

Hydrodynamics Model of Temperature Variation due to Gas Flaring activities in some parts of Niger Delta Area of Nigeria

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Abstract

This research is investigating the Hydrodynamics Model of Temperature Variation due to Gas Flaring activities in some parts of Niger Delta Area of Nigeria. The unsteady flow of radiative heat flux is represented using mathematical techniques to model the basic equations of fluid in motion by radiative transfer process using general differential forms. Laplace transform is used to obtain expressions for Temperature, Velocity, etc to solve problems of heat flow. The use the continuity, momentum, Navier-Stokes, Dasey and Energy equations is exploited. The results show that radiation parameter influences the variation of temperature distribution in the environment.

Key Words: Hydrodynamics Model, Temperature Variation, Fluid dynamics, Gas Flaring activities and Momentum Equation

Nomenclature

E = total material energy

T = temperature

φ = vision dissipation

K = thermal diffusivity/conductivity

V = fluid velocity,

ρ = density of fluid,

P = pressure and

μ = coefficient of viscosity.

1.0 General Introduction

In physics, engineering and other areas of sciences, fluid dynamics is a sub discipline of fluid mechanics that describes the flow of fluids (ie. liquids and gases), Batchelor, (1967), Chanson, (2009). It has several sub disciplines, including aerodynamics (the study of air and other gases in motion) and **hydrodynamic** (the study of liquids in motion). Basically, the solution to a fluid dynamics problem typically involves the calculation of various properties of the fluid, such as flow velocity, pressure, density, and temperature, as functions of space and time, Batchelor, (1967), .The study of Hydrodynamic Model of Temperature Variation due to Gas Flaring activities in Nigeria has stimulated considerable interest due to its important applications as functions of space and time as it affect the environment, Ogulu and Israel-Cookey (2010), Chanson, (2009).

The development of technology has led to the exploration of man's environment in a bid to increasing his standard of living. It is now very obvious, even to those who had initial doubts about the veracity of the claim by scientists of the resultant effects of gas flaring activities to the environment, which has led to an increase in temperature variation Ewona et al., (2013). Gas flaring is the burning off of gas into the atmosphere, Abdulkareem et. al., (2011) and Jike, (2004). Ewona et al., (2013) pointed out that gas flaring is a major source of climate change, which is creating increased uncertainty about the future temperature and precipitation regimes.

The phenomenon of free or natural convection arises in fluids when temperature changes cause density variations leading to buoyancy forces acting on the fluid particles. Such flows which are driven by temperature differences abound in nature and have been studied extensively because of its applications in engineering, geophysical and astrophysical environments, Batchelor, (1967), Chanson, (2009), Israel-Cookey et. al. (2010).

2.0 Mathematical Formulation

In general D/Dt denotes the rate of change of whatever quantity it operate on following the fluid.

D/Dt operator is known as the

Substantive derivative

$$\therefore \frac{DT}{Dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x}u + \frac{\partial T}{\partial y}v + \frac{\partial T}{\partial z}w \quad (2.1)$$

Can be rewritten as

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u \cdot \nabla T \quad (2.2)$$

And the substantive derivative operator in general as:

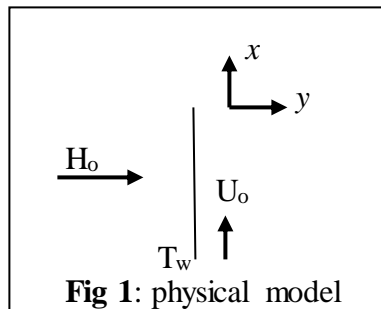
$$\frac{DT}{Dt} = \frac{\partial}{\partial t} + u \cdot \nabla \quad (2.3)$$

Exhibiting the physically obvious fact that it does not depend on a particular coordinates used.

Inference: According to Israel –Cookey et al (2003), the above relationship combines two ways in which the temperature of a fluid particle can change.

