Emigration, tax on remittances and export quality

Ganguly, Shrimoyee and Acharyya, Rajat

Department of Economics, Jadavpur University, Department of Economics, Jadavpur Universit

10 December 2019

Online at https://mpra.ub.uni-muenchen.de/100160/
MPRA Paper No. 100160, posted 06 May 2020 14:19 UTC
Emigration, Tax on Remittances and Export Quality

Shrimoyee Ganguly
Department of Economics, Jadavpur University, Kolkata

Rajat Acharyya
Department of Economics, Jadavpur University, Kolkata

Abstract
We examine implications of emigration of unskilled workers for quality of a skill-based good exported by a small open economy. This issue is relevant in the context of quality constraint faced by the developing countries like China and India, in promoting their exports, on the one hand, and significantly large emigrations of workers, particularly unskilled workers, that lower their productive capacities, on the other hand. We show that even though unskilled workers are not directly used in production of the quality-differentiated export good, their emigration would lower export quality when quality upgrading requires more intensive use of skilled workers relative to capital. This result follows from the complementarity between skilled and unskilled wages in a competitive general equilibrium model. A quality-content production subsidy in such a case can mitigate the adverse effect of emigration. Significantly large remittances received from unskilled emigrants create scope for taxing such remittances to finance the subsidy.

JEL Classification codes: F16, F20, F22, F24
Keywords: Emigration, Remittance Tax, Export Quality, Production Subsidy

An earlier version of this paper had been presented at the International Conference on Issues in International Trade and Open Economy Macroeconomics organized by The Eastern Regional Centre, Indian Council of Social Science Research. I thank Saibal Kar, Sugata Marjit, B. Ravikumar, Ajitava Raychaudhuri and other conference participants for their extremely helpful comments and suggestions. The usual disclaimer applies, however.

* Corresponding author. Email: sssganguly@gmail.com
Introduction

With an increasing number of countries globalizing during the last two decades in particular, there has been not only a surge in international exchange of commodities but also in factor movements and financial flows. Available data from different sources, such as Global Migration Data Portal, International Labour Organisation database, World Bank databank, and OECD migration data, reveal emigration of a large number of workers every year, both skilled as well as the unskilled labour type, largely from developing countries like Brazil, China and India to the developed countries. Such emigrations are driven by a host of push and pull factors including the wage differentials between the destination and origin countries. Despite the fact that this opens a source of national income in the form of remittances from these emigrated workers, the direct and immediate loss from such factor outflow is that it lowers the productive capacity of the economy since a sizeable proportion of its workforce now work abroad. The concern takes a more serious look if emigration of skilled workers compounds the problem of poor quality of goods exported by the less developed and developing countries. Many recent studies reveal that quality is an important factor for better export growth in developed-country markets (Sutton [2001], Hallak [2006], Baldwin and Harrigan [2011], Manova and Zhang [2012]).

On the other hand, a sizeable and growing empirical literature has observed robust evidences on the intensive use of domestic inputs like skilled labour and/or capital in producing higher quality export goods, in contrast to quality upgrading requiring higher intensive use of high-quality imported input (Brambilla et al. [2012], Brambilla et al. [2014], Brambilla and Porto [2016]). In such a context, for developing countries that experience growth of their exports being severely constrained by poor qualities of goods they export, allowing for emigration of skilled labour may actually worsen the situation further. The argument is pretty straightforward. A ceteris paribus outflow of skilled labour from the developing countries due to higher skilled wage abroad (the pull factor of emigration) would generate a scarcity of skilled workers there, which would push up the skilled wage. To the extent quality upgrading requires more intensive use of skilled labour, this will increase the marginal cost of raising quality and induce producers to downgrade export quality. Low quality will lower marginal willingness to pay for such exports thus sinking export promotion prospects of the country further.
A counter argument is that emigration of skilled labour has a positive impact on export promotion through networking, which is essentially a demand side explanation (Ehrhart et. al. [2014]). Migration would reduce transaction costs associated with trade and serve to complement trade. Erhart et al. have estimated such effects for the origin country in the context of Africa (effect of African migrants on African exports). Similarly, Rauch and Trindade (2002) and Felbermayr et al. (2011) found that the Chinese network increased its bilateral trade with the host country. Contrary to this pro-trade effect of migrants, Parsons (2012) found that migration promoted only the northern exports to the South.

Two comments are warranted at this point. First, these pro-trade effects work mainly through employing immigrants in trade in the host country (in contrast to employing them in production of traded goods, which has a cost reducing and corresponding export promoting effect for the host country): Immigrants employed in trade can reduce the transaction cost of international trade – imports by the host country in this instance -- by their knowledge of language, customs, and laws to conduct business with their country of birth or with similar countries (see Genc et al. [2011]). Second, such effects are stronger for differentiated products than homogeneous products. In our context of migration affecting quality of exports (gains at the extensive margin), this networking effect may be relevant in cases such as Akerlof’s (1970) lemons problem under asymmetric information of foreign buyers. However, for search goods, i.e. goods with observable quality, the cost effect of emigration might seem more relevant than the demand effect through networking. That is, where there is no problem of quality uncertainty and domestic factor costs (and the state of technology) are the important determinants of export quality, it might be imperative that emigration of skilled labour would lower export quality and thus regulating emigration may actually help promote exports at the extensive margin.

In contrast, emigration of unskilled labour, which is not directly used in quality upgrading, might appear to be a more innocuous factor flow that a country can allow for. But trade literature often talks about complementarity between wages of different skill types. For example, Jones and Marjit (1992) and Marjit and Beladi (1998) have established a complementary between

---

1 Exports of the native/origin country can also be augmented by emigration due to migrants’ preferences for goods from their country of origin, the “home preference effect” (Rauch and Casella [2003]).

2 See Bond (1984), Chiang and Masson (1989) and Acharyya (2005) for discussion on trade and export promotion under quality uncertainty and related problem of information externality.
movements in skilled and unskilled wages for a small country with a diversified trade pattern in the context of trade liberalisation. A more recent theoretical study by Acharyya, Beladi and Kar (2019) got similar results where they showed that with an exogenous increase in emigration rates, both types of wages at the equilibrium would rise. They used this complementarity relationship to show that emigration of skilled workers would lower incentives for unskilled workers to emigrate, and vice versa, regardless of whether the costs of emigration rise or fall with the flow of emigration. What all these analyses point at is that emigration of skilled and unskilled labour may have symmetric effects on skilled and unskilled wages. Hence, even a seemingly innocuous emigration of unskilled labour can have far reaching adverse implications for export quality upgrading and thus on export prospects through consequent increase in the skilled wage. This is what this paper is concerned about.

