Fuzzy Modeling and Parallel Distributed Compensation for Aircraft Flight Control from Simulated Flight Data

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A method is described that combines fuzzy system identification techniques with Parallel Distributed Compensation (PDC) to develop nonlinear control methods for aircraft using minimal a priori knowledge, as part of NASA's Learn-to-Fly initiative. A fuzzy model was generated with simulated flight data, and consisted of a weighted average of multiple linear time invariant state-space cells having parameters estimated using the equation-error approach and a least-squares estimator. A compensator was designed for each subsystem using Linear Matrix Inequalities (LMI) to guarantee closed-loop stability and performance requirements. This approach is demonstrated using simulated flight data to automatically develop a fuzzy model and design control laws for a simplified longitudinal approximation of the F-16 nonlinear flight dynamics simulation. Results include a comparison of flight data with the estimated fuzzy models and simulations that illustrate the feasibility and utility of the combined fuzzy modeling and control approach.

Nomenclature

a	=	decay rate
A, B	=	state-space matrices
b	=	wing span, ft
$\bar{c}$	=	wing mean aerodynamic chord, ft
I_x, I_y, I_z, I_{xz}	=	inertia tensor elements, slug-ft^2
J	=	cost function
K	=	control gains
m	=	mass, slugs
M	=	membership function
N	=	number of data points
p	=	vector of cell parameter estimates
P	=	Lyapunov stability matrix
q	=	body-axis pitch rate, deg/s
r	=	number of rules
R	=	rule
S	=	wing reference area, ft^2
u	=	control input
V	=	Lyapunov function
w	=	cell weight
$x_{ref}, y_{ref}, z_{ref}$	=	body-axis reference point
x_{cg}, y_{cg}, z_{cg}	=	coordinates of the center of gravity
x	=	explanatory variable, or state
X	=	matrix of regressors
z	=	modeled output
α	=	angle of attack, deg
δ_e	=	elevator deflection, deg
θ	=	vector of all parameter estimates
ϕ	=	constraint on initial conditions
μ	=	constraint on control input
superscripts		
T	=	transpose
$\hat{}$	=	estimate
$\bar{}$	=	mean
$\cdot$	=	time derivative

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I. Introduction

THE conventional paradigm for the development and flight testing of new or modified aircraft is an iterative, time-consuming process that typically involves numerous test techniques to generate an aircraft model and design a control system. The NASA Learn-to-Fly (L2F) initiative aims to facilitate this process by replacing most of the ground-based testing and human involvement with automated, efficient, onboard tools that provide in-flight aircraft modeling and learning control, as depicted in Fig. 1 [1].

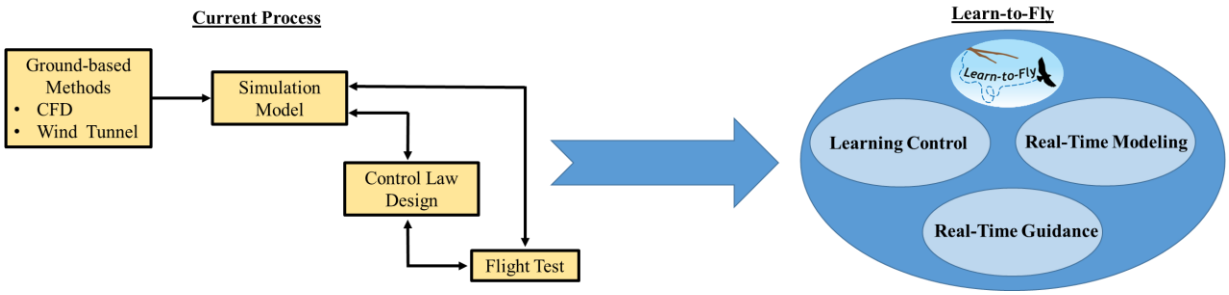


Fig. 1 Conventional aircraft development process vs. Learn-to-Fly concept.

Recent work at NASA aimed to test all components of the L2F concept experimentally in flight-test operations on an aircraft with no a priori model or knowledge of the aerodynamics [1,2]. The L2F technique of real-time global nonlinear aerodynamic modeling is based on flight data alone and consists of two main parts. First, efficient flight maneuvers must be designed to sufficiently excite the aircraft dynamics. Prior research has shown that Programmable Test Inputs (PTIs) that apply automated orthogonal optimized multi-sine perturbation inputs to the control surfaces provide rich flight data with low correlations between the explanatory variables [3,4]. Second, a recursive system identification scheme is required to estimate the aerodynamic forces and moments based on the explanatory variables measured in flight. These recent flight tests, as well as previous work, have successfully demonstrated global nonlinear aerodynamic modeling using recursive multivariate orthogonal functions (MOF). Since this method generates a global polynomial model, however, it can sometimes inadequately represent significant localized dynamics [5,6,7].

Similar to modeling, the classical approach to control is a lengthy process that involves complex simulations and tuning. Nonlinear control methods such as direct and indirect adaptive control are well-researched methods for aircraft flight control [8,9]. Indirect adaptive control methods such as nonlinear dynamic inversion (NDI) used in the recent L2F tests, are often derived from an estimated model through dynamic inversion, so the resulting control law can be sensitive to model inaccuracies. A control system that is compatible with the L2F concept must automatically adjust to a changing model while not interfering with the PTI inputs and the modeling process [10].

An alternative approach to both modeling and control, and the subject of the work presented in this paper, involves fuzzy logic. Introduced by Zadeh in Ref. [11], fuzzy logic involves a linguistic characterization of a system or process in the form of if/then statements called rules, where the consequent (output) is determined through fuzzy inference as a weighted combination of the truth values of the antecedent (input). Takagi-Sugeno (TS) fuzzy modeling was first presented in Ref. [12] as a method to represent a nonlinear dynamic system as a weighted combination of linear time invariant (LTI) subsystems.

Most fuzzy control methods generate the plant input as a weighted combination of inputs associated linguistically with the measured error, similar to a proportional-integral-derivative (PID) controller. These applications of fuzzy control do not directly rely on a mathematical model of the system, are typically based on a heuristic design process that renders them difficult to guarantee fundamental stability and performance requirements, and are often challenging to generalize across multiple platforms. Nevertheless, this fuzzy control approach has found many applications, such as in roll control and aircraft sensor failure diagnosis [13,14].

