RLS Filter Using Covariance Information and RLS Wiener Type Filter Based on Innovation Theory for Linear Discrete-Time Stochastic Descriptor Systems

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Abstract

It is known that the stochastic descriptor systemsare transformed into the conventional state equation, the observation equation and the other equation, by using the singular value decomposition. Based on the preliminary problem formulation for the linear discrete-time stochastic descriptor systems in section 2, this paper, in Theorem 1, based on the innovation theory, proposes the recursive least-squares (RLS) filter using the covariance information of the state vector in the state equation and the covariance information of the observation noise in the observation equation. The state equation and the observation equation are transformed from the descriptor systems. Secondly, in Theorem 2, based on the innovation theory, this paper proposes the RLS Wiener type filter for the descriptor systems. It might be advantageous that these filtering algorithms in this paper are derived based on the innovation theory in a unified manner.

A numerical simulation example is demonstrated to show the estimation characteristics of the proposed RLS Wiener type filtering algorithm for the descriptor systems.

Keywords: Discrete-time stochastic systems; RLS Wiener type filter; Covariance information; Descriptor systems; Innovation theory

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Type (Method/Approach): Quasi-Experimental

1.Introduction

The estimation problem for the descriptor systems has been investigated intensively ([1]-[5] and references therein).The examples of the descriptor systems are found in physical systems, e.g. a cart-pendulum system, electrical circuits [6], etc. In this paper, the filtering problem is considered for the linear discrete-time stochastic descriptor systems.In [1], the recursive filter, predictor and smoother are proposed for the descriptor systems with multiple packet dropouts for the correlated observation noise between different times.In [2], the information filters are proposed for the discrete-time descriptor systems with uncertain parameters. One filter calculates the estimate with the Riccati equations and the other filter has the form of the array algorithms.In [3], the discrete-time Kalman type filter is proposed for the descriptor systems with uncertain parameters.In [4], the discrete-time recursive filter is proposed with the optimization technique based on the data-fitting for the descriptor systems.In [5], the discrete-time robust predictor is proposed for the descriptor systems with bounded uncertainties. The robust prediction algorithm is derived by solving the optimization problem.

This paper newly proposes the recursive least-squares (RLS) filtering algorithm by using the covariance information and the RLS Wiener type filter for the descriptor systems in linear discrete-time stochastic systems. It is known that the descriptor systems are transformed into the conventional state equation, the observation equation and the other equation, by using the singular value decomposition (SVD) [1]. Based on the preliminary problem formulation for the descriptor systems in section 2, this paper, in Theorem 1, based on the innovation theory, proposes the RLS filter using the covariance information of the state vector in the state equation and the covariance information of the observation noise in the observation equation. Secondly, in Theorem 2, based on the innovation theory, this paper proposes the RLS Wiener type filter for the descriptor systems. It might be advantageous that these filtering algorithms in this paper are derived based on the innovation theory in a unified manner.

A numerical simulation example is demonstrated to show the estimation characteristics of the proposed RLS Wiener type filtering algorithm for the descriptor systems.

2. Least-squares filtering problem for linear descriptor systems

Let a state equation and an observation equation be given by

$$\begin{aligned} &ES(k+1)=FS(k)+Gw(k), E[w(k)w^T(s)]=Q\delta_K(k-s), \\ &y(k)=CS(k)+v(k), E[v(k)v^T(s)]=R\delta_K(k-s), \end{aligned} \tag{1}$$

for the linear discrete-time descriptor systems [1].Here, $S(k)$ is an n -dimensional descriptor vector, $w(k)$ denotes a q -dimensional input noise and $y(k)$ is an m -dimensional measurement output vector. The matrices E, F, G and C have dimensions $n\times n$, $n\times n$, $n\times q$, $m\times n$, respectively. Here, E is the singular matrix, i.e. $rank(E)<n$. Withorthogonal matrices U and V, the SVD of E is expressed by

$$\begin{aligned} &E=UDV^T, D=\begin{bmatrix}\Delta & 0\\ 0 & 0\end{bmatrix}, U^T=U^{-1}, V^T=V^{-1} \\ &\Delta=diag(\mu_1,\mu_2,...,\mu_l), \mu_i>0, i=1,2,...,l, \Delta>0. \end{aligned} \tag{2}$$

By introducing

$$U^TFV=\begin{bmatrix}F_{11} & F_{12}\\ F_{21} & F_{22}\end{bmatrix},\quad U^TG=\begin{bmatrix}G_1\\ G_2\end{bmatrix},\quad CV=\begin{bmatrix}C_1 & C_2\end{bmatrix},\quad S(k)=V\begin{bmatrix}\bar{S}_1(k)\\ \bar{S}_2(k)\end{bmatrix},$$

the state equation in (1) is transformed into

$$\begin{bmatrix}\Delta & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\bar{S}_1(k+1)\\ \bar{S}_2(k+1)\end{bmatrix}=\begin{bmatrix}F_{11} & F_{12}\\ F_{21} & F_{22}\end{bmatrix}\begin{bmatrix}\bar{S}_1(k)\\ \bar{S}_2(k)\end{bmatrix}+\begin{bmatrix}G_1\\ G_2\end{bmatrix}w(k). \tag{3}$$

