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Energy Efficient Resource Allocation for Multiuser Relay Networks

Keshav Singh, *Member, IEEE*, Ankit Gupta, and Tharmalingam Ratnarajah, *Senior Member, IEEE*

Abstract—In this paper, a novel resource allocation algorithm is investigated to maximize the energy efficiency (EE) in multiuser decode-and-forward (DF) relay interference networks. The EE optimization problem is formulated as the ratio of the spectrum efficiency (SE) over the entire power consumption of the network subject to total transmit power, subcarrier pairing and allocation constraints. The formulated problem is a nonconvex fractional mixed binary integer programming problem, i.e., NP-hard to solve. Further, we resolve the convexity of the problem by a series of convex transformations and propose an iterative EE maximization (EEM) algorithm to jointly determine the optimal subcarrier pairing at the relay, subcarrier allocation to each user pair and power allocation to all source and the relay nodes. Additionally, we derive an asymptotically optimal solution by using the dual decomposition method. To gain more insights into the obtained solutions, we further analyze the resource allocation algorithm in a two-user case with interference-dominated and noise-dominated regimes. In addition, a suboptimal algorithm is investigated with reduced complexity at the cost of acceptable performance degradation. Simulation results are used to evaluate the performance of the proposed algorithms and demonstrate the impacts of various network parameters on the attainable EE and SE.

Index Terms—Energy efficiency, resource allocation, multiuser, decode-and-forward, relay networks.

I. INTRODUCTION

The cooperative communication and small cell have emerged as promising future technologies for improving the network throughput, enlarging the transmission range of wireless networks and enhancing the link reliability [1]. The relay networks can swiftly increase the spectral efficiency (SE) of the network. However, the power dissipation, which is not only due to transceiver but also due to complete radio access network, increases significantly and are predicted to surge rapidly and reach to the current level of the total electricity consumption in the next 20-25 years [2]. To enhance the energy efficiency (EE) of wireless networks is of paramount importance in realizing 5G radio access solutions. Consequently, it is urgent to investigate energy-aware architecture and resource allocation techniques that prolong the network

lifespan or provide significant energy savings under the umbrella of the green communications [3].

Various relaying protocols have been proposed for cooperative networks, among which amplify-and-forward (AF), decode-and-forward (DF) [1] and compress-and-forward (CF) [4] are three common ones. In AF protocol, the relay retransmits the amplified signal to the destination, whereas in the DF protocol, the relay first attempts to decode the received signal and then forwards the re-encoded information bits to the destination. In CF protocol, the relay compresses the received signal and sends the compressed signal to the destination. The AF protocol has an advantage over others in terms of low implementation complexity. However, the DF protocol performs better than other two protocols when the channel quality of forward links, i.e., source-to-relay (SR) links is good enough. Another advantage of the DF protocol is that it is possible to use different channel coding schemes at the source and the relay nodes and the transmission can be optimized for both links. i.e., SR and relay-to-destination (RD), separately. Moreover, multiuser interference channel in a relay-assisted network becomes a major bottleneck for improving the networks performance due to the increase in the number of interfering sources. Exploiting multicarrier, resource management, and beamforming techniques, users in the network can share the resources and alleviate severe interference generated from other users. Thus, in this paper, we investigate resource allocation in multiuser DF relay interference network for enhancing the energy utilization among users, and illustrate the impact of various network parameters on the performance tradeoff between the EE and SE.

In recent years, the energy dissipation at battery-powered mobile devices has increased rapidly due to the diverse and ubiquitous wireless services. Unfortunately, we cannot increase the battery capacity of devices because of the slow advance of battery technology and size limit. Thus, the optimization of power usage in multiuser relay networks becomes a critical issue. However, the interference, energy consumption and throughput can be controlled through optimization of resource usage. The power dissipation in wireless networks are generally classified into two main categories: dynamic power dissipation and static power dissipation. The dynamic power dissipation denotes the transmit power which is allocated in response to the instantaneous channel conditions, whereas the static power dissipation covers site cooling, signal processing at each node and battery backup [5]. The EE performance of multiuser multicarrier DF relay networks highly depends on these two sources [6]. In this paper, we will focus on maximizing the EE of the DF relay interference network by

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joint optimization of resource allocation with consideration of the dynamic and the total static power consumption.

Recently, resource allocation schemes have been studied for improving the performance of the DF relay networks [7]–[14]. In [7] and [8], the optimal power allocation policies were investigated for rate maximization of the DF networks under a sum power constraint, or individual power constraints at the source and the relay nodes. However, the joint subcarrier and power allocation was not addressed in [7] and [8]. To further improve the network performance, the optimization problem for joint subcarrier and power allocation with a total network power constraint or with individual power constraints for the source and the relay nodes were formulated in [9]–[14]. The works in [9] studied the subcarrier pairing and power allocation problem for AF and DF schemes under a total power constraint and provided the optimality of ordered subcarrier pairing (OSP) without diversity. To overcome the interference, the authors in [10] jointly optimized power allocation, relay selection and subcarrier assignment. The optimal subcarrier and power allocation schemes were investigated in [11] for a two-hop DF orthogonal frequency and code division multiplexing (OFCDM) based relay network and analyzed the effects of the channel state information (CSI) under the relay and direct-link mode, whereas the subcarrier-pairing and power allocation were optimized to maximize the weighted sum rate for a point-to-point orthogonal frequency division multiplexing (OFDM) with DF relay network in [12]. Under a perfect self-interference cancellation, a joint resource allocation scheme for orthogonal frequency division multiple access (OFDMA) assisted multiuser two-way AF relay network was proposed in [14] for maximizing the total sum rate. In [9]–[14], the optimization of sum rate, power and subcarrier allocation in high signal-to-noise ratio (SNR) region was the focal point for achieving higher system throughput without concerning the energy dissipation in the networks and balancing the sum rate of different links. Many recent research works on energy efficient resource allocation have been reported in [6], [15]–[19]. The utility-based dynamic resource allocation algorithm in relay-aided OFDMA system was investigated in [15] for maximizing the average utility of all users with multiservice, whereas the issue has not been studied from the viewpoint of the EE. In [16], an energy efficient resource scheduling solution for downlink transmission in multiuser OFDMA networks was proposed under imperfect CSI, while the authors in [17] extended the work of [16] for multicarrier under perfect CSI knowledge and studied the joint subcarrier and power allocation problem under a total power constraint for downlink multiuser OFDMA system. The resource allocation problem in [16] and [17] was optimized only in downlink scenario for maximizing EE without considering the multiuser interference in the network which could be a restraining factor in the performance enhancement when the number of mobile users increases. The only power allocation policies for enhancing EE of AF networks were considered in [19]. However, the optimal joint subcarrier and power allocation schemes for DF relay networks will not be same when we consider the network EE as objective function. Therefore, there is a need to revisit the design of existing DF relay interference networks and

investigate the associated resource allocation policies in order to improve the network EE. To the best of authors knowledge, no work has been reported yet on EE maximization problem by jointly considering subcarrier pairing permutation, subcarrier allocation, and power optimization in multiuser DF relay interference network, in which the multiuser interference could be a restraining factor in the performance enhancement when the number of users increases.

In this paper, we consider a joint resource allocation problem in multiuser multicarrier DF relay networks, where all the nodes in the network are equipped with a single antenna and users communicate with each other through a single half-duplex DF relay node. The major contributions and ingenious novelty of our work are summarized as follows. We formulate EE maximization (EEM) problem in the context of a multiuser DF relay interference network as a ratio of the total achievable sum rate over the entire power consumption in the network subject to a total transmit power constraint, which jointly addresses both the subcarrier and power allocation. Unlike [9]–[14], in this paper, the primary goal is to maximize EE. Since, the original optimization problem is nonconvex fractional mixed binary integer programming problem, i.e. NP-hard to solve, we use successive convex approximation (SCA) [20] and continuous convex relaxations to transform the problem into tractable convex one. Next, the joint quasi-concavity of EE problem on joint subcarrier and power allocation matrices is derived and based on the optimization results, we obtain an asymptotically optimal solution by using dual decomposition method. Furthermore, to gain more insights into the optimal solutions, we consider energy efficient joint resource allocation for two-user scenario and analyze the optimal resource allocation algorithms in interference-dominated and noise-dominated regimes. Besides, we also propose a suboptimal algorithm with abridged complexity but acceptable performance degradation. The complexity of the proposed algorithms is further analyzed. The performance of the proposed algorithms and the impact of various network parameters on the attainable EE and SE are demonstrated through computer simulations.

The remainder of this paper is organized as follows. In Section II, we describe the network model of multiuser relay transmission, introduce the power dissipation model, and formulate the joint resource allocation optimization problem. Transformation of EE nonconvex optimization problem into convex is illustrated in Section III. An energy efficient iterative resource allocation algorithm for achieving the maximum EE is presented in Section IV. We then analyze the resource allocation policies for two-user cases under two different operating regimes in Section V. The suboptimal resource allocation algorithm is investigated in Section VI. The complexity analysis of the proposed algorithms is presented in Section VII, followed by the simulation results in Section VIII. Finally, the conclusions are given in Section IX.

Notations: The following notation conventions are considered in this paper. Boldface lowercase and uppercase letters (e.g., \mathbf{a} and \mathbf{A}) are used to represent a vector and a matrix, respectively. The notations $(\cdot)^\dagger$ and $E(\cdot)$ denote the conjugate transpose and expectation, respectively.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a multiuser DF relay network with N_{sc} subcarriers as illustrated in Fig.1, where N source nodes (S_i , for $i = 1, \dots, N$) concurrently transmit the signals to their designated destination nodes (D_i , for $i = 1, \dots, N$) by the assistance of an intermediate relay node (R). It is assumed that each node operates with only one antenna that does not transmit and receive signals simultaneously. The SR and RD channels on any subcarrier are assumed to be Rayleigh frequency flat fading. Also assume that there is no direct link between the source and the destination nodes due to the path loss and large-scale fading. For simplicity, we assume that the relay node has a perfect knowledge of CSI of each link. Further, the relay node operates in a half-duplex mode using DF scheme with two transmission phases. In the multiple access (MA) phase, all the source nodes (transmit users) transmit their signals simultaneously to the relay node, while the relay retransmits the re-encoded signals to the destination nodes (receive users) in the broadcast (BC) phase; meanwhile the source nodes remain inactive.

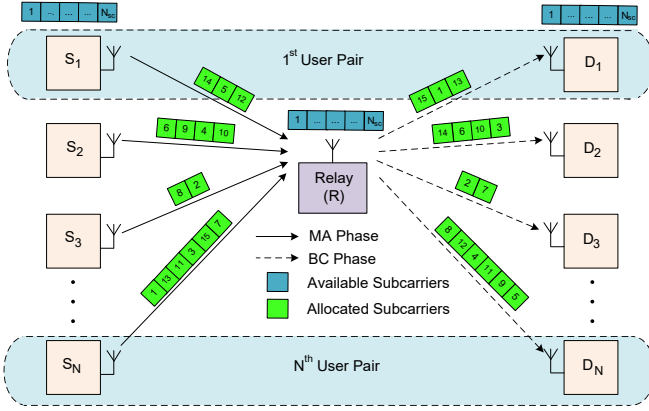


Fig. 1. A dual-hop multiuser DF relay network.

In the MA phase, the received signal at the relay node on the m -th subcarrier is given by

$$y_R^{(m)} = \sum_{i=1}^N \sqrt{P_{S,i}^{(m)}} h_{S_i R}^{(m)} x_i^{(m)} + n_R^{(m)}, \quad (1)$$

where $h_{S_i R}^{(m)}$ is the channel coefficient from the i -th source node to the relay node on the m -th subcarrier, $x_i^{(m)}$ denotes the transmitted signal from the i -th source node on the m -th subcarrier with unit power, i.e., $\mathbb{E} \left[|x_i^{(m)}|^2 \right] = 1$, $P_{S,i}^{(m)}$ represents the transmit power of the i -th source node on the m -th subcarrier, and $n_R^{(m)}$ denotes the zero-mean complex additive white Gaussian noise (AWGN) with variance $(\sigma_R^{(m)})^2$. From (1), the signal-to-interference plus noise ratio (SINR) at the relay node for the i -th source node on the m -th subcarrier can

be given as

$$\gamma_{S_i R}^{(m)} = \frac{P_{S,i}^{(m)} |h_{S_i R}^{(m)}|^2}{\sum_{j=1, j \neq i}^N P_{S,j}^{(m)} |h_{S_j R}^{(m)}|^2 + (\sigma_R^{(m)})^2}, \quad (2)$$

By assuming that the SR links are sufficiently good enough to allow the sophisticated DF relay node for successfully decoding the received signal, i.e., $\hat{x}_i^{(n)} = x_i^{(n)}$. Thus, in the BC phase, the received signal at the i -th destination node on the n -th subcarrier can be represented as

$$y_{D,i}^{(n)} = h_{RD,i}^{(n)} \sum_{i=1}^N \sqrt{P_{R,i}^{(n)}} \hat{x}_i^{(n)} + n_{D_i}^{(n)}; \\ = \underbrace{h_{RD,i}^{(n)} \sqrt{P_{R,i}^{(n)}} \hat{x}_i^{(n)}}_{\text{Desired signal}} + \underbrace{h_{RD,i}^{(n)} \sum_{j=1, j \neq i}^N \sqrt{P_{R,j}^{(n)}} \hat{x}_j^{(n)}}_{\text{Interference}} + \underbrace{n_{D_i}^{(n)}}_{\text{Noise}}, \quad (3)$$

where $\hat{x}_i^{(n)}$ is the decoded signal, $h_{RD,i}^{(n)}$ can be defined similar to $h_{S_i R}^{(m)}$ for RD links, $P_{R,i}^{(n)}$ indicates the transmit power of the relay node for the i -th source node on the n -th subcarrier, and $n_{D_i}^{(n)}$ can describe similar to $n_R^{(m)}$ but with variance $(\sigma_{D_i}^{(n)})^2$. Using (3), the SINR at the i -th destination node on the n -th subcarrier can be written as

$$\gamma_{RD,i}^{(n)} = \frac{P_{R,i}^{(n)} |h_{RD,i}^{(n)}|^2}{\sum_{j=1, j \neq i}^N P_{R,j}^{(n)} |h_{RD,i}^{(n)}|^2 + (\sigma_{D_i}^{(n)})^2}, \quad (4)$$

Define $\rho_{m,n} \in \{0, 1\}$, $\forall m, n$, as the subcarrier pairing indicator variable, where $\rho_{m,n} = 1$ if the m -th subcarrier in the MA phase is paired with the n -th subcarrier in the BC phase, and $\rho_{m,n} = 0$ otherwise. Further, $\Phi_{i,(m,n)} \in \{0, 1\}$, $\forall i, m, n$, denotes the subcarrier allocation variable, where $\Phi_{i,(m,n)} = 1$ if the (m, n) -th subcarrier pair is assigned to the i -th user pair while $\Phi_{i,(m,n)} = 0$ for the rest of the user pairs.

