(g*p)**- CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract:
In this paper, we have introduced a new class of sets called (g*p)**-closed sets which is properly placed in between the class of closed sets and the class of (g*p)**-closed sets. As an application, we introduce three new spaces namely, $gT^{**p}$, $agT^{**p}$ and $gT^{**p}$-spaces.
We have also introduced (g*p)**-continuous and (g*p)**-irresolute maps and their properties are investigated.

Keywords: (g*p)**-closed sets, (g*p)**-continuous maps, (g*p)**-irresolute maps and, $gT^{**p}$, $agT^{**p}$ and $gT^{**p}$-spaces.
1. INTRODUCTION


The purpose of this paper is to introduce the concept of \((g^*p)^*\)-closed sets, \(gT^*p\), \(\alpha g\), \(T^*p\) and \(g\times T^*p\) spaces. Further we have introduced \((g^*p)^*\)-continuous and \((g^*p)^*\)-irresolute maps.

2. PRELIMINARIES

Throughout this paper \((X, \tau)\) and \((Y, \alpha)\) represents non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset \(A\) of a \((X, \tau)\) space, \(\text{cl}(A)\) and \(\text{int}(A)\) denote the closure and the interior of \(A\) respectively.

The class of all closed subsets of a space of a space \((X, \tau)\) is denoted by \(C(X, \tau)\).

**Definition 2.1:** A subset \(A\) of a topological space \((X, \tau)\) is called

1. a pre-open [15] if \(A \subseteq \text{int}(\text{cl}(A))\) and a pre-closed if \(\text{cl}(\text{int}(A)) \subseteq A\).
2. a semi-open [11] if \(A \subseteq \text{cl}(\text{int}(A))\) and a semi-closed if \(\text{int}(\text{cl}(A)) \subseteq A\).
3. a semi-preopen [1] if \(A \subseteq \text{cl}(\text{int}(A))\) and a semi-preclosed if \(\text{int}(\text{cl}(A)) \subseteq A\).
4. an \(\alpha\)-open [18] if \(A \subseteq \text{int}(\text{cl}(A))\) and \(\alpha\)-closed [16] if \(\text{cl}(\text{int}(A)) \subseteq A\).

**Definition 2.2:** A subset \(A\) of a topological space \((X, \tau)\) is called

1. a generalized closed set (briefly g-closed) [10] if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).
2. a generalized semi-closed set (briefly \(g\alpha\)-closed) [3] if \(\text{scl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).
3. a semi-generalized closed set (briefly \(sg\)-closed) [5] if \(\text{scl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is semi-open in \((X, \tau)\).
4. an \(\alpha\)-generalized closed set (briefly \(\alpha g\)-closed) [12] if \(\text{acl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).
5. an \(\alpha\)-closed set (briefly \(g\alpha\)-closed) [13] if \(\text{acl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)-open in \((X, \tau)\).
6. an \(\alpha\)-double star closed set (briefly \(\alpha^*\)-closed) [25] if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)-open in \((X, \tau)\).
7. a \(\alpha\)-star closed set (briefly \(\alpha^*\)-closed) [24] if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(\alpha\)-open in \((X, \tau)\).
8. a \(W_g\)-closed set [17] if \(\text{cl}(\text{int}(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).
9. a generalized semi-pre closed set (briefly \(gsp\)-closed) [8] if \(\text{scl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).
10. a generalized semi-pre closed star set (briefly \(gsp^*\)-closed) [21] if \(\text{cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(gsp\)-open in \((X, \tau)\).
11. a generalized \(-pre closed set (briefly \(gp\)-closed) [14] if \(\text{pcl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is open in \((X, \tau)\).
(12) a \(g^*p\)-pre closed set (briefly \(g^*p\)-closed)[23] if \(pcl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g\)-open in \((X, \tau)\).

(13) a \(g^*\)-closed set (briefly \(g^*\)-closed)[22] if \(cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g\)-open in \((X, \tau)\).

(14) a strongly \(g^*\)-closed set (briefly strongly \(g^*\)-closed)-closed[19] if \(cl(int(A)) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \(g^*p\)-open in \((X, \tau)\).