- It can change because the whole temperature field is changing

- It can change by moving to a position where the temperature is different of which process depends on the magnitude of the spatial variations of the temperature and on the velocity, determining how quickly the fluid moves through the spatial variations.



Considering the flow of radiation of hydrodynamic fluid to the environment, which maintain at temperature

$$T_w \text{ at } t = 0, \text{ such that} \tag{2.4}$$

$$T = T_w [(1 + \epsilon H(t - t_0))]$$

Where: $H(t)$ = Heaviside step function, and T_w is high enough to initiate radiative heat transfer. A constant magnetic field H_o is maintain in the y – direction and the streamlines is uniformly along the positive x – direction with velocity U_o .

Let U^1 and H^1_x be the velocity and magnetic field component in the x – direction respectively. We assume that for times, convection have not fully developed. So the convective terms are considered to be negligible. Under this condition, the flow is governed by the equations.

$$\frac{d^2 U^*}{dy^2} - p_\infty \frac{du^1}{dt^1} + H_o \frac{dH^1}{dy^1} + p_\infty g^\beta (T^1 - T_\infty) = 0 \tag{2.5}$$

$$\frac{d^2 H^1_x}{dy^{12}} - \sigma_1 \frac{dH^1_x}{dt} + \sigma_c H_o \frac{du}{dy} = 0 \tag{2.6}$$

$$K \frac{d^2 T}{dy^2} - P_\infty C_p \frac{dT}{dt} - \nabla q_y = 0 \tag{2.7}$$

In this study, we take the differential form of the radiative heat flux q_y (Cookey & Tay 2001).

$$\nabla^2 q_y - 3 \alpha^2 q_y - 16 \alpha \sigma T^3 \nabla T = 0 \tag{2.8}$$

Now, assuming that the atmosphere is transparent ie atmosphere with relatively low density for which the optically thin grey gas is true, with this assumption the radiative heat flux given by equation (2.8) becomes

$$\nabla q^1_y = 4\sigma \alpha (T^4 - T_\infty^4) \tag{2.9}$$

Further, if the equilibrium temperature T_∞ is not much different from any other surroundings, then (Cookey & Tay 2001).