In contrast, Wang introduced a more rigorous fuzzy control method known as Parallel Distributed Compensation (PDC) that directly builds on the TS fuzzy model structure [15]. A linear control law is designed for each component LTI system simultaneously using a linear matrix inequality (LMI) formulation, and the total system input is a weighted combination of the individual control laws [16,17]. PDC combines traditional linear systems theory and Lyapunov theory to provide methods that have been shown to guarantee global asymptotic stability [16], robustness [18], optimal performance [19], and other performance constraints [20].

Prior L2F modeling work built on the fuzzy logic approach in Ref. [12] to develop a system identification routine that divided the nonlinear model into multiple linear parts that were weighted at each point in time according to the explanatory variables [6]. The fuzzy modeling method generated a global model with partitioned subsystems that accounted for localized variations more precisely than the MOF modeling approach.

Although fuzzy modeling and PDC have found a variety of applications across robotics and general process control, they have only recently been introduced to the aerospace field. Moreover, the properties of these fuzzy systems render them conducive to an improved method of aircraft control. A common approach to nonlinear aircraft control is to linearize a system at several equilibrium points and to gain schedule a linear controller at each reference condition. A curve can then be fitted through the various controllers to ensure smooth transitions, but a priori knowledge of the system as well, as extensive simulations, are typically used to guarantee global stability and performance. An advantage of the fuzzy logic approach is that the fuzzy model in Ref. [6], which can be generated automatically onboard an aircraft, is already partitioned into numerous linear subsystems, so PDC can be used to automate the design process for the linear control laws, as well as the curve fit between them, in a way that guarantees stability and performance. In the past, PDC has been applied to fuzzy models that have been built through local approximation [21] or sector nonlinearity [22], which require a priori knowledge of the system, but it has not, to the authors' knowledge, been applied to an identified fuzzy system.

The work presented in this paper builds on the fuzzy logic modeling algorithm developed in Ref. [6] to explore the feasibility of applying the PDC control approach and demonstrates the utility of this nonlinear control method through simulations. Although it is implied that the identified model can be updated recursively, and the control laws can be improved accordingly in real-time, this work performs a single batch system identification process and explores how PDC can be applied to an identified fuzzy model.

The following section describes the theory of fuzzy modeling and PDC, while Section III discusses the application to an F-16 aircraft and the resulting simulations. Finally, Section IV details conclusions drawn from this work.

II. Theory: Fuzzy Modeling and Parallel Distributed Compensation

A. Fuzzy Modeling

Fuzzy modeling is a mathematical tool that can be used to partition a nonlinear system into several linear subsystems known as cells, so that the overall nonlinear behavior of the system can be captured by a weighted combination, or fuzzy blending, of such subsystems. There are generally three ways to build a TS fuzzy model. The first method partitions the global system by approximating it at various equilibrium points to generate linearized systems that can be combined in a fuzzy framework. Although this method produces the least complex model, it is also the least accurate representation of the global model. The second and most common approach, known as sector nonlinearity, transforms an analytical model into an exact fuzzy representation of the dynamics by evaluating the nonlinear terms at their extreme values within a specified sector and deriving curves between them. The last method is system identification, in which the fuzzy cells are estimated through test data. This method can produce a reliable model in an automated fashion and has the flexibility to be used across multiple platforms. The work in this paper builds on the third fuzzy modeling approach, which will be summarized below.

The goal of the modeling process is to describe a dependent variable as a linear polynomial expansion that relates it to the measured explanatory variables. A fuzzy model is represented by a set of rules, or cells, in the form of if/then statements. Membership functions (MF), which vary from 0 to 1, are used to partition each normalized explanatory variable into weighted parts and describe the relevance of different sections across the range of the variable to each linear model.

To estimate the nonlinear function P, the ith rule is expressed as

$$R_i\text{: } IF\ x_1\ is\ M_{1,i}\ and\ \ldots and\ x_k\ is\ M_{k,i}\ THEN\ P = p_{o,i} + p_{1,i}x_1 + \cdots + p_{k,i}x_k \qquad i = 1, \ldots, r \tag{1}$$

where $x_1 \ldots x_k$ are the k explanatory variables used in the modeling, $M_{1,i} \ldots M_{k,i}$ are the associated membership functions, and $p_{o,i} \ldots p_{k,i}$ are the estimated parameters in the polynomial expansion of P_i, the ith linear cell used to describe P.

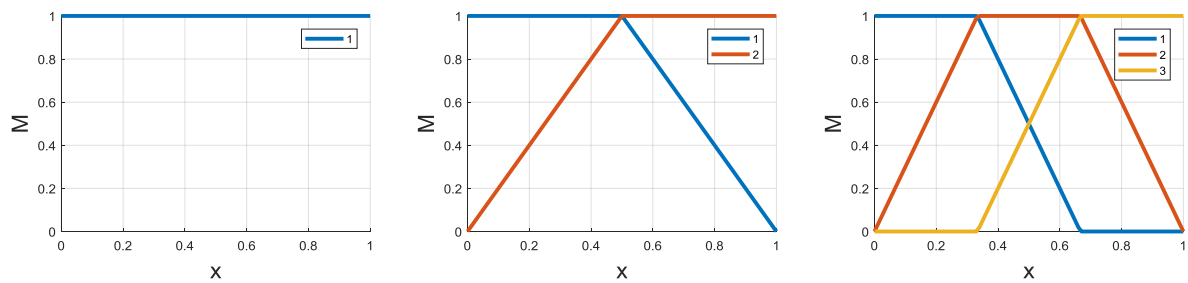
The process of designing the rule base or set of rules for a fuzzy system can be divided into three main parts: choosing explanatory variables, assigning membership functions, and estimating parameters.

1. Explanatory Variables

The explanatory variables must be chosen carefully so that there is enough information to sufficiently describe the dependent variable, while not leading to an over-parametrized and complex model. If the model structure within each cell is unconstrained, this process can be automated through a search cycle that is initialized with a (large) pool of candidate variables that is purged if it becomes apparent that a certain variable has minimal modeling value. For a restricted model structure within each cell, such as a linear state-space formulation, the states and controls can be chosen.

2. Membership Functions

The structure and complexity of the global fuzzy model are based on the shape, number, and distribution of MFs across each explanatory variable's range. The MFs can take on a variety of different shapes, but the specific shape chosen will play a significant role in the fuzzy system's ability to account for nonlinear dependencies in the variables. Figure 2 shows an example of ramp-shaped MFs that partition a single variable into one, two, and three segments.



(a) 1 MF **(b) 2 MFs** **(c) 3 MFs**

Fig. 2 Ramp-shaped membership functions.