Definition 2.3: A function \(f : (X, \tau) \to (Y, \sigma)\) is called
1. \(\alpha g\)-continuous [9] if \(f^{-1}(V)\) is an \(\alpha g\)-closed set of \((X, \tau)\) for every closed set \(V\) of \((Y, \sigma)\).
2. \(g\)-continuous [7] if \(f^{-1}(V)\) is a \(g\)-closed set of \((X, \tau)\) for every closed set \(V\) of \((Y, \sigma)\).
3. \(g\)-continuous [2] if \(f^{-1}(V)\) is a \(g\)-closed set of \((X, \tau)\) for every closed set \(V\) of \((Y, \sigma)\).
4. \(wg\)-continuous [17] if \(f^{-1}(V)\) is a \(wg\)-closed set of \((X, \tau)\) for every closed set \(V\) of \((Y, \sigma)\).
5. \(gsp\)-continuous [8] if \(f^{-1}(V)\) is a \(gsp\)-closed set of \((X, \tau)\) for every closed set \(V\) of \((Y, \sigma)\).

Definition 2.4: A topological space \((X, \tau)\) is said to be
1. a \(T_g\) \(*\) space [24] if every \(g^*\)-closed set in it is closed.
2. a \(T_b\) space [6] if every \(g\)-closed set in it is closed.
3. a \(aT_b\) space [4] if every \(\alpha g\)-closed set in it is closed.

3. BASIC PROPERTIES OF \((g^*p)^{*}\)-CLOSED SETS

We now introduce the following definition.

Definition 3.1: A subset \(A\) of a topological space \((X, \tau)\) is called a \((g^*p)^{*}\)-closed set if \(cl(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is \((g^*p)^{*}\)-open.

Proposition 3.2: Every closed set is \((g^*p)^{*}\)-closed.

Proof follows from the definition but not conversely.

Example 3.3: Let \(X = \{a, b, c\}\) and \(\tau = \{\phi, X, \{c\}, \{a, c\}\}\) and let \(A = \{b, c\}\). Then \(A\) is not closed but \((g^*p)^{*}\)-closed.

Proposition 3.4: Every \((g^*p)^{*}\)-closed set is (1) \(\alpha g\)-closed (2) \(g\)-closed (3) \(g\)-closed.

(4) \(wg\)-closed (5) \(gsp\)-closed but not conversely.

Proof: Let \(A\) be \((g^*p)^{*}\)-closed set. Let \(A \subseteq U\) and \(U\) be open. Then \(U\) is \((g^*p)^{*}\)-open.

Since \(A\) is \((g^*p)^{*}\)-closed,

1. \(\alpha cl(A) \subseteq cl(A) \subseteq U\) and hence \(A\) is \(\alpha g\)-closed.
2. \(\sigma cl(A) \subseteq cl(A) \subseteq U\) and hence \(A\) is \(g\)-closed.
3. \(p cl(A) \subseteq cl(A) \subseteq U\) and hence \(A\) is \(g\)-closed.
4. \(cl \subseteq U\) and which implies \(cl(int(A) \subseteq cl(A) \subseteq U\) hence \(A\) is \(wg\)-closed.
5. \(cl(A) \subseteq U\) and hence \(sp cl(A) \subseteq U\) therefore \(A\) is \(gsp\)-closed.
Example 3.5: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{c\}, \{a, c\}\}$ and let $A = \{a\}$. Then $A$ is $\alpha g$-closed, gs-closed, gp-closed, wg-closed, gsp-closed but it is not $(g * p)^{**}$-closed.

Proposition 3.6: Every $\alpha^{**}$-closed set is $(g * p)^{**}$-closed set but not conversely.

Proof: Let $A$ be a $\alpha^{**}$-closed set. Let $A \subseteq U$ and $U$ be $(g * p)^{*-}$-open. Then $U$ is $\alpha^{*-}$-open.

Since $A$ is $\alpha^{**}$-closed, $cl(A) \subseteq U$ therefore $A$ is $(g * p)^{**}$-closed.

Example 3.7: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{b\}, \{a, c\}\}$ and let $A = \{a, b\}$. Then $A$ is $(g * p)^{**}$-closed but it is not $\alpha^{**}$-closed.

