



Forecasting stylised features of electricity prices in the Australian National Electricity Market

A thesis submitted in fulfilment of the requirements for the degree of PhD in Economics

Taylan Akay

Masters in Applied Finance

Graduate Diploma in Business Systems

Bachelor of Economics

School of Economics Finance and Marketing

College of Business

RMIT University

April, 2016

Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis/project is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

Taylan Akay

16/09/2015

Acknowledgement

I would like to acknowledge my academic supervisors Dr Mark Stewart and Dr Ashton De Silva for their tireless efforts in guiding me through my journey. I am indebted to both.

Abstract

This thesis tests whether forecast accuracy improves when models that explicitly capture the stylised features of the Australian National Electricity Market (NEM) are employed to generate predictions. It is believed that by explicitly modelling these features of electricity wholesale spot prices, the accuracy of the price forecast models can be improved when compared to standard alternative. The stylised features identified in data are mean-reversion, sudden short-lived and consecutive jumps and heavy tails.

When employing models to capture the stylised features of electricity prices, the models necessarily become more complex and often contain a greater number of parameters which combine to mimic the characteristics observed in the price series. Throughout this thesis an adherence to the principle of parsimony (Makridakis, et al page 609) will be maintained; that is if two models effectively generate the same forecast performance the simpler model will be preferred whether it contains the stylised features or not. This is also known as Occum's Razor.

This investigation is important in terms of a better understanding of what models are more useful has the potential to lead to more accurate price forecasts which may result in less volatility in market prices leading to more efficient markets. Further, by assessing models that capture various stylised features it may be possible to infer the importance of particular features. Given that wholesale prices are a major determinant of how much end users pay for powering their homes and businesses, it is believed that a better understanding of what forecasting models work (and do not) will allow market participants to develop more successful (business) strategies for adjusting supply to meet demand and to assist with the valuation of financial assets as part of risk management. Additionally, a better understanding of the dynamics of electricity prices and its implications for successful forecasting is important for government policy makers, as Government sets the rules that govern the production and distribution of electricity.

It is believed that by explicitly modelling the stylised features of electricity wholesale prices, forecast accuracy can be improved upon baseline models commonly used in quantitative finance. This thesis investigates the forecasting ability of two distinct modelling approaches which by construction capture the stylised characteristics of electricity prices. Namely, these are linear continuous time and non-linear modelling methods. The AR-GARCH model is chosen to be the standard approach in forecasting price series (Engle, 2001) and is taken as the benchmark model in this thesis. More specifically, this thesis aims to answer the following research questions:

1. Does the application of continuous-time models in capturing the stylised features of Australian electricity wholesale spot prices improve forecasting ability upon the traditional AR-GARCH model?
2. Does the application of non-linear forecast models in capturing the stylised features of Australian electricity wholesale spot prices improve forecast ability upon traditional AR-GARCH model?

The continuous-time models examined in this thesis are; Geometric Brownian Motion (GBM), Mean-Reverting, and Mean-Reverting Jump-Diffusion processes. The inclusion of GBM in this thesis is due to it being the foundation for the Mean-Reverting and Jump-Diffusion models, which are considered in

this thesis. Continuous-time models capture some of the main stylised features of electricity prices; Mean-Reverting process captures the mean-reversion (tendency of electricity prices to revert back to its long-term average over time) characteristics of electricity prices whilst Mean-Reverting and Jump-Diffusion process models the sudden jumps prevalent in Australian electricity prices. The models are in order such that each successive model extends the one preceding it. Note that each extension addresses a stylised feature of the data therefore the a priori expectation is that the forecasting performance will improve.

The inclusion of the non-linear approach to forecasting Australian electricity prices is performed with the application of a Markov Regime-Switching model and the application of Extreme Value Theory (EVT) into electricity price modelling. The Markov Regime-Switching model is a non-linear modelling tool that is able to capture consecutive spikes prevalent in electricity prices that Mean-Reverting and Jump-Diffusion processes fail to capture. The application of EVT is included in this thesis so that heavy tails present in electricity prices can be adequately captured. Copulas are considered as a unique method that models the dependence structure of data. The forecasts based on the EVT model is built upon the application of Copula functions as these functions model the interdependence of prices within the separate regions of the Australian electricity markets.

The models examined in this thesis are:

1. AR(1)-GARCH(1)
2. Geometric Brownian Motion
3. Mean-Reverting Model
4. Mean-Reverting and Jump-Diffusion Model
5. Markov Regime-Switching Model with spike distributions modelled with
 - Gaussian distribution
 - Log-Gaussian distribution and,
6. Extreme value Theory and Copula functions

Each model under investigation mimics a known characteristic of electricity prices. Comparative performance evaluations of each model investigated in this thesis showed that the benchmark model is providing superior short-term forecasting ability.

TABLE OF CONTENTS

LIST OF FIGURES.....	10
LIST OF TABLES.....	13
RESEARCH QUESTIONS.....	14
LIST OF ABBREVIATIONS	15
CHAPTER 1 - OVERVIEW.....	16
CHAPTER 2 - BACKGROUND	22
RATIONALE	22
Continuous-Time Models.....	25
Non-Linear Models.....	26
Forecast Approach	26
A Review of Modelling Electricity Prices in Australia.....	29
AUSTRALIAN ELECTRICITY INDUSTRY	30
Electricity Consumption by Industry Type	32
Electricity Generation by Fuel Type	33
ELECTRICITY PRICES AND THE ECONOMY	35
PRICE FORMATION IN AUSTRALIAN NATIONAL ELECTRICITY MARKET	36
Australian Energy Market Operator (AEMO).....	38
CHAPTER 3 - DESCRIPTION AND SOURCES OF DATA	41
INTRODUCTION	41
AUSTRALIAN WHOLESALE SPOT ELECTRICITY PRICE SERIES	42
DESCRIPTIVE STATISTICS	47
STYLISTED FEATURES OF ELECTRICITY PRICES IN NEM	50
Normality tests.....	52
Unit root and stationary tests.....	54
Alternative tests for efficiency.....	57
CONCLUSION.....	58
CHAPTER 4 – LITERATURE REVIEW	61
INTRODUCTION	61
AUTOREGRESSIVE MODELS.....	63

MEAN-REVERTING/JUMP-DIFFUSION AND REGIME-SWITCHING MODELS.....	73
VOLATILITY MODELS	88
EMERGING MODELS.....	93
Extreme Value Theory (EVT).....	94
Periodic Autoregressive Models	95
Levy Jump-Diffusion Models	96
Continuous Time ARMA Models.....	97
Spatial Econometrics	98
Modelling Interconnected Regional Electricity Markets	99
STUDIES IN THE AUSTRALIAN NATIONAL ELECTRICITY MARKET	101
CHAPTER 5 – INTRODUCTION TO METHODOLOGY	104
THE BENCHMARK MODEL	108
CHAPTER 6 - GEOMETRIC BROWNIAN MOTION.....	116
INTRODUCTION TO BROWNIAN MOTION PROCESS	116
MODELLING ELECTRICITY PRICES WITH GBM PROCESS.....	119
PARAMETERISATION OF THE GBM PROCESS.....	120
SIMULATION OF THE GBM PROCESS	121
EVALUATION OF FORECAST PERFORMANCE	126
CHAPTER 7 - MEAN REVERTING MODEL.....	129
AN INTRODUCTION MEAN-REVERTING MODEL	130
MODELLING PRICES IN NEM WITH THE MEAN-REVERTING MODEL	132
PARAMETERISATION OF THE MEAN-REVERTING MODEL	132
SIMULATION OF THE MEAN-REVERTING MODEL	136
EVALUATION OF FORECAST PERFORMANCE	143
CHAPTER 8 – MEAN-REVERTING AND JUMP-DIFFUSION MODEL	147
MODELLING ELECTRICITY PRICES WITH MEAN-REVERTING AND JUMP-DIFFUSION MODELS.....	150
PARAMETERISATION OF THE MEAN-REVERTING AND JUMP-DIFFUSION MODEL	153
SIMULATION OF PRICES WITH MEAN-REVERTING AND JUMP-DIFFUSION MODEL	157
EVALUATION OF FORECAST PERFORMANCE	162
CHAPTER 9 MARKOV REGIME-SWITCHING MODEL.....	166

AN INTRODUCTION TO MARKOV REGIME-SWITCHING MODEL.....	166
MARKOV REGIME-SWITCHING MODELS IN ELECTRICITY PRICE MODELLING.....	169
MODELLING ELECTRICITY PRICES WITH MARKOV REGIME-SWITCHING MODELS IN NEM	171
PARAMETERISATION OF THE MARKOV REGIME-SWITCHING MODEL	174
REGIME PROBABILITIES BY NEM REGIONS	175
SIMULATION OF MARKOV REGIME-SWITCHING MODEL	200
EVALUATION OF FORECAST PERFORMANCE	207
CHAPTER 10 -MODELLING ELECTRICITY PRICES WITH EVT AND COPULA FUNCTIONS	211
INTRODUCTION	212
Price Interrelations and Non-Linearity in NEM.....	212
Correlation Analysis of Electricity Price Series in NEM	215
Non-Linearity of Electricity Prices in NEM	217
INTRODUCTION TO EXTREME VALUE THEORY (EVT)	218
AN INTRODUCTION TO COPULA FUNCTIONS	221
MODELLING ELECTRICITY PRICES WITH EVT IN NEM.....	223
Filtering Residuals via AR-EGARCH	224
Application of POT to Residuals.....	232
Parameterisation of POT and GPD Fit.....	233
SIMULATION OF THE EVT MODEL WITH COPULA FUNCTIONS.....	239
Parameterisation of the Student t-Copula.....	240
Simulation of the EVT Model with Student-T Copula	242
EVALUATION OF FORECAST PERFORMANCE	242
FORECAST PERFORMANCE STATISTICS.....	248
CONCLUSION.....	250
CHAPTER 11 -DISCUSSION AND CONCLUSION.....	252
CONTRIBUTION TO THE LITERATURE	257
COMPARATIVE FORECAST PERFORMANCE ANALYSIS	262
1-Short-term forecast accuracy of GBM Model.....	264
2-Short-term forecast accuracy of Mean-Reverting Model.....	265
3-Short-term forecast accuracy of Mean-Reverting and Jump-Diffusion Model	266

4-Short-term forecast accuracy of Markov Regime-Switching Model	268
5-Short-term forecast accuracy of Extreme Value Theory Model.....	270
OVERALL SUMMARY OF THE COMPARATIVE ACCURACIES	271
AREAS OF FUTURE RESEARCH.....	275
Simulations.....	275
Parameter estimation	276
Times Series Approach.....	279
Data Frequency	280
APPENDIX 1 KOLMOGOROV–SMIRNOV TEST	281
APPENDIX 2- OVERVIEW OF THE FORECAST ACCURACY MEASURES	282
APPENDIX 3- OVERVIEW OF STOCHASTIC CALCULUS.....	285
APPENDIX 4- SIMULATION OF STOCHASTIC DIFFERENTIAL EQUATIONS.....	287
APPENDIX 5- EXPECTATION MAXIMIZATION ALGORITHM	290
APPENDIX 6- BROCK-DECHERT-SCHEINKMAN (BDS) TEST FOR INDEPENDENCE.....	294
APPENDIX 7- CONDITIONAL STANDARD DEVIATIONS AND STANDARDISED RESIDUALS	296
APPENDIX 8- FILTERED RESIDUALS AND CONDITIONAL STANDARD DEVIATIONS	299
APPENDIX 9- COMPARISON OF GPD FITTED CDF WITH ECDF	302
APPENDIX 10- CROSS CORRELATION FUNCTIONS	305
APPENDIX 11- SAMPLE ACFs, DAILY AND SQUARED PRICES.....	307
REFERENCES.....	312

LIST OF FIGURES

Figure 1 Market Structure in NEM.....	31
Figure 2 Electricity Consumption by Sector, 2010.....	33
Figure 3 Electricity Generation by Fuel Type, 2008-09.....	34
Figure 4 Daily Wholesale Electricity Prices in NEM.....	43
Figure 5 Daily Electricity Wholesale Prices in NSW.....	44
Figure 6 Daily Electricity Wholesale Prices in VIC.....	44
Figure 7 Daily Electricity Wholesale Prices in QLD.....	45
Figure 8 Daily Electricity Wholesale Prices in SA.....	46
Figure 9 Daily Electricity Wholesale Prices in TAS.....	47
Figure 10 Simulated Price Series.....	111
Figure 11 Price Forecast with GBM Specification for All Regions of NEM.....	122
Figure 12 Price Forecast with GBM Specification for NSW.....	123
Figure 13 Price Forecast with GBM Specification for QLD.....	124
Figure 14 Price Forecast with GBM Specification for SA.....	124
Figure 15 Price Forecast with GBM Specification for TAS.....	124
Figure 16 Price Forecast with GBM Specification for VIC.....	125
Figure 17 Comparative Performance of GBM forecast.....	125
Figure 18 A Simulated Price Path with Mean-Reverting Model.....	138
Figure 19 Electricity Price Forecast with Mean-Reverting Process for All Regions of NEM.....	140
Figure 20 Electricity Price Forecast with Mean-Reverting Model for NSW.....	141
Figure 21 Electricity Price Forecast with Mean-Reverting Model for VIC.....	142
Figure 22 Electricity Price Forecast with Mean-Reverting Model for SA.....	142
Figure 23 Electricity Price Forecast with Mean-Reverting Model for QLD.....	142
Figure 24 Electricity Price Forecast with Mean-Reverting Model for TAS.....	143
Figure 25 A Simulated Price Path for Mean-Reverting and Jump-Diffusion Model.....	157
Figure 26 Price Forecast for NEM regions with Mean-Reverting and Jump-Diffusion Model.....	158
Figure 27 Price Forecast for NSW with Mean-Reverting and Jump-Diffusion Model.....	159
Figure 28 Price Forecast for VIC with Mean-Reverting and Jump-Diffusion Model.....	160
Figure 29 Price Forecast for SA with Mean-Reverting and Jump-Diffusion Model.....	160
Figure 30 Price Forecast for QLD with Mean-Reverting and Jump-Diffusion Model.....	160
Figure 31 Price Forecast for TAS with Mean-Reverting and Jump-Diffusion Model.....	161
Figure 32 Actual vs simulated prices with Mean-Reverting and Jump-Diffusion Model.....	161
Figure 33 Regime Probabilities (NSW), Gaussian spikes.....	179
Figure 34 Regime Probabilities (NSW), Log-Gaussian spikes.....	180
Figure 35 Regime Probabilities (NSW), Pareto spikes.....	181
Figure 36 Regime Probabilities (NSW), Mean-Reverting Spikes.....	182
Figure 37 Regime Probabilities (VIC), Gaussian Spikes.....	185
Figure 38 Regime Probabilities (VIC), Log-Gaussian Spikes.....	186
Figure 39 Regime Probabilities (VIC), Pareto Spikes.....	186
Figure 40 Regime Probabilities (VIC), Mean-Reverting Spikes.....	187
Figure 41 Regime Probabilities (QLD), Gaussian Spikes.....	188
Figure 42 Regime Probabilities (QLD), Log-Gaussian Spikes.....	189

Figure 43 Regime Probabilities (QLD), Pareto Spikes	190
Figure 44 Regime Probabilities (QLD), Mean-Reverting Spikes	190
Figure 45 Regime Probabilities (SA), Gaussian Spikes	192
Figure 46 Regime Probabilities (SA), Log-Gaussian Spikes	193
Figure 47 Regime Probabilities (SA), Pareto Spikes	193
Figure 48 Regime Probabilities (SA), Mean-Reverting Spikes.....	194
Figure 49 Regime Probabilities (TAS), Gaussian Spikes	195
Figure 50 Regime Probabilities (TAS), Log-Gaussian Spikes	196
Figure 51 Regime Probabilities (TAS), Pareto Spikes	197
Figure 52 Regime Probabilities (TAS), Mean-Reverting Spikes.....	198
Figure 53 A Simulated Path with a Markov Regime Switching Model.....	202
Figure 54 A Simulated Price Path with Spikes Distributed as Gaussian.....	203
Figure 55 A Simulated Price Path with Spikes Distributed as Log-Normal	203
Figure 56 Price Forecast for All Regions of NEM (Spikes dist. Gaussian).....	204
Figure 57 Price Forecast for All Regions of NEM (Spikes dist. Log-Normal)	204
Figure 58 Markov Regime-Switching Model with Spikes Dist. as Gaussian (NSW)	206
Figure 59 Markov Regime-Switching Model with Spikes Dist. as Log-Gaussian for (NSW)	206
Figure 60 Actual versus Simulated Price Paths for NSW.....	206
Figure 61 Intra-Regional Electricity Trade in NEM.....	213
Figure 62 Sample ACF of Standardised Residuals (NSW).....	229
Figure 63 Sample ACF of Squared Standardised Residuals (NSW)	229
Figure 64 Sample ACF of Standardised Residuals (QLD).....	229
Figure 65 Sample ACF of Squared Standardised Residuals (QLD).....	230
Figure 66 Sample ACF of Standardised Residuals (SA).....	230
Figure 67 Sample Autocorrelation Function of Squared Standardised Residuals for SA.....	230
Figure 68 Sample ACF of Standardised Residuals (TAS).....	231
Figure 69 Sample ACF of Squared Standardised Residuals (TAS)	231
Figure 70 Sample ACF of Standardised Residuals (VIC)	231
Figure 71 Sample ACF of Standardised Residuals (VIC)	232
Figure 72 Empirical Cumulative Distribution Function (NSW)	235
Figure 73 Empirical Cumulative Distribution Function (QLD)	235
Figure 74 Empirical Cumulative Distribution Function (SA).....	236
Figure 75 Empirical Cumulative Distribution Function (TAS).....	236
Figure 76 Empirical Cumulative Distribution Function (VIC)	237
Figure 77 Price Forecast with EVT-Copula Simulations for All Regions of NEM	243
Figure 78 A Simulated Price Path versus Actual Prices.....	244
Figure 79 Price Forecast with EVT-Copula Model (NSW).....	246
Figure 80 Price Forecast with EVT-Copula Model for VIC.....	246
Figure 81 Price Forecast with EVT-Copula Model (QLD).....	246
Figure 82 Price Forecast with EVT-Copula Model (SA)	247
Figure 83 Price Forecast with EVT-Copula Model (TAS)	247
Figure 84 RMSE Values of All Models by NEM Regions	264
Figure 85 Conditional Standard Deviations, NSW.....	296
Figure 86 Conditional Standard Deviations, VIC	296
Figure 87 Conditional Standard Deviations, QLD.....	297

Figure 88 Conditional Standard Deviations, SA	297
Figure 89 Conditional Standard Deviations, TAS	298
Figure 90 Filtered Residuals, NSW	299
Figure 91 Filtered Residuals, VIC.....	299
Figure 92 Filtered Residuals, QLD	300
Figure 93 Filtered Residuals, SA.....	300
Figure 94 Filtered Residuals, TAS.....	301
Figure 95 Empirical Cumulative Distribution Function, NSW	302
Figure 96 Empirical Cumulative Distribution Function, VIC.....	302
Figure 97 Empirical Cumulative Distribution Function, QLD	303
Figure 98 Empirical Cumulative Distribution Function, SA	303
Figure 99 Empirical Cumulative Distribution Function, TAS	304
Figure 100 Cross Correlation Functions, NSW with QLD	305
Figure 101 Cross Correlation Functions, SA with VIC.....	305
Figure 102 Cross Correlation Functions, NSW with VIC.....	306
Figure 103 Autocorrelation Functions (Daily log prices), NSW.....	307
Figure 104 Autocorrelation Functions (Squared daily log prices), NSW.....	307
Figure 105 Autocorrelation Functions (Daily log prices), QLD	308
Figure 106 Autocorrelation Functions (Squared daily log prices), QLD.....	308
Figure 107 Autocorrelation Functions (Squared daily log prices), SA.....	309
Figure 108 Autocorrelation Functions (Squared daily log prices), SA.....	309
Figure 109 Autocorrelation Functions (Daily log prices), TAS.....	310
Figure 110 Autocorrelation Functions (Squared daily log prices), TAS.....	310
Figure 111 Autocorrelation Functions (Daily log prices), VIC	311
Figure 112 Autocorrelation Functions (Squared daily log prices), VIC	311

LIST OF TABLES

Table 1 Australian Major Power Network Transfer Capabilities, 2008-09	32
Table 2 Summary Statistics of Daily Price Series in NEM	49
Table 3 Jarque-Bera Normality test	53
Table 4 Unit Root and Stationary Tests – Daily and Log-Daily Electricity Prices.....	54
Table 5 Runs Test (Mean and Median as Cut Points)	58
Table 6 AR-GARCH Model Parameters	110
Table 7 Forecast Accuracy Measures of Benchmark Model.....	112
Table 8 Comparative Forecast Performance Measures.....	126
Table 9 Parameter Estimates of the Mean-Reverting Model.....	135
Table 10 Comparative Forecast Performance Measures.....	144
Table 11 Comparative Forecast Performance Measures.....	144
Table 12 Parameter Estimates for Mean-Reverting Jump Diffusion Model.....	156
Table 13 Forecast accuracy statistics for Mean-Reverting and Jump-Diffusion Model.....	162
Table 14 Comparative Forecast Performance Measures.....	163
Table 15 Regime Probabilities, NSW region	176
Table 16 Regime Probabilities, VIC Region	183
Table 17 Regime Probabilities, QLD Region	187
Table 18 Regime Probabilities, SA Region	191
Table 19 Regime Probabilities, TAS Region.....	194
Table 20 Forecast Accuracy Statistics for Markov Regime-Switching Model	209
Table 21 Pearson Correlation Coefficients	216
Table 22 Cross Correlations	217
Table 23 Parameter Estimates for the AR-EGARCH model.....	224
Table 24 Summary Statistics for the Raw and Standardized Residuals	226
Table 25 Tail Parameter Estimates of the GPD	234
Table 26 Parameter Estimates of the Student t-Copula	241
Table 27 Parameter Estimates of the Gaussian Copula	242
Table 28 Forecast Accuracy measures for EVT-Copula Model	249
Table 29 List of Econometric Models.....	261
Table 30 Comparative Forecast Accuracy Measures of All Models by Regions of NEM	273
Table 31 One-Sample Kolmogorov-Smirnov Tests.....	281
Table 32 BDS Parameter Estimates.....	295

RESEARCH QUESTIONS

This thesis answers the following research questions:

- 1- Are forecast models generated by continuous-time models more accurate than traditional AR-GARCH model?
- 2- Are forecast models generated by non-linear models more accurate than traditional AR-GARCH model?

LIST OF ABBREVIATIONS

ABS	Australian Bureau of Statistics
ARCH	Autoregressive Conditional Heteroskedasticity
AR-GARCH Heteroskedasticity	Autoregressive-Generalised Autoregressive Conditional
ADF	Augmented Dickey-Fuller Statistic
AEMO	Australian Electricity Market Organisation
ARX	Autoregressive Exogenous
CDF	Cumulative Distribution Function
CEDF	Cumulative Empirical Distribution Function
GARCH	Generalised Autoregressive Conditional Heteroskedasticity
EM	Expectation Maximization
EVT	Extreme Value Theory
GBM	Geometric Brownian Motion
GEV	Generalised Extreme Value Distribution
GPD	Generalised Pareto Distribution
HWV	Hull-White-Vasicek Algorithm
KPSS	Kwiatkowski–Phillips–Schmidt–Shin
NEM	National Electricity Market
MATLAB	Engineering Software
ML	Maximum Likelihood
OLS	Ordinary Least Squares
PP	Philips-Peron test
POT	Peaks over Threshold
SDE	Stochastic Differential Equation

CHAPTER 1 - OVERVIEW

This thesis tests whether forecast accuracy improves when models that explicitly capture the stylised features of the market are employed to generate predictions. It is believed that by explicitly modelling these features of electricity wholesale spot prices, the accuracy of the price forecast models can be improved when compared to standard alternative. The stylised features identified in data are mean-reversion, sudden short-lived and consecutive jumps and heavy tails.

When employing models to capture the stylised features of electricity prices, the models necessarily become more complex and often contain a greater number of parameters which combine to mimic the characteristics observed in the price series. Throughout this thesis an adherence to the principle of parsimony (Makridakis, *et al* page 609) will be maintained; that is if two models effectively generate the same forecast performance the simpler will be preferred whether it contains the stylised features or not. This is also known as Occum's Razor.

This investigation is important in terms of a better understanding of what models are more useful has the potential to lead to more accurate price forecasts which may result in less volatility in market prices leading to more efficient markets. Further, by assessing models that capture various stylised features it may be possible to infer the importance of particular features.

Given that wholesale prices are a major determinant of how much end users pay for powering their homes and businesses, it is believed that a better understanding of what forecasting models work (and do not) will allow market participants to develop more successful (business)

strategies for adjusting supply to meet demand and to assist with the valuation of financial assets as part of risk management. Additionally, a better understanding of the dynamics of electricity prices and its implications for successful forecasting is important for government policy makers, as Government sets the rules which govern the production and distribution of electricity.

To investigate the accuracy of the various forecasting models two distinct modelling approaches which capture the stylised features of electricity prices in National Electricity Market (NEM) are examined¹. Namely, these are the **continuous-time** and **non-linear** modelling methods. **Continuous time** models consist of Geometric Brownian Motion (GBM), Mean-Reversion and the Mean-Reversion Jump-Diffusion processes, as they account for the mean-reversion and sudden and short-lived jumpy characteristics of the electricity prices in the NEM. The **non-linear** models of Markov Regime-Switching and Extreme Value Theory (EVT) aim to deal with consecutive jumps and non-Gaussian which is also prevalent in NEM.

This thesis is organised as follows:

Chapter 1 will outline the motivation for the thesis and describes the electricity industry in Australia. It explains that a better understanding of electricity price forecasting will help all market participants to develop more efficient price risk management strategies and improve financial asset valuations. Additionally, it is believed that accurate price forecasts will eventually result in less volatility in market prices (leading to more efficient markets) and this will clearly benefit consumers.

In Chapter 2, a background to electricity markets in Australia is presented. The importance of the electricity market in the Australian domestic economy and an overview of the institutional

¹ NEM is consisted of five separate but interconnected markets.

characteristics of electricity markets are discussed. This chapter also explains the derivation of market prices. In Australian National Electricity Markets, the delivery of electricity to customers, that is, from the point at which the energy departs the generator terminals and is delivered via the transmission and distribution network, remains a regulated monopoly (Simshauser, 2014 page 558). The wholesale market, where generators and retailers interact, is coordinated by Australian Energy Market Operator (AEMO). Clearly accurate forecasting is also of great importance to government policy makers.

In Chapter 3, the data collection, collation procedures and sources of the electricity price data used in this study are described. The summary descriptive statistics for each series are also presented. These statistics show that electricity prices tend to be mean-reverting, have sudden and short-lived jumps and are non-Gaussian. Furthermore, the importance of modelling electricity prices in NEM from a risk management point of view is emphasised in this chapter. It is shown that advanced modelling can be used by players to manage risks and explore profit taking opportunities.

In Chapter 4, the literature relevant to this thesis is presented. The approaches to modelling electricity prices are reviewed and these are used to inform the research questions in this thesis. The first section of this chapter presents Autoregressive (AR) models with examples from the literature that utilised the AR specification; whilst the section on Mean-Reverting and Mean-Reverting-Jump-Diffusion models discuss the previous work that utilised Stochastic Differential Equation (SDE) when applied to modelling electricity prices. This discussion also includes the use of Markov Regime-Switching approach. Section three describes volatility modelling of electricity prices, namely Auto Regressive Conditional Heteroskedastic (ARCH) and Generalized Auto Regressive Conditional Heteroskedastic (GARCH) models. Section four

of this Chapter then presents both the Extreme Value Theory (EVT) and Levy-diffusion models, which are thought to capture the heavy-tailed nature of electricity prices. Finally, section five shows some of the recent work conducted using electricity price data from Australia.

Chapter 5 presents an overall introduction to the separate econometric models examined in the thesis. The three continuous-time models are explained in sequence such that each successive model extends the previous one. Note that each extension addresses a stylised feature of the data, therefore it is expected that forecasting performance will improve. The inclusion of the non-linear approach to forecasting Australian electricity prices is done with the application of Markov Regime-Switching processes in combination with EVT and Copula simulations. These applications are a further extension to previously examined continuous-time models as they capture the Non-Gaussian behaviour of electricity prices in NEM and incorporate these characteristics into forecast paths.

Australian wholesale electricity price forecasts with Geometric Brownian Motion (GBM) are presented in Chapter 6. This model is included in the thesis as it is the foundation for the other Stochastic Differential Equations (SDE) based models. By construction, GBM does not capture the stylised features of electricity prices namely, mean-reversion, sudden and infrequent jumps and non-Gaussian. Consequently, forecasts based on GBM result in large forecast errors. This finding shows that capturing the stylised features of electricity prices is important for having a model with potentially more accurate forecasts.

Mean-Reverting and Mean-Reverting Jump-Diffusion models are discussed in chapters 7 and 8. After considering the poor performance of the GBM forecasts due to its inability to capture

the stylised features of electricity prices, Mean-Reverting and Mean-Reverting Jump-Diffusion models are considered in this chapter. The forecast based on these models perform much better than GBM forecasts. This is attributed to the fact that both of these models capture the dynamics of mean-reversion and the jumpy nature of the price series that is evident in NEM.

Chapter 9 presents a model based on Markov Regime-Switching dynamics of electricity prices. This model captures consecutive spikes prevalent in electricity prices. The Markov-Regime Switching model is based on the observed stochastic behaviour of a time series by two or more separate regimes with different underlying processes. The aim is to capture the mean-reversion characteristics of electricity prices similar to Mean-Reverting and Jump-Diffusion models. However the Markov process goes beyond these models in accounting for consecutive spikes that are prevalent in NEM. This model's performance is superior to other models discussed earlier in the thesis when the spike regime is modelled with a Log-Gaussian distribution.

Chapter 10 presents a unique approach to forecasting electricity prices in NEM, as with the assistance of EVT, electricity prices are simulated with Copula functions to generate forecasts. This model shifts the focus to modelling the heavy tails of the data with the aid of Copula functions; thereby capturing the dynamic interactions of the regional price inter dependencies prevalent in NEM.

Chapter 11 presents the findings of each model employed in the thesis in terms of their forecast performance. Formal comparative forecast accuracy statistics for each model are presented in this chapter. The root mean square error (RMSE) of each model is compared for the given forecast horizon. The analysis of the RMSE values shows that the forecasts generated by simulation models based on the Markov Regime-Switching process outperform other

continuous and non-linear models for each region of NEM. This model best captures all the stylised features of electricity prices i.e. mean-reverting, sudden and consecutive jumpy behaviour. However, this model was found to be generating less accurate forecasts as compared to the benchmark model. The areas that require improvement and require further research in modelling electricity prices in NEM are also discussed towards the end of this chapter, with a particular focus on the improvements in parameter estimations and model simulations.

CHAPTER 2 - BACKGROUND

RATIONALE

Electricity markets in Australia operate as a wholesale spot market in which generators and retailers trade electricity through a gross pool managed by the respective regional energy market operators. These operators aggregate and dispatch supply to meet demand. In addition to this physical wholesale market, participants in electricity markets also make use of financial markets as part of their risk management strategies. This occurs mainly in the over the counter market, however there is also an established futures market within Australian Securities Exchange (ASX) operations.

The forecasting of electricity prices is important for both the physical and financial participants in the electricity industry for the following reasons:

1. Generators are required to bid in advance, thus accurate price forecasting is necessary if optimal bidding strategies are to be formulated.
2. Generators need to plan ahead for capacity building purposes (peak and off-peak generators have varying input requirements for production) therefore efficient price forecasting allows efficiency in planning of supplies.
3. Traders need to take positions at both over the counter and the established futures trading platforms (e.g ASX), therefore accurate forecast of spot prices are important for derivatives pricing.
4. The distribution of electricity is a publicly-regulated monopoly, therefore there are also many government policy implications.

In summary, a better understanding of effective electricity price forecasting will help all market participants to develop more efficient strategies, for example it will assist firms in the industry in formulating business strategies, real and financial asset valuations and improve their price risk management. Also, price forecasting is an important aspect of the industry as generators bid to the market operator to be granted the rights to supply electricity to the grid. Ineffective bids cause revenue losses to generators. For instance, bidding a low price to get the electricity to the grid will result in lost revenue if the market price is above the bid price and *mutatis mutandis*.

The electricity market is highly regulated and controlled by government. This is likely to continue, especially given interest by policy makers to design policies to reduce carbon emissions. Therefore, having accurate electricity price forecasts is of great benefit to government policy makers. This arises as wholesale electricity prices influence the contract price at the retail level, which in turn impacts upon the final prices for consumers. It is believed that improved price forecasts will eventually result in less volatility in market prices (leading to more efficient markets) and this will result in benefits for consumers.

An important feature of wholesale electricity spot prices are that they are highly volatile due to non-storability, limited transportability, restricted arbitrage transactions and imperfect price forecasting techniques (Bunn, 2004). As such the nature of the electricity time series is not the same as traditional stock prices. In addition they are recognised as being spikier, showing extreme volatility and exhibiting more rapid mean-reverting behaviour than stock prices. These factors pose particular challenges in attempting to forecast electricity wholesale prices.

Mean-reversion, sporadic spikiness, and non-Gaussian manifesting in positive skewness and leptokurtosis are well known to be the main stylized features of electricity prices as pointed out in the literature (Kaminski 1997). Therefore any forecasting model that fails to capture these features of electricity prices will likely result in relatively larger forecast errors (albeit not always).

Consequently this thesis investigates the explicit incorporation of the stylised features of data into the forecasting models. Specifically, the features of mean-reversion, sudden and short-lived jumps, occasional consecutive jumps and non-Gaussian manifested as heavy tails are considered.

It is expected that by explicitly modelling the stylised features of the electricity wholesale spot prices, forecast accuracy can be improved when compared to baseline models commonly used in quantitative finance. The AR-GARCH² model is chosen to be the standard approach in forecasting price series (Engle, 2001) and is taken as the benchmark model in this thesis.

This thesis will employ models from two distinct model classes which by construction capture the stylised characteristics of electricity prices: linear and non-linear modelling methods. More specifically, the following research questions will be investigated:

- 1- Are forecast models generated by continuous-time models more accurate than traditional AR-GARCH model?
- 2- Are forecast models generated by non-linear models more accurate than traditional AR-GARCH model?

² Autoregressive - Generalized Autoregressive Conditional Heteroskedasticity models the variance of the series.

Continuous-Time Models

A wide variety of forecasting methods are available to businesses. These range from the naïve methods, such as the use of the most recent observation, to highly complex approaches such as neural networks and econometric systems of simultaneous equations. Companies use complex forecast models to gain a competitive edge and increase profitability.

The overwhelming majority of electricity-pricing models are adaptations of popular models for price or returns from the financial econometrics literature that have been augmented to capture the idiosyncratic time-series properties of electricity prices, albeit with varying degrees of success (e.g. Weron, 2006). Evidence suggests that using models based on Stochastic Differential Models (SDEs) otherwise known as continuous-time models provide a much better fit to electricity prices than the autoregressive models (Lucia and Schwartz 2002, Huisman and Mahieu 2003). Models based on SDEs also allow for analytical tractability and are more suitable for derivatives pricing (Weron and Misiorek, 2008). It is for these reasons, SDE based forecast models have become widely recognised in the industry.

The continuous time-models employed in this thesis are; Geometric Brownian Motion (GBM), Mean-Reverting and Mean-Reverting Jump-Diffusion processes. The inclusion of GBM in this thesis is mainly due to it being the foundation for the other continuous time models considered in this study. The other two continuous-time models capture some of the main stylised features of electricity prices. The Mean-Reverting process captures the mean-reversion (tendency of electricity prices to revert back to its long-term average over time) characteristics of electricity prices whilst Mean-Reverting and Jump-Diffusion method models the sudden jumps prevalent in electricity prices in NEM.

These models are presented such that each successive model extends the one preceding it. Note that each extension adds a stylised feature of the data, therefore the a-priori expectation is that the forecasting performance will improve.

Non-Linear Models

The inclusion of the non-linear approach to forecasting electricity prices is performed with the application of Markov Regime-Switching process and the combination of Extreme Value Theory and Copula simulations.

The Markov Regime-Switching process is a non-linear modelling tool that is able to capture consecutive spikes prevalent in Australian electricity prices that the Mean-Reverting and Jump-Diffusion processes fail to capture.

The EVT model is included in this thesis so that it captures the heavy tails present in electricity price data. Forecasts based on the EVT model build upon the application of Copula³ functions as these functions model the interdependence of prices within the separate regions of the Australian electricity markets.

Forecast Approach

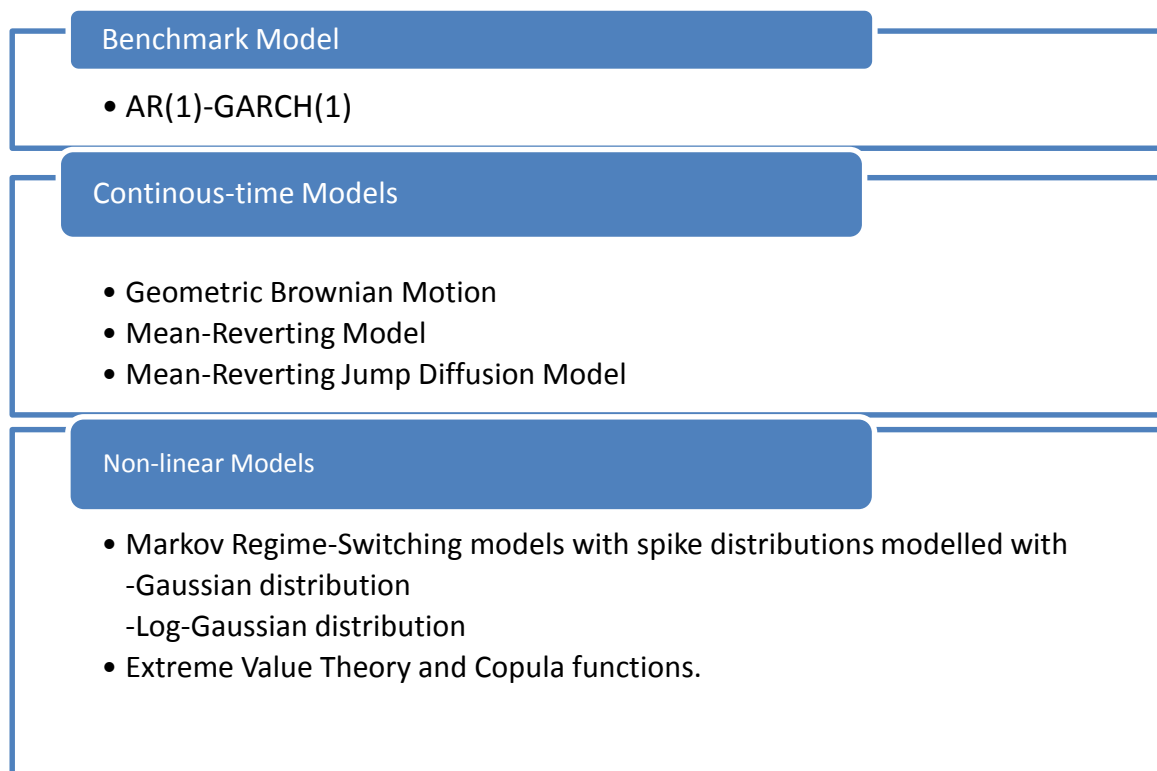
To determine whether the five forecasting approaches (each of which captures particular stylised features of the time series under consideration) are more accurate than the benchmark model, short-term forecast performances (90 days) are critically assessed. Each of these assessments are compared to the chosen (AR-GARCH) benchmark model.

³ Copulas are considered as a unique method that models the dependence structure of data.

The price data used in this study are average hourly pool price observations sourced directly from AEMO for the period of 01/06/2006 to 29/08/2010. The data from 01/06/2006 to 31/05/2010 (in-sample data) are used to estimate the parameters of the models, while the period from 01/06/2010 to 29/08/2010 (out-of-sample data) are used to derive out-of-sample forecast accuracy statistics.

The rationale behind the chosen out-of-sample period is due to the fact that most of the electricity derivatives in the market are Asian options. As known, for Asian options the payoff is determined by the average underlying price over some pre-set period of time that is most of the time is three months.

The models examined in this thesis are;



Each model under investigation mimics the known characteristics of electricity prices. The Mean-Reverting model replicates the mean reversion feature of prices series whilst Mean-Reverting and Jump-Diffusion model incorporates jumpy feature of prices series along with mean-reversion. The Markov Regime-Switching model incorporates the consecutive jumps prevalent in NEM in its formation. Finally EVT based model replicates the nonlinear, heavy tailed nature of the electricity price series.

Each of the continuous-time and the Markov Regime-Switching models are simulated using the Euler approximation method. This method simulates sample paths of correlated state variables driven by Brownian motion sources of changes over consecutive observation periods and thus approximating continuous-time stochastic processes.

EVT based forecast models on the other hand are simulated with Copula functions, returning random vectors generated from a t-copula with linear correlation parameters. This method generates a set of simulations from a bivariate t-copula and each column of the simulation sets is a sample from a uniform marginal distribution.

A set of comparative forecast performance measures is used in this thesis in measuring the relative forecast performance of each forecast model. The forecast performance measures of Root Mean Square Error (RMSE) and Theil's U are used to present the forecast errors of each model by each region of the NEM.

A Review of Modelling Electricity Prices in Australia

Limited academic work has been undertaken in Australia that provides a detailed understanding of electricity price forecast behaviour by using models that capture the stylised features of electricity prices.

Higgs and Worthington (2008) are the only researchers modelling electricity prices in Australia covering all market regions of the National Electricity Market (NEM) with the assistance of SDE based models. They applied three different SDE models to electricity price series in an attempt to determine the best spot price model applicable in all of the NEM regions. The forecast models they utilised in their study were a basic stochastic model, a mean-reverting model and a Markov Regime-Switching model. Their results showed that the Markov Regime-Switching model outperforms the basic stochastic and mean-reverting models.

This thesis extends the work of Higgs and Worthington (2008) by modelling electricity prices with Mean-Reverting and Jump-Diffusion process and compares the forecast performance of this model with GBM, Mean-Reverting, Markov Regime-Switching models, and a non-linear model based on the combination of EVT and Copula functions.

In doing so, the major contribution of this thesis to the literature is the systematic investigation of the importance of stylized facts of electricity prices to determine if their capture in model settings can improve upon the standard forecast approaches.

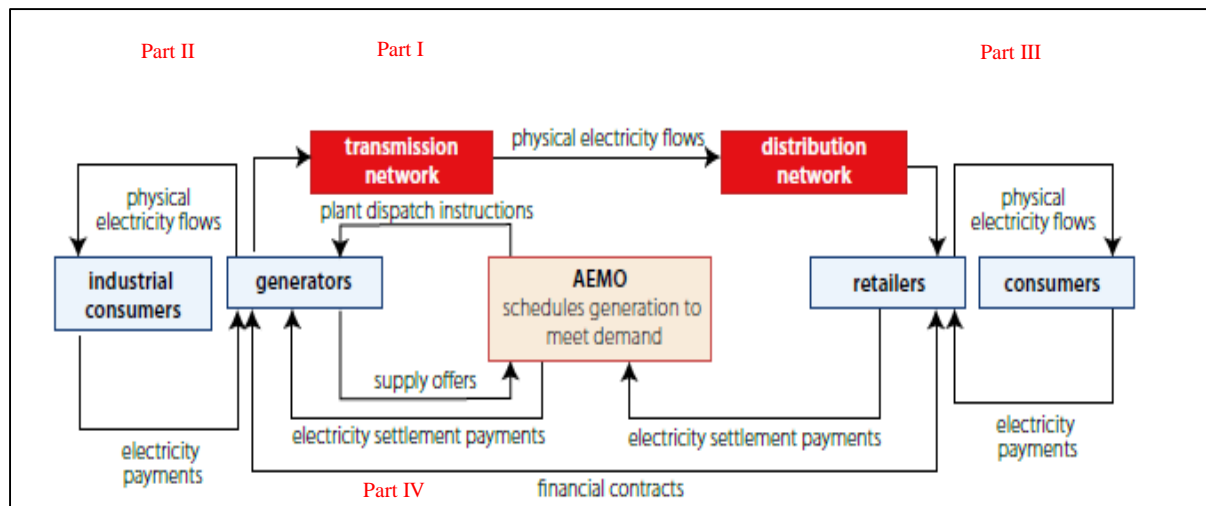
AUSTRALIAN ELECTRICITY INDUSTRY

According to a recent Government report, the electricity industry is one of Australia's largest industries, representing 1.4 per cent to total Australian industry value added in 2008–09 (Energy in Australia, 2011). The industry consists of generators, transmission and distribution networks and retailers. NEM allows market determined power flows across the Australian Capital Territory (ACT), New South Wales (NSW), Queensland (QLD), South Australia (SA), Victoria (VIC) and Tasmania (TAS).

