq \alpha_{p_{k-1}}$, the simplex method will not consider Δ_k^p until Δ_{k-1}^p hits the upper bound as the coefficients of both variables are identical in the constraints. Next, we present the reformulated model:

$$\text{minimize} \quad \sum_{p \in \mathcal{P}(g)} \sum_{k \in m_p} \alpha_k^p \Delta_k^p, \tag{4.11}$$

$$\text{subject to} \quad \sum_{p \in \mathcal{P}(i)} \sum_{k \in m_p} \Delta_k^p = d_i, \qquad i \in \mathcal{N}, \tag{4.12}$$

$$r_e \sum_{p \in \mathcal{P}(e)} \sum_{k \in m_p} \Delta_k^p = \theta_j - \theta_i, \qquad (i,j) = e \in \mathcal{E}, \tag{4.13}$$

$$s_g^{\min} \leq \sum_{p \in \mathcal{P}(g)} \sum_{k \in m_p} \Delta_k^p \leq s_g^{\max}, \qquad g \in \mathcal{G}, \tag{4.14}$$

$$f_e^{\min} \leq \sum_{p \in \mathcal{P}(e)} \sum_{k \in m_p} \Delta_k^p \leq f_e^{\max}, \qquad e \in \mathcal{E}, \tag{4.15}$$

$$\theta_i^{\min} \leq \theta_i \leq \theta_i^{\max}, \qquad i \in \mathcal{N}, \tag{4.16}$$

$$0 \leq \Delta_k^p \leq \gamma_k^p - \gamma_{k-1}^p, \qquad p \in \mathcal{P}(g),\ k = 1...m_p. \tag{4.17}$$

Note that after this reformulation, the number of constraints in the original model is preserved. However, the number of columns is considerably increased as many new decision variables are introduced. In the next section, we will discuss how to apply column generation approach to (4.11)-(4.17).

4.3 Column Generation

Column generation entails a restricted master problem (RMP) and a pricing subproblem. The master problem consists of feasible and fewer number of columns than the original problem. The idea of the column generation method is to start with a fewer number of variables in the basis and then adding the promising variables to the basis iteratively (Dantzig and Wolfe, 1960). The RMP establishes the primal feasibility. However, the dual problem may not be feasible. The infeasible constraints in the dual problem corresponds to columns that should enter the basis to improve the primal objective function value. A column with a corresponding infeasible constraint is said to have a negative reduced cost. The reduced cost of a primal variable(column) is the magnitude of the infeasibility of the corresponding dual constraint. The search for a column with negative reduced cost is carried out through a pricing subproblem. The framework of the column generation approach is given in Figure 4.2.

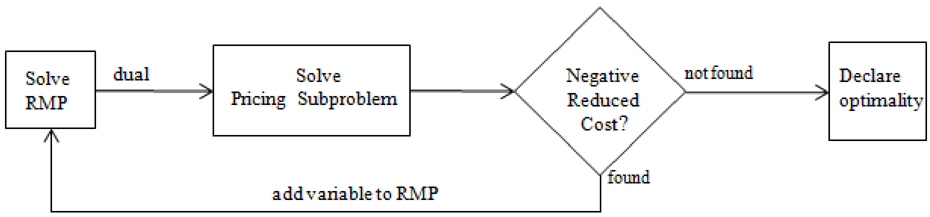

Figure 4.2: Flowchart of the column generation approach

In this thesis, the initial feasible solution for the master problem is set by using artificial variables with very high costs. The pricing subproblem is the elementary shortest path problem. This problem finds the paths that improve the objective function mostly according to their reduced costs. Then, these paths are added to the RMP in every iteration until no further negative path with a negative reduced cost is found.

The pricing subproblem searches for the paths that have negative reduced costs. Before explaining the elementary shortest path problem, the reduced cost calculation is presented. Since the reduced cost calculation is related to the dual problem, first we present the dual problem of (4.11)-(4.17):

$$\begin{aligned}
\text{maximize} \quad & \sum_{i\in\mathcal{N}} \omega_i d_i + \sum_{g\in\mathcal{G}} \lambda_g^1 s_g^{max} - \sum_{g\in\mathcal{G}} \lambda_g^2 s_g^{min} - \sum_{e\in\mathcal{E}} \alpha_e^1 f_e^{min} \\
& + \sum_{e\in\mathcal{E}} \alpha_e^2 f_e^{max} - \sum_{i\in\mathcal{N}} \mu_i^1 \theta_i^{min} + \sum_{i\in\mathcal{N}} \mu_i^2 \theta_i^{max} \qquad (4.18) \\
\text{subject to} \quad & \omega_i + \lambda_g^1 - \lambda_g^2 + \sum_{e\in p} \alpha_e^1 - \sum_{e\in p} \alpha_e^2 \\
& + \sum_{e\in p} r_e \beta_e \leq \alpha_k^p, \ p \in \mathcal{P}_g^i, \ k = 1...m_p, \ g \in \mathcal{G}, \ i \in \mathcal{N}, \qquad (4.19) \\
& \mu_i^1 - \mu_i^2, \geq 0, \ i \in \mathcal{N}, \qquad (4.20) \\
& \mu_i^1 - \mu_i^2, \geq 0, \ i \in \mathcal{N}, \qquad (4.21) \\
& \lambda_g^1, \lambda_g^2 \alpha_e^1, \alpha_e^2, \mu_i^1, \mu_i^2 \geq 0, \qquad (4.22)
\end{aligned}$$

where the dual variables $\omega_i, \lambda_g, \alpha_e, \beta_e$ and μ_n related to the constraints (4.12), (4.13), (4.14), (4.15) and (4.16) respectively. Notice that, constraints (4.14), (4.15) and (4.16) have lower bound values. Depending on the selected electric network, these values can be different than zero. In this regard, constraints (4.14), (4.15) and (4.16) are divided into two parts to make the lower bound zero. The corresponding dual variables for these constraints are defined as $\alpha_e^1, \alpha_e^2, \mu_n^1$ and μ_n^2. The reduced cost for a realizable f_p is then given by

