Estimating the Social Gap with A Game Theory Model of Lane Changing

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Abstract—**Changing lanes is a commonly-used technique for drivers to either overtake slow-moving cars or enter/exit highway ramps. Optional lane changes may save drivers travel time but increase the risk of collision with others. Drivers make such decisions based on experience and emotion rather than analysis, and thus may fail to select the best solution while in a dynamic state of flux. Unlike human drivers, autonomous vehicles can systematically analyze their surroundings and make real-time decisions accordingly. This paper develops a game theory-based lane-changing model by comparing two types of optimization methods. To realize our expectations, we need to first investigate the payoff function of drivers in discretionary lane-changing maneuvers and then quantify it in an equation of costs that trades-off safety and time-saving. After the evaluation for each alternative strategy combination, the results show that there exists a social gap in the discretionary lane-changing game. To deal with that problem, we provide some suggestions for future policy as well as autonomous vehicle controller designs, offering solutions to reduce the impact of disturbances and crashes caused by inappropriate lane changes, and also, inspire further research about more complex cases.**

Index Terms—**Discretionary lane changing, game theory, autonomous vehicles, human-driven vehicles, social dilemma**

I. Introduction

DESPITE their low frequency, lane-changing (LC) maneuvers result in 5% of all crashes and 7% of all road fatalities [1]. In the US, two-vehicle lane change crashes comprised 9% of all police-reported automobile crashes [2]. Some research finds that LC is the main reason for shockwaves and congestion reducing road efficiency [3], [4]. 'Smart' lane changes could fill gaps in traffic, weakening the impacts of blockage by slow-moving cars [5]. Ideally drivers learn from driver training and experience to decide how and when it is appropriate to change lanes by checking mirrors to observe the surroundings, but these checks are unreliable in complex or low visibility scenarios. Autonomous Vehicles (AV), with their anticipated capability of detecting traffic conditions with sensors and cameras and short response time, are expected to complete this task with fewer adverse safety outcomes [6], [7]. However, the effects on traffic flow as a whole from AV lane changes in mixed or autonomous-only traffic has yet to be determined.

For human drivers, in general, the reasoning decisions are executed due to the possibility, necessity, and desirability of LC behaviors [8]. As with many things, LC decisions must trade-off between 'greed' vs. 'fear'. Drivers are 'greedy' to save time but they also 'fear' the loss associated with crashes. Efforts to prevent crashes, such as paying attention during

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driving activities, also influence the utility [9]–[11]. The aim of these research efforts is to determine a comprehensive driver lane-changing payoff function for exploring the conditions of safe and appropriate lane changes to improve social welfare.

Lane changing factors include speed advantage (the desired lane is faster than the current one) [8], [12], [13], acceleration advantage [14], and safety issues [15], [16]. However, the hypotheses, as mentioned above, cannot be directly verified by datasets collected from real road sections because even though drivers' actual behaviors are observable, the underlying processes of their consideration are not, and actual execution remains hard to predict. At the same time, the heterogeneity of drivers (who are from different backgrounds, genders, ages, etc.) increases the complexity of prediction.

The continuous process of all driver activities like changing lanes on roads has been summarized into three main levels: strategic, tactical, and operational [17]. Game theory (GT) can explain for the first two levels [18], [19]. GT has been widely implemented in various fields for interpretation of the human decision-making process. In the domain of driver LC strategies, game theory was introduced by Kita [20], creating a new type of behavioral model. That model demonstrates the feasibility of game theory to describe the interaction of drivers in LC scenarios and inspired other similar research [21]–[24]. To facilitate this complex scenario, most of the models assumed a two-player game with two strategies and complete information for each player. The merging car has two alternatives: execute the lane change or stay in the current lane. The through car driving in the target lane can also choose to either yield to the merging car, or not yield but accelerate to block the intended lane change.

This article proposes an innovative discretionary lane-changing (DLC) framework based on classic game theory, which also considers social cooperation behaviors. Cooperative players behave in a way that benefits the system, non-cooperative players benefit only themselves. A lane-changing vehicle may behave non-cooperatively, causing vehicles in the target lane to brake, and, though we do not consider it here, congest an already congested lane while departing a less congested lane. A driver in the target lane may behave non-cooperatively in a way to make the lane changing vehicle's strategy more difficult by not yielding. Unlike other research, the model compares user-optimal (UO) behaviors (like most human drivers) and social-optimal (SO) behaviors (like hypothetical selfless drivers, but possible for AVs under specific algorithms). By comparing the results of these two models, we demonstrate in which cases drivers would 'cooperate' with each other or 'defect', behaving non-cooperatively in game

theory terms, and work selfishly. We believe this research provides a framework to promote better AV algorithm design for more efficient and safer roads.

A companion paper reviews recent GT-based models of lane changing in depth [25]. The rest of this paper is structured as follows. In section II, we further discuss and compare UO and SO models. We then introduce the methodology of the proposed model and the details of the case study in section III. The model is validated in section IV. In section V, the results from the study are examined if the model is feasible to explain LC maneuvers. In the meanwhile, by comparing the test results of the UO controller and the SO controller, we find the reasons why in some cases drivers behave aggressively but sometimes not, and try to explore ways to change the game rules. Finally, in section VI, we summarize our findings, conclude, and provide some expectations for future investigation.

II. REVIEW OF GAME THEORETIC MODELS

For the purpose of highlighting the difference between UO and SO behaviors, this section compares previous models that apply these two algorithms. Based on the priority of the client itself or the whole traffic system, a new categorization system of GT-based LC models is created including user-optimal models that promote the driver's own benefit only, social-optimal models that maximize the total welfare of the society, and hybrid user and social-optimized models that balance user experience and system efficiency.