$$T^1 = T_\infty + \psi \tag{2.10}$$

Where ψ is small temperature correction and equ (x) reduces to

$$\nabla q^1_y = 16\sigma \alpha T_\infty^3 (T^1 - T_\infty) \tag{2.11}$$

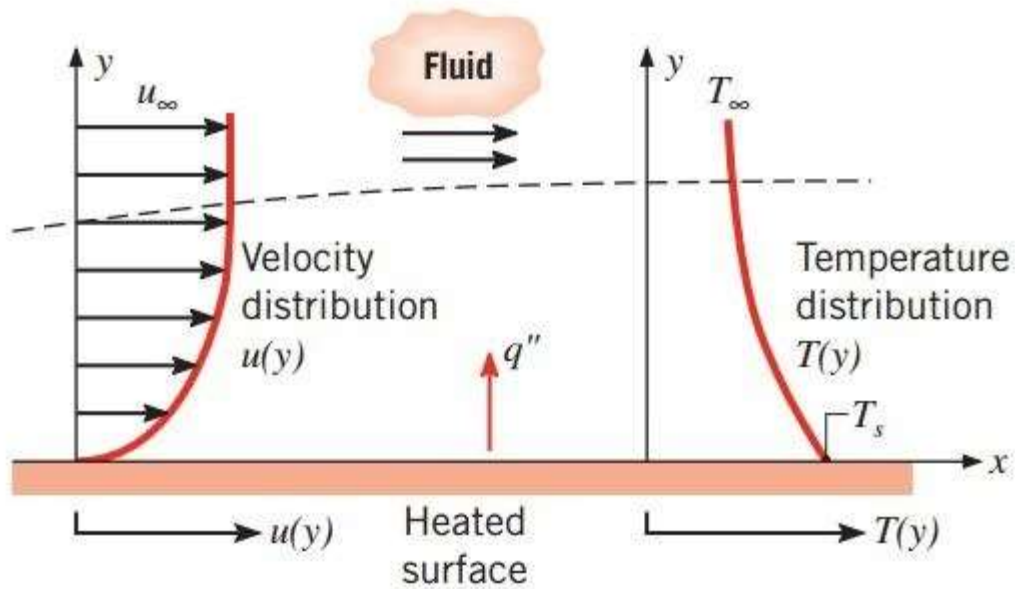


Fig. 2: Showing Fluid distribution in a medium

The differential relations for fluid particle in a medium can be written for conservation of mass, momentum and energy. In addition, there are two state relations of thermodynamic properties. They can be summarized as shown in Cooley and Omubo-Pepple (2007), Batchelor, (1967), Chanson, (2009);

Continuity: $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{V}) = 0^i$

Momentum: $\rho \frac{d\bar{V}}{dt} = \rho \bar{g} - \nabla \rho + \nabla \cdot \tau_{ij}$ (2.4)
2.12

Energy: $\rho \frac{\partial e}{\partial t} + \rho (\nabla \cdot \bar{V}) = \nabla \cdot (k \nabla T) + \Phi$

Thermodynamic state relations: $\rho = \rho(\rho, T); e = e(\rho, T)$

Here, Φ is the viscous-dissipation function, e is the internal energy and k is the thermal conductivity of the fluid. In general, the density is a variable and all these equations have 5-unknown parameters i.e. ρ, \bar{V}, ρ, e and T . In an incompressible flow with constant viscosity, the momentum equation can be decoupled from energy equation.

Thus, continuity and momentum equations are solved simultaneously for pressure and velocity Cooley et al (2007). However, there are certain flow situations, which can wipe out continuity equation by defining a suitable variable (called as *stream function*) and thereby solving the momentum equation with single variable, Chanson, (2009).

Stream Function

The idea of introducing stream function works only if the continuity equation is reduced to two terms. There are 4-terms in the continuity equation that one can get by expanding the vector equation.

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0 \quad (2.13)$$

For a steady, incompressible, plane, two-dimensional flow, this equation reduces to,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.14)$$

Here, the striking idea of stream function works that will eliminate two velocity components u and v into a single variable. So, the *stream function* $\{\psi(x, y)\}$ relates to the velocity components in such a way that continuity equation is satisfied.

$$u = \frac{\partial \psi}{\partial y}; v = -\frac{\partial \psi}{\partial x} \quad (2.15)$$

$$\text{or, } \vec{V} = \frac{\partial \psi}{\partial y} \vec{j} - \frac{\partial \psi}{\partial x} \vec{i}$$

The slope at any point along a streamline is given by

$$\frac{dy}{dx} = \frac{u}{v} \quad (2.16)$$

If we move from one point (x, y) to a nearby point $((x + dx, y + dy)$, then the corresponding change in the value of stream function is $d\psi$ which is given by

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = v dx + u dy \quad (2.17)$$

Along a line of constant

$$d\psi = -v dx + u dy = 0 \quad (2.18)$$

or

$$\frac{dy}{dx} = \frac{u}{v} \quad (2.19)$$

Equation (2.16) is same as that of equation (2.19)

Hence, it is the defining equation for the streamline. Thus, infinite number streamlines can be drawn with constant ψ . This family of streamlines will be useful in visualizing the flow patterns. It may also be noted the streamlines are always parallel to each other.