There are also theoretical studies such as those by Michael (2011) and Marjit et al. (2019) that talk about immigration of unskilled labour raising the skilled wage in the host country. In Michael (2011) this happens when skilled and unskilled labour are complementary to each other in production. In the context of a household sector, Marjit et al (2019), on the other hand, demonstrates that net skilled wage rate rises due to unskilled immigration even though the skilled wage rate falls. Even though these analyses talk about asymmetric effect of unskilled labour immigration on skilled and unskilled wages, what is apparent is that such immigration would still affect the choice of quality of exports goods (in the host country), albeit differently than when wage movements are symmetric, in production of which unskilled workers are not directly used. However, these theoretical analyses and most of the empirical studies focus on immigration of unskilled workers on welfare and skilled wage for the host country. Our focus, in contrast, is on the effect of emigration of unskilled labour on export quality and, therefore, on export prospects of the native or origin country. This shift in focus from the host to the origin country in exploring how emigration may affect exports of the origin country, is similar to Ehrhart et. al. (2014). As spelled earlier, since exports of developing countries, from which unskilled workers emigrate in large numbers, are constrained by their low qualities, it is worthwhile to address implications of emigration of unskilled workers on quality of exports of the origin country.
As we will demonstrate, emigration of unskilled workers may have adverse effects on export-quality, and in such cases policy regulations in the origin country restricting emigration of unskilled workers may seem to be a natural conclusion. But, such a policy regulation especially for a developing origin country may not be warranted for two reasons. First, emigration provides gainful employment to emigrants who would otherwise have been absorbed in the low wage informal sector of the unskilled labour market or even worse, remained unemployed. Second, if there is no open unemployment in the origin country, then emigration by raising the unskilled wage would raise the standard of living of those workers who are left behind. In such a situation, a potential policy conflict emerges between allowing for emigration of unskilled workers on the one hand, and promoting exports through incentivizing quality upgrading, on the other hand. This policy dilemma may, however, be resolved by mitigating the adverse effect of emigration through a quality-content production subsidy to producers of the quality differentiated export good. And large inflow of remittances received by developing countries like China and India creates scope for financing the production subsidy by taxing such remittances.

A quality-content production subsidy given to the producers of skill-based exports would raise the effective marginal revenue from raising quality and thus directly incentivize quality upgrading. At the same time, expansion of scale of production of the export good induced by the production subsidy will raise the demand for skilled workers and consequently push up their wages. Thus, the positive effect of a subsidy inducing export-quality upgrading and resolving the policy conflict though is quite plausible, yet is not a foregone conclusion and hence is worthwhile to examine. This policy issue is the second major concern of this paper. For the purpose, we consider a situation where the rate of emigration rises, ceteris paribus, due to, say, an external shock abroad such as an exogenous rise in the foreign unskilled wage (the pull factor of emigration). For any given tax rate, such an increase in the rate of emigration of unskilled workers would increase the total remittances received, which in turn would finance a higher rate of subsidy.

---

3That restrictions on emigration of unskilled workers by the developing countries may not be desirable policy option in the developing countries are often reflected in their push for mode 4 provisions of GATS in WTO ministerial rounds of talks.

4Latest available date from the World Bank indicates that remittances have surged from $20 billion in 2004 to $70 billion for China and $80 billion for India in 2018.
We analyse these issues theoretically by adopting the Gruen-Corden (1970) variant of a general equilibrium structure of a small open economy with a quality-differentiated export good and a set of homogeneous traded goods displaying diverse trade pattern, similar to Acharyya and Jones (2001). The small open economy under consideration produces three goods: a skill-based quality differentiated export good, a homogeneous export good and a homogeneous import competing good. The homogeneous export good is an agricultural commodity that uses land along with unskilled labour. On the other hand, the import competing good is a manufacturing good that uses unskilled labour and capital which is also shared by the skill-based quality-differentiated export good. In such a set up with the quality differentiated export good not domestically consumed in the benchmark case, we establish the following results. First, an exogenous increase in the rate of emigration of unskilled workers may downgrade export quality depending upon the technological requirement of whether higher qualities are relatively more capital intensive or skill intensive. Second, a quality content production subsidy financed by tax on remittance may mitigate the adverse effect of emigration on quality if the initial rate of subsidy is greater than a critical value as defined later. Finally, when emigration rates are endogenous, emigration of unskilled workers caused by an external shock is magnified through consequent quality variations. The quality variation, upgrading or downgrading as the case may be depending on the relative skill intensity of higher qualities of the export good, is larger as well than that due to an exogenous increase in the rate of unskilled emigration.

Our paper also contributes to the trade literature that debates over whether commodity and factor trades are substitutes or complements. The celebrated Factor Price Equalization theorem (Samuelson [1941]) and Goods Price Equalization theorem (Mundell [1950]) demonstrated that when factor endowments of trading nations are not significantly different from each other, and they share the same technology for producing the traded goods, commodity and factor trades are mutually exclusive (substitute result). However, identical technology across countries is not a reasonable approximation of the real world, particularly in the context of North-South trade. This seems to be a major reason for why do we observe coexistence of both commodity and factor trades. In such a context, trade theorists have subsequently re-examined the issue as to whether

\[\text{Ours, however, is a more generalized version in terms of the production structure.}\]
factor flows augment or lower trade volumes. Svensson (1982) and Markusen (1983), for example, found a complementary relationship. Krugman’s (1979) love of variety approach to intra-industry trade also has a built-in complemenatry relationship though at the extensive margin: labour immigration increases number of varieties produced and exported. In a more recent paper, Dutta, Kar and Marjit (2013) have studied the impact of emigration on product variety and wage inequality under oligopolistic market structure. In extending this strand of literature, we focus on the relationship between emigration of unskilled workers and export promotion through quality upgrading. Our result shows that we may have both complementarity result (when quality is upgraded) and a substitute result (when quality is downgraded) depending on the production technology.

The rest of the paper is organized as follows. In section 2 we describe the production structure of our small open economy and examine the effect of an exogenous rise in rate of emigration on the equilibrium level of export quality. Section 3 discusses how a quality content production subsidy financed out of tax on remittances can mitigate adverse effect of emigration. In Section 4 we discuss implications of causality between export quality and emigration after endogeneising the rate of emigration. Finally, Section 5 concludes the paper.

2. Exogenous emigration and export quality

2.1 The Economy
Consider a small open economy producing three goods: a skill-based export good $Z$, whose quality is observable to all and is indexed by $Q \in [0, 1]$; a homogeneous agricultural export good $X$ and a homogeneous manufacturing import-competing good $Y$. The homogeneous export and import-competing goods are produced by unskilled labour $(L)$ along with sector-specific land $(T)$ and capital $(K)$ respectively. The quality-differentiated export good uses the same capital $K$ as the import-competing good. World prices of all these goods $P^w_j$, $j = X, Y$ and $Z$, are exogenously given, though the price of good $Z$ is higher for a higher quality of it that reflects higher marginal willingness to pay for higher qualities by foreign consumers. More precisely,
\[ P^w_Z = P^w_Z(Q), \quad P^w'_Z(Q) > 0, \quad P^w''_Z(Q) > 0. \] On the cost side, higher quality of good \( Z \) requires intensive use of both skilled labour (S) and capital (K):

\[ a_{hz} = a_{hz}(Q), a'_{hz}(Q) > 0, a''_{hz}(Q) > 0, h = S, K \quad (1) \]

where, \( a_{hz} \) denote the per unit requirement of input-\( h \). Note that in (1) we assume \( a_{hz} \) to be fixed for any given quality choice, but increasing at an increasing rate with the quality level. Though both skilled labour and capital per unit of output of good \( Z \) are higher for a higher quality of it, the relative skill intensity, \( s_Z = a_{sz}(Q)/a_{KZ}(Q) \), varies as per the technological requirement.