From (2) and (4), the achievable sum rate for the i -th user pair can be evaluated as

$$R_i = \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \frac{1}{2} \rho_{m,n} \Phi_{i,(m,n)} \log_2 \left(1 + \min \left\{ \gamma_{S_i R}^{(m)}, \gamma_{RD,i}^{(n)} \right\} \right), \quad (5)$$

[bits/s/Hz],

where $\frac{1}{2}$ comes from the fact that transmission consists of two phases. Furthermore, the total sum rate of the network can be given as

$$R_{Total} = \sum_{i=1}^N R_i \\ = \sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \frac{1}{2} \rho_{m,n} \Phi_{i,(m,n)} \log_2 \left(1 + \min \left\{ \gamma_{S_i R}^{(m)}, \gamma_{RD,i}^{(n)} \right\} \right), \quad (6)$$

[bits/s/Hz],

B. Power dissipation model

The power dissipation plays a paramount role in designing of an energy efficient devices. In general, the required quality-of-service (QoS) can even be obtained by utilizing significantly less amount of power. In the network, the power is dissipated in two ways namely: 1) transmit power; and 2) static power. The transmit power depends on the instantaneous channel gains, whereas the static power includes the circuit and processing powers utilized for signal detection and processing performed by various circuitry components presented at the relay and the destination nodes, and thus it directly depends on the number of antennas [21] and remains constant for a node, irrespective of its channel conditions. Therefore, the actual power dissipation in the network after subcarrier pairing and allocation can be written as

$$P_{Total} = \underbrace{\sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} (P_{S,i}^{(m)} + P_{R,i}^{(n)})}_{\text{Dynamic Power}} + \underbrace{P_{Sc} + P_{Rc}}_{\text{Static Power, } P_c > 0} \text{ [Watts]}, \quad (7)$$

where $(P_{S,i}^{(m)} + P_{R,i}^{(n)})$ denotes the total dynamic power consumed by the source and relay nodes for transmission between i -th user pair on the m -th and n -th subcarriers, respectively, whereas P_{Sc} and P_{Rc} are the circuit and processing powers together denoting the static power ($P_c > 0$) of the network. It is observed that static power in (7) remains constant, whereas the transmit power differs.

C. Primal Problem Formulation

The main objective of this work is to maximize the EE of the relay network by jointly optimizing the subcarrier and power allocation. Using (6) and (7), the EE of the network is stated as follows.

Definition 1: The EE of the network can be defined as a ratio of the minimum achievable sum rate to the total power consumption¹:

$$\eta_{EE}(\mathbf{P}_S^{(m)}, \mathbf{P}_R^{(n)}, \boldsymbol{\rho}, \Phi) = \frac{R_{Total}(\mathbf{P}_S^{(m)}, \mathbf{P}_R^{(n)}, \boldsymbol{\rho}, \Phi)}{P_{Total}(\mathbf{P}_S^{(m)}, \mathbf{P}_R^{(n)}, \boldsymbol{\rho}, \Phi)}, \quad (8)$$

where $\mathbf{P}_S^{(m)} = [P_{S,1}^{(m)}, P_{S,2}^{(m)}, \dots, P_{S,N}^{(m)}]^T$ and $\mathbf{P}_R^{(n)} = [P_{R,1}^{(n)}, P_{R,2}^{(n)}, \dots, P_{R,N}^{(n)}]^T$ denote the source and relay power vectors for the m -th and n -th subcarriers, respectively, whereas $\boldsymbol{\rho} = \{\rho_{m,n}\}$ and $\Phi = \{\Phi_{i,(m,n)}\}$ express matrices for the subcarrier pairing and allocation, respectively. Thus, the primal

optimization problem can be formulated as follows:

$$\begin{aligned} (\mathbf{P1}) \quad & \max_{\mathbf{P}_S^{(m)}, \mathbf{P}_R^{(n)}, \boldsymbol{\rho}, \Phi} \eta_{EE}(\mathbf{P}_S^{(m)}, \mathbf{P}_R^{(n)}, \boldsymbol{\rho}, \Phi) \\ \text{s.t.} \quad & (C.1) \sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} (P_{S,i}^{(m)} + P_{R,i}^{(n)}) \leq P_{max}; \\ & (C.2) \sum_{m=1}^{N_{sc}} \rho_{m,n} = 1, \quad \forall n; \\ & (C.3) \sum_{n=1}^{N_{sc}} \rho_{m,n} = 1, \quad \forall m; \quad (9) \\ & (C.4) \sum_{i=1}^N \Phi_{i,(m,n)} = 1, \quad \forall (m,n); \\ & (C.5) \rho_{m,n} \in \{0, 1\}, \Phi_{i,(m,n)} \in \{0, 1\}, \quad \forall i, m, n; \\ & (C.6) P_{S,i}^{(m)} \geq 0, P_{R,i}^{(n)} \geq 0, \quad \forall i, m, n, \end{aligned}$$

where P_{max} is the total transmit power budget of the network. The constraint (C.1) limits the total transmit power utilized by the source and the relay nodes and the constraints (C.2) and (C.3) ensure that each subcarrier in the MA phase is paired with one and only one subcarrier in the BC phase; and the last constraint (C.4) guarantees that (m, n) -th subcarrier pair is allocated to at least one user pair. According to [22], the maximum sum rate in multiuser scenario can be achieved for the case when each subcarrier is occupied by only one user in each transmission. Therefore, each subcarrier is assigned to only one user in the designed framework. The proposed design framework can be easily extended to accommodate the scenario where each subcarrier in the first hop can be paired with one or more subcarriers in second hop and vice versa by modifying the constraints (C.2) and (C.3) in (9) as follows:

$$\begin{aligned} (C.2) \quad & 1 \leq \sum_{m=1}^{N_{sc}} \rho_{m,n} \leq N_{sc}, \quad \forall n; \\ (C.3) \quad & 1 \leq \sum_{n=1}^{N_{sc}} \rho_{m,n} \leq N_{sc}, \quad \forall m, \end{aligned}$$

The new optimization problem can be solved in a similar way as the problem (P1). However, the new optimization problem requires the update of $2N_{sc}$ Lagrangian multipliers in the master problem, and thus exhibits high computational complexity. Additionally, the Hungarian method cannot be directly applied for solving the subcarrier pairing matrix $\boldsymbol{\rho}$. Notice that there is a tradeoff between performance and computational complexity. Therefore, we consider the problem (P1) instead of new optimization problem and investigate the EEM algorithm to find the optimal resource allocation policy. In this work, the relay node is equipped with only a single antenna, however, the design framework can also be easily generalized to the scenario with multiple antennas. In this case, the SR and RD links become single input multiple output (SIMO) and multiple input single output (MISO) channels, respectively. By expertly designing receive and transmit beamforming weights for the relay node, we can derive the SINR similar to (2) and (4). In general, an increased number of antennas can offer

¹Since the function $\eta_{EE}(\mathbf{P}_S^{(m)}, \mathbf{P}_R^{(n)}, \boldsymbol{\rho}, \Phi)$ is nonconcave, the optimality here is defined in a locally optimal sense.

better interference suppression capability, but it also requires more static power consumption and skillful subcarrier pairing and allocation, thus leading to EE performance tradeoff.

III. TRANSFORMATION OF NONCONVEX OPTIMIZATION PROBLEM

The optimization problem **(P1)** is nonconvex in nature due to the fractional form of the objective function in (9) and a mixed binary integer nonlinear programming problem [23]. There is no standard technique to solve such optimization problem. Therefore, in order to determine the optimal resource allocation policies we need to transform the original optimization problem (9) into an analytically tractable form which will be convex in nature. By introducing the parameter $\Gamma_i^{(k)} \geq 0$, $k = 1, \dots, N_{sc}$, the optimization problem **(P1)** can be rewritten as

$$\begin{aligned}
 \text{(P2)} \quad & \max_{\rho, \Phi, \Gamma} \frac{\sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \frac{1}{2} \rho_{m,n} \Phi_{i,(m,n)} \log_2 \left(1 + \Gamma_i^{(k)} \right)}{\sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} \left(P_{S,i}^{(m)} + P_{R,i}^{(n)} \right) + P_c} \\
 \text{s.t.} \quad & (C.1) - (C.6); \\
 & (C.7) \quad \gamma_{S_i R}^{(k)} \geq \Gamma_i^{(k)}, \quad \forall k; \\
 & (C.8) \quad \gamma_{R D_i}^{(k)} \geq \Gamma_i^{(k)}, \quad \forall k,
 \end{aligned} \tag{10}$$

where $\Gamma^{(k)} = \{\Gamma_i^{(k)}\}$ is a auxiliary variable vector. The SINR in MA and BC phases must be greater than or equal to this auxiliary variable. Furthermore, by applying the change of variables $\hat{P}_{S,i}^{(m)} = \ln P_{S,i}^{(m)}$, $\hat{P}_{R,i}^{(n)} = \ln P_{R,i}^{(n)}$ and $\hat{\Gamma}_i^{(k)} = \ln \Gamma_i^{(k)}$, the problem **(P2)** can be equivalently written as

$$\begin{aligned}
 \text{(P3)} \quad & \max_{\rho, \Phi, \hat{\Gamma}} \frac{F(\rho, \Phi, \hat{\Gamma}^{(m)})}{\sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \frac{1}{2} \rho_{m,n} \Phi_{i,(m,n)} \log_2 \left(1 + e^{\hat{\Gamma}_i^{(m)}} \right)} \\
 & \sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} \left(e^{\hat{P}_{S,i}^{(m)}} + e^{\hat{P}_{R,i}^{(n)}} \right) + P_c \\
 \text{s.t.} \quad & (C.1) \quad \sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} \left(e^{\hat{P}_{S,i}^{(m)}} + e^{\hat{P}_{R,i}^{(n)}} \right) \leq P_{max}; \\
 & (C.2) - (C.5);
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 (C.7) \quad & \sum_{j=1, j \neq i}^N e^{\hat{\Gamma}_i^{(k)} - \hat{P}_{S,i}^{(k)} + \hat{P}_{S,j}^{(k)}} \frac{|h_{S_j R}^{(k)}|^2}{|h_{S_i R}^{(k)}|^2} \\
 & + e^{\hat{\Gamma}_i^{(k)} - \hat{P}_{S,i}^{(k)}} \frac{(\sigma_R^{(k)})^2}{|h_{S_i R}^{(k)}|^2} \leq 1, \quad \forall i, k;
 \end{aligned}$$

$$\begin{aligned}
 (C.8) \quad & \sum_{j=1, j \neq i}^N e^{\hat{\Gamma}_i^{(k)} - \hat{P}_{R,i}^{(k)} + \hat{P}_{R,j}^{(k)}} \frac{|h_{R D_j}^{(k)}|^2}{|h_{R D_i}^{(k)}|^2} \\
 & + e^{\hat{\Gamma}_i^{(k)} - \hat{P}_{R,i}^{(k)}} \frac{(\sigma_{D_i}^{(k)})^2}{|h_{R D_i}^{(k)}|^2} \leq 1, \quad \forall i, k,
 \end{aligned}$$

Because of the fractional form, the objective function in (11) is nonconvex. By utilizing the properties of nonlinear fractional programming [24] which is useful to deal with concave-over-convex fractional function in an iterative manner, we can transform the objective function in a subtractive form. To make the objective function concave-over-convex, we use SCA method to impose a lower bound on $F(\rho, \Phi, \hat{\Gamma}^{(m)})$ [6], [20]; it yields

$$\begin{aligned}
 & F(\rho, \Phi, \hat{\Gamma}^{(m)}) \\
 & \geq \sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \frac{1}{2} \rho_{m,n} \Phi_{i,(m,n)} \left(\frac{\alpha_i^{(m)}}{\ln(2)} \hat{\Gamma}_i^{(m)} + \beta_i^{(m)} \right); \\
 & \triangleq F_{LB}(\rho, \Phi, \hat{\Gamma}^{(m)}, \alpha^{(m)}, \beta^{(m)}),
 \end{aligned} \tag{12}$$

where $\alpha^{(m)} = [\alpha_1^{(m)}, \alpha_2^{(m)}, \dots, \alpha_N^{(m)}]$ and $\beta^{(m)} = [\beta_1^{(m)}, \beta_2^{(m)}, \dots, \beta_N^{(m)}]$ are the coefficients determined in the following manner [6]

$$\alpha_i^{(m)} = \frac{\varsigma_i^{(m)}}{1 + \varsigma_i^{(m)}}; \tag{13}$$

$$\beta_i^{(m)} = \log_2 \left(1 + \varsigma_i^{(m)} \right) - \alpha_i^{(m)} \log_2 \left(\varsigma_i^{(m)} \right), \tag{14}$$

for any $\varsigma_i^{(m)} > 0$. Note that the equality in (12) satisfied when $\alpha_i^{(m)} = \hat{\Gamma}_i^{(m)} / (1 + \hat{\Gamma}_i^{(m)})$ and $\beta_i^{(m)} = \log_2 \left(1 + \hat{\Gamma}_i^{(m)} \right) - \alpha_i^{(m)} \log_2 \left(\hat{\Gamma}_i^{(m)} \right)$, and the equality holds only if $(\alpha_i^{(m)}, \beta_i^{(m)}) = (1, 0)$ and $\hat{\Gamma}_i^{(m)} \rightarrow \infty$. After substituting (12) in **(P3)**, the problem in (11) can be reformulated as

$$\begin{aligned}
 \text{(P4)} \quad & \max_{\rho, \Phi, \hat{\Gamma}} \frac{F_{LB}(\rho, \Phi, \hat{\Gamma}^{(m)}, \alpha^{(m)}, \beta^{(m)})}{\sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} \left(e^{\hat{P}_{S,i}^{(m)}} + e^{\hat{P}_{R,i}^{(n)}} \right) + P_c} \\
 \text{s.t.} \quad & (C.1) - (C.5), (C.7) \& (C.8),
 \end{aligned} \tag{15}$$

Since the objective function in (15) is in a form of concave-over-convex for fixed subcarrier pairing and allocation, we can apply nonlinear programming method [24] to transform the problem into a convex optimization problem as

$$\begin{aligned}
 \text{(P5)} \quad & \max_{\hat{P}_S^{(m)}, \hat{P}_R^{(n)}, \rho, \Phi, \hat{\Gamma}^{(m)}} \mathcal{F}_{LB}(\hat{P}_S^{(m)}, \hat{P}_R^{(n)}, \rho, \Phi, \hat{\Gamma}^{(m)}, \alpha^{(m)}, \beta^{(m)}) \\
 \text{s.t.} \quad & (C.1) - (C.5), (C.7) \& (C.8),
 \end{aligned} \tag{16}$$

where $\mathcal{F}_{LB}(\hat{P}_S^{(m)}, \hat{P}_R^{(n)}, \rho, \Phi, \hat{\Gamma}^{(m)}, \alpha^{(m)}, \beta^{(m)}) = F_{LB} - \Psi \left(\sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} \left(e^{\hat{P}_{S,i}^{(m)}} + e^{\hat{P}_{R,i}^{(n)}} \right) + P_c \right)$ and Ψ is a non-negative parameter and works as a penalty factor for the resource utilization. Note that when $\Psi \rightarrow 0$, it implies that the penalty to use the resources is almost zero, and the resource allocation problem (16) is degenerated to a sum-rate maximization problem. However, for another extreme case where $\Psi \rightarrow \infty$, no resource allocation policy is good enough to maximize the objective function in (16).