$$\begin{aligned}
\overline{c_p} = \alpha_p^1 - \omega_i + \lambda_g^1 - \lambda_g^2 + \sum_{e\in p} \alpha_e^1 - \sum_{e\in p} \alpha_e^2 \\
+ \sum_{e\in p} r_e \beta_e, \ p \in \mathcal{P}_g^i, \ k = 1...m_p, \ g \in \mathcal{G}, \ i \in \mathcal{N}, \qquad (4.23)
\end{aligned}$$

where α_p^1 represents slope of the first linear piece of the cost function. The reformulated model has a slightly unusual reduced cost calculation due to the structure of the objective function. According to *Dantzig Reformulation*, the objective function consists of multiple cost components with respect to the slopes of the linear pieces. We assume an initial piece-wise convex generator cost function where the path length is not considered then use the slope of the first piece of this function in the reduced cost calculation. Recall that in equation (4.9), the second variable Δ_2^p is not included into the model until the first variable Δ_1^p hits its upper bound. This means that if the reduced cost of the variable corresponding to the piece does not improve the objective function, the others surely will not.

The pricing subproblem of column generation is the elementary shortest path problem. The standard shortest path problem is not used due to the cyclic structure of the network. When the standard shortest path problem is used as the pricing subproblem, negative cycles are encountered. As a result, we could not find any path to start with. For this reason, elementary shortest path problem is necessary to solve the model by column generation. However, finding the elementary paths in the network is an NP-hard problem (Feillet et al., 2004). In this regard, a label correcting algorithm of Feillet et al. (2004) is used which returns the elementary paths for every node in the network under a dominance rule. This rule reduces the computational time and avoids to encounter a path that contains a cycle. The notation and the elements in their work is slightly changed to adapt the structure of our problem. Consider the electric network, $G = (\mathcal{N}, \mathcal{E})$ where $\mathcal{E}$ is the set of edges and $\mathcal{N} = (i_1, ..., i_n)$ is the set of nodes in the network. The generator nodes are also included into this set. Consider that each elementary path from generator $g \in \mathcal{G}$ and $i \in \mathcal{N}$ belongs to the set $P_g^i = (X_{gi}^1, ..., X_{gi}^m)$. These paths create a label on node i as (R_i, C_i, L_i). To simplify the notation, we denote the reduced cost and the length of each elementary path as C_i and L_i respectively. Also, $R_i = (V_i^1, ..., V_i^n)$ where $(V_i^r = 1)$ if the path includes the node i_r. In this context, consider X'_{gi} and X^*_{gi} as two distinct paths between a generator node g and demand point i. The dominance rule states that X^*_{gi} dominates X'_{gi} if and only if $C_i^* \leq C_i'$, $L_i^* \leq L_i'$ and $V_i^{*k} \leq V_i'^k$ for $k = 1, ..., n$. Otherwise, the algorithm extends the labels to node i. The last part of the definition claims that if a label is a subset of another label, it is called as the dominant label. Therefore, the accumulation of the labels on the nodes is avoided by the domination rule. In addition, the dominance rule also prevents cycles.

Before presenting the algorithm, some additional notation is required. The array L represents the nodes that are waiting to be treated. The label list on node i_k is denoted as Γ_k. Furthermore, the successor set of node i_k is given by $Succ(i_k)$. The labels extended from node i_k to i_m is denoted by F_{km}. In addition, during the iterations of the algorithm we keep the labels which are to be treated and this structure is shown by $Treat(k)$. Finally, the details of the elementary shortest path algorithm is given by Algorithm 1. This algorithm is in fact adapted from Feillet et al. (2004).

Algorithm 1: Elementary Shortest Path Algorithm

for all $g \in \mathcal{G}$
INITIALIZATION
$\Gamma_g \leftarrow \{(0, ..., 0\}$
for all $i_k \in \mathcal{N} \setminus \{g\}$
  do $\Gamma_k \leftarrow \varnothing$
L $= \{p\}$
repeat
  Choose $i_k \in L$
  for all $i_m \in Succ(i_k)$
    do $F_{km} \leftarrow \varnothing$
      for all $(R_k, \overline{c_p}, l_p) \in \Gamma_k$
        do if $V_k^m = 0$
        then Extend label into F_{km}
        $Treat(k) \leftarrow (R_k, \overline{c_p}, l_p)$
        $L \leftarrow L \cup \{i_k\}$
    REDUCTION OF L
    $L \leftarrow L \setminus \{i_k\}$
  until $L = \varnothing$

Now we are ready to test our solution approach on two problems taken from the literature.

Chapter 5

Computational Study

In this chapter, we present our numerical results. We use MATLAB 12b and CPLEX 12.5 in our implementation. Two example electric networks are selected: $IEEE$ 14 bus network and $IEEE$ 118 bus network. The network data is taken from the University of Washington Power System Test Case Archive (Nanda et al., 1994; Blumsack, 2006).

5.1 IEEE 14 Bus Network

$IEEE$ 14 bus network structure is relatively simple due to the number of nodes in the network. There are 3 generators, 13 demand points and 20 undirected edges in the network. The generator, line and bus data for $IEEE$ 14 bus network is presented in *Appendix A*. Incorporating the risk function into the objective function of the path-based model results with a more condensed network where the generators satisfy demand in their vicinity. In this regard, the first implementation is done for IEEE 14 Bus Network and we present a comparison for two cases. First, (4.11)-(4.17) solved by column generation without considering the risk arising from the long distance electricity transmission. That is, the original convex-quadratic generator cost function is preserved. Second, the risk function is incorporated and (4.11)-(4.17) is solved by column generation. For the second case, we achieve to present a more condensed network where the generators satisfy the demand in their vicinity. The Figure 5.1 shows the implementation results of two cases for $IEEE$ 14 bus network where the number on the lines are the transmission line length.

