



Formulation of Plasticity Models through Symbolic Regression

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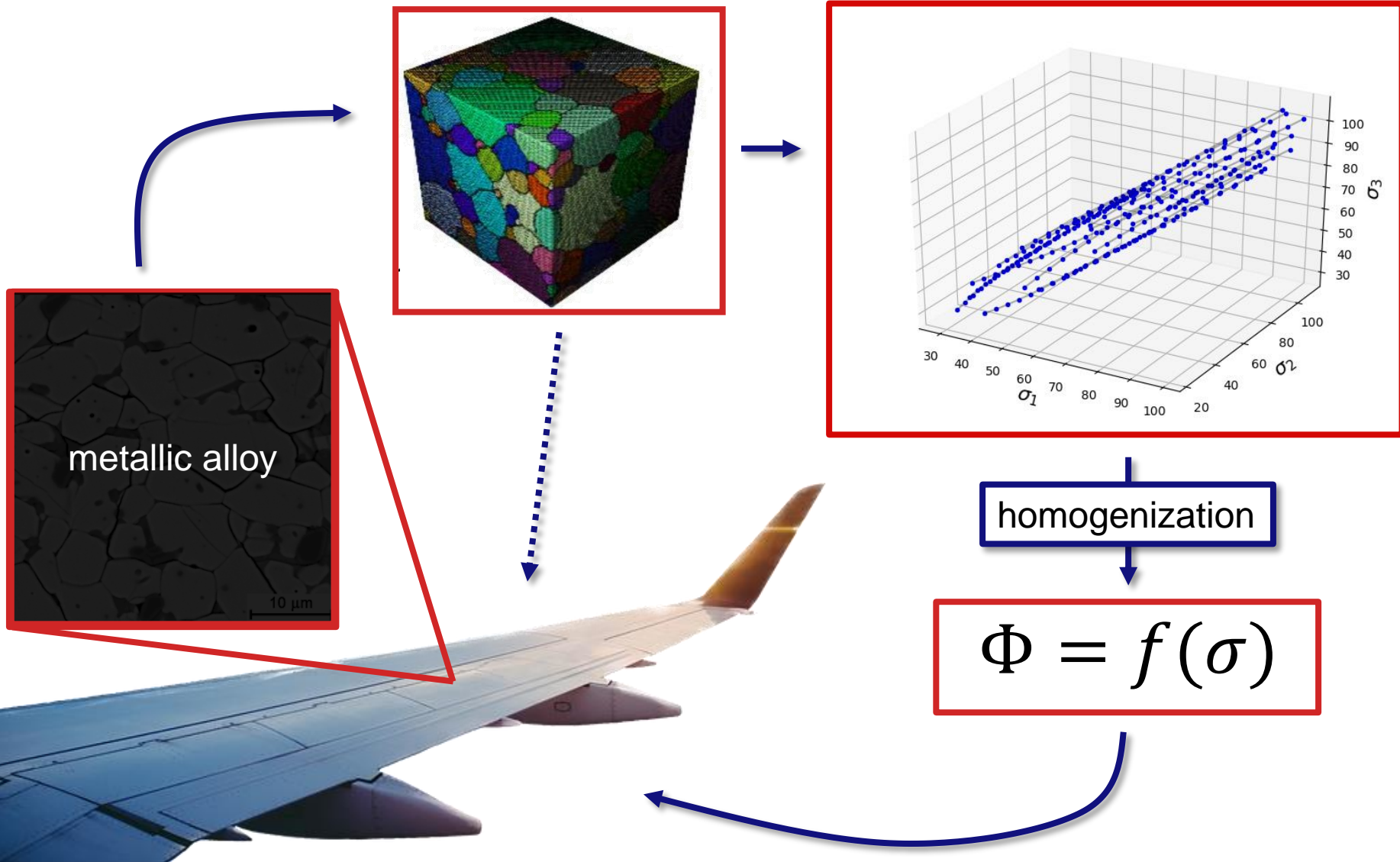
Kathryn Esham

The Ohio State University

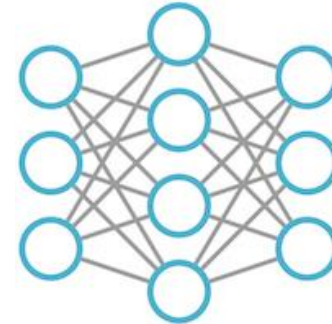
Ethan Adams

Brigham Young University Idaho

Motivation for homogenization



$$\text{Response} = f(\text{loading})$$



Human developed/derived models

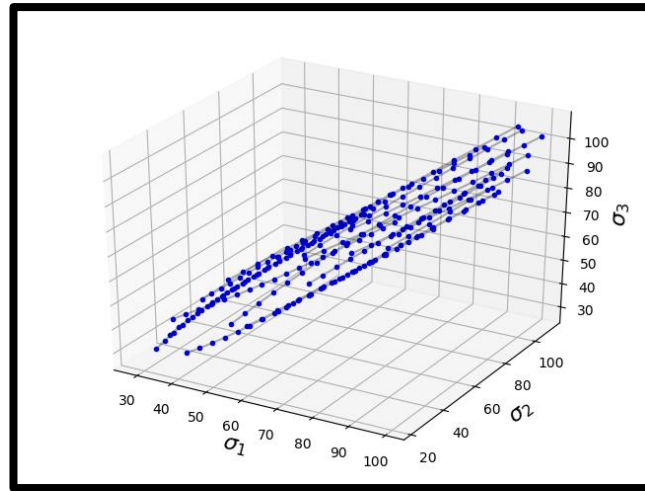
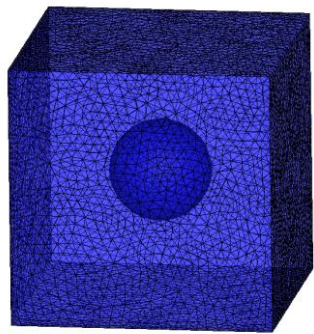
- Pros:
 - Can be physically based
 - Transferability
 - Compact and quick to evaluate
- Cons:
 - Can take decades in development

Typical machine learning models

- Pros:
 - Rapid development (training)
 - More input → more accurate
- Cons:
 - Not transferable
 - Not insightful (black box)
 - Evaluation is relatively expensive

Can we have the best of both worlds?

