Evaluation of a Multizone Impedance Eduction Method

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A computational study is used to evaluate the PyCHE impedance eduction method developed at the NASA Langley Research Center. This method combines an aeroacoustic duct propagation code based on numerical solution to the convected Helmholtz equation with a global optimizer that uses the Differential Evolution algorithm. The efficacy of this method is evaluated with acoustic pressure data simulated to represent that measured with one-zone, two-zone, and three-zone liners mounted in the NASA Langley Grazing Flow Impedance Tube. The PyCHE method has a normalized impedance error of approximately 0.2 for (uniform) one-zone liners with a length of at least 5", and produces quite reasonable results for liners as short as 2". Whereas the impedance of the liner has an effect on eduction accuracy, the amount of attenuation is shown to be the dominant parameter. Similar results are observed for two-zone liners, for which the impedance of each zone is unique. The two-zone results also indicate it is more difficult to accurately educe resistance than reactance, and a zone length of at least 6" (slightly longer than for uniform liners) is needed to limit the normalized error to 0.2. The PyCHE method is also demonstrated to successfully educe the impedances for each zone of a three-zone liner. These results are sufficiently encouraging to warrant the continued usage of the PyCHE impedance eduction method for single and multizone liners.

I. Introduction

Increasingly novel concepts are needed to meet the ever-tightening noise restrictions imposed by the International Civil Aviation Organization (ICAO) and the Federal Aviation Administration (FAA) for commercial aircraft. One of the major components of aircraft noise is the sound associated with the fan mounted in the engine nacelle. Much effort has been expended in the development of improved fan designs in order to reduce the amount of fan noise that is generated.[1,2] In addition, acoustic liners mounted in the walls of the nacelle provide significant reduction of the fan noise as it propagates through the nacelle and emanates to the surrounding community via the inlet and aft bypass duct. However, there remains significant interest in the development of advanced acoustic liners to further reduce this radiated noise. One novel liner configuration that has received renewed interest is the multizone (also referenced as multisegment) liner, which consists of two or more axial zones with distinct acoustic properties (i.e., impedance spectra).

The purpose of the current paper is to evaluate a relatively new impedance eduction method developed at the NASA Langley Research Center that is intended to be used for detailed investigation of multizone liners.[3] This method combines an aeroacoustic duct propagation code based on a numerical solution to the convected Helmholtz equation[4] with an optimizer (the Differential Evolution algorithm) included within the SciPy optimization toolkit[5] and is referred to herein as the PyCHE method. It is assumed that the lengths and locations of the respective zones are known. In general, the PyCHE method can be used for an arbitrary number of zones, but the current study examines its use with one, two, and three zone liners.

One of the concerns with any impedance eduction method is the effect of poor attenuation. Based on data acquired in the NASA Langley Grazing Flow Impedance Tube (GFIT), it has been demonstrated[6–9] that impedance eduction methods for uniform liners become less robust when the attenuation is low. If the total liner length is held constant, it seems reasonable to assume that this effect of poor attenuation

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will be exacerbated for multizone liners, as the reduced length of each zone (portion of liner with constant impedance) will result in reduced attenuation over that zone. In other words, the individual zones must remain sufficiently long to absorb enough sound such that the eduction method will remain robust.

For the current study, synthesized data for liners consisting of one, two, or three axial zones are used to evaluate a multizone impedance eduction method. Random jitter is applied to each set of acoustic pressure data to simulate the effects of measurement uncertainty typically observed in the GFIT. First, a series of uniform, one-zone, liners with lengths of 2” to 22” is used to explore the loss of impedance eduction accuracy as the liner becomes sufficiently short to produce minimal attenuation. Next, a number of two-zone liner configurations are used to determine a minimum zone length (or attenuation) required to support a robust impedance eduction process. Acoustic pressure data is generated for a number of configurations, where the respective lengths of the two zones are varied from the short/long to the long/short extremes. The PyCHE method is used to educe the impedance of each zone. Finally, a three-zone liner configuration is used to examine the accuracy of the PyCHE for this level of complexity.

The effects of impedance discontinuity are also of interest. For each multizone liner, there are impedance discontinuities at the leading and trailing edges of the liner, and at the interface between zones. Results from the earlier study[3] suggested the possibility that the strength of the discontinuity (i.e., the difference in impedance across these interfaces) might have some effect on the multizone impedance eduction results. Also, results from the earlier study suggested the possibility that the ratio of wavelength to liner zone length might have some effect on the impedance eduction results. These issues are briefly explored in this study.

Section II provides a brief overview of the aeroacoustic propagation code and optimizer used in this study, as well as a description of the PyCHE impedance eduction method based on these two elements. Computational studies with increasingly complex liner configurations (one, two, three zones) are discussed in Section III. Finally, Section IV lists the primary contributions from this study.

II. Computational Tools

Two computational tools are used for this study. The first is an aeroacoustics duct propagation code that computes the sound field throughout a flow duct based on the impedance for each zone of the liner (ranges from one to three zones in this study) and the flow conditions within the duct. The other is a global optimizer based on the Differential Evolution Algorithm. The PyCHE impedance eduction method imposes assumed impedances at the surface of each zone and uses the propagation code to predict the acoustic pressure field throughout the duct. It then uses the optimizer to iterate on these impedances until the predicted acoustic pressures are within an acceptable tolerance of those measured (or synthesized) in a waveguide such as the NASA Langley Grazing Flow Impedance Tube (GFIT, see Fig. 1). The selected impedances are taken to be the correct values for each zone.

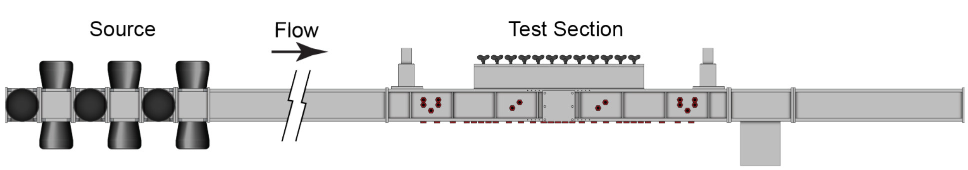


Figure 1: Sketch of the NASA Langley Grazing Flow Impedance Tube (GFIT).

A. Propagation Code - Convected Helmholtz Equation (CHE)

The acoustic field propagating through a flow duct containing a liner in the upper wall (Fig. 2), with uniform mean flow and only plane waves upstream and downstream of the liner, satisfies a convected Helmholtz equation on the acoustic pressure field (assume a time dependence of the form $e^{i\omega t}$)

$$\left(1 - M_0^2\right) \frac{\partial^2 p(x,y)}{\partial x^2} + \frac{\partial^2 p(x,y)}{\partial y^2} - 2ikM_0 \frac{\partial p(x,y)}{\partial x} + k^2 p(x,y) = 0, \qquad (1)$$

where p is the acoustic pressure field and M_0 is the uniform flow Mach number.