Lemma 1: For fixed subcarrier pairing and allocation (ρ, Φ) , the lower bound in the problem **(P5)**

$\mathcal{F}_{LB}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \boldsymbol{\rho}, \Phi, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)})$ is concavified by the change of variables $\hat{P}_{S,i}^{(m)} = \ln P_{S,i}^{(m)}$, $\hat{P}_{R,i}^{(n)} = \ln P_{R,i}^{(n)}$ and $\hat{\Gamma}_i^{(m)} = \ln \Gamma_i^{(m)}$, for any given $\alpha_i^{(m)}$, $\beta_i^{(m)}$ and Ψ .

Proof: After substituting $P_{S,i}^{(m)} = e^{\hat{P}_{S,i}^{(m)}}$, $P_{R,i}^{(n)} = e^{\hat{P}_{R,i}^{(n)}}$ and $\Gamma^{(m)} = e^{\hat{\Gamma}^{(m)}}$ in (P2), the objective function $\mathcal{F}_{LB}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \boldsymbol{\rho}, \Phi, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)})$ in (P5) can be written as

$$\begin{aligned} & \mathcal{F}_{LB}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \boldsymbol{\rho}, \Phi, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)}) \\ &= \sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \frac{1}{2} \rho_{m,n} \Phi_{i,(m,n)} \left(\frac{\alpha_i^{(m)}}{2} \hat{\Gamma}_i^{(m)} + \beta_i^{(m)} \right) \quad (17) \\ & - \Psi \left(\sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} \left(e^{\hat{P}_{S,i}^{(m)}} + e^{\hat{P}_{R,i}^{(n)}} \right) + P_c \right), \end{aligned}$$

Since, $\alpha_i^{(m)} \geq 0$, $\beta_i^{(m)} \geq 0$ and $\Psi \geq 0$, the objective function (17) forms the summation of the concave terms and linear terms (i.e. minus-exp functions) for given subcarrier pairing and allocation $(\boldsymbol{\rho}, \Phi)$, hence the objective function \mathcal{F}_{LB} is concavified by nature. ■

We can solve the problem (P5) in two steps, firstly the power allocation to each source and relay nodes is decoupled with the problem of subcarrier pairing and allocation. Next, the subcarrier allocation to each user pair is decoupled with the problem of subcarrier pairing. In case of equal subcarrier allocation and direct pairing, the total number of subcarriers are equally divided between users and first subcarrier in SR phase is paired to first one in RD phase, respectively. Further, the subcarrier pairing and allocation matrices are defined as $\boldsymbol{\rho} = \mathbf{I}_{N_{sc}}$ and $\Phi = \text{blkdiag}(\boldsymbol{\sigma}_1, \dots, \boldsymbol{\sigma}_N)$, respectively, where $\boldsymbol{\sigma}_i = \mathbf{I}_{\lfloor N_{sc}/N \rfloor}$, $\forall i = 1, \dots, N$. For the case of $N = 2$ and $N_{sc} = 2$, we can find $\gamma_{S_i R}^{(m)}$ and $\gamma_{R D_i}^{(n)}$, $m, n = 1, 2$, using (2) and (4), and the maximum achievable sum rate R_A can be computed using (6) for one-to-one mapping and user wise allocation. While in case of the optimal subcarrier pairing and allocation, the subcarriers are optimally allocated and paired using (29) – (31), respectively, and the achievable sum rate R_B is computed.

Now, we have two cases for optimally allocated powers limited by P_{max} , the sum rate R_A is calculated for fixed type of subcarrier pairing, this leads to undesired interference and noise, thereby leading to degradation of the sum rate R_A , while with the optimal power allocation, the subcarrier allocation and pairing is also done optimally and thus we can notice that $R_B > R_A$. According to [25], if R_B exceeds R_A , it is proven that we can easily treat power allocation, and subcarrier pairing and allocation as two separate problems, even for large number of subcarriers. In a similar manner, we can also prove the decoupling of subcarrier allocation and pairing for optimally calculated power, wherein the sum rate with improper mapping of subcarriers in two-hops significantly degrades as compare to that of the sum rate with optimally allocated subcarrier pairing and allocation, thereby, using [25], we can prove the decoupling of subcarrier pairing and allocation.

IV. EE RESOURCE ALLOCATION ALGORITHM

The maximization problem (P1) is transformed into a mixed binary integer nonlinear programming problem (P5), and thus an exhaustive search over all variables is required to find the global optimal solution. Consequently, the computational complexity becomes very high. However, the duality gaps between the primal problem and the dual problem in a multicarrier system approaches to zero when the number of subcarriers goes to infinity [26], [27]. The definition of duality gap and related theorem are given as follows:

Definition 2 (Duality Gap): The difference in the optimal solution of the optimization problem (P5), given by OP^* , and its dual optimization problem in (20), given by DP^* , is defined as duality gap, expressed as $D_G = OP^* - DP^*$.

Theorem 1: For sufficiently large number of subcarriers N_{sc} , the duality gap D_G tends to zero, i.e., $D_G = OP^* - DP^* \approx 0$.

Proof: Please see Appendix A. ■

Thus, we focus on solving the dual problem [23] instead of the original problem and propose an iterative EEM algorithm to find the optimal resource allocation policies that can maximize its lower bound under fixed coefficients $\alpha_i^{(m)}$ and $\beta_i^{(m)}$, followed by an update of these two coefficients that guarantees a monotonic increase in the lower bound performance.

A. Dual Problem Formulation

For fixed subcarrier pairing and allocation variables $\{\rho_{m,n}\}$ and $\{\Phi_{i,(m,n)}\}$, the optimization problem (P5) is a convex problem with given lower bound coefficients $\{\alpha_i^{(m)}\}$ and $\{\beta_i^{(m)}\}$. Thus the Lagrangian function for the relaxed optimization problem (P5) is expressed in (18), as shown on the top of the next page, where λ , $\boldsymbol{\mu}^{(k)} = [\mu_1^{(k)}, \mu_2^{(k)}, \dots, \mu_N^{(k)}]^T$ and $\boldsymbol{\nu}^{(k)} = [\nu_1^{(k)}, \nu_2^{(k)}, \dots, \nu_N^{(k)}]^T$ are Lagrangian multipliers associated with the constraints (C.1), (C.7) and (C.8), respectively. The Lagrangian dual function can therefore be expressed as

$$\begin{aligned} & g(\lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)}) \\ & \triangleq \max_{\substack{\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)} \\ \boldsymbol{\rho}, \Phi, \hat{\Gamma}^{(m)}}} \mathcal{L}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \boldsymbol{\rho}, \Phi, \hat{\Gamma}^{(m)}, \lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)}) \\ & \text{s.t.} \quad (C.2) - (C.5), \quad (19) \end{aligned}$$

Then the dual optimization problem can be written as

$$\begin{aligned} & \min_{\lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)} \geq 0} g(\lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)}) = \\ & \min_{\substack{\lambda, \boldsymbol{\mu}^{(k)} \geq 0 \\ \boldsymbol{\nu}^{(k)} \geq 0}} \max_{\substack{\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)} \\ \boldsymbol{\rho}, \Phi, \hat{\Gamma}^{(m)}}} \mathcal{L}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \boldsymbol{\rho}, \Phi, \hat{\Gamma}^{(m)}, \lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)}), \\ & \text{s.t.} \quad (C.2) - (C.5), \quad (20) \end{aligned}$$

The dual problem in (20) can be decomposed into a master problem and a subproblem, and it can be solved in an iterative manner until the convergence is reached.

$$\begin{aligned}
\mathcal{L} \left(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \boldsymbol{\rho}, \Phi, \hat{\Gamma}^{(m)}, \lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)} \right) &= \mathcal{F}_{LB} \left(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \boldsymbol{\rho}, \Phi, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)} \right) \\
&- \lambda \left(\sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} \left(e^{\hat{P}_{S,i}^{(m)}} + e^{\hat{P}_{R,i}^{(n)}} \right) - P_{max} \right) \\
&- \sum_{i=1}^N \sum_{k=1}^{N_{sc}} \mu_i^{(k)} \left(\sum_{j=1, j \neq i}^N e^{\hat{\Gamma}_i^{(k)} - \hat{P}_{S,i}^{(k)} + \hat{P}_{S,j}^{(k)}} \frac{|h_{S_j R}^{(k)}|^2}{|h_{S_i R}^{(k)}|^2} + e^{\hat{\Gamma}_i^{(k)} - \hat{P}_{S,i}^{(k)}} \frac{(\sigma_R^{(k)})^2}{|h_{S_i R}^{(k)}|^2} - 1 \right) \\
&- \sum_{i=1}^N \sum_{k=1}^{N_{sc}} \nu_i^{(k)} \left(\sum_{j=1, j \neq i}^N e^{\hat{\Gamma}_i^{(k)} - \hat{P}_{R,i}^{(k)} + \hat{P}_{R,j}^{(k)}} \frac{|h_{RD_j}^{(k)}|^2}{|h_{RD_i}^{(k)}|^2} + e^{\hat{\Gamma}_i^{(k)} - \hat{P}_{R,i}^{(k)}} \frac{(\sigma_{D_i}^{(k)})^2}{|h_{RD_i}^{(k)}|^2} - 1 \right) \left| \begin{array}{l} \sum_{m=1}^{N_{sc}} \rho_{m,n} = 1, \forall n; \\ \sum_{n=1}^{N_{sc}} \rho_{m,n} = 1, \forall m; \\ \sum_{i=1}^N \Phi_{i,(m,n)} = 1, \forall (m,n), \end{array} \right. \quad (18)
\end{aligned}$$

B. Solution of the Subproblem

From Lemma 1, the objective function in (9) is concave in $(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \boldsymbol{\rho}, \Phi, \hat{\Gamma}^{(m)})$, and thus the optimal solution can be found by solving the Lagrange dual domain because the duality gap between (16) and the dual problem is nearly zero for large number of N_{sc} , in two steps for fixed Lagrangian multipliers: 1) by using KKT [23] conditions which are first-order imperative and sufficient conditions for optimality for convex optimization problem, we can find the optimal power allocation for each source and the relay nodes for given subcarrier pairing $\boldsymbol{\rho}$ and subcarrier allocation Φ ; and 2) find the optimal $\boldsymbol{\rho}$ and Φ for the obtained power allocation in the first step.

By taking the partial derivative of (18) with respect to $P_{S,i}^{(m)}, P_{R,i}^{(n)}, \Gamma_i^{(m)}$ and equating the results to zero, we get the update equation for the power allocation and $\Gamma_i^{(m)}$ at the $(t+1)$ -th iteration as in (21)-(23), where $\varphi = \frac{1}{2}$, $\varpi = \frac{1}{2 \ln 2}$ and $[\cdot]^+ = \max\{0, \cdot\}$. Since the m -th subcarrier is allocated to the i -th source node, the transmit power allocated on the m -th subcarrier by other transmit nodes is almost close to zero, and thus under assumption of $e^{\hat{\Gamma}_i^{(m)}} \left(\sum_{j=1, j \neq i}^N e^{\hat{P}_{S,j}^{(m)}} |h_{S_j R}^{(m)}|^2 + (\sigma_R^{(m)})^2 \right) \approx \Delta_S$ and $e^{\hat{\Gamma}_i^{(n)}} \left(\sum_{j=1, j \neq i}^N e^{\hat{P}_{R,j}^{(n)}} |h_{RD_j}^{(n)}|^2 + (\sigma_{D_i}^{(n)})^2 \right) \approx \Delta_R$, the optimal power of the i -th source and the relay nodes at the $(t+1)$ -th iteration can be updated for given subcarrier pairing $\{\rho_{m,n} = 1\}$ and allocation $\{\Phi_{i,(m,n)} = 1\}$ as follows:

$$\hat{P}_{S,i}^{(m)}(t+1) = \left[\varphi \left(\ln \frac{\mu_i^{(m)} \Delta_S}{(\Psi + \lambda) |h_{S_i R}^{(m)}|^2} \right) \right]^+; \quad (24)$$

$$\hat{P}_{R,i}^{(n)}(t+1) = \left[\varphi \left(\ln \frac{\nu_i^{(n)} \Delta_R}{(\Psi + \lambda) |h_{RD_i}^{(n)}|^2} \right) \right]^+, \quad (25)$$

The optimal power allocations in (24) and (25) are indeed customized water-filling solutions and it can be observed that the water levels are not only determined by the penalty of

allocating power λ , but also on the current penalty of the power utilization to the EE given by Ψ . As can be seen from (25), the relay node also follows similar power allocation policy for the i -th source node's signal in the second hop. However, for without subcarrier pairing and allocation, the power allocation policy needs to consider the interference channel power generated from other source nodes towards the i -th source node.

