Fuzzy System and Game Theory for Green Supply Chain

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Abstract

The main issue in the for the green supply chain strategies is the ability to find exact and practical solutions (pay off function), which can motivate the main supply chain members to choose an approach under uncertainty conditions, where, any practical solution should include different mathematical techniques and use engineering sciences to make an effectiveness relation between scientific researches and trading industries. In this paper, an optimization model using fuzzy game theory for three players is developed, which is affected by customer demands in a green supply chain. The proposed model includes a practical solution to increase the confidence level of players to choose plausible green strategy. Initially, the strategies are formulated using the game theory for three players, particularly, manufacturer, costumer and government, to be able to optimize the pay-off uncertainty conditions of demands, this can be achieved by combining computational fuzzy set with ability of sensitive analysis of related fuzzy parameters to enhance the calculations and problem solving, with focus on presenting Nash equilibrium the problem-solving part.

Keywords: Fuzzy Game theory, Nash Equilibrium, Green Supply Chain, Customer pay off function, Green strategy.

I. Introduction

Recently, the growth of green supply chain approaches and sustainable development the subject of focus and study by researchers. Traditionally, most of methods and models often were presented to conceptual justify green supply chain. The main issue is to find exact and practical solutions in order to motivate the main supply chain members to choose an approach under uncertainty conditions [1]. Any practical solution should include different mathematical techniques and use engineering sciences to make an effectiveness relation between these scientific researches and trading industries [2].

In this paper development of new combination of game fuzzy analytical model in the green supply chain strategies, to make a more plausible fuzzy game model with deployment of players, decision- making parameters of pay-off functions. In which, major parameters, and players' pay-off functions are optimized in order to create more accurate analytical results for players to make a more confident strategic decision. This model aims at the government and manufacturers abilities to analyze the effectiveness of customer's decisions, their strategies and related income/cost parameters are to change status. Furthermore, an optimization model is developed to determine the optimal cost/income parameters of players pay-off (pricing, subsidiaries ...), this optimization is developed based on customer demand. Due to the vagueness in the real-world data, then the considered fuzzy based game theory model in green supply chain looked promising, in the sense that, it can put the ground to build more practical solution in order to optimize related functions and variables. Moreover, for more comprehensive analysis, a three-player game model (government, manufacturer and customer) has been considered in green supply chain management. The final results of this work show that if the green strategies are considered by players, the game fuzzy model can provide more economic results in players' pay-off than the non-fuzzy game model.

The remaining part of the paper is organized as follows; section II, lists some of the work done in the literature. Section III, the modelling of the problem is elaborated. Section IV, modelling of the problem with applying Fuzzy Set theory concepts with sensitive analysis. Section V, details the simulation and application considered for the proposed fuzzy model with optimal solution. Section VI, Numerical analysis with case study is provided. And finally, the conclusion is drawn.

II. Literature review

Since the 1980's the topics of supply chain management as programming, practical control related to selection material, production, transportation, distribution and also collaborations among members of supply chain have always been discussed in earlier studied [1]–[4]. Recently, due to reduction of resources, increase in environmental pollutions because of commercial developments, and increase of demands, green supply chain appeared as natural response to the challenge of how to improve environmental performance and long-term economic profits [5]. Where, Green supply chain management can be defined as sets of regulations and principles in Supply Chain to minimize environmental impacts from the suppliers to end users [6]. On one hand, it can be claimed as a win-win strategy via increment of long-term economic benefits, along with a decrement of environmental impacts as explained in [7], [8]. On one hand, Green supply chain management has important effects on manufacturers, especially, as growth in opportunities and challenges of green product's progress and promotion in innovation of products [9]. On the other hand, integration of clean technology in supply chain processes has another important environmental consideration to reduce industrial pollution [10]. A previous study done in [11] showed how the life cycle model of carbon emissions can be viewed as significant aid to customers for choosing products, and how it could be seen as an environmental sign in transmitting tools for manufacturers.

While Game theory can be regarded as an important technique in supply chain, especially, when conflicts of the players appears. It is essential to assist decision makers to increase effective collaboration among each other's goals, which was considered in [12]. Practical game theory is also discussed and studied often in coordination between economic stability, and effectiveness of supply chain[13]–[16]. Although, adoption of game theory in the subject of the green supply chain are being under study [17], where, an evolutional model of game theory on how governmental penalties and subsidies affect companies' environmental performance were offered. In that study, it was suggested that governments could be more effective on environmental regulations performance by companies through penalties and subsidies. In another study done in 2009 [18], where, different game models were offered to design pricing strategies that are related to the environmental regulations. In 2011 [19], an asymmetric bargaining game model was presented, in which, a search for argumentative solutions between manufacturers and suppliers of reversed logistics under governmental tax law to properly increase the bargaining power of supply chain members was considered. Furthermore in 2012 [20], a dynamic evolutionary game model was presented to study potential strategic coordination between manufacturer and retailers, which was achieved through maximizing economic benefits, that work engendered the concept of win-win to green industry activities, where the win-win condition could be considered between environment and long-term economic benefits in any green supply chain. In another work [21], an exclusive game model was presented for the mode of industry by different brands, particularly, it considered the relationship between benefit maximizing, and carbon emissions minimizing.

In [11] a game model with two players (manufacturer and government) has been represented in general cases, where, selection of green supply chain strategies between them has been analyzed. In that model, the change of traditional manufacturing methods to green supply chain approach was considered individually, and by adding penalties and subsides to decisions of government via game theory. In 2014 [22], an evolutional game model was represented via combination of the dynamic system about Chinese manufacturers, which has been analyzed for the effect of governmental subsidies on manufacturers and customers in positive performance of green supply chain. In 2015 [23], a game theory model was presented by considering competition of one green supply chain and one regular supply chain, it examined the effects of three policies of government on the chain competition. In [24], a general methodology was represented on game matrix solving for two-players case, where, it considered triangle fuzzy numbers implementation and their general extension for similar cases according to key observations. In [25], the focus thrown on the context of government intervention, which, showed that subsidies have significantly more impact than taxes on profits and sustainability, and also shown that, the green degree of green products is indifferent toward tax rate variations.

Those studies showed important cases where general models in the green supply chain using game theory implementation. In most of the cases, two-player models, and simple approach to provide conditions of studies. As the new model of this study an integration of three major players of the chain model is presented, it makes a combination of the games and strategies more different than previous studies. Moreover, it considers the effectiveness of more various parameters in pay-off function of players. In addition, for making more practical and efficient model, fuzzy sets theory and sensitivity analysis of players' pay-off function are combined to the game modelling. Therefore, the designed model is changed from a simple model of previous studies to an effective practical-analytical model in order to achieve the realized optimal results under uncertain conditions.

III. Problem modelling

In order to be able to clarify the modelling of the problem, initially a description of main parameters have to be set out to a common ground, this includes the main income and cost parameters for the *Subsidiaries* considered in this particular problem i.e., *Government*, *Manufacturer*, and *Customer*. For the income purposes, income gained from some financial facilities such as loans, world credits…etc., can be considered as sources for income for the governments. Loans, Customs duty exemption, Tax exemption, and any other special facilities can be considered as income for manufacturers. While, special discounts, special conditions of payment, after sale services…etc., are considered as incomes for customers. The sustainable development benefits can be considered as *long-term* strategic income parameter, which can be influential in decision-making process of the players especially for government and manufacturers. However, the calculation methodology of this parameter is out of scope of this work.

Costs parameters can vary, for instance: Environmental costs may include costs of negative effects of resources consumption, air pollution, and risks to human health caused by the environmental performance of supply chain. Whereas, unemployment cost; say a certain manufacturer showed a desire to change the technology, then, government must consider probable costs of unemployment for those who may be unemployed due to this change. Subsidiary cost, which, can be the cost that a player considers as subside or decline of other players. Manufacturing technology costs, which can include maintenance, energy consumption, training…etc., to be added to manufacturing direct costs. Losing credit costs, loss international or government credit in the event of non-implementation of international green industry related requirements has to be an added cost.

For the modelling of this problem, let N be the set of players and S the set of strategies are defined as:

$$N = \{\,1, 2, 3\}$$

$$S_i = \{S_1, S_2, \dots, S_k\} \quad i \in N$$

According to general model of green supply chain and if the change of green industry level is considered (see appendix A), then the main strategies of players (variables of decision making) are described in the Table 1.