The local-reacting wall boundary condition[10] is given by

$$-\frac{\partial p(x,H)}{\partial y} = ik\left(\frac{p(x,H)}{\zeta}\right) + 2M_0\frac{\partial}{\partial x}\left(\frac{p(x,H)}{\zeta}\right) + \frac{M_0^2}{ik}\frac{\partial^2}{\partial x^2}\left(\frac{p(x,H)}{\zeta}\right), \quad (2)$$

where the normalized admittance, $1/\zeta$, is taken as zero along the rigid wall portion of the upper wall and H is the duct height (2.5" for the GFIT). For a uniform liner, this boundary condition is applied uniformly along the axial extent of the liner. For a multizone liner, this boundary condition is separated into multiple components, where each zone has a unique impedance.

The normal component of the acoustic particle velocity vanishes at the rigid lower wall

$$\frac{\partial p(x,0)}{\partial y} = 0, \quad (3)$$

and the acoustic pressure at the source plane is assumed known

$$p(0,y) = p(0,0). \quad (4)$$

For the generation of synthesized data in this study, the liner impedance, ζ, and termination impedance, ζ_e, are assumed known, and the exit plane boundary condition can be written in the form

$$\frac{\partial p(L,y)}{\partial x} = \frac{-ikp(L,y)}{M_0 + \zeta_e}. \quad (5)$$

For impedance eduction, the acoustic pressure at the exit plane ($x = L$; 40" for the GFIT) is assumed known, and the exit plane boundary condition is given as

$$p(L,y) = p(L,0). \quad (6)$$

The liner impedance, $\zeta = \theta + i\chi$, is educed as described below, where θ and χ represent the normalized acoustic resistance and reactance, respectively. Throughout this paper, the impedance is assumed to be normalized by the characteristic impedance of air, ρc, where ρ and c are the density and sound speed in air, respectively.

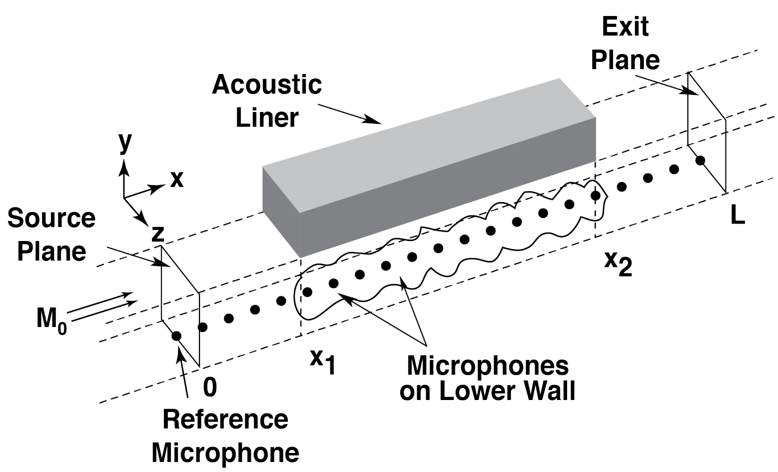


Figure 2: Sketch of flow duct computational domain.

B. Impedance Eduction Objective Function

The unknown normalized impedances (ζ_1 for a uniform liner, $\{\zeta_1, \zeta_2\}$ for a two-zone liner, $\{\zeta_1, \zeta_2, \zeta_3\}$ for a three-zone liner) are educed by minimizing the objective function, which is the sum of absolute values of the differences between the simulated and educed acoustic pressures at each of the microphone locations. This objective function is thus given by

$$F(\zeta_n) = \sum_{I=1}^{N} \left\| p(x_I, 0)_{\text{sim}} - p(x_I, 0)_{\text{edu}} \right\|, \tag{7}$$

where ζ_n denotes the unknown liner impedances (whether 1, 2, or 3 values), $||$ denotes the absolute value, N is the number of microphones, x_I is the location of the I^{th} microphone, and the subscripts 'sim' and 'edu' denote the simulated (numerically computed) and educed pressure fields, respectively.

C. Optimizer

The PyCHE impedance eduction method supports the use of a variety of optimizers contained within the SciPy optimization toolkit.[5] Previous studies[3] have considered three optimizers with this method. The first is a gradient-based optimizer that provides similar results to those achieved with the SDFP algorithm (Stewart's adaptation of the Davidon-Fletcher-Powell algorithm) that has been used extensively in earlier NASA eduction studies.[11] However, due to the occasional occurrence of multiple local optima, the use of a gradient-based optimizer necessitated multiple computations (i.e., separate eductions using multiple initial estimates of the liner impedance) in order to ensure that the optimum impedance was educed.

In an attempt to alleviate the need for multiple computations, two global optimizers (Basin-Hopping and Differential Evolution algorithms) were considered that are stochastic in nature and attempt to find the global minimum of a multivariate function. Both optimizers provided good results, but the Differential Evolution was more efficient (largely due to the particular settings used in the analysis). Thus, the results reported in this paper will be limited to those obtained with the Differential Evolution algorithm. This algorithm begins with a candidate population and with each iteration the algorithm mutates each candidate solution by mixing with other candidate solutions to create a trial candidate. The fitness of each trial candidate is assessed in relation to the objective function and the calculations continue until prescribed tolerances or maximum iterations are reached. For the current study, the population size, tolerance and maximum number of iterations were set to 50, 0.01, and 500, respectively. Also, this version of the PyCHE constrains the educed impedances to $\theta = \{0.05, 10.00\}$ and $\chi = \{-10.00, 10.00\}$ to ensure that implausible results are avoided. For the liners considered in this study, the educed impedances met the prescribed tolerances with far less than the maximum allowable number of iterations.

III. Computational Study

Simulated acoustic pressure data representative of that which would be measured in the GFIT is used to investigate the efficacy of the PyCHE impedance eduction method for use with liners consisting of one, two, and three axial zones. Each zone of a multizone liner is assumed to have a distinct impedance.

First, a series of one-zone (uniform) liners with lengths of 2" to 22" is used to determine the efficacy of the PyCHE method. Specifically, these liners explore the effects of reducing the liner length (with corresponding reduced attenuation) on the accuracy of the impedance eduction method. The results are compared with those obtained with an earlier version of this impedance eduction method.[9] Next, a number of two-zone liner configurations are used to determine a minimum zone length (or attenuation) required to support a robust impedance eduction process. Acoustic pressure data is generated for a number of configurations, where the respective lengths of the two zones are varied from the short/long to the long/short extremes, and the PyCHE method is used to educe the impedance of each zone. Finally, acoustic pressure data simulated for a three-zone liner configuration is used to examine the accuracy of the PyCHE for this level of complexity.

A. One-Zone (Uniform) Liners

Previous studies[9,11] suggested that accurate impedances could be educed from acoustic pressure data acquired in the GFIT if the liner length was at least 16". The results were attained with two impedance eduction methods, one that employed the Kumaresan and Tufts algorithm and another that used the CHE propagation code with a gradient-based optimizer. The accuracy of the educed impedances deteriorated as the liner length was reduced, but the results remained acceptable for a liner length of at least 8". Whereas the premise had been that the required liner length would be a function of the source frequency wavelength, the results suggested the key parameter to be the amount of attenuation. Indeed, the results suggested that each impedance eduction method provided sufficiently accurate results for those conditions (Mach number and frequency) where at least 5 dB attenuation was achieved. These studies were based on data measured with uniform-depth, perforate-over-honeycomb liners mounted in the GFIT. Unfortunately, those liners only provided acceptable attenuation (at least 5 dB) over a portion of the frequency range of interest. Thus, one goal of this study is to reevaluate the attenuation requirement via the use of liners specifically designed to provide nearly identical attenuation over the entire frequency range.