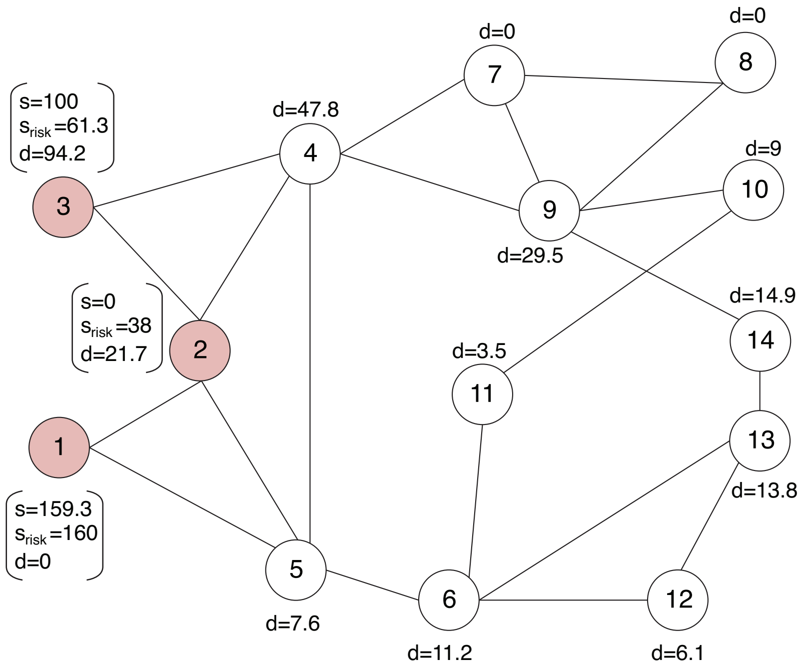

Figure 5.1: IEEE 14 Bus Network Generator Capacity for Case 1 and Case 2

In Figure 5.1, the nodes $1, 2$ and 3 indicates the generators whereas the other nodes represent the demand points in $IEEE$ 14 bus network. The amount of demand is shown below or above the demand points. The generator capacity is found under the first and second case are shown as s and s_{risk}, respectively. The results also can also be seen in the following Table 5.1.

Table 5.1: **Resulting Generator Capacities of IEEE 14 Bus Network for two cases**

Generator Number	s	s_{risk}
1	159.3	160
2	0	38
3	100	61.3

Notice that when the risk is considered, the capacity of generator 2 is increased whereas the capacity of generator 3 is decreased. The reason behind is that the path length dependent risk function promote the demand points which are closer to generator 2 than generator 3. In the first case, the demand of $12, 13$ and 14 was partially satisfied from generator 3. However, these demand points are relatively closer to generator 2. As a result, the generator capacity of generator 2 increases to satisfy the demand in its vicinity. The resulted increase is not that drastic since there are just three generators and the network is small.

Moreover, we give the same presentation for the demand side.

Table 5.2: **Average distance to satisfy demand considering the risk function for $IEEE$ 14 Bus Network**

Demand Point	25%	50%	75%	100%
1	0	0	0	0
2	500	417	10	10
3	510	407	10	10
4	461	13	13	13
5	7	7	40	40
6	100	77	110	110
7	0	0	0	0
8	0	0	0	0
9	150.5	560	0	0
10	600	95	85	85
11	107	140	140	140
12	117	117	150	150
13	160	144	144	144
14	369	141	141	141
average	220.125	151.285	60.214	60.214

We present the average distance in kilometers that is required to satisfy the 25%, 50%, 75% and 100% of the demand. First we show the results with considering the risk function in Table 5.2. Then, the risk function is not considered and the results are shown in 5.3.

Table 5.3: **Average distance to satisfy demand without the risk function for $IEEE$ 14 Bus Network**

Demand Point	25%	50%	75%	100%
1	0	0	0	0
2	416	500	500	500
3	553	510	510	510
4	355	515	515	515
5	324	598	597	597
6	494	494	494	494
7	1175	1175	1175	1175
8	0	0	0	0
9	375	555	555	555
10	431	431	431	431
11	367	1010	1010	1010
12	590	590	590	590
13	527	527	527	527
14	510	510	510	510
average	436.832	529.560	529.525	529.525

The last row of the Tables 5.2 and 5.3 show that when the risk is considered most of the demand is satisfied from closer generators. Because, the average distance found to satisfy the 75% and 100% of demand is much smaller than the average distance found to satisfy 25% and 50% of demand. However when the risk is not considered, the average distance increases. That means, considering the risk function provides a network structure where the demand is satisfied by the closer generators.

5.2 IEEE 118 Bus Network

IEEE 118 bus network can be considered as a large-scale problem due to the number of nodes and the transmission lines in it. There are 19 generators, 118 demand points and 360 edges in the network. *Appendix B* contains the generator, line and bus data tables for *IEEE* 118 Bus Network. The comparison of two cases is also given for *IEEE* 118 Bus Network. The first case does not include the risk function and the second one does. The results are presented in Figure 5.2 through a similar fashion with *IEEE* 14. However, since the *IEEE* 118 bus network is large, the figure contains the resulting generator capacities under case 1 and 2 for the selected generators 59 and 61.

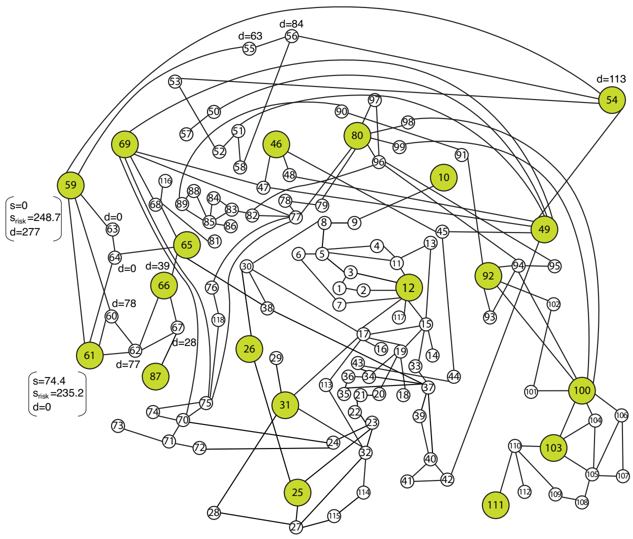

Figure 5.2: *IEEE* 118 Bus Network an example path considering the risk

The change in all of the generator capacities can be seen in the following Table 5.4.