From (3), we have

$$\begin{aligned} &\bar{S}_1(k+1)=A\bar{S}_1(k)+Bw(k), A=\Delta^{-1}(F_{11}+F_{12}\Gamma_1), B=\Delta^{-1}(F_{12}\Gamma_2+G_1), \Delta>0 \\ &y(k)=H\bar{S}_1(k)+\bar{v}(k), H=C_1+C_2\Gamma_1, \bar{v}(k)=C_2\Gamma_2 w(k)+v(k), \\ &E[\bar{v}(k)\bar{v}^T(s)]=\bar{R}\delta_K(k-s), \bar{R}=C_2\Gamma_2 Q\Gamma_2^T C_2^T+R, \\ &\bar{S}_2(k)=\Gamma_1\bar{S}_1(k)+\Gamma_2 w(k), \Gamma_1=-F_{22}^{-1}F_{21}, \Gamma_2=-F_{22}^{-1}G_2, F_{22}>0. \end{aligned} \tag{4}$$

For notational conveniences, we put $\bar{S}_1(k)$ as $x(k)=\bar{S}_1(k)$ and A as $\Phi=A$. Hence, the state equation for the state vector $x(k)$ and its observation equation can bewritten as

$$x(k+1)=\Phi x(k)+Bw(k), E[w(k)w^T(s)]=Q\delta_K(k-s),$$
$$y(k)=Hx(k)+\bar{v}(k), E[\bar{v}(k)\bar{v}^T(s)]=\bar{R}\delta_K(k-s). \quad (5)$$

The auto-covariance function of the state vector $x(k)=\bar{S}_1(k)$ is expressed in the form of the semi-degenerate function

$$K(k,s)=\begin{cases}\alpha(k)\beta^T(s), 0\le s\le k,\\ \beta(k)\alpha^T(s), 0\le k\le s,\end{cases}$$
$$\alpha(k)=\Phi^k, \beta^T(s)=\Phi^{-s}K(s,s), \quad (6)$$

where the variance $K(k,k)$ of the state vector $x(k)$ satisfies

$$K(k+1,k+1)=\Phi K(k,k)\Phi^T+BQB^T. \quad (7)$$

Let the filtering estimate $\hat{x}(k,k)$ of the state vector $x(k)$ be expressed by

$$\hat{x}(k,k)=\sum_{i=1}^{k}g(k,i)\upsilon(i) \quad (8)$$

as a linear combination of the innovation process $\upsilon(i)$, $1\le i\le k$ [7].In (8), $g(k,i)$ represents a time-varying impulse response function. The innovation process $\upsilon(k)$ is expressed by

$$\upsilon(k)=y(k)-H\Phi\hat{x}(k-1,k-1). \quad (9)$$

From the orthogonality between the filtering error $x(k)-\hat{x}(k,k)$ and the innovation process $\upsilon(s)$, $1\le s\le k$, it follows that

$$E[x(k)\upsilon^T(s)]=E[\hat{x}(k,k)\upsilon^T(s)]$$
$$=\sum_{i=1}^{k}g(k,i)E[\upsilon(i)\upsilon^T(s)] \quad (10)$$
$$=g(k,s)\Lambda(s), E[\upsilon(k)\upsilon^T(s)]=\Lambda(k)\delta_K(k-s).$$

Here, the variance of the innovation process $\upsilon(k)$ is $\Lambda(k)$. The impulse response function $g(k,s)$ is given by

$$g(k,s)=E[x(k)\upsilon^T(s)]\Lambda^{-1}(s). \quad (11)$$

$E[x(k)\upsilon^T(s)]$ is developed as follows.

$$E[x(k)\upsilon^T(s)]=E[x(k)(y(s)-H\Phi\hat{x}(s-1,s-1))^T]$$
$$=K(k,s)H^T-\sum_{i=1}^{s-1}E[x(k)\upsilon^T(i)]g^T(s-1,i)\Phi^T H^T. \quad (12)$$

From (11) and (12), the impulse response function $g(k,s)$ satisfies

$$g(k,s)\Lambda(s) = K(k,s)H^T - \sum_{i=1}^{s-1} g(k,i)\Lambda(i)g^T(s-1,i)\Phi^T H^T. \tag{13}$$

This paper, for the linear discrete-time stochastic descriptor systems, in section 3, based on the innovation theory, proposes the RLS filtering algorithm, using the covariance information of the state vector and the observation noise,and the RLS Wiener type filtering algorithm.

3. RLS filtering algorithmusing covariance information and RLS Wiener type filtering algorithm

Under the linear least-squares estimation problem for the stochastic descriptor systems in section 2, Theorem 1 presents the RLS algorithmfor the filtering estimate $\hat{S}(k,k)$ of $S(k)$ using the covariance information of the state vector $x(k) = \bar{S}_1(k)$ and the observation noise $\bar{v}(k)$, in linear discrete-time stochastic systems. Theorem 2 proposes the RLS Wiener type algorithm for the filtering estimate $\hat{S}(k,k)$ of $S(k)$. The RLS Wiener filtering and fixed-point smoothing algorithms from the output measurement are proposed in [8] for the conventional state-space models in lineardiscrete-time stochastic systems.