If all of the explanatory variables have one MF, they are each weighted with a value of 1 across their entire (normalized) range, and the result is a single linear model. However, if the function being modeled has a nonlinear dependency on a specific explanatory variable, that variable is partitioned into smaller regions as more MFs are added. Each rule combines one MF for each explanatory variable into a single cell that describes the local behavior, and the nonlinear model is a combination of these cells that are weighted according to their MF values. Note that a higher number of MFs will yield a more complex nonlinear model. The total number of cells within a fuzzy model is the product of the number of MFs for each variable.

3. Parameter Estimation

The output of each cell is weighted by the product of all of its MFs, which is defined as

$$w_i(z) = \prod_{j=1}^{k} M_{j,i}(x_j) \tag{2}$$

The final weighted output of a fuzzy model with r cells is

$$P(t) = \frac{\sum_{i=1}^{r} w_i\big(x(t)\big)\{p_{o,i} + p_{1,i}x_1(t) + \cdots + p_{k,i}x_k(t)\}}{\sum_{i=1}^{r} w_i(x(t))} \tag{3}$$

Equation (3) can be expressed in the ordinary least squares matrix form given in Eq. (4) where z is an $N \times 1$ vector of time history data for the dependent variable P, X is an $N \times (k+1) * r$ matrix of weighted regressors for all cells, and the unknown parameters $p_0 \ldots p_k$ for all of the cells are collected in a single vector θ.

$$z = X\theta \tag{4}$$

The parameter vector θ can then be estimated in an equation-error approach using least squares by defining the cost function

$$J(\theta) = \frac{1}{2}(z - X\theta)^T(z - X\theta) \tag{5}$$

which has the solution

$$\hat{\theta} = (X^T X)^{-1} X^T z \tag{6}$$

The effectiveness of the resulting model can be described by a number of modeling metrics. In particular, the coefficient of determination defined in Eq. (7) represents a model fit quality measure that varies from 0 to 1 and describes how much of the variation in the data is captured by the model.

$$R^2 = \frac{\sum_{i=1}^{N}[\hat{y}(i) - \bar{z}]^2}{\sum_{i=1}^{N}[z(i) - \bar{z}]^2} \tag{7}$$

The fuzzy model process described above is typically used to model a non-dimensional force or moment coefficient based on a wide array of potential explanatory variables; however, in the context of state feedback control through PDC, the nonlinear model is most conveniently expressed in state space form as

$$\dot{x}(t) = A(x,u)x(t) + B(x,u)u(t) \tag{8}$$

and the corresponding rules for the fuzzy model are given as

$$R_i: If\ x_1(t)\ is\ M_{1,i} \ldots and\ x_k(t)\ is\ M_{k,i}\ then\ \dot{x}(t) = A_i x(t) + B_i u(t) \quad i = 1,2,\ldots,r \tag{9}$$

Equation (9) is the same expression as Eq. (1) where the parameters $p_0 \ldots p_k$ associated with states $x(t)$ are collected into a vector A_i and those associated with controls $u(t)$ are represented by B_i. The complete fuzzy model in Eq. (3) then becomes

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i\big(x(t)\big)\{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^{r} w_i\big(x(t)\big)} \tag{10}$$

where each of the r linear systems is weighted according to the MFs associated with it. Although the least squares modeling process is performed separately on each dependent variable, when multiple state derivatives are estimated as functions of states and control inputs with the same number of MFs, a nonlinear system can be represented as a weighted combination of LTI subsystems, and Eq. (10) is expressed in matrix form.

B. Parallel Distributed Compensation

The fuzzy model in Eq. (10) that consists of multiple weighted LTI systems can be used directly for control system design via state feedback using PDC. For each linear cell in the fuzzy model, a corresponding set of control gains K_i is designed, and the total system input is the weighted combination of each cell's individual input. Each rule in Eq. (9) can then be expanded as

$$R_i: If\ z_1(t)\ is\ M_{1,i} \ldots and\ z_k(t)\ is\ M_{k,i}\ then \begin{Bmatrix} \dot{x}(t) = A_i x(t) + B_i u(t) \\ u(t) = -K_i x(t) \end{Bmatrix}\ i = 1,2,\ldots,r \tag{11}$$

and the total input to the fuzzy model is

$$u(t) = -\frac{\sum_{i=1}^{r} w_i\big(x(t)\big)K_i x(t)}{\sum_{i=1}^{r} w_i\big(x(t)\big)} \tag{12}$$

Finally, combining Eqs. (10) and (12), the closed-loop system is

$$\dot{x}(t) = \frac{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i\big(x(t)\big)w_j\big(x(t)\big)\{A_i - B_iK_j\}x(t)}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i\big(x(t)\big)w_j\big(x(t)\big)} \tag{13}$$

Traditionally, closed-loop stability for a single cell can be explored by finding a matrix P to fulfill the steady-state form of the quadratic Lyapunov inequality in Eq. (14).

$$(A - BK)^T P + P(A - BK) < 0 \tag{14}$$

In order to guarantee stability for the global closed-loop model across all of the cells, the controller design problem is expressed as a set of LMIs where Eq. (14) must be satisfied for each LTI system with a single matrix P that is common among them. If $X = P^{-1}$, and a positive definite matrix X and matrices $M_i, i = 1, \dots, r$ can be found to satisfy

$$-XA_i^T - A_iX + M_i^T B_i^T + B_iM_i > 0 \tag{15}$$

$$-XA_i^T - A_iX - XA_j^T - A_jX + M_j^T B_i^T + B_iM_j + M_i^T B_j^T + B_jM_i > 0 \tag{16}$$
$$i, j = 1, \dots, r; \;\; i < j$$

then $P = X^{-1}$ and the gains for each cell can be solved as $K_i = M_iX^{-1}$ to guarantee global asymptotic stability. The LMI formulation can then be further modified to incorporate a constraint on settling time by considering the Lyapunov function of

$$\dot{V}(x) \leq -2aV(x) \tag{17}$$

where a represents the convergence rate, and the modified LMI formulation is shown in Eqs. (18-19).

$$-XA_i^T - A_iX + M_i^T B_i^T + B_iM_i - 2\alpha X > 0 \tag{18}$$

$$-XA_i^T - A_iX - XA_j^T - A_jX + M_j^T B_i^T + B_iM_j + M_i^T B_j^T + B_jM_i - 4\alpha X > 0 \tag{19}$$
$$i, j = 1, \dots, r; \;\; i < j$$

Equations (18-19) can be solved using a generalized eigenvalue minimization to maximize the convergence rate, or the desired convergence parameter can be imposed on the problem, as it was in this work.