The CHE code was used to generate acoustic pressure data representative of that which would be measured with a one-zone (uniform) liner mounted in the GFIT. Specifically, these acoustic pressures were generated at locations corresponding to the microphone locations currently used by NASA Langley for impedance eduction. Two sets of virtual liners were explored. For the first set of liners, labeled as the 'R1' set, the acoustic resistance was set to unity ($\theta = 1$) and the acoustic reactance was chosen (varies with frequency) such that an 8"-long liner produces 5 dB attenuation at every frequency of interest. For the second set of liners, labeled as the 'X0' set, the acoustic reactance was set to zero ($\chi = 0$) and the acoustic resistance was chosen (varies with frequency) to achieve 5 dB attenuation for an 8"-long liner. The impedances for both sets of liners are provided in Table 1 for Mach numbers of 0.0 and 0.3. Clearly, these liner configurations would be difficult at best to physically achieve, but they are nonetheless useful for the purposes of this study.

Table 1: Impedances for R1 and X0 liners; Mach 0.0.

	Mach 0.0				Mach 0.3			
	R1 set		X0 set		R1 set		X0 set	
Frequency, kHz	θ	χ	θ	χ	θ	χ	θ	χ
0.4	1.00	-1.24	2.77	0.00	1.00	-0.42	1.45	0.00
0.6	1.00	-1.24	2.50	0.00	1.00	-0.45	1.44	0.00
0.8	1.00	-1.31	2.68	0.00	1.00	-0.67	1.71	0.00
1.0	1.00	-1.53	2.86	0.00	1.00	-0.88	1.82	0.00
1.2	1.00	-1.67	2.80	0.00	1.00	-0.93	1.74	0.00
1.4	1.00	-1.70	2.65	0.00	1.00	-0.95	1.71	0.00
1.6	1.00	-1.74	2.67	0.00	1.00	-1.06	1.77	0.00
1.8	1.00	-1.87	2.74	0.00	1.00	-1.16	1.77	0.00
2.0	1.00	-1.98	2.68	0.00	1.00	-1.20	1.70	0.00
2.2	1.00	-2.02	2.55	0.00	1.00	-1.25	1.66	0.00
2.4	1.00	-2.09	2.48	0.00	1.00	-1.33	1.64	0.00
2.6	1.00	-2.21	2.42	0.00	1.00	-1.38	1.54	0.00

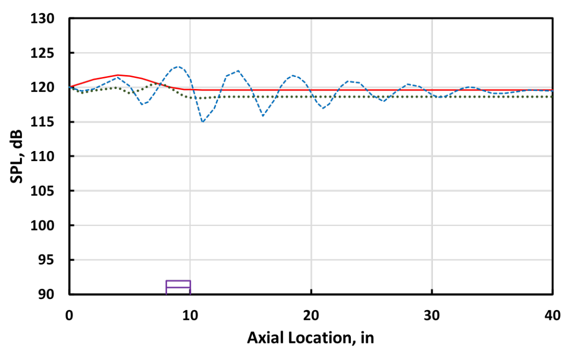
Because of the design criteria, the attenuation of these liners is a function of liner length. In the following, the attenuation parameter is explored via simulations for liner lengths of 2" to 22", for which the attenuation should range from 1.25 to 13.75 dB. Since the attenuation is designed to be frequency independent, the wavelength parameter can also be explored. Simply put, if the accuracy of the educed impedances correlates well with frequency, that would suggest the ratio of the wavelength to the liner length may be of greater importance than previously thought.

A total of 88 configurations were considered in this portion of the study (two liner types: R1 [$\theta = 1$, $\chi =$

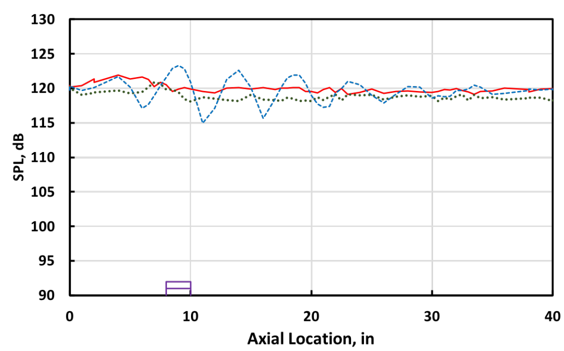
$\chi(f)$] and X0 [$\theta = \theta(f)$, $\chi = 0$]; two Mach numbers: 0.0 and 0.3; eleven liner lengths ($x_2 - x_1$): 2" to 22" in 2" increments; two datasets: Clean [no measurement error] and Random [include measurement error]). Figure 3 provides acoustic pressure profiles computed at each GFIT microphone location using the CHE propagation code for 2"-long and 22"-long samples from the X0 set, at Mach 0.3. For the sake of clarity, results for only one Mach number (0.3) and three frequencies (0.6, 1.6, and 2.6 kHz) are included. Since each liner type was independently designed with the same criteria (5 dB per 8" liner length), their respective propagation results are very similar.

The acoustic pressures in Fig. 3(a) and (c) are labeled as 'clean data' because measurement error effects are not included. Figure 3(b) and (d) includes the effects of measurement error by the addition of random jitter at each data point. Based on historical data, the sound pressure level (SPL) and phase at each microphone location are assumed to vary by up to ±0.5 dB and ±1.0 deg, respectively. The liner location and length are depicted via the sketch at the bottom of each figure. The source SPL (at $x = 0.0$") is set to 120 dB and the exit impedance is set to that of an outgoing plane wave (i.e., $\zeta_e = 1 + i\,0$) for all simulations in this study.

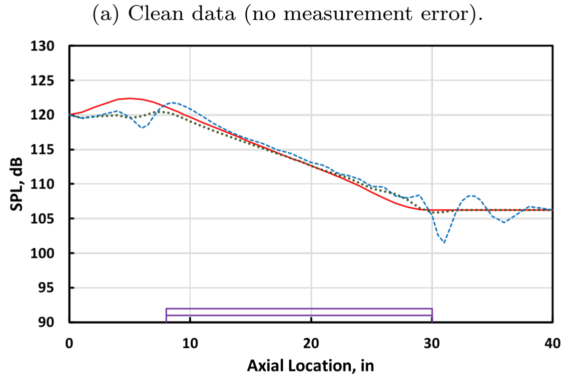
As might be expected, the effects of the 2"-long liner on the predicted acoustic pressure magnitude (depicted as SPL in these figures) are minimal, whereas the 22"-long liner produces much more attenuation. As a result, the effects of measurement error (random jitter) are more significant for the short liner, especially at 2.6 kHz. Also, note that the attenuation for all three frequencies is approximately 13.5 dB for the 22"-long liner (approximately 5 dB per 8" of liner length). Again, the impedance at each frequency was specifically chosen to achieve the same amount of attenuation.