Energy Equation (Diffusion Equation)

Most time Energy Equation written as

$$\rho \frac{\partial e}{\partial t} + \rho(\vec{v} \cdot \nabla) e = \nabla \cdot (\nabla T) + \dot{q} + \dots + \dot{q}_{Ext} \quad (2.20)$$

Where,

E = total material energy

T = temperature

φ = vision dissipation

K = thermal diffusivity/conductivity

Navier-Stokes Equation

According to Cooney et al (2007), Batchelor, (1967), Chanson, (2009); The Navier–Stokes equations of motion for a viscous isotropic fluid of constant viscosity μ in its tensorial form is given below as

$$\rho \frac{dv_i}{dt} = \rho f_i - \frac{\partial p}{\partial x_i} + \frac{1}{3} \mu \frac{\partial}{\partial x_i} \left(\frac{\partial v_j}{\partial x_j} \right) + \mu \frac{\partial^2 v_i}{\partial x_j^2}, i, j = 1, 2, 3. \quad (2.21)$$

In the vector form, the Navier-Stokes equations is

$$\rho \frac{dv}{dt} = \rho f - \nabla P + \frac{1}{3} \mu \nabla (\nabla \cdot \mathbf{v}) + \mu \nabla^2 \mathbf{v}. \quad (2.22)$$

Where

v - Fluid velocity,

ρ - Density of fluid,

P - Pressure and

μ -Coefficient of viscosity.

For an incompressible, viscous fluid the equation of continuity will be $\nabla \cdot \mathbf{v} = 0$.

Therefore, the Navier – Stokes equations for an incompressible, viscous fluid in the vector form can be expressed as

$$\rho \frac{dv}{dt} = \rho f - \nabla p + \mu \nabla^2 \mathbf{v} \quad (2.23)$$

Hydrostatics is the condition of vanishing of the total force

Hydrostatics equation is represented as:

$$-\left(\frac{2\rho}{2z} - Qg \right) dx dy dz = 0 \quad (2.24)$$

Or



$$\rho(z) = \rho(0) - \rho g z \quad (2.25)$$

With gravity = $-\nabla\phi$, then $\phi = gz$

And $p + g\phi = \text{constant}$

Hydrostatics equation in the atmosphere is represented as perfect gas (Air)

Equation of state

$$\rho = \frac{p}{RT} \quad (2.26)$$

$R = 287 \text{ J/Kg}^\circ\text{K}^\wedge$ gas constant

T = Temperature

p = Density

Hydrostatic balance in atmosphere with constant temperature

$$\frac{2\rho}{2z} = \rho g = -\frac{g}{RT} \rho \quad (2.27)$$

$$\text{So } \rho(z) = \rho(0) e^{-\left(\frac{g}{RT}\right)Z} \quad (2.28)$$

$$\frac{RT}{g} = 7.3\text{km (AIR) Scale Height}$$

Advection Diffusion Equation

Advection diffusion, given below, is the simplest model equation that can be used to test the performance of different numerical schemes for problems involving advection (convection) and diffusion phenomena.

$$\frac{\partial T}{\partial t} + \bar{v} \cdot \nabla T = k \nabla^2 T + s \quad (2.29)$$

Advection diffusion can be seen as a linearized and simplified scalar form of the Navier-Stokes equation with a single variable. It is also very similar to the energy equation solved separately by itself for incompressible flows. The scalar unknown T is advection (convection) with a known velocity field, \vec{v} which can be taken to be divergence free to simulate the constraint due to the continuity equation of incompressible flows. At the same time T is diffused with a known constant and isotropic diffusivity of k . S represents the known source term. Physically the problem corresponds to the calculation of the temperature field of a heat transfer problem of concentration field of a species transport problem with the use of a known velocity field.

3.0 METHODS OF SOLUTION

We begin our analysis considering the use of advection diffusion equation extensively to study the difficulties faced with highly convective cases and alternative solutions methods as shown in the equation.



Note that for a special case of no velocity $\vec{v} = 0$ (pure diffusion), we obtain the transient heat equation which is parabolic. If \vec{v} and $\frac{\partial T}{\partial t} = 0$, we get the steady Poisson equation which is elliptic. For the pure advection case ($k = 0$) the equation becomes hyperbolic.