A constraint on control input is enforced by including Eqs. (20-21) in the LMI,

$$\begin{bmatrix} X & M_i^T \\ M_i & \mu^2 I \end{bmatrix} \geq 0 \tag{20}$$

$$X - \phi^2 I \geq 0 \tag{21}$$

where μ is the upper bound on the control input and ϕ is the upper bound on the initial condition, such that $\|u(t)\|_2 \leq \mu$ and $\|x(0)\|_2 \leq \phi$. Despite these performance constraints imposed on the LMI, it is still posed a feasibility problem. Modifying this formulation to produce an optimal solution is the subject of ongoing work.

III. Results: Fuzzy Modeling, PDC, and Simulations for F-16 Aircraft

This section describes the automated fuzzy system identification procedure applied to data from a nonlinear F-16 simulation. The resulting model is used in conjunction with PDC to develop control laws. The purpose of this section is to show the utility of fuzzy modeling together with PDC by demonstrating their effectiveness on an approximation of the longitudinal dynamics through various simulations.

A. Aircraft

Simulated F-16 flight data was acquired from within the System IDentification Programs for AirCraft (SIDPAC) software toolbox.[23] The nominal geometry and mass properties of the F-16 used in the nonlinear simulations are summarized in Table 1. The simulated flight data consisted of a maneuver that began at a trimmed angle of attack of 4 deg at 25,000 ft. The pilot slowly increased the angle of attack by pulling back on the elevator and simultaneously excited the system with manual doublets, intending to excite the longitudinal dynamics across a wide range of angle of attack. The time histories of angle of attack (α), pitch rate (q), and elevator deflection (δ_e) are shown in Fig. 3. Two-percent Gaussian white noise was added to each of the data channels in a way that is similar to what would be seen in-flight.

Table 1. Geometry and mass properties in F-16 nonlinear simulation

length $\bar{c}$, ft	11.32
wing span b, ft	30
wing area S, ft^2	300
x_{ref}, ft	$0.35\bar{c}$
y_{ref}, ft	0.000
z_{ref}, ft	0.000
x_{cg}, ft	$0.25\bar{c}$
y_{cg}, ft	0.0
z_{cg}, ft	0.0
m, slug	647.2
I_x, slug-ft^2	9,496
I_y, slug-ft^2	55,814
I_z, slug-ft^2	63,100
I_{xz}, slug-ft^2	982

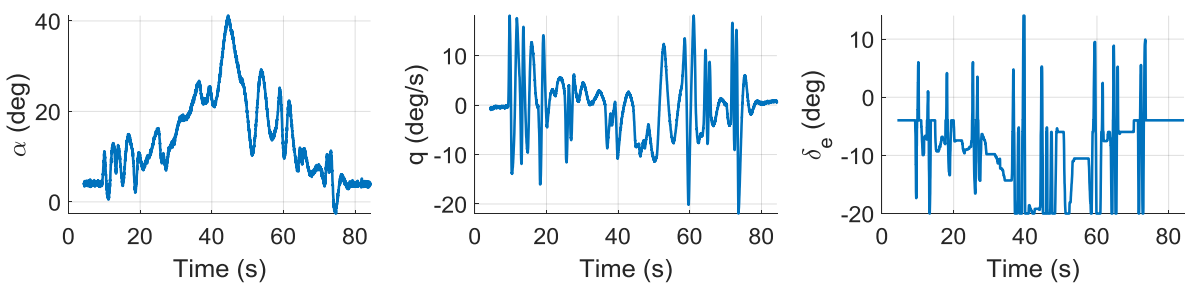


Fig. 3 F-16 simulated flight data time histories.

B. Fuzzy Modeling

Although the fuzzy modeling process is a recursive algorithm that can be updated onboard an aircraft in real-time, this work modeled the system in a batch form to focus on the effectiveness of incorporating PDC, with the understanding that PDC could be applied periodically as the model is updated. To simplify the processes of modeling and PDC, the aircraft longitudinal dynamics were approximated through the state derivatives $\dot{\alpha}$ and $\dot{q}$, which were modeled as functions of α, q, and δ_e. Each LTI subsystem is then represented in state space form by Eq. (22).

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \delta_e \\ 1 \end{bmatrix} \tag{22}$$

The state derivatives $\dot{\alpha}$ and $\dot{q}$ were modeled individually before being combined into the state space representation of the longitudinal system dynamics. The resulting constant parameters in the A and B matrices were estimated using

least squares for each fuzzy cell. B_{12} and B_{22} represent p_0 in Eq. (1), i.e., the constant bias term associated with each linear subsystem.

It is only necessary to partition an explanatory variable if $\dot{\alpha}$ or $\dot{q}$ has a nonlinear functional dependency on that variable. Batterson and Klein in Ref. [24] discuss partitioning a longitudinal aerodynamic force or moment with many crisply partitioned subsystems across the range of angle of attack alone, given that much of the nonlinearity of the longitudinal motion results from variation in angle of attack. With this in mind, the fuzzy model structure was simplified by partitioning only angle of attack with multiple MFs. The resulting number of cells in a fuzzy model is, therefore, equal to the number of angle of attack partitions.

Candidate fuzzy models for $\dot{\alpha}$ and $\dot{q}$ were estimated as the number of angle of attack partitions was increased from one up to eight, and the results of the corresponding R^2 statistics are summarized in Fig. 4.

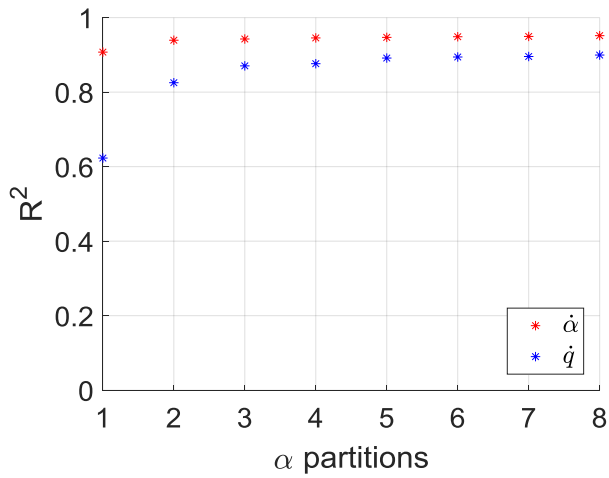


Fig. 4 Comparison of coefficient of determination for varying α partitions.