To derive the optimal subcarrier pairing and allocation, we substitute $P_{S,i}^{(m)*}, P_{R,i}^{(n)*}$, and $\Gamma_i^{(m)*}$ into the (18), the dual problem $g(\lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)})$ becomes

$$\begin{aligned}
g(\lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)}) &= \max_{\boldsymbol{\rho}, \Phi} \sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} \mathcal{B}_{i,(m,n)} \\
&+ \mathcal{C} \left(\hat{\mathbf{P}}_S^{(m)*}, \hat{\mathbf{P}}_R^{(n)*}, \hat{\Gamma}^{(m)*}, \lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)} \right) \\
&\text{s.t. } (C.1) - (C.5), \quad (26)
\end{aligned}$$

where $\mathcal{B}_{i,(m,n)}$ and \mathcal{C} are defined as

$$\begin{aligned}
\mathcal{B}_{i,(m,n)} &= \left(\varpi \alpha_i^{(m)} \hat{\Gamma}_i^{(m)*} + \varphi \beta_i^{(m)} \right) - (\Psi + \lambda) \left(e^{\hat{P}_{S,i}^{(m)*}} + e^{\hat{P}_{R,i}^{(n)*}} \right); \quad (27) \\
\mathcal{C} \left(\hat{\mathbf{P}}_S^{(m)*}, \hat{\mathbf{P}}_R^{(n)*}, \hat{\Gamma}^{(m)*}, \lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)} \right) &= -\Psi P_C + \lambda P_{max} \\
&- \sum_{i=1}^N \sum_{k=1}^{N_{sc}} \mu_i^{(k)} \left(\sum_{j=1, j \neq i}^N e^{\hat{\Gamma}_i^{(k)*} - \hat{P}_{S,i}^{(k)*} + \hat{P}_{S,j}^{(k)*}} \frac{|h_{S_j R}^{(k)}|^2}{|h_{S_i R}^{(k)}|^2} \right. \\
&\quad \left. + e^{\hat{\Gamma}_i^{(k)*} - \hat{P}_{S,i}^{(k)*}} \frac{(\sigma_R^{(k)})^2}{|h_{S_i R}^{(k)}|^2} - 1 \right) \\
&- \sum_{i=1}^N \sum_{k=1}^{N_{sc}} \nu_i^{(k)} \left(\sum_{j=1, j \neq i}^N e^{\hat{\Gamma}_i^{(k)*} - \hat{P}_{R,i}^{(k)*} + \hat{P}_{R,j}^{(k)*}} \frac{|h_{RD_j}^{(k)}|^2}{|h_{RD_i}^{(k)}|^2} \right. \\
&\quad \left. + e^{\hat{\Gamma}_i^{(k)*} - \hat{P}_{R,i}^{(k)*}} \frac{(\sigma_{D_i}^{(k)})^2}{|h_{RD_i}^{(k)}|^2} - 1 \right), \quad (28)
\end{aligned}$$

$$\hat{P}_{S,i}^{(m)}(t+1) = \left[\varphi \left(\ln \left(\mu_i^{(m)} e^{\hat{\Gamma}_i^{(m)}} \left(\sum_{j=1, j \neq i}^N e^{\hat{P}_{S,j}^{(m)}} |h_{S_j R}^{(m)}|^2 + (\sigma_R^{(m)})^2 \right) \right) - \ln \left(\sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} (\Psi + \lambda) |h_{S_i R}^{(m)}|^2 \right) \right) \right]^+; \quad (21)$$

$$\hat{P}_{R,i}^{(n)}(t+1) = \left[\varphi \left(\ln \left(\nu_i^{(n)} e^{\hat{\Gamma}_i^{(n)}} \left(\sum_{j=1, j \neq i}^N e^{\hat{P}_{R,j}^{(n)}} |h_{RD_j}^{(n)}|^2 + (\sigma_{D_i}^{(n)})^2 \right) \right) - \ln \left(\sum_{m=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} (\Psi + \lambda) |h_{RD_i}^{(n)}|^2 \right) \right) \right]^+; \quad (22)$$

$$\hat{\Gamma}_i^{(m)}(t+1) = \left[\ln \left(\sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} \varpi \alpha_i^{(m)} \right) - \ln \left(\frac{\mu_i^{(m)} e^{-\hat{P}_{S,i}^{(m)}}}{|h_{S_i R}^{(m)}|^2} \left(\sum_{j \neq i}^N e^{\hat{P}_{S,j}^{(m)}} |h_{S_j R}^{(m)}|^2 + \sigma_R^{(m)2} \right) \right) \right. \\ \left. + \frac{\nu_i^{(m)} e^{-\hat{P}_{R,i}^{(m)}}}{|h_{RD_i}^{(m)}|^2} \left(\sum_{j \neq i}^N e^{\hat{P}_{R,j}^{(m)}} |h_{RD_j}^{(m)}|^2 + \sigma_{D_i}^{(m)2} \right) \right]^+, \quad (23)$$

It can be observed that only $\mathcal{B}_{i,(m,n)}$ depends on the subcarrier pairing and allocation, whereas $\mathcal{C}(\hat{\mathbf{P}}_S^{(m)*}, \hat{\mathbf{P}}_R^{(n)*}, \hat{\mathbf{\Gamma}}^{(m)*}, \lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)})$ does not depend on any subcarrier pairing and allocation. Furthermore, the first term of $\mathcal{B}_{i,(m,n)}$ indicates the sum rate achieved by the i -th user pair on the (m,n) -th subcarrier pairing, while the second term works as penalty for power utilization.

For given subcarrier pairing matrix $\boldsymbol{\rho}$ and the optimal power allocation $(\hat{\mathbf{P}}_S^{(m)*}, \hat{\mathbf{P}}_R^{(n)*}, \hat{\mathbf{\Gamma}}^{(m)*})$ for fixed coefficients $\alpha_i^{(m)}$ and $\beta_i^{(m)}$, the optimal subcarrier allocation can be obtained by solving the following problem:

$$g(\lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)}) \\ = \max_{\Phi} \sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n}^* \Phi_{i,(m,n)} \mathcal{B}_{i,(m,n)} \\ + \mathcal{C}(\hat{\mathbf{P}}_S^{(m)*}, \hat{\mathbf{P}}_R^{(n)*}, \hat{\mathbf{\Gamma}}^{(m)*}, \lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)}) \\ \text{s.t. (C.1) \& (C.4)}, \quad (29)$$

In order to determine the optimal solution of (29), the i -th user pair can be selected that maximizes $\mathcal{B}_{i,(m,n)}$ for a given subcarrier pairing (m,n) , i.e.,

$$\Phi_{i,(m,n)}^* = \begin{cases} 1, & \text{for } i = \arg \max_i \mathcal{B}_{i,(m,n)}, \\ 0, & \text{otherwise} \end{cases} \quad (30)$$

Lastly, to find the optimal subcarrier pairing $\boldsymbol{\rho}^*$, we substitute (30) into (26), which yields

$$g(\lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)}) \\ = \max_{\rho} \sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)}^* \mathcal{B}_{i^*,(m,n)} \\ + \mathcal{C}(\hat{\mathbf{P}}_S^{(m)*}, \hat{\mathbf{P}}_R^{(n)*}, \hat{\mathbf{\Gamma}}^{(m)*}, \lambda, \boldsymbol{\mu}^{(k)}, \boldsymbol{\nu}^{(k)}) \\ \text{s.t. (C.1) - (C.3)}, \quad (31)$$

where $\mathcal{B}_{i^*,(m,n)} = \max_i \mathcal{B}_{i,(m,n)} \forall (m,n)$. The optimization problem (31) can be efficiently solved by using the standard Hungarian method [28] to find the optimal $\boldsymbol{\rho}^*$.

C. Master problem Solution: Update of Lagrangian Multipliers

By utilizing the subgradient method, the dual variables λ , $\boldsymbol{\mu}^{(k)}$ and $\boldsymbol{\nu}^{(k)}$ can be iteratively updated as in (32)-(34), where $\varepsilon_1, \varepsilon_2$ and ε_3 are positive step sizes. The optimal resource allocation $(\hat{\mathbf{P}}_S^{(m)*}, \hat{\mathbf{P}}_R^{(n)*}, \boldsymbol{\rho}^*, \Phi^*)$ of the problem (P5) can be found through the iterative procedure of (21)-(23), (30), (31), and (32)-(34) for given coefficients $\alpha_i^{(m)}$ and $\beta_i^{(m)}$ and the penalty factor Ψ . The lower bound performance $F_{LB}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \boldsymbol{\rho}, \Phi, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)})$ depends on the two coefficients $\boldsymbol{\alpha}^{(m)}$ and $\boldsymbol{\beta}^{(m)}$, and thus by carefully choosing the values of these two coefficients, the lower bound performance of $F_{LB}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \boldsymbol{\rho}, \Phi, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)})$ depends on the two coefficients $\boldsymbol{\alpha}^{(m)}$ and $\boldsymbol{\beta}^{(m)}$ can be enhanced. Next, we will provide the theorem regarding the update of $\boldsymbol{\alpha}^{(m)}$ and $\boldsymbol{\beta}^{(m)}$.

Theorem 2: If the coefficients $\alpha_i^{(m)}(t)$ and $\beta_i^{(m)}(t)$ are updated as

$$\alpha_i^{(m)}(t+1) = \frac{\Gamma_i^{(m)}(t)}{1 + \Gamma_i^{(m)}(t)}; \quad (35)$$

$$\beta_i^{(m)}(t+1) = \log_2 \left(1 + \Gamma_i^{(m)}(t) \right) \\ - \alpha_i^{(m)}(t+1) \log_2 \left(\Gamma_i^{(m)}(t+1) \right), \quad (36)$$

for the optimal solution $(\hat{\mathbf{P}}_S^{(m)*}(t), \hat{\mathbf{P}}_R^{(n)*}(t), \boldsymbol{\rho}^*(t), \Phi^*(t), \hat{\mathbf{\Gamma}}^{(m)*}(t))$ of the problem (P5) at the t -th inner iteration, then the optimal value of $F_{LB}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \boldsymbol{\rho}, \Phi, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)})$ in (16) is monotonically increased. Also, after the convergence of the coefficients $\alpha_i^{(m)}(t)$ and $\beta_i^{(m)}(t)$, the optimal solution for (P5) behaves as the local maximizer for the problem (P1).

Proof: The proof is relegated in Appendix B. ■

The penalty factor Ψ which is defined as the ratio of the achievable sum rate to the total power consumption in the network works as network EE. For a given Ψ , we first introduce an iterative EEM algorithm which is summarized

$$\lambda(t+1) = \left[\lambda(t) + \varepsilon_1(t) \left(\sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n}^* \Phi_{i,(m,n)}^* \left(e^{\hat{P}_{S,i}^{(m)*}} + e^{\hat{P}_{R,i}^{(n)*}} \right) - P_{max} \right) \right]^+ ; \quad (32)$$

$$\mu_i^{(m)}(t+1) = \left[\mu_i^{(m)}(t) + \varepsilon_2(t) \left(\sum_{j=1, j \neq i}^N e^{\hat{\Gamma}_i^{(m)*} - \hat{P}_{S,i}^{(m)*} + \hat{P}_{S,j}^{(m)*}} \frac{|h_{S_j R}^{(m)}|^2}{|h_{S_i R}^{(m)}|^2} + e^{\hat{\Gamma}_i^{(m)*} - \hat{P}_{S,i}^{(m)*}} \frac{(\sigma_R^{(m)})^2}{|h_{S_i R}^{(m)}|^2} - 1 \right) \right]^+ ; \quad (33)$$

$$\nu_i^{(n)}(t+1) = \left[\nu_i^{(n)}(t) + \varepsilon_3(t) \left(\sum_{j=1, j \neq i}^N e^{\hat{\Gamma}_i^{(n)*} - \hat{P}_{R,i}^{(n)*} + \hat{P}_{R,j}^{(n)*}} \frac{|h_{RD_j}^{(n)}|^2}{|h_{RD_i}^{(n)}|^2} + e^{\hat{\Gamma}_i^{(n)*} - \hat{P}_{R,i}^{(n)*}} \frac{(\sigma_{D_i}^{(n)})^2}{|h_{RD_i}^{(n)}|^2} - 1 \right) \right]^+ , \quad (34)$$

in Algorithm 1. We initialize the maximum number of outer iteration I_{max_1} with the iteration counter $l = 0$ and the penalty factor $\Psi(l) = 0.001$. Then, the maximum number of inner iteration I_{max_2} is initialized with the iteration counter $t = 0$ and the step sizes $\epsilon_k = 0.001$, for $k \in \{1, 2, 3\}$. Next, the subcarrier pairing ρ and allocation Φ are initialized randomly with the entity 0 or 1 such that $\sum_{m=1}^{N_{sc}} \rho_{m,n} = 1, \forall n, \sum_{n=1}^{N_{sc}} \rho_{m,n} = 1, \forall m, \sum_{i=1}^N \Phi_{i,(m,n)} = 1, \forall (m, n)$.

The lower bound coefficients are initialized as $\alpha_i^{(m)}(t) = 1$ and $\beta_i^{(m)}(t) = 0$, respectively. For given coefficients, subcarrier pairing ρ and subcarrier allocation Φ , the optimal power allocation policy $(\hat{\mathbf{P}}_{S,i}^{(m)}(t), \hat{\mathbf{P}}_{R,i}^{(n)}(t), \hat{\Gamma}_i^{(m)}(t))$ can be updated using (21), (22) and (23), respectively. The subcarrier allocation $\Phi(t)$ is updated using (30) for obtained $(\hat{\mathbf{P}}_{S,i}^{(m)}(t), \hat{\mathbf{P}}_{R,i}^{(n)}(t), \hat{\Gamma}_i^{(m)}(t))$ and given subcarrier pairing ρ , while we apply (31) to update the subcarrier pairing $\rho(t)$ after obtaining $(\hat{\mathbf{P}}_{S,i}^{(m)}(t), \hat{\mathbf{P}}_{R,i}^{(n)}(t), \hat{\Gamma}_i^{(m)}(t))$ and $\Phi(t)$. We update the dual variables $\lambda, \mu^{(k)}$ and $\nu^{(k)}$ using (32)–(34), respectively. The inner procedure is repeated until convergence or $t \leq I_{max_2}$. Moreover, in the next step, the network penalty factor $\Psi(l+1)$ is updated using (39) for updated optimal resource allocation and this outer procedure is repeated until convergence or the iteration counter $l \leq I_{max_1}$. As a result of the local optimality of the proposed penalty-based Ψ resource allocation algorithm, the optimal penalty Ψ^* obtained in this algorithm can only guarantee that the locally optimum resource allocation in (P1) with respect to Ψ^* is a local maximizer of the EE formula in (8). In fact, the EE formula is a nonconcave function in terms of $\mathbf{P}_S^{(m)}, \mathbf{P}_R^{(n)}, \rho$ and Φ , and it is in general very difficult to find the optimal penalty that can achieve the globally maximum EE.