Example: GTN porous plasticity model



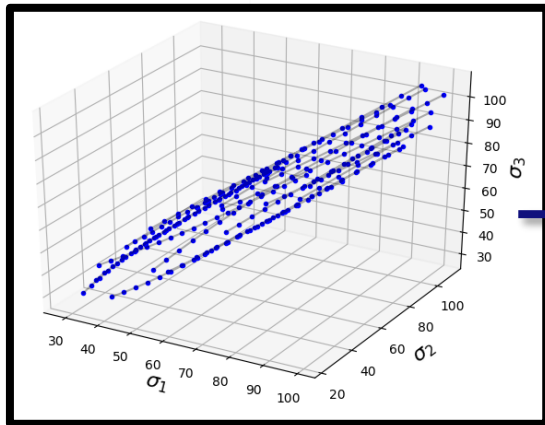
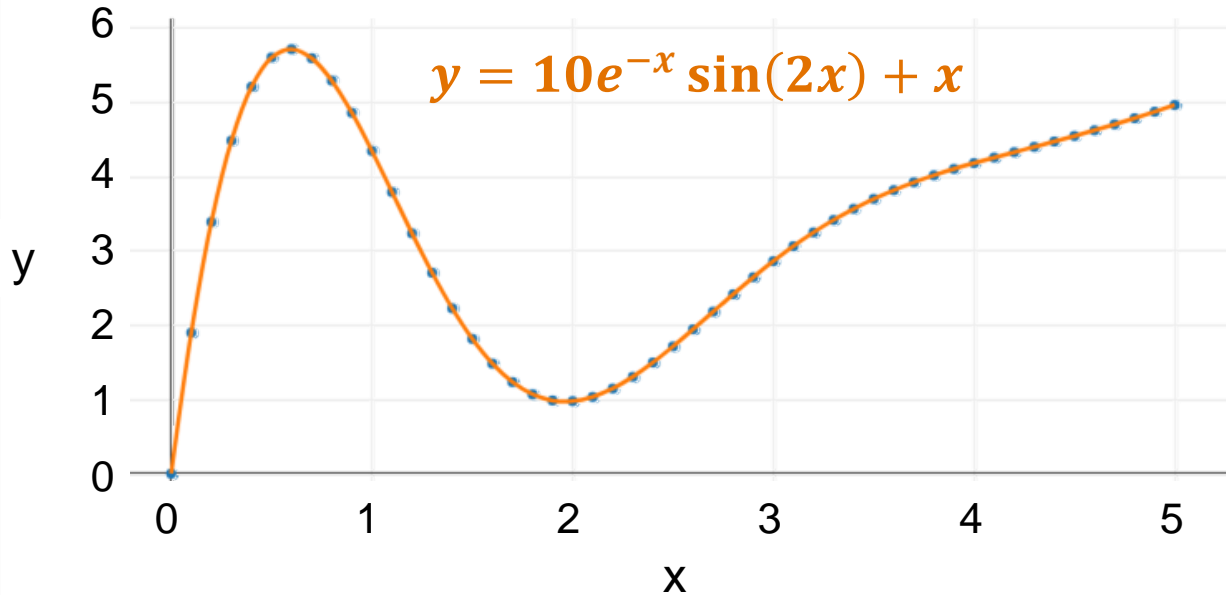
- Choose functional form
- Fit parameters (in red)
- Model misfit identified
- ~~Abuse parameters~~
- Add more physics to functional form

Time consuming step!

$$\Phi = \left(\frac{\bar{\sigma}}{\sigma_y}\right)^2 + 2q_1 f^* \cosh\left(\frac{3}{2} q_2 \frac{\sigma_h}{\sigma_y}\right) - (1 + (q_1 f^*)^2)$$

$$\Phi = \left(\frac{\bar{\sigma}}{\sigma_y}\right)^2 + 2q_1(\sigma) f^* \cosh\left(\frac{3}{2} q_2(\sigma) \frac{\sigma_h}{\sigma_y}\right) - (1 + (q_1(\sigma) f^*)^2)$$



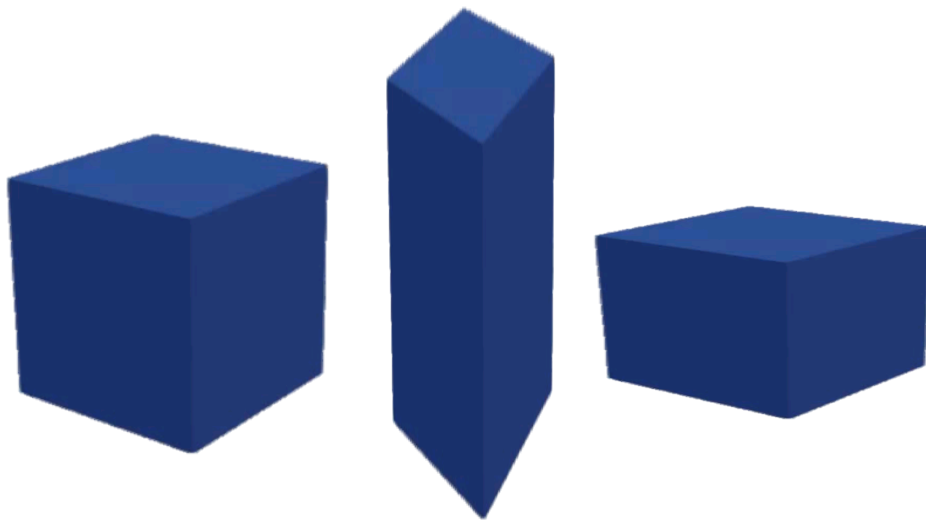
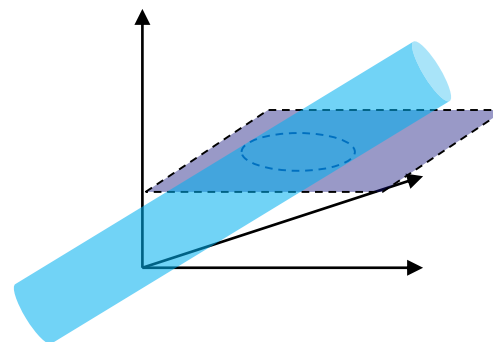
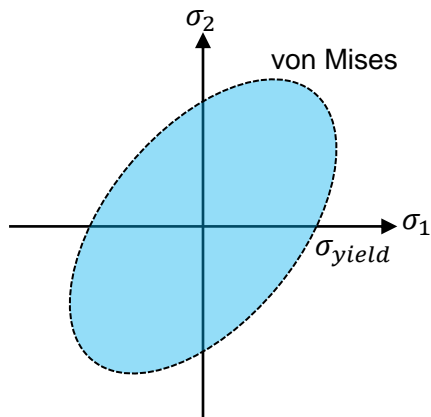


- Find **best fit functional forms** and parameters simultaneously!
 1. Decide what data to use
 2. Define fitness to data
 3. (Decide how much data to use)
- Attribute physics to portions of equations

Verification problem: von-Mises plasticity



$$\Phi = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 - 2\sigma_{yield}^2 = 0$$



RVE:

- Single element model
- von-Mises plasticity

Verification:

- Can we recover ϕ from looking at response data?