Western Australia and the Northern Territory are not connected to the rest of the regions of Australia, primarily because of their geographic distance from the East Coast. Western Australia's electricity market uses a net pool arrangement. Another distinguishing feature of the market in Western Australia is the provision of a separate capacity mechanism. On the contrary, the electricity transmission and distribution is supplied by a wholly Territory owned company Power and Water Corporation. The Western Australia's and Northern Territory's electricity market will not feature in the rest of this thesis. Its inclusion here is merely to alert the reader to its distinction.

The NEM consists of registered generators, state-based transmission networks linked by cross-border interconnectors and major distribution networks that collectively supply electricity to end-use customers. The NEM operates as a wholesale spot market in which generators and retailers trade electricity through a gross pool managed by the Australian Energy Market Operator (AEMO), which aggregates and dispatches supply to meet demand. In addition to the physical wholesale market, retailers may also contract with generators through financial markets to better manage any price risk associated with trade on the spot market. The following diagram demonstrates the interactions between the players of NEM.

Figure 1 Market Structure in NEM



Source: Energy in Australia 2011, Department of Resources, Energy and Tourism, Commonwealth of Australia

As is seen in Figure 1, AEMO receives supply offers from the generators and schedules generations to meet current demand. It achieves this by dispatching generators by matching their supply offers (Part I). Physical electricity flows to, non-industrial consumers through the transmission and distribution networks. The electricity flow between the industrial consumers occurs as a separate mechanism (Part II). Non-industrial consumers receive electricity and payments generated flow to retailers (Part III). Figure 1 also points out that financial contracts are predominant between the generators and retailers (Part IV). These financial contracts provide hedging capacity for both the retailers and generators.

NEM is the world's geographically largest interconnected power system that runs for more than 5,000 kilometres from Port Douglas in Queensland to Port Lincoln in South Australia and supplies more than \$10 billion worth of electricity annually to meet the demand of more than 8,000,000 end users (AEMO Annual Report, 2011). It is connected by seven major transmission interconnectors. These interconnectors link the electricity networks in QLD, NSW, VIC, SA and TAS. The electricity transmission and distribution networks of NEM consist of around 790,700 kilometres of overhead transmission and distribution lines and

around 113,700 kilometres of underground cables (RET, 2011). Table 1 illustrates the existing transmission lines as of 2009.

Table 1 Australian Major Power Network Transfer Capabilities, 2008-09

Interconnector	Location	Forward capability (MW)	Reverse capability (MW)
NSW to Queensland (QNI)	Armidale to Braemar	483	1,078
NSW to Queensland (Terranora)	Terranora to Mullumbimby	115	245
Snowy to NSW	Murray to Dederang	3,114	1,134
Victoria to Snowy	Borong to Red Cliffs	1,274	1,780
Victoria to SA (Murraylink)	Red Cliffs to Berri	220	180
Victoria to SA (Heywood)	Heywood to Tailem Bend	460	300
Tasmania to Victoria (Basslink)	Seaspray to Georgetown	630	480

Source: Australian Energy Market Operator, Annual Report 2010

Existing capacity of the transmission lines are an important aspect of the electricity price determination in NEM. This is due to the fact that these interconnectors allow the electricity to be traded between the separate regions of NEM when it is needed. For instance, when the demand exceeds the existing capacity in a region, adjacent regions (depending on their available capacity) offer to export electricity to the demanding region. Naturally, supply and demand mechanisms bring the price in the region back to its long-term equilibrium. This trade is restricted by the capacity of the transmission lines. Therefore the convergence in the supply and demand equilibrium is also restricted to the existing capacities of these interconnectors.

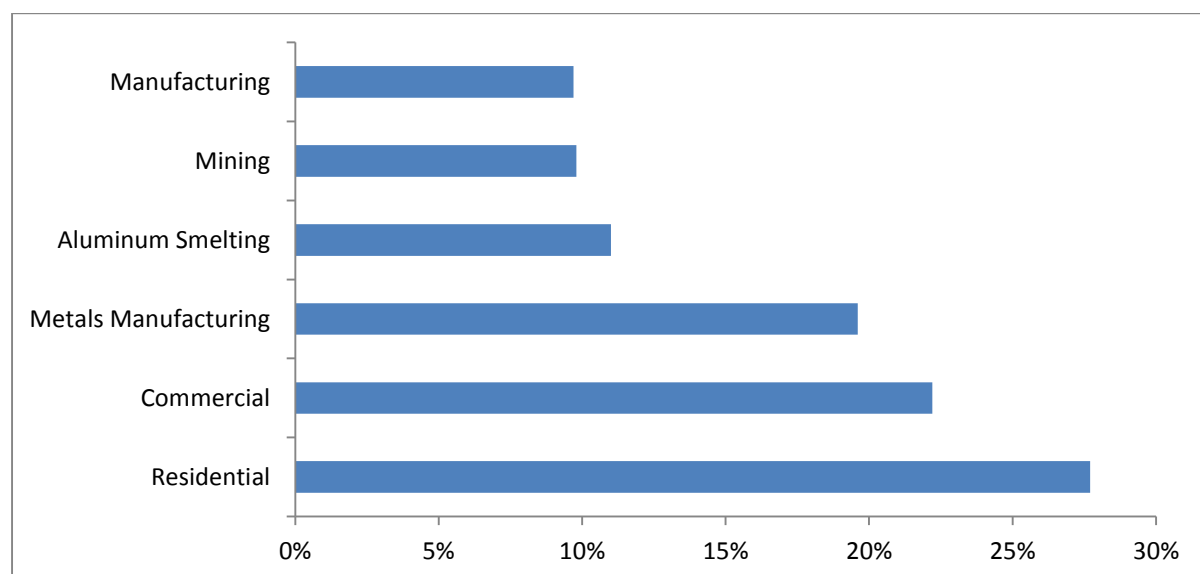
Electricity Consumption by Industry Type

Figure 2 describes the types of electricity consumers in 2010. As is seen, residential electricity users are the largest consumers of this commodity followed by commercial users, 27.7 per cent and 22.2 per cent respectively. Other larger consumers of electricity are metals (mainly steel manufacturing), aluminium smelting and mining industries.

Demand factors that play a role in household electricity consumption include: shifts in household disposable incomes and moves to improve the energy efficiency of new homes. Hence, supply factors that play a role in household electricity consumption include changes in the price of electricity and competing fuels and the availability of a wider range of fuels.

The metals manufacturing sector comprises the manufacture of iron and steel and the smelting and refining of non-ferrous metals, such as copper, lead, zinc and nickel. Aluminium production alone accounts for about 11 per cent of Australia's total electricity usage as the production of aluminium is electricity intensive. Electricity comprises an estimated 30 per cent of the operating costs of a large aluminium smelter.

Figure 2 Electricity Consumption by Sector, 2010



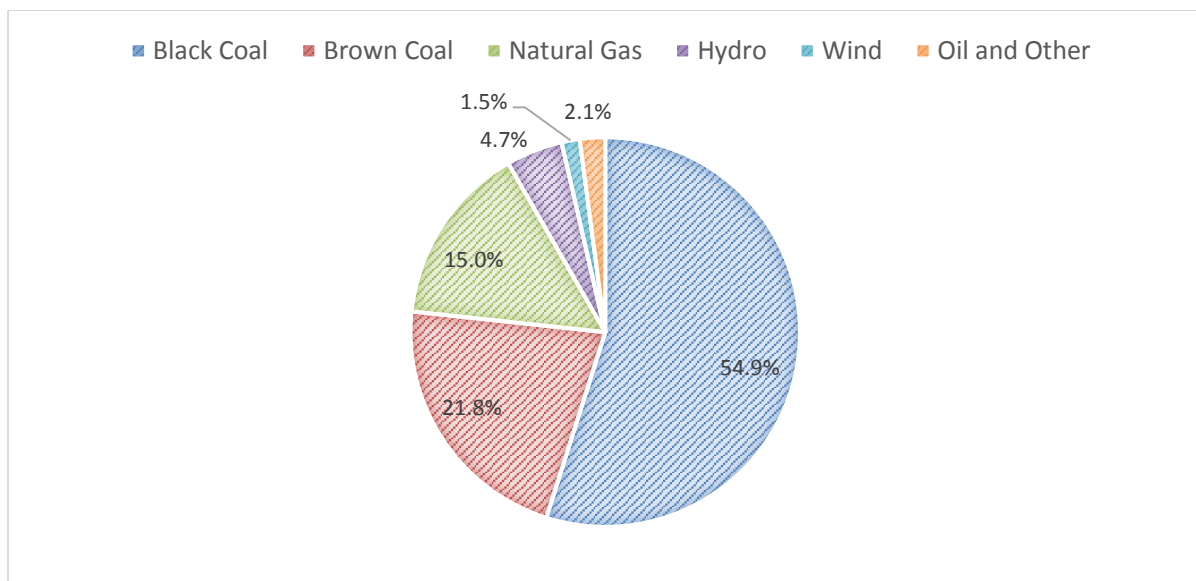
Source: Australian Energy Market Operator, Annual Report 2010

Electricity Generation by Fuel Type

Broadly speaking, there are two types of generating plants in the NEM, known as base-load and peak-plant generators. Base-load generators use coal as the primary energy input as they are the cheapest source available in Australia therefore their marginal costs are the lowest of all generators operating in NEM. Peak-plants on the other hand use mainly natural gas to

generate electricity followed by hydro and wind energy. Importantly, the reason why these plants are called peak-plants is because as the energy source they use is more expensive than the base-load generators, their marginal costs are more expensive and their operations are profitable only when the prices in the wholesale market are above a certain threshold.

Figure 3 Electricity Generation by Fuel Type, 2008-09



Source: Energy in Australia 2011, Department of Resources, Energy and Tourism, Commonwealth of Australia

A variety of fuels are used in the production of electricity. The majority of Australia's electricity generation is supplied by steam plants, using coal or natural gas as fuel. Black coal and brown coal are distinguished by differences in their energy and water content. A given tonnage of black coal contains more energy and less water than the same tonnage of brown coal. Most of Australia's black coal fuelled generation capacity is located in New South Wales and Queensland, while Queensland has the largest generation capacity of gas fuelled plants.

In 2011, about 56 per cent of all generators in NEM used black coal and about 25 per cent used brown coal to produce electricity whereas natural gas, hydro and wind energy represent a little less than 20 per cent of the total electricity generated in 2011 (Energy in Australia, 2011).

According to IBISWorld Market Research (2012), the share of electricity generated from natural gas, coal seam methane and wind has increased during the past five years at the expense of black and brown coal. The main growth fuel has been natural gas, although coal seam methane has also increased in importance. Coal seam methane is gas extracted from coal deposits. The importance of hydro-electricity tends to fluctuate depending on the availability of water reservoirs.

ELECTRICITY PRICES AND THE ECONOMY

Electricity plays an essential role in modern life, bringing benefits and progress in all sectors, including transportation, manufacturing, mining and the communication sectors. Electricity is an important input to production in these industries and more generally within the economy. In many industries like the car manufacturing industry, technical change tends to increase the relative share of electricity in the value of output as more mechanised manufacturing boosts labour productivity. These industries' productivity growth is also found to be greater as the price of electricity lowers and vice versa.

The total gross value added (chain volume measures) of the electricity industry in Australia was \$4.3 billion in March quarter 2012 (ABS, 2012). Though, electricity markets have three layers of operations in Australia; generation, transmission and distribution, each creating its own industry.

According to IBISWorld Market Research (2012), the Electricity Generation Industry produces a net profit of \$3.3 billion in 2011-12. Correspondingly, the revenues generated by the electricity generation industry totalled \$19.1 billion in 2011-12 financial year. It is also estimated that the annual growth over the past five years to 2012 has been around 5.5 per cent

but it is expected to increase to nine and a half per cent between the years of 2012 to 2017 due to the introduction of assistance provided to the industry under carbon pricing arrangements.

The Electricity Transmission Industry operates the high-voltage electricity network, linking electricity generators to the distributors that operate the low-voltage electricity supply system. The structure of the Electricity Transmission industry differs from the electricity generation and distribution industries in one fundamental area. According to IBISWorld Market Research (2012), the revenues generated by the electricity transmission industry totalled \$3.2 billion in 2011-12 financial year. It is also estimated that the annual growth over the past five years to 2012 has been around five and a half per cent but it is expected to increase to 3.1 per cent between the years of 2012 to 2017.

The Electricity Distribution Industry on the other hand involves operating low voltage power supply systems (consisting of lines, poles, meters and wires). IBISWorld Market Research (2012) points out that the revenues generated by the electricity transmission industry totalled \$50.9 billion in the financial year of 2011-12.

PRICE FORMATION IN AUSTRALIAN NATIONAL ELECTRICITY MARKET

The crucial feature of price formation in NEM is the instantaneous nature of electricity. Delivery of electricity across the transmission grid requires a synchronised energy balance between the injection of power at generating plants and the consumption at demand points (plus some allowance for transmission losses). Across the grid, production and consumption are perfectly synchronised, without any ability for storage. Electricity as a commodity differs from other commodities such as oil and grains where its storage is not possible with traditional

methods (Bunn, 2004). Therefore, modelling electricity prices with traditional models that rely on the existence of a 'convenience yield' presents its challenges.

Furthermore, end-users treat this product as a service at their convenience and as a consequence, price elasticity of demand for electricity is very low (Bohi and Zimmerman 1984, Filippini 1999, Beenstock, Goldin and Nabot 1999, King and Shatrawka 1994, King and Chatterje 2003, Reiss and White 2005, Faruqui and George 2005, Taylor, Schwarz and Cochell 2005). Price elasticity of demand measures the responsiveness, or elasticity, of the quantity demanded of electricity to a change in its price. More precisely, it gives the percentage change in quantity demanded in response to a one per cent change in price (*ceteris paribus*, i.e. holding constant all the other determinants of demand, such as income). Price elasticity is usually negative, that is, an increase in price will normally cause demand to fall, therefore it is usually quoted in absolute terms. If an own price elasticity is small, it is known as an inelastic demand; that is demand is generally unresponsive to price changes.

Low levels of absolute price elasticity of demand in NEM indicate that price increases are not associated with substantial declines in the demand for electricity. This means that when prices are higher than usual the demand does not tend to decline significantly. So when there is an unexpected spike in demand due to say, weather events, increasing prices do not cause a decline in demand, which needs to be fulfilled instantaneously. The task of the grid operator therefore is to monitor the demand process and to call on those generators who have the capacity to respond to the fluctuations in demand.

Low absolute levels of own price elasticity of demand and the operations of retailers in the NEM also means that sudden price increases are borne by the market participants at the

wholesale level rather than the end-consumer of electricity. Electricity retailers offer a fixed charge for electricity usage to their customers rather than a variable charge for electricity usage. This implies that sudden rises in prices at the wholesale level need to be borne by the retailers. Hence, this further highlights the importance of accurate price forecasting in the NEM.

Australian Energy Market Operator (AEMO)

AEMO is the grid operator in NEM. AEMO's responsibilities include; day-to-day management of wholesale and retail energy market operations and emergency management protocols; ongoing market development required to incorporate new rules, infrastructure and participants; and long term market planning through demand forecasting data and scenario analysis (AEMO Annual Report, 2011).

Day to day operation of Australian electricity markets involves dynamic trading between energy generators, wholesalers and retailers based on variable pricing levels that reflect current levels of demand. AEMO maintains the systems through which prices are set and transactions carried out and provides accurate and timely market data to participants (AEMO Annual Report, 2011).

AEMO's role in producing demand forecasts is crucial for the efficient workings of the electricity markets in NEM. Generators are called into production in line with the forecast demand produced by AEMO and furthermore they determine their bidding prices according to these forecast demand values. Therefore, any fundamental imbalances between the forecast and actual demand may cause market inefficiencies such as high/low prices in electricity at the wholesale level, increasing either the cost of electricity consumed or risk management costs of the generators.

The AEMO interconnects five regional market jurisdictions whose cooperation under the NEM is secured through Commonwealth legislation and memorandum of understandings (AEMO Annual Report, 2011). AEMO facilitates exchange between electricity producers and consumers through a pooled system where output from all generators is aggregated and scheduled to meet consumer demand. This allows for sophisticated pricing structures and load shedding arrangements that ensure security of supply despite large fluctuations in consumer demand.

Wholesale electricity trading in AEMO is conducted as a spot market where supply and demand is instantaneously matched in real-time through a centrally-coordinated dispatch process. Generators offer to supply the market with specific amounts of electricity at particular prices. Offers are submitted every five minutes of every day. From all offers submitted, AEMO determines the generators required to produce electricity based on the principle of meeting prevailing demand in the most cost-efficient way. AEMO uses the spot price as the basis for the settlement of financial transactions for all energy traded in the NEM.

The rules of electricity trading in AEMO set a maximum spot price, also known as a market price cap, of \$12,500 per megawatt hour (MWh). This is the maximum price at which generators can bid into the market and is the price automatically triggered when AEMO directs network service providers to interrupt customer supply in order to keep supply and demand in the system in balance (AEMO Annual Report, 2011).

Although the schedule of dispatch is based on submitted bids, there is scope for generators to re-bid the quantity it will produce. The volume of electricity may be changed from the original bid volume but not the price of the bid. This re-bidding mechanism has been criticised as it

may make the system vulnerable to exploitation and exposing AEMO to generator's market power (Beder, 2003, Quiggin 2004). For instance, it was argued that price spikes occur generally at times of low demand, absence of supply shortages and existence of favourable reserve plant margins⁴. Whilst Chester (2006) argued that the manipulation of prices by re-bidding strategies occurred as the result of a small number of generator's domination of the market.

In summary, this brief overview has shown that the national electricity market in Australia is highly integrated and prices are formed according to the supply and demand norms prevalent in the market. Hence, by applying complex models as opposed to more parsimonious models to the price series on the assumption that they predict better, it is believed that there will be wider benefits realised by end users of electricity.

The next chapter explores the stylised features of the electricity prices in NEM and describes the source of the data and provides descriptive analysis of the empirical data utilised in this thesis. Chapter 4 reviews the recent literature by different classes of econometric models utilised by past studies in a chronological order. Chapters 5 to 10 describe the methodology employed in this thesis while Chapter 11 discusses the findings of this thesis and it also suggests future research areas in electricity price forecast modelling in Australia.

⁴ When reserve plant margins exceeded peak demand loads by around 30 per cent.

CHAPTER 3 - DESCRIPTION AND SOURCES OF DATA

INTRODUCTION

The purpose of this chapter is to describe the data collection “collation procedures” and sources of the data used in this thesis. There are five market regions in the National Electricity Market (NEM) and this chapter analyses time series from each region of the NEM. The regions of NEM considered in this thesis are New South Wales (NSW), Victoria (VIC), Queensland (QLD), South Australia (SA), and Tasmania (TAS). The data was collated from Australian Energy Market Operator (AEMO). AEMO collates and reports average daily observations for each price for the five market regions of NEM.

The summary statistics for each region of NEM are also provided in this chapter. Summary statistics point to the stylised facts of electricity prices, namely the mean-reverting, jumpy and heavy-tailed nature of the data as well as the presence of negative prices.

A number of formal statistical tests are performed in this chapter to examine the distributional characteristics of the data i.e. Jarque-Bera and Kolmogorov–Smirnov test. Further to these tests, a number of unit root and stationary tests are also performed i.e. Augmented Dickey and Fuller (ADF) t-test, Kwiatkowski–Phillips–Schmidt–Shin (KPSS) and Lagrange Multiplier (LM)-test.

The formal normality and unit root tests confirm the non-Gaussian and stationary nature of the price series. Lastly, Runs tests were conducted to examine the efficiency of the Australian electricity prices. The tests conducted in this chapter provided a thorough understanding of the empirical data and assisted in the choice of appropriate methodology applied in this thesis.

AUSTRALIAN WHOLESALE SPOT ELECTRICITY PRICE SERIES

Electricity prices change every five minutes to match the demand with the schedule of bids offered by generators. Transactions are settled every half-hour at the spot price, which is derived as the average of the prices at which electricity supplied in the six preceding five-minute intervals. Chapter 1 described how electricity prices are set by AEMO in detail. This chapter describes the time-series characteristics of the wholesale electricity prices in NEM.

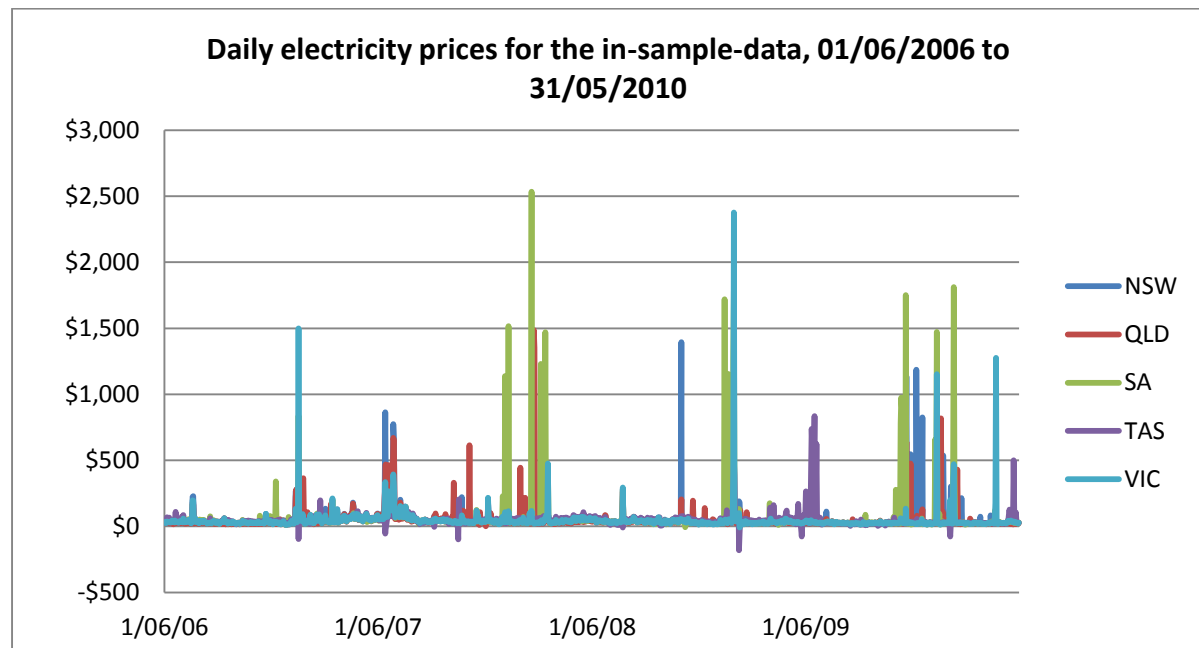
Electricity spot prices are amongst the most volatile commodity prices in the world. It has characteristics like non-storability, limited transportability and restricted arbitrage transactions. Therefore the nature of the electricity time series does not appear to be similar to traditional stock prices. Typically, they are spikier, show extreme volatility and exhibit a rapid mean-reverting pattern (Bunn 2004). Mean-reversion, the presence of jumps, and non-Gaussian manifested as positive skewness and leptokurtosis are the main stylised facts of electricity prices as pointed out in the literature (Kaminski 1997, Deng 1998).

Kaminski (1997) used random-walk jump diffusion model to capture the jumpy characteristics of electricity prices via an application of a Stochastic Differential Equation (SDE). Later, Deng (1998) considered the mean-reverting characteristics of the electricity price series. This work and the work of Kaminski (1997) opened the way for modelling electricity prices with SDEs and paved the way for research utilising SDEs in electricity price modelling.

The price data used in this study are average hourly pool price observations sourced directly from AEMO for the period of 01/06/2006 to 29/08/2010 for the regions of NSW, VIC, SA, QLD and TAS. The data from 01/06/2006 to 31/05/2010 are used to estimate the parameters of the models while the period from 01/06/2010 to 29/08/2010 are used to derive out-of-sample

forecast accuracy statistics. The rationale behind choosing this sample period is due to Tasmania's entry to AEMO towards the end of 2005. As a result, data prior to this date was not available.

Figure 4 Daily Wholesale Electricity Prices in NEM

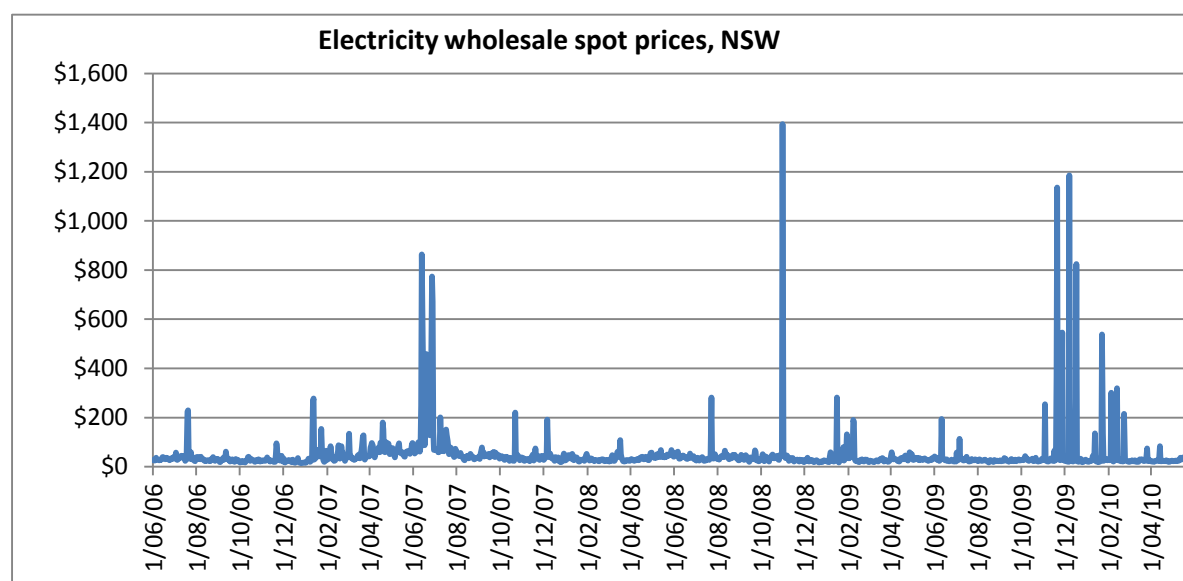


Source: Author's calculations. Data obtained from AEMO (July 2010).

Figures 5 to 9 show average daily reported price values for each region, expressed in Australian dollars per megawatt hour (MWh) for each day from June 2006 to June 2010. The figures show main features of electricity prices for all regions of NEM, namely; mean-reversion, jumpy, heavy tailed and highly volatile nature of the series.

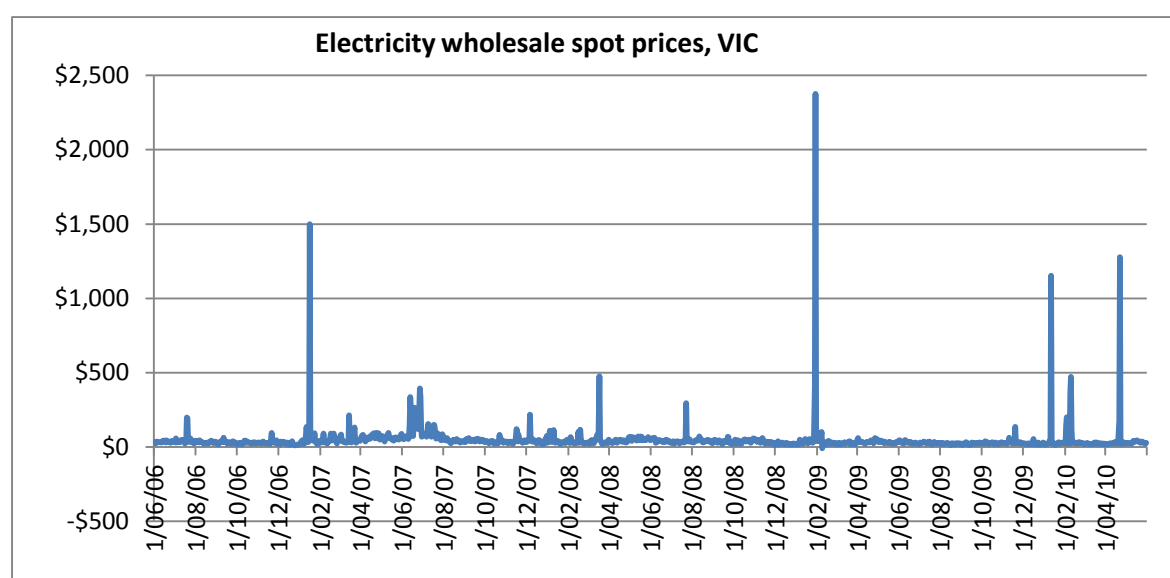
Figure 5 shows the historical wholesale spot prices in NSW. As is seen, there are large but short-lived price spikes, which tend to revert back to long-term mean levels very quickly. Generally, these spikes are observed over consecutive days leading to volatility clustering that is quite common in financial markets.

Figure 5 Daily Electricity Wholesale Prices in NSW



Source: Author's calculations. Data obtained from AEMO (July 2010).

Figure 6 Daily Electricity Wholesale Prices in VIC

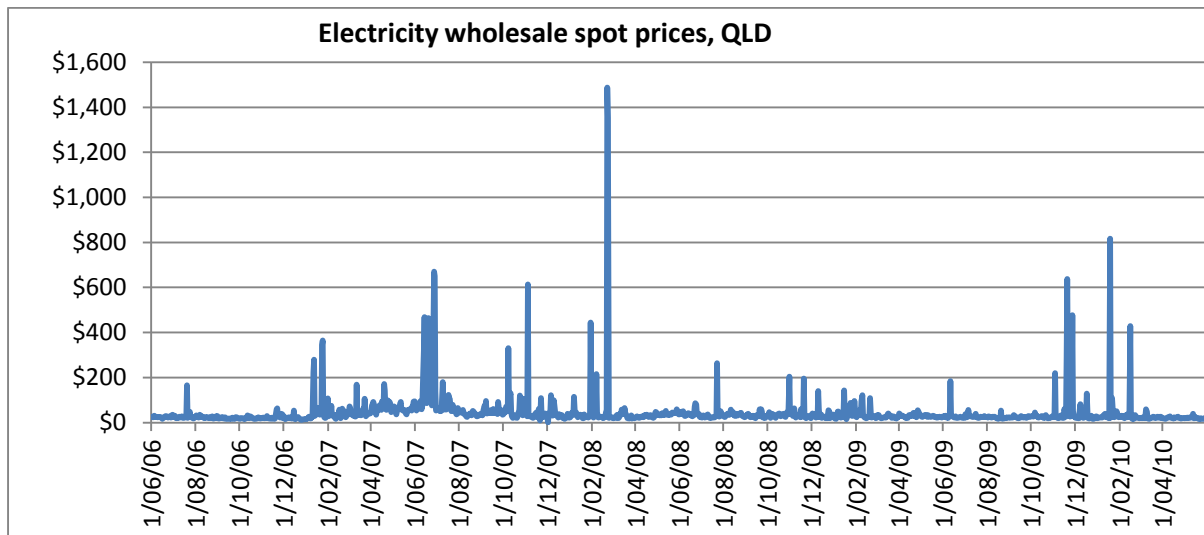


Source: Author's calculations. Data obtained from AEMO (July 2010).

Similar to prices in NSW, historical wholesale spot prices in VIC have large but short-lived price spikes, which tend to revert back to long-term mean levels very quickly. Generally, these spikes are observed over single days unlike the ones observed in NSW. Figure 6 also shows that the maximum daily prices reached in VIC are higher than the levels reached in NSW over the sample period.

Another fact that can be observed from this figure is that negative prices sometimes occur. The occurrence of negative prices in the NEM is uncommon but likely (Thomas et al, 2011). This is attributable to price bidding strategies of generators to get into the production schedule.

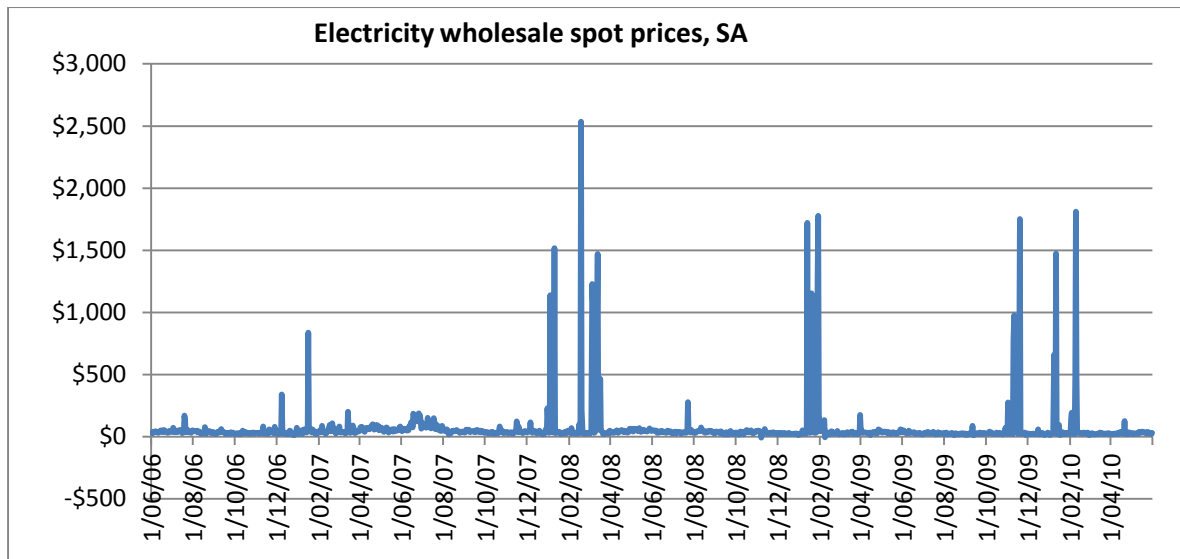
Figure 7 Daily Electricity Wholesale Prices in QLD



Source: Author's calculations. Data obtained from AEMO (July 2010).

Figure 7 shows the historical wholesale spot prices in QLD. As is seen, there are large but short-lived price spikes, which tend to revert back to long-term mean levels very quickly. Similar to the observed spikes in NSW, these spikes are observed over consecutive days leading to volatility clustering.

Figure 8 Daily Electricity Wholesale Prices in SA



Source: Author's calculations. Data obtained from AEMO (July 2010).

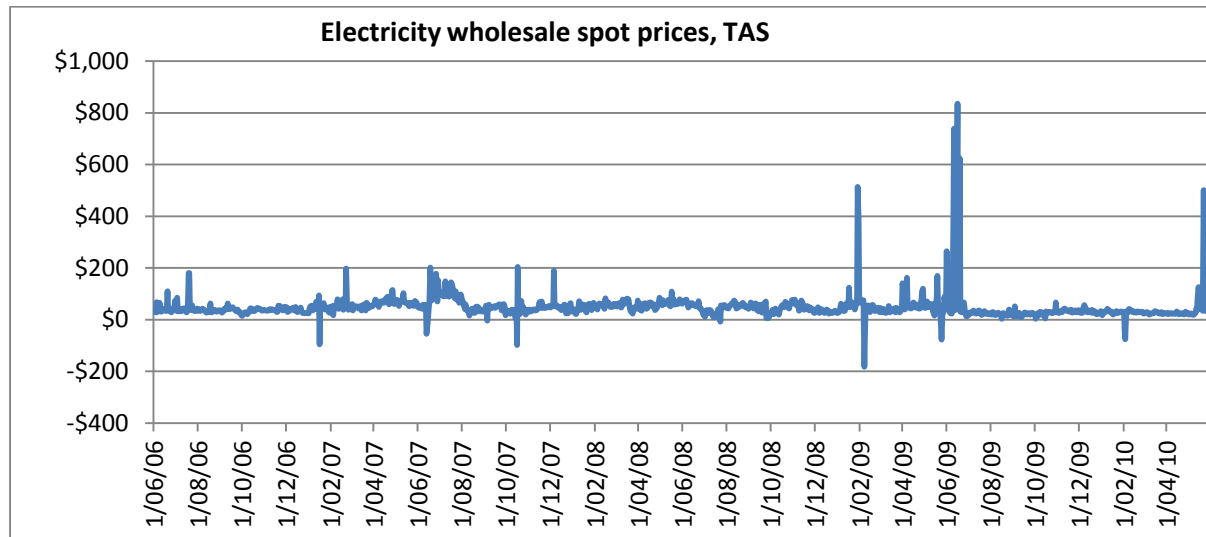
Figure 8 shows the historical wholesale spot prices in SA. As is seen, there are large but short-lived price spikes, which tend to revert back to long-term mean levels very quickly. Similarly to VIC, the average daily maximum price reached \$2,000 levels in SA over the sample period. This similarity maybe due to the weather characteristics of both regions, the capacity of the existing interconnectors and the fuel type used in generation process.

Finally, Figure 9 shows the historical wholesale spot prices in TAS, which joined the AEMO (joined in 2006) later than all other regions and therefore it is a relatively immature electricity region in NEM. As is seen, there are large but short-lived price spikes along with smaller jumps with large frequencies, which tend to revert back to long-term mean levels very quickly.

As is mentioned earlier, spikes are one of the main features of electricity prices in the NEM. According to the historical price series, which were examined in this thesis, the spikes function

as a result of a number of factors unique to each region such as weather and generation maintenance.

Figure 9 Daily Electricity Wholesale Prices in TAS



Source: Author's calculations. Data obtained from AEMO (July 2010).

DESCRIPTIVE STATISTICS

In fitting the data, log prices are used due to their convenient mathematical properties. Log specification is not defined in the presence of negative prices. This is not considered an issue as there are so few instances of negative prices in NEM. Splicing is used where log prices are not defined. For the period of time investigated in this thesis, there has been one observation of negative price occurrence in Victoria, 12 in Tasmania and two in South Australia.

This study uses average daily reported price values for each region, expressed in Australian dollars per megawatt hour (MWh) for each day. Although the highest frequency of electricity spot prices in NEM are quoted as half-hourly, daily average prices are of significant importance

to market players. The most common derivative instrument in electricity markets is Asian options, which is priced by average daily prices⁵.

Furthermore, using daily prices provides analytical tractability, commonly used in the literature. Therefore in this study, average daily prices are used rather than half-hourly price series. Most of the work on modelling electricity prices in the literature also used daily average data in order to retain simplicity and analytical tractability. This is performed despite the fact that trading in most markets is based on half-hourly intervals.

Even though use of average daily prices may lead to a loss of some important information (Ait-Sahalia et al., 2004), nevertheless as Weron et al. (2004) showed averaged time series retain the typical characteristics of electricity prices including seasonality, mean-reversion and jumps. Although this specification causes a loss of information at the left tail of the distribution, this loss is acceptable for the purpose of understanding and modelling positive spikes in electricity prices. As pointed out earlier, sudden and extremely high prices are the cause of higher electricity prices to end-users and modelling of this phenomenon is the main objective of this thesis.

The descriptive statistics by each region of the NEM (prices and log-prices) are presented in the tables that follow throughout this chapter.

⁵ Asian options are option contracts in which the payoff is linked to the average value of an underlying asset (wholesale spot electricity prices in NEM) during a defined duration. This is usually three months in electricity trading markets.

The right tailed skewness and high kurtosis indicate the fat-tailed characteristic of electricity prices in NEM. These descriptive statistics also indicate the non-Gaussian nature of the price series. Mean and median electricity prices are broadly consistent across NSW, QLD, and VIC where base generation technologies are similar and use relatively low-cost fuels. SA has the highest mean and second highest median prices per megawatt hour at \$57.94 and \$33.59 respectively. This is most likely attributable to the nature of the generation technology prevalent in this state and benefits of a direct interconnector with NSW are not available. NSW, QLD and VIC mainly rely on relatively low-cost brown and black coal fired generators for their base-load electricity needs. SA has a greater reliance on higher cost gas-turbine generators compared to low cost coal based generators in NSW, QLD and VIC.

The existence of maximum daily values of up to \$2,534 in SA where the mean price was only \$57.90, and \$2,376 in VIC with a mean price of \$45 indicates the spiky characteristics of the electricity price series in NEM.

Table 2 Summary Statistics of Daily Price Series in NEM

	NSW	VIC	QLD	SA	TAS
Mean	45.8	45.0	43.0	57.9	48.2
Median	30.6	32.2	27.7	33.6	40.7
Maximum	1,394.2	2,376.1	1,487.3	2,534.0	835.2
Minimum	13.8	-8.9	0.5	-6.9	-181.6
Standard Deviation	79.6	91.5	75.9	157.6	50.3
Skewness	10.3	17.4	11.3	9.5	8.6
Kurtosis	135.3	371.6	173.5	105.4	106.9

Source: Author's calculations.

Price increments exceeding 2.5 standard deviations of the mean are considered as spikes⁶ in this study. This filtering procedure for the in-sample data resulted in 45 spikes in NSW, 37 spikes in VIC, 34 spikes in SA, 40 spikes in QLD and 22 spikes in TAS region.

The analysis of the descriptive statistics demonstrates that the distributions of prices are significantly non-Gaussian in all regions of NEM, and this is consistent with the characteristics of electricity prices in other markets that have been analysed in the literature. The price series in all of the electricity regions are positively skewed and leptokurtic. These extreme fat-tailed characteristics are consistent with the findings of earlier studies (Huisman and Huurman 2003, Higgs and Worthington 2005, Thomas et al. 2011) and is likely to be driven by the occasional prevalence of extremely high prices.

STYLISTED FEATURES OF ELECTRICITY PRICES IN NEM

Based on the analysis of the historical spot electricity prices and their descriptive statistics presented above, this section outlines the stylised features of electricity prices in NEM.

Prices in the Australian National Electricity Market (NEM) are determined by the aggregate demand and supply functions. An unexpected event on either the supply side or demand side could either shift the supply curve to the left or the demand curve to the right, therefore causing

⁶ Literature also has other methods of identifying jumps. Jumps can be considered as price moves that are outside 90 per cent prediction intervals implied by Gaussian distribution (Borovkova and Permana, 2004) or the method applied by Geman and Roncoroni (2006) who filtered price data using different thresholds and choosing the one that leads to the best calibration of their model

a price jump. Unexpected events that either restrict supply or increase demand could cause a price jump.

Electricity prices in NEM exhibit a rapid mean-reversion process. This process occurs as the changes in either the supply or the demand curve normalise the deviations from the equilibrium levels. Mean reversion can be thought of as a modification of the random walk, where price changes are not completely independent of one another but rather are related in such a way that however much they escalate, they all always get back to long-run value known as the mean.

The jumpy and mean-reverting characteristics of electricity prices can also be explained by the microstructure elements of the electricity market. These elements are the diversity of plant technologies and fuel efficiencies at different levels of demand as different plants will be setting the market price at different market prices (fuel convergence)⁷ as explained earlier in the chapter on price formation in NEM.

While the nature of fuel convergence has a mean-reverting implication, the instantaneous production process of following a highly variable demand profile, creates significant volatility in prices. Other factors such as spikes in demand levels, system outages and congestions also contribute to the price volatility. All these elements contribute to characterisation of spot prices

⁷ The most efficient plants with lowest marginal costs (base-load) operate most of the time but during peaks in demand peak-plants operate only a few hours. The recovery of capital costs on peak-plants, through market prices, have to be achieved over a relatively few hours of operation. This will enable the construction of low capital/high operating plant for peaking purposes and the over-recovery of marginal costs in operation, with the consequence that prices are much higher in peaks.

and reflect the fundamental economic and technical nature of pricing electricity in Australia as a real-time, non-storable commodity.

As mentioned earlier, prices in NEM are sometimes negative, which is a feature not usually encountered in financial time series (Thomas et al., 2011). Generators may bid a negative price into the pool for its self-dispatch quantity as a tactical move to ensure that they are among the first to be called in to generate. The occurrence of a negative price may also be a function of the nature of technology used to generate electricity. In Victoria and New South Wales, “base load” generation capacity employs what is generally referred to as “slow-start” generation technology, this is the case with brown coal in Victoria or black coal in New South Wales fired generation plants (Thomas, 2007). The negative prices borne by these generators over the course of a day or two are generally offset with the sales revenues they generate over this time period.

The following tests verify the non-Gaussian characteristics observed in electricity price data in NEM. This finding is consistent with previous studies conducted on electricity price data elsewhere (Kaminski 1997, Deng 1998).