R^2 increases significantly from one up to three partitions, but additional MFs do not appear to improve the model quality for either $\dot{\alpha}$ or $\dot{q}$. Note that R^2 alone is a limited assessment of the model quality, and additional statistical metrics, such as the Predicted Squared Error (PSE), which penalizes model complexity, could provide further insight. However, this work explores not only the model quality itself but also how well the controller responds with varying resolution, or number of partitions, across the nonlinear dynamics. It is still worth noting, however, that the model complexity is directly parallel to that of the controller, so to improve the likelihood of finding a solution to Eq. (15-16), fewer cells are preferred.

Figures 5-6 compare the model fits for $\dot{\alpha}$ and $\dot{q}$ between one angle of attack partition (effectively a linear model), and three partitions, respectively. The aerodynamics truly vary as a function of dynamic pressure since the flight data spans a large range of angle of attack, so the inclusion of aircraft velocity as a state and therefore, as an explanatory variable, would be a good first approximation to capture this dependency and improve the model fit beyond what is shown below.

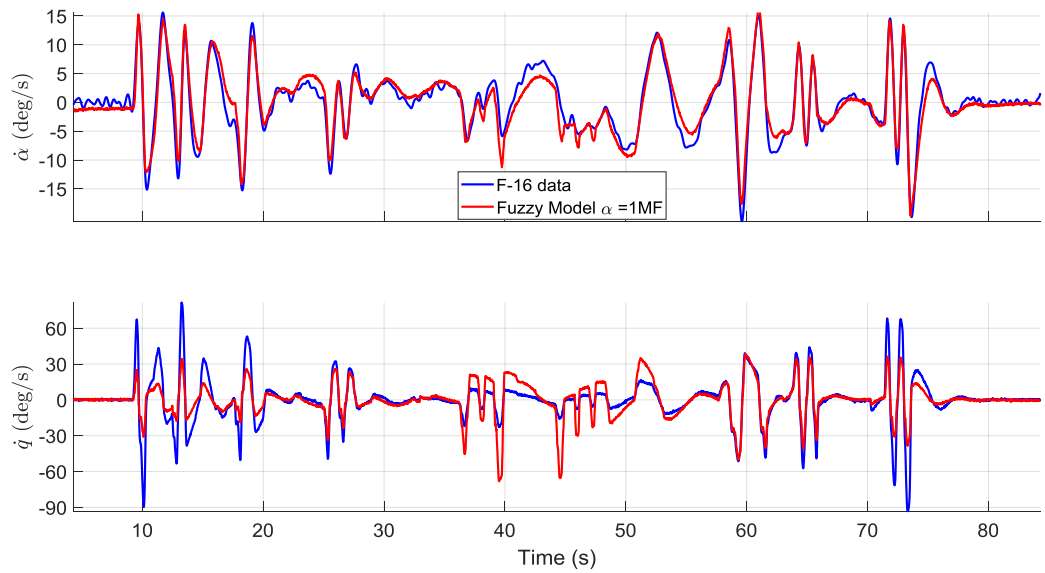


Fig. 5 Comparison of F-16 data and fuzzy model with $\alpha = 1MF$.

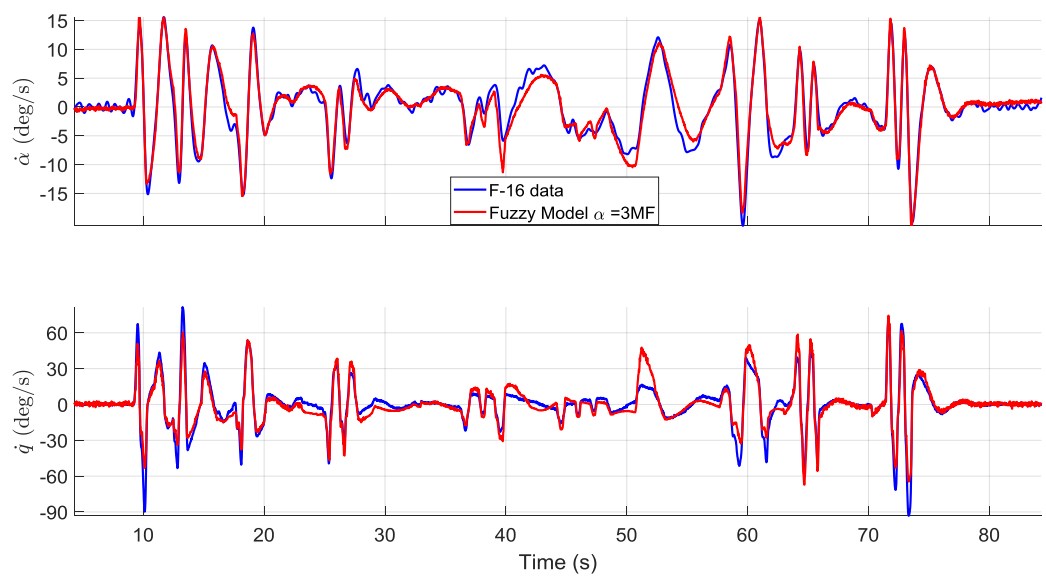
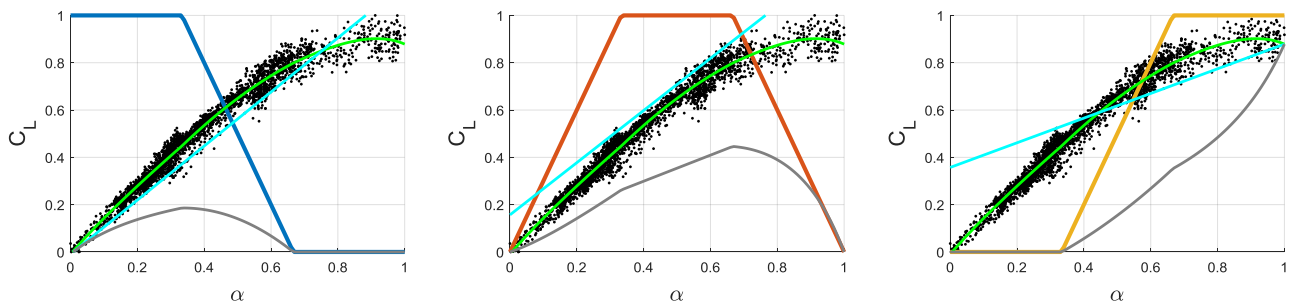


Fig. 6 Comparison of F-16 data and fuzzy model with $\alpha = 3MF$.