Sometimes, it is desirable to consider individual node power (INP) constraints in wireless networks, especially when each node in the network is operated on a different power budget. Our proposed design framework can be easily extended to accommodate this scenario by replacing the constraint (C.1) in (9) with the following transmit power constraints:

$$\sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} P_{S,i}^{(n)} \leq P_{i,max}, \quad i = 1, \dots, N;$$

Algorithm 1 Iterative EEM Algorithm

- 1: Set the maximum number of outer iterations I_{max_1} ;
 - 2: Initialize the iteration counter $l = 0$ and network penalty $\Psi(l) = 0.001$.
 - 3: Initialize ρ and Φ .
 - 4: **repeat** (Outer Loop)
 - 5: Set the maximum number of inner iterations I_{max_2} and the step sizes ϵ_1, ϵ_2 and ϵ_3 ;
 - 6: Initialize iteration counter $t = 0, \alpha_i^{(m)}(t) = 1$ and $\beta_i^{(m)}(t) = 0$;
 - 7: Initialize $\lambda(t), \mu(t)$ and $\nu(t)$.
 - 8: **repeat** (Inner Loop)
 - 9: **repeat** (Solving problem (P5))
 - 10: Update $\hat{\mathbf{P}}_{S,i}^{(m)}, \hat{\mathbf{P}}_{R,i}^{(n)}$ and $\hat{\Gamma}_i^{(m)}$ using (21)–(23).
 - 11: Update Φ using (30).
 - 12: Update ρ using (31).
 - 13: Update $\lambda, \mu^{(k)}$ and $\nu^{(k)}$ using (32)–(34).
 - 14: **until** convergence to get the optimal solutions $\hat{\mathbf{P}}_{S,i}^{(m)*}, \hat{\mathbf{P}}_{R,i}^{(n)*}, \hat{\Gamma}_i^{(m)*}, \Phi^*$ and ρ^* .
 - 15: Set $\hat{\mathbf{P}}_{S,i}^{(m)}(t+1) \leftarrow \hat{\mathbf{P}}_{S,i}^{(m)*}, \hat{\mathbf{P}}_{R,i}^{(n)}(t+1) \leftarrow \hat{\mathbf{P}}_{R,i}^{(n)*}, \hat{\Gamma}_i^{(m)}(t+1) \leftarrow \hat{\Gamma}_i^{(m)*}, \Phi(t+1) \leftarrow \Phi^*, \rho(t+1) \leftarrow \rho^*$ and $t \leftarrow t+1$.
 - 16: **until** convergence or $t > I_{max_2}$.
 - 17: Update $\Psi(l+1)$ using (39), and $l \leftarrow l+1$.
 - 18: **until** convergence or $l > I_{max_1}$.
-

$$\sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} P_{R,i}^{(n)} \leq P_{R,max},$$

where $P_{i,max}$ and $P_{R,max}$ are the maximum allowable transmit power for the i -th source node and the relay node, respectively. This new optimization problem can be solved in a similar way as in the total power constraint case, although it now requires the update of $N+1$ Lagrangian multipliers in the master problem due to the $N+1$ imposed INP constraints.

Theorem 3: If the optimal resource allocation $(\hat{\mathbf{P}}_S^{(m)*}, \hat{\mathbf{P}}_R^{(n)*}, \rho^*, \Phi^*, \hat{\Gamma}^{(m)*})$ of the problem (P5) with respect to Ψ^* satisfies the following balance equation

$$R(\hat{\mathbf{P}}_S^{(m)*}, \hat{\mathbf{P}}_R^{(n)*}, \rho^*, \Phi^*, \hat{\Gamma}^{(m)*}) - \Psi^* P_T(\hat{\mathbf{P}}_S^{(m)*}, \hat{\mathbf{P}}_R^{(n)*}, \rho^*, \Phi^*) = 0, \quad (37)$$

then Ψ^* will be the optimal penalty for the resources allocated².

Proof: The proof of Theorem 3 is similar to the proof in [6, Appendix D]. ■

D. Update procedure for penalty factor Ψ

The penalty factor Ψ is defined as

$$\Psi = \frac{F_{LB}(\boldsymbol{\rho}, \boldsymbol{\Phi}, \hat{\boldsymbol{\Gamma}}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)})}{\sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} \left(e^{\hat{P}_{S,i}^{(m)}} + e^{\hat{P}_{R,i}^{(n)}} \right) + P_c}, \quad (38)$$

It is now important to find out what will be the optimal penalty for resource allocation and also how can we update it. To tackle these two problems, we furthermore propose the following two theorems, respectively.

Theorem 4: If the penalty factor is updated for the $(l+1)$ -th iteration for the local maximizer of the problem **(P3)** i.e., $(\hat{\mathbf{P}}_S^{(m)*}(l), \hat{\mathbf{P}}_R^{(n)*}(l), \boldsymbol{\rho}^*(l), \boldsymbol{\Phi}^*(l), \hat{\boldsymbol{\Gamma}}^{(m)*}(l))$, for the penalty $\Psi(l)$ in the l -th iteration as

$$\Psi(l+1) = \frac{R(\hat{\mathbf{P}}_S^{(m)*}(l), \hat{\mathbf{P}}_R^{(n)*}(l), \boldsymbol{\rho}^*(l), \boldsymbol{\Phi}^*(l), \hat{\boldsymbol{\Gamma}}^{(m)*}(l))}{P_T(\hat{\mathbf{P}}_S^{(m)*}(l), \hat{\mathbf{P}}_R^{(n)*}(l), \boldsymbol{\rho}^*(l), \boldsymbol{\Phi}^*(l))}, \quad (39)$$

then the penalty $\Psi(l)$ is monotonically increased with l .

Proof: The proof of Theorem 4 is similar to the proof in [6, Appendix E]. ■

Theorem 5: The optimal penalty factor can be obtained at the convergence point i.e. $\Psi^* = \lim_{l \rightarrow \infty} \Psi(l)$ satisfies the balance equation.

Proof: The proof of Theorem 5 is similar to the proof in [6, Appendix F]. ■

V. ANALYSIS OF OPTIMAL RESOURCE ALLOCATION POLICIES FOR TWO-USER CASES

To get more insight into the obtained optimal solutions in the previous section, we consider the two-user cases in the following two regimes:

1) *Interference-Dominated (ID) Regime:* We assume that the network operates at very high SNR, consequently equivalent noise power at the relay and the destination nodes is diminutive as compared to the interference powers i.e., $P_{S,1}^{(m)} |h_{S_2R}^{(m)}|^2 \gg (\sigma_R^{(m)})^2$, $P_{S,2}^{(m)} |h_{S_2R}^{(m)}|^2 \gg (\sigma_R^{(m)})^2$ and $P_{R,1}^{(n)} |h_{RD_1}^{(n)}|^2 \gg (\sigma_{D_2}^{(n)})^2$, $P_{R,2}^{(n)} |h_{RD_2}^{(n)}|^2 \gg (\sigma_{D_1}^{(n)})^2$, respectively.

Case 1: When $P_{S,1}^{(m)} |h_{S_2R}^{(m)}|^2 \gg (\sigma_R^{(m)})^2$, $P_{S,2}^{(m)} |h_{S_2R}^{(m)}|^2 \gg$

²Since the original optimization problem is nonconvex, thus the optimal solution obtained for **(P5)** works as a local maximizer here.

$(\sigma_R^{(m)})^2$, the optimal power for both users are given as

$$e^{\hat{P}_{S,1}^{(m)2}} = \mu_1^{(m)} e^{\hat{\Gamma}_1^{(m)} + \hat{P}_{S,2}^{(m)}} \frac{|h_{S_2R}^{(m)}|^2}{|h_{S_1R}^{(m)}|^2} \times \left[(\Psi + \lambda) \rho_{m,n}^* \Phi_{2,(m,n)}^* + \mu_2^{(m)} e^{\hat{\Gamma}_2^{(m)} - \hat{P}_{S,2}^{(m)}} \frac{|h_{S_1R}^{(m)}|^2}{|h_{S_2R}^{(m)}|^2} \right]^{-1}; \quad (40)$$

$$e^{\hat{P}_{S,2}^{(m)2}} = \mu_2^{(m)} e^{\hat{\Gamma}_2^{(m)} + \hat{P}_{S,1}^{(m)}} \frac{|h_{S_1R}^{(m)}|^2}{|h_{S_2R}^{(m)}|^2} \times \left[(\Psi + \lambda) \rho_{m,n}^* \Phi_{2,(m,n)}^* + \mu_1^{(m)} e^{\hat{\Gamma}_1^{(m)} - \hat{P}_{S,1}^{(m)}} \frac{|h_{S_2R}^{(m)}|^2}{|h_{S_1R}^{(m)}|^2} \right]^{-1}, \quad (41)$$

In practice, the transmit power should be different from zero for all nodes. Since the m -th subcarrier is allocated to the first source node, the power allocated on the m -th subcarrier by the second source node is almost close to zero. Thus, it can be observed in (40) and (41) that the source node allocates less transmit power on the m -th subcarrier if it has better subcarrier gain among all allocated subcarriers to it in order to improve the EE, whereas in case of without subcarrier pairing and power allocation, the power allocation policy for the first(second) source node is affected by the second(first) source channel gain.

Case 2: For $P_{R,1}^{(n)} |h_{RD_1}^{(n)}|^2 \gg (\sigma_{D_2}^{(n)})^2$, $P_{R,2}^{(n)} |h_{RD_2}^{(n)}|^2 \gg (\sigma_{D_1}^{(n)})^2$, we can get the optimal power for both users similar to (40) and (41) as follows:

$$e^{\hat{P}_{S,1}^{(m)2}} = \frac{\mu_1^{(m)} \left(e^{\hat{\Gamma}_1^{(m)} + \hat{P}_{S,2}^{(m)}} \frac{|h_{S_2R}^{(m)}|^2}{|h_{S_1R}^{(m)}|^2} + e^{\hat{\Gamma}_1^{(m)}} \frac{(\sigma_R^{(m)})^2}{|h_{S_1R}^{(m)}|^2} \right)}{(\Psi + \lambda) \rho_{m,n}^* \Phi_{1,(m,n)}^* + \mu_2^{(m)} \left(e^{\hat{\Gamma}_2^{(m)} - \hat{P}_{S,2}^{(m)}} \frac{|h_{S_1R}^{(m)}|^2}{|h_{S_2R}^{(m)}|^2} \right)}; \quad (42)$$

$$e^{\hat{P}_{S,2}^{(n)2}} = \frac{\mu_2^{(n)} \left(e^{\hat{\Gamma}_2^{(m)} + \hat{P}_{S,1}^{(m)}} \frac{|h_{S_1R}^{(m)}|^2}{|h_{S_2R}^{(m)}|^2} + e^{\hat{\Gamma}_2^{(m)}} \frac{(\sigma_R^{(m)})^2}{|h_{S_2R}^{(m)}|^2} \right)}{(\Psi + \lambda) \rho_{m,n}^* \Phi_{2,(m,n)}^* + \mu_1^{(m)} \left(e^{\hat{\Gamma}_1^{(m)} - \hat{P}_{S,1}^{(m)}} \frac{|h_{S_2R}^{(m)}|^2}{|h_{S_1R}^{(m)}|^2} \right)}, \quad (43)$$

2) *Noise-Dominated (ND) Regime:* When the network operates under a very low SNR, then the disturbances produced by the noise at the relay and the destination nodes become

significantly high as compared to the interference generated at the respective nodes.

• *Relay-Noise Dominated (RND) Regime*: In this regime, the relay noise is very high as compared to the interference produced at the relay node i.e., $P_{S,1}^{(m)} |h_{S_1R}^{(m)}|^2 \ll (\sigma_R^{(m)})^2$ and $P_{S,2}^{(m)} |h_{S_2R}^{(m)}|^2 \ll (\sigma_R^{(m)})^2$, respectively. The ratio of the optimal power of first user to the second user is given as

$$\frac{e^{\hat{P}_{S,1}^{(m)*2}}}{e^{\hat{P}_{S,2}^{(m)*2}}} = \frac{\mu_1^{(m)} e^{\hat{\Gamma}_1^{(m)}} \Phi_{2,(m,n)}^* |h_{S_2R}^{(m)}|^2}{\mu_2^{(m)} e^{\hat{\Gamma}_2^{(m)}} \Phi_{1,(m,n)}^* |h_{S_1R}^{(m)}|^2}, \quad (44)$$

This result reveals a *channel-reversal power allocation policy*, in which a user with a lower SR channel gain is allocated higher transmit power.

• *Destination-Noise Dominated (DND) Regime*: The destination noise is very high in this regime as compared to the interference i.e., $P_{R,2}^{(n)} |h_{RD_2}^{(n)}|^2 \ll (\sigma_{D_1}^{(n)})^2$ and $P_{R,1}^{(n)} |h_{RD_1}^{(n)}|^2 \ll (\sigma_{D_2}^{(n)})^2$, respectively. The optimal power for both users can be obtained similar to (42) and (43).

VI. SUBOPTIMAL EE RESOURCE ALLOCATION ALGORITHM

For a large number of N_{sc} or N , the computational complexity of the proposed EEM algorithm in Section IV becomes very high. Thus, we propose a low-complexity suboptimal EE algorithm whose performance is close to that of the EEM algorithm. The suboptimal EE algorithm is sketched as follows:

A. Step 1: Optimal Subcarrier Allocation for Fixed Power Allocation

First, we equally distribute the available transmit powers over all the subcarriers among the source and the relay nodes:

$$P_{S,1}^{(m)} = P_{S,2}^{(m)} = \dots = P_{S,N}^{(m)} = \frac{P_{max}}{2NN_{sc}}, \quad \forall m; \quad (45)$$

$$P_{R,1}^{(n)} = P_{R,2}^{(n)} = \dots = P_{R,N}^{(n)} = \frac{P_{max}}{2NN_{sc}}, \quad \forall n, \quad (46)$$

Using (2) and (4), we find SINR for each user pair and then take minimum SINR in order to balance the EE of the SR and RD links as

$$\Xi_{i,(m,n)} = \min\{\gamma_{SR_i}^{(m)}, \gamma_{RD_i}^{(n)}\}; \quad (47)$$

Next we update the subcarrier allocation matrix as follows:

$$\Phi_{i,(m,n)} = \begin{cases} 1, & \text{for } i = \arg \max_i \Xi_{i,(m,n)}; \\ 0, & \text{otherwise} \end{cases} \quad (48)$$

B. Step 2: Optimal Subcarrier Pairing for Given Subcarrier Allocation

In the second step, we arrange the SR subcarriers in a descending order according to their channel gains, and the RD subcarriers are also ordered in the same way. Then, we match the corresponding subcarriers with each other in sequence. If the m -th subcarrier of the SR link is paired with the n -th subcarrier of the RD link, we set $\rho_{m,n} = 1$; otherwise $\rho_{m,n} = 0$.

C. Step 3: Optimal Power Allocation for Given Subcarrier Pairing and Allocation

In the last step, for given subcarrier allocation and pairing matrices Φ and ρ , we allocate the power to each source and the relay nodes using (24) and (25).

VII. COMPLEXITY ANALYSIS

In this section, we perform an exhaustive complexity analysis to get a better insight into the complexity reduced by the EEM and suboptimal EEM algorithms.