A. User-optimal models

Following Nash's User Equilibrium principle, drivers compete to achieve the maximum net individual benefit subject to the same selfish choices of others [22]. Most GT models choose the UO method not only due to the well-developed concept but also because of the simple computation algorithm [26]–[28]. These models retain the assumption that all drivers are rational, and they fully (with complete information) or partly (with incomplete information) understand each other.

Although the UO models perform well in many real cases and may be applied in current AV algorithms, they fail to consider the behaviors of cooperators also playing games. That may cause UO agents to wrongly predict others' strategies for interactions with selfless human drivers because they make decisions on the premise that all players are selfishly rational. Furthermore, though assuming all players in the traffic system follow these models, they may not achieve a win-win result since they reduce their opponents' benefits even if the overall situation worsens. Consequently, the models that optimize their strategies based on classic Nash Equilibrium are not appropriate to simulate the scenarios with the occurrence of the conflict between individuals and the group. There should be another type of model to describe drivers' cooperative behaviors for a better outcome with that conflict.

B. Social-optimal models

The strategies from Nash's User Equilibrium might reduce the total payoff to society because concern for individual short-term benefits ignores the group's long-term welfare [29], [30]. Evolutionary Game Theory and its social viscosity mechanisms can increase cooperation rates, and improve system efficiency [30], [31]. The mechanisms mainly aim to establish internal relationships among individuals or external relationships of groups, so players in games will gradually cooperate more frequently if they know their opponents well. More details about the reciprocity mechanisms can be seen in [32]. Nevertheless, due to its long evolutionary process and high reliance on specific conditions like initial cooperation rates and specific densities, the EGT-based models must be modified to enable more realistic LC games.

There is another problem to be solved that before knowing that the opponent is willing to cooperate to get more benefits, the player may feel afraid that the opponent will aggressively 'defect' (exploit their cooperative behavior) so the driver may in the end be unwilling to cooperate. While we cannot change their minds or stifle their human nature, it may be possible to adjust the game rules to make the competition fair to both cooperators and non-cooperators to encourage the likelihood that more socially beneficial payoffs result. Alternatively, to increase system efficiency in congested conditions, authorities can provide preferential treatments to encourage cooperative behavior, or discourage or restrict non-cooperative agents from their aggressive behavior. In uncongested scenarios, such as freely flowing sections, authorities can permit drivers to behave selfishly without penalty.

C. Hybrid (User and Social-optimal) models

A combined cost function includes both non-cooperative and cooperative costs for connected vehicles when regarding the latter as the extra cost of the control action [33] . A recent study also considered possible charges for selfish LC behaviors (like changing lanes in an over-crowded road section) to reduce the congestion designed for connected vehicles [34].

III. METHODOLOGY

The main differences of various GT-based models are the factors considered in the payoff functions and the solution(s) to optimize for either the individual or the system. This section first presents the structure of the payoff function and optimization methods of our proposed model. After that, simulation experiment tests whether the model performs well with the calibrated parameters after validation by real datasets.

A. Energy vs. Momentum Conservation Equations

The safety variables can be understood from two perspectives: the severity and the probability of crashes. Before the estimation of the overall safety cost, we suggest a new method for severity evaluation, which shows a better fit to estimate the impact of crashes than previous studies. The theory behind this method is that the kinetic energy due to the vehicle's motion suddenly declines during a collision. Although most of the energy is released in the form of heat and sound, the rest of the impact energy is absorbed by structural deformation of the vehicle's metal and by the occupants themselves. Occupants,

belted or otherwise, are finally forced to collide with the interior surface, causing injuries.

We compare this approach with the widely-used Delta-V model recommended by National Highway Traffic Safety Administration (NHTSA) [35].

$$\Delta v_1 = -\frac{m_2(v_1 - v_2)}{m_1 + m_2}, \qquad \Delta v_2 = \frac{m_1(v_1 - v_2)}{m_1 + m_2} \quad (1)$$

where:
1 and 2 identify the two vehicles;
$\Delta v_{1,2}$ is the Delta-V of vehicle 1 or 2 in this collision;
$m_{1,2}$ is the dry weight of vehicle 1 or 2;
$v_{1,2}$ is the collision speed of vehicle 1 or 2;

By combining the Energy Conservation Equation and the Momentum Conservation Equation, the energy loss W_L from a specific crash can be computed when assuming a perfectly inelastic collision (vehicles 'stick to each other' after the collision):

$$W_L = 0.5 \cdot \frac{m_1 m_2 (v_1 - v_2)^2}{(m_1 + m_2)} \quad (2)$$

The energy loss in the collision then distributes to two vehicles and finally imposes on occupants according to two effects of the mass ratio of vehicles: 'hostile effect' and 'protective effect' [36]. The hostile effect refers to the effect that the lighter vehicle tends to receive a more substantial Delta-V in a crash, as demonstrated in (1). Furthermore, another effect called the protective effect means heavy vehicles usually contain more inherent protection and other designs that help to absorb the impact energy from a crash, but it also depends on the size and structure of vehicles. Therefore, the power of the mass ratio α will be estimated later by the regression from real data.

$$\frac{W_{L1}}{W_{L2}} = \left(\frac{m_1}{m_2}\right)^{\alpha} \quad (3)$$

$$W_{L1} = 0.5 \cdot \frac{(\frac{m_1}{m_2})^{\alpha}}{1 + (\frac{m_1}{m_2})^{\alpha}} \frac{m_1 m_2 (v_1 - v_2)^2}{m_1 + m_2}$$
$$W_{L2} = 0.5 \cdot \frac{(\frac{m_2}{m_1})^{\alpha}}{1 + (\frac{m_2}{m_1})^{\alpha}} \frac{m_1 m_2 (v_1 - v_2)^2}{m_1 + m_2} \quad (4)$$

in which:
$W_{L1,2}$ is the energy absorption by vehicle 1 or 2;
α is the power of the mass ratio that reflects the individual energy absorption.