Table 1 Strategies of players

Government (1)	Manufacturer (2)	Customers (3)
Passive	Maintain technology	Demand Reduction
Supervision	Move to tolerable level	Passive
Subsidiary system	Move to acceptable level	Demand increase

Based on the above relations and variables, the pay-off function of players can be formulated as:

$$\forall S_{nm}\ ,\quad P_{nm} = \sum_{i=1}^{k}(I_{ni})_{S_{nm}} - \sum_{j=1}^{l}(C_{nj})_{S_{nm}} \tag{1}$$

$$n = 1, 2, 3$$

$$m = 1, 2, 3$$

$$i = 1, 2, \dots, k$$

$$j = 1, 2, \dots, l$$

Where, S_{nm} is the strategy (m) that is chosen by player (n), P_{nm} is pay-off of player (n) if strategy (m) is chosen, $(I_{ni})_{S_{nm}}$ is the income (i) of player (n) if strategy (m) is chosen, $(C_{nj})_{S_{nm}}$ is cost (j) of player (n) if strategy (m) is chosen, n is the number of players (1: government, 2: manufacturer, 3: customer), m is the number of players' strategy, i is the number of players' income, and j is the number of players' cost. Where details of income and cost parameters for each player are described in Table 2 and
Table **3**:

Table 2 The income parameters of pay-off functions

Government (1)	Manufacturer (2)	Customers (3)
Penalties	Sale	Subsidiary
Int. Subsidiary	Subsidiary	Sustainable Development Benefit
Sustainable Development Benefit	Sustainable Development Benefit	

Table 3 The cost parameters of pay-off functions

Government (1)	Manufacturer (2)	Customers (3)
Supervision	Investment	Purchasing
Subsidiary	Production	Environment
Environment	Subsidiary	
Losing Credit	Overhead	
Unemployment	Penalties	

If the parameter; α is defined as, the penalty rate of manufacturer according to percentage of income of selling products, the parameter β is the rate of increase in price of products in case of technology change, γ is the manufacturer's percent of share from incentive budget of government, and ε is the customers' cost if increase of demand is seen in state of no change of technology. Then according to eq.(1), Table 2 and
Table **3**, the players pay-off functions could be formulated as:

$$P_1 = (I_{11} + I_{12} + I_{13}) - (C_{11} + C_{12} + C_{13} + C_{14} + C_{15})$$

Where, $I_{11} = \alpha\,(1+\beta)(1+\rho)I_{21}$, and $C_{12} = \gamma\,C_{12} + (1-\gamma)C_{12}$

then: $P_1 = [\alpha(1+\beta)(1+\rho)I_{21} + I_{12} + I_{13}] - [(C_{11} + \gamma C_{12} + (1-\gamma)C_{12} + C_{13} + C_{14} + C_{15})]$ (2)

$P_2 = [\alpha(1+\beta)(1+\rho)I_{21} + I_{22} + I_{23}] - (C_{21} + \varphi(1+\rho)C_{22} + C_{23} + C_{24} + C_{25})$

Where $I_{22} = \gamma C_{12}$, and $C_{25} = \alpha(1+\beta)(1+\rho)I_{21}$

$P_2 = [(1+\beta)(1+\rho)I_{21} + \gamma C_{12} + I_{23}] - (C_{21} + \varphi(1+\rho)C_{22} + C_{23} + C_{24} + \alpha(1+\beta)(1+\rho)I_{21})$

then: $P_2 = [(1-\alpha)(1+\beta)(1+\rho)I_{21} + \gamma C_{12} + I_{23}] - (C_{21} + \varphi(1+\rho)C_{22} + C_{23} + C_{24})$ (3)

$P_3 = (I_{31} + I_{32}) - (C_{31} + C_{32})$, where $I_{31} = (1-\gamma)C_{12}$, and $C_{31} = (1+\beta)I_{21} + \beta\rho I_{21} + \varepsilon = (1+\beta+\beta\rho)I_{21} + \varepsilon$

then: $P_3 = [(1-\gamma)C_{12} + I_{32}] - [(1+\beta+\beta\rho)I_{21} + \varepsilon + C_{32}]$ (4)

Then, the games matrix of three players is modelled as shown in tables 4, 5 and 6. Where, the detailed analysis of the game is described in appendix B.

Table 4 Game matrix- Customer strategy = S31

	Manufacturer strategy= $\boldsymbol{S_{21}}$	Manufacturer strategy= $\boldsymbol{S_{22}}$	Manufacturer strategy= $\boldsymbol{S_{23}}$
Government strategy= $\boldsymbol{S_{11}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{11}} - \sum_{j=1}^{5}(C_{1j})_{S_{11}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{21}} - \sum_{j=1}^{5}(C_{2j})_{S_{21}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{31}} - \sum_{j=1}^{2}(C_{3j})_{S_{31}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{11}} - \sum_{j=1}^{5}(C_{1j})_{S_{11}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{22}} - \sum_{j=1}^{5}(C_{2j})_{S_{22}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{31}} - \sum_{j=1}^{2}(C_{3j})_{S_{31}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{11}} - \sum_{j=1}^{5}(C_{1j})_{S_{11}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{23}} - \sum_{j=1}^{5}(C_{2j})_{S_{23}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{31}} - \sum_{j=1}^{2}(C_{3j})_{S_{31}}$
Government strategy= $\boldsymbol{S_{12}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{12}} - \sum_{j=1}^{5}(C_{1j})_{S_{12}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{21}} - \sum_{j=1}^{5}(C_{2j})_{S_{21}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{31}} - \sum_{j=1}^{2}(C_{3j})_{S_{31}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{12}} - \sum_{j=1}^{5}(C_{1j})_{S_{12}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{22}} - \sum_{j=1}^{5}(C_{2j})_{S_{22}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{31}} - \sum_{j=1}^{2}(C_{3j})_{S_{31}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{12}} - \sum_{j=1}^{5}(C_{1j})_{S_{12}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{23}} - \sum_{j=1}^{5}(C_{2j})_{S_{23}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{31}} - \sum_{j=1}^{2}(C_{3j})_{S_{31}}$
Government strategy= $\boldsymbol{S_{13}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{13}} - \sum_{j=1}^{5}(C_{1j})_{S_{13}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{21}} - \sum_{j=1}^{5}(C_{2j})_{S_{21}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{31}} - \sum_{j=1}^{2}(C_{3j})_{S_{31}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{13}} - \sum_{j=1}^{5}(C_{1j})_{S_{13}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{22}} - \sum_{j=1}^{5}(C_{2j})_{S_{22}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{31}} - \sum_{j=1}^{2}(C_{3j})_{S_{31}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{13}} - \sum_{j=1}^{5}(C_{1j})_{S_{13}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{23}} - \sum_{j=1}^{5}(C_{2j})_{S_{23}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{31}} - \sum_{j=1}^{2}(C_{3j})_{S_{31}}$

Table 5 Game matrix- Customer strategy = S32

	Manufacturer strategy= S_{21}	Manufacturer strategy= S_{22}	Manufacturer strategy= S_{23}
Government strategy= S_{11}	$\sum_{i=1}^{3}(I_{1i})_{S_{11}}-\sum_{j=1}^{5}(C_{1j})_{S_{11}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{21}}-\sum_{j=1}^{5}(C_{2j})_{S_{21}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{32}}-\sum_{j=1}^{2}(C_{3j})_{S_{32}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{11}}-\sum_{j=1}^{5}(C_{1j})_{S_{11}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{22}}-\sum_{j=1}^{5}(C_{2j})_{S_{22}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{32}}-\sum_{j=1}^{2}(C_{3j})_{S_{32}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{11}}-\sum_{j=1}^{5}(C_{1j})_{S_{11}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{23}}-\sum_{j=1}^{5}(C_{2j})_{S_{23}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{32}}-\sum_{j=1}^{2}(C_{3j})_{S_{32}}$
Government strategy= S_{12}	$\sum_{i=1}^{3}(I_{1i})_{S_{12}}-\sum_{j=1}^{5}(C_{1j})_{S_{12}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{21}}-\sum_{j=1}^{5}(C_{2j})_{S_{21}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{32}}-\sum_{j=1}^{2}(C_{3j})_{S_{32}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{12}}-\sum_{j=1}^{5}(C_{1j})_{S_{12}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{22}}-\sum_{j=1}^{5}(C_{2j})_{S_{22}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{32}}-\sum_{j=1}^{2}(C_{3j})_{S_{32}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{12}}-\sum_{j=1}^{5}(C_{1j})_{S_{12}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{23}}-\sum_{j=1}^{5}(C_{2j})_{S_{23}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{32}}-\sum_{j=1}^{2}(C_{3j})_{S_{32}}$
Government strategy= S_{13}	$\sum_{i=1}^{3}(I_{1i})_{S_{13}}-\sum_{j=1}^{5}(C_{1j})_{S_{13}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{21}}-\sum_{j=1}^{5}(C_{2j})_{S_{21}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{32}}-\sum_{j=1}^{2}(C_{3j})_{S_{32}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{13}}-\sum_{j=1}^{5}(C_{1j})_{S_{13}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{22}}-\sum_{j=1}^{5}(C_{2j})_{S_{22}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{32}}-\sum_{j=1}^{2}(C_{3j})_{S_{32}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{13}}-\sum_{j=1}^{5}(C_{1j})_{S_{13}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{23}}-\sum_{j=1}^{5}(C_{2j})_{S_{23}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{32}}-\sum_{j=1}^{2}(C_{3j})_{S_{32}}$