Normality tests

Jarque-Bera test

To test further whether the price series are Gaussian distributed, Jarque-Bera test are applied to each region of the NEM. Jarque-Bera test statistic is defined as;

$$JB = \frac{n}{6} (S^2 + \frac{1}{4} (K - 3)^2) \quad (1)$$

where n is the number of observations; S is the sample skewness and K is the sample kurtosis. The Jarque-Bera statistic asymptotically has a chi-squared distribution with two degrees of freedom, so the statistic can be used to test the hypothesis that the data are from a Gaussian distribution⁸. The null hypothesis is a joint hypothesis of the skewness being zero and the excess kurtosis being zero. As samples from a normal distribution have an expected skewness of 0 and an expected kurtosis of 3.

As the definition of Jarque-Bera shows, any deviation from this increases the Jarque-Bera statistic and the statistics are very high for each price series tested. The Jarque-Bera statistic rejects the null hypothesis of distributional normality at the 0.01 level for all regions of the NEM as illustrated in the following table.

Table 3 Jarque-Bera Normality test

	NSW	VIC	QLD	SA	TAS
Jarque-Bera statistic	1,091,628	1,799,886	660,382	674,524	8,342,082
Probability	0.000	0.000	0.000	0.000	0.000

Source: Author's calculations.

The results from Jarque-Bera test supports the idea that non-conventional modelling techniques that do not rely on the assumption of Gaussian distribution should be used to forecast electricity prices in NEM.

⁸ A non-parametric testing for normality (Kolmogorov–Smirnov) test further substantiates the results of Jarque-Bera test. See Appendix 1 for the results of Kolmogorov–Smirnov test.

Unit root and stationary tests

Knittell and Roberts (2001) conducted unit root tests in electricity prices in the markets of Argentina, Australia (Victoria), New Zealand (Hayward), NordPool (Scandinavia), Spain and US (PJM) and found no presence of unit root.

The findings of this thesis are consistent with the views of Knittel and Roberts (2001) by confirming the non-presence of unit root in Australian electricity prices. Two alternative unit root testing procedures were performed and the results are presented in Table 4. These tests were performed in an attempt to deal with the fact that the series may not be very informative about the existence or not of a unit root.

Table 4 Unit Root and Stationary Tests – Daily and Log-Daily Electricity Prices

	NSW	VIC	QLD	SA	TAS
DAILY ELECTRICITIY PRICES					
ADF t-statistic	-6.9	-17.5	-12.3	-10.4	-31.6
Probability	0.000	0.000	0.000	0.000	0.000
KPSS LM-statistic	0.2	0.5	0.1	0.2	0.3
Probability	0.000	0.000	0.000	0.000	0.000
DAILY ELECTRICITIY LOG-PRICES					
ADF t-statistic	-5.1	-6.5	-8.5	-6.1	-6.1
Probability	0.000	0.000	0.000	0.000	0.000
KPSS LM-statistic	-23	-25.9	-25.2	-21.3	-19.6
Probability	0.000	0.000	0.000	0.000	0.000

Source: Author's calculations.

The ADF test tests the null hypothesis that a time series y_t is $I(1)$ against the alternative that it is $I(0)$ following Knittell and Roberts (2001) assuming that the dynamics in the data have an ARMA structure. The ADF test is based on estimating the test regression;

$$y_t = \beta' D_t + \phi y_{t-1} + \sum_{j=1}^p \Delta y_{t-j} + \varepsilon_t \quad (2)$$

where D_t is a vector of deterministic terms. The p lagged difference terms, Δy_{t-j} , are used to approximate the ARMA structure of the errors, and the value of p is set allowing the error ε_t to be serially uncorrelated. The specification of the deterministic terms depends on the assumed behaviour of y_t under the alternative hypothesis of trend stationarity. Under the null hypothesis, y_t is $I(1)$ which implies that $\phi = 1$.

The ADF t-statistic and normalised bias statistic are based on the least squares estimates and given by;

$$ADF_t = t_{\phi-1} = \frac{\hat{\phi}-1}{SE(\hat{\phi})} \quad (3)$$

$$ADF_n = \frac{T(\hat{\phi}-1)}{1-v_1-\dots-v_p} \quad (4)$$

The Phillip-Perron (PP) unit root test differs from ADF tests in their handling of serial correlation and heteroskedasticity in the errors. In particular, where the ADF tests use a parametric autoregression to approximate the ARMA structure of the errors in the test regression, the PP tests ignore any serial correlation in the test regression. The test regression for the PP tests is;

$$\Delta y_t = \beta' D_t + \pi y_{t-1} + u_t \quad (5)$$

where u_t is $I(0)$ and may be heteroskedastic. The PP tests correct for any serial correlation and heteroskedasticity in the errors u_t of the test regression by directly modifying the test statistics $t_{\pi=0}$ and T_π .

The ADF and PP unit root tests are for the null hypothesis that a time series y_t is $I(1)$. KPSS tests on the other hand for the null that y_t is $I(0)$. KPSS test statistic is derived by the following models;

$$y_t = \beta' D_t + \mu_t + \varepsilon_t, \text{ where } \mu_t = \mu_{t-1} + \varepsilon_t, \varepsilon_t \sim WN(0, \sigma_\varepsilon^2) \quad (6)$$

where D_t contains deterministic components, u_t is $I(0)$ and maybe heteroskedastic. The null hypothesis that y_t is $I(0)$ is formulated as $H_0: \sigma_\varepsilon^2 = 0$ implying that μ_t is a constant.

The KPSS test statistic is then the Lagrange multiplier for testing $\sigma_\varepsilon^2 = 0$ against the alternative that $\sigma_\varepsilon^2 > 0$ and is given by;

$$KPSS = (T^{-2} \sum_{t=1}^T S_t^2) / \lambda^2 \quad (7)$$

where $S_t = \sum_{j=1}^t u_j$, u_t is the residual of a regression of y_t on D_t and λ^2 is a constant estimate of the long-run variance of u_t . Under the null that y_t is $I(0)$, KPSS converges to a function of standard Brownian motion that depends on the form of the deterministic terms D_t but not their coefficient values β .

Testing of each log-price series for the presence of unit root using ADF, PP and KPSS tests is presented in this section. The respective ADF t-statistics are found to be -5.1 for NSW, -6.4 for QLD, -8.4 for SA, -6.0 for TAS and -6.1 for VIC. These tests rejected the null hypothesis of a

unit root at 0.01 level of significance, which is consistent with the earlier findings of Knittel and Roberts (2001) and Goto and Karolyi (2004).

The results of these tests indicate that the log-spot prices are stationary. An important implication of this finding is that the application of forecasting methods on electricity price forecast based on autoregressive (AR) processes and models based on Stochastic Differential Equation (SDE) family processes are appropriate in modelling electricity prices in NEM.

Alternative tests for efficiency

The implication of inefficiency for the markets is quite significant as this creates arbitrage and speculation opportunities in trading and further necessitates the accurate modelling of the electricity prices. Serletis and Bianchi (2007) and Urtskaya and Serletis (2008) investigated the efficiency of electricity markets in the literature and found that the markets are highly inefficient. By applying Wald-Wolfowitz Runs Test tests on daily log-prices, this thesis provided evidence that the electricity prices in Australia are also inefficient.

Wald-Wolfowitz Runs test assesses if the number of 'runs' in an ordering is random or not. This test assumes that the variable under consideration is continuous, and that it was measured on at least an ordinal scale (i.e., rank order). The Wald-Wolfowitz Runs test assesses the hypothesis that two independent samples were drawn from two populations that differ in some respect, i.e., not just with respect to the mean, but also with respect to the general shape of the distribution.

The null hypothesis of the Wald-Wolfowitz Runs test is that the two samples were drawn from the same population. In this respect, this test is different from the parametric *t*-test which strictly

tests for differences in locations (means) of two samples. Under the hypothesis test, if there are too few runs relative to the Gaussian mean and standard deviation, the z-value is small and it implies that the series is having a trend. If however there are a large number of runs, the z-value is large and the series contains many ups and downs.

Table 5 demonstrates for Wald-Wolfowitz Runs test z-scores for each region of the NEM both median and mean as cut points, respectively. All regions have significant negative z-scores, indicating that there exists non-randomness with large numbers of deviations from the mean. This further strengthens the importance of modelling electricity prices in Australia from a risk management point of view as in inefficient markets; players can make a difference with advanced models in managing their risks and exploring profit taking opportunities.

Table 5 Runs Test (Mean and Median as Cut Points)

	NSW	VIC	QLD	SA	TAS
MEAN AS CUT POINTS					
Z value	-24.70	-25	-24.90	-23.40	-27.30
Significance (2-tailed)	0.000	0.000	0.000	0.000	0.000
MEDIAN AS CUT POINTS					
Z value	-25.30	-24.80	-24.30	-24.20	-27.40
Significance (2-tailed)	0.000	0.000	0.000	0.000	0.000

Source: Author's calculations.

CONCLUSION

This chapter presented the times-series dynamics of the data from NEM. Electricity price series in NEM are found to exhibit extreme price spikes, fast mean-reversion, fat-tails and highly volatile. Occurrences of negative prices are found to occur from time to time. The formal

normality and unit root tests confirmed the non-Gaussian and stationary⁹ nature of the price series.

Additionally, Wald-Wolfowitz Runs test indicated the importance of modelling electricity prices in Australia in an attempt to reduce risk management costs. One appreciates that in inefficient markets, market players can make a difference with advanced models in managing their risks and exploring profit taking opportunities. In other words, existence of dependence in the prices series is likely to result in forecast models to generate accurate forecasts. The findings of this chapter points to the fact that there are features in the data if modelled results in accurate forecasts than AR-GARCH, which do not capture any other features except basic form of dependency.

An important implication of the findings described in this chapter is that the application of forecasting methods on electricity prices based on SDE and non-Gaussian techniques are appropriate. This is due to the fact that these models first of all reflect the stylised features of the electricity prices, namely the mean-reverting, jumpy and highly volatile nature of the price series. Secondly, non-presence of unit roots in the data indicates that these stochastic models are a good fit in modelling the price series. Thirdly, non-random nature of the price series as evidenced by Runs tests' results suggest players in NEM to perhaps make a difference with advanced models in managing their risks and exploring profit taking opportunities.

⁹ The stationary nature of the electricity price series allow modelling of the series without differentiating it. As Lutkepohl (2005) and Enders (2004) proposed that differentiating the series distort interesting features of the relationships between the original variables, such as the co-movements between the data or the possible co-integration relationships.

Next chapter will review the literature in electricity price modelling. The chapter will begin describing the developments in electricity price modelling with autoregressive representations. It will then describe the models based on SDEs. Modelling electricity prices with SDEs reflect the stylised features of electricity prices and there is a large literature on this area.

The literature review of SDE models covers Mean-Reverting, Mean-Reverting and Jump-Diffusion models. The application of Markov Regime-Switching models to electricity price modelling is also described in this chapter. Furthermore, incorporation of ARCH and GARCH models in electricity price modelling is also reviewed in the next chapter followed by non-linear approaches to electricity price modelling such as application of Extreme Value Theory (EVT) and Levy Jump-Diffusions. The chapter will conclude by an examination of a number of pricing models applied to data from NEM.

CHAPTER 4 – LITERATURE REVIEW

INTRODUCTION

In this chapter, a review of the literature relevant to this thesis is presented in a chronological order. As the data for electricity spot prices across major deregulated electricity markets are publicly available for researchers, a wide array of research in electricity price modelling has been developed over the past two decades.

The existing body of literature includes different aims and methodologies depending on the time period being studied. This thesis focuses on the stylised features of electricity prices and its forecasting in the short run. Structural methods of analysis, which include exogenous factors such as weather and demand, are not considered in this thesis. More specifically, this thesis is concerned with the stochastic modelling that is found to characterise the stylised features of electricity prices by early researchers in the area (Kaminski 1997, Johnson and Barz 1999).

Electricity spot prices are amongst the most volatile commodity prices in the world. They have the characteristics of non-storability, limited transportability and restricted arbitrage transactions. Thus, the basic nature of the electricity time-series is dissimilar to the traditional stock prices. Kaminski (1997) showed a number of characteristics of the electricity time-series including extreme behaviour with fast-reverting spikes, multi-scale seasonality, calendar effects, and non-Gaussian manifested as positive skewness and leptokurtosis.

Due to the fact that electricity cannot be stored, it requires a market design that achieves the best for both end-users and generators. Electricity transmission requires a synchronised energy balance between the injection of power at generators and the off-take at demand points. Across the electricity network, production and consumption are synchronised and storage is not

possible. If the two get out of balance, the frequency and voltage of the power result in fluctuations due to very low own price elasticity of demand.

Therefore, the task of the Australian Electricity Market Operator (AEMO) is to monitor continuously the demand process and to call on generators that have the capacity to respond quickly to fluctuations in demand. Generators have various technologies and fuel efficiencies hence at different levels of demand; different generators will be setting the market clearing prices.

There are two main reasons for this diversity in the system. Power plant obsolescence is one of them. As new power plants with advanced technologies come in to the production curve; prices fluctuate due to the varying efficiencies of the set plants. Secondly, base-load generators (low marginal costs) at times of high demand (during summer, winter months or weekdays as compared to weekends¹⁰), are not sufficient to meet the demand therefore generators with high marginal costs gets scheduled into production. This progression results in increasing electricity prices on the market.

The recovery of capital costs on peak operations via market prices need to be achieved in these few hours. This will favour both the construction of low-capital/high operating cost plant for peaking purposes. At the same time as more expensive generators enter the production, mean-reversion in prices occurs, bringing the electricity prices back to a level of equilibrium.

¹⁰ Seasonal factors in electricity price modelling have been included in studies by Knittell and Roberts (2001), Lucia and Schwartz (2002), Escibano et al. (2002), Guthrie and Videbeck (2007), Hadsell et al. (2004), Higgs and Worthington (2005), and Thomas et al. (2011).

Whilst the fundamental nature of price convergence has a mean-reverting implication, the instantaneous production process of following a highly variable demand profile, with a diversity of plant costs, creates the high spot price volatility. Other factors come in to play over the short term such as technical failures with power plants, congestion in the transmission system and sudden fluctuations in demand. All of these reflect in spot prices and reflect the fundamental economic nature of pricing electricity as a real-time, non-storable, commodity.

This thesis focuses on models that reflect the stylised features of the electricity prices namely its mean-reverting, jumpy and fat-tailed characteristics, addressed by autoregressive and stochastic models in addition to its high volatility, which is addressed by volatility models.

In this chapter, contributions to the literature are presented in a chronological order. The next section of this chapter presents the literature on autoregressive models while the next section focuses on jumps and regime switching models. Section 3 presents volatility models used in electricity price modelling and section 4 focuses on the emerging developments in electricity price modelling. Finally, Section 5 looks at emerging Australian literature and discusses opportunities for research emerging from the literature and the particular foci of this thesis.

AUTOREGRESSIVE MODELS

Autoregressive models (AR) and its variants are a standard modelling technique applied often in time series econometrics. In the context of electricity prices these models are widely employed. AR models traditionally are used to predict behaviour of electricity time-series from past values then such a prediction is used as a baseline to evaluate the possible importance of other variables to the system. AR modelling also contributes to the understanding of the physical system as it reveals the persistence of the physical process.

The literature has a wide array of applications of autoregressive models to exploring the price dynamics of electricity. Most of the autoregressive models aimed at explaining the stylised features of the electricity and sought to find the best fit to the empirical data. It is important to note here that a significant number of autoregressive models in the literature filtered seasonality present in electricity price data.

One of the earlier applications of AR models was performed by Knittel and Roberts (2001) who modelled electricity prices from the Californian market with a seasonal ARMA¹¹ and AR-EGARCH¹² processes. They also used a seasonal ARMA model with temperature and squared/cubed temperatures being as exogenous variables. Furthermore, Knittel and Roberts (2001) included variables to account for seasonalities and the different structure of price behaviour in peak and off-peak prices (peak and off-peak mean).

They concluded that the forecast results of their models underlined the importance of incorporating higher order autocorrelation in modelling electricity prices. They also found evidence of an inverse leverage effect as the asymmetry parameter of their AR-EGARCH process was positive and significant indicating that positive shocks to prices amplify the conditional variance of the process more so than negative shocks.

Comparison of a number of methods in modelling electricity prices from different markets was performed by Escribano et al. (2002) who estimated six different models in order to measure the relative contribution of the stylised features of electricity prices by using daily prices from

¹¹ Autoregressive–moving-average (ARMA) models provide a parsimonious description of a (weakly) stationary stochastic process in terms of two polynomials, one for the auto-regression and the second for the moving average.

¹² Generalised Autoregressive conditional heteroskedasticity (GARCH) models are used to characterize and model observed time series. They are used whenever there is reason to believe that, at any point in a series, the error terms will have a characteristic size or variance. In the EGARCH model, the conditional variance is an asymmetric function of lagged disturbances.

the electricity markets of Argentina, Australia (Victoria), New Zealand (Hayward node), NordPool, U.S. and Spain. The stylised features included were; seasonality, mean-reversion, non-constant volatility and jumps. The models considered in this study were; an AR(1) constant volatility model with no jumps (pure diffusion model); an AR(1) GARCH(1,1) model with no jumps; an AR(1) pure jump model; an AR(1), pure jump model, intensity of the Poisson process time dependent; an AR(1) GARCH(1,1) model with jumps, intensity of Poisson process constant, and finally an AR(1) GARCH(1,1) model with jumps, intensity of the Poisson process time dependent.

As in Knittel and Roberts (2001), a sinusoidal function was included in all of these processes to capture the seasonality present in the price series in each market. This study showed that electricity prices are mean-reverting and have strong volatility and jumps of time-dependent intensity even after adjusting for seasonality. This study also provided a detailed unit root analysis of electricity prices against mean reversion, jumps and GARCH errors, and proposed a new unit root procedure based on bootstrap techniques, all pointing to the stationary nature of electricity prices.

The unit root tests of Escribano et al. (2002) at $I(1)$ and $I(0)$ using Phillips-Perron and KPSS tests suggests that neither an $I(1)$ nor $I(0)$ description of the electricity price series is appropriate to capture the long-memory and mean-reversion of electricity prices as suggested by a number of researchers in the field (Atkins and Chen 2002, Haldrup and Nielsen 2006).

An alternative way of measuring long memory and mean reversion was suggested by estimation of fractionally integrated processes for the price series (Granger and Joyeux 1980, Beran 1994, Baillie 1996). It was argued that both the Phillips-Perron and the KPSS tests are

consistent against fractional alternatives if the fractional order is less than unity as shown by Lee and Shie (2004).

Electricity prices from the Alberta market were modelled by Atkins and Chen (2002) who used autoregressive fractionally integrated moving average (ARFIMA) processes to capture the long-memory feature of electricity prices. Their approach in modelling electricity prices with an ARFIMA process also aimed at capturing the irregular behaviour of the prices better than the GARCH specifications as applied by Knittel and Roberts (2001). To estimate the parameters of the ARFIMA model, authors utilised Geweke and Porter-Hudak's two step spectral regression method and approximate maximum likelihood methods¹³. Their findings indicated that ARFIMA process allows representation of dynamics in the stochastic behaviour of the electricity series.

The role of adding GARCH processes in AR models in electricity price forecasts was shown by Contreras et al. (2003) who originally applied ARMA processes with multiple seasonalities and lags to predict hourly prices in the electricity markets of Spain and California. They later improved their ARMA forecast model with the addition of GARCH (1, 3) specification and concluded that the forecast errors were around nine per cent, depending on the studied month of the year in these markets. They also argued that this model outperformed their earlier ARMA model and pointed out that adding the demand to the GARCH model as an explanatory variable

¹³ Geweke and Porter-Hudak (1983) proposed a semi-parametric procedure to obtain an estimate of the fractional differencing estimator based on a linear regression of the log periodogram on trigonometric function. In contrast, Whittle (1953) proposed a frequency domain approximate maximum likelihood method to simultaneously estimate both the short and long memory parameters of an ARFIMA model based on the minimising the approximation of the logarithm of the spectral likelihood function with respect to its parameters.

improved the performance of the method further. The importance of this study was that it strengthened the case to add GARCH processes in forecasting electricity prices.

Carnero et al. (2003) extended the ARFIMA model of Atkins and Chen (2002) by modelling different periodic extensions of regression models with ARFIMA processes for electricity prices from NordPool, Germany, France and Netherlands markets. They proposed an RegARIMA (1,0,0), seasonal ARIMA (2,0,0)×(1,0,0), periodic RegARIMA(2,0,0) and periodic seasonal ARFIMA (0,d,0) for modelling spot prices and showed that day-of-the-week periodicity and long memory are important determinants for the dynamic modelling of the conditional mean of electricity prices. This RegARIMA model is defined as a multiple regression model with ARIMA disturbances. In these models, the standard explanatory variables were polynomial, trigonometric and other periodic functions of time.

Modelling electricity prices with ARFIMA processes was further advanced with the work of Haldrup and Nielsen (2006) who estimated the fractional order of integration by specifying a multiplicative seasonal ARFIMA (SARFIMA) model where a lag polynomial order captured the within-the-day effects in NordPool. They later extended their earlier SARFIMA model to a state regime switching multiplicative SARFIMA (RS-SARFIMA) where there is an 8th order lag polynomial in the regimes determined by a Markov chain. The distinct feature of their model that differed from earlier class of regime switching models, where the Markov process generating the states is unobserved, was that all states were observable.

The literature has also applications of AR processes that treat each hour of the day in electricity price series as a separate commodity. The rationale behind modelling each hour of the price series as separate commodities is mainly the demand structure of electricity. It is thought that

each hour's demand for electricity is determined by factors specific to that hour of the day such as its temperature and consumption by different user groups.

Cuaresma et al. (2004) applied a number of AR specifications (including time varying intercepts and jumps) to forecast electricity prices in the German power market including specifications where each hour of the day was modelled separately. Their AR and MA orders included lags of 1, 23, 24 and 25 hours. They concluded that specifications where each hour of the day was modelled separately presented better forecasting properties than other AR specifications. Further, it was found that the inclusion of probabilistic processes for the arrival of jumps provides better fit to empirical data and enhanced forecast performance.

Coneja et al. (2005) proposed forecasting electricity prices from Spain with Wavelet-ARIMA technique. Their study applied a level three decomposition to hourly prices and modelled the resulting data with ARMA processes to obtain 24 hourly predicted values. This study showed that the performance of Wavelet-ARMA technique is better than of a standard ARMA process in short term forecasting.

Literature in electricity price modelling has innovative AR specifications that were found to be suitable for mimicking electricity price characteristics like sudden jumps and time varying mean reversion as well.

One of these specifications was the threshold autoregressive (TAR) model of Rambharat et al. (2005) who allowed for different rates of mean-reversion in their TAR (1) model and applied it to daily electricity prices from U.S. PJM markets. The mean reversion rates of their study were set as one for weather events, one around price jumps and another for the remainder of

the process. This model allowed both the speed of mean- reversion and the jump arrival intensity to depend on the level of the price process relative to a threshold price level. The model's AR parameter was allowed to take two values; firstly when the de-measured log price is below the threshold level, and then when it is above the threshold.

Periodic autoregressive (PAR) models are similar to AR models except that the AR coefficients take different values in different trading periods. In this aspect, the similarities of PAR models with AR models is that they treat each hour of the day as a separate commodity. The literature has a number of modelling approaches with periodic autoregressive GARCH (PAR-GARCH) and PAR-ARFIMA-GARCH models.

An earlier application of PAR-GARCH process was by Bosco et al. (2006) who used periodic ARMA models with GARCH disturbances in the Italian electricity market and compared the model's performance with more classical ARMA-GARCH processes. They built a model to account for within-year seasonal component through the low frequencies components of physical quantities. Their study revealed that much of the variability of the price series is explained by deterministic seasonalities which tend to interact with each other. Consequently, Guthrie and Videbeck (2007) applied four variants of PAR to de-seasonalised prices from electricity prices from the New Zealand market. These variants were;

1. Price in a particular trading period is regressed on the price in the same trading period on the previous day,
2. Price is regressed on the price in the previous trading period,
3. Price is regressed on the previous day's price in the same trading period and price in the previous trading period,
4. Price is regressed on all 48 prices recorded in the previous 24 hours.

They found that these models provided evidence that the intra-day dynamics are richer than can be captured by standard AR models. For instance, while a simple AR (1) process captures the low persistence evident in peak periods, it cannot simultaneously capture the greater persistence in off-peak periods, nor the fact that shocks reappear the following day.

Earlier models of PAR-GARCH with fractionally integrated moving average (ARFIMA) models were developed by Carnero et al. (2007) in four electricity markets of Europe. This study's Reg-ARFIMA-GARCH model explained the dynamics in the conditional mean and variance of electricity prices. The model specification included exogenous variables of water reservoir and demand levels and treated seasonalities by means of sinusoids and weekday dummies.

This Reg-ARFIMA-GARCH specification modelled the day-of-the-week periodic autocovariance for short run dynamics by lagged dependent variables and for long run dynamics by seasonal ARFIMA models where regressors captured yearly cycles, holiday effects and possible interventions in mean and variance. This model also included a GARCH component, which handled the volatility clustering and extreme observations present in electricity prices. One of the important findings of this study was that heteroskedasticity of prices can only be correctly represented when the conditional mean of the time-series is modelled by means of periodic autoregressive processes.

Weron and Misiorek (2008) later compared the forecast accuracy of 12 different time-series models in the California and NordPool markets. They used AR and threshold autoregressive (TAR) models with and without exogenous variables. The basic AR models only had system load as an exogenous variable in the case of California and hourly air temperature in the case

of NordPool. The AR structures also had dummy variables to account for the weekly seasonality and lagged variables. Additionally, a variable to link between bidding and price signals from the previous day and log-load forecast (California market) or actual temperature (NordPool) was included in their AR specifications.

To account for consecutive spikes in electricity prices, Weron and Misiorek (2008) used TAR models of Tong and Lim (1980) to describe the governance of regime switching between two AR processes by the value of an observable threshold variable relative to a chosen threshold level. In their TAR models, they used the same dummies and additional variables as in the simple AR models. Following Ball and Torous (1983), they modelled a mean-reverting jump diffusion model as an AR process with the addition of the same variables in their AR and TAR models.

The study of Weron and Misiorek (2008) transformed the basic AR models to semi-parametric construction. The foremost motivation for this extension stemmed from the fact that a nonparametric kernel density estimator yields better fit to empirical data than any parametric distribution. Their study used Hsieh-Manski's estimator and smoothed nonparametric maximum likelihood estimator for calibrating AR models mentioned above for semi-parametric modelling. Their study's main conclusion was that the models with the system load as the exogenous variable performed better in point forecast. Also, semi-parametric models were found to perform better than their Gaussian competitors.

A unique extension of traditional AR models in electricity price modelling was the application of Poisson autoregressive models developed by Christensen et al. (2009). Authors made it clear that the intensity of the spike process is significantly related to the historical component of the data and that this persistence need to be accounted for in electricity price modelling. They also

modelled electricity prices from Australian electricity market with zero-inflated Poisson autoregressive model, which captured the persistence in spikes and hence provided a better fit to the electricity price data. The rationale behind the use of Poisson autoregressive model was outlined earlier by Geman and Roncorni (2006). They stated that spikes occur as a result of a number of unobserved strains happening on the grid system.

K-factor generally integrated GARCH (GIGARCH) models were first introduced to the literature by Guegan (2003). Later, Dominique and Ka Diongue (2009) used a GIGARCH model to forecast one-month ahead log-hourly electricity spot prices from the German power market. They estimated three models in their comparative study:

1. One-factor GIGARCH process estimated after removing the weekly seasonality,
2. SARIMA model of Box and Jenkins (1976) with conditional heteroskedastic noise, and
3. A three-factor GIGARCH model, which takes into account a number of stylized facts observed in electricity spot prices, in particular stochastic volatility, long memory and periodic behaviours.

One of the main findings of this study was supportive of long-range dependence behaviour of electricity prices. Secondly, it also pointed to the fact that three-factor GIGARCH process provides better forecasts than models of seasonal ARMA-GARCH.

In summary, it is observed that modelling electricity prices with variants of AR processes aims to capture the stylized features of electricity prices. These features are rapid mean-reversion to long-term mean levels and occasional and sometimes consecutive jumps in prices. The inclusion of GARCH terms are proven to be effective in capturing the time-varying volatilities

present in electricity prices. Major learning that can be deducted from the established literature presented in this section is that the stylized features of electricity prices are of particular interest in modelling electricity prices and they can be captured by complex model construction where variants of AR and GARCH processes are included. As the AR-GARCH model is taken as the benchmark model in this thesis, competing models of varying stochastic differential equations are expected to mimic the features of rapid mean-reversion to long-term mean levels and occasional and sometimes consecutive jumps in prices at least as good as AR processes.

MEAN-REVERTING/JUMP-DIFFUSION AND REGIME-SWITCHING MODELS

Most of the earlier work in electricity price modelling explored the mean-reverting and jumpy characteristics of the price series in a number of markets including the Australian National Electricity Market (NEM). These earlier models were inspired by techniques used in modelling financial markets after the seminal work of Merton (1976). One of the earliest models in this arena was the extension of general random walk model of random-walk jump diffusion model of Kaminski (1997). This model captured the randomness of the prices and their spiky character via an application of a Stochastic Differential Equation (SDE).

SDEs characterize the behaviour of a continuous-time stochastic process as the sum of an ordinary Lebesgue integral and an Itô integral. A simplistic interpretation of the SDE is that in a small time interval of length, the stochastic process changes its value by an amount that is normally distributed with expectation and variance and is independent of the past behaviour of the process. This is the case because the increments of a Wiener process are independent and normally distributed. A typical SDE has two functions; a drift coefficient and a diffusion coefficient (diffusion process) and is usually a Markov process. Although the study by

Kaminski (1997) captured the jumpy characteristics of the price series, it failed to model the mean-reverting behaviour of these series.

A wave of researchers (i.e. Deng (1998) and Knittel and Roberts (2001)) modelled electricity prices after Kaminski who accounted for the mean-reverting characteristics of the prices and since then the literature on mean-reverting jump-diffusion models¹⁴ have been the focus of a wide range of studies. These studies can be classified into three approaches;

- A single jump, positive or negative followed by a mean reverting pattern,
- An initial jump, positive or negative, followed by a reverse-directed jump on the next time period and
- A cluster of jumps in a short time period which can be positive, negative, or mixed (consecutive jumps behaviour).

The initial approaches to modelling spikes utilised Poisson processes to introduce jumps to normal price dynamics. They combined either the usual Geometric Brownian Motion (GBM) or the Mean-Reverting process for the diffusion and a space-time Poisson process for the jumps such that the jump amplitudes are uniformly distributed. Deng (1998) modelled three variations of Mean-Reverting equations;

- A Mean-Reverting deterministic volatility process with two types of jumps, (one representing upward and one representing downward jump with Poisson process to model the jumps and jump intensity,

¹⁴ Most of the earlier work in modelling electricity as mean-reverting, jump diffusion and regime switching processes involved decomposing the data into a deterministic and stochastic components. Then modelling stochastic component with mean-reverting jump diffusion processes separately from the deterministic component was performed. This decomposition provided data that is easier to parameterise.

- A Markov Regime-Switching Mean-Reverting process with two types of jumps in which capturing the phenomena of spot prices switching between spike and base states was possible, and
- A Mean-Reverting stochastic volatility process with two types of jumps where the modelling of the spiky behaviour was performed by assuming that the intensity function of upward jumps is only a function of time whilst the intensity of downward type jumps is a function of the volatility.

In an attempt to describe the evolution of electricity prices from markets across California, Scandinavia, England and Wales, and Victoria (Australia), Johnson and Barz (1999) evaluated the effectiveness of four different SDE models with and without jumps. These model specifications were;

- (1) Brownian Motion,
- (2) Mean-Reversion,
- (3) Geometric Brownian motion (GBM), and
- (4) Geometric Mean Reversion.

They concluded that the Geometric Mean-Reverting model gave the best performance, and that adding jumps to each of the models improved model performance. However, it is important to note that the mathematical properties of the models utilised in particularly GBM models are unsuitable for electricity price processes as future values of price depend only on the current price, with no relationship to a long-run mean value. Also, the variance of GBM increases linearly with time, whereas electricity prices exhibit mean reversion and hence bounded variance.

Pilipovic (1998) developed a two factor Mean-Reverting model quite different from Deng's (1998) Mean-Reverting models. This model's observable factor was the price levels rather than logarithmic prices. It is noteworthy to state that most of the early work prior to Deng on stochastic modelling employed logarithmic transformations.

At the same time as the model of Pilipovic (1998), Ethier and Mount (1998) proposed a two-regime specification in which both regimes are governed by an AR(1) process varied between the regimes in modelling daily on-peak price data from the electricity market in Victoria (Australia) and three hubs in the United States (ECAR, PJM East, and SERC). This model allowed two states which meant that electricity prices can jump discontinuously between states with different state probabilities. The model parameters were estimated by the recursive filter of Hamilton (1989)¹⁵. However, the model specification of this approach imposed stationarity in the spike process, which was not appropriate for electricity prices (de Jong and Huisman, 2002).

Examining the distributional characteristics of the electricity prices with SDEs continued with the work of Knittel and Roberts (2001) with an application of a Mean-Reverting process in California market. They used an AR model as it is in continuous time being equal to the Ornstein–Uhlenbeck process Mean-Reverting process. This Mean-Reverting process captured the autocorrelations present in the price series. Though, this study ignored seasonalities present in the data (intraday, weekend/weekday and long-term seasonality) and assumed that the error

¹⁵ The Hamilton model allows stochastic jumps between regimes, where each regime is a mean reverting AR(1) process with unique mean and variance. Thus electricity prices are viewed as originating from either a high state or a low state. Regime switching is controlled by a two state Markov process, with state specific transition probabilities which allow different expected durations for each state (Ethier and Mount, 1998).

structure is independent across time. Correspondingly, its assumptions that the volatility is constant over time were a shortcoming of the model. Lastly, it was found that the normality assumption of the AR structure did not reproduce the extreme swings found in the data. Further extensions of the original model to forecast unique nature of the prices with time-varying Mean-Reverting model that included intraday, weekend and long-run seasonal effects improved the simple Mean-Reverting model but did not produce a good fit to empirical data.

The relationship between electricity spot and futures prices was examined by Lucia and Schwartz (2002) who modelled prices with one and two factor mean-reversion processes in NordPool. They used a sinusoidal in order capture the seasonal pattern of the futures and forward curve directly implied by the seasonal behaviour of spot electricity prices. Their one factor model had a deterministic function and a diffusion stochastic process represented by the Wiener process. Consequently, their two factor model extended the one-factor model by adding a second stochastic factor in the spirit of Schwartz and Smith (2000)¹⁶. They concluded that seasonal patterns play an important role in determination of spot and futures prices and their functional determination.

Extension of the modelling framework of Lucia and Schwartz (2002) by modelling daily electricity prices from Dutch power markets via a Mean-Reverting model were performed by Huisman and Mahieu (2003). This model had a deterministic component where the effects of

¹⁶ Schwartz and Smith (2000) modelled the stochastic behaviour of oil prices as having a short-term mean reverting component and a long-term equilibrium price level. An important implication of adding a second factor in the model is that changes in prices of futures contracts with different maturities are not perfectly correlated, as is the case for all one-factor models.

seasonality is captured (weekend effects) and in order to capture the stochastic component modelling by a Wiener process is included.

Later, Lucia and Schwartz (2002) used the residuals of their earlier model and applied Extreme Value Theory to assess the level of tail fatness in price series followed by comparison of the Monte Carlo simulation results based on Gaussian and Student-t distributions. Their simulation results improved upon the ones from the Gaussian distribution, as the Student-t price patterns resemble more closely the true price pattern of electricity prices. The most significant contribution of this study was that the normality assumption that researchers and practitioners often make in their simulation or valuation method was not appropriate and prone to lead to erroneous conclusions.

The study of Huisman and Mahieu (2003) was not the only work performed on Dutch electricity prices. Huisman and de Jong (2003) used a Markov Regime-Switching model switching between a mean-reversion and a pure jump regime. Their model was a two state model assuming a mean-reverting and a spike regime with the base regime being governed by AR(1) and a spike regime governed by a Gaussian distribution. This specification allowed regime independence and accommodated consecutive regimes of spike and base. They found that their model typically overstated the prices in the base regime and predicted base regime with predicted occurrences of spikes much less than the actual rate of occurrences.

The modelling of Mean-Reverting and Jump-Diffusion processes where positive jumps are always followed by a negative jump of about the same magnitude was performed by Bierbrauer et al (2003). They achieved this by letting the stochastic part to be independent of the jump component in their Mean-Reverting and Jump-Diffusion specification. This study defined that

the magnitude of jumps as a Log-Gaussian random variable and the probability of jumps as a Poisson random variable. The jump components of this model were estimated with a two-step procedure. Firstly, all jumps, defined as price increments exceeding three standard deviations of all price changes, are removed from the data and secondly the intensity and the distribution of the magnitude of the jumps were estimated from these few selected points. The parameters of the mean reverting process in this study were estimated using the Generalized Method of Moments (GMM).

Bierbrauer et al (2003) also proposed a Regime-Switching model with two-regimes, a base mean-reverting regime (governed by a mean-reverting process, e.g. given by the Vasicek SDE) and a spike regime (governed by a lognormal variable) where the price processes that linked to each of the two regimes were assumed to be independent of each other. The variable that determined the current state in their proposed model was a random variable that follows a Markov chain with two possible states, and the parameter estimation was performed using the expectation maximisation (EM) algorithm of Hamilton (1990)¹⁷. They found that both of the proposed models mirror the stylised features of electricity prices.

¹⁷ Expectation Maximisation algorithm uses an iterative procedure that consists of two steps. The first step involves filtering the regime at a time, given data assuming that the true parameter set, of the vector of parameters of the underlying stochastic processes. Then, the probability of the process that was in regime j at time t with knowledge of the complete dataset is obtained. These probabilities are referred to as smoothed inferences. In the second step, new maximum likelihood estimates for all model parameters is achieved. Starting from an arbitrary parameter set, new estimates using the smoothed inferences is calculated. With this new vector of parameters the next cycle of the algorithm starts to re-evaluate smoothed inferences and so on. Each cycle of the algorithm produces new estimates of the vector of parameters of the unknown parameter set based on the previously calculated value set. The limit of this sequence of estimates achieves a local maximum of the log-likelihood function (Weron, 2004).

Later, Weron et al. (2004) extended this two state Regime-Switching model with a base regime governed by a mean-reverting process and the spike regime governed by Gaussian, Log-Gaussian and Pareto regimes. Their model allowed for spikes to occur consecutively and assumed a number of probability distributions. This study showed that modelling the spike regime with Log-Gaussian and Pareto regimes improve the model fit.

Weron et al. (2004a) later proposed a Jump-Diffusion model and a Regime-Switching model in order to explore daily electricity prices from NordPool. Their Jump-Diffusion model allowed positive jumps being followed by a negative jump of the same magnitude each time. This is performed by allowing the stochastic part being independent of the jump component.

The estimation of the jump component in their study was done by first defining jumps by all price increments exceeding three standard deviations of all price changes and selecting those observations from the data as in Bierbrauer et al (2003). Then, the intensity and the distribution of the magnitude of the jumps were estimated from these selected observations. The Mean-Reverting parameters of this Jump-Diffusion model were estimated by GMM.

This study also proposed a two-state Markov Regime-Switching model, which distinguished between a mean-reverting and a spike regime. This model used a lognormal variable for the spike regime and estimated the parameters of the model via Expectation Maximisation (EM). It was found that the probability of remaining in the same regime was very high for the mean-reverting regime (0.98) and relatively high for the spike regime (0.63) indicating that electricity prices have consecutive spikes.

On-peak¹⁸ daily electricity spot prices from U.S. (PJM)¹⁹ markets were modelled by Mouth et al. (2006) by a Markov Regime-Switching model with time-varying parameters. In this model, the mean prices in two price regimes and the transition probabilities are specified as functions of the offered reserve margin and the system load. The most important feature of this model was that the key parameters were functions of observed explanatory variables and it was built on earlier models of Hamilton (1989), Chan and Gray (1996), Kim and Nelson (1999) and Ning (2001). The contribution of this work lies in extending the original Markov Regime-Switching models by making key parameters functions of time varying variables. Mouth et al. (2006) found that their model replicated the observed price volatility quite well.

Modelling electricity prices with SDE based models continued with the work of Geman and Roncoroni (2006) who modelled electricity prices from U.S. California, PJM and East Center Area Reliability coordination agreement (ECAR) markets. Their model mimicked the mean-reverting and jumpy characteristic of electricity prices where the jump sizes are modelled as increments of a compound jump process. Geman and Roncoroni (2006) estimated the parameters of their model by an estimator based on the exact likelihood of the unknown process with respect to a prior process chosen as a reference within the same class. The estimator is provided by the parameter vector maximizing this process over a suitable domain. The study found that the calibrated processes exhibit the expected mean reversion property fine and it was argued that this method provided two major advantages:

¹⁸ On-peak hours are weekdays 7 am to 11 pm except NERC holidays in PJM

¹⁹ PJM Interconnection LLC (PJM) is a Regional Transmission Organization (RTO) which is part of the Eastern Interconnection grid operating an electric transmission system serving all or parts of Delaware, Illinois, Indiana, Kentucky, Maryland, Michigan, New Jersey, North Carolina, Ohio, Pennsylvania, Tennessee, Virginia, West Virginia and the District of Columbia. PJM, headquartered in Valley Forge, Pennsylvania, is currently the world's largest competitive wholesale electricity market. More than 650 companies are members of PJM, which serves 51 million customers and has 167 gigawatts of generating capacity. With 1,325 generation sources, 56,000 miles of transmission lines and 6,038 transmission substations, PJM delivered 682 terawatt-hours of electricity in 2009.

- (1) Analytical form of the exact likelihood function under continuous time observations can be computed for nearly all semi-martingales through a generalized version of the Girsanov theorem,
- (2) Discrete sample estimator converges to the continuous sample and a well-established estimation theory exists in this latter case.

Hence, electricity prices from six European and two US electricity markets; NordPool, Germany, Netherlands, France, Austria, Spain, U.S. (PJM) and New England Pool (US) modelled by de Jong (2006). He divided the spot price into a predictable component and a stochastic component. The predictable component of the model accounted for predictable regularities, such as any genuine seasonal behaviour or trend, and is a deterministic function of time. The models analysed in this study were:

- A Mean-Reverting model,
- A Poisson Jump-Diffusion model,
- A model with regime switches with stochastic Poisson jumps,
- A model with regime switches with three regimes and stochastic Poisson jumps,
- A model with regime switches with exponential Poisson up and down jumps, and
- A model with regime switches with independent spikes.

In these models, GARCH (1, 1) model was found to be suitable (based on likelihood tests) to replace the constant variation to capture volatility function. The suitability of Markov Regime-Switching models to capture stylised features of electricity prices as compared to Jump-Diffusion models was also tested and it was concluded that Markov Regime-Switching specifications captured the stylised features of electricity prices better. In conclusion, four of the Markov Regime-Switching models were found to fit the empirical data well where the

model with three regimes and stochastic Poisson jumps and the model with regime switches with independent spikes provided poorer fits. It was noted that these models separated the spikes most clearly from the mean-reverting price process.

A Mean-Reverting and Affine-Jump-Diffusion (AJD) model with spikes for daily and hourly electricity prices from the Dutch electricity market was performed by Culot et al., (2006). This model had deterministic annual (a sinusoidal function to approximate the annual cycle was performed) and intraweek patterns, with coefficient vectors acting as spike risk factors. The model accounted for spike behaviour flexibly using a Markov Regime-Switching process. It was argued that the model provided closed-form solutions for various interesting contingent claims in electricity prices, in contrast to traditional mean-reverting models. The estimation of this model was performed in two steps;

- Spikes, seasonal patterns were pre-calibrated, and
- Diffusive (state-space estimated) parameters were estimated using maximum likelihood and the Kalman filter methods.

This model represented electricity market spikes well using a finite m-state Markov Regime-Switching process, which modelled spike behaviour more flexibly than by random Poisson spikes. It was suggested that the model is more amenable to efficient derivative pricing and hedging applications.

Furthermore, modelling electricity prices in the Australian NEM was performed by Higgs and Worthington (2008) who modelled daily electricity prices after dividing the price series into two components:

- (1) A deterministic function that is predictable and is represented by a known deterministic function,
- (2) A stochastic component.

Their first choice of an SDE was a simple Mean-Reverting model where spot electricity prices fluctuate around long-term equilibrium price level, which reflects the marginal cost of producing electricity following earlier models of Deng (1998). The second SDE model that was proposed was a Markov Regime-Switching model, which separated mean reversion in the normal and spike price periods. This Markov Regime-Switching model was similar to Huisman and Mahieu (2003) and it assured that on any day the electricity spot price lies in one of three regimes:

- A normal when prices follow mean-reverting electricity price dynamics,
- An initial jump regime when prices suddenly increase (decrease) during a price spike,
- A downturn regime when electricity prices revert to normal after a spike has occurred.