More precisely,

\[ \hat{s}_Z = (\gamma_{sz} - \gamma_{KZ}) \hat{Q} \]

where, \( \gamma_{hz} \equiv \frac{Q a'_{hz}(Q)}{a_{hz}(Q)} \), \( h = S, K \), are the quality elasticities of the per unit input requirement and are positive and hat over \( s_Z \) denotes its proportional change, i.e. \( \hat{s}_Z = \frac{ds_Z}{s_Z} \). Thus, quality upgrading is relatively more skill intensive if \( \gamma_{sz} > \gamma_{KZ} \), and more capital intensive otherwise. As we will see, this will be an important determinant of the effect of emigration and tax on remittances on quality choice.

Due to free entry in the three sectors, producers everywhere earn zero supernormal profits, so that in each sector the world price equals average costs:

\[ P^w_X = a_{lx} w + a_{tx} R \quad (2) \]
\[ P^w_Y = a_{ly} w + a_{ky} r \quad (3) \]
\[ P^w_Z(Q) = a_{KZ}(Q)r + a_{sz}(Q)w_s \quad (4) \]

where, \( w_s \) is the skilled wage, \( w \) is the unskilled wage, \( R \) is the rate of return to land, \( r \) is the rate of return to capital, and \( a_{ij} \), \( i = L, K, T; j = X, Y \), denote the per unit requirement of input-\( i \) in production of good-\( j \). These \( a_{ij} \)'s are essentially the least-cost choices made by the producers that depend only on the relevant factor price ratios under the assumption of CRS technology:

\[ a_{lx} = a_{lx}(w/R), \quad a_{ly} = a_{ly}(w/r) \quad (5) \]
Profit maximizing export quality $Q_0$ is the one for which the marginal revenue from quality upgrading equals the marginal cost of quality upgrading:

$$P'_Z(Q_0) = a'_{kZ}(Q_0)r + a'_{sz}(Q_0)w_s$$  \hspace{1cm} (6)

We close the model by the following full employment conditions, which are ensured by flexibility in the rate of return to capital and land, unskilled wage and skilled wage along with competitive market forces:

$$\alpha S = a_{sz}(Q)Z$$ \hspace{1cm} (7)

$$\beta L = a_{lx}X + a_{ly}Y$$ \hspace{1cm} (8)

$$T = a_{rx}X$$ \hspace{1cm} (9)

$$K = a_{kz}(Q)Z + a_{k1}Y$$ \hspace{1cm} (10)

where, $\alpha$ and $\beta$ are the proportions of skilled and unskilled workers respectively who emigrate abroad. In this section, we consider these rates as exogenously given, and shall draw implications of endogenously determined emigration rates in the last section.

Given the exogenous rates of emigrations, the equation system (2) – (10) comprising twelve equations determine the twelve variables: skilled and unskilled money wages, rates of return to capital and land, the quality level of the export good Z, four input coefficients in X and Y sectors, and the three output levels. Before proceeding further, a few properties of our small open economy deserves attention. First, the (X, Y) nugget displays a specific-factor production structure a la Jones (1971), so that the wages and rates of returns to sector-specific capital and land depend on the availability of these factors of production for this nugget. Since, given the endowment levels -- $L, T, K$ -- availability of these factors for production of X and Y depend on the rates of emigration, $\alpha$ and $\beta$, and the quality level of the export good Z, so factor returns are contingent upon emigration rates and export quality level. An (endogenous) increase in export quality, for example, will change the capital requirement to produce good Z, and consequently change capital availability for production of the import competing good. This in turn will affect the unskilled money wage and rate of returns to capital and land through change in the
composition of output levels in the (X, Y) nugget and consequent changes in relative demand for unskilled labour.

Second, skilled and unskilled wages are complementary to each other in the sense that an increase in unskilled money wage, say caused by emigration of unskilled workers, will lower the rates of return to capital and land (see eq. (2) and (3)), and the fall in $r$ will raise the skilled wage at any given quality of good Z (see eq. (4)). Similarly, any policy shock or exogenous change that causes skilled wage to rise, will lower the rate of return to capital used by Z (and Y), which in turn will raise the unskilled money wage. This complementary relationship has been similar to Margit and Beladi (1999) and Acharyya et al. (2019) and is stated more precisely in the following Lemma:

**Lemma 1:** Any exogenous shock that raises (or lowers) unskilled money wage, will raise (or lower) skilled money wage and lower rates of returns to capital and land.

Proof: By (2) and (3), we get respectively $\dot{r} = -\frac{\theta_{LY}}{\theta_{KY}} \dot{w}$ and $\dot{R} = -\frac{\theta_{LY}}{\theta_{TX}} \dot{w}$. Using this, by (4) and (6) we obtain,

$$\dot{w}_S = \frac{\theta_{KZ}}{\theta_{SZ}} \frac{\theta_{LY}}{\theta_{KY}} \dot{w}$$  \hspace{1cm} (11)

where, $\theta_i$, $i = S, K, L, j = Z, Y$, is the share of factor-$i$ in average total cost of producing good $j$.

Hence, proved. □

Thus, from Lemma 1 and the marginal condition (6), it follows that an increase in unskilled money wage will affect the choice of quality of the export good Z. However, the causality is not unidirectional. Rather, as it follows from the above discussions, the export quality and unskilled wage are inter-dependent, causing each other, and thus are to be determined simultaneously. To fix ideas, first note that by the zero-profit condition for good Z, for any given quality $Q$, the rate of return to capital and skilled money wage varies inversely due to any policy shock. Thus, by the marginal condition (6), producers will respond to a policy shock that raises the skilled money wage by downgrading quality if higher quality of good Z is more skill intensive ($\gamma_{SZ} > \gamma_{KZ}$):
\[
\hat{Q} = \frac{P_w}{\delta Q^2} \theta_{sz} (\gamma_{sz} - \gamma_{kz}) \hat{w}
\]

(12)

where, \( \delta \equiv eP_w^{''} (Q) - w_a^* a_{sz}^*(Q) - ra_{kz}^*(Q) \) < 0 by the second order condition for profit maximization.

Using (11), we can rewrite (12) indicating how an increase in unskilled money wage (due to some policy shock) affects the profit-maximizing quality of the export good Z:

\[
\hat{Q} = \theta_{kz} \frac{P_w}{\delta Q^2} \frac{\theta_{ly}}{\theta_{ky}} (\gamma_{sz} - \gamma_{kz}) \hat{w}
\]

(13)

In Figure 1, this relationship is represented by the \( QQ \) curve, which is negatively sloped if \( \gamma_{sz} > \gamma_{kz} \), and positively sloped otherwise.