Also, the energy loss equation for each vehicle can be transformed with the forms of Delta-V as,

$$W_{L1} = 0.5 \cdot \frac{(\frac{m_1}{m_2})^{\alpha-1}}{1 + (\frac{m_1}{m_2})^{\alpha}} (m_1 + m_2) \Delta v_1^2$$
$$W_{L2} = 0.5 \cdot \frac{(\frac{m_2}{m_1})^{\alpha-1}}{1 + (\frac{m_2}{m_1})^{\alpha}} (m_1 + m_2) \Delta v_2^2 \quad (5)$$

TABLE I
COMPARISON OF TWO TYPES OF SEVERITY ESTIMATED VARIABLES

	Estimate	Standard Error	$Pr(> \lvert z \rvert)$	AIC	Deviance
Delta-V	0.004161	0.001387	0.00271	3334.09	3325.59
ELVIS	5.69E-08	1.82E-08	0.00172	3333.27	3324.22

B. Expected crash cost estimation function

We hypothesize that when the value of the energy loss increases, the cost of the crash rises rapidly. The relationships are given as:

$$\frac{\partial C_L}{\partial W_L} > 0 \qquad \frac{\partial W_L}{\partial V_M} > 0 \quad (6)$$

where:
C_L denotes the expected crash cost, and
V_M is the category of Maximum Abbreviated Injury Scale (MAIS) (from 0 to 6+).

According to these relationships, we establish an energy loss-based vehicular injury severity function (ELVIS). We estimate the power ratio parameter α from the injury causation datasets of Crash Injury Research and Engineering Network (CIREN) [37]. The analysis includes two-vehicle crashes only. The parameter β unifies crash loss per collision with severity probability and time cost into the same quantity and it is calibrated with real data.

$$C_L = \beta \cdot (C_S(V_M) \cdot P_S(W_L)) \quad (7)$$

where:
C_L is the estimated crash loss from the expected collision;
$C_S(V_M)$ is the corresponding empirical cost at each level of MAIS;
$P_S(W_L)$ is the maximum severity probability of each level of MAIS based on the energy loss.

The severity measurement is represented by MAIS that judges the most serious occupant injury in an observed crash. MAIS classifies crashes into 7 categories: 0- Property Damage Only, 1-Minor Injury, 2-Moderate Injury, 3-Serious Injury, 4-Severe Injury, 5-Major Injury, and 6^+-Fatality (Untreatable cases). Because of the categorical and ordinal nature of variable MAIS, we calibrate the function using an ordered logistic regression model with a probit link function due to its better fit compared to logit.

To test the validity of the severity function, the model based on the Delta-V method will be introduced to compare with the ELVIS function by the Akaike information criterion (AIC) [38] and deviance. Moreover, P-value tests are also conducted to check whether the models are statistically significant or not. When $\alpha = -0.5$, the results show that ELVIS fits the injuries and fatalities better, as listed in Table I.

After getting the predicted probability of each MAIS from the regression model, the expected crash cost considering the lost quality of life can be estimated by choosing the maximum probability among all categories. Figure 1 shows the estimated economic costs of each level of MAIS [39].
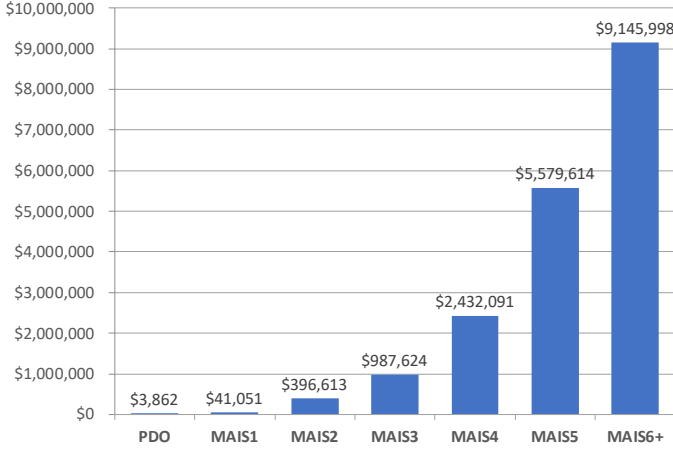
Fig. 1. Comprehensive economic unit costs of reported and unreported crashes [39]

Then, the risk probability of crashes (P_C) should depend on the speed difference and the distance between the preceding car and the following car. When the speed difference, which is also the approaching speed ($v_d = v_{following} - v_{preceding}$), becomes large or the distance is relatively small, the risk increases. The relationships are:

$$\frac{\partial P_C}{\partial v_d} > 0 \qquad \frac{\partial P_C}{\partial D} < 0 \qquad (8)$$

where:
P_C is the risk of crashes, and
D is the bumper-to-bumper distance gap between two vehicles.

To satisfy the above requirements for the crash risk prediction, we adapt a surrogate safety assessment, time-to-collision (TTC) [40], [41], to measure the crash risk and apply it for LC maneuvers. For car-following cases, the model considers rear-end collision wherein the following vehicle cannot brake sufficiently before the crash. It measures the probability of crashes [42]:

$$P_C = \begin{cases} \exp\left[-\left(\frac{\text{TTC}}{\sigma}^2\right)\right] & v_d > 0 \\ 0 & v_d \leq 0 \end{cases} \qquad (9)$$

where:
TTC is the time-to-collision safety measurement, and
σ is a constant scaling number assumed to be an average driver reaction time of 1.5 seconds.