Maximum Likelihood (ML) estimates were used to determine the parameters of the model. Higgs and Worthington (2008) concluded that the price dynamics in Australian electricity prices are captured well with this three-state regime switching model however, the main limitation of this study was that the model did not have any allowance for consecutive spikes that arise in NEM. A suggested solution to this restrictive model was a two-regime model (de Jong and Huisman 2002, Bierbrauer *et al.* 2003, de Jong 2006) which permits consecutive spikes.

Alternative to previous studies that employed price series without de-seasonalisation, Weron (2009) explored the suitability of de-seasonalised electricity prices when modelling regime switching processes. De-seasonalised process of his study included three steps:

- (1) Removing the long-term trend via wavelet filtering-smoothing,
- (2) Weekly periodicity via moving average technique and
- (3) Finally shifting the de-seasonalised prices to make the minimum of the new price process same as the original price process.

The Markov Regime-Switching model of Weron (2009) had two-regime specifications; with the base regime dynamics given by a mean-reverting Ornstein-Uhlenbeck process and the spike regime dynamics distribution of Log-Gaussian and Pareto. This model excluded a regime specification characterised by Gaussian distribution as it was found to be stable with respect to its parameter estimates. Calibration of this two-regime specification was performed with EM algorithm. This study found that the models with log-prices fit the data well as compared to models with de-seasonalised data except with the Pareto distributed model.

Consequently, the contribution of Janczura and Weron (2010) to electricity price modelling in particularly to the development of Regime-Switching models is important. They modelled de-seasonalised daily electricity prices from German, U.S. and New England power markets in an attempt to model the dynamics of the stochastic components of the electricity prices with:

- A two-regime model with shifted spike distribution and mean-reverting regime dynamics
- A two-regime model with shifted spike distributions and heteroskedastic base regime dynamics and

- A three-regime specification with heteroskedastic base regime dynamics, shifted lognormal distribution for the spike regime and inverted shifted lognormal distribution for the downward spike regime.

Weron et al. (2010) used EM algorithm in calibrating the abovementioned models. They found that the three-regime specification with heteroskedastic base regime dynamics, shifted lognormal distribution for the spike regime and inverted shifted lognormal distribution for the downward spike regime model performed superior to other models. Moreover, this model also allowed for consecutive spikes or drops in prices. Further, it was shown that this model mirrored the inverse leverage effects widely reported for electricity prices.

Almost all of the electricity models in the literature up to 2010 ignored the market capping mechanisms prevalent in some electricity regions. Janczura and Weron (2010) were pioneers in acknowledging the modelling issues with price caps. They used a number of Markov Regime-Switching models and evaluated the fit of models with standard, as well as truncated (or price-capped) spike regime distributions in the electricity markets of NSW (Australia) and New England Power Pool (U.S.) where market prices are capped. They introduced truncated spike distributions ensuring that observations do not exceed a specified level and, hence, are well suited for modelling these capped power market prices.

Their two-regime model utilised de-seasonalised prices and modelled the base regime dynamics with a mean-reverting heteroskedastic process whilst the modelling of spike regimes were performed with shifted spike regime distributions which assign zero probability to prices below a certain quantile (lognormal and truncated lognormal distributions were employed).

They found that there are significant differences between the estimated spike distributions in the truncated and non-truncated cases.

This section provided a brief review of the literature on modelling electricity prices with variants of SDEs. Similar to the learnings derived from the preceding section, this section highlights the importance of capturing the stylized features of electricity prices in modelling and forecasting. These features are rapid mean-reversion to long-term mean levels and occasional and sometimes consecutive jumps in prices. Additionally, the volatile and extreme nature of the electricity prices as manifested by fat tails in the distribution of the data is also emphasized in the literature. This asymmetric distribution in electricity price series leads to non-Gaussian. Hence, modelling electricity prices with techniques that assume normality will likely result in less than optimal forecasts. One of the major learnings that can be deduced from the established literature presented in this section is that the stylized features of the electricity prices are of particular interest in modelling electricity prices and they are best captured by variants of SDEs.

In light of the recent literature, in this thesis, SDE based models are chosen to capture the stylised features of electricity prices; Mean-Reverting, Mean-Reverting and Jump-Diffusion and Markov Regime-Switching models. GBM model is also chosen as it is the foundation block for the other SDE based models.

Particular choice of the SDE models is based on their ability to capture the stylised features of electricity prices in Australia. Mean-Reverting model is believed to mimic fast mean-reversion behaviour of the price series whereas Mean-Reverting and Jump-Diffusion model is believed to capture the sudden price spikes prevalent in Australian electricity market. Markov Regime-

Switching model on the other hand is shown to capture sudden and consecutive jumps that occur in electricity price series therefore the last SDE based model of this thesis is chosen to be a Markov Regime-Switching model.

VOLATILITY MODELS

The literature on electricity price modelling has an array of applications of Autoregressive Conditional Heteroskedasticity (ARCH) or Generalised Autoregressive Conditional Heteroskedasticity (GARCH) processes. These models allow volatility shocks to cluster and persist over time and to revert to normal levels and so may offer potentially interesting insights on the volatility observed in the electricity markets.

Many researchers investigated the conditional mean and volatility characteristics of electricity prices. The former is the result of demand, capacity margin and trading volume on volatility levels whereas the latter describes the observed clustering of stable periods (GARCH effects). Measuring volatility is an important variable in the valuation of risk management models. Hence, it is argued that heteroskedastic behaviour in prices can only be modelled correctly when the conditional mean of the time series is properly modelled.

Knittel and Roberts (2001) studied the price volatility in the Californian electricity market with an exponential GARCH (EGARCH) process in an attempt to understand the asymmetric impact of new innovations on the price volatility. This study found that the electricity prices in the Californian market have significant ARCH and GARCH effects indicating a high degree of persistence. Also, an inverse leverage effect was found to be present, indicating that positive shocks to electricity prices affect the conditional variance more than negative shocks. The

leverage effect²⁰ refers to the asymmetric behaviour of electricity prices (negative shocks tend to increase volatility more than positive shocks).

Time varying volatility of electricity prices with evidence of heteroskedasticity both in unconditional and conditional variance was described by Escribano et al. (2002) in which the measures of volatility illustrated the degree of randomness. This study specified the deterministic seasonality prevalent in a price series by sinusoidal functions before estimating six different volatility models:

- (1) A pure-Gaussian model with constant variance and without jumps,
- (2) A GARCH(1,1)-Gaussian model without jumps,
- (3) A Poisson-Gaussian models with constant variance,
- (4) A Poisson-Gaussian models with time-varying intensity for jumps,
- (5) A GARCH(1,1)-Poisson-Gaussian model with constant intensity, and
- (6) A GARCH(1,1)-Poisson-Gaussian model with time-varying intensity for jumps.

Further, modelling of daily electricity price changes from NordPool with ARMA-GARCH process was performed by Solibakke (2002) who adjusted the conditional mean equation for day-of-week and month-of-year effects and conditional variance equation with day-of-week and month-of-year effects. This model's conditional variance equation was estimated by log of the squared residuals from the conditional mean equation. Solibakke (2002) included three GARCH specifications namely, asymmetric GARCH (AGARCH), truncated GARCH (GJR) following Glosten et al. (1993) and EGARCH processes. Furthermore, this study used maximum likelihood (ML) approach based on a t-distributed log-likelihood function to account

²⁰ The connection to leverage is that a lower stock price reduces the value of equity relative to debt, thereby increasing the leverage of the firm and consequently the risk of holding the stock.

for high kurtosis and skewness present in electricity prices. The results of the study were that the volatility equation favoured GJR or AGARCH version of the univariate ARMA-GARCH lag specification. The main contribution of this paper was its incorporation of asymmetric and fat-tailed characteristics to assess the electricity price changes first reported by Knittel and Roberts (2001).

The vector autoregressive (VAR) model of Leon and Rubia (2002) applied to the Argentinian electricity market²¹ utilised a number of different conditional covariance matrices. To estimate the conditional covariance matrix (CCM) of their model, authors utilised Orthogonal GARCH (OGARCH) and a constrained multivariate GARCH (MGARCH) models. These models aimed to cope with the time-dependent volatility of time series.

The OGARCH is based on the application of the principal component analysis to identify the main sources of variation of the multivariate system associated to each eigenvalues. MGARCH model on the other hand proposed the restriction that the long run covariance matrix equals the sample covariance matrix. The main conclusion of this study was that the forecasting performance of OGARCH and MGARCH approaches yielded similar results.

Modelling volatility dynamics of electricity prices in a number of markets including Australia with a jump-reverting and GARCH processes with and without time dependency was performed by Goto and Karolyi (2004). Overall findings of their study were that the GARCH models with seasonally time dependent jumps were significant in modelling price volatility across different markets.

²¹ This market is very unique and quite different from the U.S. and European markets where mainly thermal and nuclear resources constitute the whole generation resources.

The contribution of Goto and Karolyi (2004) to the Australian electricity price modelling literature was quite important as this study found little robustness of the monthly seasonal dummies in the mean returns function. Furthermore, it showed that the jump probability coefficient is lower for the Australian data even though they occurred with lower frequency in the series as compared to other markets.

Threshold ARCH (TARCH) processes take account of asymmetric responses in electricity prices. Hadsell et al. (2004) used electricity prices from five major American markets, namely: California-Oregon Border (COB), Palo Verde, Cinergy, Entergy, and PJM in an application of TARCH model. This model incorporated seasonal effects to take account of all monthly variations in the conditional mean and variance equations. It also incorporated an asymmetric factor to take account of the different effects of positive and negative errors on the conditional variance equation. This study's main contribution to volatility modelling was the detection of downward trends in ARCH term, indicating unstable expected volatility along with a negative asymmetric effect.

The modelling of volatility in electricity prices was extended to variations of ARCH processes combined with several fundamental variables by Bunn and Karakatsani (2004). They used four alternative approaches in modelling electricity prices from the U.K. market where residual volatility is attributed to non-linear impacts of fundamentals. These models were:

- (1) GLS heteroskedasticity, asymmetric volatility responses to lagged price shocks,
- (2) A regression and TGARCH structure, evolution of the underlying price model due to market adaptation,
- (3) Time-varying regression effects and alteration of price structure during temporal market irregularities,

(4) Markov Regime-Switching regression dynamics.

These models treated stochastic volatility as a non-linear, structural specification for either the price formulation process or the random shocks. This study noted that volatility inferences were sensitive to the assumed price models developed. It also showed that GARCH effects diminish after adjusting for the time-varying price structure. This finding in particular has important applications for price modelling as it suggests that modelling the dynamic structure of the price process (e.g. an AR model with time-varying parameters) may describe volatility dynamics better than a price model with complex variance structures (e.g. AR-AGARCH).

Consequently, studying the asymmetry in electricity price series continued with the work of Knittell and Roberts (2005) who used an AR-EGARCH specification in the California market and found that the asymmetry parameter is positive and significant, suggesting the presence of an inverse leverage effect. This model has lagged variables in GARCH equation as well as variables to account for seasonalities (winter, fall, summer and weekend) and the different structure of price behaviour in peak and off-peak prices (peak and off-peak mean). The main result of the study was that AR-EGARCH model was superior to different ARMA specifications during the Californian electricity crises in the summer of 2000 whilst this specification was found to be inferior during the pre-crisis study period.

Last but not least, the study of Higgs and Worthington (2005) provided a comprehensive modelling of volatility dynamics in Australian markets. They utilised GARCH, RiskMetrics²², and Gaussian asymmetric power ARCG (APARCH), Student APARCH and skewed Student

²² Risk Metrics model is equivalent to a normal integrated GARCH (IGARCH) model where the autoregressive parameter of the conditional variance equation is pre-set to a specified value.

APARCH processes in modelling volatility dynamics in NEM. These processes also included exogenous variables of time of day, day of week, month of year effects and demand volume. This model accounted for the large positive and negative price spikes prevalent in NEM. This study concluded that skewed Student APARCH model provided the best volatility estimates for electricity prices in NEM. Another important outcome of this study was that it indicated the appropriateness of modelling conditional standard deviation equation as non-linear specification.

This section briefly reviewed the literature on volatility modelling of electricity prices. The major points that can be derived from the literature are that modelling the volatility dynamics is an inseparable part of electricity price modelling. This finding is in line with the learnings from the previous two sections of this chapter. The chosen benchmark model of this thesis models the volatility dynamics of the electricity prices in Australia. However, SDE based models that are believed to capture the stylised features of electricity prices do not specifically model volatility.

EMERGING MODELS

The literature on electricity price modelling has emerging models that combine Autoregressive, Mean-Reverting and Jump-Diffusion, and Volatility models with other econometric techniques. Application of Extreme Value Theory (EVT), spatial econometric modelling, and models based on Levy processes are such applications. The application of these models shows that as the econometric complexity of the models increase, the models get closer to mirroring true price processes in the electricity markets.

Extreme Value Theory (EVT)

It is one of the stylised facts that electricity prices exhibit heavy tail characteristics. It is this construction of the price series that consequently led to modelling electricity prices with EVT by Bystrom (2005) who examined electricity prices in NordPool.

Bystrom (2005), in order to apply EVT, first pre-filtered prices through an AR-GARCH process. This pre-filtering process achieved i.i.d. process in the model residuals. Then modelling the residuals with the assistance of Peaks-over-threshold (POT) method based on the Generalized Pareto Distribution (GPD) was conducted. The POT method is based on utilizing all exceedances of a given time-series above a specified threshold. Given the exceedances in each tail, Bystrom (2005) then optimised the negative log-likelihood function to estimate the shape and scale parameters of the GPD. This study found that the POT method accurately modelled the extreme values of the electricity prices with in-sample and out-of-sample evaluation of forecast providing strong support for the application of EVT to electricity prices.

Another application of EVT in modelling electricity prices conducted by Chan and Gray (2006) developed a Value-at-Risk (VaR) model with the aid of EVT. This study used daily electricity price returns from Victoria (Australia), NordPool, Alberta (Canada), Hayward (New Zealand) and PJM (U.S.). Authors estimated an AR-EGARCH model with a t-distribution governing the residuals and then standardising the residuals from this model before using the EVT techniques. In applying EVT, Chan and Gray (2006) used POT method to identify extreme observations (extreme standardized residuals) that exceeded a high threshold (which was derived via a combination of two popular techniques: the mean excess function and the Hill plots) and specifically modelled these exceedances separately from non-extreme observations. Compared

to a number of other parametric models and simple historical simulation based approaches, their proposed EVT-based model performed well in forecasting out-of-sample Value-at-Risk (VaR) estimates.

Periodic Autoregressive Models

The panel model of Huisman et al. (2007) examined the dynamics in hourly electricity prices for the markets of Netherlands, Germany and France via representing the price series as a panel of 24 cross-sectional hours that vary from day to day. This method of modelling electricity prices is quite similar to PAR models where each hour of the trading day can be viewed as a different commodity.

Huisman et al. (2007) divided the price series into two components, deterministic and stochastic components²³ following Huisman and Mahieu (2003). The stochastic component was designed to account for stochastic characteristics such as mean-reversion, time-varying volatility and spikes and the parameters of this model were estimated by seemingly unrelated regression method. The parameter estimation for the 24 hourly time series accounted for heteroskedasticity and contemporaneous correlations in the errors across the series. They found that hourly electricity prices mean-revert around an hourly specific mean price level, and that the speed of mean-reversion was different over the hours. Additionally, there was an apparent cross-sectional correlation pattern where prices in peak-hours correlated strongly with each other and the same held for prices in off-peak hours.

²³ The deterministic component accounted for predictable seasonality of long-term mean reversion, hourly deviations from the mean price level and days of week effects whilst the stochastic component accounted for the variation of the price around the deterministic component.

Levy Jump-Diffusion Models

Modelling electricity prices by Lévy Jump-Diffusion processes has been relatively new. The class of Lévy processes are general specifications of mean-reverting jump-diffusions of the form, which earlier researchers used in modelling electricity prices but they are not limited to those as there exist a Lévy process with infinitely many jumps in every interval.

The analytical work of Benth et al. (2007) paved the way for studies in the area of electricity price modelling by Lévy processes as it reproduced the stylised features of electricity price series while preserving analytical tractability. Authors proposed a modelling approach with sum of non-Gaussian Ornstein-Uhlenbeck processes²⁴ where each component consisted of a pure jump process with only positive jumps as a source of randomness described by Gamma distributions. These components were mean-reverting with different mean reverting rates.

Later, Meyer-Brandis and Tankov (2008) proposed an estimation model for the model of Benth et al. (2007) where filtering out the different Ornstein-Uhlenbeck process components was performed via thresholding and the adapted Potts filter methods.

Furthermore, Klüppelberg et al. (2009) developed an estimation method for the additive non-Gaussian Ornstein-Uhlenbeck model of Benth et al. (2007) and suggested that EVT may be used to identify jumps present in the price series. This model consisted of two Ornstein-Uhlenbeck Gamma processes to capture the multi-scale autocorrelation and another process to capture the spike regime where the randomness was described by Pareto distribution.

²⁴ Ornstein–Uhlenbeck is a stochastic process that describes the velocity of a massive Brownian particle under the influence of friction. The process is stationary, Gaussian, and Markov, and is the only nontrivial process that satisfies these three conditions, up to allowing linear transformations of the space and time variables. Over time, the process tends to drift towards its long-term mean: such a process is called mean-reverting.

An extension of application of Lévy processes in modelling electricity prices was performed by Borovkova and Permana (2009) who proposed modelling of electricity prices with potential Lévy diffusions. The approach taken in this study combined a diffusion driven by a Lévy process with an α -stable distribution, and with a drift given by a potential function, which forces the price process to return to its average level (given by a seasonal, ie, a deterministic, periodic function) after a large price move²⁵. They used daily electricity prices from the Netherlands, the UK and Germany to apply their model after filtering out jumps, removing yearly trend from each series by moving average technique and estimating the yearly seasonal component by the least-squares fitting of a trigonometric function. This model produced large price jumps due to its heavy-tailed property, which reflected the corresponding empirical feature of electricity prices.

Henceforth, Borovkova and Permana (2009) developed an alternative model that was a transitional model between the model based on the univariate SDE and the traditional Jump-Diffusion models. They considered a compound Poisson component while plugging in a Brownian motion with volatility instead of small jumps. The studies of Borovkova and Permana (2009) showed that these models fit empirical distributional characteristics of the data remarkably well, and certainly better than traditional Jump-Diffusion models.

Continuous Time ARMA Models

Klüppelberg et al. (2011) applied continuous time ARMA (CARMA)²⁶ processes to model electricity prices. They described the CARMA process by an equation, where the left-hand side

²⁵ The proposed models are inspired by solutions of the non-linear stochastic differential equations driven by Lévy processes, studied in Imkeller and Pavlyukevich (2006).

²⁶ CARMA models are driven by a stable non-Gaussian Levy processes.

corresponds to an AR (p) process and the right-hand side to a $(p-1)$ -dependent process. Their specification included a representation of the stable CARMA (p, q) process as the sum of dependent CAR (1) processes. They concluded that even though there is no weak MA representation of this $(p - 1)$ dependent process, for a stable CARMA process, the principles of the model can still be applied when the innovations of the driving Lévy process have finite second moment²⁷.

Spatial Econometrics

The literature also has applications of spatial econometrics in modelling electricity prices. A very unique approach to modelling electricity prices was proposed by Popova et al. (2011) whose spatial error model included forward electricity prices, system loads, air temperatures observed in each of the 16 regions of the U.S. (PJM) market, dummies for disentangling daily and weekly seasonality, lagged spot price and an error term. This study suggested that unobserved spatial correlation in electricity networks can be modelled by spatial error models and ignoring the spatial characteristics of the networks may cause biased results. Their model results indicated that electricity prices exhibit spatial correlation and this correlation fades rapidly when the congestions in the grid occur and then level out.

This study was an important development in modelling electricity prices as it provided information on the econometric structure of each node/region of the price generation process (Baltagi and Li 2006).

²⁷ The standard deviations of the estimates even decrease as the stable parameter α decreases.

Modelling Interconnected Regional Electricity Markets

The literature also has an array of applications where the notion of nonlinearity has been associated with multiplicative nonlinearity or nonlinearity in variance. ARCH/GARCH models where the conditional variance is postulated depend on the variability of recent observations, associated nonlinearity with multiplicative nonlinearity. Stochastic volatility models on the other hand associated nonlinearity with nonlinearity in variance where volatility is postulated to be a function of some unobserved or latent stochastic process (Shephard, 1996). Both ARCH/GARCH stochastic volatility models assume that the time series is a zero-mean process and when the mean of the process is nonlinear, potential problems in modelling emerges. Inaccurate conclusions of nonlinearity in variance might emerge when the prime source of serial dependence is a nonlinear structure in residuals that could not successfully be extracted by conventional linear-based time series models (Wild et al., 2010).

An examination of non-linearity in electricity prices was performed by Wild et al. (2010) who applied Portmanteau correlation, bicorrelation, and tricorrelation tests to detect nonlinear serial dependence in electricity price series in NEM. They found strong evidence of nonlinear serial dependence in electricity prices. It is argued that the presence of third- and fourth-order nonlinear serial dependence in electricity prices makes models that employ a linear structure, or assume a pure noise input.

Furthermore, to understand the characteristics of the price series in Ercot (Texas) and PJM West hubs in U.S., Kim and Powell (2011) challenged the main arguments built into most of the models by earlier researchers. They argued that filtering out the heavy-tailed fluctuations via zeroth-order smoothing can be achieved by computing the trailing median of the process

(TMP)²⁸, where the quantile function of a set is not affected by extreme events. This proposed model had the following assumptions:

- There exists a long-term median for the price process and the price process is median-reverting towards the TMP in the short term, and
- Heavy-tailed price spikes exist.

It was shown that the distribution of the error term in electricity prices of these markets is well-described by the Cauchy distribution, which ensures that the median-reversion of the price process is smooth and heavy-tailed. In traditional mean-reverting models, the distribution of error term is described as Gaussian. A very important implication of this finding is that it paves the way in utilising the empirical cumulative distribution function and the quantile function rather than the empirical mean and the variance used in traditional linear models.

In summary, these emerging models aimed to capture the stylised features of the electricity prices similar to established models reviewed earlier. These features are extreme values, heavy tails and therefore non-linearity, and the existing correlations between the nodes of electricity markets. This section reported that modelling electricity prices without capturing these features is likely to result in biased price estimates.

One of the models of this thesis chosen to be compared against the benchmark model is based on combined application of Extreme Value Theory and Copula functions. It is believed that this model captures the extreme values and heavy tails prevalent in Australian electricity price series.

²⁸ The trailing median process (TMP), which is a thin-tailed, symmetric, and a slow varying process with very low volatility, reverts towards the mean in the long term. The long-term mean of the TMP is the long-term average of the price process itself.

STUDIES IN THE AUSTRALIAN NATIONAL ELECTRICITY MARKET

There have been a number of studies in the literature that utilised data from Australian National Electricity Market (NEM). These studies confirmed the stylised features of electricity and its presence in the Australian electricity price series.

The studies of Either and Mount (1998), Johnson and Barz (1999), Escibano et al. (2002), Wilkinson and Winsen (2002) and Goto and Karolyi (2004) examined the mean-reverting and jumpy features of price series in a number of markets including Australia. These studies aimed at characterising the electricity prices by use of autoregressive and mean-reverting jump diffusion models and only used data from one market region of the Australian National Electricity Market (NEM).

The contribution of Goto and Karolyi (2004) was particularly important in the literature as they found little robustness of the monthly seasonal dummies in the mean returns function across their model specifications. Furthermore, they showed that the jump probability coefficient is lower for the Australian series even though they occurred with lower frequency in the series as compared to other markets.

Higgs and Worthington (2005) modelled electricity prices in Australia covering all market regions of the NEM. They applied a number of ARCH type models to understand volatility dynamics of the electricity prices in their first attempt to model Australian electricity prices.

Later, Higgs and Worthington (2008) applied three different variants of SDEs to the electricity price series in an attempt to determine the most robust price model applicable in all of the NEM regions. The models they utilised in this study were:

- A basic stochastic model,
- A Mean-Reverting model and
- A Markov Regime-Switching model.

The results of these models showed that the Markov Regime-Switching model outperforms the basic stochastic and mean-reverting models in providing the best fit to empirical data in NEM.

Higgs (2009) analysed the inter-relationships prevalent in electricity prices in NEM regions and found high conditional correlations between the well-connected regions. This study examined the inter-relationships of electricity prices and price volatility in four Australian electricity regions of New South Wales (NSW), Queensland (QLD), South Australia (SA) and Victoria (VIC). The study consisted of three different conditional correlation multivariate GARCH models and demonstrated that the price and price volatility inter-relationships in the regions of NEM are best described by the dynamic conditional correlation multivariate GARCH specification. An important conclusion reached in this study was that the interconnectivity and/or geographic arbitrage between the separate regions in NEM have fostered a nationally integrated electricity market. Thus, this indicates that the NEM's interconnected regions are informationally efficient.

There have also been studies to characterise the non-linear structure of electricity prices in Australia. Rozario (2002) derived VaR for Victorian half-hourly electricity returns using a threshold based Extreme Value Theory (EVT) model whereas Chan and Gray (2006) applied an AR-GARCH-EVT model using data from a number of markets including Australia with an aim to develop a VAR model.

Furthermore, Christensen et al. (2009) modelled Australian electricity prices with zero-inflated Poisson autoregressive (PAR) model, which captured the persistence in spikes and hence provided a good fit to the electricity prices.

Most recently, Wild et al. (2010) presented a unique study using Australian data in an attempt to examine nonlinearity present in NEM electricity prices. They found strong evidence of nonlinear serial dependence in the data. The main finding of this study points out that models based on Gaussian distribution will result in biased estimates.

In conclusion, previous studies in the literature that employed data from NEM examined the stylised features of electricity prices with particular emphasis on mean-reverting and jumpy features.

Additionally, most recent studies highlighted the presence of non-linearity and interconnected nature of the NEM. The existence of price and volatility interconnection amongst the physically connected regions of NEM presents unparalleled challenges in modelling electricity prices for a particular region of the NEM. A modelling approach that ignores this interconnection will result in bias estimates and inaccurate forecast.

CHAPTER 5 – INTRODUCTION TO METHODOLOGY

In this chapter an overview of the models as used in this thesis to forecast wholesale electricity prices in the Australian National Electricity Market (NEM) are presented. Mean-reversion, the presence of jumps, and non-Gaussian manifested as positive skewness and leptokurtosis are the main stylized features of electricity prices as pointed out in the literature (example Kaminski 1977).

It is believed that by explicitly modelling the stylised features of electricity wholesale prices, forecast accuracy can be improved upon baseline models commonly used in quantitative finance. This thesis investigates the forecasting ability of two distinct modelling approaches which by construction capture the stylised characteristics of electricity prices. Namely, these are linear continuous time and non-linear modelling methods. The AR-GARCH model is chosen to be the standard approach in forecasting price series (Engle, 2001) and is taken as the benchmark model in this thesis. More specifically, this thesis aims to answer the following research questions:

1. Does the application of continuous-time models in capturing the stylised features of Australian electricity wholesale spot prices improve forecasting ability upon the traditional AR-GARCH model?
2. Does the application of non-linear forecast models in capturing the stylised features of Australian electricity wholesale spot prices improve forecast ability upon traditional AR-GARCH model?

The continuous-time models examined in this thesis are; Geometric Brownian Motion (GBM), Mean-Reverting, and Mean-Reverting Jump-Diffusion processes. The inclusion of GBM in this thesis is due to it being the foundation for the Mean-Reverting and Jump-Diffusion models,

which are considered in this thesis. Continuous-time models capture some of the main stylised features of electricity prices; Mean-Reverting process captures the mean-reversion (tendency of electricity prices to revert back to its long-term average over time) characteristics of electricity prices whilst Mean-Reverting and Jump-Diffusion process models the sudden jumps prevalent in Australian electricity prices.

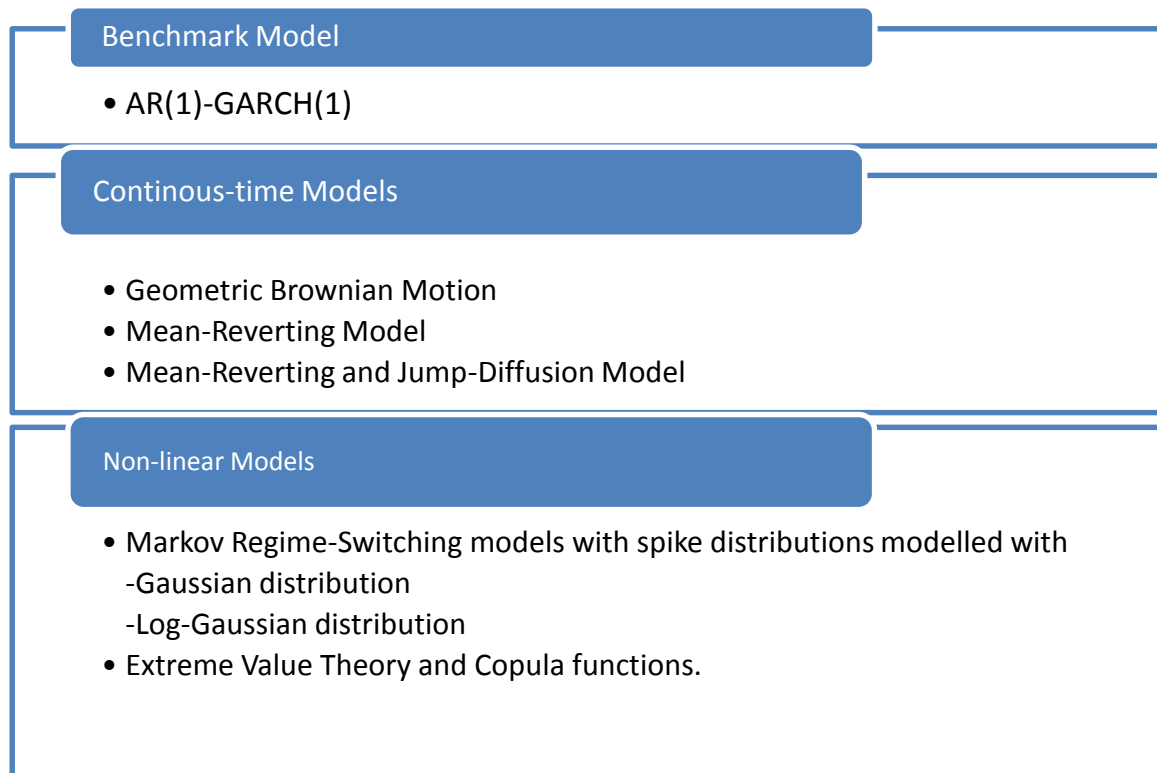
The models are in order such that each successive model extends the one preceding it. Note that each extension addresses a stylised feature of the data therefore the a priori expectation is that the forecasting performance will improve.

The inclusion of the non-linear approach to forecasting Australian electricity prices is performed with the application of a Markov Regime-Switching model and the application of Extreme Value Theory (EVT) into electricity price modelling.

The Markov Regime-Switching model is a non-linear modelling tool that is able to capture consecutive spikes prevalent in Australian electricity prices that Mean-Reverting and Jump-Diffusion processes fail to capture.

The application of EVT is included in this thesis so that heavy tails present in electricity prices can be adequately captured. Copulas are considered as a unique method that models the dependence structure of data. The forecasts based on the EVT model is built upon the application of Copula functions as these functions model the interdependence of prices within the separate regions of the Australian electricity markets.

To summarise, in total six econometric models are applied in this thesis. The models examined in this thesis are:



Each model under investigation mimics a known characteristic of electricity prices. Mean-Reverting model replicates the mean reversion feature of prices series, whilst Mean-Reverting and Jump-Diffusion model incorporates jumpy features of prices series along with mean-reversion. Markov Regime-Switching model incorporates the consecutive jumps prevalent in NEM in its formation. Finally, the EVT based model replicate the nonlinear, heavy tailed nature of the electricity price series.

To assess the relative forecast performance of these models, short-term forecast performances (90 days) are compared with each other and with the chosen benchmark model (AR-GARCH). The price data used in this study are average hourly pool price observations sourced directly from Australian Electricity Market Operator (AEMO) for the period of 01/06/2006 to

29/08/2010. The data from 01/06/2006 to 31/05/2010 (in-sample data) are used to estimate the parameters of the models while the period from 01/06/2010 to 29/08/2010 (out-of-sample data) are used to derive out-of-sample forecast accuracy statistics.

The forecast exercises involve the simulation of these models via Monte Carlo and Copula approaches. Monte Carlo simulation is a numerical method commonly used in economics to solve partial differential equations such as the Black-Scholes equation for pricing stock options. The basic principle of Monte Carlo simulation is that there is a stochastic variable in an equation that can be sampled many times over. When the result of the many simulations of a function containing the random variable is averaged, it approximates to the real mean. This is a result from the Central Limit Theorem of probability. The variable being sampled in Monte Carlo simulation usually comes from the uniform distribution as simulated by a random number generator.

Each continuous time the Markov Regime-Switching models are simulated using the Monte Carlo approach via Euler approximation method. This method simulates sample paths of correlated state variables driven by Brownian motion sources of risk over consecutive observation periods and thus approximating continuous-time stochastic processes. The model that incorporates the application of EVT on the other hand is simulated with Copula functions, returning random vectors generated from a t-copula with linear correlation parameters. This method generates a set of simulations from a bivariate t-copula and each column of the simulation sets is a sample from a uniform marginal distribution.

A set of comparative forecast performance measures is used in this thesis in measuring the relative forecast performance of each forecast model. The forecast performance measures of

Root Mean Square Error (RMSE) and Theil's U are used to present the forecast errors of each model by each region of the NEM. Appendix 2 provides an overview of the forecast accuracy measures utilised in this thesis.

These measures show that Markov Regime-Switching models have a tendency to provide better forecasting results when their spike processes are modelled with a Log-Gaussian distribution. This is due to the fact that by construction Markov Regime-Switching models capture both mean-reversion and consecutive jumps prevalent in electricity price data in a non-Gaussian model setting.

THE BENCHMARK MODEL

Autoregressive (AR) models can be considered as the benchmark modelling tool in financial modelling. These models are also widely applied to electricity price modelling in the literature. Autoregressive models (AR) and their extensions that allow incorporating exogenous factors (ARX) are the standard modelling techniques in applied econometrics. These models are widely applied to electricity price modelling literature. AR models are used to predict behavior of electricity time series from past values then such a prediction is used as a baseline to evaluate possible importance of other variables to the system. AR modelling also contributed to understanding of the physical system by revealing something about the physical process that builds persistence into the electricity series.

One of the earlier applications of AR models was performed by Knittel and Roberts (2001) who modelled electricity prices from the Californian market with seasonal ARMA and AR-EGARCH processes.

The role of adding GARCH processes²⁹ in AR models in electricity price forecasting was then shown by Contreras et al. (2003) who originally applied ARMA processes with multiple seasonalities and lags to predict hourly prices in the electricity markets of Spain and California. They later improved their ARMA forecast model with the addition of GARCH (1,3) specification and concluded that the forecast errors were around nine per cent, depending on the studied month of the year in these markets.

They also argued that this model outperformed their earlier ARMA model and pointed out that adding the demand to the GARCH model as an explanatory variable improved the performance of the method further. This study's importance in the literature lied as it strengthened the case to add GARCH processes in forecasting electricity prices.

The price data used in this model is the same as all other models studied in this thesis. The data from 01/06/2006 to 31/05/2010 are used to estimate the parameters of the models while the period from 01/06/2010 to 29/08/2010 are used to derive out-of-sample forecast accuracy statistics in line with all other models developed in this thesis.

As the benchmark model, an AR(1)-GARCH(1,1) with conditional t-distribution is fitted to electricity price series for each market of the Australian National Electricity Market (NEM). This complex conditional mean and variance model has Student's t innovation in an effort to reflect the non-Gaussian nature of electricity price series in NEM.

²⁹ For an extensive discussion of the AR-GARCH modeling in electricity price modeling, see the literature review of this thesis.

Table 6 AR-GARCH Model Parameters

		Value	Standard error	t statistic
NSW	C	0.53574	0.04044	13.2469
	AR(1)	0.84107	0.01155	72.7714
	K	0.56789	0.01205	4.7099
	GARCH(1)	0.09826	0.03851	2.5509
	ARCH(1)	0.90174	0.22236	4.0552
VIC	C	0.57203	0.04422	12.9361
	AR(1)	0.83367	0.01250	66.6603
	K	0.05587	0.00823	6.7820
	GARCH(1)	0.08821	0.05226	1.6876
	ARCH(1)	0.53787	0.1094	4.9167
QLD	C	0.5432	0.04217	12.8810
	AR(1)	0.83361	0.01240	67.2113
	K	0.04444	0.00971	4.5742
	GARCH(1)	0.28356	0.04910	5.7747
	ARCH(1)	0.71644	0.16903	4.2385
SA	C	0.70416	0.04826	14.5886
	AR(1)	0.79869	0.01355	58.9446
	K	0.11153	0.03372	3.3069
	GARCH(1)	0.1402	0.05028	2.7883
	ARCH(1)	0.8598	0.29716	2.8934
TAS	C	0.36432	0.04145	8.7883
	AR(1)	0.90162	0.01108	81.3652
	K	0.03992	0.01022	3.9051
	GARCH(1)	0.29512	0.05320	5.5470
	ARCH(1)	0.70488	0.19763	3.5667

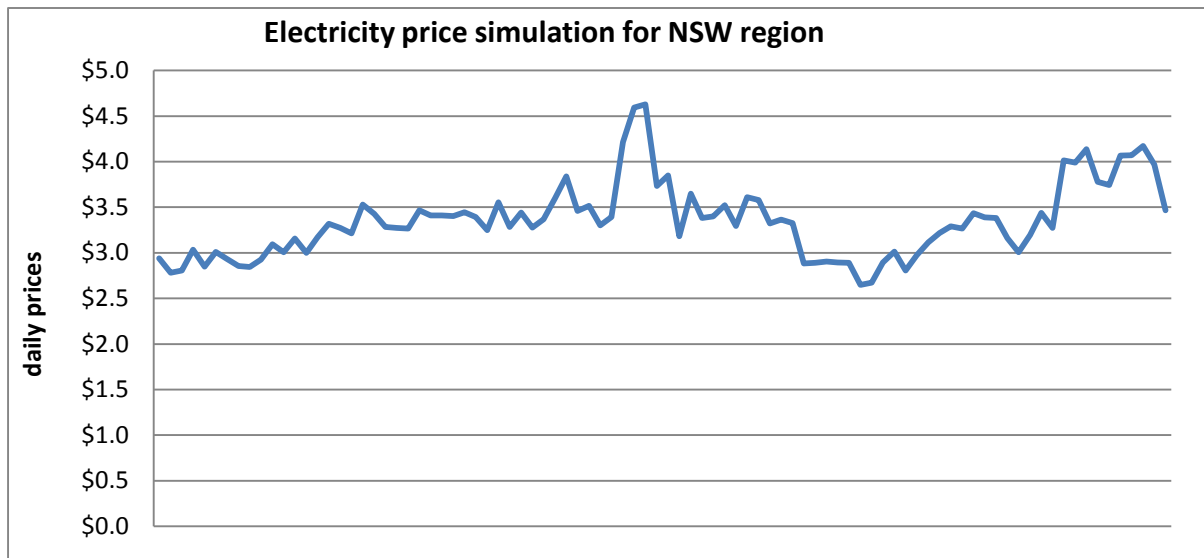
Source: Author's calculations.

Table 6 describes the estimated parameters of the model and the conditional standard deviations and standardised residuals of the model are given in the appendices. The fitted model for each of the NEM regions is simulated in order to generate the dependent stochastic process that follows the conditional mean specification of general AR-GARCH form defined by model parameters values for up to three months.

Where K is the conditional variance constant, GARCH is the coefficients related to lagged conditional variances, ARCH being the coefficients related to lagged innovations (residuals), and C is the conditional mean constant. AR represents the conditional mean autoregressive coefficients that imply a stationary polynomial.

The results of the AR(1)-GARCH(1,1) simulations over three month horizon (90 days) for each region of the NEM are illustrated in the following graph (for the period 01/06/2010 to 29/08/2010).

Figure 10 Simulated Price Series



Source: Author's calculations.

The parameterisation of the model is based on the in-sample period chosen for this study and is the same as all other models previously discussed. The price path is for 90 days horizon, the same horizon used in all type models investigated earlier, allowing easy interpretation of performance comparisons between the various models constructed. The price paths presented in the following charts are the result of average of 10,000 simulated paths.

As is seen, this model accounts for the time-varying variation present in electricity prices. The simulated prices also seem to be fluctuating around long-run mean levels. However, the model fails to capture the spikes prevalent in electricity price data.

Analysis of formal forecast accuracy measures as described in the following table reveals that the benchmark model performs surprisingly well in short-term forecasting. This model

outperforms all other models employed in this study for each market region of the NEM except South Australia (SA) measured by root mean square errors (RMSE). The AR-GARCH model used in this study has RMSE scores of 0.328 for New South Wales (NSW), 0.238 for Victoria (VIC), 0.336 for Queensland (QLD), 0.475 for SA and 0.605 for Tasmania (TAS). The Markov Regime-Switching model with spike regime modelled via Log-Gaussian distribution has superior RMSE values for the SA region. This model also performs remarkably well in terms of its Theil's U statistic indicating that the proposed model is as good as the naive model for all regions of NEM. This model has a Theil's U statistic of 0.048 for NSW, 0.035 for VIC, 0.053 for QLD 0.069 for SA and 0.085 for TAS.

Table 7 Forecast Accuracy Measures of Benchmark Model

	NSW	VIC	QLD	SA	TAS
Mean error	-0.02573	-0.09435	-0.18307	-0.12713	-0.27293
Mean square error	0.10766	0.05666	0.1129	0.22619	0.36621
Root mean square error	0.32812	0.23804	0.33600	0.4756	0.60515
Mean absolute error	0.18430	0.19122	0.20202	0.26372	0.44279
Mean percentage error	-1.44263	-3.25139	-8.0017	-36.9556	-9.78207
Mean absolute percentage error	5.12366	5.83328	8.57096	40.55399	12.77883
Theil's U	0.04849	0.03525	0.05349	0.06926	0.08525

Source: Author's calculations.

SIMULATION OF AR(1)-GARCH(1) MODEL WITH MATLAB'S GARCHSIM FUNCTION

MATLAB's garchsim function simulates sample paths of series given specifications for the conditional mean and variance of a univariate time series. It simulates a sample path with multiple observations for the series, innovations, and conditional standard deviation processes. Innovations represents a mean zero, discrete-time stochastic process whilst the series is the

dependent stochastic process and follows the conditional mean specification of general ARMA form defined in model specification.

The function generates Monte Carlo sample paths for both conditional mean and variance models. Monte Carlo simulation is the process of generating independent, random draws from a specified probabilistic model. When simulating time series models, one draw (or, realisation) is an entire sample path of specified length N, y_1, y_2, \dots, y_N . When you generate a large number of draws, say M , you generate M sample paths, each of length N . Conditional mean models specify the dynamic evolution of a process over time through the conditional mean structure.

Perform Monte Carlo simulation of conditional mean models by:

1. Specifying any required pre-sample data (or use default pre-sample data).
2. Generating an uncorrelated innovation series from the specified innovation distribution.
3. Generating responses by recursively applying the specified AR and MA polynomial operators. The AR polynomial operator can include differencing.

For example, consider an AR(2) process,

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \quad (8)$$

Given pre-sample responses y_0 and $y - 1$, and simulated innovations $\varepsilon_1, \dots, \varepsilon_N$ realizations of the process are recursively generated:

$$y_1 = c + \phi_1 y_0 + \phi_2 y_{-1} + \varepsilon_1 \quad (9)$$

$$y_2 = c + \phi_1 y_1 + \phi_2 y_0 + \varepsilon_2 \quad (10)$$

$$y_3 = c + \phi_1 y_2 + \phi_2 y_1 + \varepsilon_3 \quad (11)$$

$$y_4 = c + \phi_1 y_{N-1} + \phi_2 y_{N-2} + \varepsilon_N \quad (12)$$

For an MA (12) process, e.g.,

$$y_{t1} = c + \varepsilon_t - \emptyset_1 \varepsilon_{t-1} - \cdots - \emptyset_{12} \varepsilon_{t-12} \quad (13)$$

There is a need for 12 pre-sample innovations to initialise the simulation. By default, simulate sets pre-sample innovations equal to zero. The remaining N innovations are randomly sampled from the innovation process.

Conditional variance models specify the dynamic evolution of the variance of a process over time. Perform Monte Carlo simulation of conditional variance models by:

1. Specifying any required pre-sample data (or use default pre-sample data).
2. Generating the next conditional variance recursively using the specified conditional variance model.
3. Simulating the next innovation from the innovation distribution (Gaussian or Student's t) using the current conditional variance.