The other relationship in Figure 1, the \( ww \) curve, represents the relationship between export quality and the unskilled money wage consistent with full employment of all factors of production. An increase in export quality requires more skilled labour per unit of output of good Z. Since skilled workers are specific to this sector, so this necessitates a fall in the output of good Z, and correspondingly lowers the demand for capital by \( \gamma_{kz} \) at the margin. But, higher quality also requires additional capital at the rate \( \gamma_{kz} \) per unit of good Z. Hence, if \( \gamma_{sz} > \gamma_{kz} \), net capital requirement in Z production falls as its quality is raised. The release of capital enables a scale expansion of the import-competing good Y, which in turn requires additional unskilled workers as well. Given the rate of emigration, since unskilled workers are already fully employed, increase in demand for such workers raises their wage. If, on the other hand, \( \gamma_{sz} < \gamma_{kz} \), by a reverse logic, a rise in export quality lowers the unskilled money wage. Algebraically, this can be verified from the following (see appendix),

\[
A \hat{w} = -\frac{\lambda_{lx}}{\lambda_{ky}} \lambda_{kz} (\gamma_{kz} - \gamma_{sz}) \hat{Q}
\]

(14)

where, \( A \equiv \sigma_x \frac{\lambda_{lx}}{\lambda_{ly} \theta_{tx}} + \sigma_y \frac{1}{\theta_{ky}} ; \sigma_j, j = X, Y, \) is the factor substitution elasticity in sector j; and \( \lambda_{lj} \) is the share of sector-j in total employment of unskilled workers.
The equilibrium quality of the export good $Q_0$ and the unskilled money wage $w_0$ are thus determined simultaneously corresponding to the point of intersection between $QQ$ and $ww$ curves. The rest of the variables are determined from the zero profit and full employment conditions.

![Figure 1: Equilibrium Export Quality and Unskilled Wage](image)

### 2.2 Exogenous increase in emigration rates and export quality

In the above set up, consider an exogenous increase in emigration rate of unskilled workers. By the stroke of the pen, this raises the unskilled wage due to consequent fall in the availability of them for domestic production of homogeneous traded goods, X and Y. By Lemma 1, the increase in unskilled emigration rate will raise the skilled wage and lower rates of return to capital and land. Thus, emigration of unskilled workers will raise quality of the export good Z if its higher quality is relatively capital intensive, $\gamma_{KZ} > \gamma_{SZ}$, and lowers export quality otherwise.

In Figure 2, as emigration of unskilled workers raises unskilled wage at the initial quality of good Z, the $ww$ curve shifts up along the $QQ$ curve and at the new equilibrium quality is upgraded if $\gamma_{KZ} > \gamma_{SZ}$ and downgraded if $\gamma_{SZ} > \gamma_{KZ}$. Algebraically:

$$\hat{Q} = \frac{1}{\Delta} \theta_{KZ} \frac{P^W}{\partial Q^2} \frac{\theta_{LY}}{\theta_{KY}} (\gamma_{KZ} - \gamma_{SZ}) \varphi \beta$$

(15)

where, $\Delta = \left[ -\lambda_{KZ} \theta_{KZ} \frac{P^W}{\partial Q^2} \frac{\lambda_{LY}}{\lambda_{KY}} (\gamma_{KZ} - \gamma_{SZ})^2 - A \right] < 0$ and $\varphi = \frac{\beta}{1-\beta}$.
Hence,

**Proposition 1:** Emigration of unskilled workers leads to quality degrading if $\gamma_{SZ} > \gamma_{KZ}$.  

Proof: Follows from the above discussion. For algebraic details, see appendix. \(\square\)

![Figure 2: Emigration, Export Quality and Unskilled Wage](image)

Note that, in either case, the initial rise in the unskilled wage due to emigration is dampened to some extent by variations in quality. This follows from the reallocation of capital across Z and Y sectors due to quality upgrading or downgrading, and consequent *symmetric* changes in the demand for unskilled workers. Quality is upgraded if $\gamma_{SZ} < \gamma_{KZ}$, and, as spelled out earlier, under the same condition, the overall capital requirement in Z production rises. The reverse reasoning shows that if $\gamma_{SZ} > \gamma_{KZ}$, as emigration downgrades export quality and correspondingly raises the scale of production of good Z, larger capital requirement due to scale expansion dominates smaller capital requirement per unit due to quality degradation. Thus, in either case, whether quality is upgraded or downgraded, the overall capital requirement in Z sector rises. Since, the import-competing sector uses this capital as well, so its production must fall as a consequence of emigration induced quality variation, which in turn lowers the demand for unskilled labour and lowers the unskilled wage to some extent. Algebraically this can be verified from the following expression (see appendix),

$$\hat{K}_Z = (\gamma_{KZ} - \gamma_{SZ})\hat{Q} > 0$$  \(16\)
This result is summarized in the following Lemma 2:

**Lemma 2:** Unskilled emigration induced quality variation lowers unskilled money wage and thereby dampens the initial rise in the unskilled wage to some extent.

Proof: Follows from (16) and \[ \dot{y} = -\frac{\lambda_{kz}}{\lambda_{ky}} (\gamma_{kz} - \gamma_{sz}) \hat{Q}. \]

As we will elaborate in Section 4, this result has some far reaching implications for emigration of workers.

An exogenous increase in the rate of emigration of skilled workers will have similar effects on wages and rates of returns to capital and land. Such emigration will raise the skilled wage due to scarcity of skilled workers as lesser number of them will now be available for production of good Z. Consequent fall in production of good Z, at the initial level of quality, will deploy some capital in this sector which will lower its rate of return. Lower capital cost of producing the import-competing good Y that uses the same capital, on the other hand, will expand its production and raise the demand for unskilled workers and their wages. Thus, again the \(ww\) curve will shift upward as in Figure 3 and the export quality will rise or fall according as \(\gamma_{kz} > \gamma_{kz} \) or \(\gamma_{sz} > \gamma_{kz} \).

Therefore, emigration of either type will have similar effect on the choice of export quality, and this result follows from the complementarity between skilled and unskilled money wages as stated in Lemma 1.

3. Remittance Tax and Production Subsidy

Proposition 1 indicates a potential policy conflict when higher quality of the export good Z is relatively skill intensive: allowing emigration of unskilled workers on the one hand, and promoting exports by incentivizing quality upgrading on the other hand. But, though emigration of unskilled workers may have adverse effect on export-quality, restricting such emigration by the developing countries may be difficult for reasons mentioned earlier. Instead, the adverse effect of emigration can be mitigated through a production subsidy given to producers of the quality differentiated export good Z. A uniform subsidy \(\mu\) will not, however, incentivise quality
upgrading. Rather, it will lower quality by raising the skilled wage and consequently the marginal cost of quality upgrading. A production subsidy that is higher for higher quality of good $Z$ would be required. To fix ideas, suppose, a per unit subsidy at the rate $b$ that increases with quality upgrading at an increasing rate, is provided to the producers of good $Z$

$$
\mu(Q) = \frac{1}{2} bQ^2 \forall Q \in [0,1], \ b > 0 \tag{17}
$$

This subsidy can be financed, among the several alternatives, by revenue collected by taxing the remittance sent by unskilled emigrants:

$$
R = \tau \beta w^* L = \frac{1}{2} bQ^2 Z \tag{18}
$$

where, $\tau$ is the rate of remittance-tax, $w^*$ is the unskilled money wage in the host country, and $\beta$ is the proportion of income remitted by an emigrant.