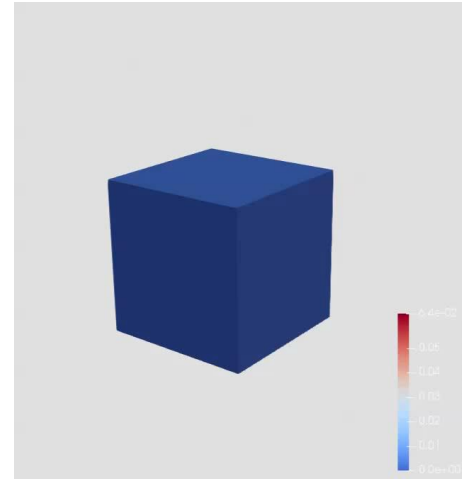
Symbolic regression problem definition



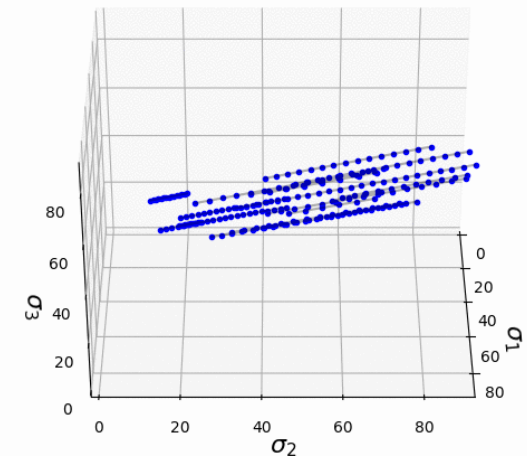
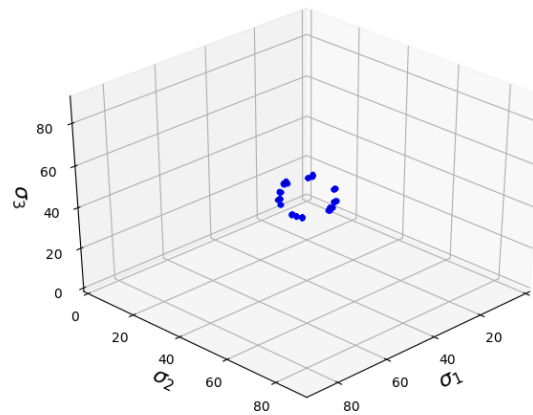
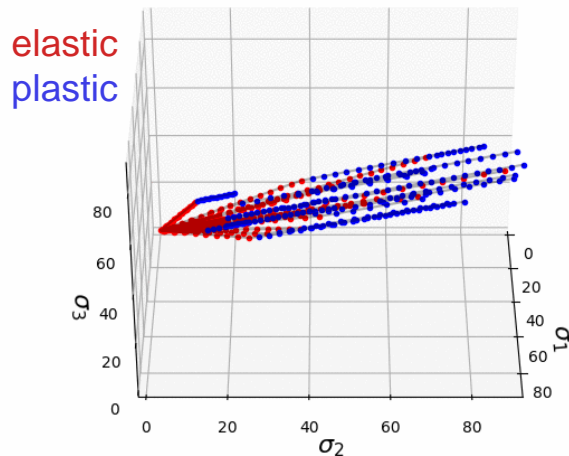
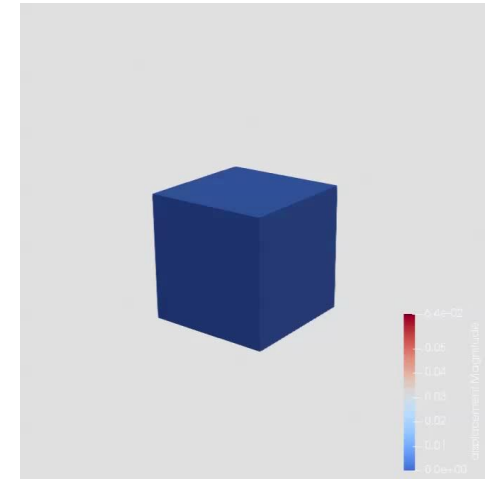
1. Decide what data to use
2. Define fitness to data

- proportional loading
- Data for each loading case:
 - Principle stresses: σ_i
 - Principle strains: ϵ_i
 - Equivalent plastic strain: ϵ^p

Loading ratio: [0.675 1.0 0.8250]



Loading ratio: [0.278 0.238 1.0]

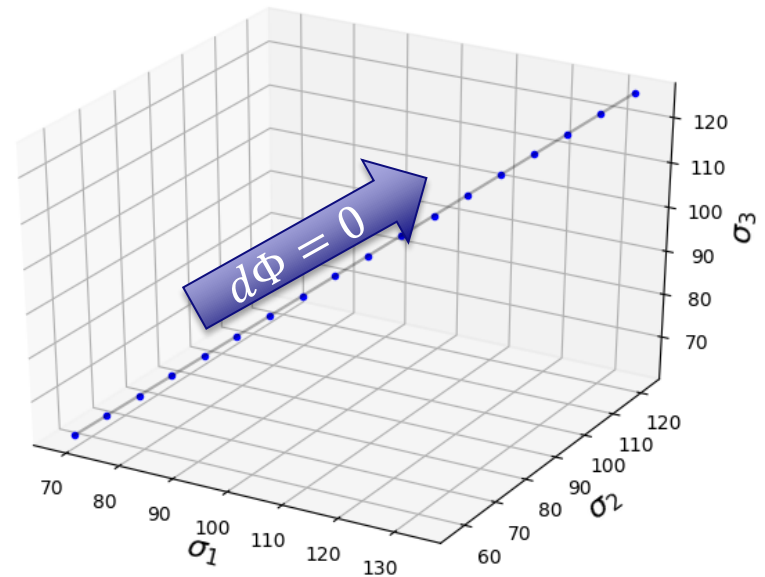
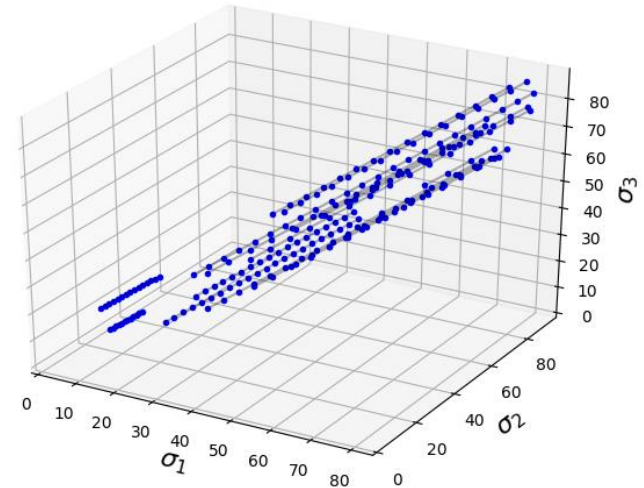