For example, consider a GARCH(1,1) process without a mean offset, $\varepsilon_t = \sigma_t z_t$ where z_t either follows a standardised Gaussian or Student's t distribution and

$$\sigma_t^2 = \kappa + \gamma_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 \quad (14)$$

Suppose that the innovation distribution is Gaussian. Given pre-sample variance σ_0^2 and pre-sample innovation ε_0 realisations of the conditional variance and innovation process are recursively generated:

$$\sigma_1^2 = \kappa + \gamma_1 \sigma_0^2 + \alpha_1 \varepsilon_0^2 \quad (15)$$

Sample ε_1 from a Gaussian distribution with variance σ_1^2

$$\sigma_2^2 = \kappa + y_1 \sigma_1^2 + \alpha_1 \varepsilon_1^2 \quad (16)$$

Sample ε_2 from a Gaussian distribution with variance σ_2^2

$$\sigma_N^2 = \kappa + y_1 \sigma_{N-1}^2 + \alpha_1 \varepsilon_{N-1}^2 \quad (17)$$

Sample ε_N from a Gaussian distribution with variance σ_N^2 .

The next chapter describes the first continuous-time model of this thesis as applied to forecasting electricity prices in NEM. As Geometric Brownian Motion (GBM) forms the part of other continuous-time models of mean-reverting and mean-reverting jump diffusion models via its Brownian motion property, its inclusion in this study is thought to be appropriate. Consequently, it is presented first.

CHAPTER 6 - GEOMETRIC BROWNIAN MOTION

Diffusion-type Stochastic Differential Equations (SDE) have been the standard approach to modelling price processes that are stochastic in nature. The Geometric Brownian Motion (GBM) is the simplest and most common example of a diffusion-type SDE. Many economic analyses have used implicit and explicit assumptions that a quantity that changes over time with uncertainty follows a GBM (i.e Black-Scholes options pricing formula).

The GBM process, which was introduced to finance by Samuelson (1965) (sometimes also called the lognormal growth process) has gained wide acceptance as a valid model for the growth in the price of financial instruments over time. Under this model, the Black-Scholes formulas for pricing European call and put options, as well as their variations for a few of the more complex derivatives, provide relatively simple analytical evaluation of asymmetric risks.

INTRODUCTION TO BROWNIAN MOTION PROCESS

A stochastic process $\{z(t), t \geq 0\}$ follows a Brownian motion process if it exhibits the following properties;

- I. The change in the value of z , (Δz) , over a time interval of length Δt is proportional to the square root of Δt where the multiplier is random. Specifically, $\Delta z = z(t + \Delta t) - z(t) = \varepsilon\sqrt{\Delta t}$, where ε is a standard normal variable. Hence values of Δz follow a Gaussian distribution with mean 0 and variance equal to the change in time over which Δz is measured.
- II. The changes in the value of $z(t)$ for any two non-overlapping intervals of time are independent.

Using ordinary calculus where it is typical to proceed from small changes to the limit as the small changes come closer to zero; the Wiener process³⁰ is the limit as $\Delta t \rightarrow 0$ of the process described above for $z(t)$.

The standard Brownian motion process has a drift rate of zero and a variance of one. The drift rate of zero means that the expected value of z at any future time is equal to the current value. The variance of one means that variance of the change in z in a time interval of length t is equal to T .

GEOMETRIC BROWNIAN MOTION PROCESS

An SDE is represented by an equation of the form:

$$dX_t = a(t, X_t)dt + b(t, X_t)dW_t \quad (18)$$

with a deterministic component defined by the function $a(t, X_t)$, the instantaneous drift is defined by function $b(t, X_t)$. The stochastic differential, dW_t , represents an infinitesimal increment of Brownian motion. Stochastic component of this differential, W_t , is called the Wiener process.

The formal definition of the process in Equation 18 comes from writing down its integral representation as follows;

³⁰ A Wiener process is a type of Markov process in which the mean change in the value of the variable is zero with the variance of change is equal to one per unit of time. A Markov process is a particular type of stochastic process where only the present value for a variable is relevant for predicting the future. In Markov process, the past history of the variable and the way the present emerged from the past are irrelevant. This process was first applied in biology to describe the motion of a particle that is subject to a large number of small molecular shocks and was called Brownian motion (Hull, 2000) and the mathematical definition of the process was later developed by Wiener (Ross, 1999).

$$X_t = X_{t_0} + \int_{t_0}^t a(s, X_s) ds + \int_{t_0}^t b(s, X_s) dW_s \quad (19)$$

This representation defines the right-hand stochastic integral in terms of a limit of sums including finite Brownian increments. This leads to the Ito calculus. A straightforward application of Ito's Lemma yields the solution;

$$X(t) = e^{\log x_0 + \hat{\mu}t + \sigma W(t)} = x_0 e^{\hat{\mu}t + \sigma W(t)} \quad (20)$$

where $\hat{\mu} = \mu - \frac{1}{2}\sigma^2$ and hence $X(t)$ is lognormally distributed, with

$$E(X(t)) = x_0 e^{\mu t} \quad (21)$$

$$VAR(X(t)) = x_0^2 e^{2\mu t} (e^{\sigma^2 t} - 1) \quad (22)$$

$$f(t, x) = \frac{1}{\sigma x \sqrt{2\pi t}} e^{-(\log x - \log x_0 - \hat{\mu}t)^2 / 2\sigma^2 t} \quad (23)$$

for $p \in [0,1]$ the p-th percentile is $x_0 e^{\hat{\mu}t + N^{-1}(p)\sigma\sqrt{t}}$

A solution to Equation 20 is called a stochastic process, which can be thought of as being indexed by t and the different realizations of Brownian motion. Hence, for a fixed t , X_t is then a random variable and for a fixed realization of Brownian motion, one obtains a sample path for X_t .

As SDEs with explicit solutions are rare, unlike Ordinary Differential Equations (ODE), there are a variety of methods for approximating solutions using discretization³¹. Euler's method is

³¹Discretization is the process of transferring continuous models and equations into discrete counterparts. This process is usually carried out as a first step toward making them suitable for numerical evaluation and implementation.

the simplest and is a straight forward extension of the ODE technique. For a discretization step size of Δt and $t_n = t_0 + n\Delta t$ the Euler's method approximation to $Y_n \cong X_{t_n}$, is given as follows;

$$Y_{n+1} = Y_n + a(t_n, Y_n)\Delta t + b(t_n, Y_n)\Delta W_n \quad (24)$$

where $Y_0 = X_0$ is the initial condition³². Increments of the Wiener process, $\Delta W_n = W_{t_{n+1}} - W_{t_n}$, can then be simulated using pseudo randomly generated normal variants, since by the defining properties of Brownian motion, $\Delta W_n^j = N(0, \Delta t)$ for $j = 1, \dots, M$. Each discretization step for Euler's method thus requires M random normals. This is essentially a simulation process.

However, generating forecast of a particular SDE via a large number of simulations has its drawbacks. The forecast value as the result of these simulations become the mean of these large number of repetitions and the meaning process naturally smooths out the fluctuations of the particular SDE's price path.

MODELLING ELECTRICITY PRICES WITH GBM PROCESS

This study applies the following GBM model to electricity prices from each region of NEM.

$$dX_t = \mu X_t dt + \sigma X_t dW_t \quad (25)$$

³² Detailed mathematical representation of Euler Scheme is described in Appendix 4.

The first term of the equation implies that X has an expected drift rate of μ per unit of time, whereas the second term can be regarded as adding variability to the path followed by X . The amount of this variability is σ times the differential of the Brownian motion process. Thus for a small interval of time, the change in the value of X is given by;

$$\Delta x = \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t} . \quad (26)$$

Note that Δx has a Gaussian distribution with mean $\mu \Delta t$ and variance $\sigma^2 \Delta t$.

Modelling electricity prices with GBM processes does not reflect the main stylised features of electricity prices series. The GBM process by construction follows a price trajectory with an increasing trend (as $dX_t = \mu X_t dt + \sigma X_t dW_t$ suggests) and it cannot capture large jumps prevalent in electricity prices. Therefore, the forecast generated with GBM processes are expected to have large errors. However, the inclusion of such processes in this thesis is appropriate as the GBM process is the foundation for other SDE processes that can capture the stylised features of electricity prices.

PARAMETERISATION OF THE GBM PROCESS

The parameters of the GBM model, namely the annualized mean and measure of volatility processes were estimated using the in-sample data of the study. The constructed GBM models for each region of the NEM produced sample paths of simulated state variables driven by Brownian motion sources of risk over consecutive observation periods.

SIMULATION OF THE GBM PROCESS

GBM like any other SDEs needs to be simulated in order to estimate the quantities of interest as its analytical solutions are rare. More technically, it can be put that the aim is to compute $E[f(X_T)]$ where X_t satisfies:

$$dX_t = \mu X_t dt + \sigma X_t dW_t \quad (27)$$

The solution to above equation is given by;

$$X_t = X_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)T + \sigma W_t\right) \quad (28)$$

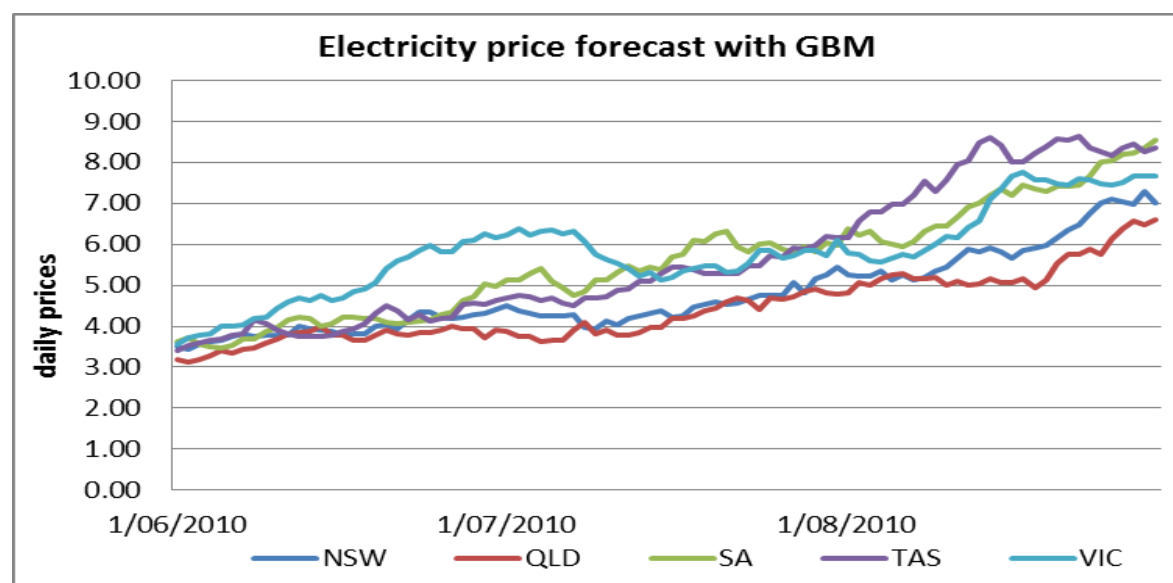
Recognize that X_t depends on the Brownian motion only through the Brownian motion's terminal value, W_t , which introduces randomness to the function. This implies that even if the analytical solution of the equation cannot be achieved, the estimation of it is possible by simulating W_t directly. For that reason, the GBM models constructed for each region of the NEM in this study are simulated via MATLAB's `simBySolution` method. MATLAB's `simBySolution`³³ method is an approximate analytic solution obtained by applying the Euler Scheme to the transformed (using Ito's formula) logarithmic process. Appendix 3 gives a brief review of stochastic calculus and Ito's Lemma along with some thoughts on solving stochastic differential equations while Appendix 4 provides an overview on simulating SDEs.

³³ The `simBySolution` method simulates n-sample paths of n-correlated state variables, driven by Brownian motion sources of risk over n-consecutive observation periods, approximating continuous-time Hull-White/Vasicek (HWV) and Geometric Brownian Motion (GBM) short-rate models by an approximation of the closed-form solution. Specifically, the architecture allows one to simulate correlated paths of any number of state variables driven by a vector-valued Brownian motion of arbitrary dimensionality, thereby approximating the underlying multivariate continuous-time process by a vector-valued stochastic difference equation.

With the assistance of this method, simulation of the state vector X_t using an approximation of the closed-form solution of diagonal-drift models is performed. It is important to note that when evaluating the expressions, Matlab's `simBySolution` assumes that all model parameters are piecewise-constant over each simulation period. In general, this is not the exact solution to the models, because the probability distributions of the simulated and true state vectors are identical only for piecewise-constant parameters. When parameters are piecewise-constant over each observation period, the simulated process is exact for the observation times at which X_t is sampled (Matlab, 2012).

The results of the GBM simulation over a three month horizon (90 days) for each region of the NEM are illustrated in the following graph (for the out-of-sample period of this study that is 01/06/2010 to 29/08/2010). The price path is for 90 days horizon, which is the typical duration of Asian options on electricity spot prices widely used in Australian electricity over-the-counter derivatives market. The price paths presented in the following charts are the result of taking the average of 10,000 GBM simulated paths.

Figure 11 Price Forecast with GBM Specification for All Regions of NEM

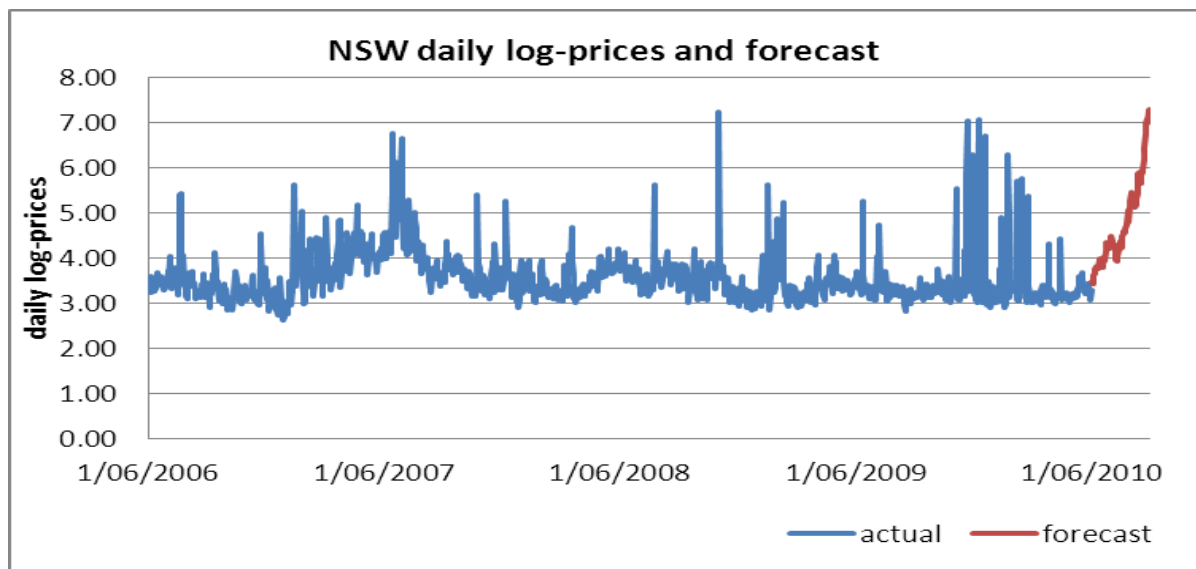


Source: Author's calculations.

As expected, simulated price paths tend to increase for each region of the NEM by time interval with some degree of randomness, which is caused by the Weiner process. This is an undesirable characteristic of any forecast model as electricity prices in NEM tend not to increase with time; in fact the prices tend to revert to its long-term mean over time. This property of the GBM based forecast can be seen in the Figures 11 to 15, which depict the forecast values of the price series by each region of the NEM.

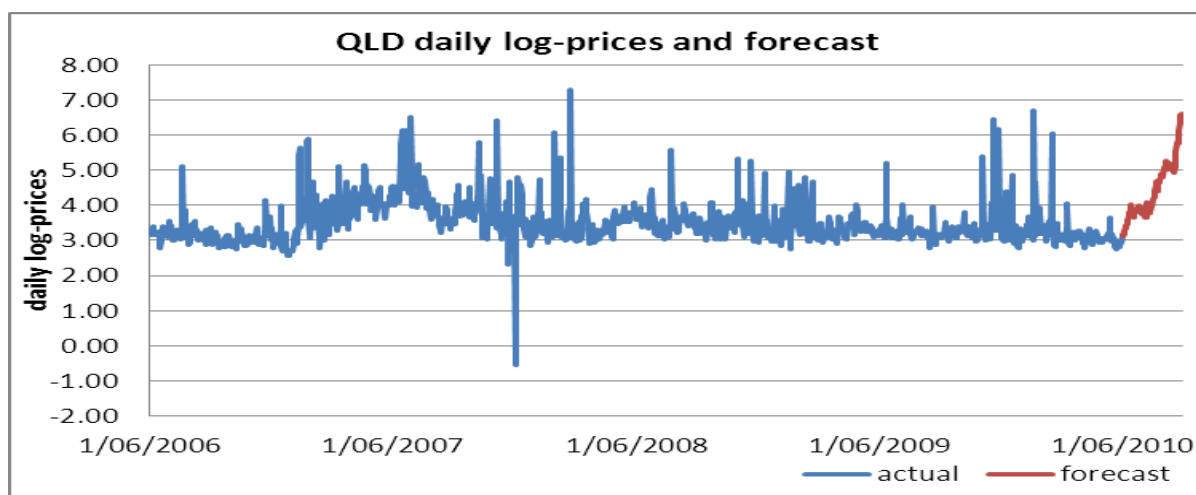
Therefore, the empirical evidence of modelling electricity prices with pure GBM process in NEM strengthens the unsuitability of modelling electricity prices with this process. This is due to the fact that simulations based on GBM over a 90 days horizon appear to overestimate the true price paths of the actual electricity prices for all regions of the NEM.

Figure 12 Price Forecast with GBM Specification for NSW



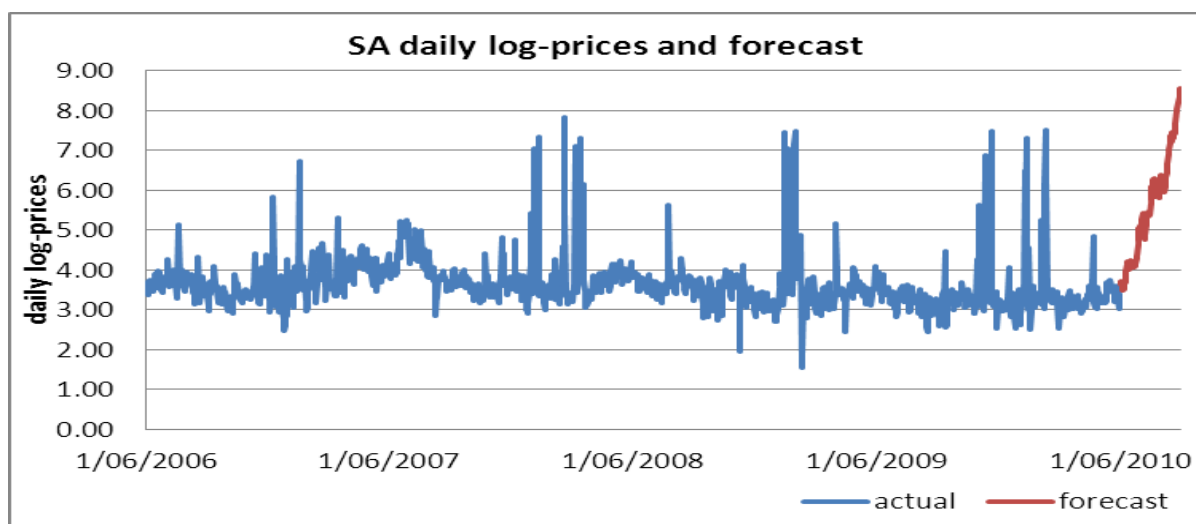
Source: Author's calculations.

Figure 13 Price Forecast with GBM Specification for QLD



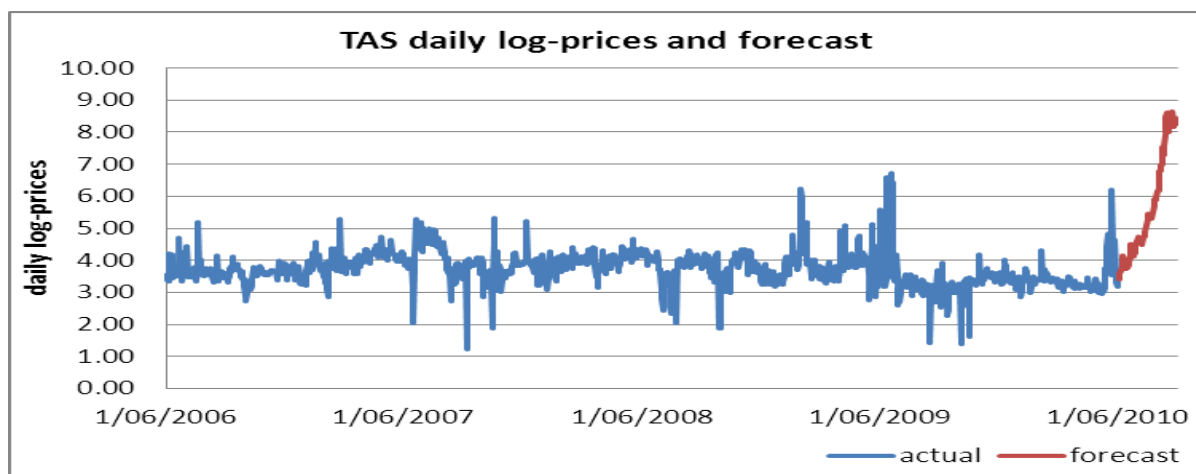
Source: Author's calculations.

Figure 14 Price Forecast with GBM Specification for SA



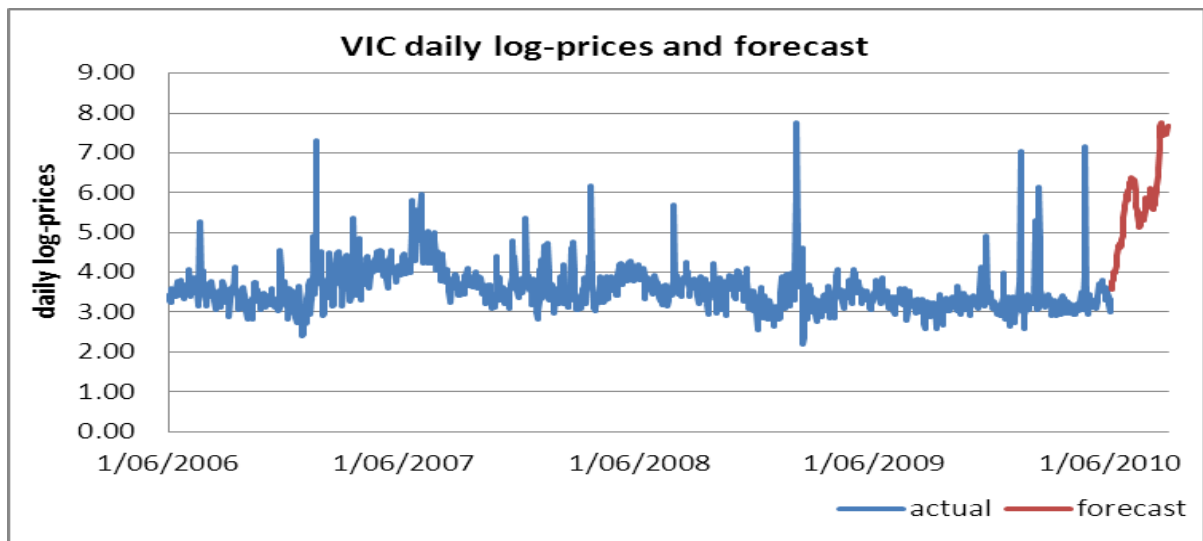
Source: Author's calculations.

Figure 15 Price Forecast with GBM Specification for TAS



Source: Author's calculations.

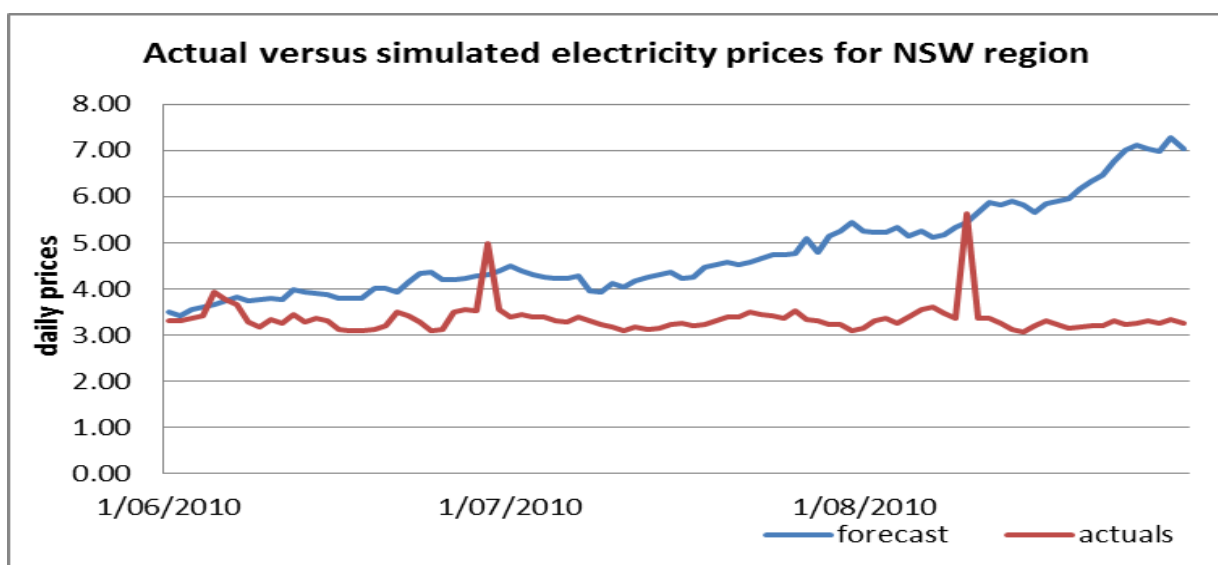
Figure 16 Price Forecast with GBM Specification for VIC



Source: Author's calculations.

Understanding this undesirable property of GBM process in forecasting electricity prices can further be enhanced by examining the following chart, which presents a visual comparison of actual log prices in the New South Wales (NSW) region along with the simulated prices based on GBM process. It is evident that the pure GBM model fails to capture two spikes in a 90 days period and tends to overshoot the actual price path over the study horizon.

Figure 17 Comparative Performance of GBM forecast



Source: Author's calculations.

EVALUATION OF FORECAST PERFORMANCE

This section evaluates the forecasting performance of the GBM model. The purpose of this section is to see if GBM model's forecast errors are within the reasonable limit of expectations or whether these errors are unreasonably large and require an improvement in the statistical models and process of producing these forecasts.

Further to graphical evidence set out earlier showing the inappropriateness of forecasting electricity prices with GBM process, Table 8 demonstrates the forecast accuracy statistics for each region of the NEM.

Table 8 Comparative Forecast Performance Measures

	GBM Model					Benchmark Model				
	NSW	VIC	QLD	SA	TAS	NSW	VIC	QLD	SA	TAS
ME	-1.420	-2.482	-1.389	-2.223	-2.280	-0.02	-0.094	-0.183	-0.127	-0.272
MSE	3.122	7.407	2.717	7.323	8.292	0.107	0.056	0.112	0.226	0.366
RMSE	1.767	2.721	1.648	2.706	2.879	0.328	0.238	0.336	0.475	0.605
MAE	1.446	2.482	1.389	2.251	2.286	0.184	0.191	0.202	0.263	0.442
MPE	-51.102	-75.794	-48.612	-135.722	-70.415	-1.442	-3.251	-8.001	-36.955	-9.782
MAPE	43.997	75.794	48.612	136.722	70.548	5.123	5.833	8.570	40.553	12.778
Theil's U	0.213	0.294	0.217	0.296	0.308	0.048	0.035	0.053	0.069	0.085

Source: Authors calculations. General rule: smaller the magnitude more accurate the model is.

It is evident that the root mean square error (RMSE) based on forecast with GBM process is the lowest for the Queensland (QLD) region (1.648) followed by the NSW region (1.767). The values of RMSE for other regions of the electricity network are found to be 2.721 for Victoria (VIC), 2.706 for South Australia (SA) and 2.879 for TAS. The RMSE values are much higher than the forecast accuracy statistics based on the simple AR(1)-GARCH(1) specification as the

benchmark model. The benchmark model's RMSE is quantitatively higher than the basic GBM model for each market region of the NEM measured by RMSE. The AR-GARCH model used in this study has RMSE scores of 0.328 for New South Wales (NSW), 0.238 for Victoria (VIC), 0.336 for Queensland (QLD), 0.475 for SA and 0.605 for Tasmania (TAS).

In brief, empirical evidence suggests that a simple GBM model lacks the ability to model two main characteristics of the electricity prices in NEM; one being the tendency to revert to long-run mean levels and the second being the inability to characterize the spikes. When a price jump occurs, the pure GBM process assumes that the new price level is a normal event and it would proceed randomly via a continuous diffusion process with no memory of prior price levels. Modelling electricity prices via pure GBM process yields a distribution of future prices that has a variance that increases without bound as the horizon increases.

These results are in line with Johnson and Barz (1999) who concluded that arithmetic and geometric Brownian processes are unsuitable in modelling electricity prices. Johnson and Barz (1999) evaluated the effectiveness of a number of SDE based models in electricity price modelling, including arithmetic and Geometric Brownian motion processes along with mean-reverting diffusion processes known as Ornstein-Uhlenbeck process (first proposed by Vasicek (1977). Johnson and Barz (1999)) concluded that the geometric mean reverting jump-diffusion model gave the best performance and all models without jumps (arithmetic and geometric Brownian processes) were inappropriate in modelling electricity prices. This was due to the fact that when a price spike occurred, GBM would assume that the new price level is a normal event and it would proceed randomly via continuous diffusion process with no consideration of prior price levels.

The first continuous-time model examined in this thesis is the GBM model. As GBM forms a part of other SDE models of mean-reverting, mean-reverting jump diffusion and regime switching models via its Brownian motion property, its inclusion in this study is thought to be appropriate. However, a more realistic and accurate model from an SDE family of models that can reflect the mean-reverting characteristic of electricity prices in NEM needs to be tested. Such a model is presented in the next chapter.

CHAPTER 7 - MEAN REVERTING MODEL

A number of studies have indicated that electricity prices tend to revert to its long-term equilibrium price level (see for example; Philipovic 1998, Clewlow and Strickland 2000, Lucia and Schwartz 2002). According to Deng (2000) the economic intuition behind the mean reverting process is that when the price of electricity is high its supply tends to increase thus putting a downward pressure on the prices whereas when the price is low the supply of electricity tends to decrease thus providing an upward lift to prices.

Hence, mean reversion can be viewed as the statistical phenomenon stating that the greater the deviation of a random variant from its mean, the greater the probability that the next measured variant will deviate far less. Kaminski (1999), Lucia and Schwartz (2002) and Huisman and Mahieu (2003) showed the existence of this phenomena in electricity prices with rescaled range analysis, de-trended fluctuation analysis, average wavelet coefficient and periodogram regression methods.

Hence, modelling electricity prices with Mean-Reverting models that have in-built mean reverting characteristics are naturally attractive as they can explicitly capture the economic phenomena outlined earlier that when prices are too high, supply of electricity will increase, producing equilibrium in prices. On the contrary, when prices are too low, there would be a decrease in supply of electricity pushing prices back to their long-term equilibrium³⁴.

³⁴ At the wholesale level, this is due to the generator diversity as explored in Chapter 2.

AN INTRODUCTION MEAN-REVERTING MODEL

Although the mean-reverting phenomenon appears to violate the definition of independent events, it simply reflects the fact that the probability density function $P(x)$ of any random variable x , by definition, is non-negative over every interval and integrates to one over the interval $(-\infty, \infty)$. Thus, as x moves away from the mean, the proportion of the distribution that lies closer to the mean than x increases continuously (Weisstein, 2012) Formally, this can be stated as;

$$\int_{\mu-1}^{\mu+i} P(x)dx > \int_{\mu-j}^{\mu+j} P(x)dx \quad (29)$$

for $i > j > 0$

Mean-Reverting models originally proposed for specifying interest rate dynamics by Vasicek (1977). Henceforth they were commonly referred to as Vasicek models. This model is also referred to as arithmetic Ornstein-Uhlenbeck process and described by the following Stochastic Differential Equation (SDE):

$$dX_t = (\mu - \beta X_t)dt + \sigma dW_t \quad (30)$$

$$= \beta(L - X_t)dt + \sigma dW_t \quad (31)$$

where W_t is a Wiener process that models the continuous inflow of randomness into the system. The standard deviation parameter, σ , determines the volatility of the price process and in a way characterises the amplitude of the instantaneous randomness inflow. The parameters β (speed of mean-reversion), L (long-term mean level) and W_t , together with X_t , characterize the price dynamics.

Mean reversion in this chapter is modelled by having a drift term that is negative if the spot electricity prices are higher than the mean reversion level and positive if it is lower. This representation is a one-factor model and it reverts to the long-term mean $L = \frac{\mu}{\beta}$ with β being the magnitude of the speed of mean-reversion whereas the second term in the above representation is the volatility of the process.

The explicit solution to the SDE represented in Equation (32) between any two periods s and t , with $0 \leq s < t$, can be derived from the solution to the general Ornstein-Uhlenbeck SDE representation as follows;

$$X_t = L(1 - e^{-\beta(t-s)}) + x_s e^{-\beta(t-s)} + \sigma e^{-\beta t} \int_s^t e^{\beta u} dW_t \quad (32)$$

and the discrete time version of this equation, on a time grid $0 = t_0, t_1, t_2, \dots$ with time step $\Delta t = t_i - t_{i-1}$ is given by;

$$x(t_i) = c + bx(t_{i-1}) + \delta \varepsilon(t_i) \quad (33)$$

where the coefficients are; $c = L(1 - e^{-\beta \Delta t})$ and $b = e^{-\beta \Delta t}$ and $\varepsilon(t)$ is a Gaussian white noise.

Finally, the long term variance (future trajectories of X_t converge around the long term mean with such variance after a long time) can be derived via Ito isometry as;

$$\delta = \sigma \sqrt{(1 - e^{-2\beta \Delta t})/2\beta} \quad (34)$$

where if one uses the Euler scheme to discretise the equation this would lead to $\delta = \sigma\sqrt{\Delta t}$, which is the same (first order in Δt) as the above.

MODELLING PRICES IN NEM WITH THE MEAN-REVERTING MODEL

Higgs and Worthington (2010) were the first to model electricity prices in NEM by mean-reverting process. The deterministic component of their model accounted for predictable regularities while the stochastic component was derived following de Jong (2006). The main conclusion of this study was the existence of strong mean reversion in electricity prices after a price spike than in a normal period, which is in parallel with international experience (i.e. Kaminski, 1997), and price volatility that is more than fourteen times higher in spike periods than in normal periods.

This chapter builds upon the study of Higgs and Worthington (2010) by modelling the electricity wholesale prices with Mean-Reverting model. Further, this chapter also presents an application of Monte Carlo simulation in generating forecast values based on the mean-reversion process. It then compares the forecast accuracy measures of the simulated model with the chosen benchmark model of this thesis.

PARAMETERISATION OF THE MEAN-REVERTING MODEL

The discrete form of the mean-reverting process is used to calibrate the model developed in this chapter. This discrete form is an exact formulation of an AR(1) process. Having $0 < b < 1$ when $0 < \beta$ implies that this AR(1) process is stationary and mean-reverting to a long-term mean given by L . This can also be confirmed by computing the mean and variance of the process as the distribution of X_t is Gaussian, therefore it is characterized by its first two

moments. The conditional mean and variance of X_t given $x(s)$, in which $x(s)$ is the initial value at period s , can be derived from the Ornstein-Uhlenbeck SDE as follows;

$$E(X_t) = L + (x_s - L)e^{-\beta(t-s)} \quad (35)$$

$$Var(X_t) = \frac{\sigma^2}{2\beta}(1 - e^{-2\beta(t-s)}) \quad (36)$$

Consequently, as time increases, the mean tends to the long-term value L and the variance remains bounded (unlike geometric Brownian motion), implying mean reversion. In other words, the long-term distribution of the Ornstein-Uhlenbeck is stationary and is Gaussian with L as mean and $\sqrt{\sigma^2/2\beta}$ as standard deviation.

Modelling electricity prices with an SDE representation that captures the mean-reversion characteristics of the price process is a relatively simple task (as compared to more advanced SDE specifications like Jump-Diffusion models) as there are only three parameters that need to be estimated. These parameters are;

1. Speed of mean reversion,
2. Long-run mean and,
3. Measure of the volatility process.

There are a number of techniques that can be used to estimate the speed of mean reversion (i.e. weighted or unweighted autoregression, quasi-maximum likelihood approach, generalised method of moments or methods based on Laplace transform) however; the ordinary least squares method is the only method that directly estimates the mean-reverting parameter whereas all other methods are based on the joint estimation of all parameters (Gourieroux and

Monfort, 2010). Therefore, in estimating the speed of mean reversion of the mean-reverting prices process, ordinary least squares method is used in this thesis.

This is attained by performing a linear regression between the log prices and their first difference as follows;

$$\frac{dX_t}{dt} = -\beta X_t + \beta L + \frac{\sigma}{dt} dW_t \quad (37)$$

The speed of mean reversion, β , is calculated from the coefficients of a linear fit (b) in Equation 25, scaled by the time interval parameter³⁵. The price data used in this study are average hourly pool price observations sourced directly from AEMO for the period of 01/06/2006 to 29/08/2010. The data from 01/06/2006 to 31/05/2010 are used to estimate the parameters of the models while the period from 01/06/2010 to 29/08/2010 are used to derive out-of-sample forecast accuracy statistics.

In this unmodified mean-reversion specification as in GBM specification, volatility term is set to a constant, despite the fact that empirical evidence suggests that electricity prices exhibit heteroskedasticity³⁶. The analysis of the descriptive statistics, as summarized in Chapter 2, demonstrates that the distributions of prices are significantly non-Gaussian for all regions of NEM. The price series in all of the electricity regions are positively skewed and leptokurtic. This extreme fat-tailed characteristic is consistent with the findings of earlier studies (Huisman

³⁵ See Yu (2009) for a detailed explanation of estimating mean-reversion parameters in continuous time.

³⁶ The speed at which electricity prices revert to their long run levels may depend on several factors such as the weather, magnitude and direction of the price shocks.

and Huurman 2003, Higgs and Worthington 2005, Thomas et al. 2011) and is likely to be driven by the prevalence of extremely high prices.

Table 9 shows the parameter estimates of the Mean-Reverting model for each region of the NEM. As is seen, the mean reversion rate, β , ranges between 99.5 to 157.3. Higher mean reversion rates indicate a region where electricity prices are relatively more volatile. Further, mean reversion parameters are positive for all regions of NEM corresponding to a price process where prices decline to their long-run mean levels after rising to very high ranges as denoted by L .

Table 9 Parameter Estimates of the Mean-Reverting Model

	NSW	VIC	QLD	SA	TAS
β	122.411	99.539	138.196	157.253	109.204
L	3.555	3.726	3.477	3.620	3.726
σ	7.783	6.883	8.652	9.783	6.883

Source: Author's calculations.

The speed of mean reversion parameter represents the annualised rate at which the underlying short-term price returns to its expected long-run equilibrium value. Hence the inverse of the speed of mean reversion rate gives the actual time scale over which mean reversion occurs. For example, a mean-reversion rate of 122.411 corresponds to an electricity price process whose price reverts to its expected value over the course of three days as is the case for NSW region³⁷. In fact, the regions of QLD and SA have the greatest mean reversion rates in NEM suggesting

³⁷ Calculated as $365/122.411=2.98$

a relatively shorter number of days for spiked prices to return to long-run mean levels as compared to other regions in NEM.

It is also interesting to note that the regions with higher mean reversion rates have the highest annualised volatility measures, which points to the fact that these regions maybe the least mature electricity markets in NEM. The main implication of this finding with regard to attaining accurate electricity forecast is that this relatively higher volatility term causes greater forecast errors in the models developed in this thesis.

SIMULATION OF THE MEAN-REVERTING MODEL

In order to estimate the future quantities of interest, this chapter uses Monte-Carlo simulations.. The need for Monte Carlo simulations to estimate a future quantity of interest is due to the difficulties of finding an explicit solution to the SDE that represents the mean-reversion phenomena (Kloeden and Platen, 1992).

Suppose that the computation of $E[f(X_T)]$ where X_t satisfies:

$$dX_t = \beta(L - X_t)dt + \sigma dW_t \quad (38)$$

where the distribution of X_t is unknown. The solution to the equation above is given by the following representation;

$$X_T = X_t + \exp(\beta T) [X_0 - X_t] + \sigma \exp(\beta T) \int_0^T \exp(\beta s) dW_s \quad (39)$$

It is important to note that unlike the case in solutions to GBM process, X_T now depends on the entire path of the Brownian motion. This means that the computation of an unbiased estimate of the future quantities of X_T by first simulating the entire path of the Brownian motion is not possible since it is only possible to simulate the latter at discrete intervals of time. In the previous chapter on forecasting electricity prices with Geometric Brownian Motion simulations, it was stated that X_T depends only on the Brownian motion' terminal value, dW_t . In the case of Mean-Reverting SDE representation, the distribution of X_T is assumed to be as Gaussian, which places the issue to the context where the future quantities of X_T can be derived by simulating X_T directly (Glasserman, 2003).

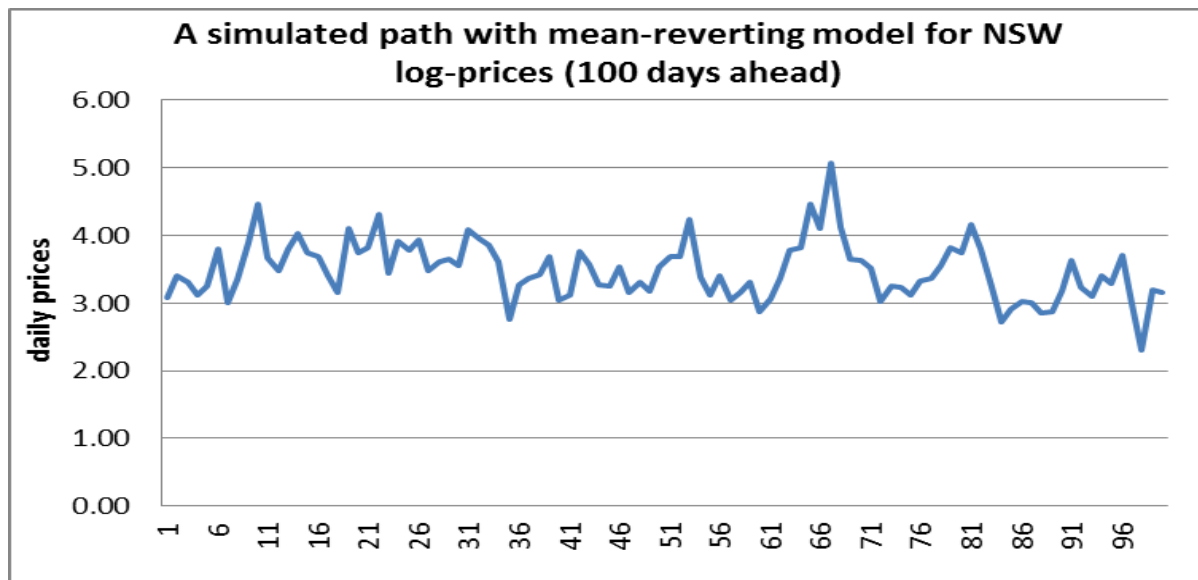
In deriving the Mean-Reverting models developed for each region of the NEM, similar to construction of GBM models as in the previous chapter, MATLAB's Hull-White-Vasicek (HWV) constructor³⁸ is used. This constructor creates and displays HWV objects, which derive from SDE with drift rate expressed in mean-reverting form classes. The state variables in this constructor are driven by Brownian motion sources of risk over consecutive observation periods, approximating continuous-time HWV stochastic processes with Gaussian diffusions (Matlab 2012). In this chapter, the Mean-Reverting models constructed by this constructor are simulated via MATLAB's `simBySolution` method (this method is explained in the preceding section on GBM modeling).

Figure 18 illustrates a sample simulated path generated via a mean-reversion model for NSW region. As is seen, the mean-reversion model generates random prices deviating from the long-run mean. This observation is very different from the earlier findings based on GBM model

³⁸ A constructor is a special function that creates an instance of the class. Typically, constructor methods accept input arguments to assign the data stored in properties and always return an initialized object.

where the existence of mean-reversion does not feature. This finding is expected as random deviations from the mean increase by time in the context of GBM model making it inefficient in modelling electricity prices.