To help visualize how the MFs are used to partition the explanatory variables, consider a familiar case of modeling the coefficient of lift $C_L = f(\alpha)$ for this F-16 data with three MFs as shown in Fig. 2c. This would lead to a fuzzy model with three linear cells. The nonlinear modeling process can be thought of as generating one linear subsystem that more accurately represents the low angle-of-attack dynamics, a second that represents the moderate range, and a third that is weighted more strongly at high values, as shown in Fig. 7. The weighted combination of the three linear systems generates a global nonlinear system with smooth transitions between the linear components.



(a) 1st α partition **(b) 2nd α partition** **(c) 3rd α partition**

Fig. 7 Fuzzy model partitions for $C_L = f(\alpha)$ with $\alpha = 3MF$, normalized.

The MF shapes from Fig. 2c plotted over the data in each partition indicate which part of the data each linear model is most closely fitted to. The cyan lines are the linear models for each partition, the grey lines are the weighted versions of each linear model across the range of angle of attack, and the green line is the weighted combination of the contributions from each partition, i.e., a nonlinear model for C_L, as computed in Eq. (10).

A significant limitation to the fuzzy model identified in this way using test data is that although it is divided into several linear subsystems, it only guarantees the *global* model itself to be physically meaningful. Since the coefficients for all of the cells are estimated in a single least-squares estimation using fuzzy regressors that are weighted according to the MFs for each cell, they cannot be considered meaningful individually. Only when all of the cells are weighted and combined as in Eq. (10) does the complete physical, nonlinear model emerge. Moreover, when the corresponding cells for the $\dot{\alpha}$ and $\dot{q}$ models are augmented in matrix form, the dynamics associated with an individual LTI subsystem cannot be considered physical. This limits the insight into how the nonlinear model is partitioned and represents a current drawback to this identification process over the other types of fuzzy models discussed at the beginning of Section IIA. This lack of physicality requires further investigation with regards to its impact on PDC; nevertheless, it is shown that PDC is still a feasible control method.

C. Parallel Distributed Compensation

While a solution to Eqs. (15-16) will ensure closed-loop stability for the fuzzy system, it will not guarantee tracking of a reference signal. To track an angle-of-attack command, each component LTI system must therefore be augmented with an integrator $\dot{z} = \alpha_c - \alpha$ to allow tracking of a reference angle-of-attack command, α_c. The open loop approximated model in Eq. (22) is modified to

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ z \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \alpha_c \tag{23}$$

With full state feedback, each cell's input is then expressed as

$$\delta_e = K\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \alpha_c - \begin{bmatrix} \alpha \\ q \\ -z \end{bmatrix} \right) = \begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix} \begin{bmatrix} \alpha_c - \alpha \\ 0 - q \\ z \end{bmatrix} \tag{24}$$

Since the feedback is associated only with input δ_e, the A_i matrices and only the first column of the B_i matrices were passed through the LMI formulation to solve for a set of gains for each cell in the fuzzy model. The inputs from each fuzzy cell are weighted according to the associated MFs to define a single system input as in Eq. (12), with the final closed-loop system expressed in Eq. (13).

The specified convergence rate in Eqs. (18-19) was set to 0.5 while the parameters μ and ϕ in Eqs. (20-21) were set to 5 and 0.01, respectively. These LMI parameters were held constant throughout all of the simulations, and each of the resulting control laws was designed automatically using the same algorithm.

D. Simulations

Simulations were performed to test the effectiveness of the PDC design algorithm on identified fuzzy models with varying numbers of angle of attack partitions. Each simulation was run at 50 Hz using SIDPAC's full F-16 nonlinear aerodynamic database and simulation tools, with elevator deflection limited to $-25 \text{ deg} \leq \delta_e \leq 25$ deg. Multistep angle of attack commands were simulated across the nonlinear angle of attack regime to test how well the system would respond.

First, to motivate this work for nonlinear aircraft control, a linear model was built using only flight data in the linear regime of angle of attack up to 12 deg. The aircraft was trimmed at 5 deg, and Fig. 8 shows that while the aircraft responds well to a step from 5 to 10 deg, a single linear controller defined over the linear aircraft dynamics is insufficient for tracking reference commands well at higher angles of attack. Although the dynamics remain stable, the response has large overshoot and oscillatory motion with slow decay.

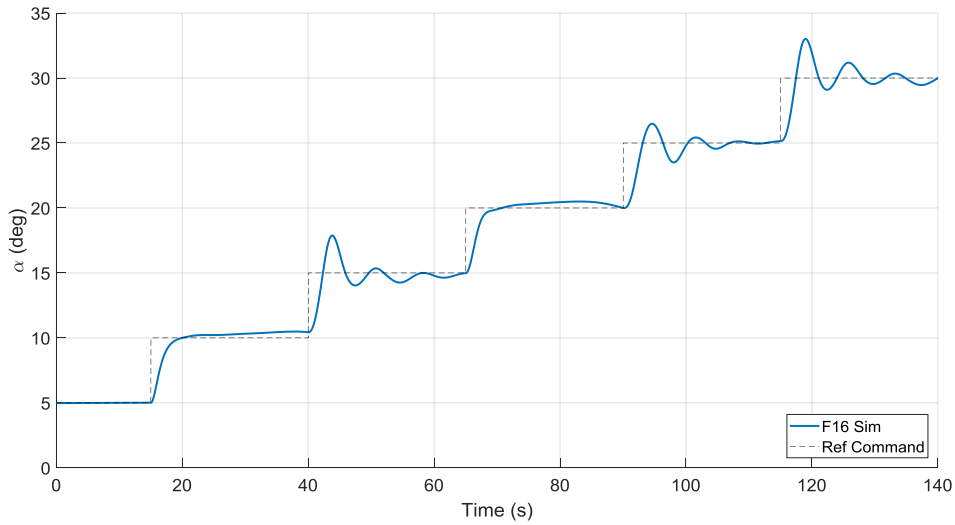


Fig. 8 Multistep response with linear controller.

To better account for the nonlinear dependency of the longitudinal dynamics on angle of attack, nonlinear controllers were designed using PDC on fuzzy models with varying numbers of angle of attack partitions. Each of the closed loop models, which have one, three, five, and seven partitions, was given multistep angle of attack commands. The responses to the multistep inputs are shown in Fig. 9. These closed-loop systems provide more consistent responses with better transient properties than the simple linear controller case shown in Fig. 8.

