Reference Temperature Dependent Thermoelastic Solid With Voids Subjected To Continuous Heat Sources

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Abstract

In the present article, the reference temperature dependency Lord–Shulman model of generalized thermoelasticity with voids subjected to a continuous heat source in a half-space is discussed. The Laplace transform together with eigenvalue approach technique is applied to find a closed-form solution for the physical variables viz. distribution of temperature, volume fraction field, deformation and stress field in the Laplace transform domain. The numerical inversions of those physical variables in the space-time domain are carried out by using the Zakian algorithm for the inversion of the Laplace transform. Numerical results are shown graphically and the results obtained are analyzed.

Keywords: Lord–Shulman Model, Thermoelastic Void Material, Eigenvalue Approach

Mathematics Subject Classification (2010) 75F05 74F15

Introduction

The investigations in the theory of thermoelastic materials with voids were first initiated by Nunziato and Cowin (1983) and Iesan (1986) to develop a nonlinear theory of elastic materials. The linear theory of elastic material with voids has been developed by Cowin and Nunziato (1983). The intended applications of the theory of elastic materials with voids are to geological materials such as rock and soils and manufactured porous materials. Ciarlatta and Chirita (2006) have pointed out that the basic concept underlying this theory is that of a material for which the bulk density is written as the product of two fields, the density field of the matrix material and the volume fraction field. Other relevant works in this field are Chirita and Scalia (2001) and Scalia et al. (2004) who enriched the theory under the assumption that the constitutive coefficients are positive definite. Lakes (1987), Lee and Lakes (1997), Caddock and Evans (1989) have outlined some of the applications of the elastic material with voids where the Poisson ratio is negative, such as, foam structures which expand laterally when stretched, in contrast to ordinary material. Some anisotropic polymer foams (which exhibit Poisson ratio exceeding 1) which can withstand high energy absorption and fracture resistance, have also been prepared. Puri and Cowin (1985) studied the propagation of plane waves in a linear elastic material with voids. Dhaliwal and Wang (1995) developed a heat-flux dependent theory of thermoelasticity in the porous material. Kumar and Leena Rani (2005) investigated the temperature and other field variables in a homogeneous isotropic, generalized thermoelastic half-space with voids due to normal, tangential force, and thermal source. Biswas (2019) studied the propagation of plane waves in an isotropic thermoelastic medium for porous materials with the linear theory of micropolar thermoelasticity. Abbas (2015a, 2015b, 2017) solved some problems in generalized thermoelasticity in Fiber–Reinforced Anisotropic Medium (2015a), hollow sphere (2015b) and hollow cylinder (2017). Some recent research considering void material has been studied by the following researchers. Hilal and Othman (2016) examined the effect of the gravity field in the propagation of plane waves. Othman and Abd–Elaziz (2015) studied the effect of thermal loading due to laser pulse on the thermoelastic medium with voids in the dual-phase lag model (DPL). Othman and Atwa (2012) discussed the deformation of micropolar thermoelastic solid with voids considering the influence of various sources acting on the plane surface. A study in the two-dimensional problem of thermoelastic rotating material with voids under the effect of the gravity and the temperature-dependent properties employing

the two-temperature generalized thermoelasticity in the context of Lord–Shulman (LS) theory has been studied by Othman and Hilal (2015). Othman and Lotfy (2010) examined a problem in micropolar generalized thermoelastic medium with voids under the influence of various sources formulated in the context of the Lord–Shulman theories. Bachhar et al. (2014) studied on the fractional-order GreenLindsay model of generalized thermoelasticity with voids subjected to instantaneous heat sources.

The present paper is devoted to formulating a reference temperature-dependent Lord–Shulman model of generalized thermoelasticity with voids subjected to a continuous heat source in a half-space. We applied this model to solve a problem of determining the distribution of temperature, the volume fraction field, the deformation and the stress field in a semi-infinite elastic medium. Laplace transforms together with eigenvalue approach is applied to find a closed-form solution in the Laplace transform domain. The numerical inversions of the physical variables in the space-time domain are carried out with the help of the Zakian algorithm (1969, 1970). Numerical results are shown graphically and the results obtained are analyzed.

1   Basic equations and formulation of the problem

Following, Iesan (1986), Sherief et al. (2010), and Lord & Shulman (1967), the governing equations for an isotropic homogeneous generalized thermoelastic material (possessing a centre of symmetry) with voids can be put in the following form:

Constitutive equations:

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e_{kk} + b\Phi - \beta\Theta]\delta_{ij}, \tag{1}$$

$$h_i = \alpha\Phi_{,i}, \tag{2}$$

$$g = -be_{kk} - \xi\Phi + m\Theta, \tag{3}$$

$$q_i + \tau_0 \frac{\partial q_i}{\partial t} = -K\Theta_{,i}, \tag{4}$$

$$\rho T_0 \eta = \rho C_E \Theta + \beta e_{kk} + m\Phi, \tag{5}$$

$$\tag{6}$$

The energy equation for the linear theory of thermoelastic material with voids in the presence of heat sources is

$$\rho T_0 \dot{\eta} = -q_{i,i} + \rho Q, \tag{7}$$

Equations of motion:

$$\sigma_{ij,j} + \rho F_i = \rho \ddot{u}_i, \tag{8}$$

Equations of equilibrated forces:

$$h_{i,i} + g + \rho l = \rho\chi\ddot{\Phi}, \tag{9}$$

where σ_{ij} are the components of the stress tensor, e_{ij} are the components of strain tensor, h_i are the components of equilibrated stress tensor, Φ is the change in volume fraction field, ρ is the density, η is the entropy per unit mass, g is the intrinsic equilibrated body force, b is the measure of diffusion effects, α, m, ξ are void material parameters, q_i are the components of heat flux vector, K is the coefficient of thermal conductivity, $\Theta = T - T_0$, T is the absolute temperature, T_0 is the temperature of the medium in its natural state assumed to be such that $|\Theta/T_0| << 1$, $F_i, (i = 1, 2, 3)$ are the components of body forces, l is the extrinsic equilibrated force, χ is the equilibrated inertia, λ, μ are Lame's constants, $\beta = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion, δ_{ij} is the Kronecker delta, u_i are the components of the displacement vector, C_E is the specific heat at constant strain, τ_0 are relaxation time parameters, Q is the internal heat sources.

From Eqs. (1)-(9), the field equations in terms of the displacement, volume fraction and temperature field, for an isotropic homogeneous generalized thermoelastic material with voids and heat transfer subjected to a heat source in the absence of body forces, and extrinsic equilibrated body forces are

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} + b\Phi_{,i} - \beta\Theta_{,i} = \rho\ddot{u}_i, \tag{10}$$

$$KΘ_{,ii} = \left(1 + τ_0 \frac{∂}{∂t}\right)\left(ρC_E \dotΘ + βT_0 \dot u_{k,k} + mT_0 \dotΦ - ρQ\right),\tag{11}$$

$$αΦ_{,ii} - bu_{k,k} - ξΦ + mΘ = ρχ\ddotΦ,\tag{12}$$

The homogeneous and isotropic infinite thermoelastic solid body is unstrained and unstressed initially but has a uniform temperature distribution T_0. Let $x = 0$ represents the plane area over which the heat sources Q are situated and the solid occupies the infinite space $-∞ < x < ∞$. From the symmetry of the problem, all the physical variables considered depending only on the space variable x and time-variable t and thus it follows that for one-dimensional problem $u_1 = u(x,t)$, $u_2 = 0$, $u_3 = 0$. Eqs. (10)-(12), and Eq. (1) maybe put in the following forms:

$$(λ + 2μ)\frac{∂^2 u}{∂x^2} + b\frac{∂Φ}{∂x} - β\frac{∂Θ}{∂x} = ρ\frac{∂^2 u}{∂t^2},\tag{13}$$

$$KΘ_{,ii} = \left(1 + τ_0 \frac{∂}{∂t}\right)\left(ρC_E \dotΘ + βT_0 \dot u_{k,k} + mT_0 \dotΦ - ρQ\right)\tag{14}$$

$$α\frac{∂^2 Φ}{∂x^2} - b\frac{∂u}{∂x} - ξΦ + mΘ = ρχ\frac{∂^2 Φ}{∂t^2},\tag{15}$$

$$σ_{xx} = (λ + 2μ)\frac{∂u}{∂x} + bΦ - βΘ.\tag{16}$$

The aim is to investigate the effect of the temperature dependence of modulus of elasticity keeping the other elastic and thermal parameters as constant. Therefore we may assume Othman (2013) that

$$[λ, μ, β, α, ξ, χ, m, K, b] = [λ_0, μ_0, β_0, α_0, ξ_0, χ_0, m_0, K_0, b_0]f(T)\tag{17}$$

where $λ_0, μ_0, β_0, α_0, ξ_0, χ_0, m_0, K_0, b_0$ are constants, $f(T)$ is a given non–dimensional function of temperature. In case of a temperature-independent modulus of elasticity, $f(T) = 1$, such that $f(T) = (1 - α^* T_0)$ where $α^*$ is called empirical material constant, in the case of the reference temperature independent of modulus of elasticity and thermal conductivity $α^* = 0$.

Under the above assumption equation (13)-(16) becomes,

$$\left[(λ_0 + 2μ_0)\frac{∂^2 u}{∂x^2} + b_0\frac{∂Φ}{∂x} - β_0\frac{∂Θ}{∂x}\right]f(T) = ρ\frac{∂^2 u}{∂t^2},\tag{18}$$

$$\left[K_0\frac{∂^2 Θ}{∂x^2}\right]f(T) = \left(1 + τ_0 \frac{∂}{∂t}\right)\left(ρC_E \dotΘ + β_0 T_0 \dot u_{k,k} f(T) + m_0 T_0 \dotΦ f(T) - ρQ\right)\tag{19}$$

$$α_0\frac{∂^2 Φ}{∂x^2} - b_0\frac{∂u}{∂x} - ξ_0 Φ + m_0 Θ = ρχ_0\frac{∂^2 Φ}{∂t^2},\tag{20}$$

$$σ_{xx} = \left[(λ_0 + 2μ_0)\frac{∂u}{∂x} + b_0 Φ - β_0 Θ\right]f(T).\tag{21}$$

To transform Eqs. (18)-(21) in non–dimensional forms, we will use the following non–dimensional variables

$$(x', u') = \frac{\widetildeω}{c_1}(x, u),\ (t', τ_0') = \widetildeω(t, τ_0),\ σ_{xx}' = \frac{σ_{xx}}{β_0 T_0},$$

$$\Phi' = \frac{\widetilde{\omega}^2 \chi_0}{c_1^2} \phi, \ \alpha' = \frac{\alpha}{\rho c_1^2 \chi_0}, \ \Theta' = \frac{\Theta}{T_0}, \ Q' = \frac{K_0 Q}{\rho c_1^2 c_e^2 T_0}, \varepsilon' = \frac{\beta_0 c_1^2}{K_0 \widetilde{\omega}},$$

where $\widetilde{\omega} = \frac{\rho C_E c_1^2}{k_0}$ and $c_1^2 = \frac{\lambda_0 + 2\mu_0}{\rho}$.