• *EEM Algorithm*: First the complexity of the EEM algorithm is analyzed as follows. To find the optimal power allocation of (P5) for N user pairs with N_{sc} subcarriers in each hop, we need to solve NN_{sc}^2 subproblems. The optimal power allocation solution $(\hat{P}_{S,i}^{(m)*}, \hat{P}_{R,i}^{(n)*})$ can be found using the exhaustive search (ES) approach which searches over $\hat{P}_{S,i}^{(m)}$ and $\hat{P}_{R,i}^{(n)}$ under assumption that each takes discrete values [29]. Therefore, K denotes the number of power levels that can be taken by each of $\hat{P}_{S,i}^{(m)}$ and $\hat{P}_{R,i}^{(n)}$. Each subcarrier pairing $\rho_{m,n}$ is allocated to a particular user pair and each maximization in (30) has a complexity of $\mathcal{O}(N)$, hence the total complexity for subcarrier allocation becomes $\mathcal{O}(NN_{sc}^2)$. Furthermore, the Hungarian method is used to determine optimal subcarrier pairing in (31) with a complexity of $\mathcal{O}(N_{sc}^3)$. The complexity of updating a dual variable is $\mathcal{O}((2N)^\varrho)$ (for example, $\varrho = 2$ if the ellipsoid method is used [26]). Therefore, the total complexity for updating dual variables is $\mathcal{O}(3(2N)^\varrho)$. Let us suppose if the dual objective function $g(\lambda, \mu, \nu)$ and the penalty factor Ψ converge in Z and L iterations, respectively, the total complexity for EEM algorithm is $\mathcal{O}(3(2N)^\varrho N_{sc}^2 ZL (N(K^3 + 2) + N_{sc}))$, whereas with equal subcarrier power allocation (ESPA), the total complexity of EEM algorithm becomes $\mathcal{O}(5(2N)^\varrho N_{sc}^2 ZL (N(K^3 + 4) + N_{sc}))$.

• *Without subcarrier allocation and pairing (WSAP)*: In this algorithm only power is allocated optimally without considering the subcarrier allocation and pairing, respectively. Thus we update the power and the dual variables with complexity of $\mathcal{O}(NN_{sc}^2 Z (K^3 + 1))$ and $\mathcal{O}(3(2N)^\varrho)$, respectively. Thus the total complexity for this algorithm becomes $\mathcal{O}(3(2N)^\varrho NLN_{sc}^2 (K^3 + 1))$.

• *Suboptimal Algorithm*: The implementation of the subcarrier allocation in step 1 as stated in (48) requires a complexity of $\mathcal{O}(2NN_{sc})$, whereas the subcarrier pairing in step 2 has a complexity of $\mathcal{O}(2N_{sc})$. Further, the power allocation in the last step adds a complexity of $\mathcal{O}(N_{sc}(K^3 + 1))$ and the total complexity for updating dual variables is $\mathcal{O}(3(2N)^\varrho)$. If the dual objective function $g(\lambda, \mu, \nu)$ and the penalty factor Ψ converge in Z and L iterations, respectively, then the suboptimal algorithm has a complexity of $\mathcal{O}(3(2N)^\varrho N_{sc} ZL (2N + K^3 + 3))$.

• *Exhaustive search (ES)*: As a benchmark, we compare the complexity of the proposed algorithms with the exhaustive search (ES) algorithm, which gives the optimal solution after searching over all variables. However, the complexity of ES increases very quickly as the size of the problem increases. Therefore, ES search is typically used when the

TABLE I
COMPLEXITY ANALYSIS FOR DIFFERENT ALGORITHMS

Complexity Comparison	
Algorithm	Complexity
EEM	$\mathcal{O}(3(2N)^e N_{sc}^2 ZL(N(K^3+2)+N_{sc}))$
Suboptimal EEM	$\mathcal{O}(3(2N)^e N_{sc} ZL(2N+K^3+3))$
ESPA	$\mathcal{O}(5(2N)^e N_{sc}^2 ZL(N(K^3+4)+N_{sc}))$
WSAP	$\mathcal{O}(3(2N)^e N_{sc}^2 ZL(N(K^3+1)))$
ES	$\mathcal{O}(3(2N)^e ZLN_{sc}^2 (K^3+1))$

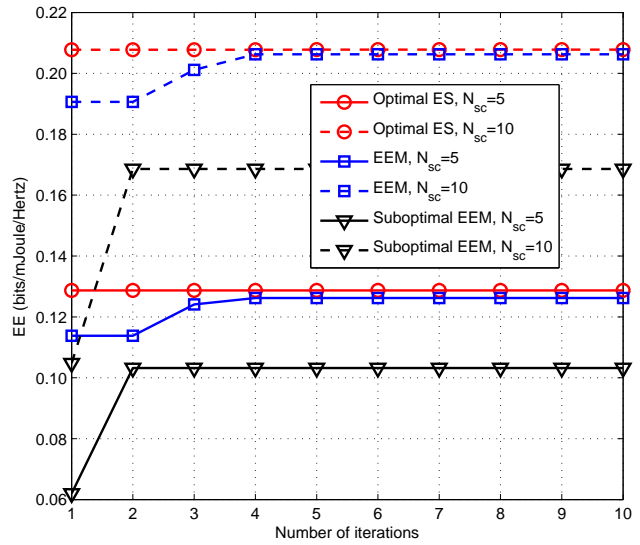
problem size is limited. The complexity of ES can be given by $\mathcal{O}(3(2N)^e ZLN_{sc}^2 (K^3+1))$. The complexity of the proposed algorithms and ES is summarized in Table I.

VIII. SIMULATION RESULTS

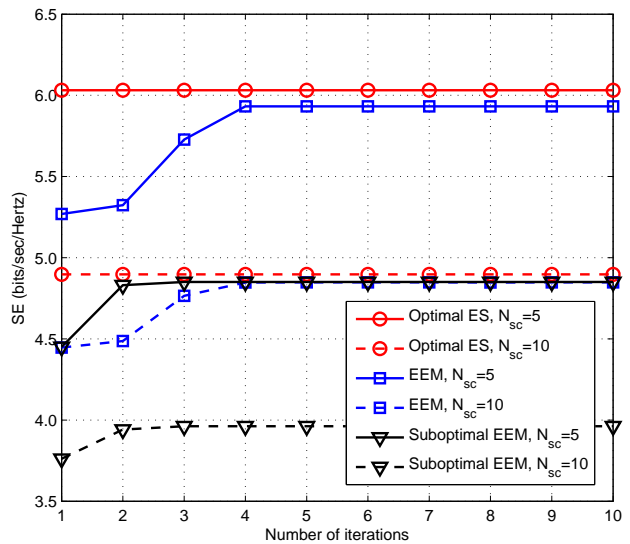
In this section, we evaluate the performance of the proposed energy efficient resource allocation algorithms through computer simulations. The path loss model stated by $131.1+42.8 \times \log_{10}(d)$ dB (d : distance in km) is adopted in our simulations [30]. Moreover, the log-normal shadowing $\sim \ln \mathcal{N}(0, 8dB)$ and independently and identically distributed (i.i.d.) Rayleigh fading effects $\sim \mathcal{CN}(0, 1)$ for all links in the considered framework are taken into consideration, respectively. It is assumed that the circuit and processing power dissipation per antenna at each node is 14 dBm, respectively [6]. The subcarrier spacing and thermal noise density are given by 12 kHz and -174 dBm/Hz, respectively. The maximum number of inner and outer iterations I_{max_1} and I_{max_2} for proposed algorithms are set as 10, and the convergence tolerance value is 10^{-5} . The constant step sizes ϵ_1 , ϵ_2 , and ϵ_3 are used in Algorithm with value 0.01. The penalty factor Ψ that shows a tradeoff between the EE and the SE is $\Psi = 0.001$. We define a distance ratio as $r_d = d_{SR}/(d_{SR} + d_{RD})$, where d_{SR} and d_{RD} are used to present the distance from all source nodes to the relay node and from the relay node to all destination nodes, respectively. The ES algorithm which gives the optimal solution of the considered problem, the proposed EEM algorithm without (w/o) subcarrier pairing and allocation (SPA) and the spectral efficiency maximization (SEM) algorithm are also implemented for performance comparison.

Fig. 2 shows the convergence behavior of the ES and the proposed algorithms for a single channel realization, where $N = 2$, $N_{sc} = \{5, 10\}$, and $d_{SR} = d_{RD} = 200$ m. The maximum allowable transmit power is $P_{max} = 10$ dBm. It can be observed that the EE and the SE performance of the ES and the proposed algorithms are monotonically increased with the number of iterations, and the proposed algorithms are converged in less than five iterations. The attaining EE and SE performance of the EEM algorithm is very close to that of the ES algorithm.

In Fig. 3, we compare the performance of various algorithms for $N_{sc} = 32$. For EEM-INP, we set $P_{i,max} = P_{R,max} = \frac{P_{max}}{N+1}$ for a fair comparison with the total power constraint case. It is shown in Fig. 3 that both the average EE and SE performances of the EEM-INP algorithm are slightly worse than those using the total power constraint when P_{max} is small, while the performance gap gradually reduces to zero as P_{max} increases and the EEM algorithm performs much



(a) EE versus Number of iterations

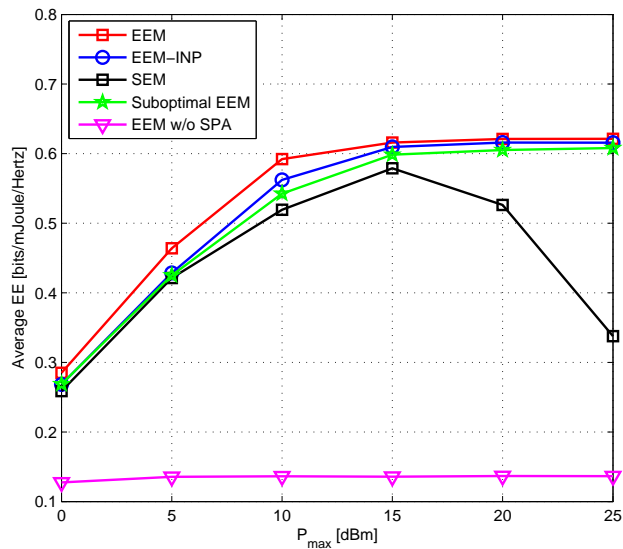
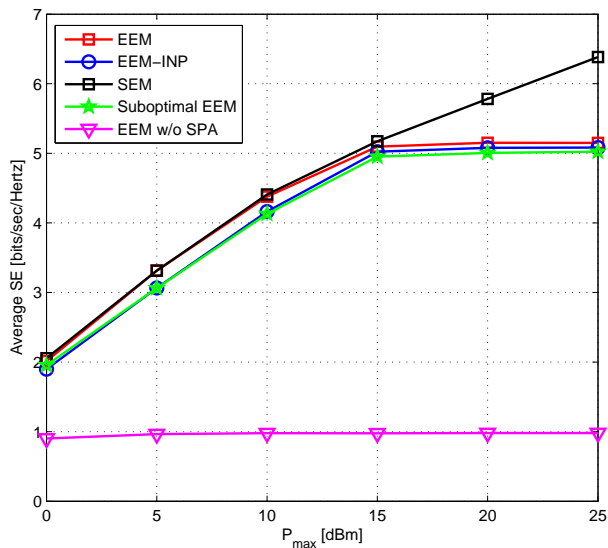


(b) SE versus Number of iterations

Fig. 2. Convergence behavior of the proposed algorithms ($N = 2$ and $d_{SR} = d_{RD} = 200$ m).

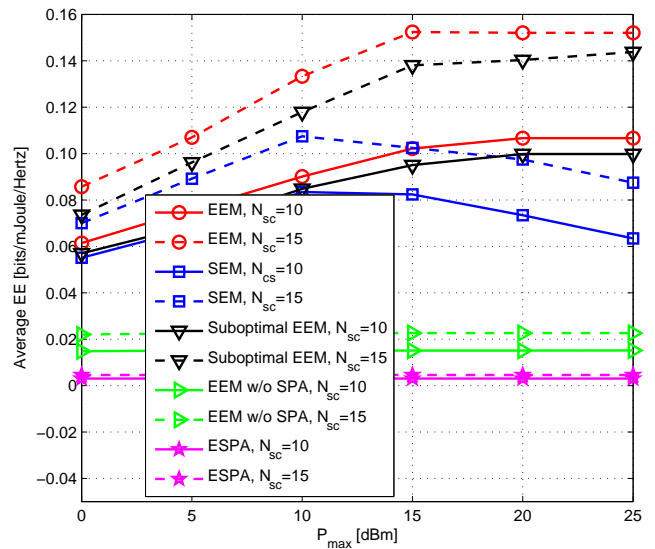
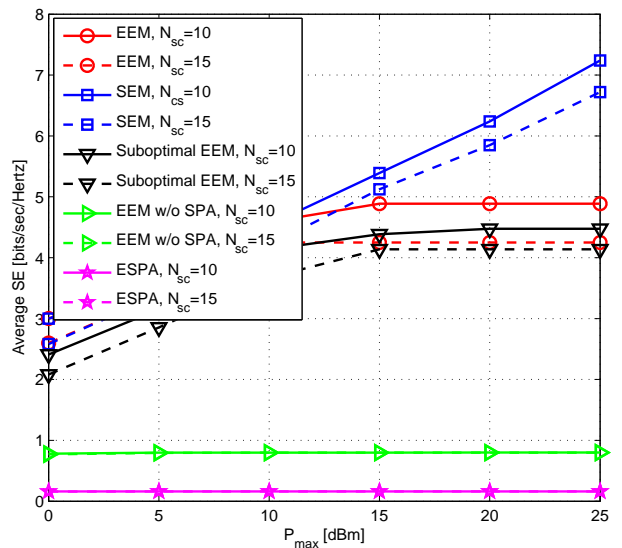
better than the other algorithms in terms of both the average SE and EE.

The effect of the number of subcarriers N_{sc} on the average EE and SE of the proposed algorithms is shown in Fig. 4, where $N = 5$ and $d_{SR} = d_{RD} = 200$ m. The ESPA algorithm which equally allocates the available power among users and relay node over all the subcarriers is also simulated for comparison. We can observe that the average EE and SE can be significantly improved as P_{max} increases. When $P_{max} \leq 10$ dBm, the EEM, suboptimal and SEM algorithms exhibit almost identical EE or SE performance due to limited power budget. However, in a rich power budget i.e., $P_{max} > 10$ dBm, the average EE performance of the EEM and suboptimal algorithms become constant, whereas that of

(a) Average EE versus P_{max} (b) Average SE versus P_{max} Fig. 3. Performance comparison of different algorithms ($N = 2$, $N_{sc} = 32$ and $d_{SR} = d_{RD} = 200$ m).

the SEM algorithm quickly declines as P_{max} increases. On the other hand, the average SE of the proposed algorithms is slowly saturated for $P_{max} > 10$ dBm, while the performance of the SEM algorithm is continuously improved as P_{max} increases. As expected, as N_{sc} increases, the average EE improves due to the frequency diversity. The EEM w/o SPA performs worst than the EEM and SEM in terms of both the average EE and SE. We can also observed that the average EE and SE performance of the EEM and SEM algorithms with total power constraint is much better than the ESPA.