Figure 10 allows a closer inspection to compare the responses at each step for each of the individual controllers. The controller with the single partition, denoted $\alpha = 1MF$ differs from the linear case shown in Fig. 8 since a linear model was fit to the entire data set, including the angle of attack variations up to 40 deg. While the linear model was a good fit for the linear region of the data, this model would be fit across the entire data set, but would lose its accuracy in any particular region. Nevertheless, it responds fairly well, but does not appear to converge completely within 25 s, and the response at each step is inconsistent with the others. The $\alpha = 3MF$ case performs better with faster settling times and more consistent responses. The $\alpha = 5MF$ case performs best with overshoot limited to just above 1 deg at all steps, rapid settling in all cases, and consistent trends in the responses. While the responses in the $\alpha = 7MF$ case are fairly consistent at each step, the overshoot increases significantly from all the other cases to above 3 degrees at the high alpha-of-attack step. While each controller provides a feasible option to track angle of attack commands across the flight regime, an optimization procedure would have to be designed to select the specific number of partitions that would prioritize certain performance requirements to more directly influence the nature of the response.

Figures 11-12 show similar plots for multistep responses beginning at a trimmed 30 deg condition and stepping down to 5 deg. The comparisons between controllers are consistent with the first cases, but the responses do differ between the increasing and decreasing reference angle of attack commands. In particular, the $\alpha = 5MF$ case responds better in the downward steps, with overshoot limited to less than 0.5 deg except for the first step.

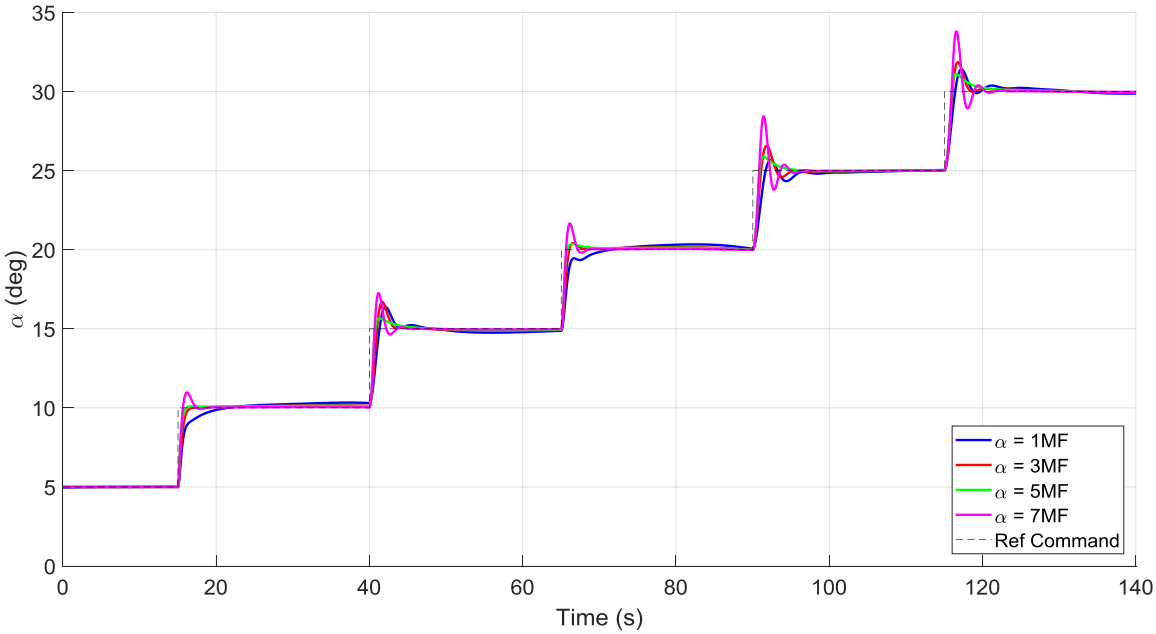


Fig. 9 Multistep responses with fuzzy controllers, increasing angle of attack reference commands.

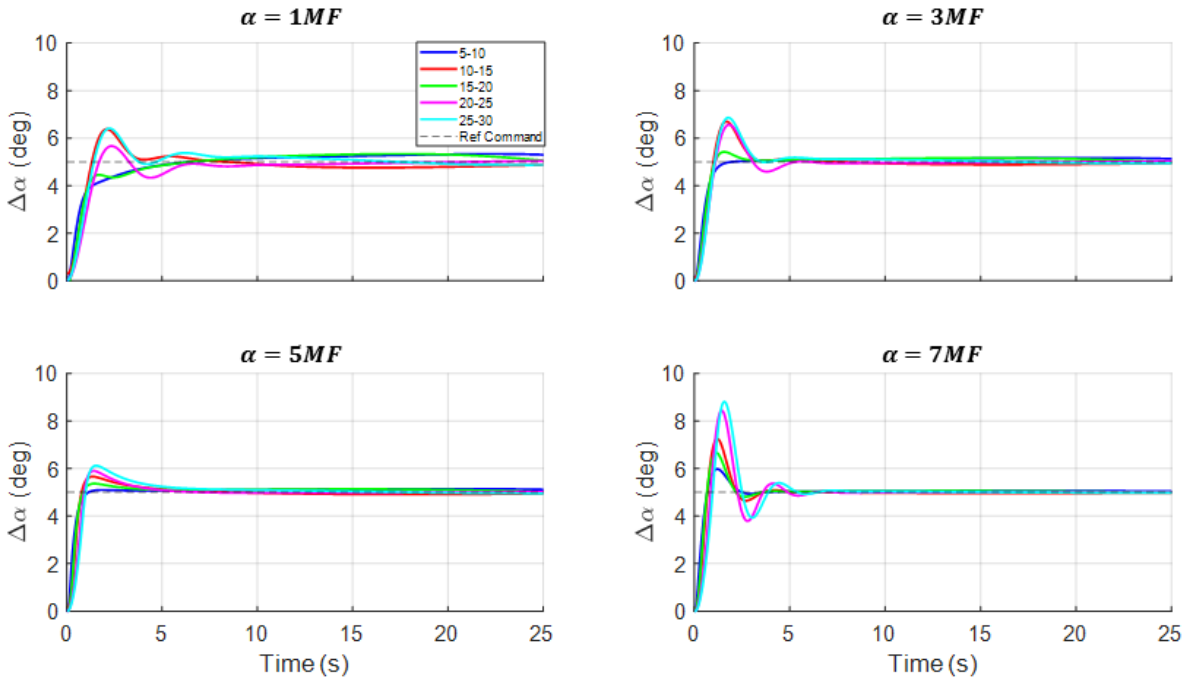


Fig. 10 Multistep responses for each fuzzy controller, increasing angle of attack reference commands.