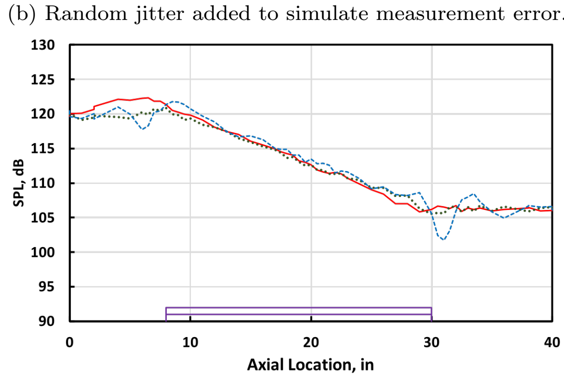
(a) Clean data (no measurement error).



(b) Random jitter added to simulate measurement error.



(c) Clean data (no measurement error).



(d) Random jitter added to simulate measurement error.

Figure 3: Acoustic pressure profile for 2"-long (a, b) and 22"-long (c, d) uniform liners. X0 set, Mach 0.3; red solid line, blue dashed line, and green dotted line present results for 0.6, 1.6, and 2.6 kHz, respectively.

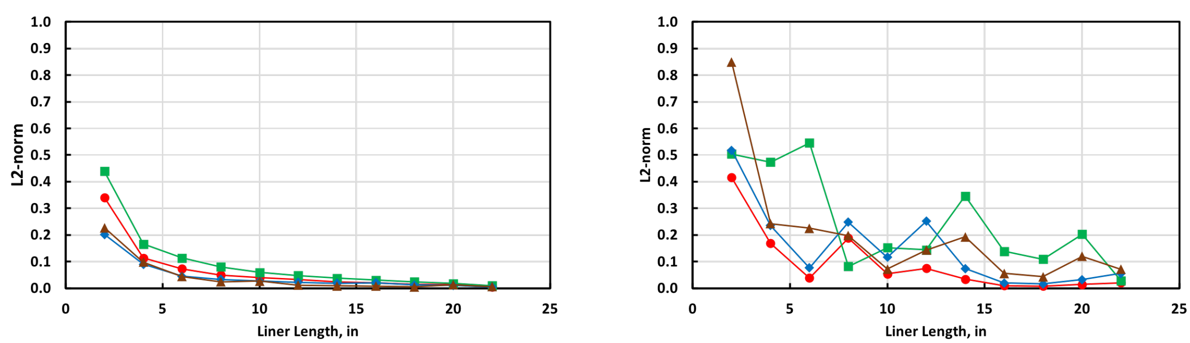
The simulated acoustic pressure profiles for each liner configuration were used as input to the PyCHE impedance eduction method described earlier. In an attempt to quantify the accuracy of the impedance eduction process, L2 error norms were computed for the errors in each impedance component and for each liner configuration as follows. The input impedance, ζ_I, for each configuration is given in Table 1. The L2 error norm, L_2, is computed as

$$L_{2,\theta} = \left\{ \sum_{i=1}^{12} (\theta_{I,i} - \theta_{E,i})^2 \right\}^{0.5} \; ; \; L_{2,\chi} = \left\{ \sum_{i=1}^{12} (\chi_{I,i} - \chi_{E,i})^2 \right\}^{0.5} , \tag{8}$$

where θ and χ represent the normalized acoustic resistance and reactance, respectively. The subscripts 'I' and 'E' refer to the input and educed values, respectively, and summation is performed over the 12 frequencies listed in Table 1. To first order, the L2 error norm provides an estimate of the average error in the educed value (whether resistance or reactance, in units of normalized impedance) at each frequency.

Results for the Mach 0.3 condition are provided in Figure 4. When clean data (no random jitter) is used, the L2 error norms for both liner sets asymptote to zero in a consistent manner, and are nearly identical to zero for a liner length of 22". For liner lengths of at least 4", the L2 error norms for resistance and reactance and for each liner set are less than 0.2. Clearly, if a sufficient amount of acoustic pressure data were to be used for this process (i.e., at a finer axial resolution than is achieved with the current microphone locations), the eduction error should reduce to zero since the same propagation code is used to generate the input data and to educe the impedance from that data. Thus, Fig. 4(a) provides an indication of the limitations of the PyCHE method caused by limiting the data to axial locations corresponding to GFIT microphone locations.

When random jitter is added to the propagation data to simulate measurement error [Fig. 4(b)], the L2 error norms continue to asymptote toward zero, but experience significantly increased variability. In general, less error (lower L2-norms) was associated with the R1 set of liners than the X0 set. Given the fact that both sets of liners were designed with the same attenuation criterion, this suggests that there are other parameters that affect the accuracy of this eduction method. Although not shown here, the propagation data for the R1 set contain slightly stronger standing waves upstream of the liner due to the impedance discontinuity at the liner leading edge. It is therefore conjectured that distinctive features in the propagation data result in improvements to the eduction process. It seems reasonable to assume that there are a limited number of impedances that will produce specific distinctive features in the acoustic pressure profile, thereby improving the likelihood that the eduction method will converge to one of these impedances.



(a) Clean data (no measurement error).

(b) Random jitter added to simulate measurement error.

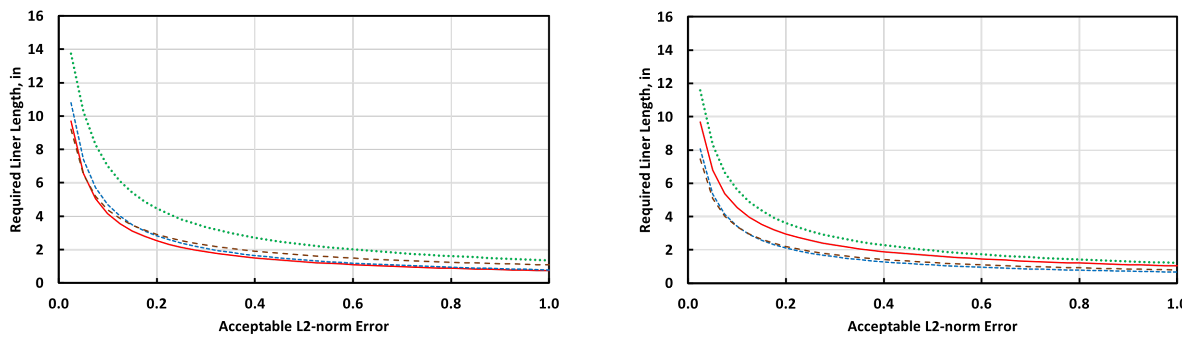
Figure 4: Effects of liner length on L2 error norms; Mach 0.3; red circles and blue diamonds represent $L_{2,\theta}$ and $L_{2,\chi}$ for R1 set, and green squares and brown triangles represent $L_{2,\theta}$ and $L_{2,\chi}$ for X0 set, respectively.