Proposition 3.8: Every $(gsp)^{*}$-closed set is $(g * p)^{**}$-closed set but not conversely.

Proof: Let $A$ be a $(gsp)^{*}$-closed set. Let $A \subseteq U$ and $U$ be $(g * p)^{*-}$-open. Then $U$ is gsp-open.

Since $A$ is $(gsp)^{*}$-closed, $cl(A) \subseteq U$ therefore $A$ is $(g * p)^{**}$-closed.

Example 3.9: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{b\}, \{a, c\}\}$ and let $A = \{c\}$. Then $A$ is $(g * p)^{**}$-closed but it is not $(gsp)^{*}$-closed.

Proposition 3.10: Every $g^{*}$-closed set is $(g * p)^{**}$-closed set.

Proof: Let $A$ be a $g^{*}$-closed set. Let $A \subseteq U$ and $U$ be $(g * p)^{*-}$-open. Then $U$ is $g^{*-}$open.

Since $A$ is $g^{*-}$closed, $cl(A) \subseteq U$ therefore $A$ is $(g * p)^{**}$-closed.

Remark 3.11: $g\alpha$-closedness is independent of $(g * p)^{**}$-closedness.

Example 3.12: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and let $A = \{a, b\}$. Then $A$ is $(g * p)^{**}$-closed but it is not $g\alpha$-closed.

Example 3.13: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{c\}, \{a, c\}\}$ and let $A = \{c\}$. Then $A$ is $g\alpha$-closed but it is not $(g * p)^{**}$-closed.

Remark 3.14: $sg$-closedness is independent of $(g * p)^{**}$-closedness.

Example 3.15: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{c\}, \{a, c\}\}$ and let $A = \{a\}$. Then $A$ is $sg$-closed but it is not $(g * p)^{**}$-closed.

Example 3.16: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{c\}\}$ and let $A = \{a, c\}$. Then $A$ is $(g * p)^{**}$-closed but it is not $sg$-closed.

Remark 3.17: strongly $g^{*}$-closedness is independent of $(g * p)^{**}$-closedness.

Example 3.18: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{c\}, \{a, c\}\}$ and let $A = \{a\}$. Then $A$ is strongly $g^{*}$-closed but it is not $(g * p)^{**}$-closed.

Example 3.19: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and let $A = \{a, b\}$. Then $A$ is $(g * p)^{**}$-closed but it is not strongly $g^{*}$-closed.

Remark 3.20: $g^{*} p$-closedness is independent of $(g * p)^{**}$-closedness.

Example 3.21: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{c\}, \{a, c\}\}$ and let $A = \{a\}$. Then $A$ is $g^{*} p$-closed but it is not $(g * p)^{**}$-closed.

Example 3.22: Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and let $A = \{a, c\}$. Then $A$ is
(g * p)**-closed but it is not g * p - closed.

**Proposition 3.23**: If A and B are (g * p)**-closed sets, then A ∪ B is also a (g * p)** - closed set.

Proof follows from the fact that cl (A ∪ B) = cl (A) ∪ cl (B).

The above results can be represented in the following figure.

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**CONTINUOUS MAPS AND (g * p)**-IRRESOLUTE MAPS

We introduce the following definitions.

**Definition: 4.1**: A map f : (X, τ) → (Y, σ) is called (g * p)**-continuous if the inverse image of every closed set in (Y, σ) is (g * p)**-closed in (X, τ).

**Definition: 4.2**: A map f : (X, τ) → (Y, σ) is said to be a (g * p)**-irresolute map if f⁻¹(V) is a (g * p)**-closed set in (X, τ) for every (g * p)**-closed set V of (Y, σ).
Theorem 4.3: Every continuous map is \((g * p)^{**}\)-continuous.

Proof: Let \(f : (X, \tau) \rightarrow (Y, \sigma)\) be a continuous map and let \(F\) be a closed set in \((Y, \sigma)\). Then \(f^{-1}(F)\) is closed in \((X, \tau)\). Since every closed set is \((g * p)^{**}\)-closed, \(f^{-1}(F)\) is \((g * p)^{**}\)-closed. Then \(f\) is \((g * p)^{**}\)-continuous.