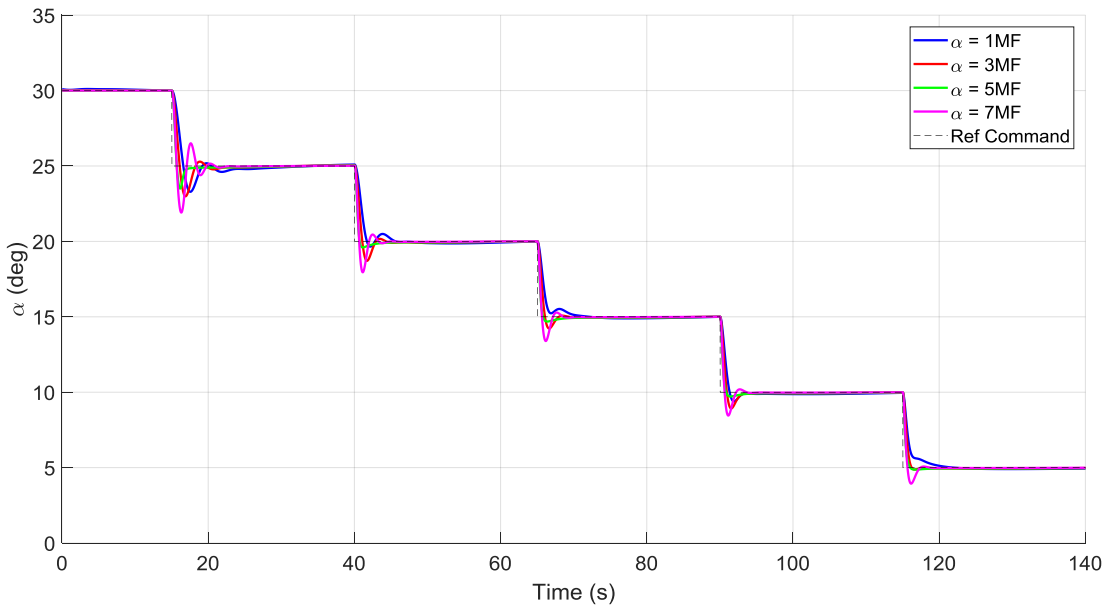


Fig. 11 Multistep responses with fuzzy controllers, decreasing angle of attack reference commands.

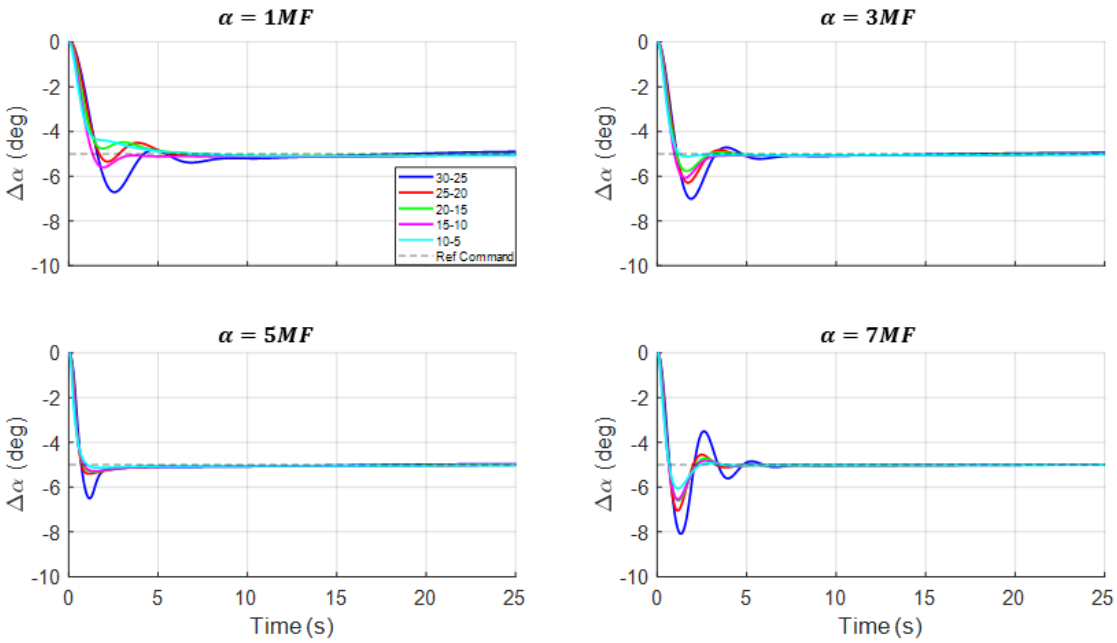


Fig. 12 Multistep responses for each fuzzy controller, decreasing angle of attack reference commands.

IV. Conclusions

This work applied PDC in a novel way to a fuzzy model that has been identified using simulated flight data. In the past, PDC has been applied to fuzzy models built using an analytical model of the system. This work represented a feasibility study of the application of PDC to an aircraft model identified in flight in order to track angle of attack commands across the nonlinear flight regime. In support of the L2F concept, the fuzzy model can be generated onboard an aircraft with minimal a priori knowledge of the aerodynamics, and PDC can be used to automate the control law design based on the fuzzy model structure to guarantee stability and performance requirements.

A fuzzy modeling routine was used to build an approximated longitudinal model for the F-16 aircraft with varying complexity in the number of angle-of-attack partitions, and PDC was applied to track reference angle-of-attack commands across the nonlinear flight regime. Modeling results were shown for fuzzy models with one and three angle-of-attack partitions, and despite the lack of dynamic pressure information, the model fits were sufficient for control purposes.

The automated control law design through PDC incorporated analytical guarantees for stable responses, tracking of angle-of-attack reference commands, and other performance constraints on convergence rate and control input. Despite the limited fidelity of the fuzzy model used to build these control laws, satisfactory responses were generated for a series of simulated angle-of-attack commands throughout the flight envelope. Four different controllers were compared with varying complexities based on the number of angle-of-attack partitions, and the controller built on a model with five partitions generated the most consistent responses across the angle-of-attack regime. This work demonstrated the feasibility and utility of applying PDC to an identified fuzzy model.

Acknowledgements

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