Symbolic regression problem definition



1. Decide what data to use
2. Define fitness to data

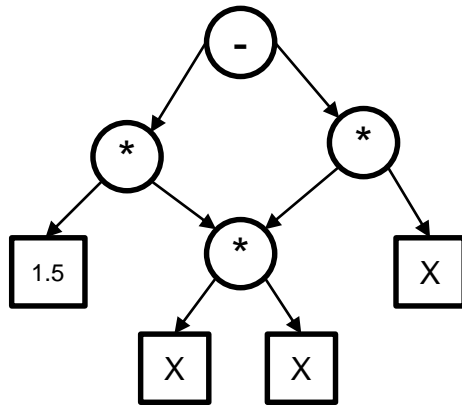
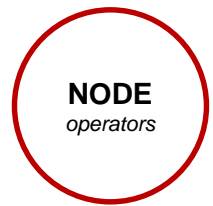
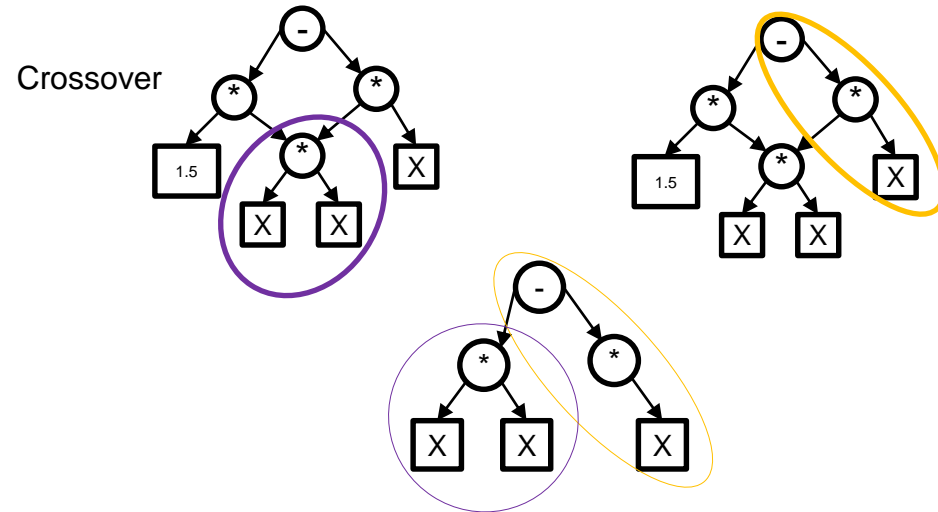
- $\Phi = f(\sigma) = 0$ (on yield surface)
- Implicit regression
- $E = \sum \frac{\frac{df(\sigma)}{d\sigma} \cdot \frac{d\sigma}{dt}}{\left\| \frac{df(\sigma)}{d\sigma} \cdot \frac{d\sigma}{dt} \right\|} \rightarrow 0$
- $\Phi(\sigma) = \text{constant for each loading case}$



Solving the symbolic regression problem

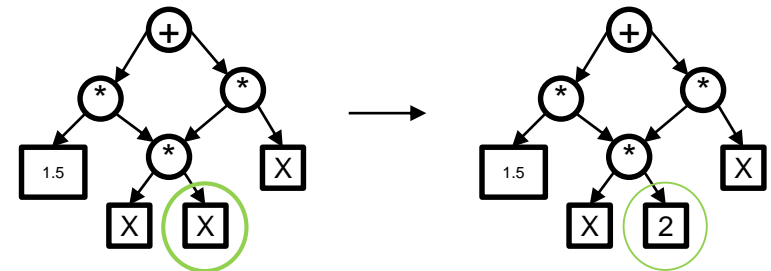


- Using genetic programming
 - (Genetic algorithms of computer programs)
- Equations evolve until they fit the data
- In-house code: bingo



$$y = 1.5x^2 - x^3$$

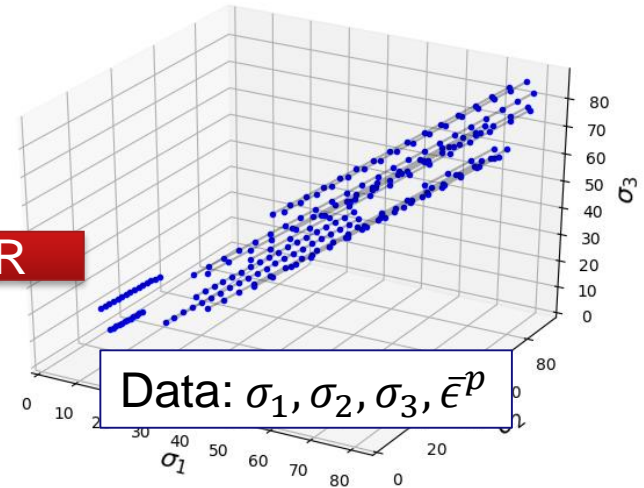
Mutation



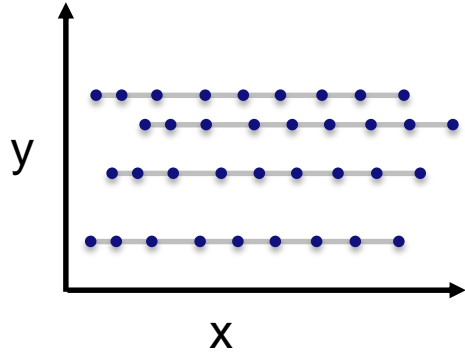
Early results

- Looking for yield surface: $\Phi(\sigma) = 0$
- $\Phi(\sigma) = \text{constant}$ for each loading case

$$\sigma_1 - (\sigma_3 + \sigma_2) + \sigma_1 = \text{constant}$$

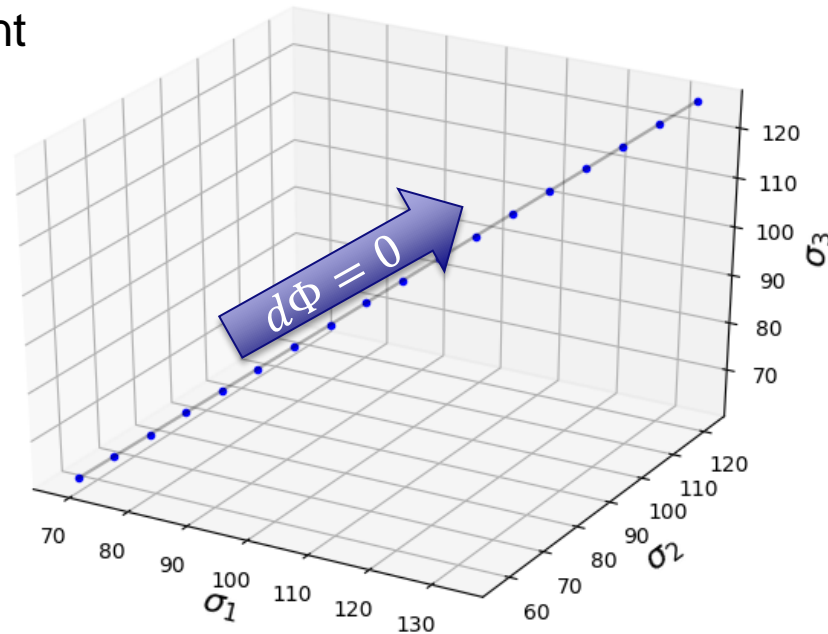


- Issue:** all loading cases are parallel!