Example 4.4: Let \(X = Y = \{a, b, c\}, \tau = \{\phi, X, \{c\}, \{a, c\}\}\) and \(\sigma = \{\phi, Y, \{a\}\}\). Then \(f : (X, \tau) \rightarrow (Y, \sigma)\) is the identity map. The inverse image of all closed sets of \((Y, \sigma)\) are \((g * p)^{**}\)-closed in \((X, \tau)\). Therefore \(f\) is \((g * p)^{**}\)-continuous but not continuous.

Theorem 4.5: Every \((g * p)^{**}\)-continuous map is \(\alpha g\) -continuous, \(gs\) -continuous, \(gp\) -continuous, \(wg\) -continuous and \(GSP\) -continuous but not conversely.

Proof: Let \(f : (X, \tau) \rightarrow (Y, \sigma)\) be a \((g * p)^{**}\)-continuous map. Let \(V\) be a closed set in \((Y, \sigma)\). Since \(f\) is \((g * p)^{**}\)-continuous, \(f^{-1}(V)\) is \((g * p)^{**}\)-closed in \((X, \tau)\). Then \(f^{-1}(V)\) is \(\alpha g\) -closed, \(gs\) -closed, \(gp\) -closed, \(wg\) -closed and \(GSP\) -closed set of \((X, \tau)\).

Example 4.6: Let \(X = Y = \{a, b, c\}, \tau = \{\phi, X, \{c\}, \{a, c\}\}\) and \(\sigma = \{\phi, Y, \{b, c\}\}\). Then \(f : (X, \tau) \rightarrow (Y, \sigma)\) is the identity map. Then \(f^{-1}(a) = [a]\) is not \((g * p)^{**}\)-closed in \((X, \tau)\). But \(a\) is \(\alpha g\) -closed set, \(gs\)-closed set. Then \(f\) is \(\alpha g\) -continuous, \(gs\) - continuous but not \((g * p)^{**}\)-continuous.

Example 4.7: Let \(X = Y = \{a, b, c\}, \tau = \{\phi, X, \{c\}, \{a, c\}\}\) and \(\sigma = \{\phi, Y, \{a, c\}\}\). Then \(f : (X, \tau) \rightarrow (Y, \sigma)\) is defined as \(f(a) = b, f(b) = c, f(c) = a\). Then \(f^{-1}([c]) = [a]\) is \(gp\) -closed but not \((g * p)^{**}\)-closed. Then \(f\) is \(gp\) - continuous but not \((g * p)^{**}\)-continuous.

Example 4.8: Let \(X = Y = \{a, b, c\}, \tau = \{\phi, X, \{c\}, \{a, c\}\}\) and \(\sigma = \{\phi, Y, \{a, b\}\}\). Then \(f : (X, \tau) \rightarrow (Y, \sigma)\) is defined as \(f(a) = c, f(b) = a, f(c) = b\). Then \(f^{-1}([b]) = [a]\) is \(wg\) - closed but not \((g * p)^{**}\)-closed in \((X, \tau)\). Hence \(f\) is \(wg\) - continuous but not \((g * p)^{**}\)-continuous.

Example 4.9: Let \(X = Y = \{a, b, c\}, \tau = \{\phi, X, \{c\}, \{a, c\}\}\) and \(\sigma = \{\phi, Y, \{b, c\}\}\). Then \(f : (X, \tau) \rightarrow (Y, \sigma)\) is defined as \(f(a) = a, f(b) = c, f(c) = b\). Then \(f^{-1}([a]) = [a]\) is not \((g * p)^{**}\)-closed in \((X, \tau)\). Hence \(f\) is \(gsp\) - continuous but not \((g * p)^{**}\)-continuous.

Theorem 4.10: Every \((g * p)^{**}\)-irresolute map is \((g * p)^{**}\)-continuous.

Proof follows from the definitions of \((g * p)^{**}\)-irresolute map and \((g * p)^{**}\)-continuous.

Theorem 4.11: Every \((g * p)^{**}\)-irresolute map is \(\alpha g\) - continuous, \(gs\) - continuous, \(gp\) - continuous, \(wg\) - continuous and \(gsp\) - continuous.