C. Driver payoff function

To apply game theory to reveal the driver's decision-making process in LC maneuvers, the first challenge is knowing the payoff function that drivers may gain from their choices. Generally speaking, this depends on human characteristics, that is, drivers from different backgrounds, genders, ages, etc. value different factors in diverse ways. Nevertheless, the common point is that they are greedy to get more travel time savings while they also fear the possible risks of loss. Considering the

TABLE II
THE PAYOFF TABLE OF EACH STRATEGY IN LC GAMES

		Player I	
		Give way (q)	Not yield ($1-q$)
Player J — Change lanes (p)		$U_I(A,A), U_J(A,A)$	$U_I(A,B), U_J(A,B)$
Player J — Stay ($1-p$)		$U_I(B,A), U_J(B,A)$	$U_I(B,B), U_J(B,B)$

If $U_I(A, A) + U_J(A, A)$ is the minimum total cost and $U_I(A, B)$ and $U_J(A, B)$ are pure Nash equilibrium solutions, the cell in red represents the social-optimal scenario while the cell in blue indicates the user-optimal scenario.

aggressiveness of different drivers, we introduce a factor of γ ranging from 0 to 1 to weight the importance of saving time vs. reducing the risk [21], [43]. We also assume a Value of Time (VOT) to evaluate the time cost component of different scenarios. An unobserved explanatory variable is added to present the factors that are not currently investigated in order to increase the error tolerance of this model.

As for the extra time spent in different strategies, we create parallel worlds to calculate the travel time difference T_s of worlds with different consequences when players conclude their LC games [34]. The product of VOT and the time saving is the estimated cost of extra time spent, which contributes to the total cost before drivers choose one of the strategies.

Here, for simplification, we ignore the unobservable explanatory variable (ε) for this research. Now, by combining the expression of the safety cost part into the function, we can construct the overall driver payoff function as:

$$C_{z,total} = \gamma \times P_{z,C} \cdot C_{z,L} + (1 - \gamma) \times V_{z,T} \cdot T_{z,s} \qquad (10)$$

where:
z is the indicator of players (1 or 2);
$V_{z,T}$ is the Value of Time of player z;
$T_{z,s}$ means the extra time spent compared to the fastest case. It equals 0 when the scenario is the fastest one.

D. Optimization methods

A 2×2 game for LC maneuvers, and the payoff table for LC games is displayed in Table II. For this model, we design from two considerations: the user-optimal (UO), Nash Equilibrium based model and the social-optimal (SO) model. They adopt different algorithms to find the solution(s). It should be noted that the costs of driver choices are a kind of disutility. Drivers prefer to earn more benefits and also reduce their loss or costs. Therefore, the lower the expected costs, the more the willingness for the corresponding strategies.

For the SO solution, by following Pareto Optimality theory [44], we can directly resolve at which point(s) the total cost U_t reaches the minimum value and then find its corresponding pure strategies for two players. That means one strategy will not be a cooperative strategy all the time. For a specific strategy, it could be the cooperative strategy in the current situation but may switch to the non-cooperative behavior depending on

the varying conditions. The social-optimal algorithm for Player I is given by (Player J is identical but for subscripts):

$$U_{I,SO} = \begin{cases} U_I(A, A|B) & \text{if } U_t(A, A|B) = \min(U_t(A, A|B), U_t(B, A|B)) \\ U_I(B, A|B) & \text{if } U_t(B, A|B) = \min(U_t(A, A|B), U_t(B, A|B)) \end{cases}$$
(11)

The classic Nash Equilibrium can realize the user optimization because it has been proven that for every finite game at least one NE exists when considering mixed strategies [45]. When a strategy meets certain requirements, this strategy is the pure strategy adopted by one player during the whole game.

However, when it fails to satisfy the conditions, players tend to apply mixed strategies according to the computed possibilities of each strategy. To compare the payoffs of mixed strategies to pure strategies, we calculate the expected payoff of mixed strategies. For example, if the probability for Player I to change lanes is p and for Player J to give way is q, the overall user optimization algorithm for Player I will be:

$$U_{I,UO} = \begin{cases} U_I(A, A|B) & \text{if } U_I(A, A) < U_I(B, A) \text{ and } U_I(A, B) < U_I(B, B) \\ U_I(B, A|B) & \text{if } U_I(A, A) > U_I(B, A) \text{ and } U_I(A, B) > U_I(B, B) \\ pqU_I(A, A) + p(1-q)U_I(A, B) + (1-p)qU_I(B, A) \\ +(1-p)(1-q)U_I(B, B) & \text{otherwise} \end{cases}$$
(12)

The algorithm of Player J is quite similar to the expression above. These two optimization algorithms will then be deployed to determine which strategy will be selected depending on two different modes in each LC game. The payoff of every designed game will be plotted into the contour to check the payoff difference between two models.

IV. MODEL VALIDATION

To examine whether real cases align with models and assumptions introduced earlier, we adopt the 45-minute trajectory dataset, including lane records collected from southbound Highway US-101 in June 2005 [46]. The flow chart (Figure 2) shows the filtered available LC information, so 744 cases in total are used for validation. Due to missing information that this dataset cannot provide, it is assumed that the expected MAIS of two vehicles are the same, 3-Serious Injury. In addition, both drivers are assumed to neutrally value reducing crash risks and reducing travel time ($\gamma = 0.5$), and V_T aligns with the assumption for every driver ($V_T = 25\,\$$/hour). Note these simplifications limit the predictive capabilities of models.

We extract the speed profiles of two competing vehicles from trajectories and then present examples of different scenarios in Figure 3. In these profiles, 'LC point' refers to the time when two vehicles start to take action. To identify the strategies that real drivers adopt, we set acceleration and deceleration rate thresholds as $2\,\text{m/s}$ and $-3\,\text{m/s}$. Therefore, behaviors with significant acceleration or deceleration values that exceed thresholds can be judged as specific strategies, for example, Change lanes ($a > 2\,\text{m/s}$) and Stay ($a < -3\,\text{m/s}$).