Table 6 Game matrix- Customer strategy = S33

	Manufacturer strategy= S_{21}	Manufacturer strategy= S_{22}	Manufacturer strategy= S_{23}
Government strategy= S_{11}	$\sum_{i=1}^{3}(I_{1i})_{S_{11}}-\sum_{j=1}^{5}(C_{1j})_{S_{11}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{21}}-\sum_{j=1}^{5}(C_{2j})_{S_{21}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{33}}-\sum_{j=1}^{2}(C_{3j})_{S_{33}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{11}}-\sum_{j=1}^{5}(C_{1j})_{S_{11}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{22}}-\sum_{j=1}^{5}(C_{2j})_{S_{22}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{33}}-\sum_{j=1}^{2}(C_{3j})_{S_{33}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{11}}-\sum_{j=1}^{5}(C_{1j})_{S_{11}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{23}}-\sum_{j=1}^{5}(C_{2j})_{S_{23}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{33}}-\sum_{j=1}^{2}(C_{3j})_{S_{33}}$
Government strategy= S_{12}	$\sum_{i=1}^{3}(I_{1i})_{S_{12}}-\sum_{j=1}^{5}(C_{1j})_{S_{12}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{21}}-\sum_{j=1}^{5}(C_{2j})_{S_{21}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{33}}-\sum_{j=1}^{2}(C_{3j})_{S_{33}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{12}}-\sum_{j=1}^{5}(C_{1j})_{S_{12}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{22}}-\sum_{j=1}^{5}(C_{2j})_{S_{22}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{33}}-\sum_{j=1}^{2}(C_{3j})_{S_{33}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{12}}-\sum_{j=1}^{5}(C_{1j})_{S_{12}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{23}}-\sum_{j=1}^{5}(C_{2j})_{S_{23}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{33}}-\sum_{j=1}^{2}(C_{3j})_{S_{33}}$
Government strategy= S_{13}	$\sum_{i=1}^{3}(I_{1i})_{S_{13}}-\sum_{j=1}^{5}(C_{1j})_{S_{13}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{21}}-\sum_{j=1}^{5}(C_{2j})_{S_{21}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{33}}-\sum_{j=1}^{2}(C_{3j})_{S_{33}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{13}}-\sum_{j=1}^{5}(C_{1j})_{S_{13}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{22}}-\sum_{j=1}^{5}(C_{2j})_{S_{22}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{33}}-\sum_{j=1}^{2}(C_{3j})_{S_{33}}$	$\sum_{i=1}^{3}(I_{1i})_{S_{13}}-\sum_{j=1}^{5}(C_{1j})_{S_{13}}$ $\sum_{i=1}^{3}(I_{2i})_{S_{23}}-\sum_{j=1}^{5}(C_{2j})_{S_{23}}$ $\sum_{i=1}^{2}(I_{3i})_{S_{33}}-\sum_{j=1}^{2}(C_{3j})_{S_{33}}$

IV. Modelling with fuzzy sets and sensitive analysis

As mentioned earlier, due to nature to the problem, the decision-making variables and major uncertain parameters can be considered as fuzzy sets [26]. The basic idea of fuzzy sets was raised by Zadeh in 1965 [27] to provide a perfect tool for modelling imprecise and vague concepts, especially knowledge containing linguistic descriptions. Moreover, it is shown to be a very powerful tool for modelling imprecise dependencies (Rules) among various system inputs and/or outputs. The origin of the imprecision usually comes from experts' knowledge, or representation of information extended from inherently (imprecise) data.

One may ask, what fuzzy set is? Initially to answer this question, we first need to recap a bit about classical (crisp) set, here the belongingness of any element to a certain set can be described by the following characteristic function:

$$m_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases} \Rightarrow m_A(x) \in \{0,1\}$$

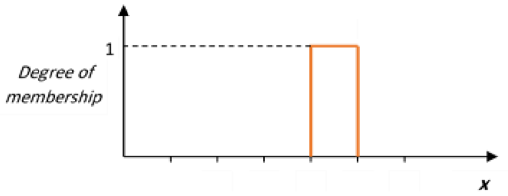


Fig. 1 Crisp Set

Fig. 1 depicts the graphical representation of the ordinary characteristic function exemplified through the above equation, which emphasizes a binary state: either an element belongs to that set or does not belong at all.

On the other hand, a fuzzy set can describe this belongingness relation with various states, where the ordinary set $m_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases} \Rightarrow m_A(x) \in \{0,1\}$) is only a particular case; namely, the following membership function μ_A of fuzzy set $\boldsymbol{A}$ takes values in interval [0,1] instead of {0,1}:

$$\mu_A(x) \in [0,1]$$

It assigns a degree of membership to each element x of universe of discourse $\boldsymbol{X}$ rather than (exists or not exists). This degree of membership can be any value within [1, 0] interval. Fig. 2 provides an example of fuzzy set $\boldsymbol{A}$.

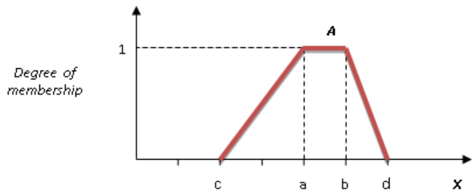


Fig. 2 Fuzzy Set A

We can see that elements between $\boldsymbol{a}$ and $\boldsymbol{b}$ strongly belong to the fuzzy set with degree of 1, any element less than $\boldsymbol{a}$ or greater than $\boldsymbol{b}$ belongs to the same set with gradually and linearly decaying value (smaller than 1) depending on its occurrence in the set [28].

Fuzzy System is the broad title that can be used to describe any system that deals with imprecise, uncertain or vague knowledge. Accurately those systems could be referred as Fuzzy inference systems. Basically, a fuzzy inference system composed of five functional blocks Fig. 3:

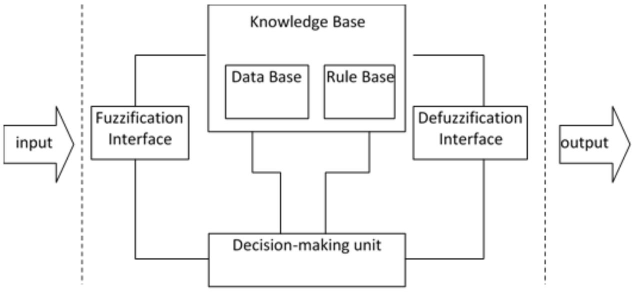


Fig. 3 Fuzzy Inference System

- *Rulebase*; containing a number of fuzzy if-then rules, of the form: "$if\ A\ then\ B$", for example:

$$If\ \text{“}temperature\ is\ low\text{”}\ then\ \text{“}heater\ is\ ON\text{”}$$

- *Database*; defines the membership functions of the fuzzy sets used in the fuzzy rules.

Usually, the rule base and the database are jointly referred to as knowledge base.

- *Decision-making unit*; performs the inference operations on the rules.

This is main interest in fuzzy systems, where, it helps us drawing conclusions of combining certain rules, drawing decision based on certain fuzzy knowledge given in the rule base. In another words, if we have a fuzzy set with certain degree of membership $\mu_A(x)$, and given that $A \rightarrow B$, we need to achieve a membership degree $\mu_B(y)$ of output variable y to fuzzy set B. But we know the degree of membership of medium temperature, we know that, still we can draw weak conclusion based on the available overlapping between "low" and "$medium$" fuzzy sets.

- *Fuzzification*; transforms the crisp inputs into degrees of match with linguistic values.
- *Defuzzification*; transforms the fuzzy results of the inference into crisp output.

Coming back to our problem, then, we can claim that, the model and the achieved results are definitely going to be more efficient and effective in order to achieve optimal result. The consideration of fuzzy set can increase of confidence level in decision making for each of the players to change the status. Among of the all existing variables and parameters in the problem, variables of customers' decision-making said to be of more importance. In real conditions, if the variables of customer decision-making and its major parameters in pay-off function are considered as the certain states, then the exact optimal results will not be achieved. In fact, if the rate of considered subsidiaries for customers is not interested enough (not zero, necessarily), their decisions will not be changed, even if their pay-off function is higher than what it was before. Likewise, some other parameters (including interests resulting from sustainable development), if it is not reflecting the effective value for customers, this parameter cannot affect positively the customers' pay-off function. Accordingly, and based on these notices, we can have:

$$if\ I_{31} + I_{32} - \partial C_{31} < B_1\ then\ S_{31}$$

$$if\ I_{31} + I_{32} - \partial C_{31} < B_1\ then\ S_{32}$$

$$if\ I_{31} + I_{32} - \partial C_{31} < B_1\ then\ S_{33}$$

Where B_1 and B_2 are attraction levels of sum of incomes resulting from facilities and long-term interests of sustainable development for decision-making and $B_2 > B_1$. In fact, we cannot expect that all players (especially customers), have a theoretical and precise view to details and exactly, decide according to result of *pay-off* function with any achieved value. Therefore, customers will analyze their pay-off function results qualitatively, not numerically.

Where fuzzy theory sets can be very applicable to this kind of analysis. If the parameters of customer's pay-off function are considered as the fuzzy sets, customer behavior will be predictable because of parameters change. This effective model motivates the government and manufacturer to take the optimized amount for their related parameters (such as subsidiaries or production price) to change customer demand positively or keep it constant. This case will be explained more in the following paragraphs and as part of sensitivity analysis of customer behavior.

To make the game model more applicable, to increase effectiveness of problem solving method, and to achieve optimal results, a fuzzy model is planned by the following pattern, which is adapted from [29]:

Step 1: determination of fuzzy parameters and variables:

Fuzzy Parameters and variable: $C_{12}, C_{23}, \partial C_{31}, I_{32}, P_3$ (5)

Step 2: planning fuzzy logical relations: $C_{12}, C_{23}, \partial C_{31}, I_{32} \rightarrow \partial P_3$ (6)

Step 3: planning linguistic variables of fuzzy parameters.

Verbal variables set of each parameter can be like below: $X = \{x_1, x_2, \dots, x_n\}$ (7)

In this research, according to analysis of the type of parameters, these variables will be defined by the following fuzzy sets:

$$C_{12} = \{Unfavorable\ (UF), Favorite\ (F)\}$$

$$C_{23} = \{Unfavorable\ (UF), Favorite\ (F)\}$$

$$\partial C_{31} = \{Low\ (L), High\ (H)\}$$

$$I_{32} = \{Low\ (L), High\ (H)\}$$

$$P_3 = \{Negative\ (N), Zero\ (Z), Positive\ (P)\} \tag{8}$$

According to above sets: negative linguistic variable in pay-off function means negative reaction to change demand, zero means passive decision making and, positive means positive reaction to change demand.