$\Phi(x, y) = y = \text{constant}$

- Solution: more complex loading cases

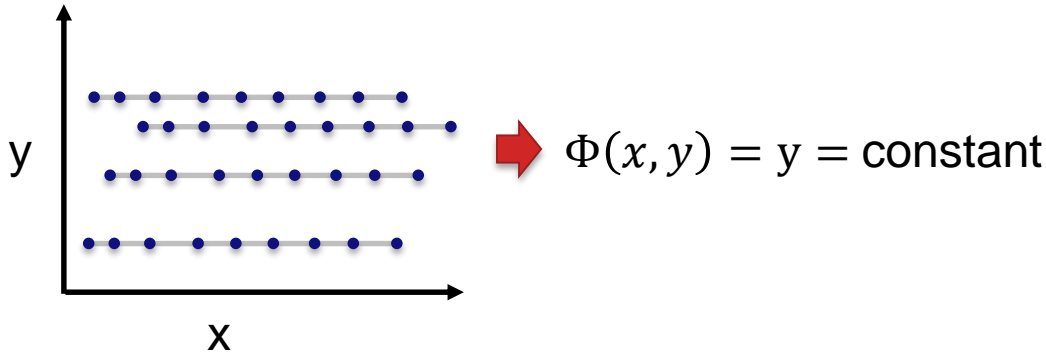


Early results

- Looking for yield surface: $\Phi(\sigma) = 0$
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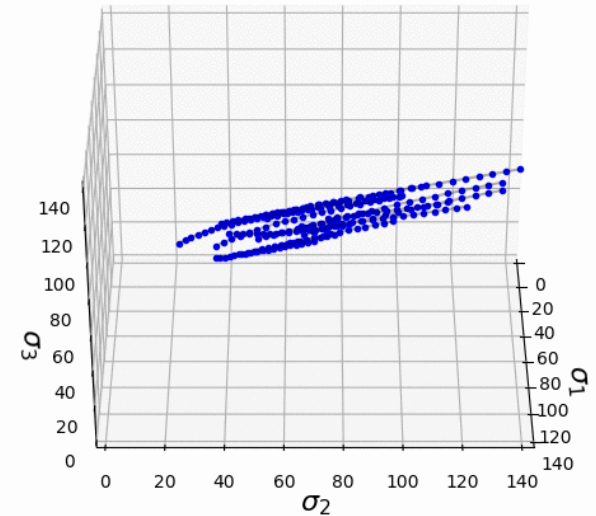
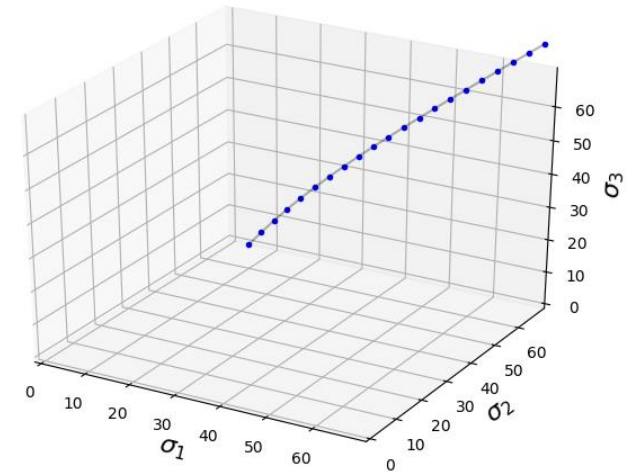
- Solution: more complex loading cases

Stage 1

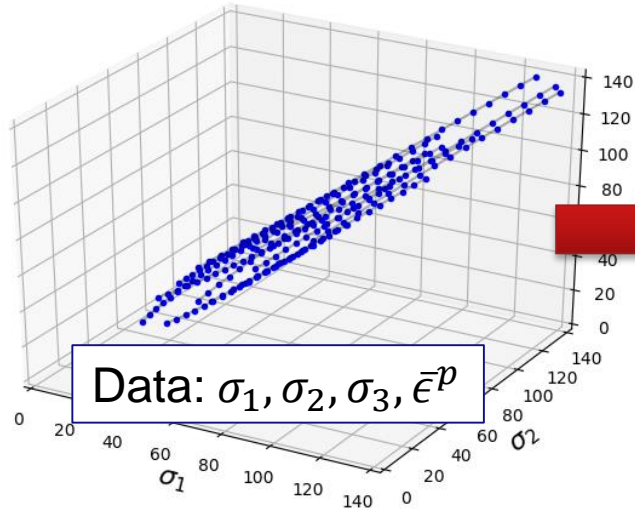
Loading ratio: [0.675 1.0 0.825]

Stage 2

Loading ratio: [1.0 0.825 0.675]



Yield surface from 2 stage loading data



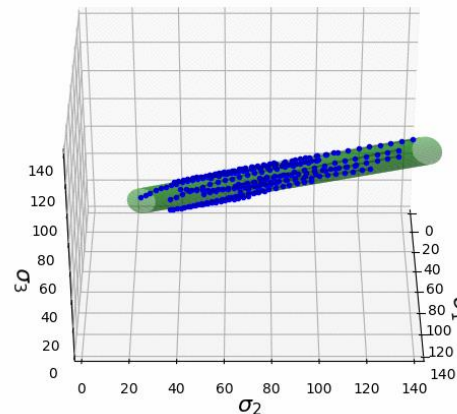
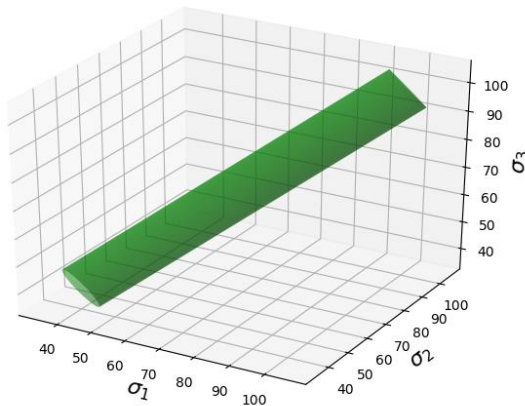
Computation time: 32s on 40 processors



$$(\sigma_3 - \sigma_2)(\sigma_3 - \sigma_2) - (\sigma_2 - \sigma_1)(\sigma_3 - \sigma_1) = \text{constant}$$

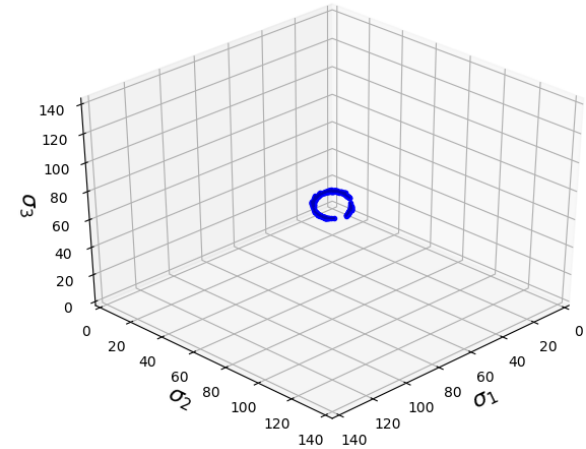
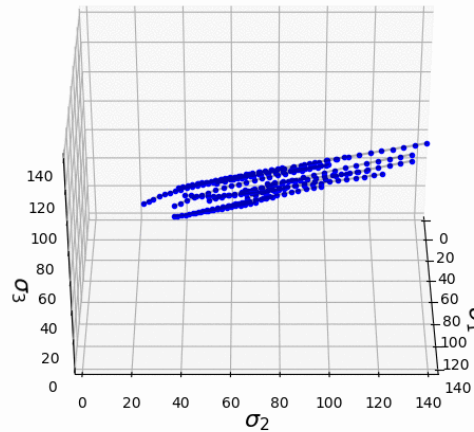
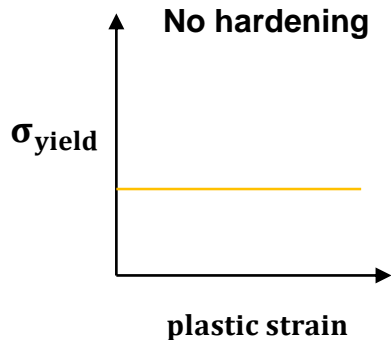