Proof follows from theorems (4.4) and (4.11).

The converse of the above theorem need not be true in general as seen in the following examples.

Example 4.12: Let \(X = Y = \{a, b, c\}, \tau = \{\phi, X, \{c\}, \{a, c\}\}\) and \(\sigma = \{\phi, Y, \{c\}\}\). Let \(f : (X, \tau) \rightarrow (Y, \sigma)\) be the identity map. \(\phi, Y, \{a, b\}\) are closed sets of \(Y\). \(f^{-1}(a) = [a, b]\) is \(gs\)-closed, \(gp\)-closed, \(wg\)-closed, \(gsp\)-closed. Hence \(f\) is \(gsp\) - continuous, \(gs\) - continuous, \(wg\) - continuous and \(gsp\) - continuous. \((g * p)^{**}\)-closed sets of \(Y\) are \(\phi, Y, \{a\}, \{b\}, \{a, b\}, \{c, a\}, \{b, c\}\). \(f^{-1}(a) = [a]\) is not \((g * p)^{**}\)-closed in \((X, \tau)\). Hence \(f\) is not a \((g * p)^{**}\)-irresolute.

Example 4.13: Let \(X = Y = \{a, b, c\}, \tau = \{\phi, X, \{a\}\}\) and \(\sigma = \{\phi, Y, \{c\}\}\). Let \(f : (X, \tau) \rightarrow (Y, \sigma)\) be the identity map. \(\phi, Y, \{a, b\}\) are closed sets of \(Y\). \(f^{-1}(a) = [a, b]\) is
αg-closed. Hence f is αg-continuous. (g*p)**-closed sets of Y are \( \phi, Y, \{ a \}, \{ b \}, \{ a, b \}, \{ a, c \}, \{ b, c \} \) \( f^{-1}(\{ a \}) = \{ a \} \) is not (g*p)**-closed in \((X, \tau)\). Hence f is αg-continuous but not a (g*p)**-irresolute.

The above results can be represented in the following figure.

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5. APPLICATION OF (g*p)**-CLOSED SETS

We introduce the following definitions.

**Definition 5.1:** A space \((X, \tau)\) is called a \( T_{**} \)-space if every set (g*p)**-closed set is closed.

**Definition 5.2:** A space \((X, \tau)\) is called a \( p \)-\( T_{**} \)-space if every αg-closed set is (g*p)**-closed.

**Definition 5.3:** A space \((X, \tau)\) is called a \( g \)-\( T_{**} \)-space if every (g*p)**-closed set is gsp-continuous.

**Theorem 5.4:** Every \( T_{**} \)-space is a \( T_{**} \)-space.

**Proof:** Let \((X, \tau)\) be a \( T_{**} \)-space. Let A be a g*p-closed set. Since every g*p-closed set is (g*p)**-closed, A is (g*p)**-closed. Since \((X, \tau)\) is \( T_{**} \)-space, A is closed. Hence \((X, \tau)\) is a \( T_{**} \)-space.

**Theorem 5.5:** Every \( T_{b} \) space is a \( T_{**} \)-space but not conversely.

**Proof:** Follows from the definitions of \( T_{b} \) space and \( T_{**} \)-space.

**Example 5.6:** Let \( X = \{ a, b, c \} \) and \( \tau = \{ \phi, X, \{ a, c \} \} \). Here (g*p)**-closed sets are \( \phi, X, \{ a \}, \{ b \}, \{ a, b \}, \{ a, c \}, \{ b, c \} \) and the gsp-closed sets are \( \phi, X, \{ a \}, \{ b \}, \{ a, b \}, \{ a, c \}, \{ b, c \} \). Since every (g*p)**-closed set is closed, the space \((X, \tau)\) is a \( T_{**} \)-space. A = \{ a, c \} is gsp-closed but not closed. Therefore the space \((X, \tau)\) is not a \( T_{b} \)-space.
Theorem 5.7: Every \(a T_b\)-space is a \(T_{**}^p\) -space

Proof follows from the definitions of \(T_b\)-space and \(T_{**}^p\)-space. The converse is not true.