The zero profit condition and the marginal condition for the skill based export good under quality-content production subsidy now changes to,

$$
P_Z^w (Q) + \frac{1}{2} bQ^2 = a_{sz}(Q)w_s + a_{kz}(Q)r \tag{19}
$$

$$
P_Z^{w'} (Q) + bQ = a_{sz}'(Q)w_s + a_{kz}'(Q)r \tag{20}
$$

It is immediate that a production subsidy will raise the demand for both skilled workers and capital, and skilled labour being specific to this sector, its larger demand would raise its wage unambiguously. Thus, at initial $w$ and $r$, this will act as a disincentive for quality upgrading as the marginal cost of quality will rise unambiguously. To be precise, the skilled wage would vary with the rate of subsidy per unit of quality as follows:

$$
\dot{w}_s = \frac{\mu_z}{\theta_{sz}} \hat{b} \tag{21}
$$

where, $\mu_z = \frac{bQ^2}{2P_z^w}$ is the per unit subsidy as a proportion to the unit production cost. From the marginal condition (20) then, given (21) and at initial $w$ and $r$, the change in the quality level due to an increase in the rate of subsidy can be obtained as,
\[ \hat{Q} = \frac{b}{2\delta} (\gamma_{sz} - 2) \hat{b} \]  
(22)

That is, a quality-content production subsidy will raise export quality conditionally. More precise statement is made in the following Lemma:

**Lemma 3:** Quality-content production subsidy is worthwhile if \( \gamma_{sz} < 2 \).

**Proof:** By (22), given that \( \delta < 0 \) by the second order condition for profit maximization, an increase in the rate of production subsidy will raise export quality if \( \gamma_{sz} < 2 \). □

A one percent increase in the rate of subsidy raises the effective marginal revenue from quality upgrading by one percent. On the other hand, by the quadratic form of the subsidy function, a one percent rise in the rate of production subsidy raises the marginal cost of quality upgrading due to consequent increase in the skilled wage by \( \frac{\gamma_{sz}}{2} \) percent. Hence, the quality content production subsidy raises the marginal revenue from quality upgrading more than the marginal cost if \( \gamma_{sz} < 2 \), and thereby incentivizes producers to raise quality.

By Lemma 3, as a quality-content production subsidy raises export quality for any level of unskilled wages, so the \( QQ \) curve in Figure 3 will also shift to the right as a consequence emigration of unskilled workers. Note that, for any given remittance-tax rate, an exogenous increase in the rate of emigration will generate larger remittances and hence larger remittance-tax revenue. This will enable the local government to finance a proportionately higher rate of quality-content production subsidy at the initial level of quality. If the consequent quality increase is large enough, it can outweigh the adverse effect of emigration. This is shown in Figure 3 by a larger shift of the \( QQ \) curve to the right than the (leftward) shift of the \( WW \) curve. As shown in the appendix, this will be the case if the initial rate of production subsidy was larger than a critical value:

\[ b > \bar{b} = \frac{\lambda_{KY} \varphi 2\delta}{(\gamma_{sz} - 2)(\gamma_{sz} - \gamma_{KZ}) \lambda_{LY} \lambda_{KZ} + \lambda_{KY} \varphi (\gamma_{sz} - 2)^2} \]  
(23)
Thus, given the above condition, even when $\gamma_{sz} > \gamma_{kz}$, emigration of unskilled workers can upgrade export quality if a quality-content production subsidy is given to the producers of Z, which is financed by taxes on remittances. Note that even when the initial rate of subsidy is lower than the critical rate defined in (23) above, the adverse effect of emigration can be dampened to some extent through remittance-tax financed production subsidy.

**4. Endogenous Emigration and Export Quality**

From Lemma 2 it appears that if we allow for endogeneity of emigration with the rate of emigration responding to both the source country factors (push factors) and the host or destination country factors (pull factors), then quality variations due to any policy or exogenous shock will cause the rate of emigration to change as well. That is, emigration and export quality will cause each other and thus are to be determined simultaneously. In this section, we see whether and how such endogeneity and two-way causality alter our results stated in Proposition 1. According to Acharyya et al. (2019), the decision to emigrate is an endogenous function of the wage differentials across the two countries and the costs associated to emigration. Assuming costs of migration to be negligible, as it is not the primary focus of this analysis, emigration of either skilled or unskilled labour will be driven entirely by the wage differentials. Let $w^*$ and $w_s^*$ denote, respectively, unskilled wage and skilled wage at the destination country. So an unskilled (skilled) worker will emigrate if $w^* (w_s^*)$ exceeds $w (w_s)$ that she would have earned in
the source country. Otherwise, he will stay back. So, given Lemma 2, the emigration equilibrium conditions can be written as:

\[
\begin{align*}
    w(\beta, Q) &= w^* \\
    w^*_s(\alpha, Q) &= w^*_s
\end{align*}
\]  

(24)  
(25)

In the above, we equate the gain from migration (the right hand side) to its opportunity cost which is the domestic wages forgone. To ensure that the emigration equilibrium exists and is unique, it must be that, \( w(0, Q) < w^* \) and \( w^*_s(0, Q) < w^*_s \; \forall \; Q \in [0,1] \). Since we have established above that either type of labour emigration has the same impact on the factor prices in the source country, let us focus on unskilled emigration for our endogeneity analysis. Note that by the small country assumption, any change in rates of emigration from the source country will have negligible effect on the world wage rates. So in the \((w, \beta)\) plane, the right hand side of (24) will be a horizontal line at some fixed value of \( w^* \). On the other hand, a ceteris paribus rise in emigration will raise wages at the source, such that the wage differential for each type decreases with the rate of emigration. Therefore, the incentive for emigration based on wage differential decreases with the rate of emigration itself. So the left hand side of (24) will be a positively sloped line which we denote as the \( ww \) schedule and its intersection with the \( w^* \) line will give the initial equilibrium value of \( \beta = \beta_0 \).