$$\Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 = \text{constant}$$

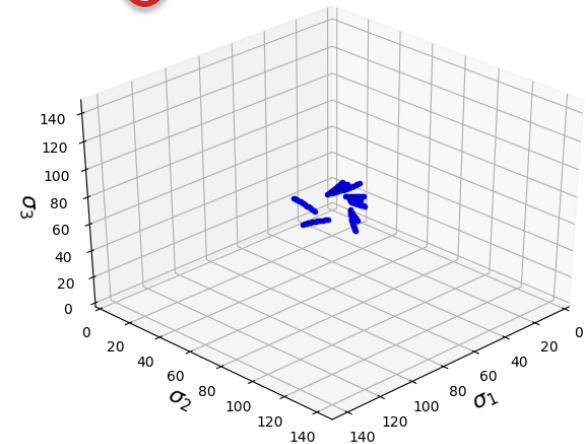
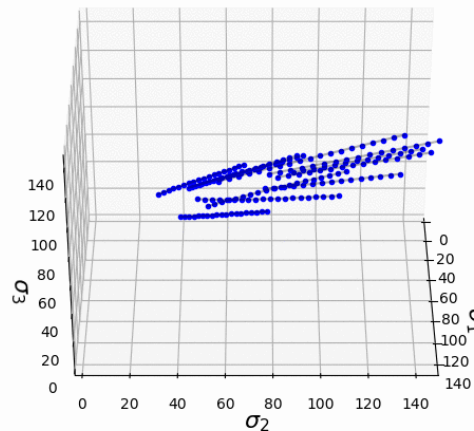
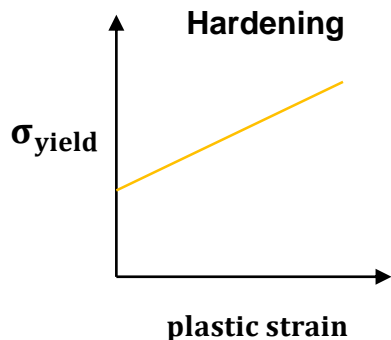


Adding hardening

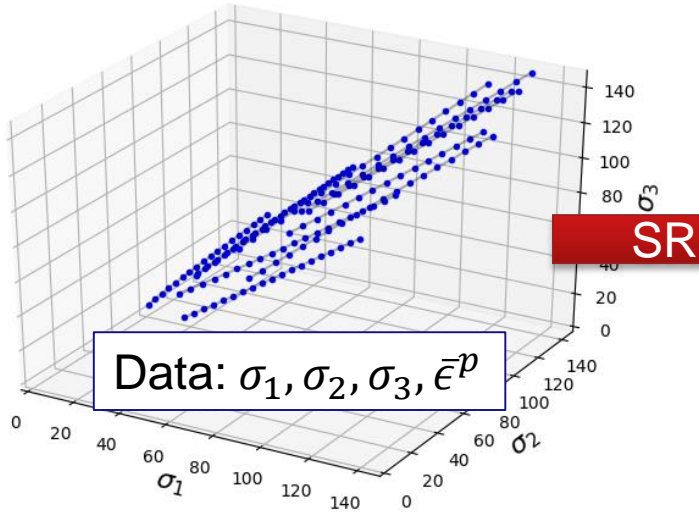
$$\Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 = \text{constant}$$



$$\Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 - c_1 \bar{\epsilon}^p - c_2 (\bar{\epsilon}^p)^2 = \text{constant}$$



Hardening yield surface from 2 stage loading data



Computation time: 1.5h on 160 processors



$$((\sigma_1 - \sigma_3 + \sigma_3 - \sigma_2)(\sigma_3 - \sigma_2 + \sigma_1) + 47602 + \sigma_1 + (\sigma_3 - \sigma_1 - \sigma_3)(\sigma_3 + \sigma_3 -$$



$$\Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 - 19797\bar{\epsilon}^p - 980185(\bar{\epsilon}^p)^2 = constant$$

$$\Phi = (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 - c_1\bar{\epsilon}^p - c_2(\bar{\epsilon}^p)^2 = constant$$



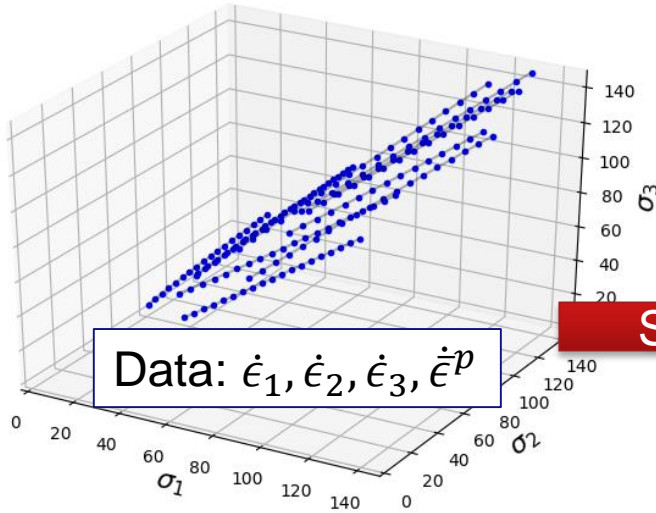
Yield surface now depends on a state variable!
Now it needs a state evolution equation



Hardening yield surface from 2 stage loading data



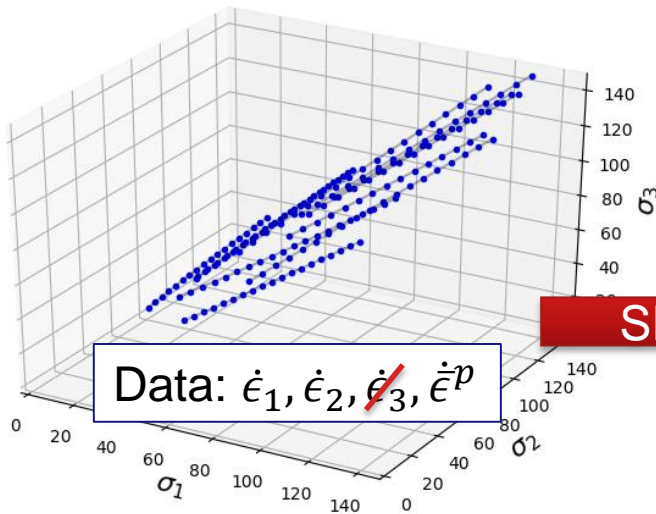
Assuming incremental elastic strains are small



$$\text{Goal: } \dot{\epsilon}^p = \sqrt{\frac{2}{9} [(\dot{\epsilon}_1 - \dot{\epsilon}_2)^2 + (\dot{\epsilon}_2 - \dot{\epsilon}_3)^2 + (\dot{\epsilon}_3 - \dot{\epsilon}_1)^2]}$$

$$\dot{\epsilon}_1 + \dot{\epsilon}_2 + \dot{\epsilon}_3 = \text{constant}$$

$$\dot{\epsilon}_1 + \dot{\epsilon}_2 + \dot{\epsilon}_3 = 0 \quad \text{deviatoric plastic strains}$$



$$\text{Goal: } \dot{\epsilon}^p = \sqrt{\frac{4}{3} [\dot{\epsilon}_1^2 + \dot{\epsilon}_1\dot{\epsilon}_2 + \dot{\epsilon}_2^2]}$$

$$\dot{\epsilon}_1^2 + \dot{\epsilon}_1\dot{\epsilon}_2 + \dot{\epsilon}_2^2 - (\dot{\epsilon}^p)^2 = \text{constant}$$