Figure 18 A Simulated Price Path with Mean-Reverting Model



Source: Author's calculations.

Figure 19 shows the electricity price forecast generated by the Mean-Reverting model for all regions of NEM for the same time horizon. Price forecast are based on an implementation of Monte Carlo simulations where the result of the many simulations of the mean-reversion process containing the random variable is averaged, approximating to the real mean.

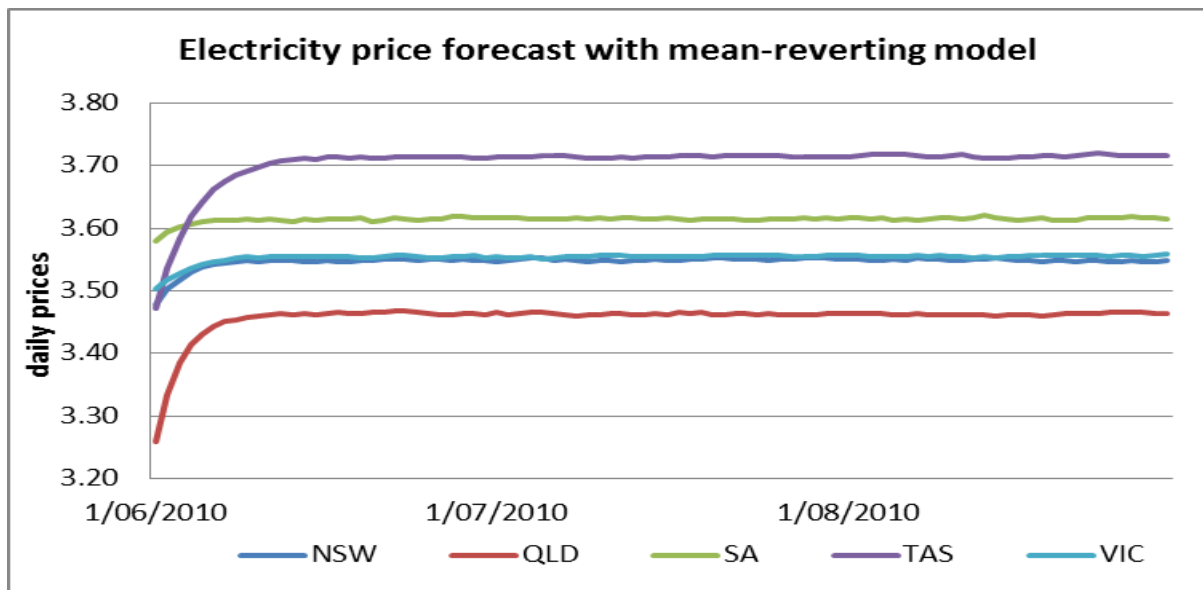
Monte Carlo simulation is a numerical method used to construct probability distributions based on underlying distributions. In a Monte Carlo simulation, random variables are simulated with a random number generator and expected values are approximated by computed averages. It is often employed in finance to solve stochastic differential equations such as the Black Scholes equation for pricing stock options.

The basic tenet of Monte Carlo simulation is that there is a stochastic variable in an equation that can be sampled many times over. In Monte Carlo simulation, one generates a set of suitable sample paths X_T on $[0, T]$. For each sample path, there is a sample path solution to the stochastic differential equation on $[0, T]$. One then estimates the X_T by computing the mean-sum over the large finite set of approximate sample solutions. Appendix 3 gives an overview of Monte Carlo simulations in the context of stochastic differential equations.

Figure 19 shows that simulated price series quickly revert to their long-run equilibrium as indicated by the upward convex move. The upward convex move at the beginning of the forecast horizon shows how long it takes prices to reach their long-run mean levels. As expected, this upward convex is greatest for the regions of QLD and TAS where the mean-reversion parameters were estimated to be the highest.

The main reason why this upward move occurs in the forecast horizon is due to the fact that the in-sample period of the empirical data ends in a period (31/05/2010, Monday) where electricity prices were trading at a lower rate than their long-run mean. Once the predicted prices reach their long-run mean levels, they tend to present rather a constant price level (albeit with minor fluctuations).

Figure 19 Electricity Price Forecast with Mean-Reverting Process for All Regions of NEM



Source: Author's calculations.

The empirical evidence of modelling electricity prices in NEM with Mean-Reverting model strengthens the unsuitability of modelling electricity prices with this stochastic diffusion process. This is due to the fact that simulations based on a mean-reverting model over 90 days horizon seem not to represent the true price paths of the electricity prices as the simulated paths fail to capture the jumps prevalent in the price series (for a visual inspection see Figure 19).

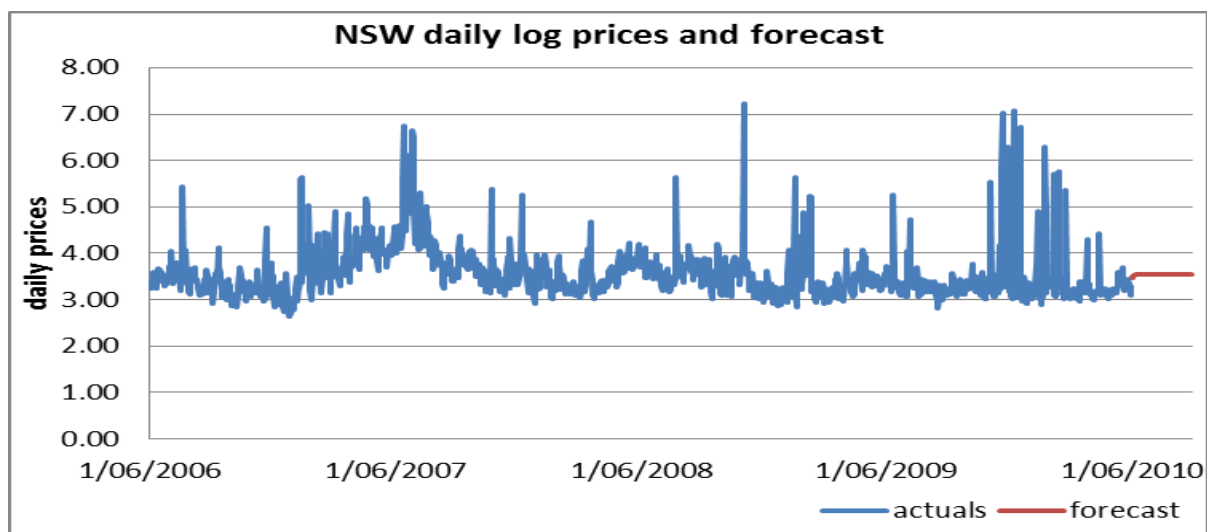
This is attributable to the assumptions of the mean-reverting process where the conditional distribution of the electricity prices is Gaussian, which rather underestimates the large movements in the price series. Another shortcoming of predicting electricity prices with mean-reverting process is that it assumes non-negativity of the prices. The literature in Australian electricity prices as well as international markets pointed out the fact that there are negative price occurrences in the markets.

It may be useful at this point to analyse the historic prices for each region of NEM along with forecast prices depicted in the same graph. The following graphs illustrate the actual and

simulated prices for each region of the NEM. In these figures, the average of 10,000 simulated paths is taken as the point in time forecast value.

As is seen, Mean-Reversion model, first of all, fails to simulate the observed time series characteristics of electricity prices in generating spikes. Secondly, the model also tends to overestimate the general price trend as the forecast values are mostly above the actual prices. The second point is the function of time horizon where forecasts are being based. In other words, rate of mean reversion will result in forecasts to be above mean when the general price levels are relatively lower such as in winter times³⁹.

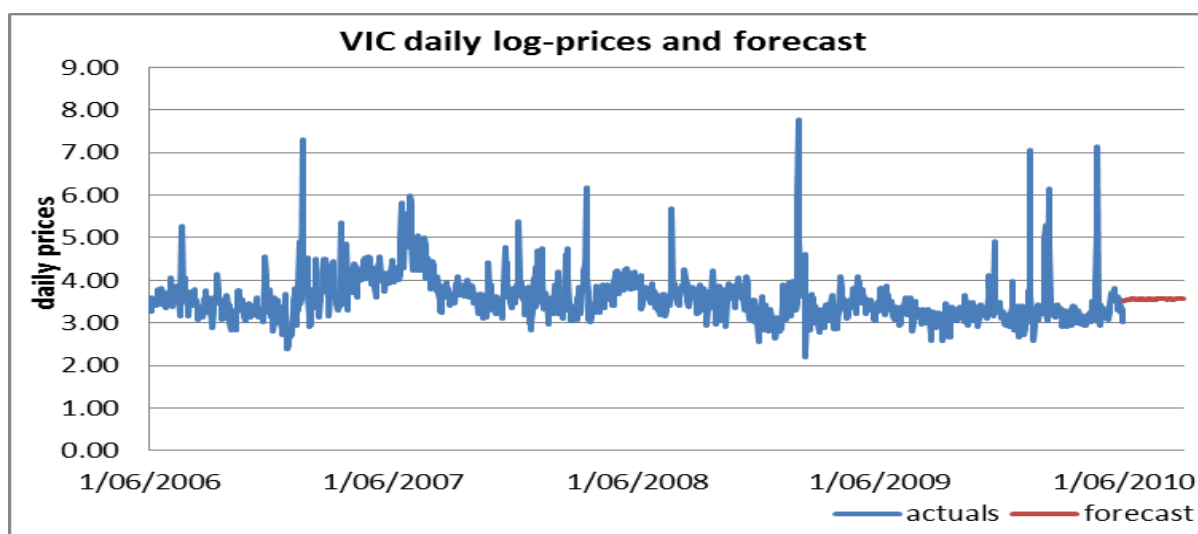
Figure 20 Electricity Price Forecast with Mean-Reverting Model for NSW



Source: Author's calculations.

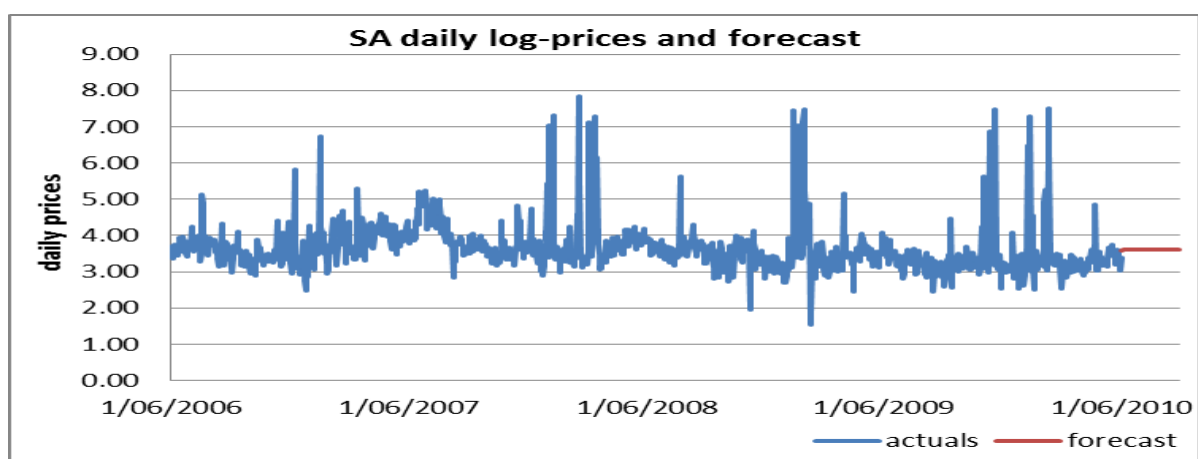
³⁹ It was discussed earlier in this thesis that summer months in Australia coincides with relatively high price levels due to widespread use of air-conditioning by end users and vice versa.

Figure 21 Electricity Price Forecast with Mean-Reverting Model for VIC



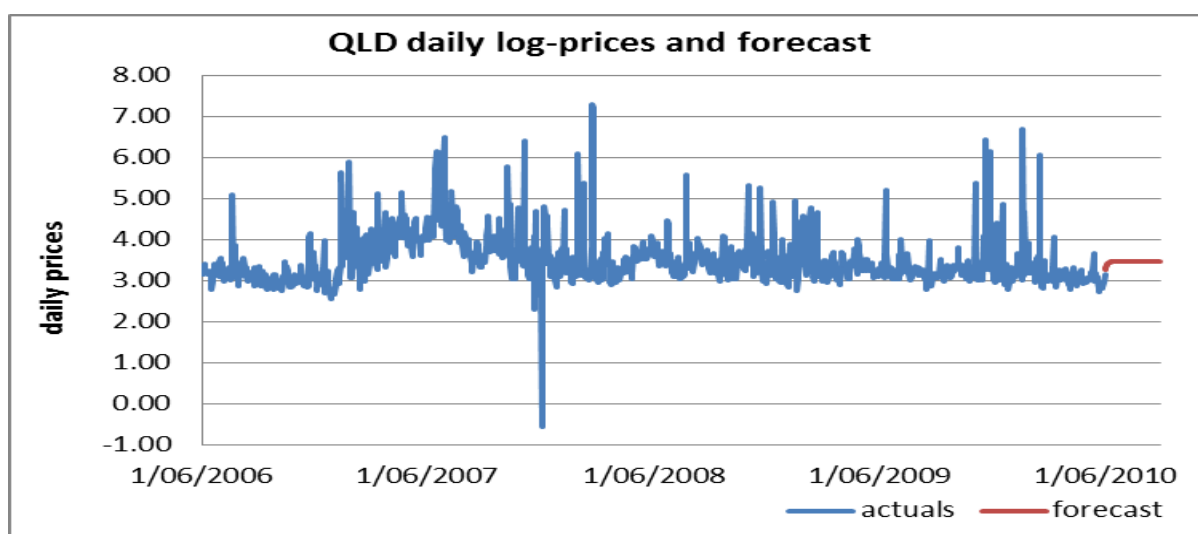
Source: Author's calculations.

Figure 22 Electricity Price Forecast with Mean-Reverting Model for SA



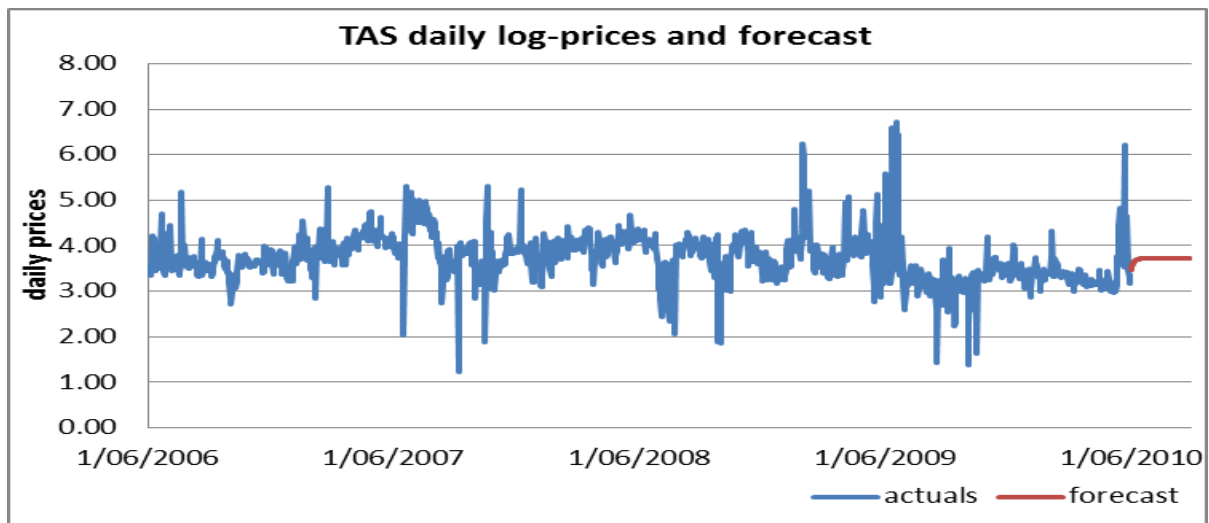
Source: Author's calculations.

Figure 23 Electricity Price Forecast with Mean-Reverting Model for QLD



Source: Author's calculations.

Figure 24 Electricity Price Forecast with Mean-Reverting Model for TAS



Source: Author's calculations.

EVALUATION OF FORECAST PERFORMANCE

In this section evaluation of forecast performance of the Mean-Reverting model is presented. The aim of this section is to observe if the Mean-Reverting model's forecast errors are within the reasonable limit of expectations or whether these errors are unreasonably large and require an improvement in the statistical models and process of producing these forecasts.

Significant improvements can be noticed in this model's fit as compared to the forecast generated with GBM model in the previous chapter. RMSE decreased to about one-sixth in some regions and MPE and MAPE also showed dramatic improvements as compared to the GBM model.

Table 10 Comparative Forecast Performance Measures

	GBM Model					Mean-Reverting Model				
	NSW	VIC	QLD	SA	TAS	NSW	VIC	QLD	SA	TAS
ME	-1.420	-2.482	-1.389	-2.223	-2.280	-0.185	-0.228	-0.414	-0.259	-0.312
MSE	3.122	7.407	2.717	7.323	8.292	0.145	0.101	0.242	0.248	0.382
RMSE	1.767	2.721	1.648	2.706	2.879	0.381	0.318	0.492	0.498	0.618
MAE	1.446	2.482	1.389	2.251	2.286	0.280	0.269	0.414	0.309	0.475
MPE	-51.102	-75.794	-48.612	-135.722	-70.415	-6.249	-7.309	-15.628	-41.385	-10.958
MAPE	43.997	75.794	48.612	136.722	70.548	8.170	8.348	15.628	42.648	13.782
Theil's U	0.213	0.294	0.217	0.296	0.308	0.055	0.046	0.075	0.071	0.086

Source: Authors calculations. General rule: smaller the magnitude more accurate the model is.

To formally assess the accuracy of the forecast generated by the Mean-Reverting model as compared to the benchmark model, accuracy statistics are presented in Table 11 below.

Table 11 Comparative Forecast Performance Measures

	Mean-Reverting Model					Benchmark Model				
	NSW	VIC	QLD	SA	TAS	NSW	VIC	QLD	SA	TAS
ME	-0.185	-0.228	-0.414	-0.259	-0.312	-0.02	-0.094	-0.183	-0.127	-0.272
MSE	0.145	0.101	0.242	0.248	0.382	0.107	0.056	0.112	0.226	0.366
RMSE	0.381	0.318	0.492	0.498	0.618	0.328	0.238	0.336	0.475	0.605
MAE	0.280	0.269	0.414	0.309	0.475	0.184	0.191	0.202	0.263	0.442
MPE	-6.249	-7.309	-15.628	-41.385	-10.958	-1.442	-3.251	-8.001	-36.955	-9.782
MAPE	8.170	8.348	15.628	42.648	13.782	5.123	5.833	8.570	40.553	12.778
Theil's U	0.055	0.046	0.075	0.071	0.086	0.048	0.035	0.053	0.069	0.085

Source: Authors calculations. General rule: smaller the magnitude more accurate the model is

The benchmark AR(1)-GARCH(1) model has been modelled based on the in-sample data of the study reported in this chapter. The results of the AR(1)-GARCH(1) simulations over a three-month horizon (90 days) for each region of NEM are determined as the result of an average of 10,000 simulated paths. The AR(1)-GARCH(1) model has RMSE scores of 0.328

for NSW, 0.238 for VIC, 0.336 for QLD, 0.475 for SA and 0.605 for TAS. This model also performed better in terms of Theil's U statistic, indicating that the proposed model is as good as the naïve model in all the regions of NEM. This model has Theil's U statistic of 0.048 for NSW, 0.035 for VIC, 0.053 for QLD, 0.069 for SA and 0.085 for TAS.

The Mean-Reverting model presented in this chapter reflects findings by Pilipovic (1998), Clewlow and Strickland (2000), Lucia and Schwartz (2002) and Huisman and Mahieu (2003), amongst others, that electricity prices tend to fluctuate around some long-term equilibrium price level, reflecting the marginal cost of producing electricity.

Stochastic diffusion models of this kind that incorporate mean reversion go a long way in capturing the nature of electricity prices; notably their tendency to randomly oscillate away from, and over time back towards a price level determined by the cost of production. The advantages of mean reversion spot forecast and option pricing models over their Black-Scholes counterparts have the potential as traders and risk managers are able to assign greater accuracy to their models or at a minimum to their model assumptions.

In conclusion, evidence found in this chapter suggests that Mean-Reverting model has superior performance than a simple GBM model in forecasting electricity spot prices in NEM measured by widely accepted forecast evaluation methods. For instance, the RMSE statistics showed significant drops in mean-reverting model as compared to GBM model in all regions of the NEM. The drop in the values of this statistic was about eight times in some instances (the RMSE value dropped about 4.6 times for NSW and about 8.5 times for VIC). However, modelling electricity prices with a pure mean-reverting model lacks the ability to model a main characteristic of the electricity prices in NEM; the inability to characterize price spikes.

Therefore, the next chapter will examine a continuous-time model that incorporates the stylised feature of electricity prices, namely sudden and infrequent jumps.

.....

CHAPTER 8 – MEAN-REVERTING AND JUMP-DIFFUSION MODEL

The main stylized features of electricity prices are; mean reversion, jumpy and highly volatile nature of the series (Kaminski, 1997). Previous chapters of this thesis show that both Geometric Brownian Motion (GBM) and Mean-Reverting models result in a degree of bias in forecasting electricity prices. GBM by construction is not an adequate specification in modelling electricity prices as it trends upward unlike the spot electricity prices. Whilst the Mean-Reverting model captures the mean reverting dynamics of the electricity price series adequately, it nevertheless fails to account for infrequent and large jumps prevalent in electricity prices.

Modelling electricity prices with Mean-Reverting model are intuitive as they explicitly capture the economic mechanism where electricity supply is intrinsically linked to prices. That is when prices are below equilibrium, supply of electricity decreases, pushing prices back to their long-term equilibrium and vice-versa⁴⁰.

As described in the Introduction Chapter, the electricity market is made of different suppliers. The types of suppliers that are most sensitive to changes in prices are peak-plants. The recovery of capital costs on peak-plants, through market prices, have to be achieved over a relatively few hours of operation. This will enable the construction of low capital/high operating plants for peaking purposes and the over-recovery of marginal costs in operation, with the consequences that prices are much higher in peaks.

The presence of price jumps is an important characteristic of electricity price series in NEM. On a half-hourly price interval, the maximum price of electricity per megawatt-hour can go up

⁴⁰ At the wholesale level, this is due to the generator diversity as explored in Chapter 1.

to \$12,500 from its mean level of around \$40. Jumps in the electricity price series are due to fluctuations in the amount of electricity load demanded by consumers. These fluctuations are generally caused by extreme weather conditions, generation outages, or transmission failures. These jumps are as short lived, as when the weather phenomenon or the outage is over, the prices fall back to their normal levels (Simonsen et al., 2004).

The Mean-Reverting and Jump-Diffusion model evaluated in this chapter captures both the jumpy behaviours and the mean-reverting nature of the price series. Understanding the nature of spikes at the wholesale level is of great importance for the market players including consumers of electricity as these spikes lead to price increases for those who use it as an end product. Thus incorporation of jumps in electricity price modelling is expected to result in more adequate forecast models. Hence, wholesalers and end-users of electricity would benefit from enhanced forecast models.

Early modelling approaches involved modifications of models that allowed for the spiky feature of the price series (Kaminski 1997, Johnson and Barz 1999). These models utilised the mean-reverting specifications similar to the one described in the previous chapter with an addition of a jump component; a Poisson process⁴¹ with given intensity and Log-Gaussian distribution of jump sizes.

⁴¹ The Poisson process is a stochastic process which counts the number of events and the time that these events occur in a given time interval. The time between each pair of consecutive events has an exponential distribution with an intensity parameter and each of these inter-arrival times is assumed to be independent of other inter-arrival times.

A general specification of a jump diffusion model involves a Stochastic Differential Equation (SDE), governing the dynamics of the price process. This general process can be represented as;

$$dP_t = \mu(X_t, t)dt + \sigma(X_t, t)dW_t + dq(X_t, t) \quad (39)$$

Brownian motion process (W_t) in this representation accounts for random fluctuations around the long term mean ($\mu(X_t, t)dt$) whilst the jump process ($q(X_t, t)$) accounts for infrequent and large jumps. The jump process is defined as a compound Poisson process⁴² with given intensity and severity of spikes independent of Brownian motion process (W_t).

The price data used in this study are average hourly pool price observations sourced directly from Australian Electricity Market Operator (AEMO) for the period of 01/06/2006 to 29/08/2010. The data from 01/06/2006 to 31/05/2010 are used to estimate the parameters of the models while the period from 01/06/2010 to 29/08/2010 are used to derive out-of-sample forecast accuracy statistics.

⁴² The standard Poisson processes has limitations in developing realistic electricity price models as its jumps are of constant size. Compound Poisson processes allow jump processes to have random jump sizes. Let $(Z_k) k \geq 1$ denote an *i.i.d.* sequence of random variables with probability distribution $v(dy)$, independent of the Poisson process.

$$P(Z_k \in [a, b]) = \int_a^b v(dy)$$

where $-\infty < a \leq b < \infty$

Then the process $Y_t = \sum_{k=1}^{N_t} Z_k$ is called a compound Poisson process.

In this chapter, model parameters of the Mean-Reverting and Jump-Diffusion model are estimated by using the in-sample data (01/06/2006 to 31/05/2010) consistent with the presentation of previous chapters. In order to estimate the future quantities of interest, this chapter uses Monte-Carlo simulations. The need for Monte Carlo simulations to estimate a future quantity of interest is due to the difficulties of finding an explicit solution to SDE that represents the Mean-Reverting and Jump-Diffusion specification.

The Monte Carlo simulation is a numerical method to construct probability sampling distributions based on underlying distributions. In a Monte Carlo simulation, random variables are simulated with a random number generator and expected values are approximated by computed averages. It is often employed in finance to solve stochastic differential equations such as the Black Scholes equation for pricing stock options. The basic tenet of Monte Carlo simulation is that there is a stochastic variable in an equation that can be sampled many times over. In Monte Carlo simulation, one generates a set of suitable sample paths X_T on $[0, T]$. For each sample path, there is a sample path solution to the stochastic differential equation on $[0, T]$. One then estimates the X_T by computing the mean-sum over a large finite set of approximate sample solutions. Appendix 4 gives an overview of Monte Carlo simulations in the context of stochastic differential equations.

MODELLING ELECTRICITY PRICES WITH MEAN-REVERTING AND JUMP-DIFFUSION MODELS

Consistent with the representation of earlier continuous-time models developed in the previous chapters, the Mean-Reverting and Jump-Diffusion model of this chapter takes the following form:

$$dX_t = (\mu - \beta X_t)dt + \sigma dW_t \quad (40)$$

$$dX_t = \beta(L - X_t)dt + \sigma dW_t + dq(X_t, t) \quad (41)$$

where W_t is a Wiener process that models the continuous inflow of randomness into the system. The standard deviation parameter, σ , determines the volatility of the price process and in a way characterizes the amplitude of the instantaneous randomness inflow. The parameters β (speed of mean-reversion), L (long-term mean level) and σ (random variation), together with d_q (jump component) characterize the price dynamics.

The explicit solution to the SDE represented in Equation (41) above between any two periods s and t , with $0 \leq s < t$, can be derived from the solution to the general Ornstein-Uhlenbeck SDE representation as follows;

$$X_t = L(1 - e^{-\beta(t-s)}) + x_s e^{-\beta(t-s)} + \sigma e^{-\beta t} \int_s^t e^{\beta u} dW_u \quad (42)$$

and the discrete time version of this equation, on a time grid $0 = t_0, t_1, t_2, \dots$ with time step $\Delta t = t_i - t_{i-1}$ is given by; $x(t_i) = c + b x(t_{i-1}) + \delta \varepsilon(t_i)$ where the coefficients are; $c = L(1 - e^{-\beta \Delta t})$ and $b = e^{-\beta \Delta t}$ and $\varepsilon(t)$ is a Gaussian white noise.

Finally, the long term variance (future trajectories of X_t converge around the long term mean with such variance after a long time) can be derived via Ito isometry as;

$$\delta = \sigma \sqrt{(1 - e^{-2\beta \Delta t}) / 2\beta} \quad (43)$$

where if one uses the Euler scheme to discretise the equation this would lead to $\delta = \sigma\sqrt{\Delta t}$, which is the same (first order in Δt) as the above (Menaldi, 2008).

The first term of the Equation (41) implies that X has an expected drift rate of $\beta(L - X_t)dt$ per unit of time whereas the second term can be regarded as adding variability to the path followed by X_t . The amount of this variability is σ times the differential of the Brownian motion process. The third term of the equation $dq(X_t, t)$ generates jumps based on compound Poisson process.

Mean reversion part of the Equation (41), $X_t = \beta(L - X_t)dt + \sigma dW_t$, is modelled based on jump-free data. The jump component of the model is estimated from the log-prices by a two-step procedure. Firstly, all jumps are removed from the price series. Following Weron (2006)⁴³, price increments exceeding 2.5 standard deviations of the mean are considered as spikes. Then, secondly, the intensity and the distribution of the magnitude of the jumps are estimated from these few selected points in deriving the Poisson process of the Equation (41), $dq(X_t, t)$. The filtering procedure adopted resulted in 43 spikes in NSW, 37 spikes in VIC, 34 spikes in SA, 40 spikes in QLD and 22 spikes in TAS region. These spikes constituted around one to two and a half percentage of the whole sample.

The mean reversion part of the above Equation (41) is modelled by having a drift term that is negative if the spot electricity prices are higher than the mean reversion level and positive if it is lower. This representation is a one-factor model and it reverts to the long-term mean $L = \frac{\mu}{\beta}$

⁴³ Previous investigations have used other methods of identifying jumps. One of the approaches is to consider jumps as price moves that are outside 90 per cent prediction intervals implied by Gaussian distribution (Borovkova and Permana, 2004) or the method applied by Geman and Roncoroni (2006) that filtered price data using different thresholds, choosing the one that leads to the best calibration of their model.

with β being the magnitude of the speed of mean-reversion whereas the second term in Equation (41), σdW_t , is the volatility of the price process.

The main assumption of the Mean-Reverting and Jump-Diffusion model as presented in this chapter is that positive jumps are always followed by a negative jump of the same magnitude. This assumption is similar to the one taken in Weron et. al (2004). When modelling electricity prices in NEM, this assumption is a satisfactory approximation as spikes do not typically last more than a day. This assumption of positive jumps are always followed by a negative jump of the same magnitude is fulfilled by letting the stochastic part in Equation (33), $\beta(L - X_t)dt + \sigma dW_t$, be independent of the jump component, $dq(X_t, t)$. The process of separating the jump component from the stochastic component can be represented in the following form:

$$d_t = J_t dq_t + X_t \quad (44)$$

where X_t is the mean-reverting stochastic diffusion specification of Equation (41). The random variable responsible for spike severity (J_t) is set to be a lognormal random variable $\log J_t \sim N(\mu, \rho^2)$ and q_t to be a Poisson random variable with intensity, κ . In this specification, jumps that occur at time t disappear immediately in the next period generating a spiky behaviour and do not require a high mean-reversion rate to bounce back (Weron et al, 2004).

PARAMETERISATION OF THE MEAN-REVERTING AND JUMP-DIFFUSION MODEL

There are a relatively large number of parameters to be estimated for the Mean-Reverting and Jump-Diffusion model as compared to models of GBM and Mean-Reverting models of the

previous chapters. According to Weron (2006), the estimation of these parameters from a small number of observations with a Maximum-Likelihood (ML) method is rather problematic. ML estimates based on small numbers of observations for the jump component will potentially be unstable as it tends to converge on the smallest and most frequent spike component of the price series. However, in electricity price modelling, the aim is to capture the irregular, large jump components.

Therefore, following Weron (2006) this chapter takes a hybrid approach in calibrating the parameters of the Mean-Reverting and Jump-Diffusion model. Weron (2006) extracted the jump events' frequency (intensity) by simple counting and its distributional parameters (mean and standard deviation) after filtering the jumps from the mean-reverting component by the filtering procedure. This process described the severity of the jumps. The data points left out by jumps are then filled in by the mid-points of neighbouring data points. This process provided a filtered (jump-free) price series.

The calibration of the mean reverting parameters is then performed by conducting a linear regression between the log prices and their first difference, as follows;

$$\frac{dX_t}{dt} = -\beta X_t + \beta L + \frac{\sigma}{dt} dW_t \quad (45)$$

The speed of mean reversion, β , is calculated from the coefficients of a linear fit (b) in Equation 35, scaled by the time interval parameter⁴⁴. In this specification, the volatility term is set to a

⁴⁴ See Yu (2009) for a detail explanation of estimating mean-reversion parameters in continuous time.

constant, despite the fact that empirical evidence suggests that electricity prices exhibit heteroskedasticity.

Table 12 shows the parameter estimates of the model for each region of the NEM. The speed of mean reversion (β) and the long-run mean level (L) are calculated from the coefficients of a linear fit scaled by the time interval parameter. The speed of the mean reversion parameter represents the annualised rate at which the underlying short-term price returns to its expected long-run equilibrium value. Hence the inverse of the speed of mean reversion rate gives the actual time scale over which mean reversion occurs.

For example, a mean-reversion rate of 72.033 corresponds to an electricity price process whose price reverts to its expected value over the course of five days as is the case for NSW region⁴⁵. In fact, the regions of QLD and SA have the greatest mean reversion rates in NEM suggesting relatively shorter number of days for spiked prices to return to long-run mean levels as compared to other regions in NEM. These mean-reversion rate estimates are consistent with the parameter estimates derived in the preceding chapter.

The mean reversion parameters are found to be positive for all regions of NEM and ranges from 65.3 to 89.3, highlighting the evolution of mean-reversion in electricity price series. The fact that they are positive for all regions of NEM implies that prices decline to their long-run mean levels after jumps occur. Higher mean reversion rates indicate a region where electricity prices are relatively more volatile.

⁴⁵ $365/72.033=5.067$

As is seen in Table 12, the highest intensity parameter is found in NSW region followed by QLD region. TAS region on the other hand has the lowest intensity of jump statistic⁴⁶, indicating that the probability of price spikes occurring in any given day seems to be lower than it occurring in other regions of the NEM. The jump component is represented by the Poisson process with an intensity parameter, which is well approximated by a simple binary probability ($q = \text{intensity} \cdot dt$) of a jump and $(1-q)$ for no jump.

It is also interesting to note that the regions with higher mean reversion rates have the highest annualised volatility measures, which points to the fact that these regions may be the least mature electricity markets in NEM. High volatility indicates a relatively immature market where suppliers' infrastructure is not yet able to meet the changing demand levels instantly.

Table 12 Parameter Estimates for Mean-Reverting Jump Diffusion Model

	NSW	VIC	QLD	SA	TAS
β	72.033	69.168	85.575	89.303	65.274
L	3.506	3.528	3.430	3.562	3.712
σ	4.565	4.639	5.278	5.295	4.731
κ	0.0308	0.0253	0.0273	0.0232	0.0150
μ	4.7825	5.8090	4.4318	5.5788	5.6725
q	0.961	1.116	0.707	1.818	2.826

Source: Authors calculations.

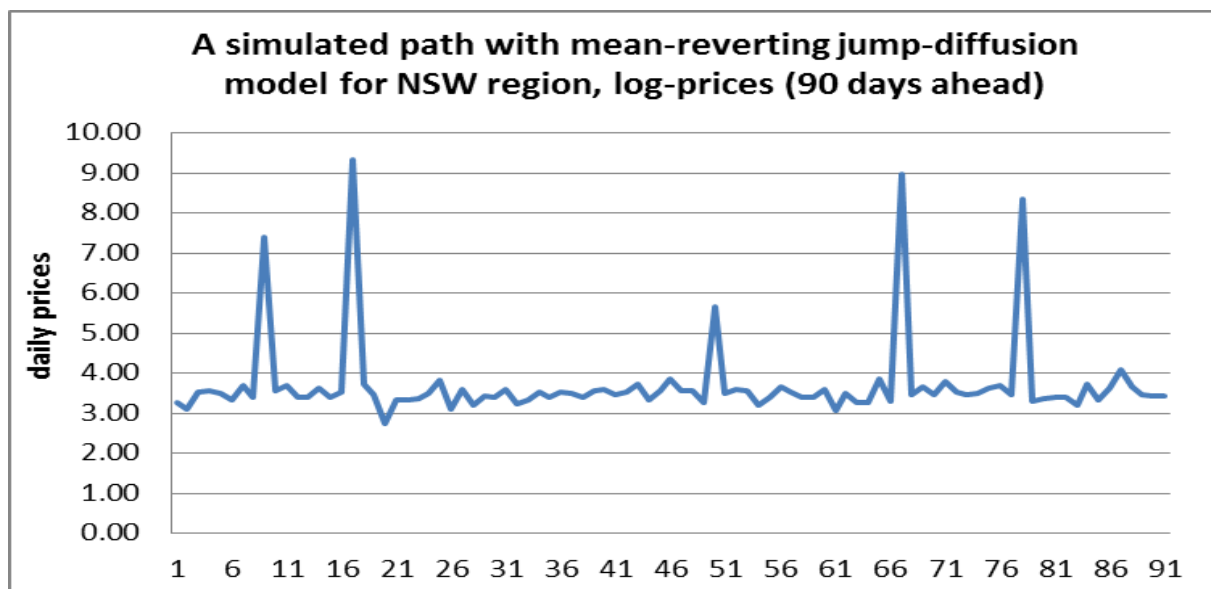
⁴⁶ The intensity of the jump parameter (κ) has the dimensions of $1/dt$, and it identifies the mean number of jumps per time interval, which in the context of this study, is $1/1461$.

SIMULATION OF PRICES WITH MEAN-REVERTING AND JUMP-DIFFUSION MODEL

In order to estimate the future quantities of interest, this chapter uses Monte-Carlo simulations. The need for Monte Carlo simulations to estimate a future quantity of interest is due to the difficulties of finding an explicit solution to the SDE that represents the mean-reversion phenomena (Kloeden and Platen, 1992). This approach is similar to the approach taken in previous chapters.

Figure 25 shows a sample path generated via the Mean-Reverting Jump-Diffusion model for the NSW region. As is seen, the mean-reverting component of the model generates random prices deviating from the long-run mean but on average these prices revert back to the long-run mean levels. The jump component of the model on the other hand generates spikes that last for a day at varying degrees of magnitude.

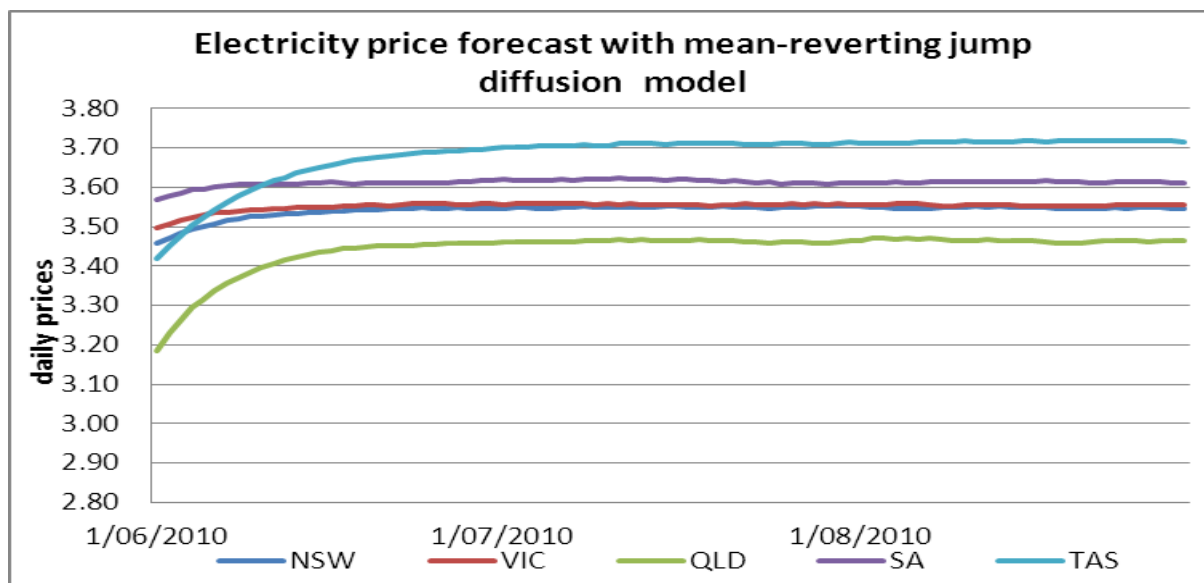
Figure 25 A Simulated Price Path for Mean-Reverting and Jump-Diffusion Model



Source: Authors calculations.

Figure 26 on the other hand shows the electricity price forecast generated by the Mean-Reverting Jump-Diffusion model for all regions of the NEM for up to three months. The forecast values are an average of 10,000 simulated paths. As is seen, simulated price series quickly revert to their long-run equilibrium similar to mean-reverting model described in the previous chapter.

Figure 26 Price Forecast for NEM regions with Mean-Reverting and Jump-Diffusion Model



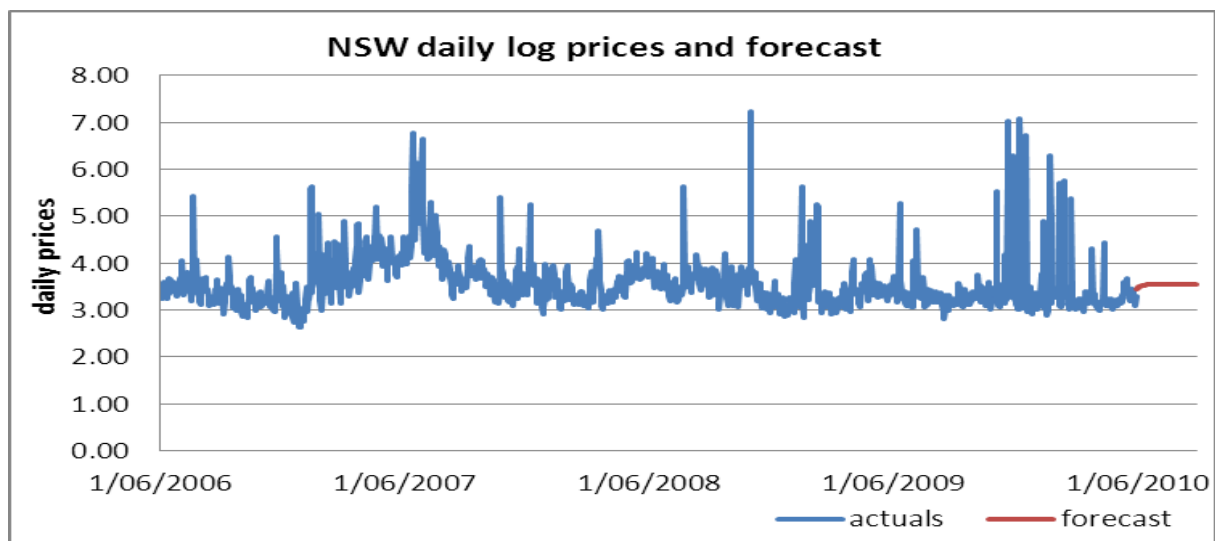
Source: Authors calculations.

The upward convex move at the beginning of the forecast horizon shows how long it takes prices to reach their long-run mean levels. This upward move is the natural result of seasonal dynamics of electricity prices in NEM. The spot price at the last day of the in-sample-period forms the starting value in the following figure. The simulated values present an immediate upward move to their long run mean levels as the last month of the in-sample-period is a winter month (the electricity prices tend to be lower in winter months as compared to summer months).

Figures 27 to 31 on the other hand show the actual and simulated prices for each of the NEM regions. As is seen, Mean-Reverting and Jump-Diffusion model generates forecast values that resemble the forecast values generated by the Mean-Reverting model from the previous chapter. This may suggest that the Mean-Reverting and Jump-Diffusion model, despite its complex calibration process and theoretical advantage, does not perform better on face value than the simpler Mean-Reverting model.

