Lanczos Potential for The Weyl Tensor

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Abstract:

For arbitrary spacetimes with Petrov types O, N and III, we indicate general results about the Lanczos potential for the corresponding Weyl tensor.

Keywords: Conformal tensor, Lanczos generator, Newman-Penrose formalism, Petrov classification, Weyl-Lanczos equations, 2-spinors, Spin coefficients.

1. Introduction

We shall employ the notation and quantities explained in [1-6]. The Lanczos potential K_{abc} [7-12] is a generator for the Weyl tensor in four dimensions; in [9] was used the Newman-Penrose (NP) formalism [4, 6, 13-17] to determine the Lanczos spin tensor for any spacetime of Petrov types [12-15, 18-21] N, O, and III, thus:

$$S_{abc} = K_{abc} + i\ {}^{*}K_{abc} = \frac{2q}{3}[V_{ab}(-3\nu\, l_c - \pi\, n_c + 3\lambda\, m_c + \mu\, \overline{m}_c) + \qquad (1)$$

$$+ U_{ab}(\tau\, l_c + 3\kappa\, n_c - \rho\, m_c - 3\sigma\, \overline{m}_c) + M_{ab}(-\mu\, l_c + \rho\, n_c + \pi\, m_c - \tau\, \overline{m}_c)],$$

with $q = \frac{1}{2}$ and 1 for the types O, N, and III, respectively; besides [22]:

$$V_{ab} = l_a\ x\ m_b\,, \qquad U_{ab} = \overline{m}_a\ x\ n_b\,, \qquad M_{ab} = m_a\ x\ \overline{m}_b + n_a\ x\ l_b\,, \qquad (2)$$

for the corresponding canonical null tetrad [15, 21]:

$$l^a \ \leftrightarrow\ o^A\, o^{\dot{B}}\,, \qquad n^a \ \leftrightarrow\ \iota^A\, \iota^{\dot{B}}\,, \qquad m^a \ \leftrightarrow\ o^A\, \iota^{\dot{B}}\,, \qquad \overline{m}^a \ \leftrightarrow\ \iota^A\, o^{\dot{B}}\,, \qquad o_A\, \iota^A = 1. \qquad (3)$$

In Sec. 2 we obtain the Lanczos spinor $L_{ABC\dot{D}}$ [2, 23-30] associated to the tensorial result (1):

$$L_{ABC\dot{D}} = \frac{1}{4}\sigma^{a}{}_{A\dot{E}}\,\sigma^{b}{}_{B}{}^{\dot{E}}\,\sigma^{c}{}_{C\dot{D}}\,S_{abc}\,, \qquad (4)$$

where $\sigma^{r}{}_{F\dot{G}}$ are the Infeld-van der Waerden symbols [19, 29-31], in accordance with [28].

We note that a better understanding of the Lanczos potential permits to know more about the Liénard-Wiechert field, for example, to obtain the physical meaning of the Weert generator [32-34] and to construct [35] a Petrov classification [12-15, 18-21] for the electromagnetic field produced by a point charge in arbitrary motion. The Lanczos spintensor is known for arbitrary types O, N and III 4-spaces [9], Kerr geometry [36-41], Gödel cosmological model [42-44], plane gravitational waves [45], and several spacetimes [46-50] of interest in general relativity. The deduction of $K_{\mu\nu\alpha}$ for arbitrary types I, II and D is an open problem. Lanczos [7] determined his potential for weak gravitational fields, and in the corresponding calculations showed up the Dirac equation for spin-1/2 without the mass term, hence he had hoped that K_{abc} may be important in a future quantum gravity theory.

In Sec. 3, for arbitrary metrics of Petrov types III, N, O, and D (empty), we determine the Andersson-Edgar's generator [51, 52] for the Lanczos spinor.