Results similar to those presented in Figure 4 were computed for both Mach numbers based on data that included the simulated effects of measurement error. Each individual L2 error norm was then curve-fit with a power law ($L_2 = Ax^B$). If we accept the results achieved with the 22"-long liner as the desired result, D, the length of liner, l_x, required to achieve an error (L2 error norm) less than ϵ can be computed as

$$D = A(22)^B \; \rightarrow \; D + \epsilon = A(l_x)^B \; \rightarrow \; l_x = \left(\frac{D+\epsilon}{A} \right)^{1/B} . \tag{9}$$

It is perhaps useful to note that the error norm for a given liner length in Figure 4(b) is the result of an eduction process for a single instance of random jitter applied to the simulated microphone responses at that liner length. If multiple simulations were conducted with independent randomizations, it is assumed that the averaged L2 error norms at a given length would approach those computed with the curve-fit process described via Equation (9).

As shown in the Mach 0.0 results of Fig. 5, less error is computed for the R1 liners (both resistance and reactance components) than for the X0 liners, and an 8” liner limits the error to approximately 0.1. At Mach 0.3, the reactance component has less error than the resistance component, for both the R1 and X0 liners, and an 8” liner limits the error to approximately 0.05. Although not nearly as good as the 8” (or longer) liners, surprisingly good results are computed for the shortest liners (2”-long). It is also interesting to note that the Mach 0.3 results are slightly better (less error for a given liner length) than those for Mach 0.0. Overall, these results are consistent with those observed in the earlier studies.[9,11] For comparison with two-zone liner results to be discussed later, it is noted that a liner length of at least 5” (corresponding to slightly more than 3 dB of attenuation for these liners) is required to limit the error to 0.2.



(a) Mach 0.0.

(b) Mach 0.3.

Figure 5: Liner length needed to achieve selected error (L2 error norm). Red and blue lines represent θ and χ results for R1 set, and green and brown lines represent θ and χ results for X0 set, respectively.

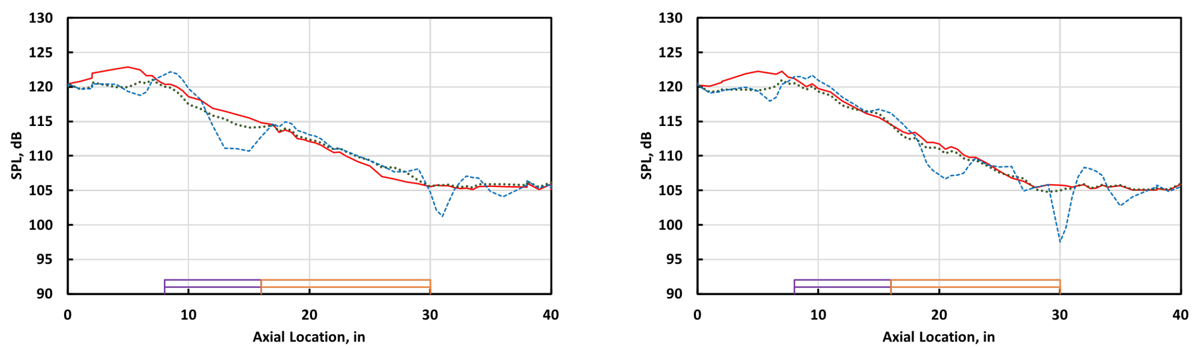
Recall that the maximum number of iterations was set to 500 for the Differential Evolution algorithm used with the PyCHE impedance eduction method. For the uniform liner configurations used in this study, the average number of iterations needed at each frequency was approximately 25 and 17 for the Mach 0.0 and 0.3 flow conditions, respectively. These values remained constant whether or not random jitter was added to the simulated acoustic pressures to incorporate the effects of measurement error. It is interesting to note that the number of iterations is less for the Mach 0.3 condition. Since the mean flow and sound are in the same direction, an increase in flow speed causes the wavelength to be increased for each frequency. Therefore, this result could be interpreted to suggest that an increase in the wavelength to liner length ratio may improve the accuracy of the educed impedances. However, the need for less iterations at Mach 0.3 is nearly independent of frequency, i.e., approximately the same number of iterations are needed over the entire frequency range. This latter result clearly suggests that the wavelength is not a dominant parameter. Further study is needed to understand whether this Mach number effect will apply for other liner configurations, or if this result applies only to the liner configurations used in this study.

As an aside, it should be noted that there may be multiple impedances that will produce a specified acoustic pressure profile (SPL and phase) at a finite number of locations to within a desired tolerance. Thus, even if the tolerance were set to zero, the ability of the eduction method to educe the correct impedance may be limited by the number of microphones used in the analysis. The fact that the shorter liners (for which the attenuation is correspondingly less) have larger impedance eduction errors is possibly an indication that this acoustic pressure profile non-uniqueness is more evident for shorter liners.

B. Two-Zone Liners

The CHE propagation code was next used to generate acoustic pressure data representative of that which would be measured with 22"-long, two-zone liners mounted in the GFIT. Two sets of liners were considered. For the first set, labeled R1X0, the impedance of the two axial zones were set to match those labeled as R1 and X0, respectively, in Table 1. The orientation was reversed for the second set, labeled X0R1 (i.e., X0 upstream and R1 downstream). Ten liners were simulated for each set (per Mach number), with the respective lengths of the two zones varied from the short/long (2" for zone 1, 20" for zone 2) to the long/short (20" for zone 1, 2" for zone 2) extremes in 2" increments. As noted earlier, the effects of measurement error (± 0.5 dB, ± 1.0 deg) were simulated by the addition of random jitter on each acoustic pressure profile. The PyCHE method was then used to educe the impedance of both zones for each liner configuration.

Figure 6 provides acoustic pressure profiles for one configuration from each liner set (R1X0 and X0R1). Again, the liner length and location are depicted via the sketch at the bottom of each figure. For each configuration, the two zones have lengths of 8" and 14", respectively. These lengths were chosen to ensure that the effects of each zone (i.e., each impedance) could be readily observed. The data are presented for the Mach 0.3 condition. For the R1X0 configuration, the effects (standing wave pattern) of the changes in impedance at the interface between the two zones and at the trailing edge of the liner are clearly evident at 1.6 kHz, but they are subdued for the other frequencies. For the X0R1 configuration, the effects remain prominent for the trailing edge interface, but not so for the interface between the two zones.



(a) R1X0 configuration.

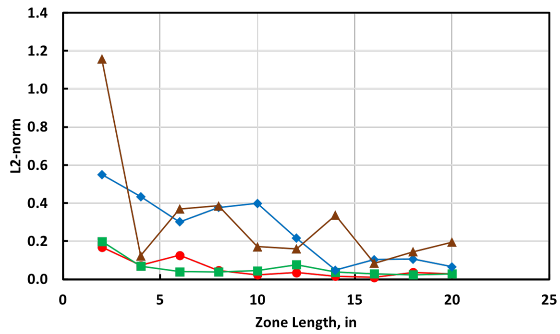
(b) X0R1 configuration.

Figure 6: Acoustic pressure profiles for two-zone liners. Lengths of the two zones are 8" and 14", respectively. Mach 0.3, with random jitter added; red solid line, blue dashed line, and green dotted line present results for 0.6, 1.6, and 2.6 kHz, respectively.