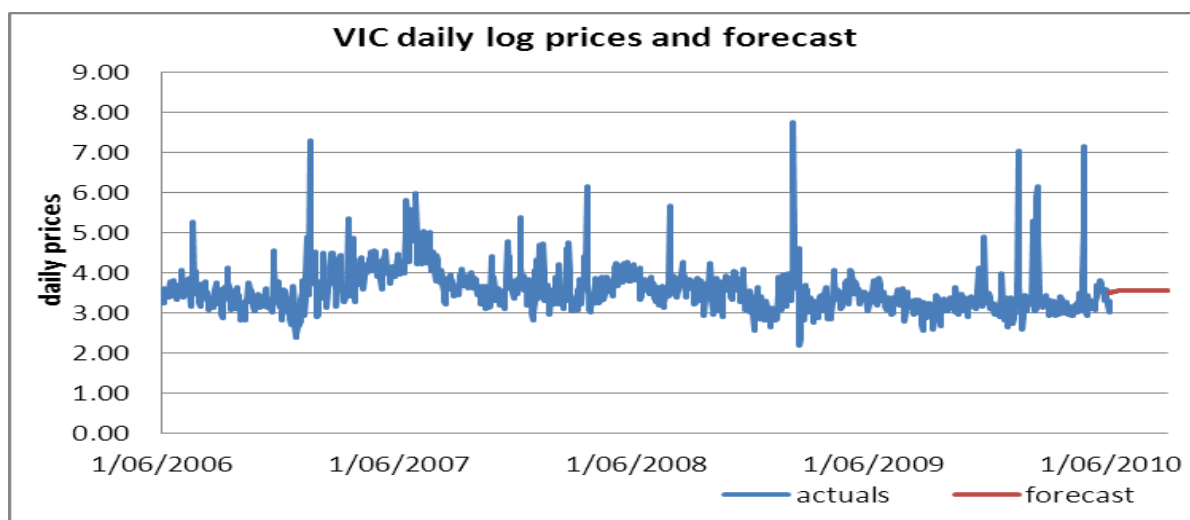
However, it is expected that this model reduces the variance of the forecasts and performs better in terms of its forecast accuracy measures i.e. root mean squared error (RMSE). It is also important to acknowledge here that the forecast prices are bias estimators of the actuals.

Figure 27 Price Forecast for NSW with Mean-Reverting and Jump-Diffusion Model



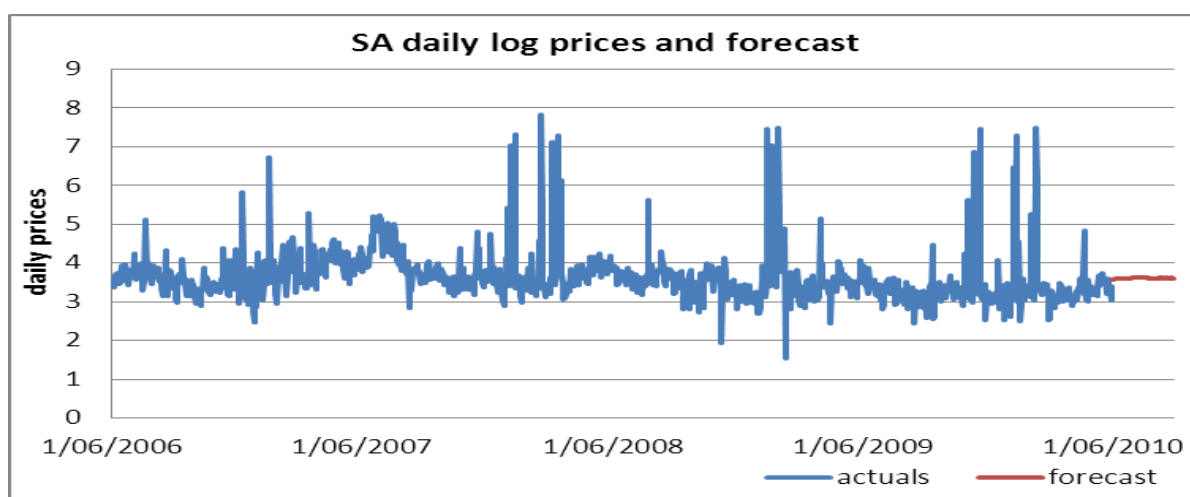
Source: Author's calculations.

Figure 28 Price Forecast for VIC with Mean-Reverting and Jump-Diffusion Model



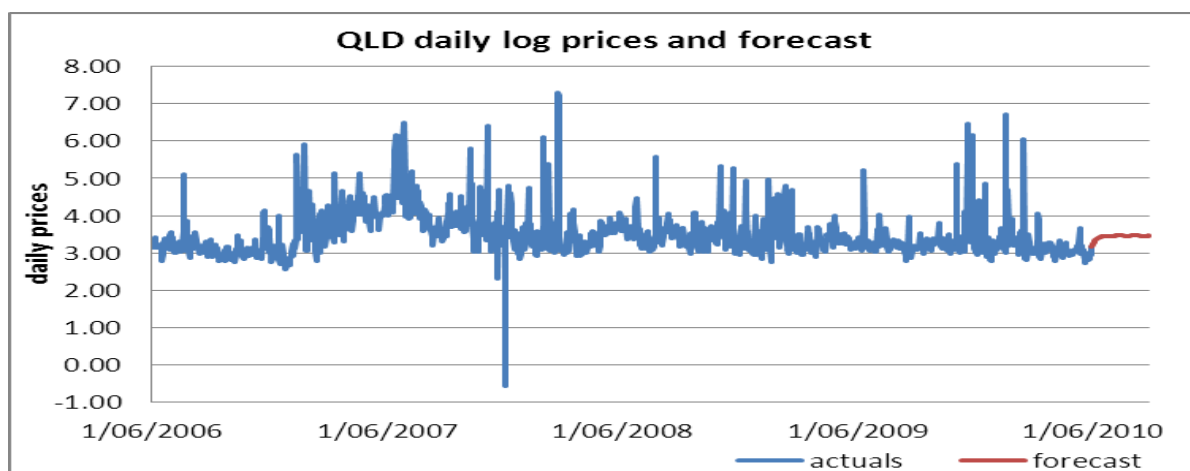
Source: Author's calculations.

Figure 29 Price Forecast for SA with Mean-Reverting and Jump-Diffusion Model



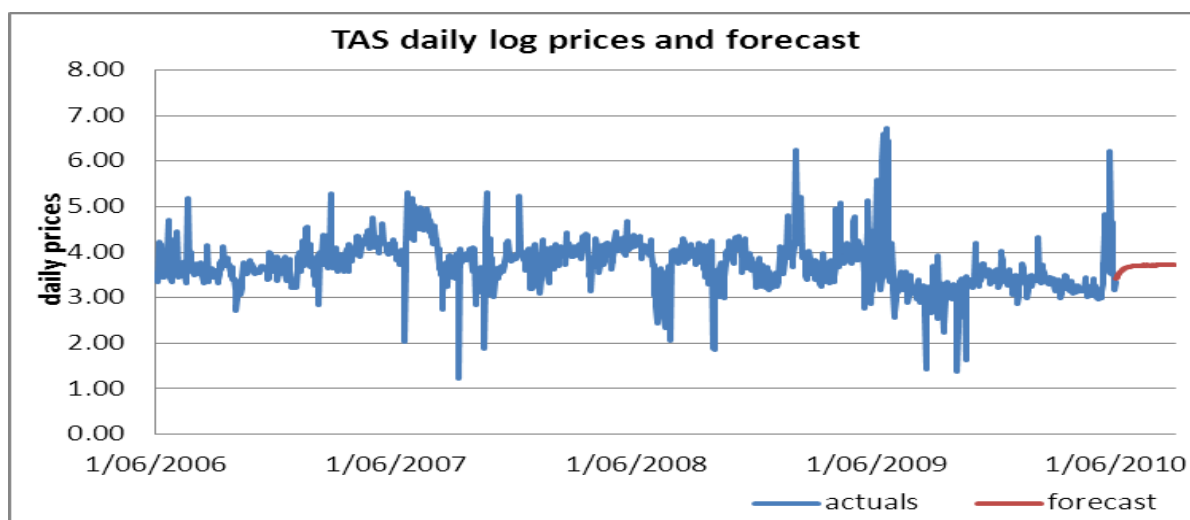
Source: Author's calculations.

Figure 30 Price Forecast for QLD with Mean-Reverting and Jump-Diffusion Model



Source: Authors calculations.

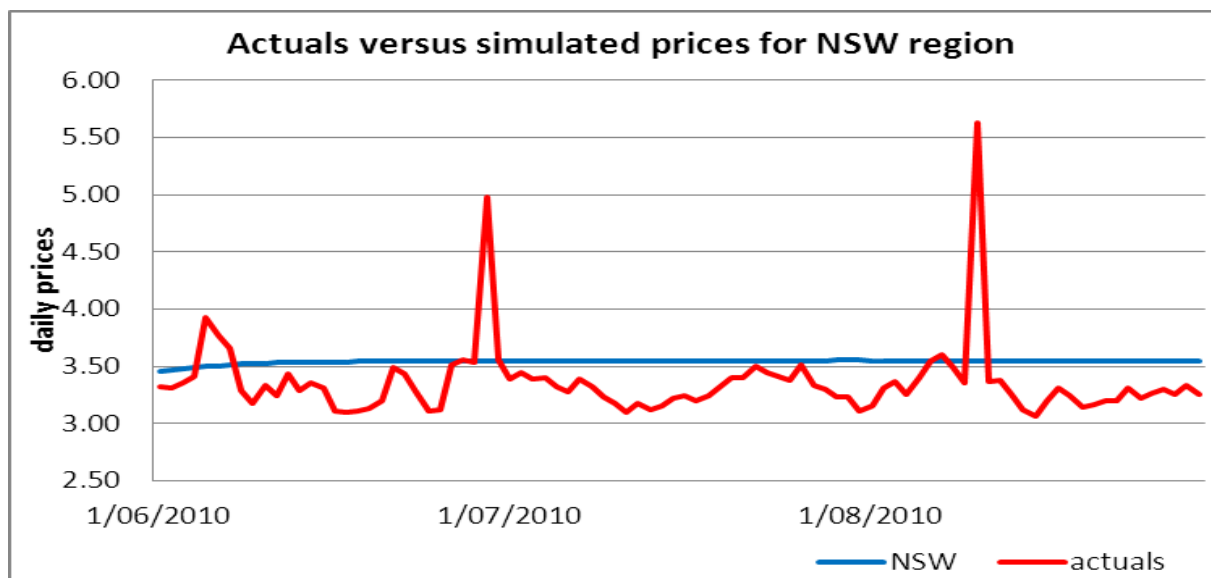
Figure 31 Price Forecast for TAS with Mean-Reverting and Jump-Diffusion Model



Source: Authors calculations.

The expected superior performance of the Mean-Reverting and Jump-Diffusion model over GBM and Mean-Reverting models is in its ability to capture the price spikes prevalent in electricity prices in NEM. However, the magnitude and timing of these spikes do not necessarily align with the actual price spikes that occur. This can be clearly seen in Figure 32.

Figure 32 Actual vs simulated prices with Mean-Reverting and Jump-Diffusion Model



Source: Authors calculations.

EVALUATION OF FORECAST PERFORMANCE

In this chapter evaluation of the forecasting performance of the Mean-Reverting and Jump-Diffusion model is presented. The aim of this chapter is to observe if the model's forecast errors are within the reasonable limit of expectations or whether these errors are unreasonably large and require an improvement in the statistical models and process of producing these forecasts. To formally assess the accuracy of the forecast generated by the Mean-Reverting and Jump-Diffusion model, forecast accuracy statistics are produced for each region of the NEM.

Table 13 shows the comparative forecast measures of the Mean-Reverting and Jump-Diffusion model with the Mean-Reverting model from the previous chapter. As is observed, the comparative analysis of these models shows a mixed picture. RMSE of the model shows declines in the regions of NSW and QLD as compared to the RMSE based on the mean-reverting model by about 0.7 per cent and 3 per cent, respectively. However, in the regions of VIC, SA and TAS, the application of Mean-Reverting Jump-Diffusion model does not result in improvements in RMSE values. RMSE values in these regions actually show deterioration in the range of 0.3 per cent to 4 per cent.

Table 13 Forecast accuracy statistics for Mean-Reverting and Jump-Diffusion Model

	NSW	VIC	QLD	SA	TAS	NSW	VIC	QLD	SA	TAS
	Mean-Reverting Model					Mean-Reverting and Jump-Diffusion Model				
ME	-0.185	-0.228	-0.414	-0.259	-0.312	-0.197	-0.247	-0.398	-0.266	-0.330
MSE	0.145	0.101	0.242	0.248	0.382	0.143	0.110	0.227	0.263	0.384
RMSE	0.381	0.318	0.492	0.498	0.618	0.378	0.332	0.476	0.512	0.620
MAE	0.280	0.269	0.414	0.309	0.475	0.286	0.285	0.398	0.335	0.484
MPE	-6.249	-7.309	-15.628	-41.385	-10.958	-6.607	-7.950	-14.899	-38.497	-11.555
MAPE	8.170	8.348	15.628	42.648	13.782	8.396	8.913	14.899	40.153	14.314
Theil's U	0.055	0.046	0.075	0.071	0.086	0.054	0.048	0.073	0.073	0.087

Source: Authors calculations. General rule: smaller the magnitude more accurate the model is.

Furthermore, RMSE values of the model are found to be indicating a poorer performance as compared to the RMSE values of the benchmark model. A simple AR(1)-GARCH(1) model has been modelled based on the in-sample data of the study reported in this chapter. The results of the AR(1)-GARCH(1) simulations over a three-month horizon (90 days) for each region of NEM are determined as the result of an average of 10,000 simulated paths.

The benchmark model has RMSE scores of 0.328 for NSW, 0.238 for VIC, 0.336 for QLD, 0.475 for SA and 0.605 for TAS. Therefore, its RMSE is quantitatively higher than that of other models. Benchmark model also performs better in terms of Theil's U statistic, indicating that it is as good as the naïve model in all the regions of NEM. This model has Theil's U statistic of 0.048 for NSW, 0.035 for VIC, 0.053 for QLD, 0.069 for SA and 0.085 for TAS.

Table 14 Comparative Forecast Performance Measures

	Mean-Reverting and Jump-Diffusion Model					Benchmark Model				
	NSW	VIC	QLD	SA	TAS	NSW	VIC	QLD	SA	TAS
ME	-0.197	-0.247	-0.398	-0.266	-0.330	-0.02	-0.094	-0.183	-0.127	-0.272
MSE	0.143	0.110	0.227	0.263	0.384	0.107	0.056	0.112	0.226	0.366
RMSE	0.378	0.332	0.476	0.512	0.620	0.328	0.238	0.336	0.475	0.605
MAE	0.286	0.285	0.398	0.335	0.484	0.184	0.191	0.202	0.263	0.442
MPE	-6.607	-7.950	-14.899	-38.497	-11.555	-1.442	-3.251	-8.001	-36.955	-9.782
MAPE	8.396	8.913	14.899	40.153	14.314	5.123	5.833	8.570	40.553	12.778
Theil's U	0.054	0.048	0.073	0.073	0.087	0.048	0.035	0.053	0.069	0.085

Source: Authors calculations. General rule: smaller the magnitude more accurate the model is.

In conclusion, forecasting electricity prices in NEM with a Mean-Reverting and Jump-Diffusion model performs better than the GBM model but it provides mixed results when compared to relatively simpler Mean-Reverting model. However, Mean-Reverting and Jump-

Diffusion model has theoretical superiority over the simpler Mean-Reversion model. In forecasts based on the Mean-Reverting and Jump Diffusion model, one identifies large and small jumps with some jump clusters. The GBM and the Mean-Reverting models fail to simulate these jumpy characteristics prevalent in NEM.

The Mean-Reverting and Jump-Diffusion model performs better as it identifies jumps and the magnitude of these jumps resembles the observed time series. Nevertheless, closer examination of the empirical price trajectories reveals that the differences between the magnitude jumps and timing are quite different when compared to actual price series.

These differences can be attributed to a number of areas in the model development. First of all, Mean-Reverting and Jump-Diffusion model assumes the diffusion process to be independent of the Poisson component, but this is not the case for electricity prices in NEM. Prices in NEM are unlikely to spike at off-peak periods where demand and price are very low. A solution to this problem could be forcing the jump sizes proportional to the current spot prices. Secondly, it is known that electricity prices are seasonal therefore; applying a homogenous compound Poisson process to model the jump component may not be optimal. Using a non-homogenous Poisson process with a deterministic periodic intensity function could be a solution to this issue. However, the problem with the application of this method is the scarcity of spikes identified by the filtering procedures.

Overall, empirical evidence suggests that the Mean-Reverting and Jump-Diffusion model's performance is superior to basic GBM and Mean-Reverting models in forecasting electricity spot prices despite the fact that the jump component of this SDE based forecast model fails to capture the magnitude and timing of the spikes in the electricity price series. Therefore, an

application of a model that captures the dynamics of the jumpy and mean-reverting characteristics of the price series better than the Mean-Reverting and Jump-Diffusion model is the aim of the next chapter. Markov Regime-Switching models are one way to capture such specifications that capture both the mean-reverting and the jumpy characteristics of the prices series. In the next chapter, results of a comparison with the Mean-Reverting and Jump-Diffusion model is likely to provide valuable insight in choosing the most effective forecast model in NEM.

.....

CHAPTER 9 MARKOV REGIME-SWITCHING MODEL

AN INTRODUCTION TO MARKOV REGIME-SWITCHING MODEL

The fundamental idea behind a regime-switching process is to model the observed stochastic behaviour of a time-series by two or more separate regimes with different underlying processes. The separate regimes of a Markov Regime-Switching model are determined by an unobservable variable where one cannot be certain that a particular regime has occurred at a particular point in time, but one can assign probabilities of their occurrences. The switching process between the regimes is captured by time varying estimates of the conditional probability of each state and an estimate of a constant matrix of state transition probabilities.

The regime variables are assumed to follow a first order Markov chain where the transition probabilities for the two regimes are assumed to be constant. Denoted by (Q) the probability of switching from regime i at time t to regime j at time $t+1$, for $i, j = \{1,2\}$, the matrix of transition probabilities (q_{ij}) can be written as;

$$Q = (q_{ij}) = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix} = \begin{pmatrix} q_{11} & 1 - q_{11} \\ 1 - q_{22} & q_{22} \end{pmatrix} \quad (46)$$

Markov property suggests that the current state j at time t of a Markov chain depends on the past only through the most recent value i :

$$P(R_t = j | R_{t-1} = i, R_{t-2}) = P(R_t = j | R_{t-1} = i) \quad (47)$$

This is also known as first order Markov process. This means that the probability of being in the i^{th} regime at time t depends only on the regime at time $t-1$ and not on the regimes that occurred at i^{th} times such as $t-2$, $t-3$, etc. The transition probabilities determine the probability

of being in a certain regime at time t given a certain regime $t-1$. By the Markov property the regimes at times $t-2$, $t-3$, etc. are irrelevant for this transition property.

In contrast to linear model specifications that assume stationarity, Markov Regime-Switching models are based on a mixture of parametric distributions that depend on unobserved state variables. In this context, the Regime-Switching or Markov Regime-Switching models seem to be an adequate non-linear alternative to linear time series models. This model involves multiple structures (equations) that can characterize the time series behaviours in different regimes. By permitting switching between these structures, this model is able to capture more complex dynamic patterns.

Markov Regime-Switching models also differ from the models of structural changes. While the former allows for frequent changes at random time points, the latter admits only occasion and exogenous changes. The Markov Regime-Switching model is therefore suitable for describing correlated data that exhibit distinct dynamic patterns during different time periods.

Goldfield and Quandt (1973) were the first to introduce Markov-switching regressions. Hamilton (1990, 1994) calculated all the variables of interest of the Markov Regime-Switching processes as a by-product of an iterative algorithm similar to a Kalman filter algorithm⁴⁷. Hamilton's seminal work popularised the application of the Markov Regime-Switching processes in macro-economic modelling. In this study, the Markov Regime-Switching process was applied to model the probability of a recession in the U.S. economy, which was assumed to be alternating between two unobserved states of high growth and slow growth according to

⁴⁷ The Kalman filter is a recursive estimator. This means that only the estimated state from the previous time step and the current measurement are needed to compute the estimate for the current state.

a Markov chain process. This model assumed constant transition probabilities for the unobserved states, which in turn imply constant expected durations in the various regimes.

Markov Regime-Switching models also allow the study of the general characterizations of moment and stationary conditions. For instance, Tjostheim (1986), Yang (2000), Timmermann (2000) and Francq and Zakoian (2001) studied the general characterisations of moment and stationary conditions of the Markov Regime-Switching processes.

Markov Regime-Switching models have also been employed in modelling heteroskedasticity in time series. For instance, Hamilton and Susmel (1994), Gray (1996), and Klaassen (2002) combine Markov Regime-Switching and ARCH specifications into a switching-ARCH (SWARCH) that embeds ARCH models within different regimes. The advantage of these models is that they capture conditional heteroskedasticity that is not captured by traditional GARCH specifications.

The Markov Regime-Switching specifications allow more accurate estimation of electricity price dynamics than the ones considered so far. The findings of this study are quite similar to earlier findings in the literature. For example, the two-regime model of Weron et al. (2004) included log-normally distributed spikes whereas Bierbrauer et al. (2004) found that using Pareto distributed spikes overestimated the spike sizes. This study concluded that the best model is the one with Log-Gaussian spikes in terms of its statistical properties, closely followed by the model with Gaussian spikes. The findings of this study also support the earlier findings of the literature as the Markov Regime-Switching specification with Gaussian and Log-Gaussian distributed spikes have variances much smaller than the specification with Pareto distributed spikes of the spike regimes. It is important to note that Weron & Misiorek (2008)

showed that in models based on log-prices (like the models developed in this study), the calibration scheme generally assigns all extreme prices to the spike regime, no matter whether they truly are spikes or only artificial sudden drops (due to taking the logarithm of small values).

MARKOV REGIME-SWITCHING MODELS IN ELECTRICITY PRICE MODELLING

The previous chapter demonstrated the inability of Mean-Reverting and Jump-Diffusion models in capturing consecutive price spikes in NEM. These models generally do not allow consecutive spikes. In particular, they allow for spikes that last for more than just one time period (an hour, a day), without the disadvantage of rapid mean-reversion after a jump as generally observed in Jump-Diffusion models. Markov Regime-Switching processes on the other hand are capable of modelling consecutive price jumps.

However, it is important to understand that capturing the exact timing of a spike is not possible with these processes similar to previously examined models. These processes merely generate spikes in the forecast horizon and as a consequence minimise the variance of the conditional mean of the series accurately. That is to say that Markov Regime-Switching models like the previously investigated continuous-time based processes are likely to succeed in generating spikes and mean-reversion characteristics of the electricity price series but it is not likely to capture the exact timing of the spike occurrences.

Markov Regime-Switching models have widely been recognised in the electricity modelling literature. Either and Mount (1998) applied a Markov Regime-Switching model in a two state specification in which both regimes were governed by AR(1) processes with homoscedasticity and heteroskedasticity. They used data from Victoria (Australia) and United States electricity

markets and confirmed the existence of different means and variances in the two regimes. Huisman and Mahieu (2003) applied a three regimes model to electricity prices.

The base regime of this model described the normal electricity prices dynamics while the initial jump regime attempted to capture the sudden increase or decrease in prices. The jump reversal regime as the third regime of this model described how the prices move back to the normal regime after the initial jump. In fact, this three regime specification was quite similar to the simpler jump-diffusion specification developed in the previous section where the prices tend to revert to their long-run mean levels after the initial spike. However, the difference is in how the prices revert to their long-run mean levels. The previous chapter's Jump-Diffusion specification forces prices to revert to pre-specified long-run mean levels whereas the three-regime specification of Huisman and Mahieu (2001) models the parameters of this reversal probabilistically. Therefore they have the potential to capture the dynamics of mean-reversion after jumps occur.

The transition matrix of this regime-switching specification is illustrated as;

$$Q = (q_{ij}) = \begin{pmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{pmatrix} \quad (48)$$

This model, however, did not allow for consecutive spikes, which are commonly observed in electricity price data. Huisman and de Jong (2003) later relaxed this specification and proposed a model with two regimes; a mean-reverting AR(1) and a spike regime. The third regime in this approach is unnecessary as the prices are assumed to be independent of each other in two regimes. They suggested that the spike regime can be modelled with a Gaussian distribution for which the mean and variance are higher than those of mean-reverting base regime.

However, the heavy-tailed nature of electricity prices required extension of these models. Weron (2004) and Bierbrauer et al. (2004) modelled the spike regimes with Log-Gaussian and Pareto distributions, while de Jong (2006) considered Poisson driven spike regime dynamics.

The adequacy of these models in terms of their forecasting ability has been rarely tested. Haldrup and Nielsen (2006) found that a Markov Regime-Switching Seasonal Autoregressive Fractionally Integrated Moving Average (ARFIMA) model outperforms a seasonal ARFIMA model when applied to electricity prices in NordPool. Kosater and Mosler (2006) compared to a regime switching specification (driven by two AR(1) processes) to an AR(1) model using electricity prices from the German EEX market. This study found that in the short term both models performed alike but over a long run Markov Regime-Switching model outperforms the model with the single AR(1) specification. These results were also similar to findings of Misiorek et al. (2006).

MODELLING ELECTRICITY PRICES WITH MARKOV REGIME-SWITCHING MODELS IN NEM

In this chapter, electricity prices in NEM are modelled with Markov Regime-Switching models with four separate spike regimes. These spike regimes are characterised by Gaussian, log-Normal, Pareto⁴⁸ distributions and a mean-reverting process. The switching mechanism between the states in these specifications is assumed to be governed by an unobserved random variable that has the Markov property.

⁴⁸ Since spikes happen very rarely but usually are of great magnitude the use of heavy-tailed distributions (Log-Normal, Pareto) are also considered in the literature (see Weron and Janczura, 2010).

Markov Regime-Switching models are frequently discussed in the literature however in this chapter; the focus goes beyond the mere estimation process and takes account of the forecasting ability of the Markov Regime-Switching models in NEM.

This forecast process aims at generating forecast future values of electricity prices that minimise the variance of the mean. In other words, Markov Regime-Switching models like the previously investigated continuous-time models are likely to succeed in generating spikes and mean-reversion characteristics of the electricity price series but they are not likely to capture the exact timing of the spike occurrences.

Extending the previously developed model of Mean-Reverting and Jump-Diffusion model with Markov Regime-Switching approach is theoretically sound as Markov Regime-Switching specification allows for consecutive spikes, which are not allowed in the jump-diffusion model of the previous section. Another difference between the Mean-Reverting and Jump-Diffusion forecast model employed in the preceding chapter and the Markov Regime-Switching model developed here is that the probability of jumps is no longer fixed, but dependent on the current regime processes. In practice, the current regime is not directly observable, but determined through an adaptive probabilistic process. That is to say that the Markov Regime-Switching model of this section is an extension of the Mean-Reverting and Jump-Diffusion model of the previous chapter in the sense that it has Markov Regime-Switching probabilities that stochastically adapt themselves to the previously observed prices.

There are a variety of possibilities in fitting a Markov Regime-Switching model to electricity price data. This is due to the fact that there could be a number of regimes (two, three or more)

and a number of different stochastic processes for the price series in each of these regimes. The literature has many examples that choose alternative distributions for the spike regime.

The modelling of electricity prices in this section are performed as follows;

$$dY_{t,1} = (c_1 - \beta_1 Y_{t,1})dt + \sigma_1 dW_t \quad (49)$$

where c_1 is the long-run mean level, β is the mean-reversion rate and σ_1 is the volatility term of the process. The Brownian motion process is represented by W_t in the base-regime dynamic. This model is assumed to have normally distributed homoscedastic errors.

The dynamics of this model in the spike regime follows three different distributions:

1. Gaussian $Y_{t,2} \sim N(c_2, \sigma_2^2)$ (50)

2. Log-Gaussian $\log(Y_{t,2}) \sim N(c_2, \sigma_2^2)$ (51)

3. Pareto $Y_{t,2} \sim F_{Pareto}(c_2, \sigma_2^2) = 1 - (\frac{c_2}{x})^{\sigma_2^2}$ (52)

These distributions incorporate the heavy-tailed features of electricity prices in NEM. The heavy-tailed characteristics of electricity prices in NEM were investigated in Chapter 3 of this thesis. This chapter also models the spike regime as a mean-reverting process as:

$$dY_{t,2} = (c_2 - \beta_2 Y_{t,2})dt + \sigma_2 dW_t \quad (53)$$

where c_2 is the long-run mean level, β_2 is the mean-reversion rate and σ_2 is the volatility term of the process. The Brownian motion process is represented by W_t in the base-regime dynamic. This model is assumed to have normally distributed homoscedastic errors.

When interpreting the results of Markov Regime-Switching models it is important to note that the regime that governs the process is not observable and must be inferred from the available data. It is not possible to observe which regime currently governs the process and then adopt

the estimate of Markov regime property for that regime as being the one that is commensurate with the prevailing conditions in the market. Rather, the identity of the regime in question is inferred from the observable data. Inferences about which regime governs the process at a particular point in time are summarised in the form of regime probabilities. Regime probabilities are discussed in the next section.

PARAMETERISATION OF THE MARKOV REGIME-SWITCHING MODEL

Calibration of Markov Regime-Switching models is non-trivial. The regimes are latent hence they are not directly observable. The challenge in parameterising the model lies in the unobservable nature of the regimes. Hamilton (1990) introduced the application of Expectation-Maximization (EM) algorithm where the whole set of parameters θ is estimated by an iterative two-step procedure.

This study follows Hamilton's approach in calibrating the parameters of the models. The EM algorithm can be described as follows;

Step 1- The EM algorithm involves the conditional probabilities $P(R_t = j | P_1, \dots, P_T; \theta)$ for the process being in regime j at time t to be derived based on starting values of $\hat{\theta}^{(0)}$ for the parameter vector θ of the underlying stochastic processes. This process involves Maximum Likelihood (ML) estimation.

Step 2- Estimating new and more accurate ML estimates θ for all model parameters by using the parameters derived in step one occurs in this step. Each iteration of the EM algorithm generates new estimates $\hat{\theta}^{(n+1)}$ and each iteration cycle increases the log-likelihood function. Therefore the limit of this sequence of estimates reaches a maximum of the log-likelihood

function. The EM algorithm is simplified considerably when the errors are assumed to be normally distributed in each regime.

The ML estimation is complicated by the fact that the estimation of conditional regime probabilities⁴⁹ as it requires a sub-iteration at each step of the EM algorithm to maximize the log-likelihood function. Considering the complexity of the log-likelihood function and the relatively large number of parameters to be estimated, the selection of starting values are critical for the convergence of the likelihood estimation (Alexander, 2008).

In calibrating the Markov Regime-Switching models in this chapter, this study utilised the EM algorithm as applied by Hamilton (1990). Appendix 5 provides a detailed description of the EM algorithm.

REGIME PROBABILITIES BY NEM REGIONS

Two-regime model for NSW

Table 15 presents the estimated parameters and transition probabilities of the Markov Regime-Switching model for the NSW region. From left to right in Table 11, the parameter estimates are presented. The mean-reversion parameter for the base regime dynamics is denoted as β_i and α_i and σ_i^2 are the mean and standard deviation of both the base and spike regime dynamics. Panel B in Table 11 presents the transition probabilities of the Markov Regime-Switching model along with its expected values and its variances where $E(Y_{t,i})$ is the expected value and $Var(Y_{t,i})$ denotes the variance of the $E(Y_{t,i})$. The term q_{ii} is the probability of remaining in the same regime in the next time step whilst $P(R = i)$ is the unconditional probability of being in regime i .

⁴⁹ The literature generally sets regression coefficients and error standard deviations to be equal to their values from a standard linear regression at their starting values and also sets the transition probabilities to 0.5 i.e. Weron (2006). This section applies a similar approach in setting the initial values of the EM algorithm.

Table 15 Regime Probabilities, NSW region

Panel A				Panel B			
Parameter estimates				Statistics			
Regime	β_i	α_i	σ_i^2	$E(Y_{t,i})$	$Var(Y_{t,i})$	q_{ii}	$P(R = i)$
Two regime model with Gaussian spikes							
Base	0.26264	1.04591	0.20859	3.98225	0.45714	0.93099	0.54594
Spike	-	3.24289	0.02287	3.24289	0.02287	0.91702	0.45406
Two regime model with Log-Gaussian spikes							
Base	0.12840	0.43870	0.02651	3.41663	0.11031	0.95143	0.86027
Spike	-	1.41089	0.03912	4.18058	0.69733	0.70096	0.13973
Two regime model with Pareto spikes							
Base	0.12178	0.42256	0.02957	3.46995	0.12927	0.95320	0.88732
Spike	-	2.31062	2.62973	4.63621	29.94805	0.63148	0.11268
Two regime model with mean-reverting process for spikes							
Base	0.13450	0.46011	0.02701	3.42092	0.10764	0.95411	0.86550
Spike	0.77423	3.27204	0.79544	4.22617	0.83816	0.70472	0.13450

Source: Author's calculations.

In the base regime the mean-reversion parameter, denoted by β_i , is positive for all specifications considered in this study and ranges from 0.12178 (two-regime model with Pareto spikes) to 0.26264 (two-regime model with Gaussian spikes). The mean-reversion parameters found in this section are consistent with the parameters derived in previously discussed models of Mean-Reverting and Mean-Reverting and Jump Diffusion models. Once again, this reveals the importance of mean-reversion in electricity price dynamics and the quicker the return of prices from some extreme position to equilibrium. This explanation is consistent with Higgs and Worthington (2009).

The estimated volatility coefficients, denoted by σ_i^2 , in the base regime range from 0.02651 for the two-regime model with Log-Gaussian spikes to 0.20859 for the two-regime model with Gaussian spikes. This indicates that volatility in electricity markets, once price spikes are excluded, is actually quite low. This is in line with the findings of Higgs and Worthington (2009).

In all specifications, the probability of remaining in the base regime in the next time step is very high. However the probability of remaining in the spike regime in the next time step, denoted by q_{ii} in Table 11, is also relatively high. The probabilities of being in the spike regimes in the next time step for each model are 0.93099 for the specification with Gaussian spikes, 0.95143 for the specification with Log-Gaussian spikes, 0.95320 for the specification with Pareto spikes and 0.95411 for the specification with mean-reverting spikes. The probability of a spike therefore varies from 6.901 per cent for the specification with Gaussian spikes, 4.857 per cent for the specification with Log-Gaussian spikes, 4.68 per cent for the specification with Pareto spikes and 4.589 per cent for the specification with mean-reverting spikes.

It is important to note that the model with Pareto spikes gives the lowest probability of being in the spike regime for the next step. Hence it can be said that a heavy tailed distribution such as Pareto gives lower probabilities for being in the spike regime and a higher variance as evidenced in Table 15. This finding is in line with Weron and Misiorek (2008).

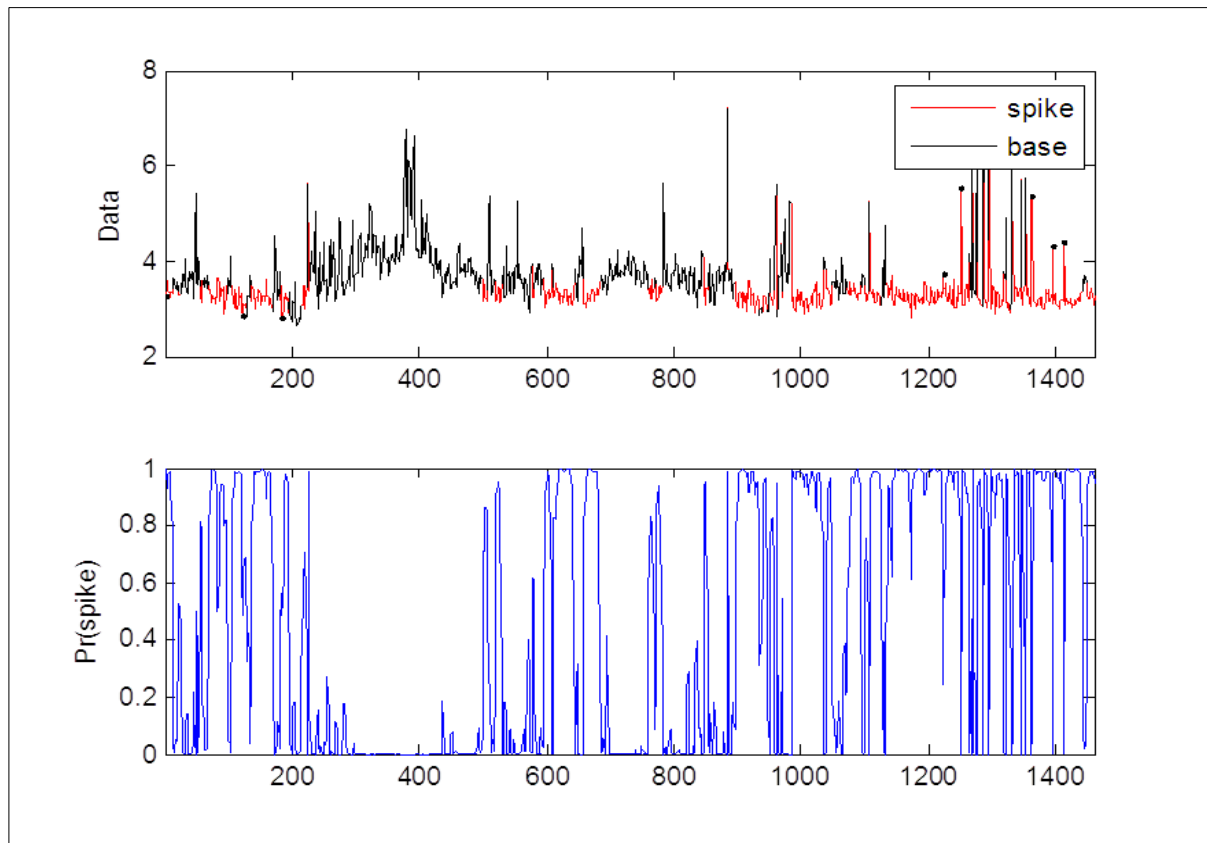
The unconditional probability of being in the spike regime, denoted by $P(R = i)$, is smaller than the unconditional probabilities of being in the base regimes of each specification. However, the unconditional probabilities of being in the spike regimes are quite low for the specifications with log-Normal, Pareto and Mean-Reverting spikes as compared to the specification with Gaussian spikes. The last column of Table 15 presents the unconditional probabilities of being in the base and spike regimes.

The following figures illustrate the transition probabilities of all the specifications described in the Table 15. The bottom panels of these figures reveal the probabilities that an observation

comes from the spike regime. The top presents the data identified as belonging to the spike regime (i.e. having $\Pr(\text{spike}) > 0.5$). These figures can be interpreted through a recursive process. Supposing there has been a series of high prices over the previous days then the model would assign a high probability to the process being governed by the spike regime. In other words, it is much more likely that a series of high prices would be generated by the spike regime than by the base regime.

In the case of average prices being observed the next day, there could be two possibilities; (1) the process remains in the spike regime and this just happens to be a low volatility observation or, (2) there has been a switch to the base regime. Because it is difficult to be certain on the basis of one observation, the probability of being in the spike regime is likely to reduce marginally. If the following observation is a large price increase, the model will conclude that no switch has occurred and the process remains in the spike regime. If, however, the moderate price increase is followed by more moderate observations, the model will conclude that a switch has occurred and that the process is unlikely to be in the spike regime.

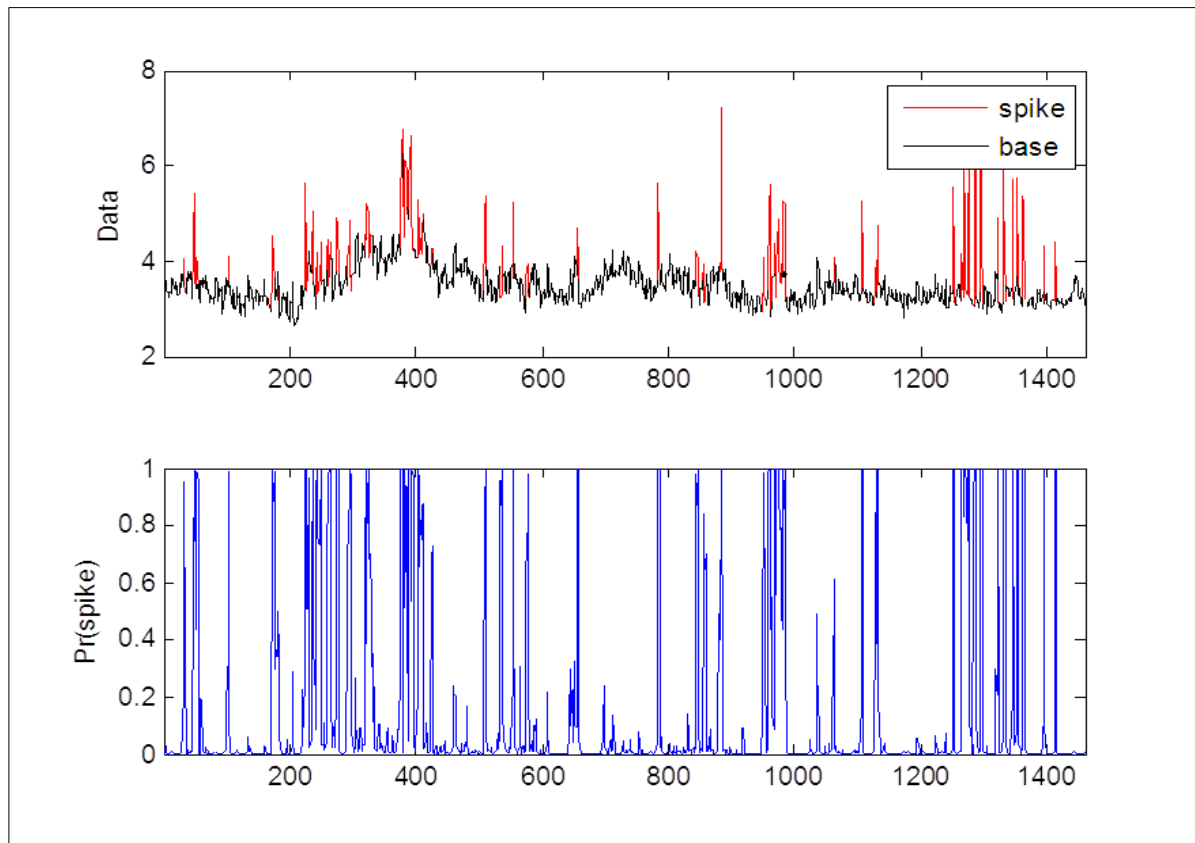
Figure 33 Regime Probabilities (NSW), Gaussian spikes



Source: Author's calculations.

The estimated regimes of electricity prices in the NSW region shows that the model with Gaussian spikes tends to underestimate the spike severity but it overestimates the number of spikes. It incorrectly identifies the data as being in the spike regime when in fact the data may be in a base regime. There are also instances where the model does not identify large movements in the prices as being in a spike regime. Perhaps, the most important observation based on this model is that the mean of the price series in the base regime is greater than the mean of the prices series in the spike regime. This is contrary to the stylised characteristics of electricity prices in NEM.

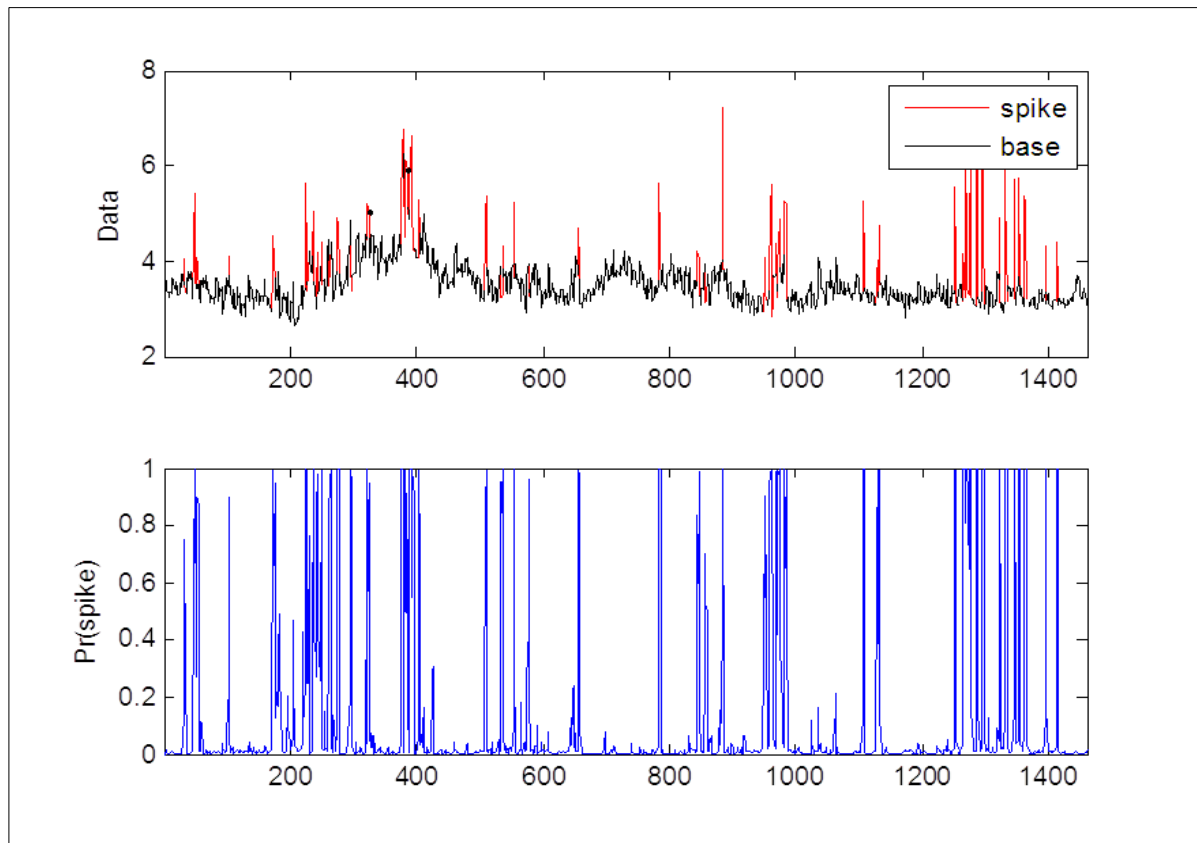
Figure 34 Regime Probabilities (NSW), Log-Gaussian spikes



Source: Author's calculations.