Now following the discussion in section 2.2, a ceteris paribus rise in export quality say from \( Q_0 \) to \( Q \) will lower the domestic unskilled wage when higher qualities are relatively more capital intensive, and will raise the unskilled wage otherwise. This is also evident from (14). The fall in domestic wage, when \( \gamma_{KZ} > \gamma_{SZ} \), incentivises more workers to emigrate. The \( ww \) line shifts to the right and we have a higher value of \( \beta \) corresponding to a higher quality of good \( Z: \beta_1(Q_i) > \beta_0(Q_0) \). In the alternative case, i.e. when \( \gamma_{SZ} > \gamma_{KZ} \), a rise in quality raises the domestic unskilled wage and induces lesser out-migration of workers. So \( ww \) line shifts left and \( \beta \) falls: \( \beta_1(Q_i) < \beta_0(Q_0) \). The two panels of Figure 4 depict the above two cases respectively:
From this emerges a relationship between the level of export quality and the rate of emigration. In Figure 5, this relationship is represented by the $\beta(Q)$ curve. From the emigration equilibrium condition (24), it can be shown (see appendix) that $\beta(Q)$ is negatively sloped if $\gamma_{SZ} > \gamma_{KZ}$, and positively sloped otherwise:

$$\frac{d\beta}{dQ_{MM}} = -\frac{\hat{\partial}w}{\hat{\partial}Q} = -\lambda_{LY} \lambda_{KZ} (\gamma_{KZ} - \gamma_{SZ}) \beta \frac{\beta}{\lambda_{KY} \phi Q}$$  \hspace{1cm} (26)

The other side of the two-way causation, i.e. how a ceteris paribus change in rate of emigration of unskilled labour affects the level of export quality chosen, has already been derived in Section 2.2 and stated in Proposition 1. To recall, an exogenous increase in emigration rate of unskilled workers raises raises quality of the export good $Z$ if its higher quality is relatively capital intensive, $\gamma_{KZ} > \gamma_{KZ}$, and lowers export quality otherwise. This relationship between $Q$ and $\beta$ where the causation is now from the latter to the former is depicted in Figure 5 by the $Q(\beta)$ schedule. Algebraically, this can be verified from the slope of $Q(\beta)$ from (15):

$$\frac{d\beta}{dQ_{QQ}} = \frac{\Delta}{\theta_{KZ} p_{z}^w \theta_{Ly} \theta_{KY} (\gamma_{KZ} - \gamma_{SZ})} \beta \frac{\beta}{Q}$$  \hspace{1cm} (27)
Given these two relationships, equilibrium rate of emigration and export quality are determined at the intersection of $\beta(Q)$ and $Q(\beta)$ schedules at point E in Figure 5. 

Now consider a ceteris paribus rise in the destination country unskilled wage rate ($\hat{w}^*>0$). As a pull factor of migration, at the stroke of the pen, this will induce a greater rate of emigration of unskilled workers from the source country. This is captured by an upward shift of the $w^*$ line whose intersection with the initial $ww$ curve (in Figure 4) gives us a higher value of $\beta$. Note that this change in $\beta$ occurs at the initial level of quality and is also independent of the relative skill intensity of higher quality varieties of $Z$. So this will be reflected through an upward shift of the $\beta(Q)$ schedule along the $Q(\beta)$ line in Figure 5. While the rate of emigration rises unconditionally, export quality will rise if higher qualities are relatively more capital intensive and fall otherwise, as in the case of exogenously given emigration rates.

Figure 5: Effect of rise in $w^*$ on $\beta$ and $Q$

The most important thing to note here is that a change in rate of emigration affecting the level of export quality does not end the story. This means that the overall changes that the two variables

\[ \gamma_{KZ} > \gamma_{SZ} \]

\[ \gamma_{SZ} < \gamma_{KZ} \]

$\hat{w}^*$

Note that by stability requirements, $Q(\beta)$ schedule must be steeper than $\beta(Q)$ schedule.
go through are not one shot. To explain further, consider the case when higher qualities are relatively more capital intensive (the left hand side panel in Figure 5). Increased emigration of unskilled workers in response to higher wages in the destination country raises unskilled wage in home country, lowers the rate of return to capital and raises the skilled wage. This will induce producers of Z to upgrade export quality. Higher level of quality raises capital requirement in Z sector and the consequent change in composition of output in the (X, Y) nugget dampens the initial rise in w. More workers will emigrate as a consequence of export quality being upgraded. This leads to a second round rise in value of $\beta$. Once again this will change the domestic factor prices in such a way that given the relative skill intensity ranking of higher quality varieties, quality will again be upgraded. In this way, rounds of feedback effects of one variable on the other will continue, the magnitude of changes in both $\beta$ and Q petering out with every successive rounds until the new equilibrium values of the two variables are reached at the intersection of $Q(\beta)$ and the shifted up dotted $\beta(Q)$. At the end, there will be even larger rate of emigration than was initially caused by the increase in the unskilled wage in the host country, and a larger rise in export quality than an increase in the exogenously given emigration rate would have led to. By similar logic, when $\gamma_{KZ} < \gamma_{SZ}$, the fall in export quality will be magnified by multiplier expansion in the rate of emigration. All these are summarized in Proposition 2 below:

Proposition 2: Quality variations are magnified when emigration of unskilled workers is endogenous.

Proof: Follows from the above discussion. □

5. Conclusion

In this paper we have shown that emigration of unskilled workers may adversely affect export prospects of a small open economy with a diversified export basket by downgrading quality of its skill-based export good. This would be the case when higher qualities are relatively more skill intensive. Similar result follows when skilled workers emigrate since skilled and unskilled wages are complementary. Such adverse effect, however, can be mitigated through a quality-content production subsidy financed by taxes on the remittances received from emigrants. Increase in the
rate of emigration will generate larger remittances and hence larger remittance-tax revenue enabling the local government to provide a larger quality-content production subsidy.

The quality upgrading or downgrading, as the case may be, gets magnified when emigration rate is endogenous and increases due to a wage increase in the host country. The rate of emigration gets magnified as well by the consequent quality variation.

Appendix

A.1. Exogenous rise in rate of emigration

A.1.1: Relation between quality and skilled wage.

Total differentiation of the marginal condition (6) for quality choice as given in text yields,

\[ P_z''(Q)dQ = a_{sz}''(Q)dqw_s + a_{sz}'(Q)dw_s + ra_{kz}''(Q)dQ + a_{kz}'(Q)dr \]

\[ \Rightarrow \left[ P_z''(Q) - ra_{kz}''(Q) - w_s a_{sz}'(Q) \right] dQ = ra_{kz}'(Q)\dot{r} + w_s a_{sz}'(Q)\dot{w}_s \]

\[ \Rightarrow \dot{Q}Q = \frac{a_{kz}(Q)r}{P_z} \frac{Qa_{kz}'(Q)}{Q} \dot{r} + \frac{a_{sz}(Q)w_s}{P_z} \frac{Qu_{sz}'(Q)}{a_{sz}(Q)} \dot{w_s} \]

\[ \Rightarrow \dot{Q}Q = \frac{P_z''}{Q} \left[ \theta_{kz}\gamma_{kz} \dot{r} + \theta_{sz}\gamma_{sz} \dot{w}_s \right] \]

(A1.1)

From the zero profit condition in Z sector given by (5), at initial Q, proportional change in skilled wage is given by, \( \dot{r} = -\frac{\theta_{sz}}{\theta_{kz}} \dot{w}_s \). Substituting this in (A1.1) yields,

\[ \Rightarrow \dot{Q}Q = \frac{P_z''}{Q} \left[ \theta_{kz}\gamma_{kz} \left( -\frac{\theta_{sz}}{\theta_{kz}} \dot{w}_s + \theta_{sz}\gamma_{sz} \dot{w}_s \right) \right] \rightarrow \dot{Q} = \frac{P_z''}{Q} \left( \theta_{sz}\gamma_{sz} - \gamma_{kz} \right) \dot{w}_s \]

(A1.2)

\[ \Rightarrow \frac{dQ}{dw_s} > 0 \text{ when } \gamma_{kz} > \gamma_{sz} \]

A.1.2: Determination of \( QQ \) schedule.