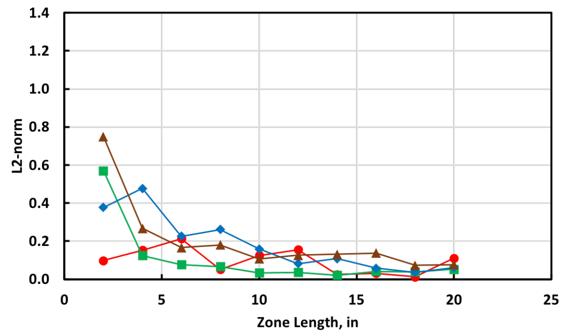
L2 error norms are again used to evaluate the efficacy of the PyCHE method (see Fig. 7). As an example, the red circles in Fig. 7(a) represent $L_{2,\theta}$ for zone 1 of the R1X0 liners, i.e., the resistances are compared against the R1 set from Table 1. The blue diamonds represent $L_{2,\theta}$ for zone 1 of the X0R1 liners, i.e., the resistances are compared against the X0 set from Table 1. The brown triangles and green squares represent $L_{2,\theta}$ for zone 2 of the R1X0 and X0R1 liners, respectively. These latter results provide comparisons of the educed resistances against those from the X0 and R1 liner sets, respectively.

The results are generally better (lower L2 error norm) for the zone designed with R1 impedances, regardless of where that zone is placed (zone 1 for R1X0, zone 2 for X0R1). This applies for both Mach numbers (0.0 and 0.3). Recall that the impedances for each zone were selected to ensure a constant amount of attenuation (5 dB per 8" length) at all frequencies and both Mach numbers. Thus, the accuracy of the educed impedance is not solely controlled by the attenuation rate. These results suggest that the eduction process is at least somewhat affected by the particular impedance of each individual zone, and perhaps by the impedance discontinuity between adjacent zones (as well as at the leading and trailing edges of the liner).

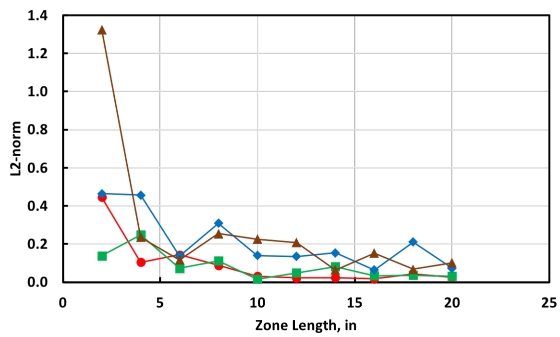
Looking back at some of the results from the earlier study,[3] there is some evidence that this impedance discontinuity may influence the ability to converge to a correct solution. However, recall the earlier observation that distinctive features in the one-zone liner acoustic pressure profiles appeared to lead to improved impedance eduction. This apparent dichotomy of results may be the subject of a future study. Nonetheless, the results suggest that the attenuation is the dominant parameter.



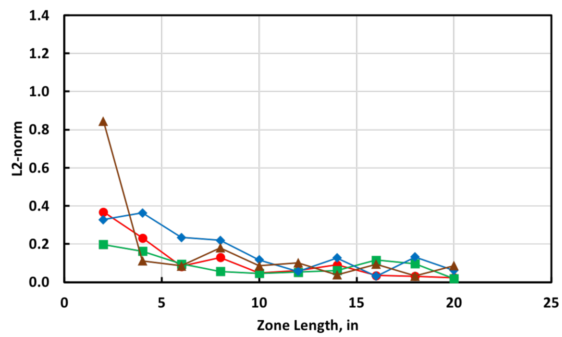
(a) Resistance, Mach 0.0.



(b) Reactance, Mach 0.0.



(c) Resistance, Mach 0.3.



(d) Reactance, Mach 0.3.

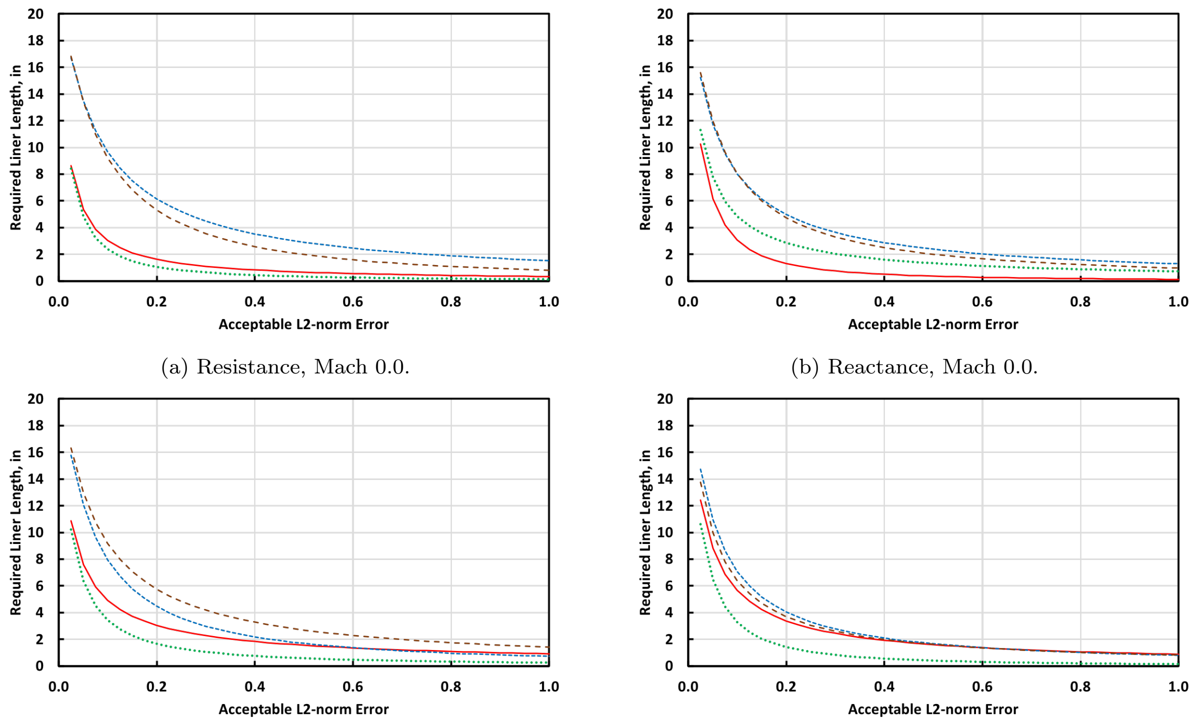
Figure 7: Effects of zone length on L2 error norms. Red circles and blue diamonds represent zone 1 results for the R1X0 and X0R1 liners, and brown triangles and green squares represent zone 2 results for the R1X0 and X0R1 liners, respectively.

Again, power laws were used to curve-fit the L2 error norm results (see Fig. 8). These results provide an indication of the zone length required to keep the error (L2 error norm) below a selected value. For both Mach 0.0 and 0.3, the reactance component provides less error (lower L2 error norm) than the resistance component; i.e., it is more difficult to accurately educe the resistance. Regardless, all of the data suggest that a zone length of at least 6" will constrain the error to less than 0.2 for both the resistance and reactance components. Recall that the one-zone liner results suggested that a 5" liner length is needed to constrain the error to less than 0.2. Although not a significant effect, it is perhaps useful to note that the length needed to constrain the error to 0.2 is less for the one-zone liner than for a single zone of a two-zone liner.