Table 5.4: **Resulting Generator Capacities of IEEE 118 Bus Network for two cases**

Generator Number	s	s_{risk}
10	550	541.6
12	30	127.8
25	320	318.3
26	307.6	412.5
31	17	1.5
46	11.3	2
49	304	117.4
54	30	2.3
59	0	248.7
61	74.4	235.2
65	491	169.4
66	492	448.4
69	805.2	804
80	577	575.3
87	8.5	7.3
92	5	4
100	352	346
103	139.5	132
111	4.5	25.3

As it can be seen from the Figure 5.2 and the Table 5.4 the resulting capacity of generators changes when the risk is incorporated into the model. The generators 59 and 61 are two example generators whose capacities increase drastically when the risk arising from long distance electricity transmission is considered. These generators are closer to demand points 54,55,56,60,62,67,63,64 and 65. When the model is solved with the path-length dependent risk function the demand of these points mostly satisfied by generators 59 and 61. The same interpretation can be given for the remaining generators where their capacity is changed with the risk function.

In the following tables, we also present the results through the demand point perspective. In Table 5.5 and 5.6, the average distances are shown that 25%, 50%, 75% and 100% of the demand is satisfied with and without considering the risk function. The average distance is given considering all of the demand points in the network at the last row of the tables.

As it can be seen from the tables when the risk is considered, the demands are mostly satisfied from closer generators. Because, the average distance to satisfy 75% and 100% are much smaller than 25% and 50% in Table 5.5. However, in Table 5.6 the average distances found are larger than the values in Table 5.5 as expected. In this case, the demand is mostly satisfied from distant generator points. There are demand points that has the same average distance values in all columns. The reason behind is that all of the demand is satisfied by a single generator.

Table 5.5: **Average distance to satisfy demand considering the risk function for** *IEEE* 118 **Bus Network**

Demand Point	25%	50%	75%	100%	Demand Point	25%	50%	75%	100%	Demand Point	25%	50%	75%	100%
1	717	876	876	876	41	230	230	370	370	81	0	0	0	0
2	638	635	635	635	42	683	683	767	767	82	94	94	94	94
3	354	749	834	834	43	476	476	476	476	83	452	349	349	349
4	870	545	545	545	44	586	586	586	586	84	288	288	288	288
5	0	0	0	0	45	910	897	897	897	85	1083	1083	252	252
6	896	740	740	740	46	1055	194	194	194	86	1140	1140	295	295
7	824	824	824	824	47	993	993	971	971	87	0	0	0	0
8	985	401	401	401	48	0	0	0	758	88	902	282	282	282
9	0	0	0	0	49	529	803	803	803	89	0	0	0	0
10	0	0	0	0	50	279	279	249	249	90	895	895	895	895
11	838	838	838	838	51	244	244	244	100	91	1205	1205	1205	1205
12	397	615	615	615	52	981	981	981	981	92	536	819	819	819
13	657	657	657	378	53	1050	1050	1050	1000	93	446	1038	1038	1038
14	794	794	794	794	54	970	970	970	100	94	709	186	186	186
15	861	700	700	700	55	923	898	898	898	95	0	0	0	96
16	892	752	741	741	56	1056	312	312	312	96	838	718	358	358
17	563	563	563	563	57	959	178	178	178	97	0	0	0	324
18	751	726	726	726	58	968	959	959	959	98	332	332	1038	1038
19	855	826	826	826	59	1016	595	357	357	99	83	83	83	83
20	755	755	755	755	60	948	351	351	351	100	240	691	691	691
21	640	640	622	622	61	0	0	0	0	101	1164	1164	455	455
22	686	643	643	643	62	710	988	988	988	102	127	127	127	127
23	706	706	706	706	63	0	0	0	0	103	141	141	141	141
24	600	600	600	600	64	0	0	0	0	104	808	294	294	294
25	0	0	0	0	65	0	0	0	0	105	1131	529	136	136
26	0	0	0	0	66	802	453	453	420	106	652	170	170	170
27	693	693	693	693	67	807	836	879	879	107	734	952	952	200
28	789	789	692	692	68	0	0	0	0	108	568	568	568	568
29	659	659	582	582	69	0	0	0	0	109	969	697	697	697
30	0	0	0	0	70	365	349	349	349	110	1137	504	501	501
31	677	677	677	677	71	0	0	0	0	111	0	0	0	0
32	696	647	647	647	72	565	565	565	565	112	249	249	249	249
33	0	0	0	543	73	471	471	471	471	113	843	774	774	774
34	772	620.0	620.0	620.0	74	726	256	256	256	114	710	710	683	683
35	0	0	0	490	75	953	366	366	366	115	980	701	701	701
36	772	551	551	551	76	446	212	212	212	116	269	53	53	53
37	0	0	0	0	77	360	296	296	296	117	674	674	674	674
38	0	0	0	0	78	38	38	38	38	118	536	260	260	260
39	637	637	704	704	79	28	28	28	28	average	542.40	465.51	444.05	444.75
40	610	414	414	414	80	359	359	359	359					

Table 5.6: **Average distance to satisfy demand without the risk function for *IEEE* 118 Bus Network**