2. Lanczos spinor

From (2) and (3) are immediate the relations:

$$o_A\, o_B = \frac{1}{2}\sigma^a{}_{A\dot{E}}\,\sigma^b{}_B{}^{\dot{E}}\, V_{ab}\,, \quad \iota_A\, \iota_B = \frac{1}{2}\sigma^a{}_{A\dot{E}}\,\sigma^b{}_B{}^{\dot{E}}\, U_{ab}\,, \quad o_A\, \iota_B + o_B\, \iota_A = -\frac{1}{2}\sigma^a{}_{A\dot{E}}\,\sigma^b{}_B{}^{\dot{E}}\, M_{ab}\,, \qquad (5)$$

then (1), (4) and (5) imply:

$$L_{ABC\dot{D}} = \frac{q}{3}[o_A\, o_B\big((\mu\, \iota_c - 3\nu\, o_c)\, o_{\dot{D}} + (3\lambda\, o_c - \pi\, \iota_c)\, \iota_{\dot{D}}\big) + \qquad (6)$$

$$+\,\iota_A\, \iota_B\big((\tau\, o_C - 3\sigma\, \iota_C)\, o_{\dot{D}} + (3\kappa\, \iota_C - \rho\, o_C)\, \iota_{\dot{D}}\big) + (o_A\, \iota_B + o_B\, \iota_A)\big((\mu\, o_C + \tau\, \iota_C)\, o_{\dot{D}} - (\pi\, o_C + \rho\, \iota_C)\, \iota_{\dot{D}}\big),$$

which can be written in compact form; in fact, it is simple to deduce the following expression [4]:

$$\nabla_{C\dot{D}}\,(o_A\, \iota_B) = o_A\, o_B\, [(\nu\, o_C - \mu\, \iota_C)\, o_{\dot{D}} + (\pi\, \iota_C - \lambda\, o_C)\, \iota_{\dot{D}}] +$$

$$+\, \iota_A\, \iota_B\, [(\sigma\, \iota_C - \tau\, o_C)\, o_{\dot{D}} + (\rho\, o_C - \kappa\, \iota_C)\, \iota_{\dot{D}}]\,, \qquad (7)$$

therefore (6) acquires the structure [2]:

$$L_{ABC}{}^{\dot{D}} = -q\, \nabla_{(C}{}^{\dot{D}}\, o_A\, \iota_{B)}\,, \qquad q = \frac{1}{2}\,, 1\,. \qquad (8)$$

The equation (8) gives the Lanczos potential in terms of the spin coefficients associated to the corresponding canonical null tetrad for the Petrov types O, N, and III; we can verify (8) via its connection with the Weyl tensor [3, 28, 29]:

$$\psi_{ABCE} \equiv \psi_0\, \iota_A \iota_B \iota_C \iota_E - 4\psi_1\, o_{(A}\iota_B\iota_C\iota_{E)} + 6\psi_2\, o_{(A}o_B\iota_C\iota_{E)} - 4\psi_3\, o_{(A}o_B o_C \iota_{E)} + \psi_4\, o_A o_B o_C o_E\,, \qquad (9)$$

$$= 2\, \nabla_{(E}{}^{\dot{D}}\, L_{ABC)\dot{D}}\,. \qquad (10)$$

We know [30] the formula $\nabla_E{}^{\dot{D}}\, \nabla_{C\dot{D}} = \frac{1}{2}\varepsilon_{CE}\, \square - \square_{EC}$, hence:

$$-\nabla_E{}^{\dot{D}}\, \nabla_{(C|\dot{D}|}\, \big(o_A\, \iota_{B)}\big) = \square_{(EC}\big(o_A\, \iota_{B)}\big); \qquad (11)$$

thus (8), (10) and (11) imply:

$$\psi_{ABCE} = 2q\, [o^F\, \psi_{F(ABC}\, \iota_{E)} + \iota^F\, \psi_{F(ABC}\, o_{E)}]\,,$$

$$= 2q\, \big[-\psi_0\, \iota_A\, \iota_B\, \iota_C\, \iota_E + 2\psi_1\, o_{(A}\iota_B\iota_C\iota_{E)} - 2\psi_3\, o_{(A}o_B o_C \iota_{E)} + \psi_4\, o_A o_B o_C o_E\big], \qquad (12)$$

whose comparison with (9) gives $q = \frac{1}{2}$ and 1 for the Petrov types O, N, and III, respectively, about the canonical tetrad [15, 21].