For the 2-zone liner configurations used in this study, the average number of iterations needed at each frequency was approximately 56 for both the Mach 0.0 and 0.3 flow conditions when clean data were used. When random jitter was added, this decreased to approximately 34 iterations for both flow conditions. Whereas the results were Mach number dependent for the uniform liners, this was not the case for the 2-zone liners. It is somewhat surprising that the number of iterations decreased significantly when random jitter was added to the simulated acoustic pressure profiles. This would seem to strengthen the argument that the inclusion of distinctive features in the acoustic pressure profile, such as those that would be generated by the addition of random jitter, might simplify the search for the liner impedance.

C. Three-Zone Liner

Finally, the CHE propagation code was used to generate acoustic pressure data representative of that which would be measured with a 22"-long, three-zone liner mounted in the GFIT (including effects of measurement error). Two liners were considered, one designed for each Mach number (0.0 and 0.3). For both liners, the lengths of the three zones were 6.0”, 7.2”, and 8.8”, respectively. The impedances of these zones are provided in Table 2. Note that the impedances of the first and third zones are based on the R1 and X0 sets, respectively, from the earlier discussion. As such, they are expected to provide sufficient attenuation for accurate impedance eduction. The second zone, however, assumes a rather arbitrary impedance pattern that does not assume a particular attenuation rate. Instead, the normalized resistance is increased from 0.50 to 1.60 in 0.10 increments and the normalized reactance is decreased from 2.00 to -0.75 in 0.25 increments over the frequency range of interest. Because of this, the amount of attenuation across this zone is expected to be frequency dependent.



(a) Resistance, Mach 0.0.

(b) Reactance, Mach 0.0.

(c) Resistance, Mach 0.3.

(d) Reactance, Mach 0.3.

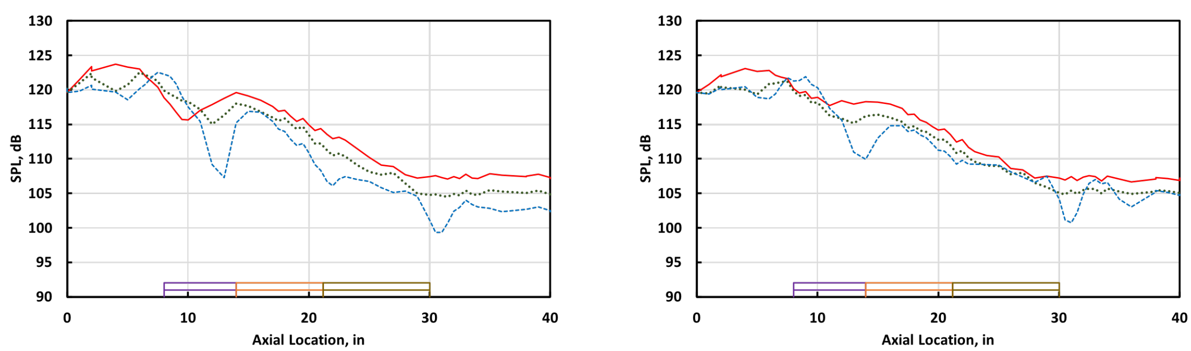
Figure 8: Liner length needed to achieve selected erorr (L2 error norm). Red and blue lines represent zone 1 results for R1X0 and X0R1 liners, and brown and green lines represent zone 2 results for R1X0 and X0R1 liners, respectively.

Figure 9 provides acoustic pressure profiles for each of these liners with the effects of measurement error simulated via the addition of random jitter (±0.5 dB, ±1.0 deg). The liner length and location are depicted via the sketch at the bottom of each figure. Similar to the results observed for the two-zone liners, the acoustic pressure profile at 1.6 kHz exhibits more sensitivity to the changes in impedance at each interface than is observed with the other two frequencies.

The PyCHE method was again used to educe the impedance for each zone. Figure 10 provides the error (difference between educed and input impedances) as a function of frequency. It is quite encouraging to note that the maximum error has a magnitude less than 0.2 for all frequencies and for each zone. The error for the third zone (based on X0 set) is larger than that for the other two zones for the liner designed for a Mach 0.0 flow condition, but the error magnitude is generally less than 0.1. For the liner designed for a Mach 0.3 flow condition, the worst results generally occur for zone 2, where the impedance was arbitrarily chosen. Although the errors are slightly larger for this flow condition, the error magnitude remains less than 0.2.

Table 2: Impedances for each zone of three-zone liner.

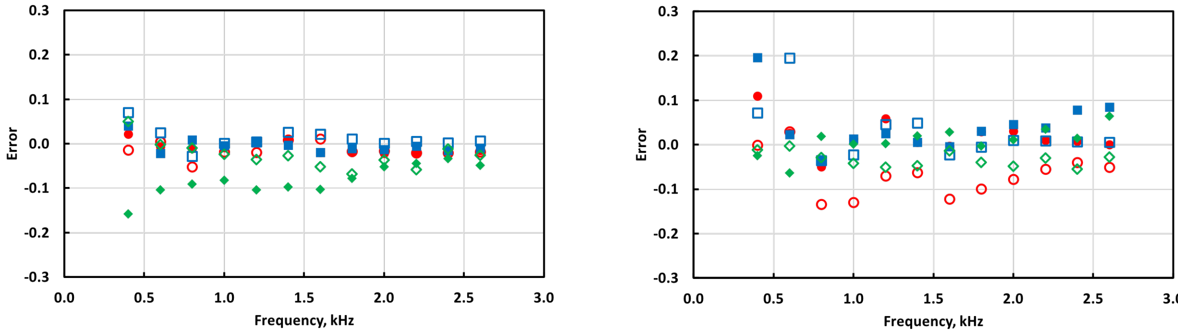
Frequency, kHz	Mach 0.0						Mach 0.3					
	Zone 1		Zone 2		Zone 3		Zone 1		Zone 2		Zone 3	
	θ	χ	θ	χ	θ	χ	θ	χ	θ	χ	θ	χ
0.4	1.00	-1.24	0.50	2.00	2.77	0.00	1.00	-0.42	0.50	2.00	1.45	0.00
0.6	1.00	-1.24	0.60	1.75	2.50	0.00	1.00	-0.45	0.60	1.75	1.44	0.00
0.8	1.00	-1.31	0.70	1.50	2.68	0.00	1.00	-0.67	0.70	1.50	1.71	0.00
1.0	1.00	-1.53	0.80	1.25	2.86	0.00	1.00	-0.88	0.80	1.25	1.82	0.00
1.2	1.00	-1.67	0.90	1.00	2.80	0.00	1.00	-0.93	0.90	1.00	1.74	0.00
1.4	1.00	-1.70	1.00	0.75	2.65	0.00	1.00	-0.95	1.00	0.75	1.71	0.00
1.6	1.00	-1.74	1.10	0.50	2.67	0.00	1.00	-1.06	1.10	0.50	1.77	0.00
1.8	1.00	-1.87	1.20	0.25	2.74	0.00	1.00	-1.16	1.20	0.25	1.77	0.00
2.0	1.00	-1.98	1.30	0.00	2.68	0.00	1.00	-1.20	1.30	0.00	1.70	0.00
2.2	1.00	-2.02	1.40	-0.25	2.55	0.00	1.00	-1.25	1.40	-0.25	1.66	0.00
2.4	1.00	-2.09	1.50	-0.50	2.48	0.00	1.00	-1.33	1.50	-0.50	1.64	0.00
2.6	1.00	-2.21	1.60	-0.75	2.42	0.00	1.00	-1.38	1.60	-0.75	1.54	0.00



(a) Configuration designed for Mach 0.0.