Substituting \( \dot{w}_s = \frac{\theta_{kz}}{\theta_{sz}} \frac{\theta_{LY}}{\theta_{KY}} \dot{w}_n \) in (A1.2) we get the equation that relates change in quality to the rise in emigration induced change in domestic unskilled wage as:

\[ \dot{Q} = \frac{P_z''}{Q^2} \theta_{kz}\gamma_{kz}\frac{\theta_{LY}}{\theta_{KY}} \left( \gamma_{sz} - \gamma_{kz} \right) \dot{w} \]

(A1.3)
A.1.2: Determination of \(ww\) schedule.

Total differentiation of (10), (9) and (8) yields respectively,

\[
\hat{K} = 0 = \lambda_{KY}(\hat{Y} + \hat{a}_{KY}) + \lambda_{KZ}(\hat{Z} + \gamma_{KZ}\hat{Q}) \quad (A1.4)
\]
\[
\hat{X} = -\hat{a}_{TX} \quad (A1.5)
\]
\[
-\varphi\hat{\beta} = \lambda_{LY}(\hat{X} + \hat{a}_{LY}) + \lambda_{LY}(\hat{Y} + \hat{a}_{LY}) \quad (A1.6)
\]

where \(\varphi = \frac{\beta}{1-\beta}\).

Under the assumption that the rate of skilled emigration does not change, total differentiation of (7) yields,

\[
\hat{Z} = -\gamma_{SZ}\hat{Q} \quad (A1.7)
\]

Substituting (A1.7) in (A1.4) we get:

\[
\hat{Y} = -\left(\hat{a}_{KY} + \frac{\lambda_{KZ}}{\lambda_{KY}}(\gamma_{KZ} - \gamma_{SZ})\hat{Q}\right) \quad (A1.8)
\]

Substituting (A1.5) and (A1.8) in (A1.6) we get:

\[
-\varphi\hat{\beta} = \lambda_{LY}(\hat{X} - \hat{a}_{TX}) + \lambda_{LY}\left(\hat{a}_{LY} - \hat{a}_{KY} - \frac{\lambda_{KZ}}{\lambda_{KY}}(\gamma_{KZ} - \gamma_{SZ})\hat{Q}\right) \quad (A1.9)
\]

Note that by the condition for least-cost choice of inputs we have (see Jones [1965]):

\[
\hat{a}_{TX} = \hat{a}_{TX} - \theta_{TX}\hat{a}_{TX} = \theta_{TX}(\hat{a}_{TX} - \hat{a}_{TX}) \Rightarrow \hat{a}_{TX} = \theta_{TX}(\hat{a}_{TX} - \hat{a}_{TX})
\]

\[
\Rightarrow \hat{a}_{TX} = \sigma_{X}\theta_{TX}(\hat{w} - \hat{R})
\]

where \(\sigma_{j} = \frac{\hat{a}_{Kj} - \hat{a}_{Lj}}{\hat{w} - \hat{r}_{j}}\). \(\hat{a}_{LY}\) and \(\hat{a}_{LY}\) can be expressed similarly.

We can rewrite (A1.9) using the above as:

\[
-\varphi\hat{\beta} = \lambda_{LY}\sigma_{X}(\hat{R} - \hat{w}) - \lambda_{LY}\sigma_{Y}(\hat{w} - \hat{r})(\hat{a}_{KY} - \hat{a}_{LY}) - \frac{\hat{\lambda}_{LY}\lambda_{KZ}}{\lambda_{KY}}(\gamma_{KZ} - \gamma_{SZ})\hat{Q}
\]

\[
\Rightarrow \varphi\hat{\beta} = A\hat{w} + \frac{\hat{\lambda}_{LY}\lambda_{KZ}}{\lambda_{KY}}(\gamma_{KZ} - \gamma_{SZ})\hat{Q} \quad (A1.10)
\]

where \(A = \frac{\hat{\lambda}_{LY}\sigma_{X}}{\theta_{TX}} + \frac{\hat{\lambda}_{LY}\sigma_{Y}}{\theta_{KY}} > 0\)

22
A.1.4: Proof of Proposition 1

(A1.3) and (A1.10) constitute the system of equations (written below in matrix notation) to solve for changes in quality and unskilled money wage:

\[
\begin{bmatrix}
A & \frac{\lambda_{LZ}^L \lambda_{ZK}^L}{\lambda_{KY}^L} (\gamma_{ZK} - \gamma_{SZ}) \\
\frac{P_Z^w}{\partial Q^2} \theta_{KY} \theta_{LZ} (\gamma_{SZ} - \gamma_{LZ}) & -1
\end{bmatrix}
\begin{bmatrix}
\hat{\dot{W}} \\
\hat{\dot{Q}}
\end{bmatrix}
= \begin{bmatrix}
\phi \hat{\beta} \\
0
\end{bmatrix}
\]

\[
\hat{Q} = \frac{1}{\Delta} \theta_{LZ} \frac{P_Z^w}{\partial Q^2} \theta_{KY} (\gamma_{ZK} - \gamma_{SZ}) \phi \hat{\beta}
\]

(A1.11)

where, \( \Delta = \left[ \frac{\lambda_{ZK}^L \theta_{ZK} \frac{P_Z^w}{\partial Q^2} \theta_{KY}^L (\gamma_{ZK} - \gamma_{SZ})^2}{\lambda_{KY}^L} - A \right] < 0 \).

A.1.4: Proof of \( \dot{K}_Z = (\gamma_{ZK} - \gamma_{SZ}) \dot{Q} > 0 \)

Total capital requirement in the Z sector: \( K_Z(Q) = a_{KZ}(Q)Z \)

Using full employment condition for skilled labour as given in the text by (7), we can rewrite the above as: \( K_Z(Q) = a_{KZ}(Q) \frac{(1 - \alpha)S}{a_{SZ}(Q)} \).

With \( \hat{\alpha} = 0 \), total differentiation of this gives us: \( \dot{K}_Z(Q) = \dot{a}_{KZ}(Q) - \dot{a}_{SZ}(Q) = (\gamma_{ZK} - \gamma_{SZ}) \dot{Q} \).