Demand Point	25%	50%	75%	100%	Demand Point	25%	50%	75%	100%	Demand Point	25%	50%	75%	100%
1	1072	877	877	877	41	2331	1400	976	976	81	0	0	0	0
2	2238	773	773	773	42	733	0	0	0	82	1510	953	953	953
3	856	2284	2284	2284	43	816	816	816	816	83	1701	1072	1072	1072
4	2750	343	343	343	44	1204	1204	706	706	84	2024	499	499	499
5	0	0	0	0	45	621	557	557	557	85	1277	1277	1896	1896
6	1739	780	780	780	46	1686	1869	1869	1869	86	557	557	557	557
7	2982	877	877	877	47	590	590	590	590	87	0	0	0	0
8	2558	2006	2006	2006	48	760	760	760	760	88	1952	2168	2168	2168
9	0	0	0	0	49	790	790	790	790	89	0	0	0	0
10	0	0	0	0	50	1403	1403	489	489	90	1340	809	809	809
11	2057	1348	1348	1348	51	1806	1987	1987	1987	91	3074	644	644	644
12	2373	1678	1678	1678	52	1051	1051	914	914	92	2404	764	764	764
13	2191	2191	2191	2191	53	1994	1994	632	632	93	518	518	477	477
14	963	963	908	908	54	1379	886	886	886	94	747	747	747	747
15	1446	1622	1622	1622	55	523	523	523	523	95	2993	1015	1015	1015
16	1345	1345	1345	1345	56	1618	1618	1618	1618	96	2756	1727	1727	1727
17	2980	2980	2300	2300	57	1266	1266	512	512	97	2944	2944	387	387
18	1412	1540	1540	1540	58	2696	2878	1355	1355	98	823	823	823	823
19	2499	1171	1171	1171	59	1313	1313	1313	1313	99	1409	1409	1409	1409
20	1545	2061	551	551	60	1634	2729	688	688	100	1610	1610	1610	1610
21	819	819	819	819	61	0	0	0	0	101	3481	918	509	509
22	937	937	937	937	62	698	698	698	698	102	489	489	489	489
23	1659	2099	2099	2099	63	0	0	0	0	103	1790	699	699	699
24	510	510	510	510	64	0	0	0	0	104	1882	422	422	422
25	0	0	0	0	65	0	0	0	0	105	3032	1653	1653	1653
26	0	0	0	0	66	1758	2073	2073	2073	106	1404	677	677	677
27	991	991	991	991	67	1737	518	518	518	107	1693	576	576	576
28	2369	2369	1344	1344	68	0	0	0	0	108	1711	1711	1719	1719
29	1121	1121	1121	1121	69	0	0	0	0	109	796	796	796	796
30	0	0	0	0	70	1144	1144	1144	1144	110	1443	402	402	402
31	1110	1110	1110	1110	71	0	0	0	0	111	0	0	0	0
32	1867	1125	1449	1449	72	1007	1007	1185	1185	112	2082	238	238	238
33	898	898	1782	1782	73	524	524	524	524	113	1089	1089	1089	1089
34	897	897	897	897	74	1019	1019	1019	1019	114	1878	1878	1878	1878
35	1043	931	931	931	75	1015	901	901	901	115	2285	1046	622	622
36	827	827	827	827	76	1958	1974	1974	1974	116	1415	602	602	602
37	0	0	0	0	77	1179	2318	2318	2318	117	2314	2314	762	762
38	0	0	0	0	78	512	452	452	452	118	892	892	892	1321
39	1928	643	575	575	79	649	649	649	649	average	1281.86	995.87	877.56	881.20
40	1540	1540	1540	1540	80	1010	1010	1010	1010					

5.3 Sensitivity Analysis

We present a sensitivity analysis for different values of the constant element in the risk coefficient function. Recall that, in Chapter 3 the risk function is given as the multiplication of the original convex quadratic function and the risk coefficient. The risk coefficient function $\beta(l_p)$ has a constant element which is given as t. In this section, a sensitivity analysis is given for three different values of the constant element t. The constant element is taken as 2,3 and 4 and implementation is done for $IEEE$ 14 and $IEEE$ 118 Bus Networks. The values in the tables shows the average distance between the generators and the demand points to satisfy the demand. We use the following calculation which is:

$$\text{average distance} = \frac{\sum_{p\in\mathcal{P}(g)} l_p f_p}{\sum_{p\in\mathcal{P}(g)} f_p},\ g \in \mathcal{G}.$$

The last row of the tables shows the average distance which considers all of the generators in the network. This row indicates the result that when the risk coefficient becomes larger, generators satisfy the demand of the closer points. In both of the tables, the average distance to satisfy the demand becomes smaller when t is increased. In addition, There are cases of generators that shows an average distance increase when t increases. The reason is due to the capacity limit over the generators.

Table 5.7: **Sensitivity Analysis for $IEEE$ 14 Bus Network**

Generator Number	$2 - e^{l_p}$	$3 - e^{l_p}$	$4 - e^{l_p}$
1	478	475	451
2	36	63	62
3	91	90	92
average	326.04	324.74	308.56

Table 5.8: **Sensitivity Analysis for** $IEEE$ 118 **Bus Network**

Generator Number	$2-e^{l_p}$	$3-e^{l_p}$	$4-e^{l_p}$
10	1032	956	672
12	775	734	841
25	552	710	733
26	969	845	862
31	915	1001	775
46	272	506	384
49	589	506	209
54	769	408	776
59	718	710	551
61	630	572	631
65	413	504	628
66	549	581	510
69	355	342	304
80	346	295	362
87	672	547	728
92	1089	615	824
100	356	500	462
103	348	395	865
111	726	651	425
average	589.051	579.324	565.637

Chapter 6

Conclusion and Future Work

This thesis incorporates the risk arising from the long distance electricity transmission into an electric power optimization model. As mentioned, electricity transmission can disrupted by many unexpected outside factors. This disruption creates risk of encountering a situation where the demand may not be met. This risk becomes more crucial when the distance between supply and demand is large. The risk function that we consider considers both of these facts. As a result, we achieve to present a more condensed network structure where the generators satisfy the demand in their vicinity. We achieve this result by the risk function that considers path length and path flow. Our work differs from the works in the literature through the incorporation of the risk function into the power flow optimization model. We utilize one example of the outside factor which is the incurred voltage drop due to line losses. However, the risk function can be improved to obtain a more accurate and realistic function that contains all of the risk factors for a future study.