To check differences between observations and predictions, we fit the observed and predicted choices of drivers with their payoffs in LC games. By bringing the median values of two probability distributions closer, the model parameter β is calibrated as 3.2×10^{-4}. The realistic behaviors generated by the model heavily rely on the exact estimation of β.
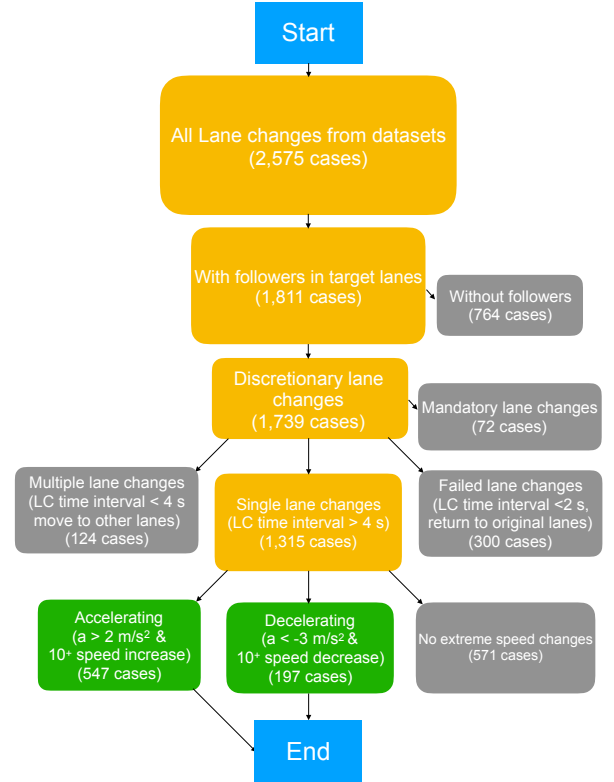

Fig. 2. The validation process filtering available data

Other advanced estimating methods would help improve the practicality of predictions in future models, but we have only found this estimation valid so far.

Figure 4 demonstrates two probability distributions from model outcomes and reality, the left side of the figure is for the merging cars, the right side for the through cars in the target lane. They present how the strategy probabilities are distributed throughout the proportion of cars (density) dropping in that probability. The yellow vertical represents the model prediction, which is concentrated at 100% for merging cars, indicating a high payoff for accelerating by the lane changing vehicle, which the median value observed is about 74% as shown in gray, indicating actual drivers are less willing to accelerate than predicted. The yellow bar is concentrated at 0% for most through cars, indicating they are more likely not to yield from a UO perspective, but about 12% do yield, as shown in gray.

Agents with the UO model would always aim to maximize individual benefit from games but not care about others. That is why the probability distribution of UO seeks non-cooperative strategies for both merging and through vehicles.