Quick Recap



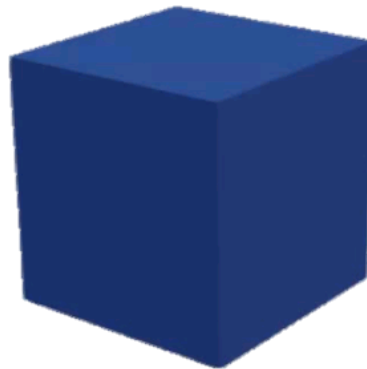
- Verification of von-Mises plasticity

- 👍 Non-hardening yield surface

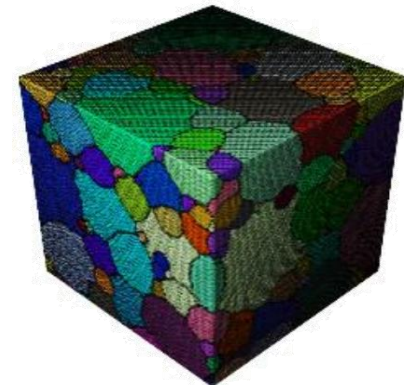
- 👍 Hardening yield surface

- 👍 State evolution

- Moving Forward:



Seconds (on single processor)



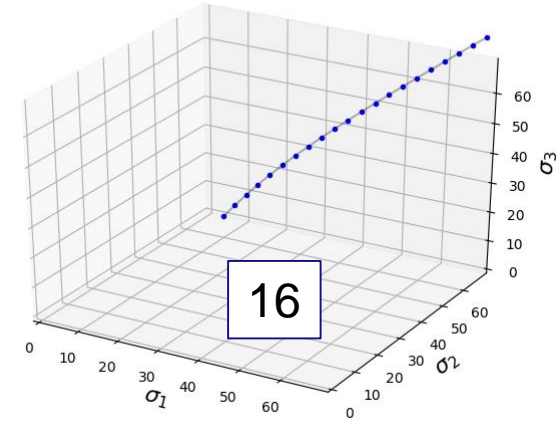
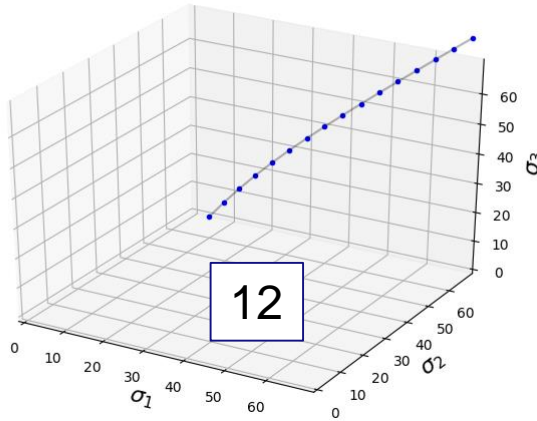
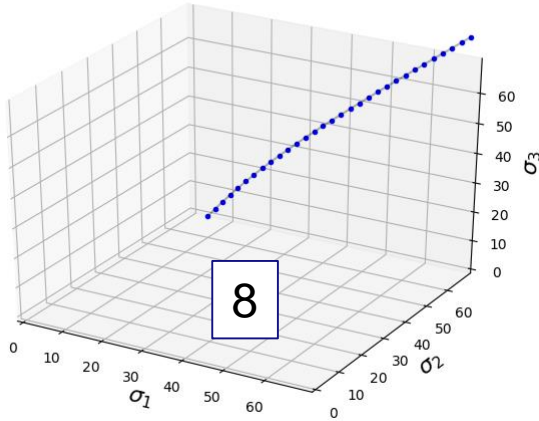
hours (on multiple processors)

- How much data is required?

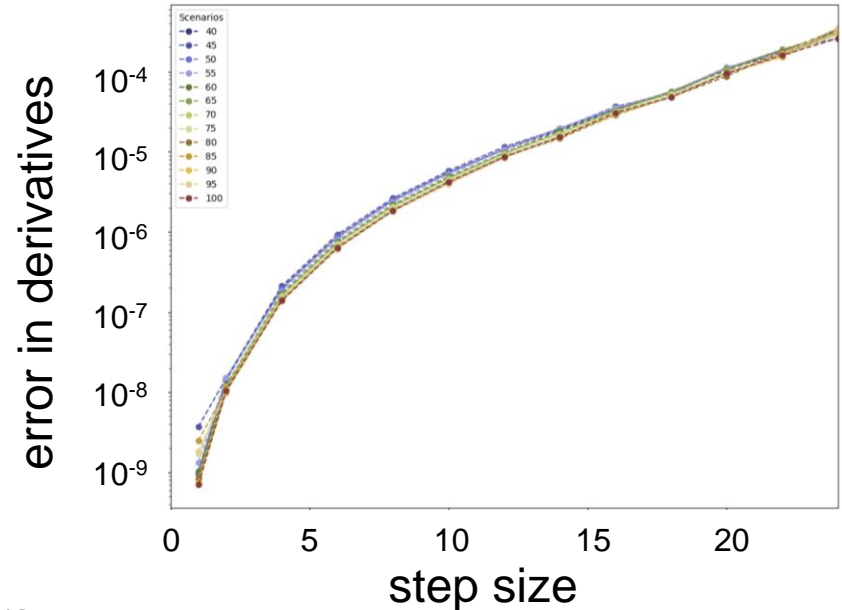
How much data is needed?



- **Step size**

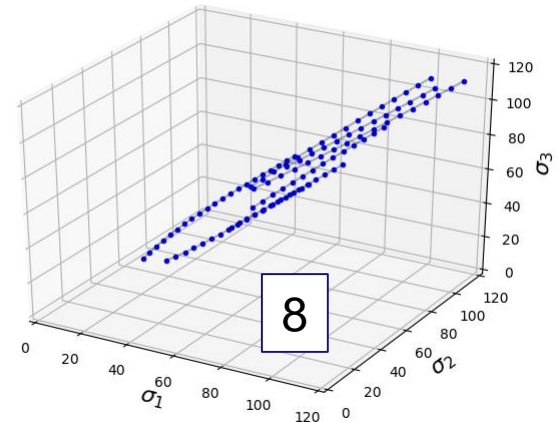
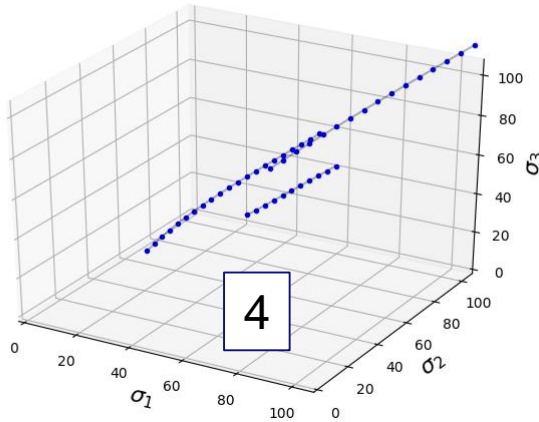
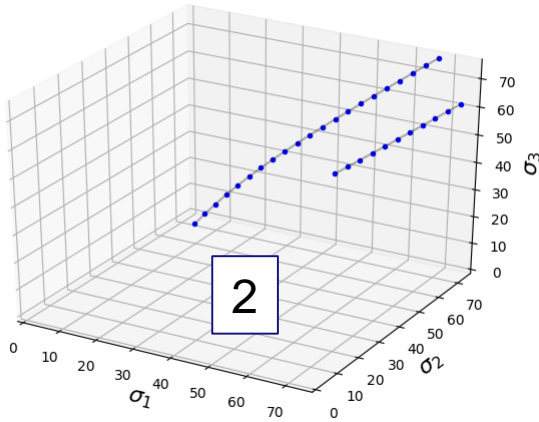