The dimensionless form of the advection diffusion equation can be considered using the solution in equation (2.29), which becomes

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial x^2} \tag{3.1}$$

Using a characteristic length L , a characteristic velocity u_0 and a characteristic time L/u_0 following dimensionless variables can be defined as

$$T^* = \frac{T}{\Delta T}, \quad x^* = \frac{x}{L}, \quad u^* = \frac{u}{u_0}, \quad t^* = \frac{t}{L/u_0} \tag{3.2}$$

Where ΔT is the characteristic driving temperature difference for a heat transfer problem, using these dimensionless variables the following nondimensional form of the advection diffusion equation can be derived as

$$\frac{\partial T^*}{\partial t^*} + u^* \frac{\partial T^*}{\partial x^*} = \frac{1}{Pe} \frac{\partial^2 T^*}{\partial x^{*2}} \tag{3.3}$$

Where Peclet number $pe = u_0 L/k$ is similar to the Reynolds number of the Navier-Stokes equation. It represents a ratio between the ‘strength’ of advection and diffusion process. Convection dominated flows are characterized by high Peclet values. Note that for diffusion problems using a characteristic time L^2/k is more appropriate.

According to Sigalo et al (2001) assuming that radiation is affected by these increase related to the effective surface temperature T , as

$$L = 4[R^2 \sigma T^4]$$

Where: R heat source and σ is the Stefan’s constant of radiation. Obviously, if we keep R constant and decreases T , then L will decrease. On the other hand, the cooling or heating of the environment depends on the heat source R , a small change in temperature dT and small change in the heat source R will generate the following relationship

$$\frac{dR}{R} = -2 \frac{dT}{T} \tag{3.4}$$

4.0 DISCUSSION

Based on the research, as the fluid particles flows with distances, there is a clear temperature variation. Temperature observes depend on the heat source. Of primary importance is the temperature within a astrophysical environment. The temperature profiles given by equation (2.4 to 2.11), (2.29) and 3.1 to 3.4) 92.4 to 2.11), (2.29) (3.1 to 3.4) invoke the mathematical representation on how the environment becomes.

For purpose of hydrodynamics model of temperature variation equation which represent a continuity equation

$$\frac{\partial p}{\partial t} + \nabla(ev) = 0 \tag{4.1}$$



For a physical quantity $T = F(x,t)$ temperature we must actually take care of distinguish two different time derivatives. By dF/dt , as equation (4.1), we mean the rate of change of T at a particular point that is fixed in space. Considering the rate of change of T in a hydrodynamic third as it moves along its trajectory $X = x(t)$ in the flow.

This define the material (or substantive) derivative

$$\frac{DT}{dt} = \frac{d}{dt} T(x(t), y(t), z(t), t) \quad (4.2)$$

$$\frac{DT}{Dt} = \frac{dT}{dt} + \frac{dx}{dt} \frac{dT}{dx} + \frac{dy}{dt} \frac{dT}{dy} + \frac{dz}{dt} \frac{dT}{dz} \quad (4.3)$$

$$\frac{DT}{Dt} = \frac{dT}{dt} + u \frac{dT}{dx} + v \frac{dT}{dy} + w \frac{dT}{dz} \quad (4.4)$$

$$\frac{DT}{Dt} = \frac{dT}{dt} + \mathbf{V} \cdot \nabla T \quad (4.5)$$

5.0 Conclusion

This research is investigating the Hydrodynamics Model of Temperature Variation due to Gas Flaring activities in some parts of Niger Delta Area of Nigeria. And it's established mathematical techniques to model the basic equations of fluid in motion by radiative transfer process using differential models. The use of Diffusion advection equation to solve problems of heat flow under the influence of internal and external medium. The result shows that the applications using the continuity, momentum, Navier-Stokes, Dasey and Energy equations are true.

Equation (4.5) above conveys that intuitively obvious fact that even in a time independent flow field ($\frac{dt}{dt} = 0$ everywhere), any given element can suffer changes in T (via $\mathbf{V} \cdot \nabla T$) as it moves from place to place.

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