We input all initial conditions in two models, and again, compare observed and predicted outcomes when translating strategies into the binary values of 1 (Change lanes or Yield) or 0 (Stay or Not Yield). Note we exploit the mathematical
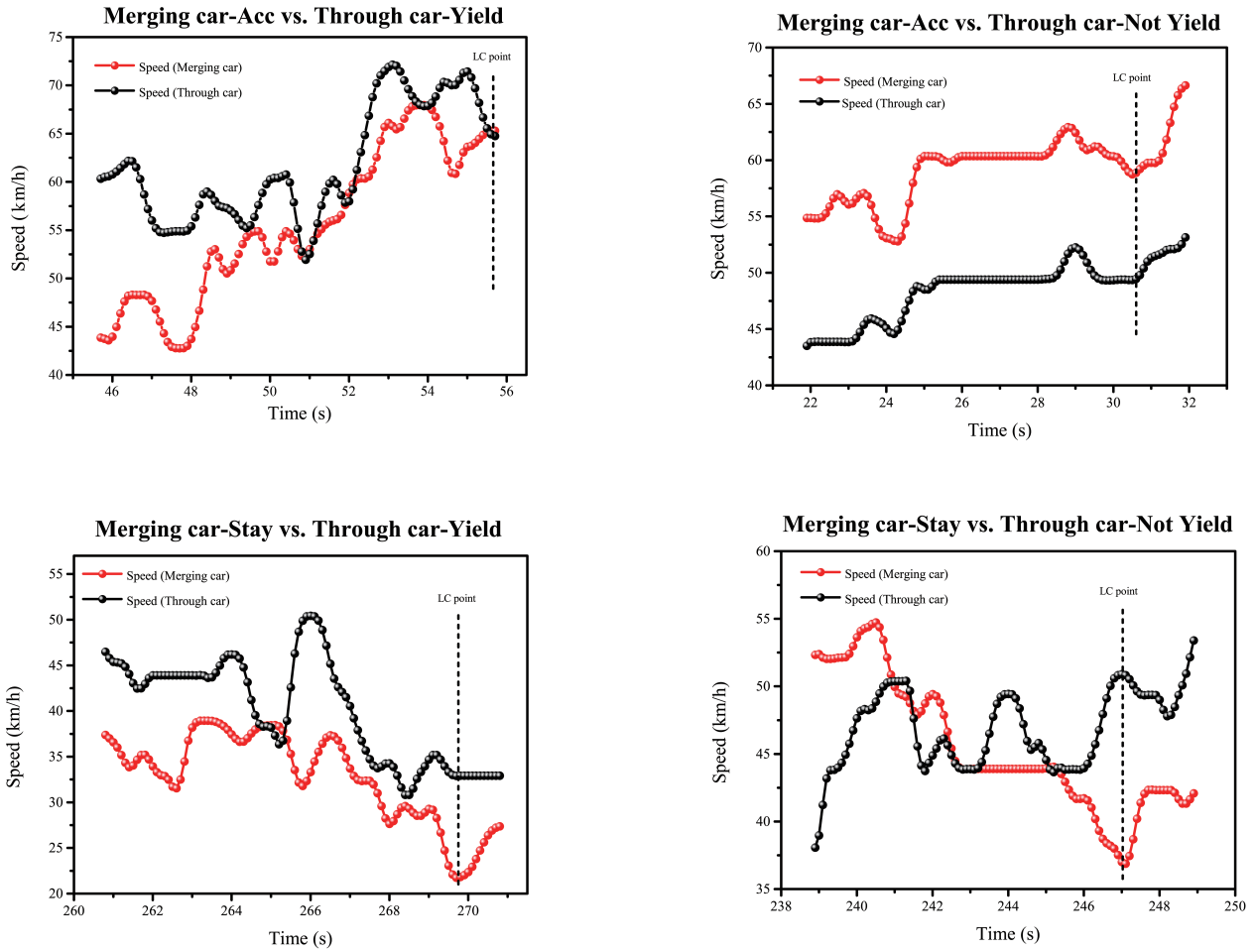
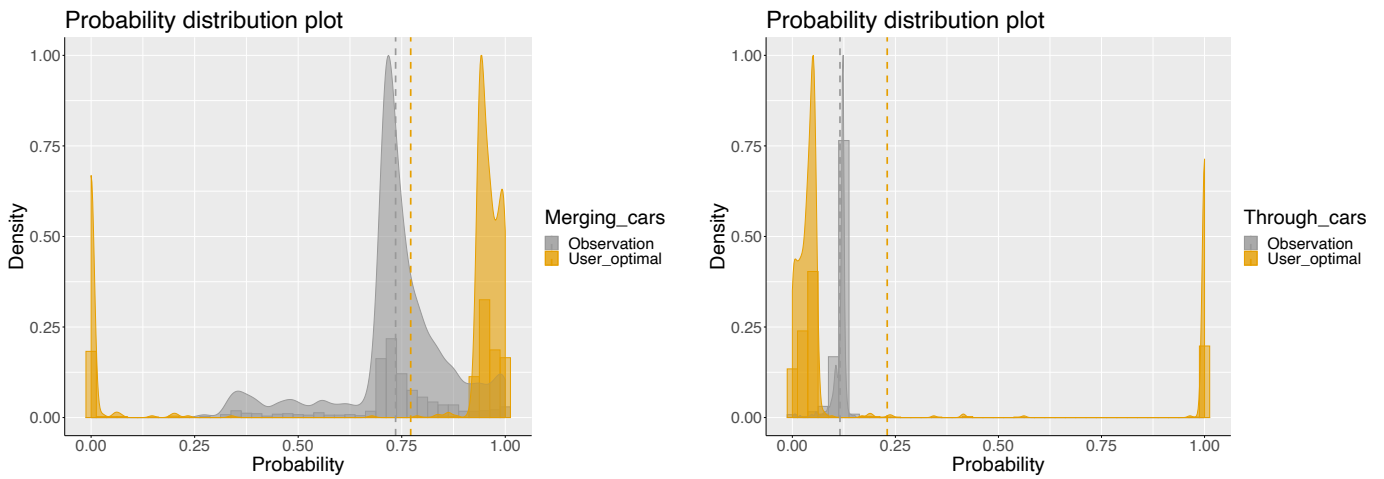Fig. 3. Speed profiles of vehicles in lane-changing maneuvers, example cases

Fig. 4. Probability distribution of strategy choices in simulation and in reality. For merging cars (left), 1.0 = probability of accelerating, 0.0 = probability of decelerating. For through cars in the target lane (right), 1.0 = probability of yielding, 0.0 = probability of not yielding.

TABLE III
VALIDATION RESULTS OF TWO MODELS

The user-optimal model	Merging car	Through car
Calibrated model parameter β	3.2×10^{-4}	3.2×10^{-4}
No. of validation cases	744	744
RMSE	0.5143	0.5479
MAE	0.2697	0.3045

expectation (as same as the possibilities of choosing '1' in this case) to represent outcomes with mixed NE strategies. Two metrics, Root Mean Square Root (RMSE) and Mean Absolute Error (MAE), are used in this validation. The results are shown in Table III.

The validation results indicate that GT-based models predict decisions of human drivers in LC maneuvers with an approximate 30% false alarm rate and reasonable values of RMSE. Our results compare favorably with the accuracy of validation studies in [24], 72.4% false predictions for discretionary LC scenarios. However, given the simplifying parameter assumptions required by the available data, we cannot replicate the actual strategies with high accuracy using GT models. One of the reasons could be that real LC scenarios are more complicated than two players with two simple choices. Also, not all drivers are entirely rational (in game theory, all players are assumed to behave rationally). Thus, they may make decisions depending on more factors (such as their aggressiveness) that have not been explored in this validation process. Further investigations for the decision-making process of human drivers would help in this regard.

In summary, there remain gaps between game-theoretical models and real scenarios, arguing for future model development and better data sets, but the model prediction errors are within an acceptable range, and so will be employed for the next step, which should be considered as stylized results.

V. CASE STUDY

A. Input parameters and assumptions

The lane changing game is designed to operate on a straight highway section without any intersections. It is simplified as a two-player game with complete information, consistent with previous studies. Two autonomous agents make decisions following the two algorithms mentioned before, and their final payoff is recorded after the LC maneuvers finish. Table IV summarizes the related specifications and parameters of two agents involved in the following calculations. Player I, identified as an SUV, acts as the through car, and Player J, modeled as a sedan, acts as the merging car. In terms of aggressiveness factors, they are assumed as Player I behaves aggressively, and Player J is more polite.