3. Andersson-Edgar's potential for the Lanczos spinor

In [46] was obtained the Lanczos generator for an arbitrary empty spacetime of Petrov type D, with the following Newman-Penrose (NP) components:

$$\Omega_2 = \pi\, \psi_2^{-\frac{2}{3}}\,, \qquad \Omega_6 = \mu\, \psi_2^{-\frac{2}{3}}\,, \qquad \Omega_r = 0,\ r \neq 2, 6, \tag{13}$$

in terms of the spin coefficients associated to the canonical null tetrad, hence for the type D the Lanczos spinor is given by [53]:

$$L_{ABC\dot{D}} = \psi_2^{-\frac{2}{3}} (o_A o_B \iota_C + (o_A * \iota_B) o_C)(-\mu\, o_{\dot{D}} + \pi\, \iota_{\dot{D}}). \tag{14}$$

Similarly [9]:

$$\Omega_0 = q\,\kappa\,, \qquad \Omega_3 = -q\,\lambda\,, \qquad \Omega_4 = q\,\sigma\,, \qquad \Omega_7 = -q\,\nu\,, \tag{15}$$

$$\Omega_1 = \frac{q}{3}\rho\,, \qquad \Omega_2 = -\frac{q}{3}\pi\,, \qquad \Omega_5 = \frac{q}{3}\tau\,, \qquad \Omega_6 = -\frac{q}{3}\mu\,,$$

for the types N ($q = \frac{1}{2}$) and III ($q = 1$), in the corresponding canonical tetrad, with the Lanczos spinor:

$$L_{ABC\dot{D}} = \iota_A\, \iota_B\, \iota_C (\Omega_0\, \iota_{\dot{D}} - \Omega_4\, o_{\dot{D}}) + (\iota_A\, \iota_B\, o_C + (o_A * \iota_B)\, \iota_C)\,(-\Omega_1\, \iota_{\dot{D}} + \Omega_5\, o_{\dot{D}}) + \tag{16}$$

$$+ (o_A\, o_B\, \iota_C + (o_A * \iota_B)\, o_C)\,(\Omega_2\, \iota_{\dot{D}} - \Omega_6\, o_{\dot{D}}) + o_A\, o_B\, o_C(-\Omega_3\, \iota_{\dot{D}} + \Omega_7\, o_{\dot{D}}).$$

For the type O we may employ $q = \frac{1}{2}$ or $q = 1$.

On the other hand, Andersson-Edgar [51, 52] proved that any Lanczos spinor can be generated via the relation:

$$L_{ABC\dot{D}} = \nabla^E{}_{\dot{D}}\, T_{ABCE}\,, \qquad T_{ABCE} = T_{(ABC)E}\,, \tag{17}$$

then we shall construct T_{ABCE} for the cases (14) and (16). Thus, in (17) we use the expansion:

$$T_{ABCE} = \iota_A\, \iota_B\, \iota_C (\Lambda_0\, \iota_E - \Lambda_4\, o_E) + (\iota_A\, \iota_B\, o_C + (o_A * \iota_B)\, \iota_C)\,(-\Lambda_1\, \iota_E + \Lambda_5\, o_E) + \tag{18}$$

$$+ (o_A\, o_B\, \iota_C + (o_A * \iota_B)\, o_C)\,(\Lambda_2\, \iota_E - \Lambda_6\, o_E) + o_A\, o_B\, o_C(-\Lambda_3\, \iota_E + \Lambda_7\, o_E),$$

to obtain the set of NP equations:

$$\Omega_0 = \bar{\delta}\Lambda_0 - D\Lambda_4 + (\pi - 4\alpha)\,\Lambda_0 + 3\rho\,\Lambda_1 + (2\varepsilon + \rho)\,\Lambda_4 - 3\kappa\,\Lambda_5\,,$$

$$\Omega_1 = \bar{\delta}\Lambda_1 - D\Lambda_5 - \lambda\,\Lambda_0 + (\pi - 2\alpha)\,\Lambda_1 + 2\rho\,\Lambda_2 + \pi\,\Lambda_4 + \rho\,\Lambda_5 - 2\kappa\,\Lambda_6\,,$$

$$\Omega_2 = \bar{\delta}\Lambda_2 - D\Lambda_6 - 2\lambda\,\Lambda_1 + \pi\,\Lambda_2 + \rho\,\Lambda_3 + 2\pi\,\Lambda_5 + (\rho - 2\varepsilon)\,\Lambda_6 - \kappa\,\Lambda_7\,,$$

$$\Omega_3 = \bar{\delta}\Lambda_3 - D\Lambda_7 - 3\lambda\,\Lambda_2 + (2\alpha + \pi)\,\Lambda_3 + 3\pi\,\Lambda_6 + (\rho - 4\varepsilon)\,\Lambda_7\,, \tag{19}$$

$$\Omega_4 = \Delta\Lambda_0 - \delta\Lambda_4 + (\mu - 4\gamma)\,\Lambda_0 + 3\tau\,\Lambda_1 + (2\beta + \tau)\,\Lambda_4 - 3\sigma\,\Lambda_5\,,$$

$$\Omega_5 = \Delta\Lambda_1 - \delta\Lambda_5 - \nu\,\Lambda_0 + (\mu - 2\gamma)\,\Lambda_1 + 2\tau\,\Lambda_2 + \mu\,\Lambda_4 + \tau\,\Lambda_5 - 2\sigma\,\Lambda_6\,,$$

$$\Omega_6 = \Delta\Lambda_2 - \delta\Lambda_6 - 2\nu\,\Lambda_1 + \mu\,\Lambda_2 + \tau\,\Lambda_3 + 2\mu\,\Lambda_5 + (\tau - 2\beta)\,\Lambda_6 - \sigma\,\Lambda_7\,,$$

$$\Omega_7 = \Delta\Lambda_3 - \delta\Lambda_7 - 3\nu\,\Lambda_2 + (2\gamma + \mu)\,\Lambda_3 + 3\mu\,\Lambda_6 + (\tau - 4\beta)\,\Lambda_7,$$

hence (19) implies (15) for $\Lambda_2 = -\Lambda_5 = \frac{q}{3}$, $\Lambda_r = 0$, $r \neq 2, 5$, that is:

$$T_{ABCE} = \frac{q}{3}[(o_A\, o_B\, \iota_C + (o_A * \iota_B)\, o_C)\, \iota_E - (\iota_A\, \iota_B\, o_C + (o_A * \iota_B)\, \iota_C)\, o_E], \tag{20}$$

is a generator of the Lanczos spinor (16) for arbitrary spacetimes of Petrov types O, N, and III, in the canonical null tetrad.

Let's remember that for type D vacuum geometries [3, 15]:

$$\kappa = \sigma = \lambda = \nu = 0, \qquad \psi_2 \neq 0, \qquad \psi_r = 0, \quad r \neq 2, \tag{21}$$

$$D\psi_2 = 3\rho\,\psi_2\,, \qquad \Delta\psi_2 = -3\mu\,\psi_2\,, \qquad \delta\psi_2 = 3\tau\,\psi_2\,, \qquad \bar{\delta}\psi_2 = -3\pi\,\psi_2\,,$$

then (13) is consequence from (19) for the values $\Lambda_2 = -\frac{3}{2}\Lambda_5 = \frac{3}{5}\psi_2^{-2/3}$, therefore:

$$T_{ABCE} = \frac{1}{5}\,\psi_2^{-2/3}[3(o_A\,o_B\,\iota_C + (o_A * \iota_B)\,o_C)\,\iota_E - 2(\iota_A\,\iota_B\,o_C + (o_A * \iota_B)\,\iota_C)\,o_E], \tag{22}$$

is a potential for the Lanczos spinor (14).

The construction of $L_{ABC\dot{D}}$ for arbitrary 4-spaces of Petrov types I, II, and D, is an open problem, and we consider that the equations (19) are important in such research.

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