Theorem 1 Let an n-dimensional state equation and an m-dimensional observation equation be given by (1) for the descriptor systems. Let an auto-covariance function $K(k,s)$ of the state vector $x(k) = \bar{S}_1(k)$ beexpressed by (6). Then the RLS algorithm for the filtering estimate $\hat{S}(k,k)$ of $S(k)$,using the covariance information of the state vector $x(k) = \bar{S}_1(k)$ and the observation noise $\bar{v}(k)$,consists of (14)-(21) in linear discrete-time stochastic systems.

Filtering estimate of $S(k)$: $\hat{S}(k,k)$

$$\hat{S}(k,k) = V\begin{bmatrix} \hat{\bar{S}}_1(k,k) \\ \hat{\bar{S}}_2(k,k) \end{bmatrix} \tag{14}$$

Filtering estimate of $\bar{S}_1(k)$: $\hat{\bar{S}}_1(k,k)$

$$\hat{\bar{S}}_1(k,k) = \hat{x}(k,k) \tag{15}$$

Filtering estimate of $\bar{S}_2(k)$: $\hat{\bar{S}}_2(k,k)$

$$\hat{\bar{S}}_2(k,k) = \Gamma_1 \hat{\bar{S}}_1(k,k),\ \Gamma_1 = -F_{22}^{-1}F_{21}, F_{22} > 0 \tag{16}$$

Filtering estimate of the state vector $x(k)$: $\hat{x}(k,k)$

$$\hat{x}(k,k) = \alpha(k)e(k), \quad \alpha(k) = \Phi^k \tag{17}$$

Recursive equation for $e(k)$:

$$e(k) = e(k-1) + J(k)(y(k) - H\Phi\hat{x}(k-1,k-1)), e(0) = 0 \tag{18}$$

Equation for $J(k)$:

$$J(k)=(\beta^T(k)H^T-r(k-1)\alpha^T(k-1)\Phi^T H^T)\Lambda^{-1}(k),\beta^T(k)=\Phi^{-k}K(k,k) \tag{19}$$

Recursive equation for $r(k)$:

$$r(k)=r(k-1)+J(k)\Lambda(k)J^T(k),r(0)=0 \tag{20}$$

Variance of innovation process: $\Lambda(k)=E[\upsilon(k)\upsilon^T(k)]$

$$\begin{aligned}&\Lambda(k)=K(k,k)+\bar{R}-H\Phi\alpha(k-1)r(k-1)\alpha^T(k-1)\Phi^T H^T,\\&\Phi=\Delta^{-1}(F_{11}+F_{12}\Gamma_1),H=C_1+C_2\Gamma_1,\Gamma_1=-F_{22}^{-1}F_{21},\\&\bar{R}=C_2\Gamma_2Q\Gamma_2^T C_2^T+R,\Gamma_2=-F_{22}^{-1}G_2,F_{22}>0\end{aligned} \tag{21}$$

Proof of Theorem1 is deferred to the appendix.

Theorem 2 Let an n-dimensional state equation and an m-dimensional observation equation be given by (1) for the descriptor systems. Let an auto-covariance function $K(k,s)$ of the state vector $x(k)=\bar{S}_1(k)$ beexpressed by (6). Then the RLS Wiener type algorithm for the filtering estimate $\hat{S}(k,k)$ of $S(k)$ consists of (22)-(28) in linear discrete-time stochastic systems.

Filtering estimate of $S(k)$: $\hat{S}(k,k)$

$$\hat{S}(k,k)=V\begin{bmatrix}\hat{\bar{S}}_1(k,k)\\ \hat{\bar{S}}_2(k,k)\end{bmatrix} \tag{22}$$

Filtering estimate of $\bar{S}_1(k)$: $\hat{\bar{S}}_1(k,k)$

$$\hat{\bar{S}}_1(k,k)=\hat{x}(k,k) \tag{23}$$

Filtering estimate of $\bar{S}_2(k)$: $\hat{\bar{S}}_2(k,k)$

$$\hat{\bar{S}}_2(k,k)=\Gamma_1\hat{\bar{S}}_1(k,k),\quad \Gamma_1=-F_{22}^{-1}F_{21},F_{22}>0 \tag{24}$$

Filtering estimate of the state vector $x(k)$: $\hat{x}(k,k)$

$$\hat{x}(k,k)=\Phi\hat{x}(k-1,k-1)+G(k)(y(k)-H\Phi\hat{x}(k-1,k-1)),\hat{x}(0,0)=0 \tag{25}$$

Variance of innovation process: $\Lambda(k)=E[\upsilon(k)\upsilon^T(k)]$

$$\begin{aligned}&\Lambda(k)=HK(k,k)H^T+\bar{R}-H\Phi\Omega(k-1)\Phi^T H^T,\\&\Phi=\Delta^{-1}(F_{11}+F_{12}\Gamma_1),H=C_1+C_2\Gamma_1,\Gamma_1=-F_{22}^{-1}F_{21},\\&\bar{R}=C_2\Gamma_2Q\Gamma_2^T C_2^T+R,\Gamma_2=-F_{22}^{-1}G_2,F_{22}>0\end{aligned} \tag{26}$$

Filter gain: $G(k)$

$$G(k)=(K(k,k)H^T-\Phi\Omega(k-1)\Phi^T H^T)\Lambda^{-1}(k) \tag{27}$$

Variance of filtering estimate $\hat{x}(k,k)$: $\Omega(k)$

$$\Omega(k) = \Phi\Omega(k-1)\Phi^T + G(k)\Lambda(k)G^T(k), \Omega(0) = 0 \tag{28}$$

Proof of Theorem 2 is also deferred to the appendix.