Example 5.8: Let \(X = \{a,b,c\}\) and \(\tau = \{\phi, X, \{c\}, \{a,c\}\}\). \((g \ast p)\)-closed sets are \(\phi, X, \{b\}, \{a,b\}, \{b,c\}\) and \(\alpha g\)-closed sets are \(\phi, X, \{a\}, \{b\}, \{a,b\}, \{b,c\}\). Since every \((g \ast p)\)-closed set is \(\alpha g\)-closed, the space \((X, \tau)\) is a \(T_{**}^p\)-space.

Example 5.9: Let \(X = \{a,b,c\}\) and \(\tau = \{\phi, X, \{b\}, \{a,c\}\}\). \((g \ast p)\)-closed sets are all the subsets of \(X\) and \(\alpha g\)-closed sets are \(\alpha g\)-closed sets are \(\phi, X, \{b\}, \{a,c\}\). Since every \((g \ast p)\)-closed set is closed, the space \((X, \tau)\) is a \(T_{**}^p\)-space.

Theorem 5.9: Every \(T_{**}^p\)-space is a \(T_{**}^p\)-space.

Proof follows from the definitions of \(T_{**}^p\)-space and \(T_{**}^p\)-space. The converse is not true.

Example 5.10: Let \(X = \{a,b,c\}\) and \(\tau = \{\phi, X, \{b\}, \{a,c\}\}\). \((g \ast p)\)-closed sets are all the subsets of \(X\) and \(\alpha g\)-closed sets are \(\phi, X, \{a\}, \{b\}, \{a,c\}\}. Since every \((g \ast p)\)-closed set is closed, the space \((X, \tau)\) is a \(T_{**}^p\)-space.

Example 5.11: Let \(X = \{a,b,c\}\) and \(\tau = \{\phi, X, \{a\}\}\). \((g \ast p)\)-closed sets are \(\phi, X, \{b\}, \{c\}, \{a,b\}, \{a,c\}\{b,c\}\) and \(\alpha g\)-closed sets are \(\alpha g\)-closed sets are \(\phi, X, \{a\}, \{b\}, \{a,c\}\}. \(A = \{b\}\) is \((g \ast p)\)-closed but not closed. Therefore the space \((X, \tau)\) is not a \(T_{**}^p\)-space.

Theorem 5.11: Every \(a T_b\)-space is a \(a T_{**}^p\)-space.

Proof: Let \((X, \tau)\) be a \(T_{**}^p\)-space. Let \(A\) be \(\alpha g\)-closed. Then \(A\) is \(\alpha g\)-closed. Since the space is \(a T_{**}^p\)-space, \(A\) is closed and hence \(A\) is \((g \ast p)\)-closed. Therefore the space \((X, \tau)\) is a \(a T_{**}^p\)-space.

Example 5.12: Let \(X = \{a,b,c\}\) and \(\tau = \{\phi, X, \{a\}\}\). \((g \ast p)\)-closed sets are \(\phi, X, \{b\}, \{c\}, \{a,b\}, \{a,c\}\{b,c\}\). \(A = \{b\}\) is \((g \ast p)\)-closed and also \(\alpha g\)-closed. Therefore the space \((X, \tau)\) is a \(a T_{**}^p\)-space.

Example 5.13: Every \(T_b\)-space is a \(T_{**}^p\)-space but not conversely.

Proof follows from the definitions of \(T_b\)-space and \(T_{**}^p\)-space.

Example 5.14: Let \(X = \{a,b,c\}\) and \(\tau = \{\phi, X, \{a\}\}\). Here \((g \ast p)\)-closed sets are \(\phi, X, \{b\}, \{b\}\), \(gs\)-closed sets are \(\phi, X, \{a\}, \{b\}, \{a,b\}\). \(A = \{b,c\}\) is \((g \ast p)\)-closed and also \(gs\)-closed. Therefore the space \((X, \tau)\) is a \(gs T_{**}^p\)-space.

Therefore the space \((X, \tau)\) is not a \(T_b\)-space.

The above results can be represented in the following figure.
where $A \implies B$ represents $A$ implies $B$ and $B$ need not imply $A$.

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