Using the above defined non–dimensional variables, Eqs. (18)-(21) take the following forms (omitting the primes for convenience):

$$\frac{\partial^2 u}{\partial x^2} + g_1 \frac{\partial \Phi}{\partial x} - g_2 \frac{\partial \Theta}{\partial x} = \frac{\partial^2 u}{\partial t^2} \tag{22}$$

$$\frac{\partial^2 \Theta}{\partial x^2} = \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \left[\frac{\dot{\Theta}}{1 - \alpha^* T_0} + \varepsilon \frac{\partial \dot{u}}{\partial x} + g_7 \dot{\Phi} - \frac{Q}{1 - \alpha^* T_0}\right] \tag{23}$$

$$g_3 \frac{\partial^2 \Phi}{\partial x^2} - g_4 \frac{\partial u}{\partial x} - g_5 \Phi + g_6 \Theta = \frac{\partial^2 \Phi}{\partial t^2} \tag{24}$$

$$\sigma_{xx} = g_8 \frac{\partial u}{\partial x} + g_9 \Phi - \Theta. \tag{25}$$

where

$$g_1 = \frac{b_0}{\rho \chi_0 \widetilde{\omega}^2}, \ g_2 = \frac{\beta_0 T_0}{\rho c_1^2}, \ g_3 = \alpha_0, \ g_4 = \frac{b_0}{\rho c_1^2}, \ g_5 = \frac{\xi_0}{\rho \chi_0 \widetilde{\omega}^2},$$

$$g_6 = \frac{m_0 T_0}{\rho c_1^2}, \ g_7 = \frac{m_0 c_1^4}{K_0 \chi_0 \widetilde{\omega}^3}, \ g_8 = \frac{\rho c_1^2}{\beta_0 T_0}, \ g_9 = \frac{b_0 c_1^2}{\beta_0 \chi_0 \widetilde{\omega}^2 T_0}.$$

If the heat source is continuous and acts on the surface $x = 0$, we may represent it as $Q(x,t) = Q_0 \delta(x) H(t)$, where $\delta(x)$ is the Dirac delta function, $H(t)$ is the Heaviside unit step function, and Q_0 is a constant.

2   Solution in the Laplace transform domain: Eigenvalue approach

Taking the Laplace transform of parameter s, defined by

$$L[f(x,t)] = \int_0^\infty \exp(-st) f(x,t) dt = \bar{f}(x,s) \quad (\text{Re}(s) > 0), \tag{26}$$

on both sides of the Eqs. (17)-(20) (assuming the homogeneous initial conditions), we get

$$D^2 \bar{u} = s^2 \bar{u} - g_1 D\bar{\Phi} + g_2 D\bar{\Theta}, \tag{27}$$

$$D^2 \bar{\Phi} = \frac{g_4}{g_3} D\bar{u} + \left(\frac{g_5 + s^2}{g_3}\right) \bar{\Phi} - \frac{g_6}{g_3} \bar{\Theta}, \tag{28}$$

$$D^2 \bar{\Theta} = s(1 + \tau_0 s) \left[\varepsilon D\bar{u} + g_7 \bar{\Phi} + \frac{\bar{\Theta}}{(1 - \alpha^* T_0)} - \frac{Q_0 \delta(x)}{s^2(1 - \alpha^* T_0)}\right], \tag{29}$$

$$\bar{\sigma}_{xx} = g_8 D\bar{u} + g_9 \bar{\Phi} - \bar{\Theta}. \tag{30}$$

Following Sarkar and Lahiri (2012) and Sarkar (2013) , Eqs. (27)-(29) can be written in a *vector–matrix differential equation* as follows:

$$D\tilde{v}(x,s) = \mathcal{A}(s)\tilde{v}(x,s) + \tilde{f}(x,s), \tag{31}$$

where

$$D = \frac{d}{dx}, \quad \tilde{v}(x,s) = \begin{pmatrix} \bar{u} \\ \bar{\Phi} \\ \bar{\Theta} \\ D\bar{u} \\ D\bar{\Phi} \\ D\bar{\Theta} \end{pmatrix}, \quad \mathcal{A}(s) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ C_{41} & 0 & 0 & 0 & C_{45} & C_{46} \\ 0 & C_{52} & C_{53} & C_{54} & 0 & 0 \\ 0 & C_{62} & C_{63} & C_{64} & 0 & 0 \end{pmatrix}, \quad \tilde{f}(x,s) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\bar{Q} \end{pmatrix},$$

$$C_{41} = s^2, \quad C_{45} = -g_1, \quad C_{46} = g_2, \quad C_{52} = \frac{g_5 + s^2}{g_3}, \quad C_{53} = -\frac{g_6}{g_3}, \quad C_{54} = \frac{g_4}{g_3},$$

$$C_{62} = g_7 s(1 + \tau_0 s), \quad C_{63} = \frac{s(1 + \tau_0 s)}{1 - \alpha^* T_0}, \quad C_{64} = \varepsilon s(1 + \tau_0 s), \quad \bar{Q} = \frac{Q_0(1 + \tau_0 s)\delta(x)}{s(1 - \alpha^* T_0)}.$$

Following the solution methodology through eigenvalue approach Sarkar and Lahiri (2012, 2013) and Sarkar (2013), we now proceed to solve the *vector-matrixx differential equation* (31). The characteristic equation of the matrix $\mathcal{A}(s)$ can be written as

$$k^6 - Pk^4 + Qk^2 - R = 0, \tag{32}$$

where

$$P = C_{41} + C_{52} + C_{63} + C_{45}C_{54} + C_{46}C_{64},$$
$$Q = C_{41}(C_{52} + C_{63}) + C_{52}C_{63} - C_{53}C_{62} + C_{46}(C_{52}C_{64} - C_{54}C_{62})$$
$$\quad - C_{45}(C_{53}C_{64} - C_{54}C_{63}),$$
$$R = C_{41}(C_{52}C_{63} - C_{53}C_{62}).$$

Let k_1^2, k_2^2 and k_3^2 be the roots of the above characteristic Eq. (32) with positive real parts. Then all the six roots of the above characteristic equation which are also the eigenvalues of the matrix $\mathcal{A}(s)$ are of the form

$$k = \pm k_1, \quad \pm k_2, \quad \pm k_3,$$

where

$$k_1^2 = \frac{1}{3}(2p\sin q + P),$$
$$k_2^2 = \frac{-1}{3}\left(p[\sqrt{3}\cos q + \sin q] - P\right),$$
$$k_3^2 = \frac{1}{3}\left(p[\sqrt{3}\cos q - \sin q] + P\right),$$

and

$$p = \sqrt{P^2 - 3Q}, \quad q = \frac{\sin^{-1} r}{3}, \quad r = \frac{9PQ - 2P^3 - 27R}{2p^3}.$$

Suppose $\mathcal{X}(k)$ be a right eigenvector corresponding to the eigenvalue k of the matrix $\mathcal{A}(s)$. Then after some simple manipulations, we get

$$\mathcal{X}(k) = \begin{pmatrix} k\left[C_{45}C_{53} - C_{46}(C_{52} - k^2)\right] \\ \left[k^2 C_{46}C_{54} - C_{53}\left(C_{41} - k^2\right)\right] \\ \left[(C_{41} - k^2)(C_{52} - k^2) - k^2 C_{45}C_{54}\right] \\ k^2\left[C_{45}C_{53} - C_{46}(C_{52} - k^2)\right] \\ k\left[k^2 C_{46}C_{54} - C_{53}\left(C_{41} - k^2\right)\right] \\ k\left[(C_{41} - k^2)(C_{52} - k^2) - k^2 C_{45}C_{54}\right] \end{pmatrix}. \tag{33}$$

We can easily calculate the eigenvector $\mathcal{X}_j$ $(j = 1, 2, 3)$ corresponding to the eigenvalue $\pm k_j$ $(j = 1, 2, 3)$ from (33). For our further reference, we shall use the following notations:

$$\mathcal{X}_1 = \mathcal{X}(k_1), \quad \mathcal{X}_2 = \mathcal{X}(-k_1), \quad \mathcal{X}_3 = \mathcal{X}(k_2),$$
$$\mathcal{X}_4 = \mathcal{X}(-k_2), \quad \mathcal{X}_5 = \mathcal{X}(k_3), \quad \mathcal{X}_6 = \mathcal{X}(-k_3). \tag{34}$$

We assume the inverse of the matrix $V = \begin{pmatrix} \mathcal{X}_1, & \mathcal{X}_2, & \mathcal{X}_3, & \mathcal{X}_4, & \mathcal{X}_5, & \mathcal{X}_6 \end{pmatrix}$ as

$$V^{-1} = (w_{ij}), \quad i,j = 1,2,...,6.$$

Hence using the expression for $\tilde{f}(s)$, we can calculate the expression for Q_r as (see Sarkar (2013) for details):

$$Q_r = \sum_{j=1}^{6} w_{rj} f_j = w_{r6} f_6, \ \ f_6 = -\frac{Q_0(1+s)\delta(x)}{s}, \quad r = 1,2,...6. \tag{35}$$

The solution of the *vector–matrix differential equation* (31) can be written as

$$\tilde{v} = X_2 y_2 + X_4 y_4 + X_6 y_6, \tag{36}$$

where

$$y_r = e^{k_r x} \left[y_r e^{-k_r x} \right]_{x=-\infty} + e^{k_r x} \int_{-\infty}^{x} Q_r e^{-k_r x} dx, \ x > 0. \tag{37}$$

Since $y = V^{-1}\tilde{v}$ and the field variables in $\tilde{v}$ vanish at $x = +\infty$, we neglect the first term on the right-hand side of (37), and we get

$$y_2 = -\frac{Q_0(1+s)}{s} w_{26} e^{-k_1 x}, y_4 = -\frac{Q_0(1+s)}{s} w_{46} e^{-k_2 x}, y_6 = -\frac{Q_0(1+s)}{s} w_{66} e^{-k_3 x} \ (x>0), \tag{38}$$

since y_1, y_3 and y_5 are neglected from the physical considerations of the problem.

Thus, we get

$$
\begin{pmatrix} \bar{u} \\ \bar{\Phi} \\ \bar{\Theta} \end{pmatrix} = - \begin{pmatrix} -k_1 \left[C_{45}C_{53} - C_{46}(C_{52} - k_1^2) \right] \\ \left[k_1^2 C_{46}C_{54} - C_{53}\left(C_{41} - k_1^2\right) \right] \\ \left[(C_{41} - k_1^2)(C_{52} - k_1^2) - k_1^2 C_{45}C_{54} \right] \end{pmatrix} \frac{Q_0(1+s^\zeta) w_{26}}{s} e^{-k_1 x}
$$
$$
- \begin{pmatrix} -k_2 \left[C_{45}C_{53} - C_{46}(C_{52} - k_2^2) \right] \\ \left[k_2^2 C_{46}C_{54} - C_{53}\left(C_{41} - k_2^2\right) \right] \\ \left[(C_{41} - k_2^2)(C_{52} - k_2^2) - k_2^2 C_{45}C_{54} \right] \end{pmatrix} \frac{Q_0(1+s^\zeta) w_{46}}{s} e^{-k_2 x}
$$
$$
- \begin{pmatrix} -k_3 \left[C_{45}C_{53} - C_{46}(C_{52} - k_3^2) \right] \\ \left[k_3^2 C_{46}C_{54} - C_{53}\left(C_{41} - k_3^2\right) \right] \\ \left[(C_{41} - k_3^2)(C_{52} - k_3^2) - k_3^2 C_{45}C_{54} \right] \end{pmatrix} \frac{Q_0(1+s^\zeta) w_{66}}{s} e^{-k_3 x}
$$

The expressions for $\bar{u}(x,s)$, $\bar{\phi}(x,s)$ and $\bar{\Theta}(x,s)$ can now be written as

$$\bar{u}(x,s) = \frac{Q_0(1+s^\zeta)}{s} [k_1\{C_{45}C_{53} - C_{46}(C_{52} - k_1^2)\} w_{26} e^{-k_1 x}$$
$$+ k_2\{C_{45}C_{53} - C_{46}(C_{52} - k_2^2)\} w_{46} e^{-k_2 x}$$
$$+ k_3\{C_{45}C_{53} - C_{46}(C_{52} - k_3^2)\} w_{66} e^{-k_3 x}], \tag{39}$$

$$\bar{\Phi}(x,s) = -\frac{Q_0(1+s^\zeta)}{s} [\{k_1^2 C_{46}C_{54} - C_{53}\left(C_{41} - k_1^2\right)\} w_{26} e^{-k_1 x}$$
$$+ \{k_2^2 C_{46}C_{54} - C_{53}\left(C_{41} - k_2^2\right)\} w_{46} e^{-k_2 x}$$
$$+ \{k_3^2 C_{46}C_{54} - C_{53}\left(C_{41} - k_3^2\right)\} w_{66} e^{-k_3 x}], \tag{40}$$

$$\bar{\Theta}(x,s) = -\frac{Q_0(1+s^\zeta)}{s} [\{(C_{41} - k_1^2)(C_{52} - k_1^2) - k_1^2 C_{45}C_{54}\} w_{26} e^{-k_1 x}$$
$$+ \{(C_{41} - k_2^2)(C_{52} - k_2^2) - k_2^2 C_{45}C_{54}\} w_{46} e^{-k_2 x}$$
$$+ \{(C_{41} - k_3^2)(C_{52} - k_3^2) - k_3^2 C_{45}C_{54}\} w_{66} e^{-k_3 x}. \tag{41}$$

Using Eqs. (39)-(41) in the Eq. (30), the stress component $\bar{\tau}_{xx}(x,s)$ can be determined as

$$\bar{\sigma}_{xx}(x,s) = -\frac{g_8 Q_0(1+s^\zeta)}{s}[k_1^2\{C_{45}C_{53} - C_{46}(C_{52} - k_1^2)\}w_{26}e^{-k_1 x}$$
$$+k_2^2\{C_{45}C_{53} - C_{46}(C_{52} - k_2^2)\}w_{46}e^{-k_2 x}$$
$$+k_3^2\{C_{45}C_{53} - C_{46}(C_{52} - k_3^2)\}w_{66}e^{-k_3 x}]$$
$$-\frac{g_9 Q_0(1+s^\zeta)}{s}[\{k_1^2 C_{46}C_{54} - C_{53}(C_{41} - k_1^2)\}w_{26}e^{-k_1 x}$$
$$+\{k_2^2 C_{46}C_{54} - C_{53}(C_{41} - k_2^2)\}w_{46}e^{-k_2 x}$$
$$+\{k_3^2 C_{46}C_{54} - C_{53}(C_{41} - k_3^2)\}w_{66}e^{-k_3 x}]$$
$$+\frac{Q_0(1+s^\zeta)}{s}[\{(C_{41} - k_1^2)(C_{52} - k_1^2) - k_1^2 C_{45}C_{54}\}w_{26}e^{-k_1 x}$$
$$+\{(C_{41} - k_2^2)(C_{52} - k_2^2) - k_2^2 C_{45}C_{54}\}w_{46}e^{-k_2 x}$$
$$+\{(C_{41} - k_3^2)(C_{52} - k_3^2) - k_3^2 C_{45}C_{54}\}w_{66}e^{-k_3 x}. \quad (42)$$

3   Numerical results and discussions

To illustrate and compare the theoretical results obtained in Section 3, we now present some numerical results which depict the variations of temperature, volume fraction field, displacement, and stress component. The material chosen for numerical evaluations is magnesium crystal, for which we take the following values of the different physical constants Dhaliwal and Singh (1980):

$$\lambda = 2.17 \times 10^{10} \ Nm^{-1}, \ \mu = 3.278 \times 10^{10} \ Nm^{-1}, \ \rho = 1.74 \times 10^3 \ kgm^3, \ T_0 = 298^o \ K,$$

$$C_E = 1.04 \times 10^3 \ Jkg^{-1}deg^{-1}, \ k = 1.7 \times 10^2 \ Wm^{-1}deg^{-1}, \ \beta = 2.68 \times 10^6 \ Nm^{-2}deg^{-1}.$$

The void parameters are

$$\chi = 1.753 \times 10^{-15} \ m^2, \ \alpha = 3.688 \times 10^{-5} \ N, \ \xi = 1.475 \times 10^{10} \ Nm^{-2},$$

$$b = 1.13849 \times 10^{10} \ Nm^{-2}, \ m = 2 \times 10^6 \ Nm^{-2}deg^{-1}.$$

The non-dimensional relaxation time is $\tau_0 = 0.02$.

The computations are carried out for $\alpha^* = 0.002, 0.003$ and $t = 1$. The numerical technique of Zakian (1969, 19970) is used to invert the Laplace transforms in (35)-(39), providing the temperature Θ, the volume fraction field Φ, the displacement u and the stress τ_{xx} distributions in the physical domain. The results are represented graphically for different positions of x. The case $\alpha^* = 0$ indicates the case where the elastic module does not depend on the reference temperature.

Figs. 1, 2, 3 and 4 are drawn for a non-dimensional time $t=1$. These figures exhibit the spatial variations of the field quantities in the context of reference temperature-dependent generalized thermoelasticity for different values of empirical material constant α^*.

Fig. 1 depicts the variations of temperature Θ with distance x for different values of α^* and it is noticed that in all the cases (i.e., $\alpha^* = 0$, $\alpha^* = 0.002$ and $\alpha^*=0.003$), Θ attains its maximum value on the boundary of the half-space $x \geq 0$. We observe significant differences in the values of Θ and all the series approach to zero as x increases further.

Fig.2 shows the variations of volume fraction field Φ with x for different values of α^*. It is evident from the figure that all the three series have a similar trend, that is, starting from a maximum value converge to zero finally.

Fig.3 displays the variations of displacement component u for different values of α^*. In all the cases (i.e., $\alpha^* = 0, 0.002, 0.003$), the displacement component attains maximum value at $x = 0.06$, and then continuously decreases to zero. Hence, displacement component has similar trend for all the values of α^*.

Fig.4 shows the variations of stress component σ with x for different values of α^*. It is evident from the figure that all the three series started from a maximum magnitude decreases to zero which ensures the compressive nature of the stress field. The difference is significant.

In all of these four figures, it is observed that all the field variables exhibit its greater magnitude for the smaller value of empirical material constant α^*.
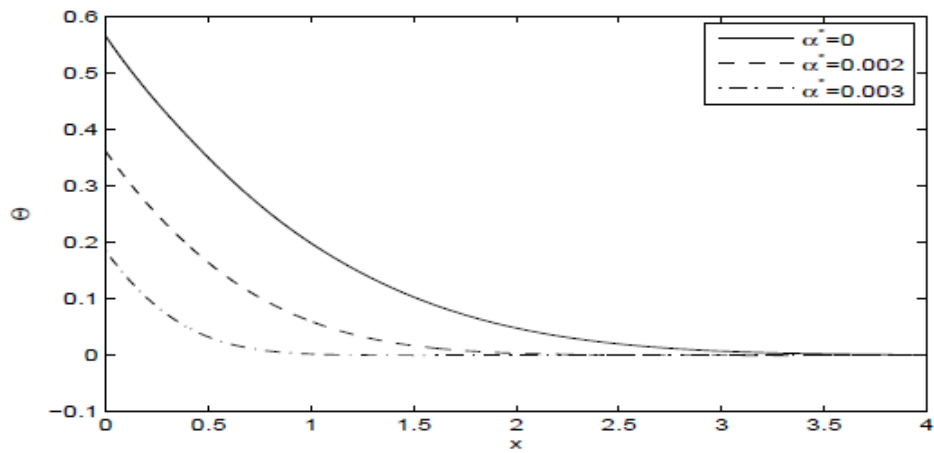

Fig. 1 Temperature distribution Θ at $t = 1$ for different values of α^*.
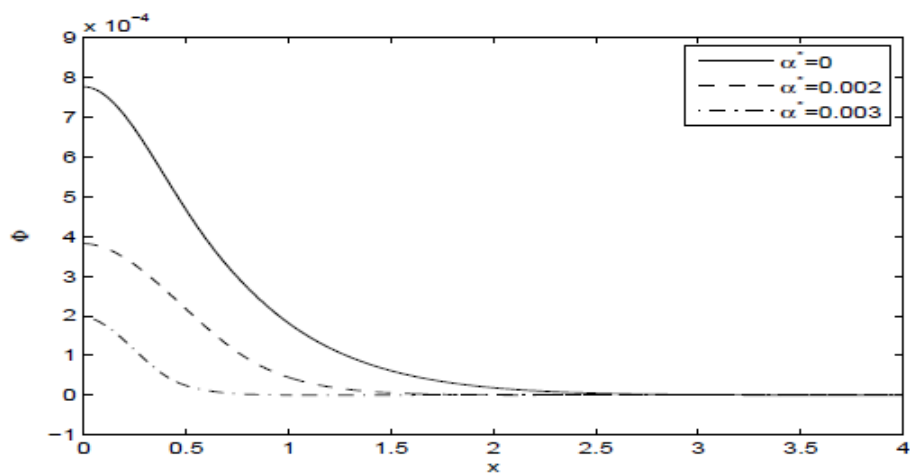

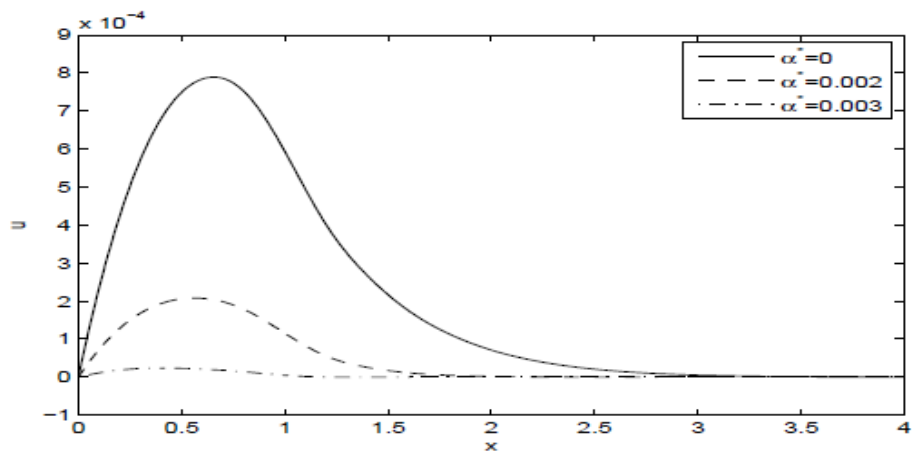Fig. 2 Volume fraction field distribution Φ at $t = 1$ for different values of α^*.

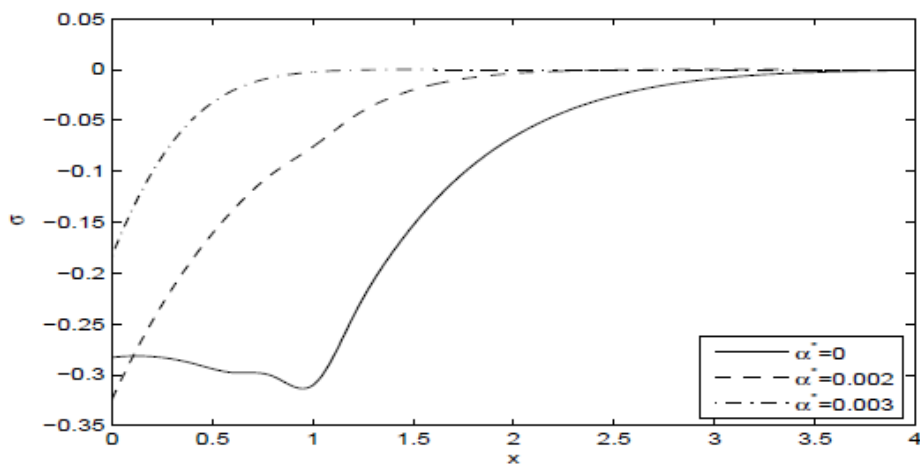Fig. 3 Displacement distribution$u$ at $t = 1$ for different values of α^*.

Fig. 4 Stress distributionσ_{xx} at $t = 1$ for different values of α^*.
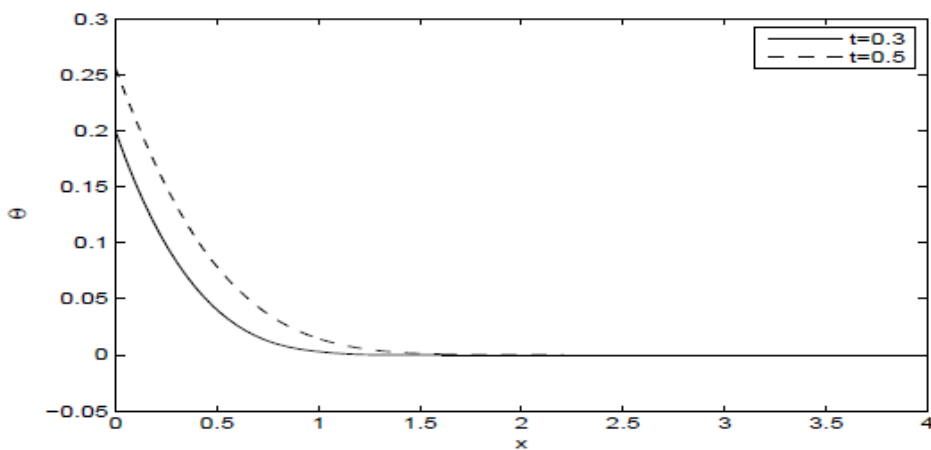

Fig. 5 Temperature distribution Θ at $\alpha^* = 0.002$ for different t.
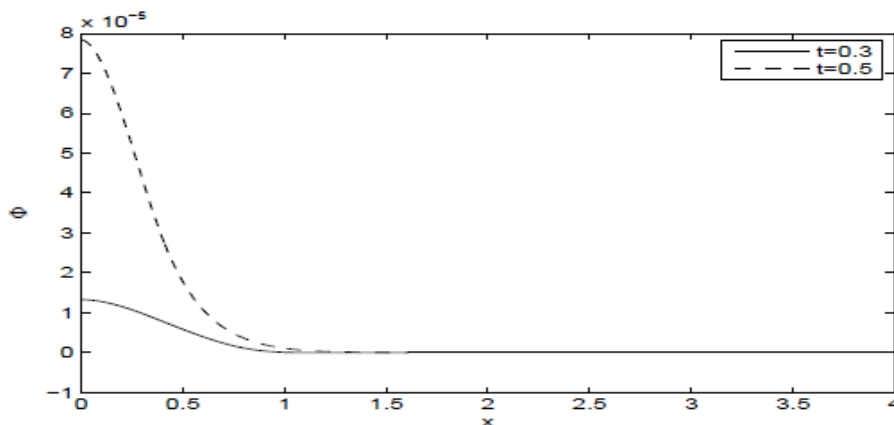
Fig. 6 Volume fraction field distribution Φ at $\alpha^* = 0.002$ for different t.
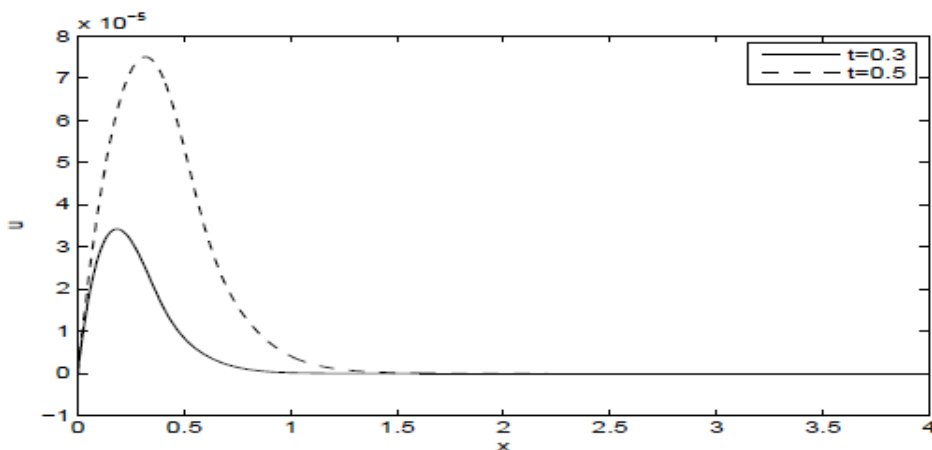

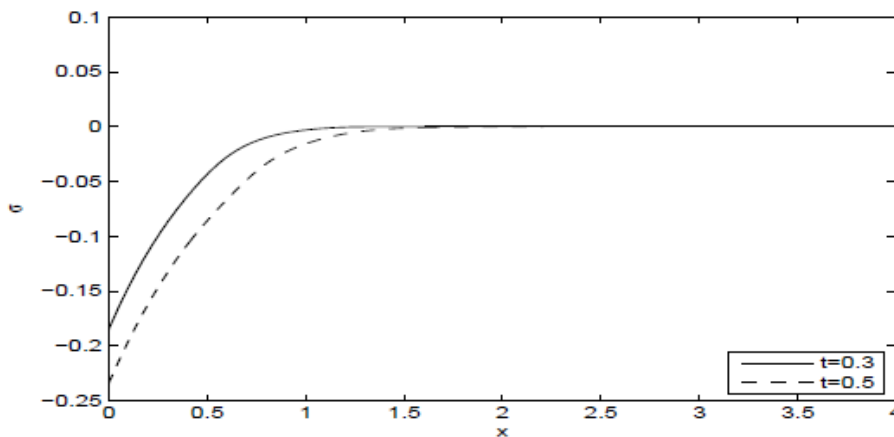Fig. 7 Displacement distribution$u$ at $\alpha^* = 0.002$ for different t.

Fig. 8 Stress distributionσ_{xx} at $\alpha^* = 0.002$ for different t.

Figs. (5)–(8) display the temperature, volume fraction field, displacement, and stress distributions for a wide range of x $(0.0 \leq x \leq 4.0)$ at $\alpha^* = 0.002$ for different values of the time $t = 0.3, 0.5$ and we have noticed that the time parameter t play significant role in all the studied fields. The increasing of the value of t causes increasing of the values

of all the studied fields and makes the speed of the waves propagation vanishes more rapidly.

4   Concluding remarks

(i) The results of this work presents the reference temperature-dependent generalized thermoelasticity theory with voids as a new improvement and progress in the field of the thermoelasticity with voids subjected to a continuous heat source.

(ii) The method *eigenvalue approach* reduced the problem on *vector-matrix differential equation* to algebraic eigenvalue problems and the solutions for the field variables were achieved by determining the eigenvalues and the corresponding eigenvectors of the coefficient matrix. In this method, the physical quantities are directly involved in the formulating of the problem and as such the boundary and initial conditions can be applied directly. This is not in other methods, like *State–Space–Approach.*

(iii) The phenomenon of finite speeds of propagation is observed in all depicted figures. This is expected since the thermal wave travels with finite speed.

(iv) The effects of empirical material constant α^* on all the studied fields are very significant.

(v) The value of time t has also an essential role in changing the value of the distributions.

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