For the stability of the RLS Wiener type filtering algorithm in Theorem 2, the following conditions are required.

1. All the real parts in the eigenvalues of the matrix $\Phi - G(k)H\Phi$ are negative.

2. Positive definite condition $HK(k,k)H^T + \bar{R} - H\Phi S(k-1)\Phi^T H^T > 0$ holds.

3. All the real parts in the eigenvalues of the state transition matrix Φ are negative.

4. Positive definite condition $F_{22} > 0$ holds.

4. Filtering error variance function

This section discusses on the existence of the filtering estimate by the RLS Wiener type filter of Theorem 2. Since the filtering estimate $\hat{\bar{S}}_2(k,k)$ of $\bar{S}_2(k)$ is given by (24) in terms of the filtering estimate $\hat{\bar{S}}_1(k,k)$ of $\bar{S}_1(k)$, let us consider on the existence of the filtering estimate $\hat{\bar{S}}_1(k,k)$ of $\bar{S}_1(k)$. From $\hat{\bar{S}}_1(k,k) = \hat{x}(k,k)$ of (23), the existence of the filtering estimate $\hat{\bar{S}}_1(k,k)$ is guaranteed by proving the existence of the filtering estimate $\hat{x}(k,k)$ of the state vector $x(k)$ .The filtering error variance function $P(k)$ for the state vector $x(k)$ is given by

$$\begin{aligned} P(k) &= E[(x(k) - \hat{x}(k,k))(x(k) - \hat{x}(k,k))^T] \\ &= K(k,k) - E[\hat{x}(k,k)\hat{x}^T(k,k)]. \end{aligned} \tag{29}$$

The variance of the filtering estimate $\hat{x}(k,k)$ is denoted by $\Omega(k)$. Hence, from (29), the inequality $0 \le \Omega(k) \le K(k,k)$ holds. This indicates that the variance $\Omega(k)$ of the filtering estimate $\hat{x}(k,k)$ is lower bounded by the zero matrix and upper bounded by the variance of the state vector $x(k)$. From this fact, it is clear that the filtering estimate $\hat{\bar{S}}_1(k,k)$ of $\bar{S}_1(k)$ exists.Henceforth, the filtering estimate $\hat{\bar{S}}_2(k,k)$ of $\bar{S}_2(k)$ also exists.

5. A numerical simulation example

Let a state equation and an observation equation be given by

$$\begin{aligned} &ES(k+1) = FS(k) + Gw(k), E[w(k)w(s)] = Q\delta_K(k-s), \\ &y(k) = CS(k) + v(k), E[v(k)v(s)] = R\delta_K(k-s), \\ &S(k) = \begin{bmatrix} S_1(k) \\ S_2(k) \\ S_3(k) \end{bmatrix}, \\ &E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, F = \begin{bmatrix} 2 & -0.85 & 1 \\ 1 & 0 & 0.5 \\ 0 & 0.5 & 1.5 \end{bmatrix}, G = \begin{bmatrix} 0.5 \\ 1 \\ 0.2 \end{bmatrix}, \\ &C = \begin{bmatrix} 0.5 & 0.9 & 0.6 \end{bmatrix}, Q = 0.5^2, \end{aligned} \tag{30}$$

in the linear stochastic descriptor systems [1]. The SVD of E is given by

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = U \begin{bmatrix} \Delta & 0 \\ 0 & 0 \end{bmatrix} V^T, \Delta = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

From the problem formulation in section 2, the following relationshipsfollow.

$$S(k) = \begin{bmatrix} S_1(k) \\ S_2(k) \\ S_3(k) \end{bmatrix} = V \begin{bmatrix} \bar{S}_1(k) \\ \bar{S}_2(k) \end{bmatrix} = V \begin{bmatrix} x(k) \\ \bar{S}_2(k) \end{bmatrix} = \begin{bmatrix} x_1(k) \\ x_2(k) \\ \bar{S}_2(k) \end{bmatrix}, x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

The state equation for $x(k)$ and the observation equation are described as

$$\begin{aligned} & x(k+1) = \Phi x(k) + Bw(k), E[w(k)w(s)] = Q\delta_K(k-s), \\ & y(k) = Hx(k) + \bar{v}(k), E[\bar{v}(k)\bar{v}(s)] = \bar{R}\delta_K(k-s), \\ & \Phi = \begin{bmatrix} 2 & -1.1833333 \\ 1 & -0.1666666 \end{bmatrix}, B = \begin{bmatrix} 0.3666666 \\ 0.9333333 \end{bmatrix}, H = [0.5 \quad 0.7], \\ & Q = 0.5^2, \bar{R} = 0.08^2 Q + R. \end{aligned} \tag{31}$$