- More dense data =
 - more computation time
 - more accurate derivative calculations
- Density needed will depend on
 - complexity of loading scenarios
 - Complexity of yield surface



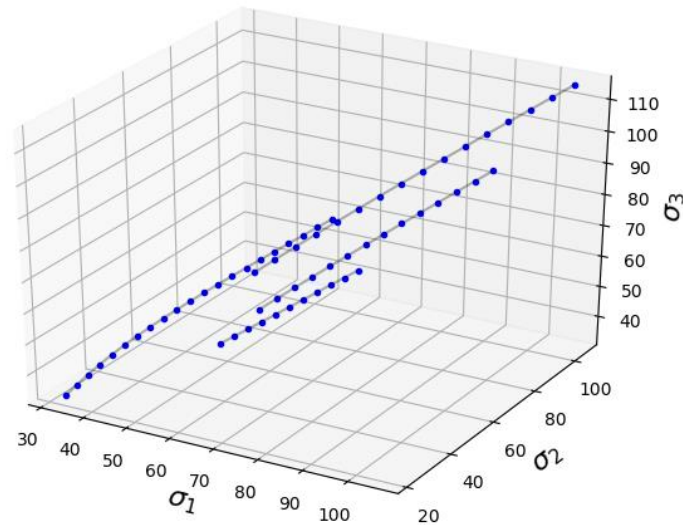
How much data is needed?

- Number of loading scenarios



- No real trend (except very low values)

- Minimum case found:

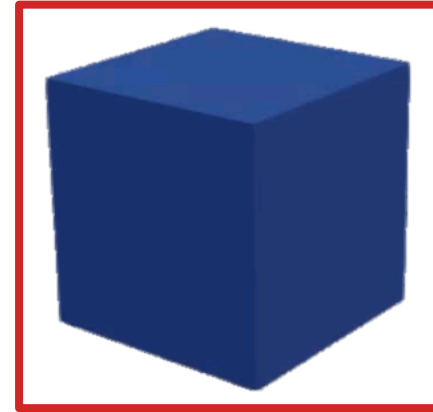


Summary

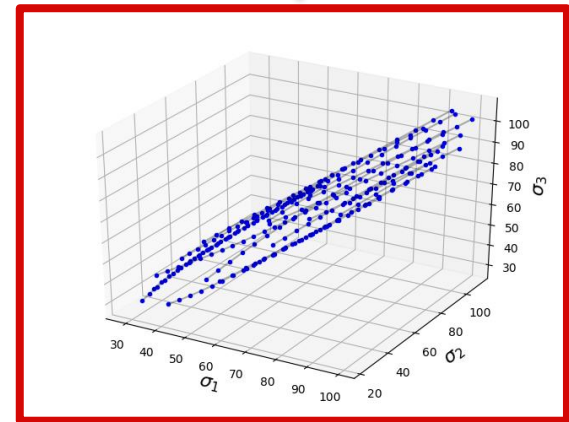


- Set up framework for SR formulation of plasticity models
- Implicit symbolic regression of yield surface
- Use non-proportional loading
- Von-Mises verification problem
👍 👍 👍
- Surprisingly little data needed
 - will depend on complexity of yield surface

RVE



Data



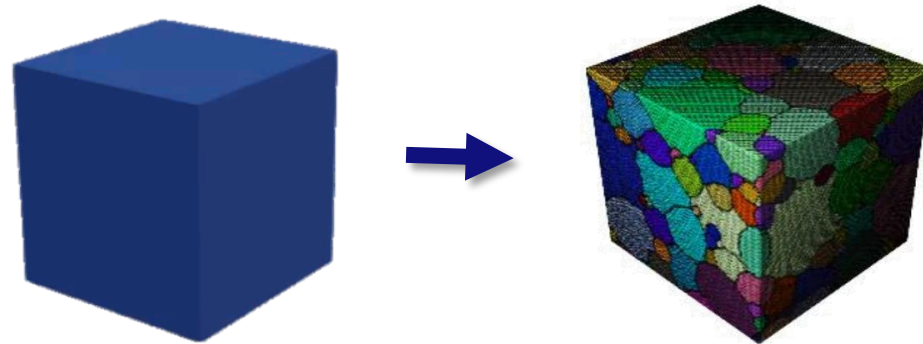
Symbolic
Regression

homogenization

$$\Phi = f(\sigma)$$

Future work

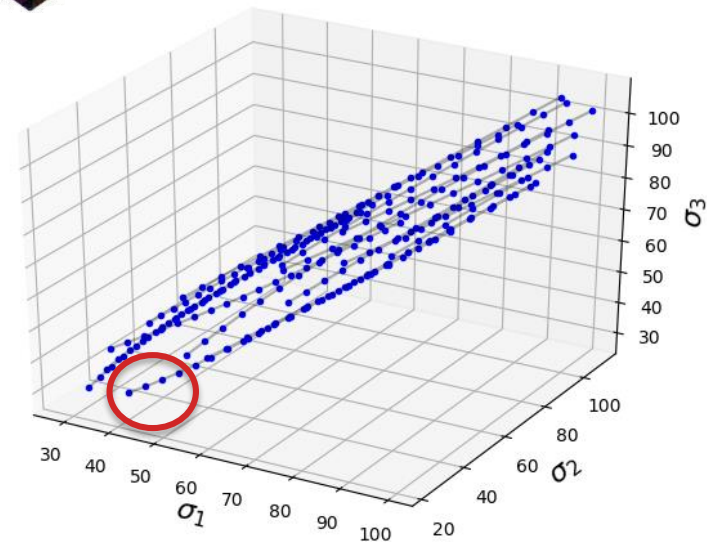
- application to real materials

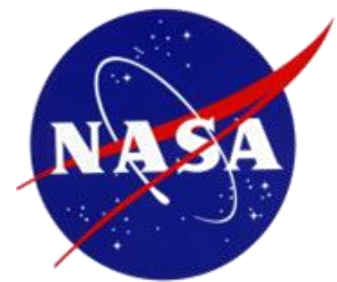


- adaptive data generation
- bingo (soon to be open source)
 - python & c++
 - Features:

- Coevolution of fitness predictors
- Island parallelization scheme
- Acyclic graph representation
- Constant optimization
- Age-fitness Pareto selection

} performance
 } robustness





thank you!

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