Step 4: determination of membership function for every linguistic variable of fuzzy set as:

$$\mu_{C_{12}(UF)} = \mu_1(x)$$

$$\mu_{C_{12}(F)} = \mu_2(x)$$

$$\mu_{C_{23}(UF)} = \mu_3(x)$$

$$\mu_{C_{23}(F)} = \mu_4(x)$$

$$\mu_{\partial C_{31}(L)} = \mu_5(x)$$

$$\mu_{\partial C_{23}(H)} = \mu_6(x)$$

$$\mu_{I_{32}(L)} = \mu_7(x)$$

$$\mu_{I_{32}(H)} = \mu_8(x)$$

$$\mu_{P_3(N)} = \mu_9(x)$$

$$\mu_{P_3(Z)} = \mu_{10}(x)$$

$$\mu_{P_3(P)} = \mu_{11}(x) \tag{9}$$

The X-axis of membership function of customers pay-off function represents the value of ascending change of customers pay-off function as the percentage.

Step 5: defining the fuzzy inference: Among the available fuzzy inference systems, Mamadani inference [29] is selected to solve fuzzy problem, where: the inputs: C_{12},C_{23},∂C_{31}, and I_{32}, and the output is P_3, then Rule_i can be defined as:

$$if\ C_{12i}, C_{23i}, \partial C_{31i}, I_{32i}\ then\ P_{3i}$$

$$a_i = \min(\mu_i(C_{12}), \mu_i(C_{23}), \mu_i(\partial C_{31}), \mu_i I_{32})$$

$$\mu_i = \min(a_i, \mu_{P_{3i}})$$

The overall system output can be calculated using the union operator as:

$$\mu = \cup\ \mu_i \tag{10}$$

Where μ is member function of fuzzy parameters, a_i is the alpha cut amount for fuzzy parameters. Where finally the Defuzzification process should take place.

Step 6: planning logical relation: Pattern of rational relations should be determined between the result and customers' decisions to anticipate costumers' final decision. This pattern could be generally defined as:

$$If\ 0 \leq\ \theta \leq l\ \rightarrow\ S_{31}$$

$$If\ \ l <\ \theta \leq m\ \ \rightarrow S_{32}$$

$$If\ m <\ \theta \leq h\ \ \rightarrow S_{33}$$

$$0 < l < m < h \tag{11}$$

Where θ is the fuzzy result of the problem, l is the minimum level of qualitative value of Pay-off function change for customer that shows dissatisfaction and possibility of decrease in demand, m is the medium level of qualitative value of Pay-off function change for customer that shows neutralism and demand rate is unchanged, and h is the maximum level of qualitative value of Pay-off function change for customer that shows satisfaction and possibility of increase in demand. This pattern analysis outperforms the sensitivity analysis of customers pay-off function which is described in the next part.

Step 7: sensitivity analysis of customers' pay-off function and selection of optimal parameters:

To analyze customers, pay-off function with different combinations of input parameters, we assume that a, b, c and, d are four input parameters with different values. In this case there will be $a\ \times b \times c\ \times d$ possible states for sensitivity analysis of customers decision and pay-off function. In a collaborative game and after achieving results of sensitivity analysis for customers' behavior, government and manufacturer undertake the optimized value of governmental incentive facilities parameters for customers, the optimized value of incentive facilities of manufacturer, and the optimized value of increase in products price due to change technology levels. This combined fuzzy game model optimizes the players pay-off based on prediction of customers' strategies. Which aims at the government and manufacturer to achieve the optimal cost parameters or price of products, where by changing related fuzzy parameters (such as incentive cost, facilities, price of product, ...), this may result to avoid extra costs by prediction of customers' strategies.

V. Simulation of the proposed fuzzy model and optimal solution

To evaluate the proposed model, a Matlab program was constructed to compute the defuzzified output of this fuzzy model, the results as extracted to Microsoft excel sheet as shown in Fig. 4. Which contains the major parts of the system, first the input vector, Rule Base for the inference system which consists of set of 16 if-then rules as explained in step5, the membership value for each rule with respect to the input vector, the aggregated membership function value for the output, and finally computing the defuzzified result, which will be used by experts to make the decision. According to the example in Fig. 4, if $I_{32} = 0.5, \partial C_{31} = 1.5, C_{23} = 3$, and $C_{12} = 2$, then, the computed final answer is 8% which reflects the customers conceive to change in the related pay-off function. Of course, this 8% is not only a quantitative value, but also a qualitative data that we can realize whether this is enough for decision making of customers or not.

The optimal model of decision-making problem for the players can be computed as:

$$\max_{s_i \in S_i} u_i \ (S_i, S_{-i}) \tag{12}$$

Games are seeking of the optimal pay-off function for player (i) in front of combination of strategies related to this player strategy (S_i) with other strategies of players except player$(S_{-i})i$. Nash Equilibrium [27], [28] is approved as the optimal result for this type of problems, where Nash stated that "an n-tuples such that each player's mixed strategy maximizes his pay-off if the strategies of the others are held fixed. Thus each player's strategy is optimal against those of the others" [32].

	C_{12}	C_{23}	∂C_{31}	I_{32}
Input Data:	2	3	1.5	0.5

						μ Result for inputs				
						C_{12}	C_{23}	∂C_{31}	I_{32}	MIN
R1:	U	U	H	L	N	0	0	1	1	0
R2:	U	U	H	H	N	0	0	1	0	0
R3:	U	U	L	L	Z	0	0	0	1	0
R4:	U	U	L	H	Z	0	0	0	0	0
R5:	U	F	H	L	N	0	1	1	1	0
R6:	U	F	H	H	Z	0	1	1	0	0
R7:	U	F	L	L	Z	0	1	0	1	0
R8:	U	F	L	H	Z	0	1	0	0	0
R9:	F	U	H	L	N	1	0	1	1	0.25
R10:	F	U	H	H	Z	1	0	1	0	0.25
R11:	F	U	L	L	Z	1	0	0	1	0.25
R12:	F	U	L	H	Z	1	0	0	0	0.25
R13:	F	F	H	L	Z	1	1	1	1	0.5
R14:	F	F	H	H	Z	1	1	1	0	0.25
R15:	F	F	L	L	P	1	1	0	1	0.33
R16:	F	F	L	H	P	1	1	0	0	0.25

	P_C Rate (%)																
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
μ Result for Rule1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule9	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.167	0	0	0	0	0	0	0	0
μ Result for Rule10	0	0	0	0	0.2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	0	0	0	0
μ Result for Rule11	0	0	0	0	0.2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	0	0	0	0
μ Result for Rule12	0	0	0	0	0.2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	0	0	0	0
μ Result for Rule13	0	0	0	0	0.2	0.4	0.5	0.5	0.5	0.5	0.5	0.25	0	0	0	0	0
μ Result for Rule14	0	0	0	0	0.2	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	0	0	0	0
μ Result for Rule15	0	0	0	0	0	0	0	0	0.2	0.333	0.333	0.333	0.333	0.333	0.333	0	0
μ Result for Rule16	0	0	0	0	0	0	0	0	0	0.25	0.25	0	0	0.25	0.25	0.25	0.25
Membership Degree Function Result (max μ_i)	0.25	0.25	0.25	0.25	0.25	0.4	0.5	0.5	0.5	0.5	0.5	0.333	0.333	0.333	0.333	0.25	0.25
Final Result (Deffuzification)	8%																

Fig. 4 Program Sample results

In our case, there is a combination of strategies among all combinations in game model at least that players are not interested to change that in a logical condition. Since the game is a dynamic with the complete information, if the Nash equilibrium is iteratively earned more than once, then the optimal result would be achieved via complete equilibrium or backward Nash equilibrium[31]. This optimal result can be achieved by the analysis of the resultant Nash equilibrium. One of the techniques, which can be used to obtain this result is by converting the original game model to a secondary game model. Finally, gaining final result by all results of the secondary game. In the next part, Nash Equilibrium and Backward Nash Equilibrium will be elaborated for two states of game modelling and numerical analysis, first using the non-fuzzy version, and second using combination of fuzzy model to the problem results.

VI. Numerical analysis

Taking numerical example can ease the description of the process. Assume a steel making company is going to study and review to change production technology of its pelletizing plant in order to reduce greenhouse and flow gas of output of this factory. After 6 months study by the R&D department of this company, the investment, income, cost and other requirement parameters are determined as Table 7. Accordingly, a problem is constructed to test the performance of the mentioned model and its solving method.