In this example, the filtering estimate $\hat{S}(k,k)$ of $S(k)$ is related to $\hat{x}(k,k)$, $\hat{\bar{S}}_1(k,k)$ and $\hat{\bar{S}}_2(k,k)$ as

$$\begin{aligned} & \hat{S}(k,k) = \begin{bmatrix} \hat{S}_1(k,k) \\ \hat{S}_2(k,k) \\ \hat{S}_3(k,k) \end{bmatrix} = V \begin{bmatrix} \hat{\bar{S}}_1(k,k) \\ \hat{\bar{S}}_2(k,k) \end{bmatrix} = V \begin{bmatrix} \hat{x}(k,k) \\ \hat{\bar{S}}_2(k,k) \end{bmatrix}, \\ & \hat{\bar{S}}_2(k,k) = \Gamma_1 \hat{\bar{S}}_1(k,k), \Gamma_1 = [0 \quad -0.333333]. \end{aligned} \tag{32}$$

The variance $K(k,k)$ of the state vector $x(k)$ is calculated by

$$K(k+1,k+1) = \Phi K(k,k)\Phi^T + BQB^T \tag{33}$$

as

$$K(k,k) = K(0) = \begin{bmatrix} 23.343685 & 19.948466 \\ 19.948466 & 17.395173 \end{bmatrix}.$$

By substituting Φ, $K(k,k)$, H, $\bar{R}$ into the RLS Wiener type filtering algorithm of Theorem 2, the filtering estimates $\hat{S}_1(k,k)$ of $S_1(k)$, $\hat{S}_2(k,k)$ of $S_2(k)$ and $\hat{S}_3(k,k)$ of $S_3(k)$ are calculated recursively. Fig.1 illustrates $S_1(k)$ and the filtering estimate $\hat{S}_1(k,k)$ vs. k for the white Gaussian observation noise $v(k)$, $N(0,R), R=1$. Here, $\bar{R}$ is evaluated as $\bar{R} = 1.0016$. Fig.2 illustrates $S_2(k)$ and the filtering estimate $\hat{S}_2(k,k)$ vs. k for the white Gaussian observation noise $v(k)$, $N(0,R), R=1$. Fig.3 illustrates $S_3(k)$ and the filtering estimate $\hat{S}_3(k,k)$ vs. k for the white Gaussian observation noise $v(k)$, $N(0,R), R=1$. Fig.1, Fig.2 and Fig.3 show that the RLS Wiener type filter estimates $S_1(k)$, $S_2(k)$ and $S_3(k)$ vs. $k$surely. The mean-square values (MSVs) of the filtering errors vs. the standard deviation $\sqrt{R}$ of the observation noise $v(k)$ are illustrated in Fig.4.Fig.4 indicates, as the standard deviation $\sqrt{R}$ increases, the MSVs of the filtering errors increase and the estimation accuracies for $S_1(k)$, $S_2(k)$ and $S_3(k)$ become worse.

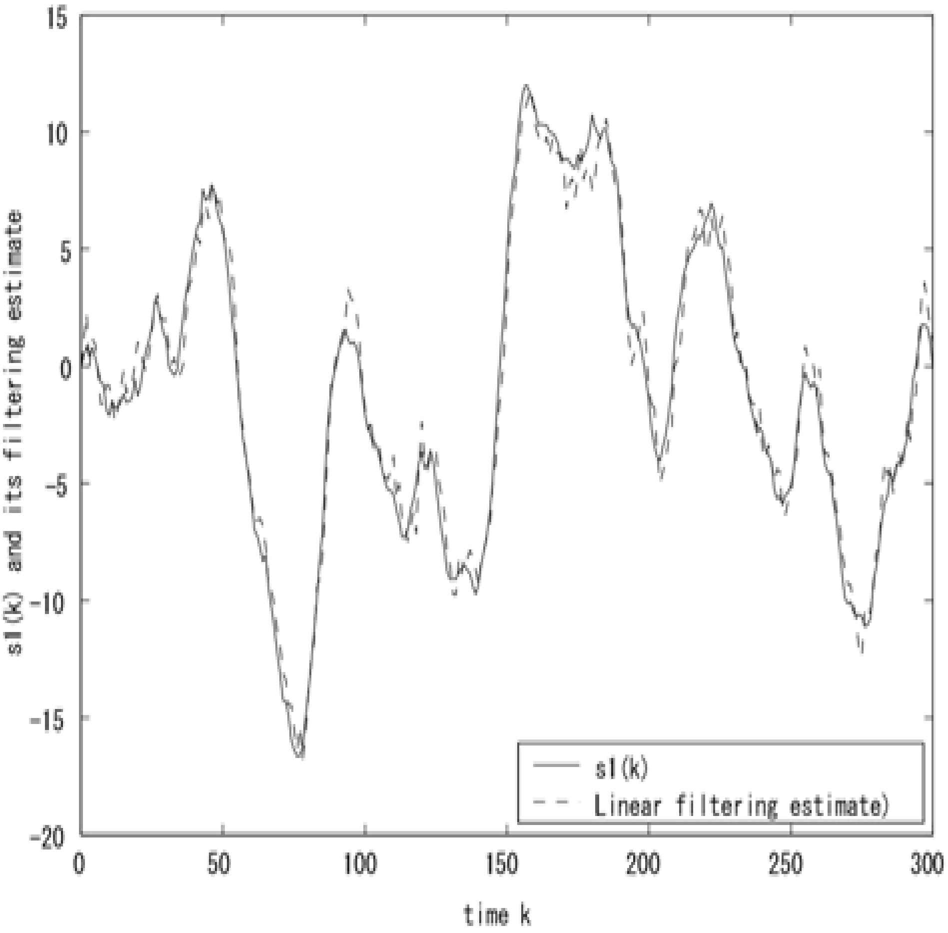

Fig.1 $S_1(k)$ and the filtering estimate $\hat{S}_1(k,k)$ vs. k for the white Gaussian observation noise $v(k)$, $N(0,R), R=1$.

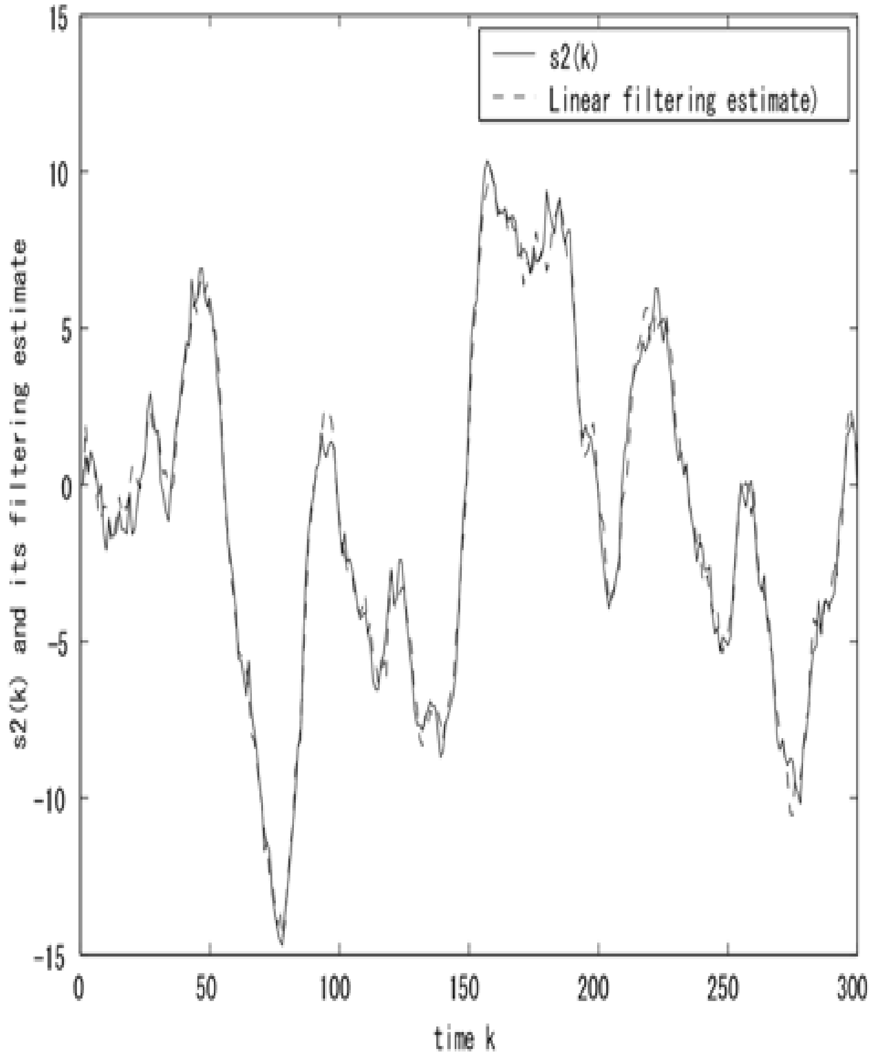

Fig.2 $S_2(k)$ and the filtering estimate $\hat{S}_2(k,k)$ vs. k for the white Gaussian observation noise $v(k)$, $N(0,R), R=1$.

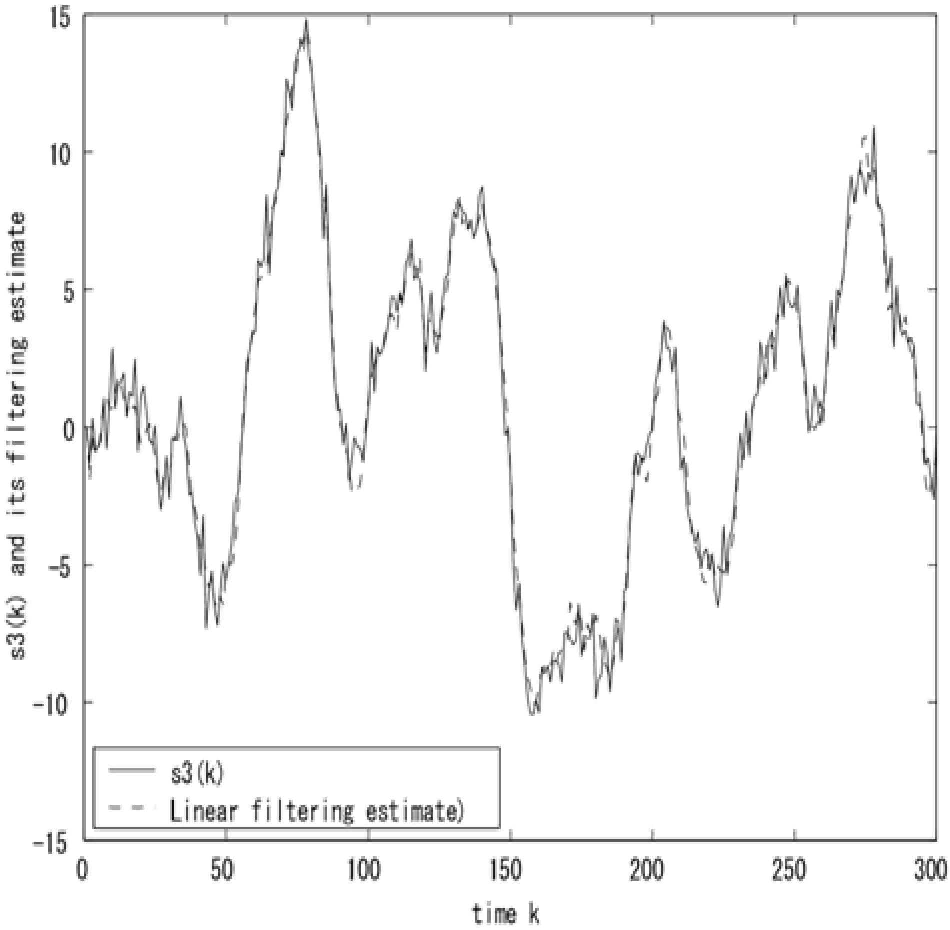

Fig.3 $s_3(k)$ and the filtering estimate $\hat{s}_3(k,k)$ vs. k for the white Gaussian observation noise $v(k)$, $N(0,R), R=1$.

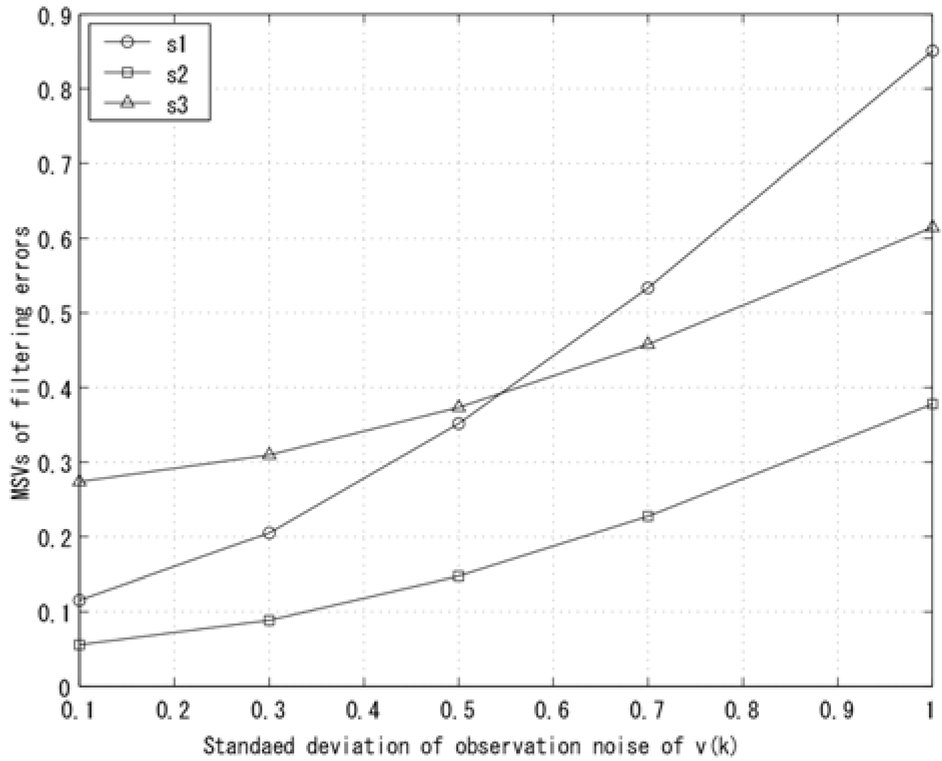

Fig.4 Mean-square values of the filtering errors vs. the standard deviation $\sqrt{R}$ of the observation noise $v(k)$.

Here, the MSVs of the filtering errors are evaluated by $\sum_{i=1}^{500}(S_1(i)-\hat{S}_1(i,i))^2/500$, $\sum_{i=1}^{500}(S_2(i)-\hat{S}_2(i,i))^2/500$ and $\sum_{i=1}^{500}(S_3(i)-\hat{S}_3(i,i))^2/500$.

6. Conclusions

With regard to the linear least-squares estimation problem for the descriptor systems, this paper in Theorem 1 presents the RLS algorithm for the filtering estimate $\hat{S}(k,k)$ of $S(k)$ using the covariance information of the state vector $x(k)=\overline{S}_1(k)$ and the observation noise $\overline{v}(k)$ in linear discrete-time stochastic systems. Theorem 2 proposes the RLS Wiener type algorithm for the filtering estimate $\hat{S}(k,k)$ of $S(k)$. A numerical simulation example has shown that the RLS Wiener type filtering algorithm has the feasible estimation characteristicsfor the discrete-time stochastic descriptor systems.