Recall that, the linear flow-based model is decomposed into a path-based model to incorporate the risk function. This incorporation can also be presented for the flow-based problem. A similar risk function that considers the risk on each of the transmission line can be incorporated into the objective function of the model. However, due to structure of the risk function, flow-based problem will result in an overestimation of risk. This overestimation can be observed in flow-based problem in a future study.

In this thesis, we use an electric network optimization model where the objective is to minimize the convex quadratic risk function subject to linear constraints. We employ column generation method to solve the path-based model. First, the convex-quadratic objective function is approximated by piece-wise linear functions. Afterwards, an equivalent linear programming model for the piece-wise model is given to apply the column generation method. The *Dantzig Reformulation* method avoids the increase in number of rows which improves the computational time of the column generation. For future work, the solution methodology which is proposed by Muter et al. (2013) can be applied and solution performance can be compared.

In addition; the linear programming network optimization model that we use is an approximation of the original non-linear and non-convex optimal power flow model. A more realistic approach should have been given, if a nonlinear model is used. In this thesis, we neglect some of the power flow equations due to using the DC approximated model. However, the selected scope and the given time to propose such a model was not enough.

In conclusion, we successfully incorporate the risk function into the path-based model and also present a more condensed network for IEEE 14 Bus Network and IEEE 118 Bus Network. This implementation can be done for larger networks where the number of nodes can be between 250-3000. The power optimization in electric networks has a wide research area. There are still voids in model where these voids can be fulfilled with further improvement in implementation and also in the mathematical modeling.

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Appendix A

The data of IEEE 14 Bus Network is shown in the following tables. The minimum and maximum vales for phase angles is taken as $-45°$ and $-45°$. The data does not cover all of the information about the transmission system. Some part of the data where it is taken from Nanda et al. (1994). The line length values are randomly generated.

Table 1: **Generator Capacity and Cost Coefficients**

Generator Number	$s_g^{\min}$	$s_g^{\max}$	a	b	c
1	10	160	0.005	2.45	105
2	20	150	0.005	3.51	44.1
3	20	100	0.005	3.89	40.6

Table 2: **Line Data**

Line Number	From Bus	To Bus	Resistance *p.u.*	$F_e^{\min}$	$F_e^{\max}$	Line Length (km)
1	1	2	0.01938	-220	220	300
2	1	5	0.05403	-220	220	400
3	2	3	0.04699	-220	220	10
4	2	4	0.05811	-220	220	20
5	2	5	0.05695	-220	220	30
6	3	4	0.06701	-220	220	5
7	4	5	0.01335	-220	220	2
8	4	7	0	-220	220	60
9	4	9	0	-220	220	40
10	5	6	0	-220	220	70
11	6	11	0.09498	-220	220	30
12	6	12	0.12291	-220	220	40
13	6	13	0.06615	-220	220	50
14	7	8	0	-220	220	60
15	7	9	0	-220	220	10
16	9	10	0.03181	-220	220	40
17	9	14	0.12711	-220	220	100
18	10	11	0.08205	-220	220	100
19	12	13	0.22092	-220	220	5
20	13	14	0.17093	-220	220	10

Table 3: **Bus Data**

Bus Number	Real Power Demand	$\theta_i^{\min}$	$\theta_i^{\max}$
1	0	-45	45
2	21.7	-45	45
3	94.2	-45	45
4	47.8	-45	45
5	7.6	-45	45
6	11.2	-45	45
7	0	-45	45
8	0	-45	45
9	29.5	-45	45
10	9	-45	45
11	3.5	-45	45
12	6.1	-45	45
13	13.8	-45	45
14	14.9	-45	45

Appendix B

The data of IEEE 14 Bus Network is shown in the following tables. The minimum and maximum vales for phase angles is taken as 180−° and −180°. The data does not cover all of the information about the transmission system. Some part of the data is taken from Blumsack (2006) and Washington University Test Case Archive. The generator cost data is randomly generated.

Table 4: **Generator Capacity and Cost Coefficients**

Generator Number	$s_g^{\min}$	$s_g^{\max}$	a	b	c
10	0	550	0.009	7.24	89.5
12	0	185	0.003	6.42	72.8
25	0	320	0.002	7.17	81.8
26	0	414	0.002	4.68	50.0
31	0	107	0.006	3.26	81.0
46	0	119	0.006	4.40	9.6
49	0	304	0.010	7.30	21.9
54	0	148	0.007	9.94	25.9
59	0	255	0.007	6.77	46.8
61	0	260	0.007	7.91	45.9
65	0	491	0.007	1.71	71.0
66	0	492	0.010	0.27	17.8
69	0	805.2	0	8.00	53.1
80	0	577	0.001	9.04	16.8
87	0	104	0.003	0.25	76.9
92	0	100	0.007	4.92	92.8
100	0	352	0.008	5.26	60.9
103	0	140	0.009	5.96	15.0
111	0	136	0.010	0.52	49.0