Table 7 parameters and assumptions data

$n = 10$ (years)	$i = 10\%$	$\alpha = 0.03$
$\beta_1 = 0.02$	$\beta_2 = 0.03$	$\rho_1 = -0.2$
$\rho_2 = 0.1$	$I_{12} = 0.45\, C_{12}$	$I_{13(A)} = 2m\$$
$I_{13(B)} = 5m\$$	$C_{11} = 1.25m\$$	$\gamma C_{12} = 0.40\, C_{21}$
$(1-\gamma)C_{12} = [0,2.5]\ m\$$		$C_{13(A)} = 5m\$$
$C_{13(B)} = 2m\$$	$C_{14(A)} = 3m\$$	$C_{14(B)} = 1m\$$
$C_{15(A)} = 1m\$$	$C_{15(B)} = 2m\$$	I_{21}: 65 $m\$$
$I_{23(A)} = 1m\$$	$I_{23(B)} = 3m\$$	$C_{21(A)} = 40m\$$
$C_{21(B)} = 60m\$$	$C_{22(A)} = 50m\$$	$C_{22(B)} = 48m\$$
$C_{22(C)} = 47m\$$	$C_{23(A)} = [0,5]m\$$	$C_{24(A)} = 3m\$$
$C_{24(B)} = 1m\$$	$C_{24(C)} = 0.65m\$$	$\partial C_{22} = 0.5\, \rho * C_{22}$
$I_{32(A)} = 0.5m\$$	$I_{32(B)} = 1m\$$	$C_{32(A)} = 3m\$$
$C_{32(B)} = 1m\$$	$\varepsilon = 0.01\ m\$$	

The following assumptions can be made to solve this problem:

In the non-fuzzy model, government and manufacturer, in technology change, tolerable level use half of subsidiary budget for customers and in change to the acceptable level use maximum of subsidiary budget for customers.

In the non-fuzzy model, manufacturer considers the maximum of product price.

In the state of reduction of customer demand, production purchasing cost parameter for the loyal customers is given based on new rate and for the missed customer is based on production purchasing cost in intolerable level.

Subsidiary costs of government and manufacturer for customers are floating according to their demand.

As the same way, customer incomes from facilities and sustainable development are floating according to their demand.

In this defined model, customer is considered as a set.

The symbols of A, B and C are respectively, three levels of intolerable, tolerable and acceptable in environment performance.

For integration of calculations, costs of investment for technology change is converted to annual invariable costs using the following formula:

$$A = P(A/P\,, i\,\%\,, n) \rightarrow \; A = P(A/P\,, 10\%, 10)$$

The problem solving in non-fuzzy model is detailed in appendix C. In the following, the problem is modelled and resolved in fuzzy case, and they are compared together at the end.

According to the relations described earlier from eq. (5-9), the results of membership function are shown in Fig. 5, while the detailed mathematical values of membership functions are given in appendix D.

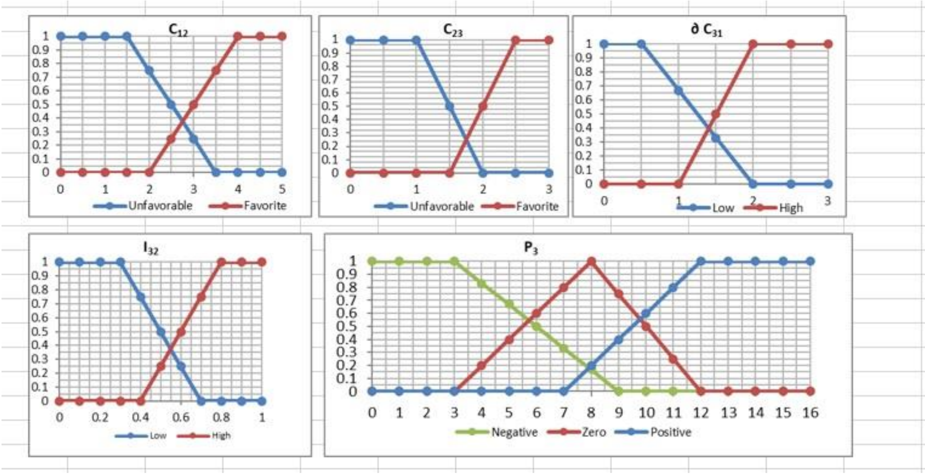
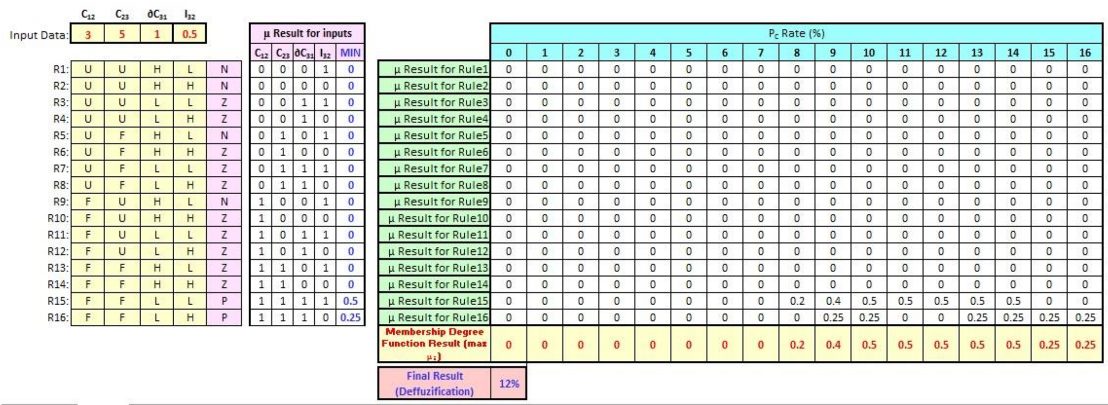


Fig. 5 The results membership functions



	C_{12}	C_{23}	∂C_{31}	I_{32}
Input Data:	3	5	1	0.5

						μ Result for inputs				
						C_{12}	C_{23}	∂C_{31}	I_{32}	MIN
R1:	U	U	H	L	N	0	0	0	1	0
R2:	U	U	H	H	N	0	0	0	0	0
R3:	U	U	L	L	Z	0	0	1	1	0
R4:	U	U	L	H	Z	0	0	1	0	0
R5:	U	F	H	L	N	0	1	0	1	0
R6:	U	F	H	H	Z	0	1	0	0	0
R7:	U	F	L	L	Z	0	1	1	1	0
R8:	U	F	L	H	Z	0	1	1	0	0
R9:	F	U	H	L	N	1	0	0	1	0
R10:	F	U	H	H	Z	1	0	0	0	0
R11:	F	U	L	L	Z	1	0	1	1	0
R12:	F	U	L	H	Z	1	0	1	0	0
R13:	F	F	H	L	Z	1	1	0	1	0
R14:	F	F	H	H	Z	1	1	0	0	0
R15:	F	F	L	L	P	1	1	1	1	0.5
R16:	F	F	L	H	P	1	1	1	0	0.25

	P_C Rate (%)																
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
μ Result for Rule1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ Result for Rule15	0	0	0	0	0	0	0	0	0.2	0.4	0.5	0.5	0.5	0.5	0.5	0	0
μ Result for Rule16	0	0	0	0	0	0	0	0	0	0.25	0.25	0	0	0.25	0.25	0.25	0.25
Membership Degree Function Result (max μi)	0	0	0	0	0	0	0	0	0.2	0.4	0.5	0.5	0.5	0.5	0.5	0.25	0.25
Final Result (Deffuzification)	12%																

Fig. 6 Results of the fuzzy inference for the problem

The results of eq. (10) and also the previous step, with combination of various parameters along with the results of fuzzy problem (costumers' variable) is calculated. And, a single calculation is shown in Fig. 6 and in table 8 some results are presented which is calculated by this program.

According to the relation 12, rational relations between results of fuzzy problem and customers' decision making are determined as follows:

$$if\ 0 \leq \theta \leq 5\% \quad \rightarrow S_{31}$$

$$if\ 5\% < \theta < 10\% \quad \rightarrow S_{32}$$

$$if\ 10 \leq \theta \quad \rightarrow S_{33}$$

If different combinations of input data in the presented model in Fig. 5 are put and analyzed with sensitivity analysis of customers' pay-off function, then, by considering its adaption with above relation, optimal parameters of the problem will be as follows:

In technology change to the tolerable or acceptable level, if government is not going to use incentive facilities, then optimal results are calculated as follows:

$$C_{12} = 0, C_{23} = 0, \partial C_{31} = 1\% , I_{32} = 0.5, \partial P_3 = 8\%$$

In technology change to the tolerable level, if government is going to use incentive facilities, optimal results described as follows: $C_{12} = 1.5, C_{23} = 0, \partial C_{31} = 1\% , I_{32} = 0.5, \partial P_3 = 8\%$

In technology change to the acceptable level, if government is going to use incentive facilities, optimal results are: $C_{12} = 3, C_{23} = 3, \partial C_{31} = 1.5\% , I_{32} = 1, \partial P_3 = 10\%$

Where ∂P_3 shows increase in Customers' pay-off function than the previous level in that. Some results of fuzzy problem solving and sensitivity analysis of Customers' pay-off function are in Table 8:

Table 8 sensitivity analysis of Customers' pay-off function

C_{12}	C_{23}	∂C_{31}	I_{32}	P_3
0.0	0.0	2.0	0.5	4.0%
1.0	4.0	1.5	0.5	5.0%
3.0	2.5	2.5	0.5	5.0%
3.0	3.0	2.5	0.5	6.0%
2.0	3.0	3.0	0.5	6.0%
3.0	2.5	1.5	0.5	7.0%
2.0	2.5	1.5	0.5	7.0%
0.0	1.0	0.0	0.5	8.0%
0.0	0.0	1.0	0.5	8.0%
1.5	0.0	1.0	0.5	8.0%
2.5	2.5	1.0	0.5	9.0%
3.0	3.0	1.5	1.0	10.0%
3.0	4.0	1.5	0.5	10.0%
3.0	3.0	1.0	0.5	11.0%
3.0	3.0	0.0	0.5	11.0%
3.0	5.0	1.0	0.5	12.0%

According to above, the matrices of problem's games are given in Table 9.