Substituting expression for change in quality as derived in (A1.11) in the above, we can prove that \( \dot{K}_Z > 0 \) irrespective of whether quality is raised or lowered:

\[\dot{K}_Z = \frac{1}{\Delta} \theta_{LZ} \frac{P_Z^w}{\partial Q^2} \theta_{KY} (\gamma_{ZK} - \gamma_{SZ}) \phi \hat{\beta} \]

which is positive for \( \hat{\beta} > 0 \).

A.2: Remittance tax and production subsidy:

Condition under which quality content production subsidy will outweigh the adverse effect of emigration on quality choice:

Total differentiating the new marginal condition as given in the text by (20) under quality content production subsidy case yields:

\[ P_Z^{w''}(Q)dQ + bQ + dQ + db = a''_{SZ}(Q)dQw_s + a'_{SZ}(Q)dw_s + ra''_{KZ}(Q)dQ + a'_{KZ}(Q)dr \]
From the zero profit condition in the Z sector we get:

\[ P'_Z (Q) dQ + \frac{1}{2} b 2Q dQ + \frac{1}{2} Q^2 d\hat{b} = ra'_{KZ} (Q) dQ + a_{KZ} (Q) dr + a_{sz} dw_s + w_s a'_{sz} (Q) dQ \]

Using the marginal condition reproduced below,

\[ P'_Z (Q) + b Q = a'_{sz} (Q) \dot{w}_s + a'_{KZ} (Q) r \]

the above expression boils down to:

\[ 0 = \theta_{KZ} \hat{r} + \theta_{sz} \hat{w}_s - \mu \hat{b} \quad (A2.2) \]

Substituting \( \hat{r} = -\frac{\theta_{LY}}{\theta_{KY}} \hat{\tilde{w}} \) as can be derived from the zero profit condition of good Y, the above boils down to:

\[ \hat{w}_s = \frac{\theta_{KZ}}{\theta_{sz}} \frac{\theta_{LY}}{\theta_{KY}} \hat{\tilde{w}} + \frac{\mu}{\theta_{sz}} \hat{b} \quad (A2.3) \]

Substituting (A2.3) and \( \hat{r} = -\frac{\theta_{LY}}{\theta_{KY}} \hat{\tilde{w}} \) once again in (A2.1) we get:

\[ \hat{Q} = \frac{P'_Z}{\delta Q^2} \left( \frac{\theta_{LY}}{\theta_{KZ}} \gamma_{sz} - \gamma_{KZ} \right) \hat{\tilde{w}} - \frac{b}{2\delta} (\gamma_{sz} - 2) \hat{b} \quad (A2.4) \]

Total differentiation of \( R = \frac{1}{2} b Q^2 \frac{\tilde{s}}{a_{sz} (Q)} \) gives,

\[ a_{sz} (Q) dR + Ra'_{sz} (Q) dQ = b Q d\tilde{s} + \frac{1}{2} Q^2 \tilde{s} d\hat{b} \]

\[ \Rightarrow a_{sz} (Q) R \left( \hat{R} + \frac{a'_{sz} (Q) Q}{a_{sz} (Q)} \hat{Q} \right) = \frac{1}{2} b Q^2 (2 \hat{\tilde{Q}} + \hat{b}) \Rightarrow \hat{b} = \hat{R} - (2 - \gamma_{sz}) \hat{Q} \quad (A2.5) \]

On the other hand, total differentiation of \( R = \tau \rho \beta w^* L \) yields \( \hat{R} = \hat{\beta} + \hat{\tau} \), which by substituting in (A2.5), we get,

\[ \hat{b} = \hat{\beta} + \hat{\tau} - (2 - \gamma_{sz}) \hat{Q} \quad (A2.6) \]
Substituting (A2.6) in (A2.4) and rearranging the terms we get the equation for the new QQ schedule as follows:

\[
\left(1 - \frac{b}{2\delta}(\gamma_{sz} - 2)^2\right)\hat{Q} + \frac{P^w}{\theta_{kz}} \frac{\partial \theta_{ky}}{\partial \theta_{kz}} (\gamma_{kz} - \gamma_{sz}) \hat{w} = \frac{b}{2\delta} (\gamma_{sz} - 2)(\hat{\beta} + \hat{\tau}) \tag{A2.7}
\]

Finally, quality will go up if the magnitude of horizontal shift of this QQ curve for a rise in unskilled emigration \(\hat{\beta} > 0\) exceeds that of the \(ww\) schedule:

\[
dQ_{QQ} = \frac{b}{2\delta} (\gamma_{sz} - 2)(\hat{\beta} + \hat{\tau}) \left(1 - \frac{b}{2\delta}(\gamma_{sz} - 2)^2\right) > dQ_{ww} = -\frac{\lambda_{ky} \phi \hat{\beta}}{\lambda_{ly} \lambda_{kz} (\gamma_{kz} - \gamma_{sz})}
\]

At the initial tax rate, \((\hat{\tau} = 0)\), rearranging the above we can get a critical value of \(b = \bar{b}\) as given the text, for which \(\hat{Q} = 0\) i.e. \(dQ_{QQ} = dQ_{ww}\). Then \(\forall \ b > \bar{b}, \hat{Q} > 0\)

### A.3: Endogenous Emigration and Export Quality

**Derivation of slope of \(\beta(Q)\):**

Total differentiation of the unskilled migration equilibrium, \(w(\beta, Q) = w^*\), we get:

\[
\frac{\partial w}{\partial Q} dQ + \frac{\partial w}{\partial \beta} d\beta = dw^* = 0
\]

(A3.1)

Now the effect of a change in quality on the unskilled money wage, given everything else, is nothing but the slope of the \(ww\) curve which has been derived in (A1.10) and restated as under:

\[
\phi \hat{\beta} = A\hat{w} + \frac{\lambda_{ly} \lambda_{kz}}{\lambda_{ky}} (\gamma_{kz} - \gamma_{sz}) \hat{Q} ; \text{ where } A > 0 \Rightarrow \frac{\partial w}{\partial Q} = -\frac{\lambda_{ly} \lambda_{kz} (\gamma_{kz} - \gamma_{sz})}{\lambda_{ky} A}
\]

On the other hand, \(\frac{dw}{d\beta}\) is the effect of a ceteris paribus change in rate of unskilled emigration on the unskilled wage, at initial level of quality, which again from (A1.10) can be derived as follows: \(\frac{\partial w}{\partial \beta} = \frac{\phi}{A}\). Substituting these expressions back in (A3.1) will give us the slope of the unskilled migration equilibrium condition:
\[
\frac{\partial w}{\partial Q} dQ + \frac{\partial w}{\partial \beta} d\beta = dw^* = 0
\]
\[
\Rightarrow -\frac{\lambda_{LY} \lambda_{KZ} (\gamma_{KZ} - \gamma_{SZ})}{\lambda_{KY} A} dQ + \varphi d\beta = 0 \Rightarrow \left| \frac{d\beta}{dQ} \right| = \frac{\lambda_{LY} \lambda_{KZ} (\gamma_{KZ} - \gamma_{SZ})}{\lambda_{KY} \varphi}
\]

References:


