Construction of Codes By Hyper KU-Valued Functions

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Abstract

Coding Theory is a mathematical domain with many applications in Information theory. Various type of codes and their connections with other mathematical objects have been intensively studied. One of these applications, namely connections between binary block codes and BCK-algebras, was recently studied in (Jun, Song, Flaut).In this paper, we will focus to one of the recent applications of KU-algebras in the coding theory, namely the Construction of codes by hyper KU-valued functions. First, we shall introduce the notion of hyper KU -valued functions, on a set and investigate some of it's related properties. Moreover the codes generated by a hyper K U- valued function are constructed and several Examples are given. Furthermore, Example with graphs of binary block code constructed from a hyper KU-algebra and hyper KU-algebra are constructed.

Keywords: Hyper KU-Valued Function, Binary Block Code Of Hyper KU-Valued Functions.

AMS Classification. 06F35

1. Introduction

C.Prabpayak and U.Leerawat ([1], [2]) introduced a new algebraic structure, which is called KU-algebra. They gave the concept of homomorphisms of KU - algebras and investigated some related properties. The hyper structure theory (called also multialgebras) is introduced in 1934 by F. Marty [3] at the 8th congress of Scandinvian Mathematiciens. Around the 40's, several authors worked on hyper groups, especially in France and in the United States, but also in Italy, Russia and Japan. Hyper structures have many applications to several sectors of both pure and applied sciences. In [4], S. M. Mostafa et al. applied the hyper structures to KU-algebras, and introduced the concept of a hyper KU-algebra which is a generalization of a KU-algebra and investigated some related properties. They also introduced the notion of a hyper KU-ideal, a weak hyper KU-ideal and gave relations between hyper KU-ideals and weak hyper KU-ideals. These algebras form an important class of logical algebras and have many applications to various domains of mathematics, such as, group theory, functional analysis, fuzzy sets theory, probability theory, topology, etc. Coding theory is a very young mathematical topic. It started on the basis of transferring information from one place to another. For instance, suppose we are using electronic devices to transfer information (telephone, television, etc.). Here, information is converted into bits of 1's and 0's and sent through a channel, for example a cable or via satellite. Afterwards, the 1's and 0's are reconverted into information again. Due to technical problems, one can assume that while the bits are sent through the channel, there is a positive probability p that single bits are being changed. Thus the received bits could be wrong. The idea of coding theory is to give a method of how to convert the information into bits, such that there are no mistakes in the received information, or such that at least some of them are corrected. On this account, encoding and decoding algorithms are used to convert and reconvert these bits properly. One of the recent applications of BCK/KU-algebras was given in the Coding theory [5,6 ,7]. In Coding Theory, a block code is an error-correcting code which encodes data in blocks. In the paper [6], the authors introduced the notion of BCK-valued functions and investigate several properties. Moreover, they established block-codes by using the notion of BCK-valued functions. they show that every finite BCK-algebra determines a block-code constructed a finite binary block-codes associated to a finite BCK-algebra. In [5,8] provided an algorithm which allows to find a BCK-algebra starting from a given binary block code.

In [8] the authors presented some new connections between BCK- algebras and binary block codes.

In [7] the authors established block-codes by using the notion of KU-valued functions.

In this paper, we will focus to one of the recent applications of KU-algebras in the coding theory, namely the Construction of codes by hyper KU-valued functions. First, we shall introduce the notion of hyper KU -valued functions, on a set and investigate some of it's related properties. Moreover the codes generated by a hyper K U- valued function are constructed and several Examples are given. Furthermore, Example with graphs of binary block code constructed from a hyper KU-algebra and hyper KU-algebra are constructed..

2. Preliminaries

Now, we will recall some known concepts related to hyper KU-algebra from the literature, which will be helpful in further study of this article.

Let H be a nonempty set and $P^*(H) = P(H) \setminus \{\phi\}$ the family of the nonempty subsets of H. A multi valued operation (said also hyper operation) "$\circ$" on H is a function, which associates with every pair $(x, y) \in H \times H = H^2$ a non empty subset of H denoted $x \circ y$ .An algebraic hyper structure or simply a hyper structure is a non empty set H endowed with one or more hyper operations.

We shall use the $x \circ y$ instead of $x \circ \{y\}$, $\{x\} \circ y$ or $\{x\} \circ \{y\}$.

Definition3.1[5] Let H be a nonempty set and "$\circ$" a hyper operation on H, such that $\circ : H \times H \to P^*(H)$. Then H is called a hyper KU-algebra if it contains a constant "0" and satisfies the following axioms: for all $x, y, z \in H$

(HKU_1) $[(z \circ y) \circ (z \circ x)] << y \circ x$

(HKU_2) $x \circ 0 = \{0\}$

(HKU_3) $0 \circ x = \{x\}$

(HKU_4) $if \;\; x << y, \;\; y << x \; implies \;\; x = y$.

where x << y is defined by $0 \in y \circ x$ and for every $A, B \subseteq H$, $A << B$ is defined by $\forall a \in A, \exists b \in B \;\; such \; that \;\; a << b$. In such case, we call "<<" the hyper order in H.

Note that if $A, B \subseteq H$, then by $A \circ B$ we mean the subset $\bigcup\limits_{a \in A, b \in B} a \circ b \;\; of \;\; H$.

Example 3.2.[5] Let $H=\{0,a,b\}$ be a set. Define hyper operation $\circ$ on H as follows:

$\circ$	0	a	b
0	$\{0\}$	$\{a\}$	$\{b\}$
a	$\{0\}$	$\{0,a\}$	$\{a,b\}$
b	$\{0\}$	$\{0,a\}$	$\{0,a,b\}$

Then $(H,\circ,0)$ is a hyper KU-algebra.

Example 3.3. .[5] (i) Define the hyper operation " $\circ$ " on Z as follows:

$$x \circ y = \begin{cases} \{y\} & if \quad x=0 \\ Z & otherwise \end{cases}$$

Then $(Z,\circ,0)$ is a hyper KU-algebra.

(ii) Let $H=\{0,1,2,3\}$ be a set. The hyper operations $\circ_i$ on H are defined as follows.

$\circ_1$	0	1	2	3
0	{0}	{1}	{2}	{3}
1	{0}	{0}	{1}	{3}
2	{0}	{0}	{0}	{0,3}
3	{0}	{0}	{1}	{0,3}

$\circ_2$	0	1	2	3
0	{0}	{1}	{2}	{3}
1	{0}	{0}	{1}	{3}
2	{0}	{0}	{0,1}	{0,3}
3	{0}	{0}	{1}	{0,3}

Then $(H,\circ_i,0)$, i=1,2 are hyper KU-algebras.

Proposition 3.4. .[5] Let H be a hyper KU-algebra. Then for all $x,y,z\in H$, the following statements hold:

(P_1) *A* $\subseteq$ *B* implies $A<<B$, for all nonempty subsets *A*, *B* of *H*.

(P_2) $0\circ 0=\{0\}$.

(P_3) $0<<x$.

(P_4) $z<<z$.

(P_5) $x\circ z<<z$

(P_6) $A\circ 0=\{0\}$.

(P_7) $0 \circ A = A$.

(P_8) $(0 \circ 0) \circ x = \{x\}$ and $(x \circ (0 \circ x)) = \{0\}$.

Lemma 3.5. [5] In hyper KU-algebra $(X, \circ, 0)$, the following hold:

$$x << y \quad \text{imply} \quad y \circ z << x \circ z \quad \text{for all } x, y, z \in X.$$

Lemma 3.6. .[5] In hyper KU-algebra $(X, \circ, 0)$, we have

$$z \circ (y \circ x) = y \circ (z \circ x) \quad \text{for all } x, y, z \in X.$$

Lemma 3.7. .[5] For all $x, y, z \in H$, the following statements hold:

(i) $x \circ y << z \Leftrightarrow z \circ y << x$,

(ii) $0 << A \Rightarrow 0 \in A$,

(iii) $y \in (0 \circ x) \Rightarrow y << x$.

3.Hyper KU-valued functions

In what follows let ***A*** and ***X*** denote a nonempty set and hyper KU-algebra respectively, unless otherwise specified.

Definition 3.1 A mapping $\widetilde{A} : A \to X$ is called hyper KU-valued function (briefly, hyper KU-function) on A.

Definition 3.2 A cut function of $\widetilde{A}$, for $\{q\} \in X$ is defined to be a mapping

$\widetilde{A}_{\{q\}} : A \to \{0,1\}$ such that $(\forall x \in A)\widetilde{A}_{\{q\}}(x) = 1 \Leftrightarrow \widetilde{A}(x) \circ \{q\} = \{0\}$.

Obviously, $\widetilde{A}_{\{q\}}$ is the characteristic function of the following subset of A, called a cut subset or a

{q}- cut of $\widetilde{A}$ i.e $\widetilde{A}_{\{q\}}(x) := \left\{ x \in A : \widetilde{A}(x) \circ \{q\} = \{0\} \right\}$.

Example 3.3 Let A ={x, y, z} and let X = {0, a, b, c} is a hyper KU-algebra with the following Cayley table:

$\circ$	0	a	b	c
0	{0,a}	{a}	{0,b}	{c}
a	{0,a}	{0, a}	{0,b}	{ c}
b	{0,a}	{0,a}	{0,a,b}	{c}
c	{0,a}	{0,a}	{0,a,b}	{0,a,c}

The function $\widetilde{A}: A \to X$ given by $\widetilde{A} = \begin{pmatrix} x & y & z \\ \{a\} & \{b\} & \{c\} \end{pmatrix}$ is hyper KU-function on A, and its cut subsets are: $A_{\{0\}} = \Phi$, $A_{\{a\}} = \{x\}$, $A_{\{b\}} = \{x, y\}$, $A_{\{c\}} = A$.

Lemma 3.4 On hyper KU-algebra (*X; *; 0*). We define a binary relation $<<$ on *X* by putting *x* $<<$ *y* if and only if $0 \in y \circ x$. Then (*X;*<<) is a partially ordered set and 0 is its smallest element.

Proof. Let X be hyper KU-algebra $\forall a,b,c \in X$, we have

1. $<<$ is reflexive as $a << a$.

2. if $a << b,\ b << a$, then by (HKU_4), we have $a = b$. Hence $<<$ is anti-symmetric.

3. if $a << b,\ b << c$,then we want to prove that $a << c$.

Since $c \circ a = 0 \circ (c \circ a) = (c \circ b) \circ (c \circ a) << b \circ a = 0$ we have $c \circ a = 0 \Rightarrow a << c,$ then $<<$ is transitive. Hence $(X, <<)$ is partial order set.

Proposition 3.5 Every hyper KU-function $\widetilde{A}: A \to X$ on A is represented by the infimum of the set $\{\ \{q\} \in X,\ A_{\{q\}}(x) = 1\}$, that is $\forall x \in X$: $\widetilde{A}(x) = \inf\{\ \{q\} \in X,\ \widetilde{A}_{\{q\}}(x) = 1\ \}$.

Proof. For any $x \in A$. Let $\widetilde{A}(x) = \{q\} \in X$, then $\widetilde{A}(x) \circ \{q\} = \{0\}$ *and so* $\widetilde{A}_{\{q\}}(x) = 1 \quad \forall \{q\} \in X$.

Assume that $\widetilde{A}_{\{r\}}(x) = 1$ *for* $\{r\} \in X$, then $\widetilde{A}(x) \circ \{r\} = \{0\} = \{q\} \circ \{r\}$, *i.e* $\{r\} << \{q\}$.

Since $q \in \{\{r\} \in X\ ,\ \widetilde{A}_{\{r\}}(x) = 1\}$, for $x \in A$, $\{r\} \in X$, it follows that $\widetilde{A}(x) = \{q\} = \inf\{\ \{r\} \in X, \widetilde{A}_{\{r\}}(x) = 1\}$.This completes the proof.

Proposition 3.6 Let $\widetilde{A}: A \to X$ be a hyper KU-function on A . If $\{q\} \circ \{p\} = \{0\}$ for all $\{p\}$, $\{q\} \in X$,

we get $A_{\{p\}} \subseteq A_{\{q\}}$.

Proof. Let $\{p\}$, $\{q\} \in X$, be such that $\{q\} \circ \{p\} = \{0\}$ and $x \in A_{\{p\}}$, then $\{0\} = \widetilde{A}(x) \circ \{p\}$

Using (Hku_1) and (Hku_2), we have

$\{0\} = \overbrace{(\{q\} \circ \{p\}) \circ (\widetilde{A}(x) \circ \{P\})}^{(\mathrm{HKU}_2)} = (\widetilde{A}(x) \circ \{q\})$, and so $x \in A_{\{q\}}$. Therefore $A_{\{p\}} \subseteq A_{\{q\}}$. This completes the proof

Proposition 3.7 Let $\widetilde{A}: A \to X$ be hyper KU-function on A. Then

$$(\forall x, y \in A)(\widetilde{A}(x) \neq \widetilde{A}(y) \Leftrightarrow A_{\widetilde{A}(x)} \neq A_{\widetilde{A}(y)}$$

Proof. (1) The sufficiency is obvious. Assume that $A_{\widetilde{A}(x)} \neq A_{\widetilde{A}(y)}$ for all $x, y \in A$.Then

$A_{\widetilde{A}(y)} \circ A_{\widetilde{A}(x)} \neq \Phi$ *or* $A_{\widetilde{A}(x)} \circ A_{\widetilde{A}(y)} \neq \Phi$.Thus

$$A_{\widetilde{A}(x)} = \left\{ \ z \in A, \ \widetilde{A}(z) \circ \widetilde{A}(x) = \{0\} \ \right\} \neq \left\{ \ z \in A, \widetilde{A}(z) \circ \widetilde{A}(y) = \{0\} \ \right\} = A_{\widetilde{A}(y)}$$

Corollary 3.8 Let $\widetilde{A} : A \to X$ be hyper KU-function on A.Then

$$(\forall x, y \in A)(\widetilde{A}(x) \circ \widetilde{A}(y) = \{0\} \Leftrightarrow A_{\widetilde{A}(y)} \subseteq A_{\widetilde{A}(x)}).$$

Proof. Straightforward.

For a hyper KU-function $\widetilde{A} : A \to X$, consider the following sets:

$$A_x = \left\{ \ A_{\{q\}} : \{q\} \in X \right\}, \quad \widetilde{A}_x = \left\{ \ \widetilde{A}_{\{q\}} : \{q\} \in X \right\} .$$

Proposition 3.9 Let $\widetilde{A} : A \to X$ be hyper KU-function on A. Then

$$(\forall Y \subseteq X) \quad A_{\inf(\{q\}:\{q\} \in Y)} = \bigcup \{A_{\{q\}} : \{q\} \in Y\}.$$

Proof. Let $(Y \subseteq X), \quad x \in A_{\inf(\{q\}:\{q\} \in Y)}$. We have

$$x \in A_{\inf(\{q\}:\{q\} \in Y)} \Leftrightarrow \widetilde{A}(x) \circ \inf \{\{q\} : \{q\} \in Y\} = \{0\} \Leftrightarrow (\forall \{r\} \in Y)(\widetilde{A}(x) \circ \{r\} = \{0\}) \Leftrightarrow (\forall \{r\} \in Y)(x \in A_{\{r\}}) \Leftrightarrow$$
$$x \in \bigcup \{A_{\{q\}} : \{q\} \in Y\}.$$
This completes the proof.

Corollary 3.10 Let $\widetilde{A} : A \to X$ be KU-function on A, where X is a bounded hyper KU-algebra, then $\forall S \subseteq X$, $A_{\inf(\{q\}:\{q\} \in S)} = \bigcup \{A_{\{q\}} : \{q\} \in S\}$.

Corollary 3.11 Let $\widetilde{A} : A \to X$ be hyper KU-function on A, assume that for any $Y \subseteq X$, there exists a infimum of Y such that ($\forall \{p\}, \{q\} \in Y$), we have $A_{\{p\}} \cup A_{\{q\}} \in A_X$.

The following example shows that the converse of the corollary 3.10 may not true in general.

$\circ$	0	a	b	c	d

Example 3.12.

0	{0}	{a}	{b}	{c}	{d}
a	{0}	{0}	{b}	{b}	{a}
b	{0}	{a}	{0}	{a}	{d}
c	{0}	{0}	{0}	0	{a}
d	{0}	{0}	{b}	{b}	{0}

Let $A=\{x,y\}$ *be a set and let* $X=\{0,a,b,c,d\}$ be hyper KU-algebra with the following Cayley table:

The function $\widetilde{A}:A\to X$ given by

$\widetilde{A}=\begin{pmatrix} x & y \\ \{a\} & \{b\} \end{pmatrix}$ is a hyper KU-function on A, then

	x	y
	{a}	{b}
$\widetilde{A}_{\{0\}}$	0	0
$\widetilde{A}_{\{a\}}$	1	0
$\widetilde{A}_{\{b\{}$	0	1
$\widetilde{A}_{\{c\}}$	1	1
$\widetilde{A}_{\{d\}}$	1	0

And its cut subsets are $A_{\{0\}} = \Phi$, $A_{\{a\}} = \{x\}$, $A_{\{b\}} = \{y\}$, $A_{\{c\}} = \{x, y\}$, $A_{\{d\}} = \{x\}$

Note that $A_{\{a\}} \cup A_{\{b\}} = \{x\} \cup \{y\} \notin A_X$, but $\inf \{\{a\},\{b\}\}$ does exists in X .

Proposition 3.13Let $\tilde{A} : A \to X$ be hyper KU-function on A, then

$$\bigcap \left\{ A_{\{q\}} \{|q\} \in X \right\} = A$$

Proof. Obviously, $\bigcap \left\{ A_{\{q\}} |\{q\} \in X \right\} \subseteq A$.For every $x \in A$, let $\tilde{A}(x) = \{q\} \in X$ .Then $x \in A_{\{q\}}$ and hence

$x \in \bigcap \left\{ A_{\{q\}} |\{q\} \in X \right\}$ .Thus $A \subseteq \bigcap \left\{ A_{\{q\}} |\{q\} \in X \right\}$ .Therefore the result is valid.

Proposition 3.14 Let $\tilde{A} : A \to X$ be hyper KU-function on A, then

$$(\forall x \in A)(\bigcup \left\{ A_{\{q\}} \middle| x \in A_{\{q\}} \right\} \in A_X)$$

Proof. Note that for any $x \in A, x \in A_{\{q\}} \Leftrightarrow \tilde{A}_{\{q\}}(x) = 1$,

From Proposition 3.7 we get the following

$\bigcup \left\{ A_{\{q\}} \middle| x \in A_{\{q\}} \right\} = \bigcup \left\{ A_{\{q\}} \middle| \tilde{A}_{\{q\}}(x) = 1 \right\} = A_{\inf \left\{ \{q\} \middle| \tilde{A}_{\{q\}}(x) = 1 \right\}} \in A_{\{q\}}$. This completes the proof.

Let $\tilde{A} : A \to X$ be KU-function on A and Θ be a binary hyper operation on X defined by $\forall \{p\},\{q\} \in X (\{p\}\Theta\{q\} \Leftrightarrow A_{\{p\}} = A_{\{q\}})$. Then Θ is clearly an equivalence relation on X.

Let $\tilde{A}(A) = \left\{ \{q\} \in X \middle| \tilde{A}(x) = \{q\} \ \ for \ \ some \ x \in A \right\}$ and for $\{q\} \in X$, $(q] = \left\{ x \in X \middle| x \circ q = \{0\} \right\}$.

Proposition 3.15 For a KU-function $\tilde{A} : A \to X$ on A, we have

$$\forall \{p\},\{q\} \in X (\{p\}\Theta\{q\} \Leftrightarrow (\{p\}] \cup \tilde{A}(A) = (\{q\}] \cup \tilde{A}(A)$$

Proof. We have $\{p\}\Theta\{q\} \Leftrightarrow A_{\{p\}} = A_{\{q\}}$

$$\begin{aligned} &\Leftrightarrow (\forall x \in A) \left\{ \tilde{A}(x) \circ \{p\} = \{0\} \Leftrightarrow \tilde{A}(x) \circ \{q\} = \{0\} \right\} \\ &\Leftrightarrow \left\{ x \in A \middle| \tilde{A}(x) \in (\{p\}] \right\} = \left\{ x \in A \middle| \tilde{A}(x) \in (\{q\}] \right\} \\ &\Leftrightarrow (\{p\}] \cup \tilde{A}(A) = (\{q\}] \cup \tilde{A}(A). \end{aligned}$$

This completes the proof.

Example 3.16 Let $X = \{\{a_n\}; n = 1,2,3,......,9\}$ and define a binary operation $\circ$ on X as follows

$(\forall a_i, a_j \in X) \ (\{a_i\} \circ \{a_j\} = \{a_k\})$, where $k = \dfrac{j}{(i,j)}$ $\ and \ (i,j)$ is the least common divisor of $i \ \ and \ \ j$.

Then $(X; \circ, a_i)$ is a hyper KU-algebra. Its Cayley table is as follows:

Let

$\circ$	$\{a_1\}$	$\{a_2\}$	$\{a_3\}$	$\{a_4\}$	$\{a_5\}$	$\{a_6\}$	$\{a_7\}$	$\{a_8\}$	$\{a_9\}$
$\{a_1\}$	$\{a_1\}$	$\{a_2\}$	$\{a_3\}$	$\{a_4\}$	$\{a_5\}$	$\{a_6\}$	$\{a_7\}$	$\{a_8\}$	$\{a_9\}$
$\{a_2\}$	$\{a_1\}$	$\{a_1\}$	$\{a_3\}$	$\{a_2\}$	$\{a_5\}$	$\{a_3\}$	$\{a_7\}$	$\{a_4\}$	$\{a_9\}$
$\{a_3\}$	$\{a_1\}$	$\{a_2\}$	$\{a_1\}$	$\{a_4\}$	$\{a_5\}$	$\{a_2\}$	$\{a_7\}$	$\{a_8\}$	$\{a_3\}$
$\{a_4\}$	$\{a_1\}$	$\{a_1\}$	$\{a_3\}$	$\{a_1\}$	$\{a_5\}$	$\{a_3\}$	$\{a_7\}$	$\{a_2\}$	$\{a_9\}$
$\{a_5\}$	$\{a_1\}$	$\{a_2\}$	$\{a_3\}$	$\{a_4\}$	$\{a_1\}$	$\{a_6\}$	$\{a_7\}$	$\{a_8\}$	$\{a_9\}$
$\{a_6\}$	$\{a_1\}$	$\{a_1\}$	$\{a_1\}$	$\{a_2\}$	$\{a_5\}$	$\{a_1\}$	$\{a_7\}$	$\{a_4\}$	$\{a_3\}$
$\{a_7\}$	$\{a_1\}$	$\{a_2\}$	$\{a_3\}$	$\{a_4\}$	$\{a_5\}$	$\{a_6\}$	$\{a_1\}$	$\{a_8\}$	$\{a_9\}$
$\{a_8\}$	$\{a_1\}$	$\{a_1\}$	$\{a_3\}$	$\{a_1\}$	$\{a_5\}$	$\{a_3\}$	$\{a_1\}$	$\{a_1\}$	$\{a_9\}$
$\{a_9\}$	$\{a_1\}$	$\{a_2\}$	$\{a_1\}$	$\{a_4\}$	$\{a_5\}$	$\{a_2\}$	$\{a_7\}$	$\{a_8\}$	$\{a_1\}$

$A=\{a,b,c,d,e\}$ and $\tilde{A}:A\rightarrow X$ be hyper KU-function defined by:

$\tilde{A}=\begin{pmatrix} a & b & c & d & e \\ \{a_4\} & \{a_6\} & \{a_7\} & \{a_1\} & \{a_2\} \end{pmatrix}$ Then

$\circ$	a $\{a_4\}$	b $\{a_6\}$	c $\{a_7\}$	d $\{a_1\}$	e $\{a_2\}$
$\tilde{A}_{a_1}$	0	0	0	1	0
$\tilde{A}_{a_2}$	0	0	0	1	1
$\tilde{A}_{a_3}$	0	0	0	1	0
$\tilde{A}_{a_4}$	1	0	0	1	1
$\tilde{A}_{a_5}$	0	0	0	1	0
$\tilde{A}_{a_6}$	0	1	0	1	1
$\tilde{A}_{a_7}$	0	0	1	1	0
$\tilde{A}_{a_8}$	1	0	0	1	1

$\tilde{A}_{a_9}$	0	0	0	1	0

and cut sets of Ã are as follows:

$$\tilde{A}_{\{a_1\}} = \tilde{A}_{\{a_3\}} = \tilde{A}_{\{a_5\}} = \tilde{A}_{\{a_9\}} = \{d\}, \tilde{A}_{\{a_2\}} = \{d,e\}, \tilde{A}_{\{a_4\}} = \tilde{A}_{\{a_8\}} = \{a,d,e\}, \tilde{A}_{\{a_6\}} = \{b,d,e\}, \tilde{A}_{\{a_7\}} = \{c,d\}.$$

4. Codes generates by hyper KU-functions

Let $x/_{\Theta} = \{\, y \in A;\ \ x\Theta y \,\}$; for any $x \in A$, $x/_{\Theta}$ is called equivalence class containing x .

Lemma 4.1 Let $\tilde{A} : A \to X$ be a hyper KU- function on A . For every $x \in A$, we have $\tilde{A}(x) = \inf \{x/_{\Theta}\}$, that is $\tilde{A}(x)$ the least element of the Θ to which it belongs.

Proof. Straightforward.

Let $A = \{1,2,3,.....,n\}$ and X be a finite hyper KU-algebra. Then every hyper KU-function

$\tilde{A} : A \to X$ on A determines a binary block code V of length n in the following way: To every $x/_{\Theta}$, where $x \in A$, there corresponds a codeword $V_x = x_1\, x_2 .....x_n$

Such that $x_i = x_j \Leftrightarrow \tilde{A}_x(i) = j \ \ for\ \ i \in A\ \ and\ \ j \in \{0,1\}$.

Let $V_x = x_1\, x_2 .....x_n$, $V_y = y_1\, y_2 .....y_n$ be two code words belonging to a binary block-code V .

Define an order hyper relation $<<_c$ on the set of code words belonging to a binary block- code V as follows: $V_x <<_c V_y \Leftrightarrow x_i << y_i \ \ for\ \ i = 1,2,....,n$ ...... (4.1)

Example 4.2 Let $X = \{0,a,b,c\}$ be hyper KU-algebra with the following Cayley table**:**

∘	0	a	b	c
0	{0}	{a}	{b}	{c}
a	{0}	{0}	{a}	{c}
b	{0}	{0}	{0}	{c}
c	{0}	{a}	{b}	{0}

Let $\tilde{A} : X \to X$ be hyper KU-function on X given by

$$\tilde{A} = \begin{pmatrix} \{0\} & \{a\} & \{b\} & \{c\} \\ \{0\} & \{a\} & \{b\} & \{c\} \end{pmatrix}.$$

Then

$\tilde{A}_{\{x\}}$	{0}	{a}	{b}	{c}
$\tilde{A}_{\{0\}}$	1	0	0	0
$\tilde{A}_{\{a\}}$	1	1	0	0
$\tilde{A}_{\{b\}}$	1	1	1	0
$\tilde{A}_{\{c\}}$	1	0	0	1

$V = \{1000, 1100, 1110, 1001\}$. See Figure (1)

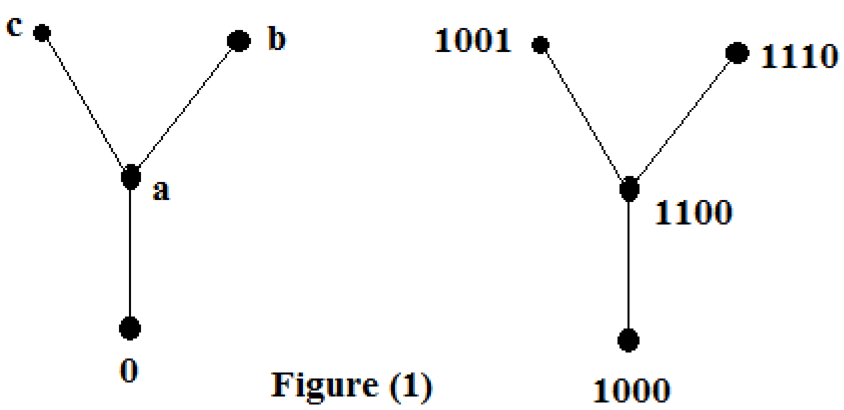

Figure (1)

$(X, <<)$ $(V, <<)$

Generally, we have the following theorem.

Theorem 4.3 Every finite hyper KU-algebra X determines a block-code V such that $(X, <<)$ is isomorphic to $(X, <<_c)$.