Table 9 results of fuzzy games model- Customer Strategy= S31

	S_{21}	S_{22}	S_{23}
S_{11}	-8, 4, -68	-3, 2.82, -66	2, 2.81, -64.7
S_{12}	-7.7, 2.4, -68	-3, 2.82, -66	2, 2.81, -64.7
S_{13}	-7.7, 2.4, -68	-2.7, 6.1, -65	0.1, 5.55, -59.4

Table 10 and
Table **11**, where the optimal result of each element is calculated by sensitive analysis on fuzzy program. The final optimal result of problem is achieved by Nash Equilibrium and Backward Nash Equilibrium according to formulas (2, 3, 4 and 12), and tables 4, 5 and 6.

Table 10 results of fuzzy games model- Customer Strategy= S_{32}

	S_{21}	S_{22}	S_{23}
S_{11}	-8, 12, -68	-3, 11.1, -66.2	2, 10.6, -64.7
S_{12}	-7.3, 10, -68	-3, 11.1, -66.2	2, 10.6, -64.7
S_{13}	-7.3, 10, -68	-3, 14.4, -66	-0.5, 14, -59

Table 11 results of fuzzy games model- Customer Strategy= S_{33}

	S_{21}	S_{22}	S_{23}
S_{11}	-8, 16, -68.01	-3, 15.3, -66.2	2, 15.4, -64.7
S_{12}	-7.1, 13.8, -68.01	-3, 15.3, -66.2	2, 15.4, -64.7
S_{13}	-7.1, 13.8, -68.1	-3, 18.6, -65	-0.8, 15.7, -57.1

According to the above matrices, the optimal result in fuzzy model is determined as below:

$$N(G)_1 = (S_{13}, S_{22}, S_{31}) = (-2.7, 6.1, -65)$$

$$N(G)_2 = (S_{13}, S_{22}, S_{33}) = (-3, 18.6, -65)$$

$$SPE = (S_{13}, S_{22}, S_{33}) = (-3, 18.6, -65) \rightarrow \text{optimal result in the game fuzzy model}$$

According to Appendix C (the problem solving by non-fuzzy model), the optimal result is as follows:

$$N(G)_1 = (S_{12}, S_{21}, S_{31}) = (-7.7, 2.4, -68)$$

$$N(G)_2 = (S_{12}, S_{21}, S_{32}) = (-7.3, 10, -68)$$

$$N(G)_3 = (S_{13}, S_{22}, S_{32}) = (-3, 11.9, -62.8)$$

$$SPE(G) = (S_{13}, S_{22}, S_{32}) = (-3, 11.9, -62.8) \rightarrow \text{optimal result in the non-fuzzy model}$$

By comparing the results of the problem solving in both of fuzzy and non-fuzzy modes, it is obvious in fuzzy-game model, results of Nash equilibrium and Backward Nash equilibrium are different of non-fuzzy model problem solving (see appendix C) and it also defines different strategies in optimal result of the problem. The optimal result in the fuzzy model, government considers strategy of subsidiary and supervisory system, manufacturer change its technology from intolerable to the tolerable level and customers' demand will have positive reaction to this change status. However, in the non-fuzzy model, the optimal result shows that the customer selects the passive strategy and both of the pay offs of manufacturer and customer are decreased. That is obvious that the optimal results in fuzzy model (especially in pay off results), makes motivate of both manufacturer and customer to move the green industry. That is more optimization in pay-off functions of manufacturer and customers than non-fuzz games model. In most of matrix elements, especially, in the parts that technology is changed, the pay-off functions of manufacturer and customers are more optimal and augmented than non-fuzzy mode.

VII.Conclusion

The strategies among players of a green supply chain have been modelled by game theory in strategic form. To make problem's model more practical, initially, the model combined by fuzzy logic relations between customer's strategies and main parameters of players' pay-off function and then, developed with analytical pattern in order to analyze the sensitivity of customers' pay-off function results by changing fuzzy parameters. In addition, the method of problem solving is presented by Nash equilibrium. finally, a numerical analysis was done and problem modelling and solving accomplished in both of fuzzy and non-fuzzy model and their results were compared with each other. The evaluation of results showed that the fuzzy model optimizes players' pay-off function more than non-fuzzy and it causes to change players' strategies and thus motivates to move green strategy.

Finally, in case of three players the game fuzzy model of green supply chain can be efficient, especially when customers' strategies and fuzzy parameter are combined with the model. This can help government and manufacturer to choose optimal amounts of their income/cost parameters (such as price, subsidiaries, ...), that not only considering the effect of keeping or increasing customer's demands positively, but also to make pay-off function more economical effective towards choosing green strategies.

References

1. D. Blanchard, *Supply chain management best practices*, 2nd ed. John Wiley & Sons, 2010.
2. T. Hines, *Supply chain strategies: customer driven and customer focused*. Elsevier Butterworth-Heinemann, 2004.
3. A. Harrison and R. I. van. Hoek, *Logistics management and strategy: competing through the supply chain*. Pearson/Financial Times Prentice Hall, 2011.
4. R. K. Oliver and M. D. Webber, “Supply-chain management: logistics catches up with strategy,” in *The roots of logistics: a reader of classical contributions to the history and conceptual foundations of the science of logistics*, Springer, 2012, pp. 183–194.
5. J.-B. Sheu, Y.-H. Chou, and C.-C. Hu, “An integrated logistics operational model for green-supply chain management,” *Transp. Res. Part E Logist. Transp. Rev.*, vol. 41, no. 4, pp. 287–313, Jul. 2005.
6. R. Basu, *Total supply chain management.* Routledge, 2016.
7. Q. Zhu and R. P. Cote, “Integrating green supply chain management into an embryonic eco-industrial development: a case study of the Guitang Group,” *J. Clean. Prod.*, vol. 12, no. 8–10, pp. 1025–1035, Oct. 2004.
8. Q. Zhu, J. Sarkis, and K. Lai, “Confirmation of a measurement model for green supply chain management practices implementation,” *Int. J. Prod. Econ.*, vol. 111, no. 2, pp. 261–273, Feb. 2008.
9. H.-F. Wang and S. M. Gupta, *Green supply chain management: product life cycle approach*. McGraw

Hill, 2011.

10. H.-F. Wang, *Web-based green products life cycle management systems: reverse supply chain utilization*. Information Science Reference, 2009.
11. R. Zhao, P. Deutz, G. Neighbour, and M. McGuire, “Carbon emissions intensity ratio: an indicator for an improved carbon labelling scheme,” *Environ. Res. Lett.*, vol. 7, no. 1, p. 014014, Mar. 2012.
12. G. P. Cachon and S. Netessine, *Game Theory in Supply Chain Analysis*, Vol 74. Springer, Boston, MA, 2004.
13. M. Esmaeili, M.-B. Aryanezhad, and P. Zeephongsekul, “A game theory approach in seller–buyer supply chain,” *Eur. J. Oper. Res.*, vol. 195, no. 2, pp. 442–448, Jun. 2009.
14. S. X. Li, Z. Huang, J. Zhu, and P. Y. K. Chau, “Cooperative advertising, game theory and manufacturer–retailer supply chains,” *Omega*, vol. 30, no. 5, pp. 347–357, Oct. 2002.
15. M. Nagarajan and G. Sošić, “Game-theoretic analysis of cooperation among supply chain agents: Review and extensions,” *Eur. J. Oper. Res.*, vol. 187, no. 3, pp. 719–745, Jun. 2008.
16. J. Yue, J. Austin, M.-C. Wang, and Z. Huang, “Coordination of cooperative advertising in a two-level supply chain when manufacturer offers discount,” *Eur. J. Oper. Res.*, vol. 168, no. 1, pp. 65–85, Jan. 2006.
17. Q. ZHU and Y. DOU, “Evolutionary Game Model between Governments and Core Enterprises in Greening Supply Chains,” *Syst. Eng. - Theory Pract.*, vol. 27, no. 12, pp. 85–89, Dec. 2007.
18. Y. J. Chen and J.-B. Sheu, “Environmental-regulation pricing strategies for green supply chain management,” *Transp. Res. Part E Logist. Transp. Rev.*, vol. 45, no. 5, pp. 667–677, Sep. 2009.
19. J.-B. Sheu, “Bargaining framework for competitive green supply chains under governmental financial intervention,” *Transp. Res. Part E Logist. Transp. Rev.*, vol. 47, no. 5, pp. 573–592, Sep. 2011.
20. Sikhar Barari, Gaurav Agarwal, W.J.(Chris) Zhang, Biswajit Mahanty, and M.K. Tiwari, “A decision framework for the analysis of green supply chain contracts: An evolutionary game approach,” *Expert Syst. Appl.*, vol. 39, no. 3, pp. 2965–2976, Feb. 2012.
21. A. Nagurney and M. Yu, “Sustainable fashion supply chain management under oligopolistic competition and brand differentiation,” *Int. J. Prod. Econ.*, vol. 135, no. 2, pp. 532–540, Feb. 2012.
22. Y. Tian, K. Govindan, and Q. Zhu, “A system dynamics model based on evolutionary game theory for green supply chain management diffusion among Chinese manufacturers,” *J. Clean. Prod.*, vol. 80, pp. 96–105, Oct. 2014.
23. A. Hafezalkotob, “Competition of two green and regular supply chains under environmental protection and revenue seeking policies of government,” *Comput. Ind. Eng.*, vol. 82, pp. 103–114, Apr. 2015.
24. S. Chandra and A. Aggarwal, “On solving matrix games with pay-offs of triangular fuzzy numbers: Certain observations and generalizations,” *Eur. J. Oper. Res.*, vol. 246, no. 2, pp. 575–581, Oct. 2015.
25. D. Yang and T. Xiao, “Pricing and green level decisions of a green supply chain with governmental interventions under fuzzy uncertainties,” *J. Clean. Prod.*, vol. 149, pp. 1174–1187, Apr. 2017.
26. H.-J. Zimmermann, “Fuzzy mathematical programming,” *Comput. Oper. Res.*, vol. 10, no. 4, pp. 291–298, 1983.
27. L. A. Zadeh, “Fuzzy sets as a basis for a theory of possibility,” *Fuzzy Sets and Systems*, vol. 1, no. 1. pp. 3–28, 1978.
28. W. Siler and J. J. Buckley, *Fuzzy Expert Systems and Fuzzy Reasoning*. Hoboken, NJ, USA: John Wiley & Sons, Inc., 2004.