(b) Configuration designed for Mach 0.3.

Figure 9: Acoustic pressure profiles for three-zone liners. Lengths of the three zones are 6.0", 7.2", and 8.8", respectively. Random jitter added; red solid line, blue dashed line, and green dotted line present results for 0.6, 1.6, and 2.6 kHz, respectively.



(a) Mach 0.0.

(b) Mach 0.3.

Figure 10: Effects of frequency on error. Closed and open symbols represent resistance and reactance components, respectively. Results for zones 1, 2, and 3 are depicted as red circles, blue square, and green triangle, respectively.

For the 3-zone liner configurations used in this study, the average number of iterations needed at each frequency was approximately 94 and 99 for the Mach 0.0 and 0.3 flow conditions, respectively, when clean data were used. This is a significant increase as compared to the results for the 1-zone and 2-zone liners. When random jitter was added, this decreased to approximately 56 iterations for both flow condition. This is nearly identical to the results for the 2-zone liner.

IV. Concluding Remarks

This paper has presented the results of a computational study used to evaluate a relatively new impedance eduction method developed at the NASA Langley Research Center. The PyCHE method combines an aeroacoustic propagation code based on the convected Helmholtz equation with an optimizer included within the SciPy optimization toolkit. The efficacy of this method was evaluated with acoustic pressure data simulated to represent that which would typically be measured in the NASA Langley Grazing Flow Impedance Tube. Data were generated for a number of one-zone, two-zone, and three-zone liner configurations. Primary conclusions are as follows:

1. For one-zone liners, the accuracy of the impedance eduction method depends on the particular impedance of the liner. The results suggest that distinctive features in the acoustic pressure profile (e.g., standing waves created by the impedance discontinuity at the liner leading edge) may actually improve the eduction accuracy. Impedances educed for liners with lengths of at least 8”, for which the attenuation is approximately 5 dB, have errors (L2 error norms) that are nominally less than 0.1 at Mach 0.0. At Mach 0.3, there is generally less error for the reactance component than for the resistance component, and an 8” liner limits the error to approximately 0.05. Although not nearly as good as the 8” (or longer) liners, surprisingly good results are achieved with the shortest liners (2”-long), for which the attenuation is reduced to approximately 1 dB. Also, the Mach 0.3 results are slightly better (less error for a given liner length) than those for Mach 0.0. These results are consistent with those observed in earlier studies. There is conflicting evidence regarding the effect of wavelength on the impedance eduction process. The ratio of the wavelength to the liner length appears to correlate with the impedance eduction results for one criterion, but does not for a different criterion. In any event, this effect is limited, but perhaps it warrants additional investigation.

2. For two-zone liners, the accuracy of the PyCHE method again depends on the impedance of the particular zone, regardless of whether that impedance is placed in the upstream or downstream zone. As observed for the one-zone (uniform) liners, the accuracy is at least in part affected by the uniqueness of the acoustic pressure profile. In other words, distinctive features caused by impedance discontinuities have some effect on the eduction process. Nonetheless, the results suggest that attenuation is the dominant parameter. Again, the reactance component provides less error than the resistance component;

i.e., it is more difficult to accurately educe the resistance. Regardless, the data suggest that a zone length of at least 6" will constrain the error to less than 0.2 for both the resistance and reactance components, as compared with a required length of 5" for the one-zone liners. In other words, slightly more attenuation is needed to ensure quality impedance eduction for a single zone of a 2-zone liner.

3. For the three-zone liners considered herein, the maximum error has a magnitude less than 0.2 for all frequencies and for each zone. Only two three-zone liners were considered, so further investigation is warranted. Regardless, it is very encouraging to note that the new impedance eduction method can be successfully used with liners consisting of up to three distinct axial zones.

It should be noted that the results presented herein are for a particular set of liner configurations, and are therefore not necessarily representative of those that would be achieved with other liner configurations. Nevertheless, the results are sufficiently encouraging to warrant the continued usage of the PyCHE impedance eduction method. This method is therefore intended to be used with the variety of acoustic liners tested in the GFIT for the foreseeable future, and additional features will be reported as they are encountered.

Acknowledgements

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References

[1] Envia, E. E., Huff, D. L., and Morrison, C. R., "Analytical assessment of stator sweep and lean in reducing rotor-stator tone noise," AIAA Paper 1996-1791, May 1996.

[2] Woodward, R. P., Elliott, D. M., Hughes, C. E., and Berton, J. J., "Benefits of Swept-and-Leaned Stators for Fan Noise Reduction," *Journal of Aircraft*, Vol. 38, No. 6, 2001, pp. 1130–1138.

[3] Jones, M. G., Watson, W. R., Nark, D. M., and Howerton, B. M., "Impedance Eduction for Multisegment Liners," AIAA Paper 2018-3441, June 2018.

[4] Watson, W., Jones, M., and Parrott, T., "A Quasi-3-D Theory for Impedance Eduction in Uniform Grazing Flow," AIAA Paper 2005-2848, May 2005.

[5] Jones, E., Oliphant, T., Peterson, P., et al., "SciPy: Open source scientific tools for Python," URL: http://www.scipy.org/, 2001–.

[6] Watson, W. R., Jones, M. G., and Parrott, T. L., "Investigation of an Anomaly Observed in Impedance Eduction Techniques," AIAA Paper 2008-3013, May 2008.

[7] Watson, W. R. and Jones, M. G., "Explanation of Anomalous Behavior Observed in Impedance Eduction Techniques Using Measured Data," AIAA Paper 2010-3766, May 2010.

[8] Jones, M. G. and Watson, W. R., "Validation of an Improved Experimental Method for Use in Impedance Eduction," *AIAA Journal*, Vol. 51, No. 1, January 2013, pp. 186–199.

[9] Jones, M. G., Watson, W. R., and June, J. C., "Optimization of Microphone Locations for Acoustic Liner Impedance Eduction," AIAA Paper 2015-3271, June 2015.

[10] Myers, M. K., "On the Acoustic Boundary Condition in the Presence of Flow," *Journal of Sound and Vibration*, Vol. 71, No. 3, 1980, pp. 429–434.

[11] Watson, W. R. and Jones, M. G., "A Comparative Study of Four Impedance Eduction Methodologies Using Several Test Liners," AIAA Paper 2013-2274, May 2013.