Proof. Let $X = \{a_i; i = 1,2,3,......,n\}$ be a finite KU-algebra in which a_1 is the least element and let $\tilde{A}: X \to X$ be identify KU-function on X .The decomposition of $\tilde{A}$ provides a family $\{\tilde{A}_{\{q\}} | \{q\} \in X\}$ which is the desired code under the order $V_x <<_c V_y \Leftrightarrow x_i << y_i \ \ for \ \ i = 1,2,....,n$

Let $f : X \to \{\widetilde{A}_{\{q\}}; \{q\} \in X\}$ be a function defined by $f(\{q\}) = \widetilde{A}_{\{q\}}$ for all $\{q\} \in X$ .By lemma 4.1, every Θ class contains exactly one element. So, f is one to one. Let $x, y \in X$ be such that $\{y\} \circ \{x\} = \{a_1\}$ $i.e$ $x << y$. Then $A_x \subseteq A_y$ (by Proposition 3.5), which means that $\widetilde{A}_x \subseteq \widetilde{A}_y$. Therefore f is an isomorphism. This completes the proof.

Example 4.4

Consider hyper KU-algebra $X = \{\{a_n\}; n = 1,2,3,\ldots\ldots,9\}$ which is considered in example 3.15.

Let $\widetilde{A} : X \to X$ be a hyper KU-function on X given by

$$\widetilde{A} = \begin{pmatrix} \{a_1\} & \{a_2\} & \{a_3\} & \{a_4\} & \{a_5\} & \{a_6\} & \{a_7\} & \{a_8\} & \{a_9\} \\ \{a_1\} & \{a_2\} & \{a_3\} & \{a_4\} & \{a_5\} & \{a_6\} & \{a_7\} & \{a_8\} & \{a_9\} \end{pmatrix}.$$

Then

$\circ$	$\{a_1\}$	$\{a_2\}$	$\{a_3\}$	$\{a_4\}$	$\{a_5\}$	$\{a_6\}$	$\{a_7\}$	$\{a_8\}$	$\{a_9\}$
$\widetilde{A}_{\{a_1\}}$	1	0	0	0	0	0	0	0	0
$\widetilde{A}_{\{a_1\}}$	1	1	0	0	0	0	0	0	0
$\widetilde{A}_{\{a_2\}}$	1	0	1	0	0	0	0	0	0
$\widetilde{A}_{\{a_3\}}$	1	1	0	1	0	0	0	0	0
$\widetilde{A}_{\{a_4\}}$	1	0	0	0	1	0	0	0	0
$\widetilde{A}_{\{a_5\}}$	1	1	1	0	0	1	0	0	0
$\widetilde{A}_{\{a_6\}}$	1	0	0	0	0	0	1	0	0
$\widetilde{A}_{\{a_7\}}$	1	1	0	1	0	0	0	1	0
$\widetilde{A}_{\{a_8\}}$	1	0	1	0	0	0	0	0	1
$\widetilde{A}_{\{a_9\}}$	1	0	1	0	0	0	0	0	1

Thus V= {100000000, 110000000, 101000000, 110100000, 100010000, 11100100,100000100, 110100010, 101000001}. See Figure (2)

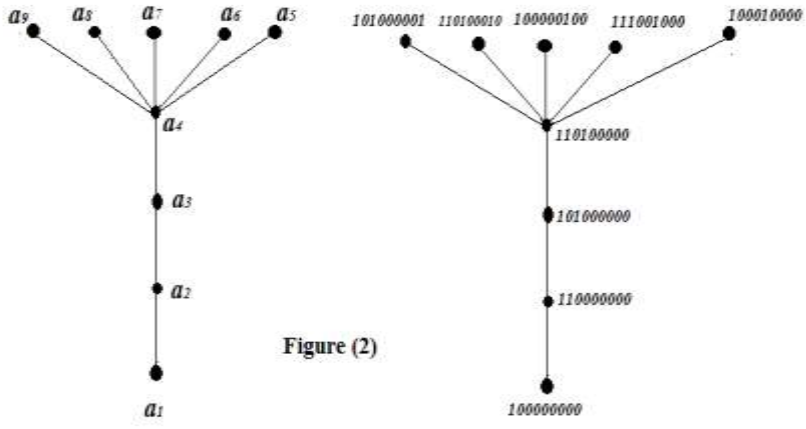

Figure (2)

$(X, <<)$ $(V, <<)$

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