29. C. Kahraman, *Fuzzy applications in industrial engineering*. Springer-Verlag, 2006.

30. S. Hart and A. Mas-Colell, "Stochastic uncoupled dynamics and Nash equilibrium," *Games Econ. Behav.*, vol. 57, pp. 286–303, 2006.

31. A. M. Colell, "Bargaining Games," in *Cooperation: Game-Theoretic Approaches*, Berlin, Heidelberg: Springer Berlin Heidelberg, 1997, pp. 69–90.

32. S. Hart, "Nash Equilibrium and Dynamics," 2008.

33. R. Zhao, G. Neighbour, J. Han, M. McGuire, and P. Deutz, "Using game theory to describe strategy selection for environmental risk and carbon emissions reduction in the green supply chain," *J. Loss Prev. Process Ind.*, vol. 25, no. 6, pp. 927–936, Nov. 2012.

Appendix A

Green model of green supply chain

Regarding to Fig. 7 adopted from [33], moving in green industry way has three major areas that are described below:

Intolerable Region: an area that industry is in intolerable level because it does not regard green supply chain requirements (red region).

Tolerable Region: an area industry is in tolerable level regard the minimum requirements of green supply chain (mixed red and pale-green region).

Acceptable Region: an area that industry is in an acceptable level to regard green supply chain requirements (green region).

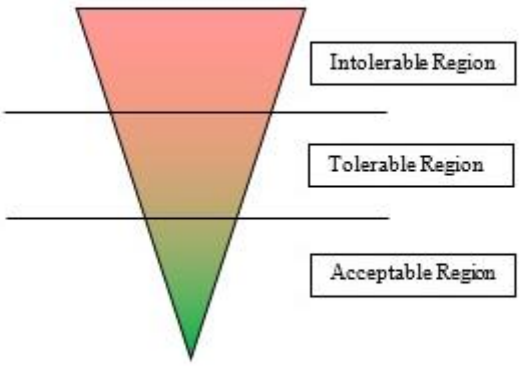


Fig. 7 Regions of Green Supply Chain according to [33]

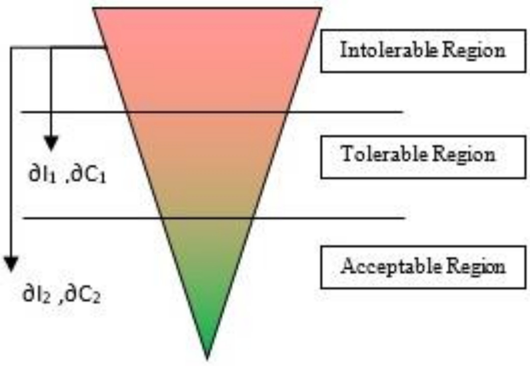


Fig. 8 Profits and Costs in Different Regions according to [33]

Industry's motion from the intolerable area to tolerable and the acceptable area is the aim of green supply chain that in which formulation and analysis of the problem will be modelled in other parts. As it is shown in Fig. 8 adopted from [33], motion of every players from intolerable level to the ones (tolerable and acceptable) brings income difference (∂I) and cost difference (∂C) to status quo for each one. Each of them can cause positive or negative outcome. But the essential point is that in benefit (pay-off) function every player logically must realize true parameters of income and cost then, apply them in calculations. Manufacturer's traditional interest function will be the following function:

$$PF = I_S - C_P \qquad (A_1)$$

Which revenue is (I_S) and direct and indirect costs of production are (C_P). Absolutely, interest function of manufacturer with approach of green supply chain differs from above. In this approach, manufacturer is defined as the following:

$$PF' = PF - C_g \qquad (A_2)$$

Where C_g is a cost of environmental negative effect (fines, hygiene and health costs, resources exploitation, job hazards, and ...) because of the use of traditional technologies. If a manufacturer decides to change the technology, he can add new income and cost resultant to his function, which would be completely a suitable impetus for that decision, according to Fig. 8. In this state and two other states, there are changes from the intolerable level to the tolerable level and from the intolerable level to the acceptable level:

$$PF'_1 = PF - C_g + \partial I_1 - \partial C_1$$

$$PF'_2 = PF - C_g + \partial I_2 - \partial C_2 \qquad (A_3)$$

Appendix B

Analysis of Games and Pay-off Function of Players:

In the following equations and logical relations if each of the strategies is chosen by the player, it would affect income and cost parameters, eventually, the result of Pay-off function.

Government:

$$P_1 = \Sigma_{i=1}^{3}(I_{1i})_{S_{13}} - \Sigma_{j=1}^{5}(C_{1j})_{S_{13}} \Rightarrow (I_{11} + I_{12} + I_{13}) - (C_{11} + C_{12} + C_{13} + C_{14} + C_{15}) \qquad (B_1)$$

Strategy: $S_{11} \rightarrow I_{11} = 0, I_{12}, I_{13} \geq 0, C_{11}, C_{12} = 0, C_{13}, C_{14}, C_{15} \geq 0 \Rightarrow P_1 = (I_{12} + I_{13}) - (C_{13} + C_{14} + C_{15})$

Strategy: $S_{12} \rightarrow I_{12}, I_{13} \geq 0, C_{12} = 0, C_{13}, C_{14}, C_{15} \geq 0 \;\Rightarrow\; P_1 = (I_{11} + I_{12} + I_{13}) - (C_{11} + C_{13} + C_{14} + C_{15})$

Strategy: $S_{13} \rightarrow I_{11,}\, I_{12}, I_{13}, C_{11}, C_{13}, C_{14}, C_{15} \geq 0 \;\Rightarrow\; P_1 = (I_{11} + I_{12} + I_{13}) - (C_{11} + C_{12} + C_{13} + C_{14} + C_{15})$

Generally, if resultant of credits and incomes of green industry and sustainable development are not interesting, on the contrary of regulatory and incentive costs or decrement of environmental costs, normally, approach of neutralism would be chosen by government. This selection is principally for governments who lack a strategic view to the sustainable development, which is why governments' non-strategic view to interests of sustainable development causes these parameters to be deleted from their interest function. If a government chooses approach of green industry and resultant of incomes against costs are positive, regulatory and incentive policies could be undertaken as non-passive strategies according to amount of amount of the resultant.

Totally, if the following inequalities were confirmed, we would have:

$$(I_{11} + I_{13}) - (C_{11} + C_{13} + C_{14} + C_{15}) < A,$$

$$(I_{11} + I_{12} + I_{13}) - (C_{11} + C_{12} + C_{13} + C_{14} + C_{15}) < A \rightarrow strategy: S_{11}$$

$$(I_{11} + I_{12} + I_{13}) - (C_{11} + C_{13} + C_{14} + C_{15}) > A,$$
$$(I_{11} + I_{12} + I_{13}) - (C_{11} + C_{12} + C_{13} + C_{14} + C_{15}) < A \rightarrow strategy\ : S_{12}$$

$$(I_{11} + I_{12} + I_{13}) - (C_{11} + C_{13} + C_{14} + C_{15}) > A, \quad (I_{11} + I_{12} + I_{13}) - (C_{11} + C_{12} + C_{13} + C_{14} + C_{15}) > A,$$
$$A' > A'' \rightarrow strategy\ : S_{12}$$

$$(I_{11} + I_{12} + I_{13}) - (C_{11} + C_{13} + C_{14} + C_{15}) > A, \quad (I_{11} + I_{12} + I_{13}) - (C_{11} + C_{12} + C_{13} + C_{14} + C_{15}) > A,$$
$$A' < A'' \rightarrow strategy\ : S_{13}$$

Where A is value of Pay-off function for government in the state of non-regulatory system, A' is value of Pay-off function for government with the state of triggering regulatory system and A'' is value of Pay-off function for government with the state of regulatory and incentive system.