The game will be repeated with all parameters fixed except the initial distance and the speed difference between two players. The initial distance can be changed by moving the original position of one player but within the range of 15 to 30 m. Various initial speeds for Player J ($v(J)$) will also be tested to check whether the speed difference influences the trend or not when determining a fixed initial distance between players. Note that this study is not a repeated game,

TABLE IV
SPECIFICATIONS AND RELATED PARAMETERS OF TWO TEST OBJECTS

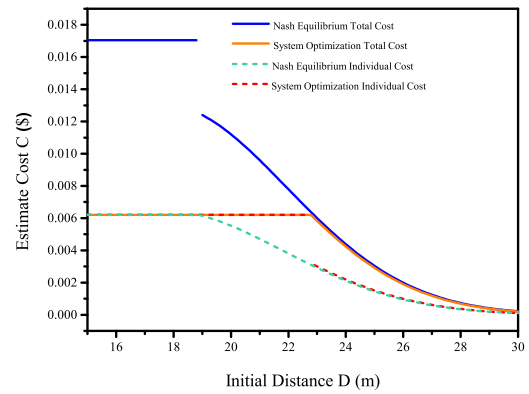| Parameters	Player I (SUV)	Player J (Sedan)
Length (m)	4.932	4.594
Weight (kg)	1,725	1,528
Initial Speed (m/s)	$v(I) = 15$	$v(J) = [20, 22]$
Position (m)	$X(I) = [15, 30]$	0
Acceleration (m^2/s)	2.5	2.5
Deceleration (m^2/s)	5	5
Maximum Speed (m/s)	30	30
Minimum Speed (m/s)	10	10
Calibrated model parameter β	3.2×10^{-4}	3.2×10^{-4}
Aggressiveness Factor γ	0.6	0.4
Value of Time V_T ($/hour)	25	25

Fig. 5. The payoff contour of user-optimal and social-optimal models for the merging car ($v(J) = 20\,\mathrm{m/s}$)

as described in classic game theory. A repeated game would use the same players in the game multiple times, enabling them to learn from their previous play and accordingly adjust their strategies. This experiment assumes that players are memoryless.

Each LC game, starts when we begin to count the first time step. Throughout the whole game, all vehicles are assumed to apply the same acceleration of $2.5\,\mathrm{m}^2/\mathrm{s}$ and deceleration of $5\,\mathrm{m}^2/\mathrm{s}$. In this experiment, the range of the speed is constrained between 10 and $30\,\mathrm{m/s}$.

B. Simulation test

The first test checks the effect of varying initial distance on agents' strategies and their payoff. The payoff contour for two different models is plotted for the merging agent in Figure 5. At the beginning of the contour (below $19.2\,\mathrm{m}$), the total cost when agents make choices according to Nash Equilibrium (NE) is much higher than the social optimal (SO) strategy.

When the initial distance is relatively short (e.g., from 19.2 to $22.8\,\mathrm{m}$), the UO agent (plotted by the blue dashed line) incurs less expense when it behaves aggressively, while the SO agent (plotted by the orange dash line) incurs a higher personal cost due to its cooperative behaviors. Therefore, their UO total cost (solid blue line) is much higher than the SO total

cost (solid orange line). However, when the distance increases, these two dashed lines effectively coincide with each other and continue to decline.

At the early stage, agents do not have a pure strategy based on Nash Equilibrium but use a mixed strategy instead to respond to others. Agents incur a considerable cost when adopting the high-cost strategy even for a small probability. That is the most critical case because they randomly select one of the strategies when their opponents' behaviors are hard to predict. When agents keep a relatively short distance between each other, the one that refuses to cooperate obtains relatively more individual benefits from its selfish behavior (change lanes or do not yield); however, the system cost becomes higher when more drivers decide not to cooperate. The difference between NE and SO costs at the second stage is defined as the 'social gap' that occurs in every time interval where the social dilemma exists. It is similar to the 'price of anarchy' in the route choice literature [47], which is a ratio, rather than a difference.

In contrast, for the low-density scenarios, the social dilemma effectively disappears, and the NE model quickly starts to follow the SO behavior. That is because the distance between players is large enough for their non-cooperative behaviors to pose minimal potential risk. When we check the actual strategies used by agents, it is found that the merging car finally decides to change lanes, and the through car chooses not to yield, which are typically regarded as 'aggressive' behaviors. That means the expected crash cost has been decreased to an acceptable range (defined as a 'dilemma-free' distance) for drivers to take less risk but save more travel time for themselves.

The second set of tests changes the initial speed difference of two drivers and compares their respective payoff contours. To display results more clearly, only individual costs are plotted with five levels of merging speeds. The results in Figure 6 indicate that the trends are quite similar among scenarios with various speed differences. The individual costs of players with a large speed difference are less than the costs with a small speed difference. Meanwhile, the social gap also reduces with the enlargement of their speed difference. For instance, comparing to the maximum social gap with $v(J) = 20$ m/s as \$0.0031, the one with $v(J) = 22$ m/s is \$0.0018. However, the range of the most critical stage is extended due to the large speed difference, as shown in the first stage of Figure 5, where driver strategies are highly unpredictable.

Therefore, when the speed differences are small, extra costs are incurred to finish the LC maneuver compared to situations with large speed differences. The reason is that the large speed difference reduces the time required to finish the whole LC maneuver. That means the large speed difference accelerates the completion of the game to mitigate the costs that players suffer. However, it also leads to a wide range in which mixed strategies exist, so players adopt high-cost strategies more often in this case.