Fig. 5 shows the average EE and SE performance for different numbers of users, where $N_{sc} = 15$ and $d_{SR} = d_{RD} = 200$ m. As can be seen, as N increases the average EE performance of all the algorithms deteriorates due to the increases in the

(a) Average EE versus P_{max} (b) Average SE versus P_{max} Fig. 4. Effect of number of subcarriers on the average EE and SE ($N = 5$ and $d_{SR} = d_{RD} = 200$ m).

static power, whereas the average SE can be enhanced via multiuser diversity (see Fig. 5(a) and Fig. 5(b)). In higher power budget, the average SE of the SEM algorithm improves at the cost of a degradation in the average EE, while the average EE and SE are remained steady for the EEM and suboptimal algorithms.

Fig. 6 and Fig. 7 show the impact of the relay position on the average EE and SE for different N_{sc} and N , respectively. The total transmit power, P_{max} , and the distance from the source to the destination, $d_{SR} + d_{RD}$, are set as 15 dBm and 1000 m, respectively. As can be seen from Fig. 6, the average EE of the EEM algorithm is improved as N_{sc} increases and the algorithm achieves largest average EE when the relay nodes is placed in the middle of the source and the destination. Fig. 7 shows

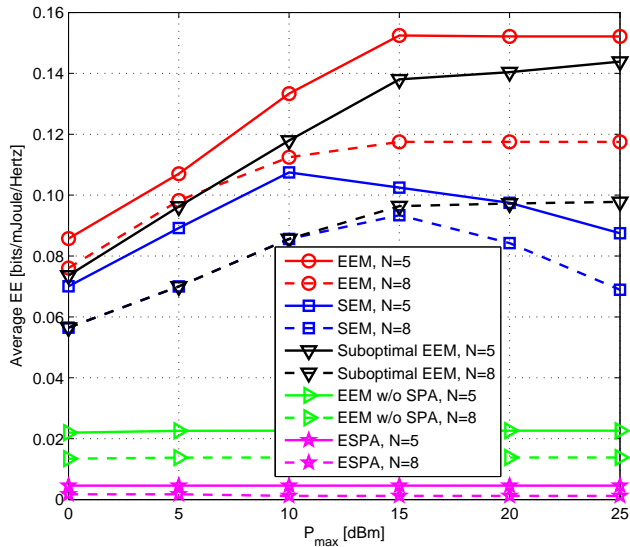
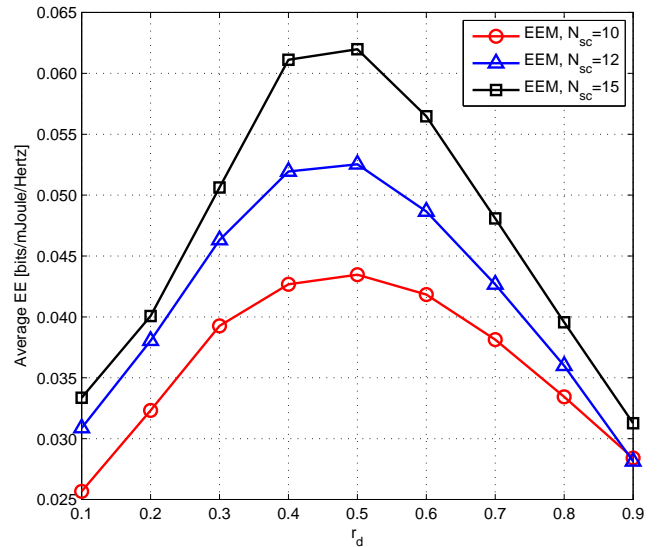
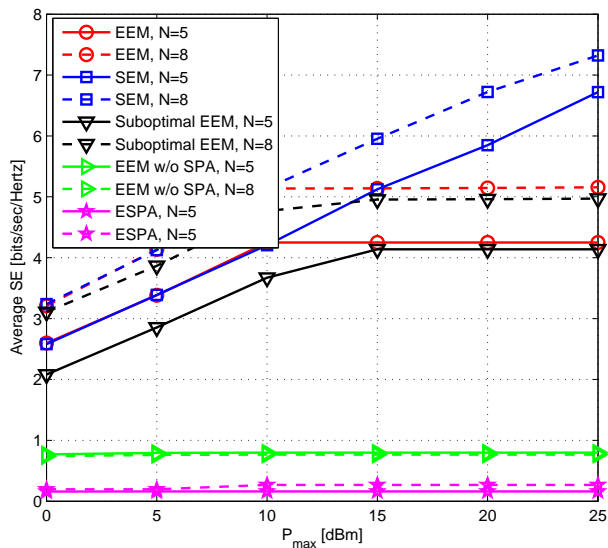
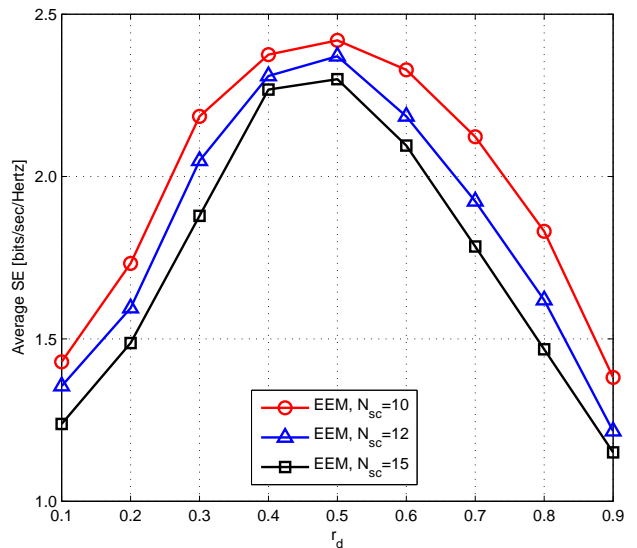
(a) Average EE versus P_{max} (a) Average EE versus P_{max} (b) Average SE versus P_{max} (b) Average SE versus P_{max}

Fig. 5. Effect of number of users on the average EE and SE ($N_{sc} = 15$ and $d_{SR} = d_{RD} = 200$ m).

a reverse trend. The EEM algorithm gives highest average EE(SE) when relay node is in middle and it's performance decreases(increases) with increasing N .

IX. CONCLUSIONS

In this paper, we investigated the problem of joint power and subcarrier allocation in relay-assisted multiuser networks from a green energy perspective. To improve the EE of the network a penalty based approach has been adopted and a balance is created between the maximum achievable network EE and SE. The primal problem was nonconvex due to a form of fractional and mixed binary integer nonlinear programming. We transformed this problem into a convex problem by a series of transformations. Further, the dual decomposition method is

Fig. 6. Effect of the relay position with different N_{sc} on the average EE and SE ($N = 5$ and $P_{max} = 15$ dBm).

adopted to find the optimal power and subcarrier pairing and allocation matrices, respectively. Moreover, the convergence behaviour of the EEM algorithm is proved theoretically. To reduce the complexity of the proposed iterative EEM algorithm, we further demonstrated a suboptimal algorithm by exchanging minimal network performance. We compared the performance of the proposed algorithms with that of the ES and SEM algorithms and demonstrated the effects of various network parameters such as number of subcarriers, users, and the relay's position, on the average EE and SE performance. Simulation results confirmed that a higher number of subcarriers, although marginally decreases the SE, can improve the EE. Improvements in a SE come by increasing the number of users at a certain expense of the EE of the network. This

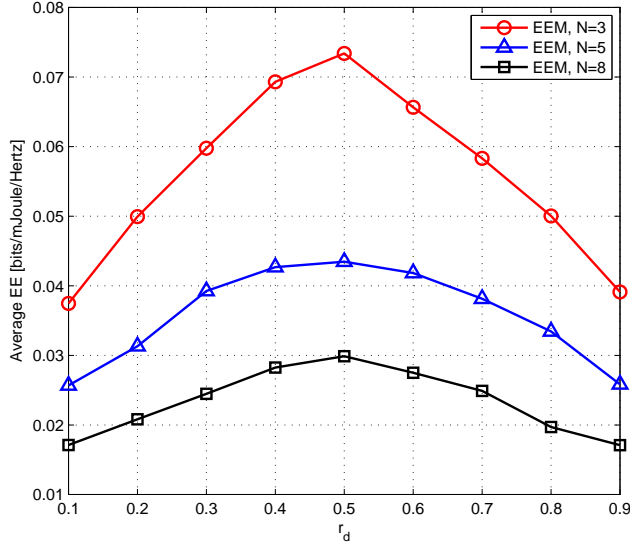
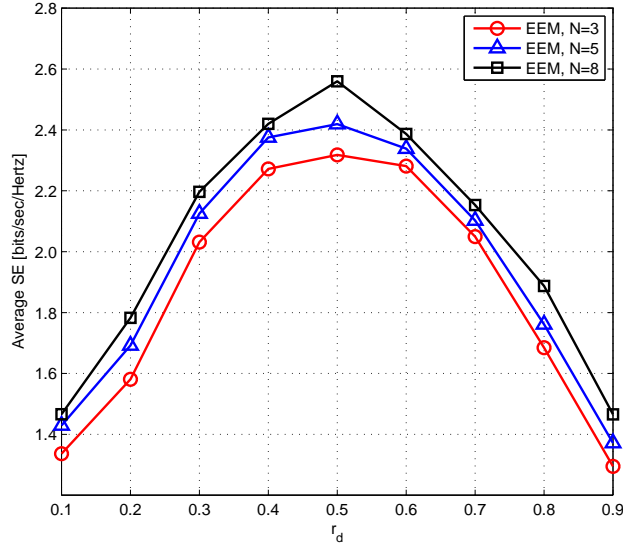
(a) Average EE versus P_{max} (b) Average SE versus P_{max}

Fig. 7. Effect of the relay position with different N on the average EE and SE ($N_{sc} = 10$ and $P_{max} = 15$ dBm).

design framework provides a useful model for developing future energy efficient multiuser relay networks.

APPENDIX A PROOF OF THEOREM 1

The objective function in **(P5)** can be rewrite as:

$$\begin{aligned} \mathcal{F}_{LB}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \boldsymbol{\rho}, \Phi, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)}) \\ = \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \Lambda_{m,n}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)}), \quad (\text{A.1}) \end{aligned}$$

where the term $\Lambda_{m,n}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)})$ is explicitly defined as

$$\begin{aligned} \Lambda_{m,n}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)}) \\ = \sum_{i=1}^N \frac{1}{2} \rho_{m,n} \Phi_{i,(m,n)} \left(\frac{\alpha_i^{(m)}}{\ln(2)} \hat{\Gamma}_i^{(m)} + \beta_i^{(m)} \right) \\ - \Psi \sum_{i=1}^N \rho_{m,n} \Phi_{i,(m,n)} \left(e^{\hat{P}_{S,i}^{(m)}} + e^{\hat{P}_{R,i}^{(n)}} \right) - \frac{\Psi}{N_{sc}^2} P_c, \quad (\text{A.2}) \end{aligned}$$

wherein $\{\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \hat{\Gamma}^{(m)}\} \in \mathbb{W}^N$ and $\Lambda_{m,n}(\cdot) : \mathbb{W}^N \rightarrow \mathbb{R}$, are not necessarily convex in nature.

In a similar manner, the constraints (C.1) – (C.5), (C.7) and (C.8) can be expressed as

$$\sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \Omega_{m,n}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)}) \leq 0, \quad (\text{A.3})$$

where $\Omega_{m,n}(\cdot) : \mathbb{W}^N \rightarrow \mathbb{R}^T$ and $T = 7$ is the total number of constraints limiting the optimization problem **(P5)**. Hence, the optimization problem **(P5)** can be written as $OP^* \triangleq$

$$\begin{aligned} \max_{\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \hat{\Gamma}^{(m)}} \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \Lambda_{m,n}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)}) \\ \text{s.t.} \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \Omega_{m,n}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)}) \leq \mathbf{0}, \quad (\text{A.4}) \end{aligned}$$

where $\mathbf{0} \in \mathbb{R}^L$. Next, we define a perturbation function $\omega(\mathbf{U})$ to prove $D_G \approx 0$, as follows:
 $\omega(\mathbf{U}) \triangleq$

$$\begin{aligned} \max_{\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \hat{\Gamma}^{(m)}} \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \Lambda_{m,n}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)}) \\ \text{s.t.} \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \Omega_{m,n}(\hat{\mathbf{P}}_S^{(m)}, \hat{\mathbf{P}}_R^{(n)}, \hat{\Gamma}^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)}) \leq \mathbf{U}, \quad (\text{A.5}) \end{aligned}$$

where $\mathbf{U} \in \mathbb{R}^L$ is a perturbation vector. From [26] and [27], it is evident that duality gap tends to zero $D_G \approx 0$, when the time-sharing condition is satisfied. Moreover, it is also shown that the time-sharing condition will be satisfied if the optimal value of the optimization problem **(P5)** is a concave function of the constraints. Therefore, if $\omega(\mathbf{U})$ is a concave function of \mathbf{U} , then $D_G \approx 0$. Next, we define time sharing property as follows:

Definition 3: If $(\hat{\mathbf{P}}_{S_i}^{(m)*}, \hat{\mathbf{P}}_{R_i}^{(n)*}, \hat{\Gamma}_i^{(m)*})$, $i = 1, 2$, denotes the optimal solution of (A.5) and represented by $\omega(\mathbf{U}_1)$ and $\omega(\mathbf{U}_2)$, respectively, then there always exists a solution

$(\hat{\mathbf{P}}_{S_3}^{(m)*}, \hat{\mathbf{P}}_{R_3}^{(n)*}, \hat{\Gamma}_3^{(m)*})$ that satisfies the following condition:

$$\sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \Omega_{m,n} \left(\hat{\mathbf{P}}_{S_3}^{(m)}, \hat{\mathbf{P}}_{R_3}^{(n)}, \hat{\Gamma}_3^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)} \right) \leq \Delta \mathbf{U}_1 + (1 - \Delta) \mathbf{U}_1; \quad (\text{A.6})$$

$$\sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \Lambda_{m,n} \left(\hat{\mathbf{P}}_{S_3}^{(m)}, \hat{\mathbf{P}}_{R_3}^{(n)}, \hat{\Gamma}_3^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)} \right) \geq \Delta \Lambda_{m,n} \left(\hat{\mathbf{P}}_{S_1}^{(m)*}, \hat{\mathbf{P}}_{R_1}^{(n)*}, \hat{\Gamma}_1^{(m)*} \right) + (1 - \Delta) \Lambda_{m,n} \left(\hat{\mathbf{P}}_{S_2}^{(m)*}, \hat{\mathbf{P}}_{R_2}^{(n)*}, \hat{\Gamma}_2^{(m)*} \right), \quad (\text{A.7})$$

where $0 \leq \Delta \leq 1$.

Next, we need to prove that $\omega(\mathbf{U})$ is a concave function of \mathbf{U} . For some Δ , it is easy to find out \mathbf{U}_3 that satisfies $\mathbf{U}_3 = \Delta \mathbf{U}_1 + (1 - \Delta) \mathbf{U}_2$. We assume $(\hat{\mathbf{P}}_{S_1}^{(m)*}, \hat{\mathbf{P}}_{R_1}^{(n)*}, \hat{\Gamma}_1^{(m)*})$, $(\hat{\mathbf{P}}_{S_2}^{(m)*}, \hat{\mathbf{P}}_{R_2}^{(n)*}, \hat{\Gamma}_2^{(m)*})$ and $(\hat{\mathbf{P}}_{S_3}^{(m)*}, \hat{\mathbf{P}}_{R_3}^{(n)*}, \hat{\Gamma}_3^{(m)*})$ are the optimal solutions subject to the constraints of $\omega(\mathbf{U}_1)$, $\omega(\mathbf{U}_2)$ and $\omega(\mathbf{U}_3)$, respectively. Time-sharing condition points out that there exists a value $(\hat{\mathbf{P}}_{S_3}^{(m)}, \hat{\mathbf{P}}_{R_3}^{(n)}, \hat{\Gamma}_3^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)})$ that satisfying (A.6) and (A.8). $(\hat{\mathbf{P}}_{S_3}^{(m)*}, \hat{\mathbf{P}}_{R_3}^{(n)*}, \hat{\Gamma}_3^{(m)*})$ is the optimal solution to $\omega(\mathbf{U}_3)$, thus

$$\begin{aligned} & \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \Lambda_{m,n} \left(\hat{\mathbf{P}}_{S_3}^{(m)*}, \hat{\mathbf{P}}_{R_3}^{(n)*}, \hat{\Gamma}_3^{(m)*} \right) \\ & \geq \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \Lambda_{m,n} \left(\hat{\mathbf{P}}_{S_3}^{(m)}, \hat{\mathbf{P}}_{R_3}^{(n)}, \hat{\Gamma}_3^{(m)}, \boldsymbol{\alpha}^{(m)}, \boldsymbol{\beta}^{(m)} \right) \\ & \geq \Delta \Lambda_{m,n} \left(\hat{\mathbf{P}}_{S_1}^{(m)*}, \hat{\mathbf{P}}_{R_1}^{(n)*}, \hat{\Gamma}_1^{(m)*} \right) \\ & \quad + (1 - \Delta) \Lambda_{m,n} \left(\hat{\mathbf{P}}_{S_2}^{(m)*}, \hat{\mathbf{P}}_{R_2}^{(n)*}, \hat{\Gamma}_2^{(m)*} \right), \quad (\text{A.8}) \end{aligned}$$

Then, the concavity of $\omega(\mathbf{U})$ is proved.

Since, $\omega(\mathbf{U})$ is concave, it is possible to show that (A.4) satisfies the time-sharing property. When the number of subcarriers goes to infinity, the time-sharing condition always holds for multicarrier systems [29]. Let us assume $(\hat{\mathbf{P}}_{S_1}^{(m)*}, \hat{\mathbf{P}}_{R_1}^{(n)*}, \hat{\Gamma}_1^{(m)*})$ and $(\hat{\mathbf{P}}_{S_2}^{(m)*}, \hat{\mathbf{P}}_{R_2}^{(n)*}, \hat{\Gamma}_2^{(m)*})$ be two feasible solutions. Then, Δ percentage of total subcarriers N_{sc} are allocated with the solution $(\hat{\mathbf{P}}_{S_1}^{(m)*}, \hat{\mathbf{P}}_{R_1}^{(n)*}, \hat{\Gamma}_1^{(m)*})$, whereas the rest of $(1 - \Delta)$ percentage of the total subcarriers N_{sc} are allocated with the solution $(\hat{\mathbf{P}}_{S_2}^{(m)*}, \hat{\mathbf{P}}_{R_2}^{(n)*}, \hat{\Gamma}_2^{(m)*})$. Further, $\sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \Lambda_{m,n} \left(\hat{\mathbf{P}}_{S_3}^{(m)}, \hat{\mathbf{P}}_{R_3}^{(n)}, \hat{\Gamma}_3^{(m)} \right)$ is a linear combination, expressed as $\Delta \Lambda_{m,n} \left(\hat{\mathbf{P}}_{S_1}^{(m)*}, \hat{\mathbf{P}}_{R_1}^{(n)*}, \hat{\Gamma}_1^{(m)*} \right) + (1 - \Delta) \Lambda_{m,n} \left(\hat{\mathbf{P}}_{S_2}^{(m)*}, \hat{\mathbf{P}}_{R_2}^{(n)*}, \hat{\Gamma}_2^{(m)*} \right)$. Hence the constraints are linear combinations, therefore, it is proved that (A.4) satisfies the time-sharing property. Henceforth, $\omega(\mathbf{U})$ is a concave function of \mathbf{U} , and the duality gap $D_G \approx 0$. This proof of theorem is completed.

APPENDIX B PROOF OF THEOREM 2

This theorem is proved in two steps: 1) we need to prove that the network EE performance is increased monotonically along with the update of two coefficients $\alpha_i^{(m)}(t)$ and $\beta_i^{(m)}(t)$; and 2) the optimal solution for (P5) is also a local maximizer for (P1) when the update has converged. To prove first part, let $(\hat{\mathbf{P}}_S^{(m)*}(t), \hat{\mathbf{P}}_R^{(n)*}(t), \boldsymbol{\rho}^*(t), \boldsymbol{\Phi}^*(t), \hat{\Gamma}^{(m)*}(t))$ is the optimal solution in the t -th iteration for the coefficients $\alpha_i^{(m)}(t)$ and $\beta_i^{(m)}(t)$. If we update the coefficients in accordance with (35) and (36), then from (12)-(14), we have (B.1) and also from the problem (P5) it directly implies the condition defined in (B.2). From (B.1) and (B.2), we can conclude that the lower bound performance improves with the update of coefficients $\alpha_i^{(m)}(t)$ and $\beta_i^{(m)}(t)$. Next, we need to prove that the optimal solution of (P5) is a local maximizer for (P1) when the update has converged. This can be done by proving that the optimal solutions which satisfies the KKT conditions of (P5) also satisfies the KKT conditions of (P1). Therefore, we take the partial derivative of the Lagrangian function in (18) with respect to $\hat{P}_{S,i}^{(m)}$, $\hat{P}_{R,i}^{(n)}$ and $\Gamma_i^{(m)}$ at $(\hat{\mathbf{P}}_S^{(m)*}, \hat{\mathbf{P}}_R^{(n)*}, \boldsymbol{\rho}^*, \boldsymbol{\Phi}^*, \Gamma^{(m)*}, \boldsymbol{\lambda}^*, \boldsymbol{\mu}^*, \boldsymbol{\nu}^*)$ and equate the results to zero. Furthermore, by using Lemma 1, we get

$$P_{S,i}^{(m)*} = \frac{\mu_i^{(m)*} \left(\sum_{j=1, j \neq i}^N \frac{\Gamma_i^{(m)*} P_{S,j}^{(m)*} |h_{S_j R}^{(m)}|^2}{P_{S,i}^{(m)*} |h_{S_i R}^{(m)}|^2} + \frac{\Gamma_i^{(m)*} (\sigma_R^{(m)})^2}{P_{S,i}^{(m)*} |h_{S_i R}^{(m)}|^2} \right)}{\sum_{n=1}^{N_{sc}} \rho_{m,n}^* \Phi_{i,(m,n)}^* (\boldsymbol{\lambda}^* + \Psi)}; \quad (\text{B.3})$$

$$P_{R,i}^{(n)*} = \frac{\nu_i^{(n)*} \left(\sum_{j=1, j \neq i}^N \frac{\Gamma_i^{(n)*} P_{R,j}^{(n)*} |h_{RD_j}^{(n)}|^2}{P_{R,i}^{(n)*} |h_{RD_i}^{(n)}|^2} + \frac{\Gamma_i^{(n)*} (\sigma_{D_i}^{(n)})^2}{P_{R,i}^{(n)*} |h_{RD_i}^{(n)}|^2} \right)}{\sum_{m=1}^{N_{sc}} \rho_{m,n}^* \Phi_{i,(m,n)}^* (\boldsymbol{\lambda}^* + \Psi)}; \quad (\text{B.4})$$

$\Gamma_i^{(m)*}$ can also be computed in a similar manner. For the optimal solutions obtained in (B.3) and (B.4), the dual function is given as

$$\begin{aligned} & g(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^{(m)*}, \boldsymbol{\nu}^{(m,n)*}) \\ & = \max_{\boldsymbol{\rho}, \boldsymbol{\Phi}} \sum_{i=1}^N \sum_{m=1}^{N_{sc}} \sum_{n=1}^{N_{sc}} \rho_{m,n} \Phi_{i,(m,n)} \mathcal{B}'_{i,(m,n)} + \mathcal{C}' \\ & \quad \text{s.t.} \quad (\text{C.1}) - (\text{C.5}), \quad (\text{B.5}) \end{aligned}$$

where $\mathcal{B}'_{i,(m,n)}$ and \mathcal{C}' are defined as

$$\begin{aligned} \mathcal{B}'_{i,(m,n)} & = \left(\varpi \alpha_i^{(m)*} \Gamma_i^{(m)*} + \varphi \beta_i^{(m)*} \right) \\ & \quad - (\Psi + \boldsymbol{\lambda}^*) \left(P_{S,i}^{(m)*} + P_{R,i}^{(n)*} \right), \quad (\text{B.6}) \end{aligned}$$

$$\begin{aligned}
& \mathcal{F}_{LB} \left(\hat{\mathbf{P}}_S^{(m)*}(t), \hat{\mathbf{P}}_R^{(n)*}(t), \boldsymbol{\rho}^*(t), \boldsymbol{\Phi}^*(t), \boldsymbol{\Gamma}^{(m)*}(t), \boldsymbol{\alpha}^{(m)}(t), \boldsymbol{\beta}^{(m)}(t) \right) \\
& \leq \mathcal{F} \left(\hat{\mathbf{P}}_S^{(m)*}(t), \hat{\mathbf{P}}_R^{(n)*}(t), \boldsymbol{\rho}^*(t), \boldsymbol{\Phi}^*(t), \boldsymbol{\Gamma}^{(m)*}(t) \right) \\
& = \mathcal{F}_{LB} \left(\hat{\mathbf{P}}_S^{(m)*}(t), \hat{\mathbf{P}}_R^{(n)*}(t), \boldsymbol{\rho}^*(t), \boldsymbol{\Phi}^*(t), \boldsymbol{\Gamma}^{(m)*}(t), \alpha_i^{(m)}(t+1), \beta_i^{(m)}(t+1) \right); \tag{B.1}
\end{aligned}$$

$$\begin{aligned}
& \mathcal{F}_{LB} \left(\hat{\mathbf{P}}_S^{(m)*}(t), \hat{\mathbf{P}}_R^{(n)*}(t), \boldsymbol{\rho}^*(t), \boldsymbol{\Phi}^*(t), \boldsymbol{\Gamma}^{(m)*}(t), \alpha_i^{(m)}(t+1), \beta_i^{(m)}(t+1) \right) \\
& \leq \mathcal{F}_{LB} \left(\hat{\mathbf{P}}_S^{(m)*}(t+1), \hat{\mathbf{P}}_R^{(n)*}(t+1), \boldsymbol{\rho}^*(t+1), \boldsymbol{\Phi}^*(t+1), \boldsymbol{\Gamma}^{(m)*}(t+1), \alpha_i^{(m)}(t+1), \beta_i^{(m)}(t+1) \right), \tag{B.2}
\end{aligned}$$

$$\begin{aligned}
C' &= -\Psi P_C + \lambda^* P_{max} \\
& - \sum_{i=1}^N \sum_{k=1}^{N_{sc}} \mu_i^{(k)*} \left(\sum_{j=1, j \neq i}^N \frac{(\Gamma_i^{(k)*} + P_{S,j}^{(k)*}) |h_{S_j R}^{(k)}|^2}{P_{S,i}^{(k)*} |h_{S_i R}^{(k)}|^2} \right. \\
& \quad \left. + \frac{\Gamma_i^{(k)*} (\sigma_R^{(k)})^2}{P_{S,i}^{(k)*} |h_{S_i R}^{(k)}|^2} + 1 \right) \\
& - \sum_{i=1}^N \sum_{k=1}^{N_{sc}} \nu_i^{(k)*} \left(\sum_{j=1, j \neq i}^N \frac{(\Gamma_i^{(k)*} + P_{R,j}^{(k)*}) |h_{RD_j}^{(k)}|^2}{P_{R,i}^{(k)*} |h_{RD_i}^{(k)}|^2} \right. \\
& \quad \left. + \frac{\Gamma_i^{(k)*} (\sigma_{D_i}^{(k)})^2}{P_{R,i}^{(k)*} |h_{RD_i}^{(k)}|^2} + 1 \right), \tag{B.7}
\end{aligned}$$

Similar to (29) the optimal subcarrier allocation is determined for a given subcarrier pairing as

$$\Phi_{i,(m,n)}^* = \begin{cases} 1, & \text{for } i = \arg \max_i \mathcal{B}'_{i,(m,n)}, \\ 0, & \text{otherwise} \end{cases} \tag{B.8}$$

and the optimal subcarrier pairing matrix for obtained power and subcarrier allocation can be found similar to (31) where \mathcal{B}' is defined as

$$\mathcal{B}' = \begin{bmatrix} \mathcal{B}'_{i^*,(1,1)} & \cdots & \mathcal{B}'_{i^*,(1,K)} \\ \vdots & \ddots & \vdots \\ \mathcal{B}'_{i^*,(K,1)} & \cdots & \mathcal{B}'_{i^*,(K,K)} \end{bmatrix} \tag{B.9}$$

Moreover, the optimal solution of **(P1)** can be derived from the KKT conditions that the partial derivatives of the Lagrangian function of the problem **(P1)** with respect to $P_{S,i}^{(m)}$ and $P_{R,i}^{(n)}$ are identical to (B.3) and (B.4), respectively. This is also true for the subcarrier pairing and allocation matrix. Hence, the theorem is proved.

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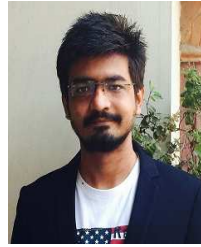
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