Appendix (Proof of Theorem 1 and Theorem 2)

The impulse response function $g(k,s)$ satisfies (13). Introducing

$$J(s)\Lambda(s)=\beta^T(s)H^T-\sum_{i=1}^{s-1}J(i)\Lambda(i)g^T(s-1,i)\Phi^T H^T, \qquad \text{(A-1)}$$

we have

$$g(k,s)=\alpha(k)J(s)\,. \tag{A-2}$$

From (8), the filtering estimate $\hat{x}(k,k)$ of $x(k)$ is expressed by

$$\hat{x}(k,k)=\alpha(k)e(k) \tag{A-3}$$

in terms of a function $e(k)$ given by

$$e(k)=\sum_{i=1}^{k}J(i)\upsilon(i)\,. \tag{A-4}$$

Subtracting $e(k-1)$ from $e(k)$, we have

$$e(k)-e(k-1)=J(k)(y(k)-H\Phi\hat{x}(k-1,k-1)),e(0)=0. \tag{A-5}$$

From (A-1) $J(k)$ satisfies

$$J(k)\Lambda(k)=\beta^T(k)H^T-\sum_{i=1}^{k-1}J(i)\Lambda(i)g^T(k-1,i)\Phi^T H^T\,. \tag{A-6}$$

Introducing a function

$$r(k)=\sum_{i=1}^{k}J(i)\Lambda(i)J^T(i)\,, \tag{A-7}$$

we have an expression for $J(k)$ as

$$J(k)=(\beta^T(k)H^T-r(k-1)\alpha^T(k-1)\Phi^T H^T)\Lambda^{-1}(k)\,. \tag{A-8}$$

Subtracting $r(k-1)$ from $r(k)$, we have the recursive equation for $r(k)$ as

$$r(k)-r(k-1)=J(k)\Lambda(k)J^T(k)\,,\quad r(0)=0\,. \tag{A-9}$$

$\Lambda(k)$ is the variance of the innovation process $\upsilon(k)$. $\Lambda(k)$ is developed as follows.

$$\begin{aligned}\Lambda(k)&=E[\upsilon(k)\upsilon^T(k)]\\&=K(k,k)+\bar{R}-H\Phi\sum_{i=1}^{k-1}g(k-1,i)\Lambda(i)g^T(k-1,i)\Phi^T H^T\\&=K(k,k)+\bar{R}-H\Phi\alpha(k-1)r(k-1)\alpha^T(k-1)\Phi^T H^T\end{aligned} \tag{A-10}$$

From (4), $\bar{S}_2(k)=\Gamma_1\bar{S}_1(k)+\Gamma_2 w(k)$, in the calculation of the filtering estimate $\hat{\bar{S}}_2(k,k)$ of $\bar{S}_2(k)$, the filtering estimate $\hat{w}(k,k)$ of $w(k)$ is necessary. $\hat{w}(k,k)$ is formulated as

$$\hat{w}(k,k)=\sum_{i=1}^{k}E[w(k)\upsilon^T(i)]\Lambda^{-1}(i)\upsilon(i)\,. \tag{A-11}$$

From (5), the state equation for $x(k)$ is given by

$$x(k+1) = \Phi x(k) + Bw(k), E[w(k)w^T(s)] = Q\delta_K(k-s),$$
$$y(k) = Hx(k) + \bar{v}(k), E[\bar{v}(k)\bar{v}^T(s)] = \bar{R}\delta_K(k-s).$$

$E[w(k)\upsilon^T(s)]$, $1 \le s \le k$, is given by

$$E[w(k)\upsilon^T(s)] = E[w(k)(y(s) - H\Phi\hat{x}(s-1,s-1))^T]$$
$$= E[w(k)y^T(s)] - E[w(k)\hat{x}^T(s-1,s-1)]\Phi^T H^T. \quad \text{(A-12)}$$

From (A-11) and (A-12) it is clear that $\hat{w}(k,k) = 0$. Hence, the filtering estimate $\hat{\bar{S}}_2(k,k)$ of $\bar{S}_2(k)$ is given by

$$\hat{\bar{S}}_2(k,k) = \Gamma_1 \hat{\bar{\bar{S}}}_1(k,k). \quad \text{(A-13)}$$

Substituting (A-5) into (A-3) and putting the filter gain $G(k)$ as

$$G(k) = \alpha(k)J(k), \quad \text{(A-14)}$$

we obtain (25). By putting

$$\Omega(k) = \alpha(k)r(k)\alpha^T(k), \quad \text{(A-15)}$$

$\Omega(k)$ satisfies the recursive equation (28). From (A-10) the variance $\Lambda(k) = E[\upsilon(k)\upsilon^T(k)]$ of the innovation process $\upsilon(k)$ is calculated by (26). The filtering estimate $\hat{S}(k,k)$ of $S(k)$ is calculated by (22). The filtering estimate $\hat{\bar{S}}_1(k,k)$ of $\bar{S}_1(k)$ is given by (23). The filtering estimate $\hat{\bar{S}}_2(k,k)$ of $\bar{S}_2(k)$ is calculated by (24). (Q.E.D.)

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