a review of An exactification of the monoid of primitive recursive functions by Lambek, Joachim; Scott, Philip

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Lambek, Joachim; Scott, Philip

An exactification of the monoid of primitive recursive functions. (English) Zbl 1096.03048

Stud. Log. 81, No. 1, 1-18 (2005).

Let $\mathcal{R}$ be a partially ordered category with involution $*$, assuming that the hom-sets are $\wedge$-semilattices. The category $\mathcal{R}$ is to be regarded as a category of relations, in which we can find a non-null subcategory $\mathcal{C}$ of functions, namely, relations $f : A \to B$ with $ff^* \leq 1_B$ and $1_A \leq f^*f$. The authors are interested in two special cases. The first case is that $\mathcal{C}$ is the monoid $\mathcal{N}$ of primitive recursive functions $\mathbb{N} \to \mathbb{N}$ and $\mathcal{R}$ is the category of recursively enumerable relations on $\mathbb{N}$. The second case is that $\mathcal{C}$ is a regular category and $\mathcal{R}$ is the category of relations constructed from spans in $\mathcal{C}$. The authors note a similarity between the construction $\widetilde{\mathcal{N}}$ from $\mathcal{N}$ in the first case and the exact completion of $\mathcal{C}$ in the second case, so that they want to bring these two constructions under one hat. One possible way with the first case would be to first embed $\mathcal{N}$ in its regular completion and then apply the methods in the second case, which was already implicit in [*P. J. Freyd* and *A. Scedrov*, Categories, allegories. North-Holland Mathematical Library, 39. Amsterdam etc.: North-Holland. (1990; Zbl 0698.18002)]. This article aims to handle the first case by a one-step construction, which is very akin to the construction of the category of modest sets (*G. Rosolini*, [Int. J. Found. Comput. Sci. 1, No. 3, 341–353 (1990; Zbl 0729.18003)] and *E. S. Bainbridge* [Theor. Comput. Sci. 70, No. 1, 35–64 (1990; Zbl 0717.18005)]) as well as the idempotent splitting construction (Karoubi envelope) used in [*J. Lambek* and *P. J. Scott*, Introduction to higher order categorical logic. Cambridge Studies in Advanced Mathematics, 7. Cambridge etc.: Cambridge University Press. (1986; Zbl 0596.03002)].

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MSC:

03D20 Recursive functions and relations, subrecursive hierarchies
03G30 Categorical logic, topoi

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References:

[1] Bainbridge, E. S., P. Freyd, A. Scedrov, and P. J. Scott,'Functorial Polymorphism', Theoretical Computer Science 70 (1990), 35–64. · Zbl 0717.18005 · doi:10.1016/0304-3975(90)90151-7

[2] Barr, M., et. al., Exact categories and categories of sheaves, Springer Lecture Notes in Mathematics 236, 1971. · Zbl 0223.18009

[3] Borceux, F., Handbook of Categorical Algebra 1, Cambridge, 1994. · Zbl 0803.18001

[4] Calenko, M. S., et. al., Ordered Categories with involution, Dissertaiones Mathematicae 227, 1984.

[5] Carboni, A., and E. M. Vitale, 'Regular and Exact Completions', Journal of Pure and Applied Algebra 125 (1998), 29–117. · Zbl 0891.18002 · doi:10.1016/S0022-4049(96)00115-6

[6] Cutland, N. J., Computability: An Introduction to Recursive Function Theory, Cambridge, 1980. · Zbl 0448.03029

[7] Freyd, P., and A. Scedrov, Categories, Allegories, North-Holland, 1990.

[8] Hofstra, P., Completions in Realizability, Phd Thesis, Utrecht, 2003.

[9] Kleene, S. C., Introduction to Metamathematics, Van Nostrand, New York, 1952. · Zbl 0047.00703

[10] Lambek, J., 'Goursat's theorem and the Zassenhaus lemma', Can. J. Math. 10 (1957), 45–56. · Zbl 0079.25202 · doi:10.4153/CJM-1958-005-6

[11] Lambek, J., 'Least fixpoints of endofunctors of cartesian closed categories', Mathematical Structures in Computer Science 3 (1993), 229–257. · Zbl 0788.18006 · doi:10.1017/S0960129500000190

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[12] Lambek, J., 'The butterfiy and the serpent', in Agliano et. al., (eds.), Logic and Algebra, Marcel Dekker, New York, 1996, pp. 161–179. · Zbl 0868.08002

[13] Lambek, J., 'Diagram chasing in ordered categores with involution', J. Pure and Applied Algebra 143 (1999), 293–307. · Zbl 0954.18001 · doi:10.1016/S0022-4049(98)00115-7

[14] Lambek, J., and P. J. Scott., Introduction to Higher Order Categorical Logic, Cambridge Studies in Advanced Mathematics 7, Cambridge University Press, 1986. · Zbl 0596.03002

[15] Longo, G., and E. Moggi, 'Constructive Natural Deduction and its -set interpretation', Mathematical Structures in Comp. Sci (1991), 215–254. · Zbl 0756.03028

[16] Lounsbury, F.G., 'Another view of Trobriand kinship categories', in E. A. Hammel, (ed.), 'Formal Semantics II', American Anthropologist 67 (1965), 142–185.

[17] Malinowski, B., Sexual life of savages, Routledge and Kegan Paul, London, 1932.

[18] McLarty, C., Elementary Categories, elementary toposes, Clarendon Press, Oxford, 1992. · Zbl 0762.18001

[19] Rosolini, G., 'About modest sets', Internat. J. Found. Comp. Sci. 1 (1990) · Zbl 0729.18003

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