Manufacturer:

$$P_2 = \sum_{i=1}^{3} (I_{2i})_{S_{21}} - \sum_{j=1}^{5} (C_{2j})_{S_{21}} \Rightarrow (I_{21} + I_{22} + I_{23}) - (C_{21} + C_{22} + C_{23} + C_{24} + C_{25}) \qquad (B_2)$$

Strategy: $S_{21} \;\rightarrow\; I_{22}, I_{23} = 0, C_{21}, C_{23} = 0, C_{25} \geq 0 \;\Rightarrow\; P_2 = (I_{21}) - (C_{22} + C_{24} + C_{25})$

Strategy: $S_{22} \;\rightarrow\; I_{22} \geq 0,\ C_{23},\ C_{25} \geq 0 \;\Rightarrow\; P_2 = (I_{21} + I_{22} + I_{23}) - (C_{21} + C_{22} + C_{23} + C_{24} + C_{25})$

Strategy: $S_{23} \;\rightarrow\; I_{22} \geq 0,\ C_{23},\ C_{25} \geq 0 \;\Rightarrow\; P_2 = (I_{21} + I_{22} + I_{23}) - (C_{21} + C_{22} + C_{23} + C_{24} + C_{25})$

Of the three major players of the supply chain, manufacturer is one of them that have a so accurate view to interest function for decision-making. Resultant of costs of technology change is certainly influential in manufacturers' strategic decision-makings, in face of interior facilities, production sale increase and decrease of costs, related to environmental crimes (if government pledges regulatory policy). Generally, if we consider B as amount of manufacturer's interest in status quo (traditional manufacturing), then if the following inequalities were true, we would have:

$$(I_{21} + I_{22} + I_{23}) - (C_{21} + C_{22} + C_{23} + C_{24} + C_{25}) < B \rightarrow strategy: S_{21}$$

$$(I_{21} + I_{22} + I_{23}) - (C_{21} + C_{22} + C_{23} + C_{24} + C_{25}) > B \rightarrow strategy: S_{22}, S_{23}$$

$$B' > B'' \rightarrow strategy: S_{22}$$

$$B' < B'' \rightarrow strategy: S_{23}$$

Where B is value of manufacturer's Pay-off Function in the state of traditional technology, B' is value of manufacturer's Pay-off Function in state of change to the tolerable technology and B'' is value of manufacturer's Pay-off Function in state of change to the acceptable technology.

Customers:

$$P_3 = \sum_{i=1}^{2}(I_{3i})_{S_{31}} - \sum_{j=1}^{2}(C_{3j})_{S_{31}} \Rightarrow (I_{31} + I_{32}) - (C_{31} + C_{32}) \qquad (B3)$$

Strategy: $S_{31} \rightarrow I_{31}, I_{32} = 0, \Rightarrow P_3 = -(C_{31} + C_{32})$

Strategy: $S_{32} \rightarrow I_{31}, I_{32} \geq 0 \Rightarrow P_3 = (I_{31} + I_{32}) - (C_{31} + C_{32})$

Strategy: $S_{33} \rightarrow I_{31}, I_{32} \geq 0,\ C_{32} \geq 0 \Rightarrow P_3 = (I_{31} + I_{32}) - (C_{31} + C_{32})$

In the state of S_{31} strategy selection due to different reasons, customers are not inclined to buy green products. Traditional approach and confrontation with change, lack of enough information, short-time life cycle of industry (lack of sustainable development approach), absence of incentive impetuses, possible change in price of productions, etc., are some of the most important reasons that cause this reaction by the customers. In industries that we face this approach by customers, establishing incentive policies and advertising costs to change this approach are the most serious decisions of government and manufacturers (if they play cooperatively).

It is clear, if customers take the negative reaction approach certainly, they pay hidden costs (environmental costs) more than cost of products in the time of payment. But if the customers see industry with view of Green Supply Chain and the following condition is ruled, the second and the third strategies could be selected, undoubtedly:

$$(I_{31} + I_{32}) - (C_{31} + C_{32}) \geq -(C'_{31} + C'_{32}) \qquad (B4)$$

The left side parameters of the inequality are related to interest function with approach of green industry and the right-side parameters are related to interest function with traditional approach.

Another important point that should be paid attention is, when manufacturer is not going to change a technology but customers have positive reaction, amount of ε should be added to the customers' costs. Because new customers are added to the former set and the latter set do not earn new advantages, they would have been imposed general –even a little- costs.

Appendix C

Problem solving of games in non-fuzzy mode

In non-fuzzy state and according to the following relations, problem's matrix of games with result of pay-off function of every player for combination of each strategy is:

$$P_1 = [\alpha\,(1+\beta)(1+\rho)I_{21} + I_{12} + I_{13}] - [(C_{11} + \gamma C_{12} + (1-\gamma)C_{12} + C_{13} + C_{14} + C_{15})]$$

$$P_2 = [(1-\alpha)(1+\beta)(1+\rho)I_{21} + \gamma C_{12} + I_{23}] - [(C_{21} + \varphi(1+\rho)C_{22} + C_{23} + C_{24})]$$

$$P_3 = [(1-\gamma)C_{12} + I_{32}] - [(1+\beta+\beta\rho)I_{21} + \varepsilon + C_{32}]$$

According to above, the matrices of game are given in Table 12:

Table 12-A. Non- fuzzy game model of Customer Strategy= S_{31}

	S_{21}	S_{22}	S_{23}
S_{11}	-8, 4, -68	-3, 1.3, -64.6	2, -0.1, -61.4
S_{12}	-7.7, 2.4, -68	-3, 1.3, -64.6	2, -0.1, -61.4
S_{13}	-7.7, 2.4, -68	-2.7, 3.9, -63.4	0.1, 3.8, -59.2

Table 12-B. Non- fuzzy game model of Customer Strategy= S_{32}

	S_{21}	S_{22}	S_{23}
S_{11}	-8, 12, -68	-3, 9.3, -64.3	2, 7.6, -60.3
S_{12}	-7.3, 10, -68	-3, 9.3, -64.3	2, 7.6, -60.3
S_{13}	-7.3, 10, -68	-3, 11.9, -62.8	-0.5, 11.5, -57.3

Table 12-C. Non- fuzzy game model of Customer Strategy= S_{33}

	S_{21}	S_{22}	S_{23}
S_{11}	-8, 16, -68.01	-3, 13.5, -64.4	2, 11.9, -60.4
S_{12}	-7.1, 13.8, -68.01	-3, 13.5, -64.4	2, 11.9, -60.4
S_{13}	-7.1, 13.8, -68.01	-3, 16.1, -62.9	-0.8, 15.8, -57.4

According to the achieved result of matrices above, by earning Nash Equilibrium and Backward Nash Equilibrium, the optimal result of problem is calculated as below:

$$\max_{s_i \in S_i} u_i(S_i, S_{-i})$$

$$N(G)_1 = (S_{12}, S_{21}, S_{31}) = (-7.7, 2.4, -68)$$

$$N(G)_2 = (S_{12}, S_{21}, S_{32}) = (-7.3, 10, -68)$$

$$N(G)_1 = (S_{13}, S_{22}, S_{32}) = (-3, 11.9, -62.8)$$

$SPE(G) = (S_{13}, S_{22}, S_{32}) = (-3, 11.9, -62.8)$ $\rightarrow$ the optimal result in the non-fuzzy model

Appendix D

Membership mathematical functions:

$$C_{12}(Favorite): \mu = \begin{cases} 0, & 0 \leq x \leq 1.5 \\ x - 1.5, & 1.5 < x \leq 2.5 \\ 1, & 2.5 < x \leq 3 \end{cases}$$

$$C_{23}(Unfavorable): \mu = \begin{cases} 1, & 0 \leq x \leq 1.5 \\ -\frac{1}{2}x + \frac{7}{4}, & 1.5 < x \leq 3.5 \\ 0, & 3.5 < x \leq 5 \end{cases}$$

$$C_{23}(Favorite): \mu = \begin{cases} 0, & 0 \le x \le 2 \\ \frac{1}{2}x - \frac{7}{4}, & 2 < x \le 4 \\ 1, & 4 < x \le 5 \end{cases}$$

$$\partial C_{31}(low): \mu = \begin{cases} 1, & 0 \le x \le 0.5 \\ -\frac{2}{3}x + \frac{4}{3}, & 0.5 < x \le 2 \\ 0, & 2 < x \le 3 \end{cases}$$

$$\partial C_{31}(High): \mu = \begin{cases} 0, & 0 \le x \le 1 \\ x - 1, & 1 < x \le 2 \\ 1, & 2 < x \le 3 \end{cases}$$

$$I_{32}(low): \mu = \begin{cases} 1, & 0 \le x \le 0.3 \\ -2.5x + 1.75, & 0.3 < x \le 0.7 \\ 0, & 0.7 < x \le 1 \end{cases}$$

$$I_{32}(High): \mu = \begin{cases} 0, & 0 \le x \le 0.4 \\ 2.5x - 1, & 0.4 < x \le 0.8 \\ 1, & 0.8 < x \le 1 \end{cases}$$

$$P_3(Negative): \mu = \begin{cases} 1, & 0 \le x \le 3 \\ -\frac{1}{6}x + \frac{3}{2}, & 9 < x \le 16 \\ 0, & 0 \le x \le 3 \end{cases}$$

$$P_3(Zero): \mu = \begin{cases} 0, & 0 \le x \le 3 \\ \frac{1}{5}x - \frac{3}{5}, & 3 < x \le 8 \\ -\frac{1}{4}x + 3, & 8 < x \le 12 \\ 0, & 12 < x \le 16 \end{cases}$$

$$P_3(Positive): \mu = \begin{cases} 0, & 0 \le x \le 7 \\ \frac{1}{5}x - \frac{7}{5}, & 7 < x \le 12 \\ 1, & 12 < x \le 16 \end{cases}$$