Table 5: **Line Data**

Line Number	From Bus	To Bus	Resistance $p.u.$	$F_e^{\min}$	$F_e^{\max}$	Line Length (km)
1	1	2	0.0303	-220	220	48
2	1	3	0.0129	-220	220	20.4
3	2	12	0.0187	-220	220	29.6
4	3	5	0.0241	-220	220	43.7
5	3	12	0.0484	-220	220	76.8
6	4	5	0.00176	-220	440	3.2
7	4	11	0.0209	-220	220	33.1
8	5	6	0.0119	-220	220	21.7
9	5	11	0.0203	-220	220	32.4
10	6	7	0.00459	-220	220	8.4
11	7	12	0.00862	-220	220	14.7
12	8	9	0.00244	-220	1100	90.5
13	8	5	0	-220	880	90.5
14	8	30	0.00431	-220	220	154.3
15	9	10	0.00258	-220	1100	95.6
16	11	12	0.00595	-220	220	9.4
17	11	13	0.02225	-220	220	35.2
18	12	15	0.0215	-220	220	34
19	12	17	0.0212	-220	220	36.2
20	12	117	0.0329	-220	220	58.3
21	13	15	0.0744	-220	220	117.8
22	14	15	0.0595	-220	220	94.1
23	15	17	0.0132	-220	440	21
24	15	19	0.012	-220	220	19
25	15	33	0.038	-220	220	60.1
26	16	17	0.0454	-220	220	77.9
27	17	19	0.0123	-220	220	21.4
28	17	31	0.0474	-220	220	75.2
29	17	113	0.00913	-220	220	14.5
30	18	19	0.01119	-220	220	20.8
31	19	20	0.0252	-220	220	46.5
32	19	34	0.0752	-220	220	119
33	20	21	0.0183	-220	220	33.8
34	21	22	0.0209	-220	220	38.6
35	22	23	0.0342	-220	220	63.2
36	23	24	0.0135	-220	220	22.3
37	23	25	0.0156	-220	440	30.3
38	23	32	0.0317	-220	220	52.3
39	24	70	0.00221	-220	220	176.5
40	24	72	0.0488	-220	220	84.2

Line Number	From Bus	To Bus	Resistance $p.u.$	$F_e^{\min}$	$F_e^{\max}$	Line Length (km)
41	25	27	0.0318	-220	440	61.7
42	26	25	0	-220	220	61.7
43	26	30	0.00799	-220	660	274.2
44	27	28	0.01913	-220	220	34.7
45	27	32	0.0229	-220	220	36.3
46	27	115	0.0164	-220	220	29.9
47	28	31	0.0237	-220	220	29.9
48	29	31	0.0108	-220	220	40.7
49	30	17	0	-220	660	16.6
50	30	38	0.00464	-220	220	165.7
51	31	32	0.0298	-220	220	47.3
52	32	113	0.0615	-220	220	97.5
53	32	114	0.0135	-220	220	24.6
54	33	37	0.0415	-220	220	66.8
55	34	36	0.00871	-220	220	13.4
56	34	37	0.00256	-220	440	4.2
57	34	43	0.0413	-220	220	71.7
58	35	36	0.00224	-220	220	4.1
59	35	37	0.011	-220	220	4.1
60	37	39	0.0321	-220	220	50.9
61	37	40	0.0593	-220	220	88.7
62	38	37	0	-220	660	88.7
63	38	65	0.00901	-220	440	311.8
64	39	40	0.0184	-220	220	29.1
65	40	41	0.0145	-220	220	23
66	40	42	0.0555	-220	220	88
67	41	42	0.041	-220	220	65
68	42	49	0.0715	-220	220	130.3
69	43	44	0.0608	-220	220	105.1
70	44	45	0.0224	-220	220	38.7
71	45	46	0.04	-220	220	64.1
72	45	49	0.0684	-220	220	100.8
73	46	47	0.038	-220	220	60.6
74	46	48	0.0601	-220	220	93.5
75	47	49	0.0191	-220	220	30.2
76	47	69	0.0844	-220	220	133.7
77	48	49	0.0179	-220	220	26.7
78	49	50	0.0267	-220	220	39.8
79	49	51	0.0486	-220	220	72.5
80	49	54	0.073	-220	220	125.1

Line Number	From Bus	To Bus	Resistance $p.u.$	$F_e^{\min}$	$F_e^{\max}$	Line Length (km)
81	49	66	0.018	-220	440	34.8
82	49	69	0.0985	-220	220	156
83	50	57	0.0474	-220	220	70.8
84	51	52	0.0203	-220	220	30.6
85	51	58	0.0255	-220	220	38.1
86	52	53	0.0405	-220	220	70
87	53	54	0.0263	-220	220	48.5
88	54	55	0.0169	-220	220	29.7
89	54	56	0.00275	-220	220	4.5
90	54	59	0.0503	-220	220	92
91	55	56	0.00488	-220	220	7.5
92	55	59	0.04739	-220	220	86.7
93	56	57	0.0343	-220	220	51.2
94	56	58	0.0343	-220	220	51.2
95	56	59	0.0825	-220	220	126.7
96	59	60	0.0317	-220	220	58.1
97	59	61	0.0328	-220	220	60.1
98	60	61	0.00264	-220	440	5.1
99	60	62	0.0123	-220	220	22.5
100	61	62	0.00824	-220	220	15.1
101	62	66	0.0482	-220	220	87.9
102	62	67	0.0258	-220	220	47.1
103	63	59	0	-220	440	47.1
104	63	64	0.00172	-220	440	61.4
105	64	61	0	-220	220	61.4
106	64	65	0.00269	-220	440	94.3
107	65	66	0	-220	220	94.3
108	65	68	0.00138	-220	220	49.2
109	66	67	0.0224	-220	220	40.9
110	68	69	0	-220	440	40.9
111	68	81	0.00175	-220	220	62.2
112	68	116	0.00034	-220	440	12.3
113	69	70	0.03	-220	440	53
114	69	75	0.0405	-220	440	62
115	69	77	0.0309	-220	220	48.8
116	70	71	0.00882	-220	220	15.2
117	70	74	0.0401	-220	220	63.6
118	70	75	0.0428	-220	220	67.8
119	71	72	0.0446	-220	220	77.1
120	71	73	0.00866	-220	220	17