The results from these tests demonstrate that the increase of initial distance (the distance when the LC game starts) is able to weaken the social dilemma effect. In addition, the
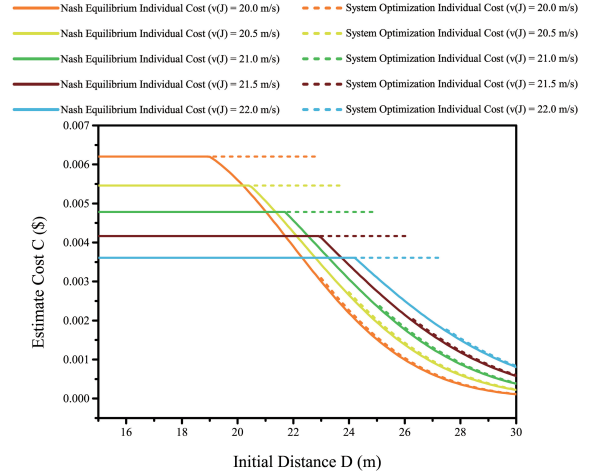

Fig. 6. The payoff contour of user-optimal and social-optimal models with multiple $v(J)$

large speed difference can reduce the required LC duration or distance so that drivers can finish their maneuver swiftly as desired, but it also increases the probability of high-cost strategy adoption. Future AV controllers aiming for an SO strategy could consider this in their LC module design.

VI. CONCLUSIONS AND OUTLOOK

To better describe intelligent discretionary lane-changing behaviors, we introduce a classic game theory model with the assumption that two drivers (players) compete in a game under the premise of their full understanding of their opponent's strategy. Meanwhile, to fulfill the core requirement (the payoff function) of the GT application, players' real costs when making decisions in LC games are quantified with a payoff function, including safety and time-efficiency factors. A validation approach, based on human driver lane-recording datasets, delimits the prediction capability of this simplified model. Further progress of the validation process is left for subsequent studies.

This paper compares social-optimal (SO) and user-optimal (UO) (based on Nash Equilibrium (NE)) models and investigates the motivation of drivers to behave selfishly so that a series of case studies are conducted. In the first set of simulations, the initial distances of two vehicles are varied to check if there is a situation that the social dilemma can be eliminated with other parameters fixed. The second set of simulations investigates the impact of different levels of the speed difference.

When the initial gap is fairly short, drivers may adopt less-safe strategies due to having no pure choices. After the initial distance increases to a minimum distance threshold, drivers can behave non-cooperatively to obtain more personal benefits. In that case, they maximize their selfish gains though non-cooperative behaviors increase the extra social cost to the cost of both cooperators and the whole system. Finally, when the initial distance reaches the so-called 'dilemma-free' distance, the SO model aligns with the UO model. Due to the long distance to be considered, drivers are more likely to engage

in selfish strategies. In other words, it makes their aggressive behaviors less risky so that the overall costs for those behaviors effectively, and then most drivers choose to apply SO. As for the second test, the large speed difference leads to a lower cost and social gap, which promotes the fast completion of the LC process. However, it also extends the interval where possible costly choices are adopted by players, which worsens the total system benefit.

To mitigate the social dilemma effect, the social gap in congested conditions could be addressed either by policy restrictions that charge drivers for inappropriate behaviors or by modifications of AVs and Advanced Driver Assistance System (ADAS) algorithms to consider this gap on the basis of the current algorithm for better interaction with human drivers and other AVs. Additionally, AVs are expected to avoid getting involved into such situations as the critical first stage, so they can appropriately approach the speeds of their opponents to decrease the probability of an incident.

In future research, an effective car-following model, together with real parameters collected from highways, will replace the values assumed here for a more realistic estimation. Extensions that improve the estimation of the cost scale parameter β may yield supplemental insights. More scenarios that drivers may encounter in LC maneuvers (not only including the specific situation in this study) should also be investigated for other possible conclusions; for example, the merging car is ahead of the through car but drives at a lower speed. That should enrich the applications of this model for different complex conditions for the possible deployment in other simulation tools or on real roads.

Moreover, it may be useful to explore the three player-game for LC behaviors, which should be more realistic in cases where a third player significantly impacts on LC maneuvers. Additionally, when discussing the interaction between human drivers and AVs for the possible scenario with specific penetration of AVs, we can explore the situations of all human drivers, all AVs, and mix of them playing the LC game.

LC models rely on valid datasets from reliable sources. These include vehicle trajectories and crash reports. In particular police reports may miss many unreported incidents. It is expected that with the deployment of AVs and widespread vehicle sensors, more detailed LC-related datasets will be published. These will enable researchers to not only modify current LC models but also promote the development of new models for future research. Future work can verify the model with other AV datasets. While some limitations exist in the current proposed model, we believe it provides insights and structures our understanding of how rational and cooperative human drivers choose appropriate strategies and suggests solutions to support safe and socially efficient travel behavior.

NOMENCLATURE

α the power of mass ratio, calibrated from crash injury data

β the scale parameter of two costs, calibrated from trajectory datasets

Δv Delta-V of one vehicle in a collision

γ the aggressiveness factor of drivers, ranged from 0 to 1

σ the constant scaling number related to driver reaction time

a the current acceleration rate of vehicles

C_L the estimated crash cost from the expected collision

C_S the corresponding empirical value of life at each level of MAIS

C_{total} overall cost for one player in games

D bumper-to-bumper distance gap between two vehicles

m the dry weight of vehicles

p the probability of Player J changes lanes

P_C the probability of getting involved in crashes

P_S the probability of injury severity at each level of MAIS

q the probability of Player I gives way

T_s the extra time spent compared to the fastest one

TTC the time to collision safety measurement

U the estimated payoff of drivers

v the current speed of vehicles

v_d the approaching speed, $= v_{following} - v_{preceding}$

V_M the category of MAIS (from 0 to 6+)

V_T the value of time of drivers

W_L the energy loss in a collision

X the current position of vehicles

z the ID of players (1 or 2)

MAIS   maximum abbreviated injury scale

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