Line Number	From Bus	To Bus	Resistance $p.u.$	$F_e^{\min}$	$F_e^{\max}$	Line Length (km)
121	74	75	0.0123	-220	220	19.5
122	75	77	0.0601	-220	220	95.6
123	75	118	0.0145	-220	220	23
124	76	77	0.0444	-220	220	70.7
125	76	118	0.0164	-220	220	26.1
126	77	78	0.00376	-220	220	6
127	77	80	0.017	-220	440	25.5
128	77	82	0.0298	-220	220	44.7
129	78	79	0.00546	-220	220	9.9
130	79	80	0.0156	-220	220	28.4
131	80	96	0.0356	-220	220	69
132	80	97	0.0183	-220	220	35.4
133	80	98	0.0238	-220	220	43.5
134	80	99	0.0454	-220	220	82.9
135	81	80	0	-220	220	82.9
136	82	83	0.0112	-220	220	17.7
137	82	96	0.0162	-220	220	25.6
138	83	84	0.0625	-220	220	74.6
139	83	85	0.043	-220	220	69.4
140	84	85	0.0302	-220	220	36.2
141	85	86	0.035	-220	220	57
142	85	88	0.02	-220	220	38.7
143	85	89	0.0239	-220	220	169.1
144	86	87	0.02828	-220	220	201.5
145	88	89	0.0139	-220	440	27
146	89	90	0.0518	-220	660	85.5
147	89	91	0.0099	-220	220	85.5
148	89	92	0.0099	-220	220	67.8
149	90	91	0.0254	-220	660	40.2
150	91	92	0.0387	-220	220	61.3
151	92	93	0.0258	-220	220	40.9
152	92	94	0.0481	-220	220	76.1
153	92	100	0.0648	-220	220	118.5
154	92	102	0.0123	-220	220	22.5
155	93	94	0.0223	-220	220	35.3
156	94	95	0.0132	-220	220	20.9
157	94	96	0.0269	-220	220	42.3
158	94	100	0.0178	-220	220	28.1
159	95	96	0.0171	-220	220	26.8
160	96	97	0.0173	-220	220	33.5

Line Number	From Bus	To Bus	Resistance $p.u.$	$F_e^{\min}$	$F_e^{\max}$	Line Length (km)
161	98	100	0.0397	-220	220	72.3
162	99	100	0.018	-220	220	32.8
163	100	101	0.0277	-220	220	50.7
164	100	103	0.016	-220	440	25.3
165	100	104	0.0451	-220	220	82
166	100	106	0.0605	-220	220	101.6
167	101	102	0.0246	-220	220	45
168	103	104	0.0466	-220	220	74.8
169	103	105	0.0535	-220	220	82.1
170	103	110	0.03906	-220	220	72.1
171	104	105	0.00994	-220	220	16.7
172	105	106	0.014	-220	220	23.9
173	105	107	0.053	-220	220	85.6
174	105	108	0.0261	-220	220	38.3
175	106	107	0.053	-220	220	85.6
176	108	109	0.0105	-220	220	15.5
177	109	110	0.0278	-220	220	41.1
178	110	111	0.022	-220	220	35.5
179	110	112	0.0247	-220	220	35.8
180	114	115	0.0023	-220	220	4.2

Bus Number	Real Power Demand	$\theta_i^{\min}$	$\theta_i^{\max}$
1	51	-180	180
2	20	-180	180
3	39	-180	180
4	39	-180	180
5	0	-180	180
6	52	-180	180
7	19	-180	180
8	28	-180	180
9	0	-180	180
10	0	-180	180
11	70	-180	180
12	47	-180	180
13	34	-180	180
14	14	-180	180
15	90	-180	180
16	25	-180	180
17	11	-180	180
18	60	-180	180
19	45	-180	180
20	18	-180	180
21	14	-180	180
22	10	-180	180
23	7	-180	180
24	13	-180	180
25	0	-180	180
26	0	-180	180
27	71	-180	180
28	17	-180	180
29	24	-180	180
30	0	-180	180
31	43	-180	180
32	59	-180	180
33	23	-180	180
34	59	-180	180
35	33	-180	180
36	31	-180	180
37	0	-180	180
38	0	-180	180
39	27	-180	180
40	66	-180	180

Table 6: **Bus Data**

Bus Number	Real Power Demand	$\theta_i^{\min}$	$\theta_i^{\max}$
41	37	-180	180
42	96	-180	180
43	18	-180	180
44	16	-180	180
45	53	-180	180
46	28	-180	180
47	34	-180	180
48	20	-180	180
49	87	-180	180
50	17	-180	180
51	17	-180	180
52	18	-180	180
53	23	-180	180
54	113	-180	180
55	63	-180	180
56	84	-180	180
57	12	-180	180
58	12	-180	180
59	277	-180	180
60	78	-180	180
61	0	-180	180
62	77	-180	180
63	0	-180	180
64	0	-180	180
65	0	-180	180
66	39	-180	180
67	28	-180	180
68	0	-180	180
69	0	-180	180
70	66	-180	180
71	0	-180	180
72	12	-180	180
73	6	-180	180
74	68	-180	180
75	47	-180	180
76	68	-180	180
77	61	-180	180
78	71	-180	180
79	39	-180	180
80	130	-180	180

Bus Number	Real Power Demand	$\theta_i^{\min}$	$\theta_i^{\max}$
81	0	-180	180
82	54	-180	180
83	20	-180	180
84	11	-180	180
85	24	-180	180
86	21	-180	180
87	0	-180	180
88	48	-180	180
89	0	-180	180
90	440	-180	180
91	10	-180	180
92	65	-180	180
93	12	-180	180
94	30	-180	180
95	42	-180	180
96	38	-180	180
97	15	-180	180
98	34	-180	180
99	42	-180	180
100	37	-180	180
101	22	-180	180
102	5	-180	180
103	23	-180	180
104	38	-180	180
105	31	-180	180
106	43	-180	180
107	50	-180	180
108	2	-180	180
109	8	-180	180
110	39	-180	180
111	0	-180	180
112	68	-180	180
113	6	-180	180
114	8	-180	180
115	22	-180	180
116	184	-180	180
117	20	-180	180
118	33	-180	180