While the Markov Regime-Switching model with Gaussian spikes generates a higher mean for the prices series in the base regime than the spike regime, the model with Log-Gaussian spikes tend to be more accurate and produce a mean of the price series in the spike regime higher than the mean of the prices series in the base regime. This model also tends to identify large price moves as spikes in the NSW region more accurately than the previous model.

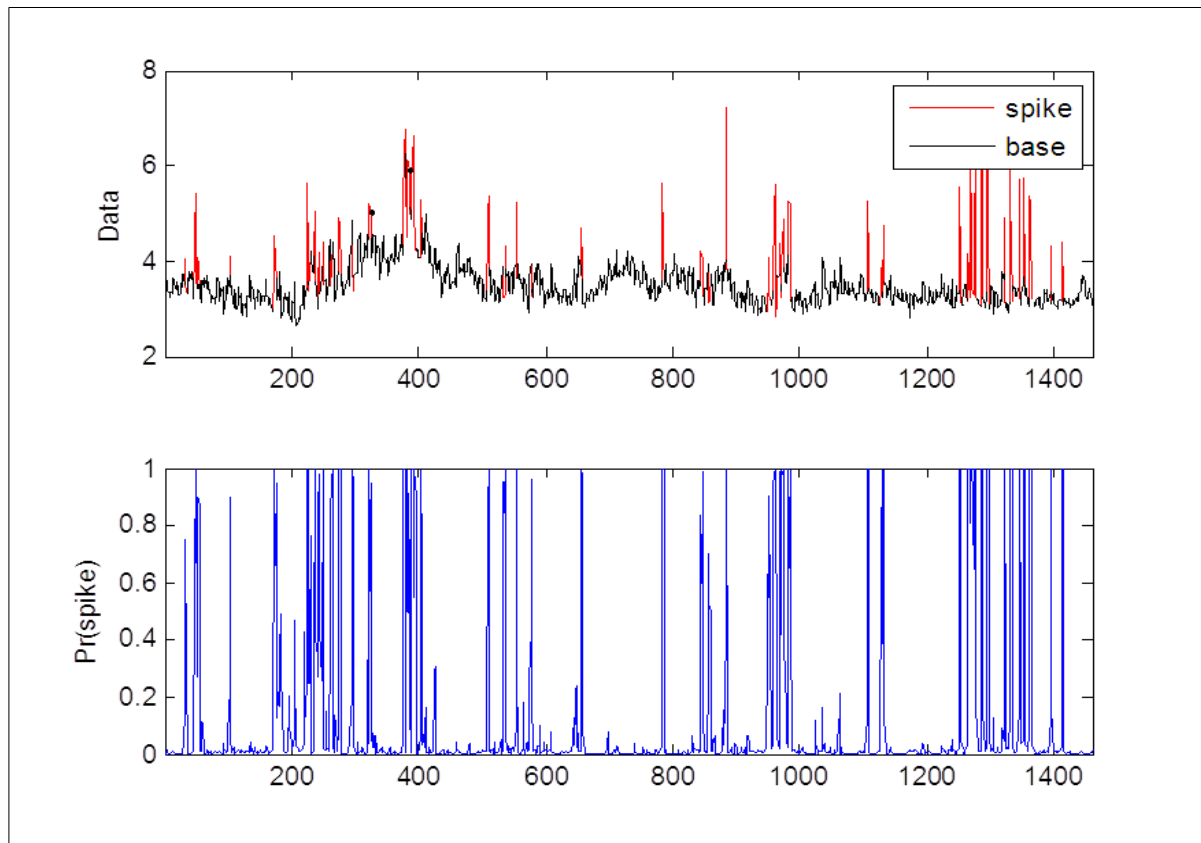
Figure 35 Regime Probabilities (NSW), Pareto spikes



Source: Author's calculations.

The Markov Regime-Switching model specification with Pareto spikes performs relatively well in identifying the large price moves as spikes and assigns these observations as being in the spike regime. This model by its nature identifies less numbers of spikes as compared to the model specification with Log-Gaussian spikes. This is due to the fact that it identifies more extreme price moves as a spike. Therefore the mean of the price series identified as being in the spike regime is higher than the price series identified by the model specification with Log-Gaussian spikes.

Figure 36 Regime Probabilities (NSW), Mean-Reverting Spikes



Source: Author's calculations.

The estimated regimes of electricity prices in the NSW region shows that the model with mean-reverting spikes tends to identify the price series being in either of the base and spike regimes similar to the model specification with Log-Gaussian spikes. The model generates a mean of the price series in the spike regime higher than the mean of the prices series in the base regime. This model also tends to identify large price moves as spikes in the NSW region more accurately than the previous model specification with Gaussian spikes.

As is seen, the visual representation of the Markov Regime-Switching specifications illustrates the essential features of these models were unlike jump-diffusion models, these models allow for consecutive spikes in a natural way.

Two-regime model for VIC

Estimated parameters and transition probabilities of the Markov Regime-Switching model for the VIC region is presented in Table 16. As defined before, the parameter estimates are presented in Panel A where β_i is the mean-reversion parameter for the base regime dynamics, α_i and σ_i^2 are the mean and standard deviation of both the base and spike regime dynamics. Panel B of Table 16 has the transition probabilities of the Markov Regime-Switching model with its expected values and variances similar to Table 15.

Table 16 Regime Probabilities, VIC Region

Panel A				Panel B			
Parameter estimates				Statistics			
Regime	β	c_i	σ_i^2	$E(Y_{t,i})$	$Var(Y_{t,i})$	q_{ii}	$P(R = i)$
Two regime model with Gaussian spikes							
Base	0.28484	1.06916	0.16614	3.75348	0.34007	0.97267	0.68731
Spike	-	3.23473	0.02891	3.23473	0.02891	0.93993	0.31269
Two regime model with Log-Gaussian spikes							
Base	0.15592	0.54018	0.04073	3.46446	0.14167	0.97095	0.91677
Spike	-	1.43350	0.04382	4.28626	0.82296	0.67999	0.08323
Two regime model with Pareto spikes							
Base	0.15963	0.55659	0.04413	3.48681	0.15021	0.97650	0.94074
Spike	-	1.52049	2.19054	6.39912	56.16433	0.62685	0.05926
Two regime model with mean-reverting process for spikes							
Base	0.15839	0.54897	0.04099	3.46591	0.14051	0.97250	0.91908
Spike	0.77783	3.37063	0.91146	4.33336	0.95878	0.68763	0.08092

Source: Author's calculations.

In the base regime the mean-reversion parameter, denoted by β_i , is significant and positive for all specifications considered in this study and ranges from 0.15839 (two-regime model with mean-reverting spikes) to 0.28484 (two-regime model with Gaussian spikes). The estimated volatility coefficients, denoted by σ_i^2 , in the base regime range from 0.04073 for the two-regime model with Log-Gaussian spikes to 0.16614 for two-regime model with Gaussian spikes. The magnitudes of volatilities are smaller than the volatilities presented for the NSW region. This implies a less volatile spot electricity market for the VIC region.

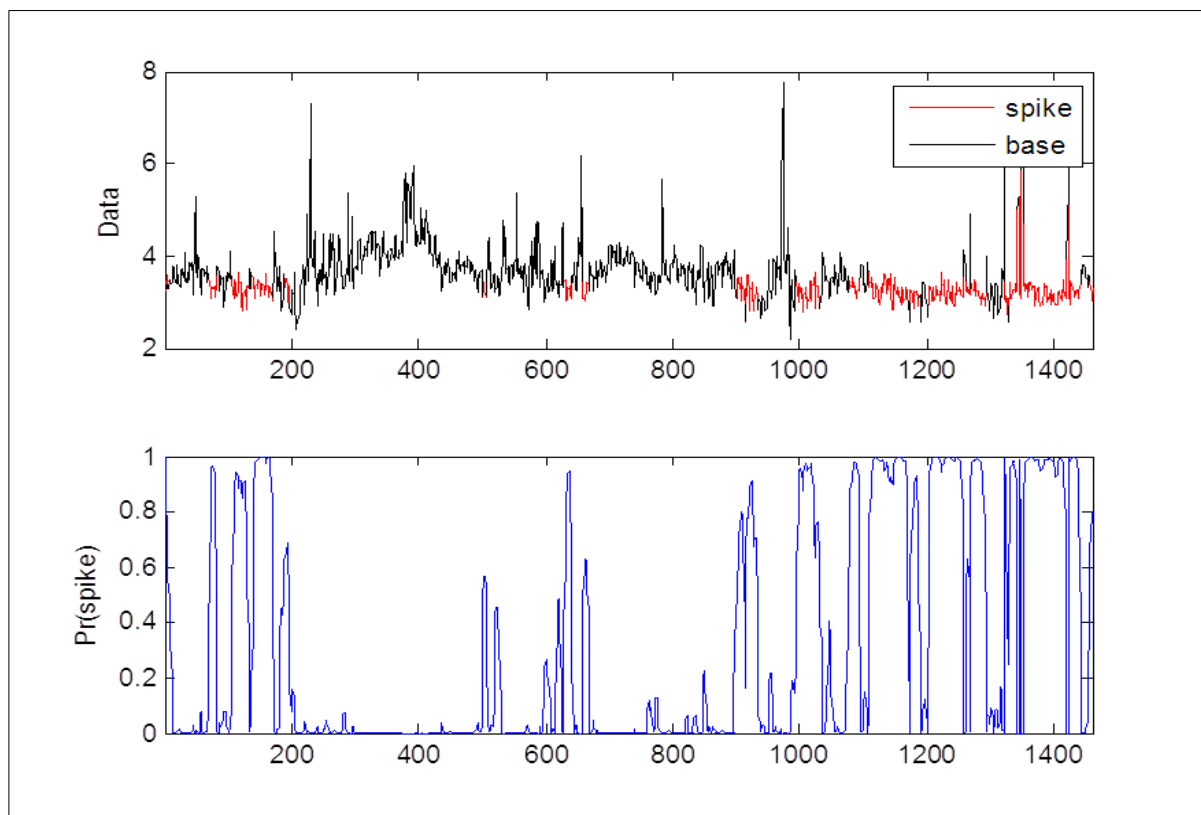
In all specifications, the probability of remaining in the base regime in the next time step is very high. However the probability of remaining in the spike regime in the next time step, denoted by q_{ii} in Table 16 is also relatively high. The probabilities of being in the spike regimes in the next time step for each model are 0.93993 for the specification with Gaussian spikes, 0.67999 for the specification with Log-Gaussian spikes, 0.62685 for the specification with Pareto spikes and 0.68763 for the specification with mean-reverting spikes. The probability of a spike therefore varies between 2.733 per cent for the specification with Gaussian spikes, 2.905 per cent for the specification with Log-Gaussian spikes, 2.35 percent for the specification with Pareto spikes and 2.75 per cent for the specification with mean-reverting spikes. The unconditional probability of being in the spike regime, denoted by $P(R = i)$, is smaller than the unconditional probabilities of being in the base regimes of each specification. However, the unconditional probabilities of being in the spike regimes are quite low for the specifications with log-Normal, Pareto and Mean-Reverting spikes as compared to the specification with Gaussian spikes.

Analogous to graphs plotted earlier for the NSW region, the following graphs reveal the probabilities that an observation comes from the spike regime at the bottom panels whilst the top panels are reserved for the time series displayed with observations identified as belonging to the spike regime for the VIC region. The following figures illustrate the estimated regimes of electricity prices in the VIC region. Figure 37 shows that the model with Gaussian spikes tends to underestimate the spike severity but it overestimates the number of spikes similar to the case for the NSW region. It incorrectly identifies the data as being in the spike regime when in fact the data may be in a base regime. While the Markov Regime-Switching model with Gaussian spikes generates a higher mean for the prices series in the base regime than the spike regime, the model with Log-Gaussian spikes tend to be more accurate and produce a mean of

the price series in the spike regime higher than the mean of the prices series in the base regime. This model also tends to identify large price moves as spikes in the VIC region more accurately than the previous model. The Markov Regime-Switching model specification with Pareto spikes performs relatively well in identifying the large price moves as spikes and assigns these observations as being in the spike regime.

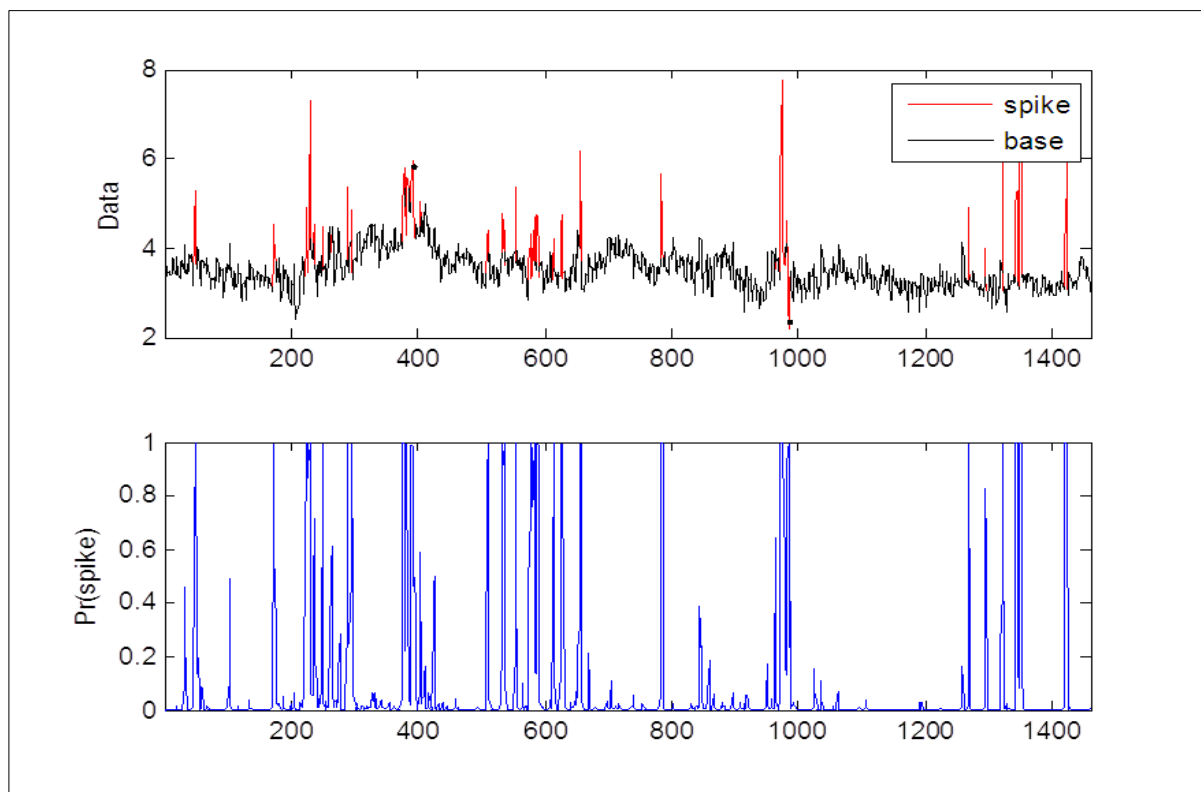
The regimes of electricity prices in the VIC region shows that the model with mean-reverting spikes tends to identify a price series being in either of the base and spike regimes similar to the model specification with Log-Gaussian spikes. The model tends to identify large price moves as spikes in the VIC region more accurately than the previous model specification with Gaussian spikes.

Figure 37 Regime Probabilities (VIC), Gaussian Spikes



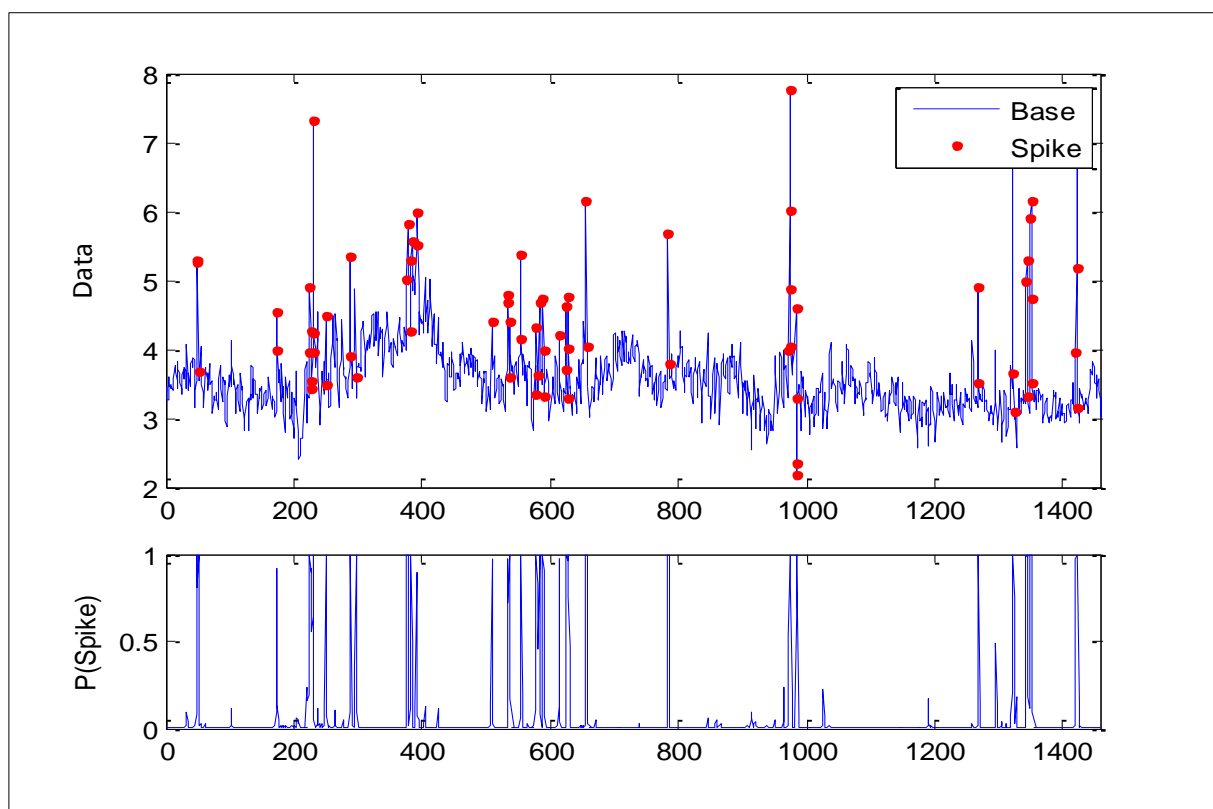
Source: Author's calculations.

Figure 38 Regime Probabilities (VIC), Log-Gaussian Spikes



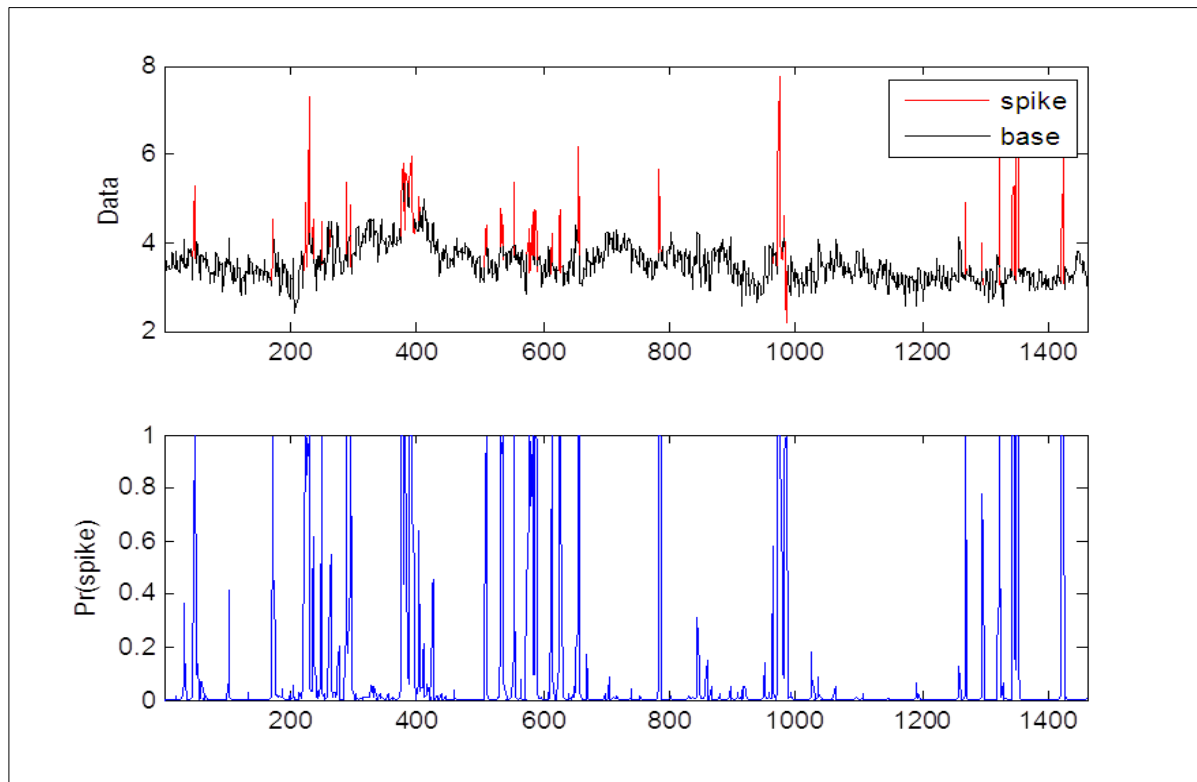
Source: Author's calculations.

Figure 39 Regime Probabilities (VIC), Pareto Spikes



Source: Author's calculations

Figure 40 Regime Probabilities (VIC), Mean-Reverting Spikes



Source: Author's calculations.

Two-regime model for QLD

Table 17 has the same interpretation as previous tables of this section.

Table 17 Regime Probabilities, QLD Region

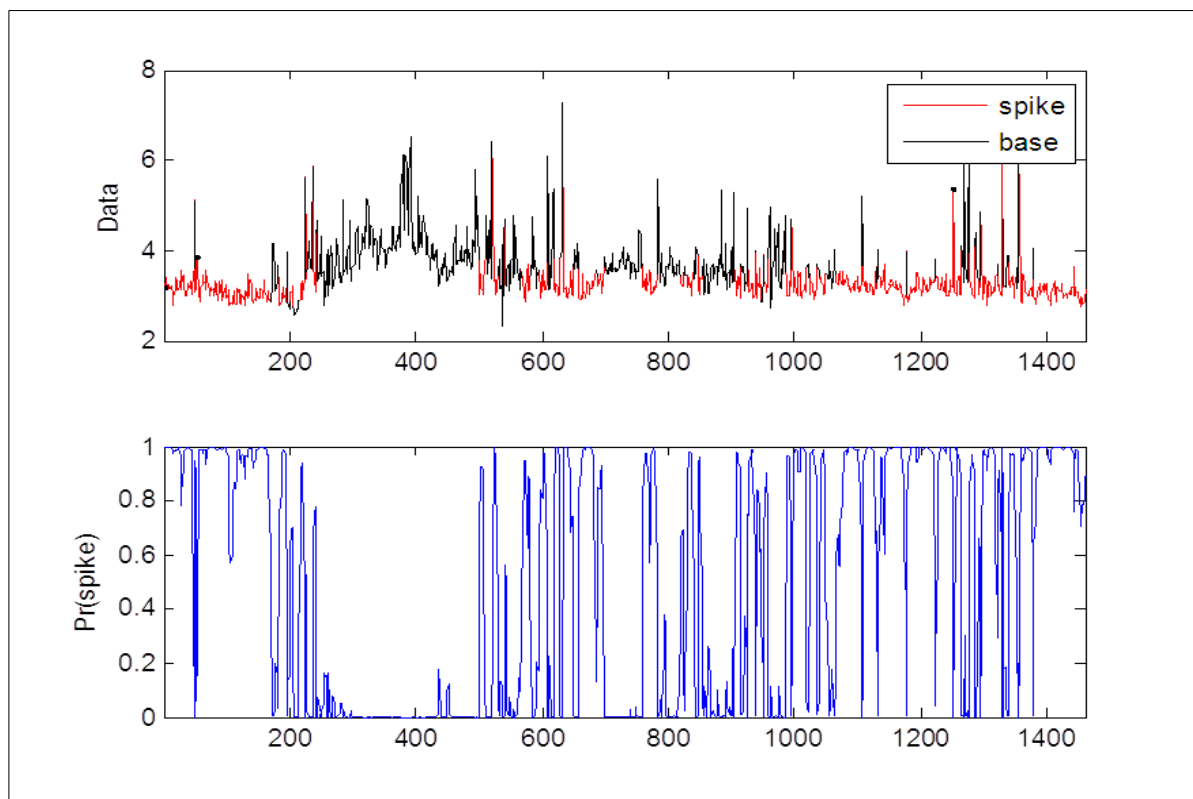
Panel A				Panel B			
Parameter estimates				Statistics			
Regime	β	c_i	σ_i^2	$E(Y_{t,i})$	$Var(Y_{t,i})$	q_{ii}	$P(R = i)$
Two regime model with Gaussian spikes							
Base	0.39312	1.53176	0.27629	3.89645	0.43737	0.92859	0.49326
Spike	-	3.15728	0.02852	3.15728	0.02852	0.93049	0.50674
Two regime model with Log-Gaussian spikes							
Base	0.10432	0.34459	0.02487	3.30311	0.12576	0.92693	0.78337
Spike	-	1.35463	0.03565	3.94503	0.56483	0.73578	0.21663
Two regime model with Pareto spikes							
Base	0.13174	0.44593	0.03422	3.38485	0.13902	0.94587	0.86344
Spike	-	1.82675	2.30358	5.08991	81.85773	0.65774	0.13656
Two regime model with mean-reverting process for spikes							
Base	0.12334	0.40750	0.02514	3.30377	0.10859	0.93294	0.78453
Spike	0.69581	2.77532	0.63532	3.98860	0.70011	0.75582	0.21547

Source: Author's calculations.

The following figures illustrate the estimated regimes of electricity prices in QLD region. They show that the model with Gaussian spikes tends to underestimate the spike severity but it overestimates the number of spikes. It incorrectly identifies the data as being in the spike regime when in fact the data may be in a base regime.

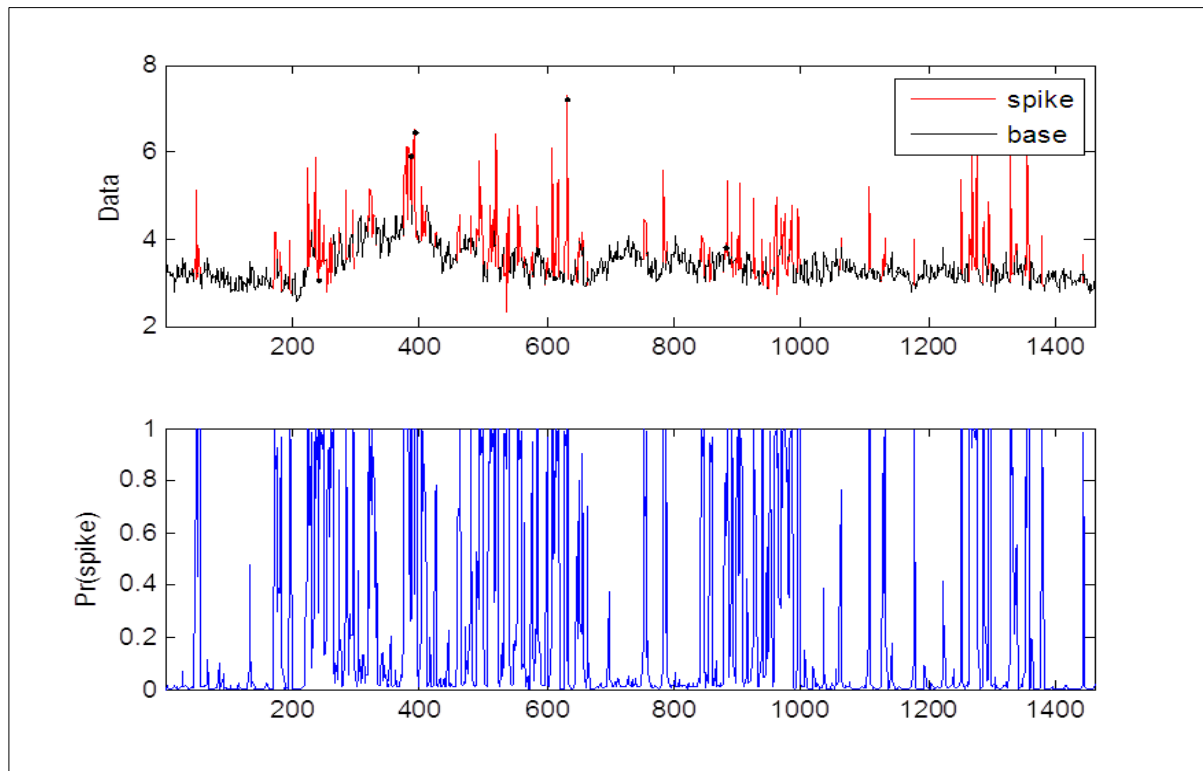
While the Markov Regime-Switching model with Gaussian spikes generates a higher mean for the prices series in the base regime than the spike regime, the model with Log-Gaussian spikes tends to be more accurate and produce a mean of the price series in the spike regime higher than the mean of the prices series in the base regime. This model also tends to identify large price moves as spikes in the QLD region more accurately than the previous model.

Figure 41 Regime Probabilities (QLD), Gaussian Spikes



Source: Author's calculations.

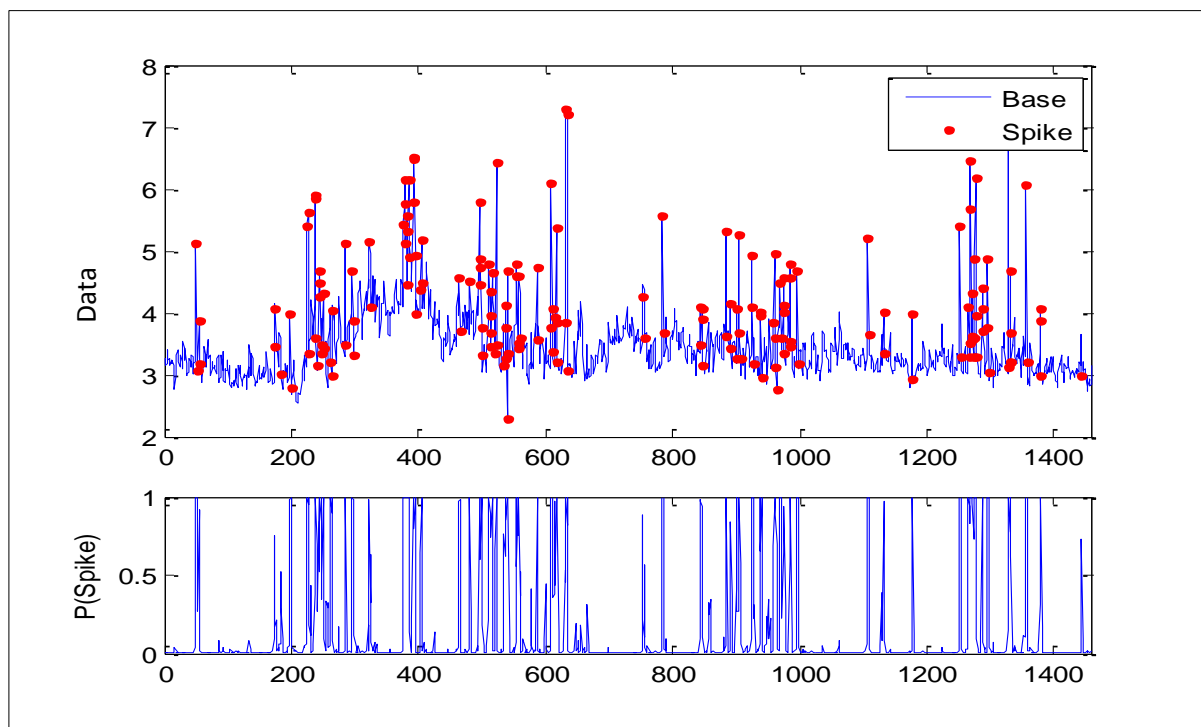
Figure 42 Regime Probabilities (QLD), Log-Gaussian Spikes



Source: Author's calculations.

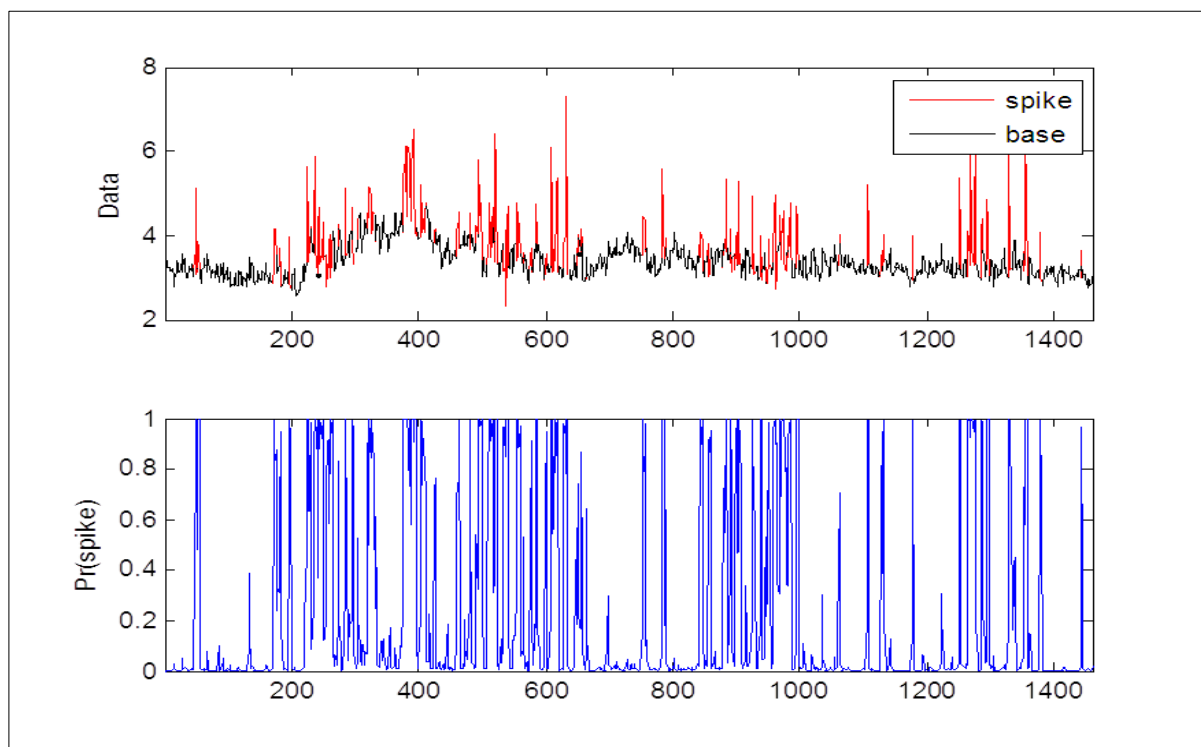
The Markov Regime-Switching model specification with Pareto spikes performs a relatively good performance in identifying the large price moves as spikes and assigns these observations as being in the spike regime. The regimes of electricity prices in the QLD region shows that the model with mean-reverting spikes tends to identify the price series being in either of the base and spike regimes similar to the model specification with Log-Gaussian spikes. The model tends to identify large price moves as spikes in the QLD region more accurately than the previous model specification with Gaussian spikes.

Figure 43 Regime Probabilities (QLD), Pareto Spikes



Source: Author's calculations.

Figure 44 Regime Probabilities (QLD), Mean-Reverting Spikes



Source: Author's calculations

Two-regime model for SA region

Table 18 has the same interpretation as previous tables of this section.

Table 18 Regime Probabilities, SA Region

Panel A				Panel B			
Parameter estimates				Statistics			
Regime	β	c_i	σ_i^2	$\mathbb{E}(Y_{t,i})$	$Var(Y_{t,i})$	q_{ii}	$\mathbb{P}(R = i)$
Two regime model with Gaussian spikes							
Base	0.44463	1.89545	0.72784	4.26297	1.05245	0.85222	0.25270
Spike	-	3.44800	0.07424	3.44800	0.07424	0.95003	0.74730
Two regime model with Log-Gaussian spikes							
Base	0.14354	0.50507	0.04056	3.51860	0.15222	0.95468	0.88424
Spike	-	1.40443	0.07697	4.23302	1.43373	0.65386	0.11576
Two regime model with Pareto spikes							
Base	0.13174	0.44593	0.03422	3.38485	0.13902	0.94587	0.86344
Spike	-	1.82675	2.30358	5.08991	81.85773	0.65774	0.13656
Two regime model with mean-reverting process for spikes							
Base	0.15335	0.53942	0.04145	3.51748	0.14639	0.95873	0.89273
Spike	0.80509	3.47740	1.64159	4.31926	1.70641	0.65654	0.10727

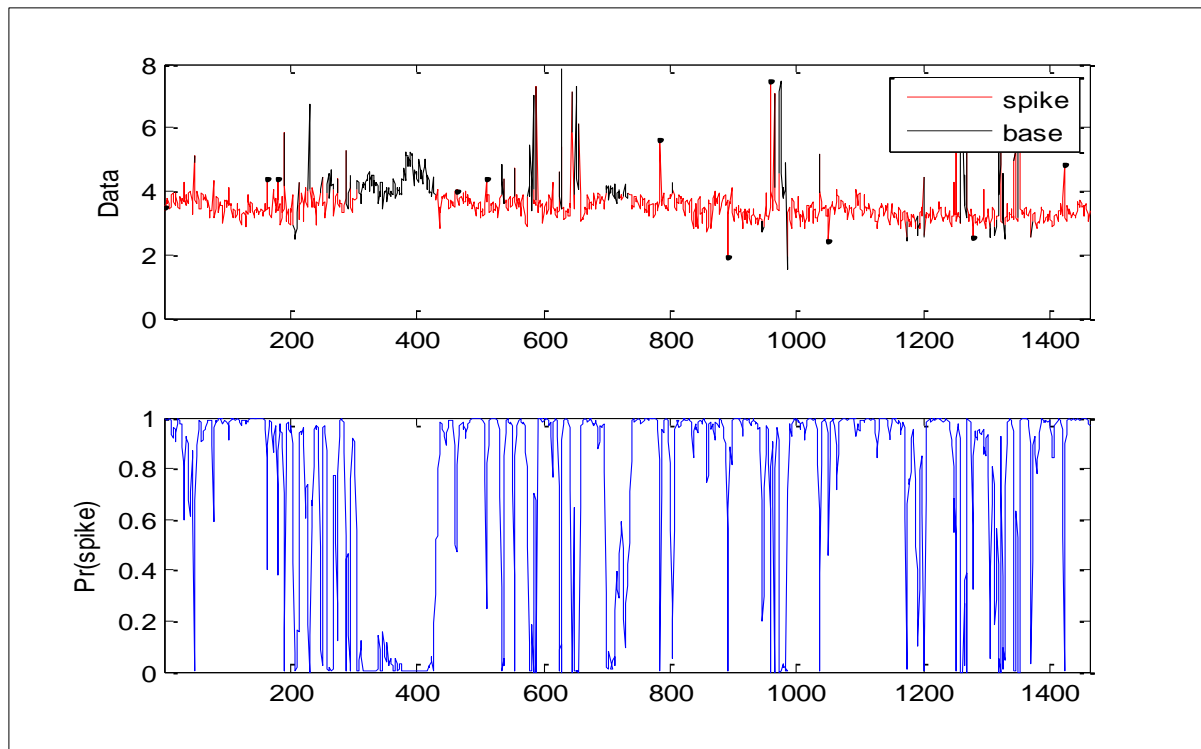
Source: Author's calculations

The following figures illustrate the estimated regimes of electricity prices in the SA region. They show that the model with Gaussian spikes tends to underestimate the spike severity but it overestimates the number of spikes. It incorrectly identifies the data as being in the spike regime when in fact the data may be in a base regime. While the Markov Regime-Switching model with Gaussian spikes generates a higher mean for the prices series in the base regime than the spike regime. The model with Log-Gaussian spikes tends to be more accurate and produces a mean of the price series in the spike regime higher than the mean of the prices series in the base regime. This model also tends to identify large price moves as spikes in the SA region more accurately than the previous model.

The Markov Regime-Switching model specification with Pareto spikes performs a relatively solid performance in identifying the large price moves as spikes and assigns these observations as being in the spike regime. The regimes of electricity prices in the SA region shows that the

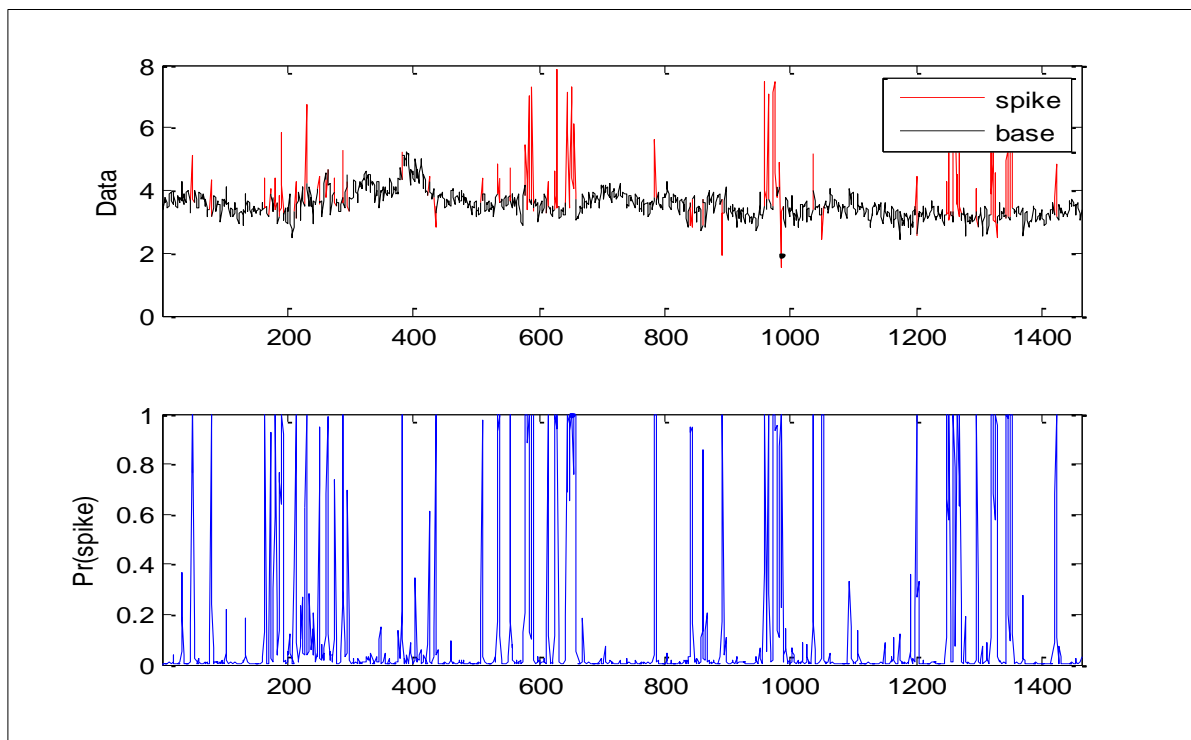
model with mean-reverting spikes tends to identify the price series being in either of the base and spike regimes similar to the model specification with Log-Gaussian spikes. The model tends to identify large price moves as spikes in the SA region more accurately than the previous model specification with Gaussian spikes.

Figure 45 Regime Probabilities (SA), Gaussian Spikes



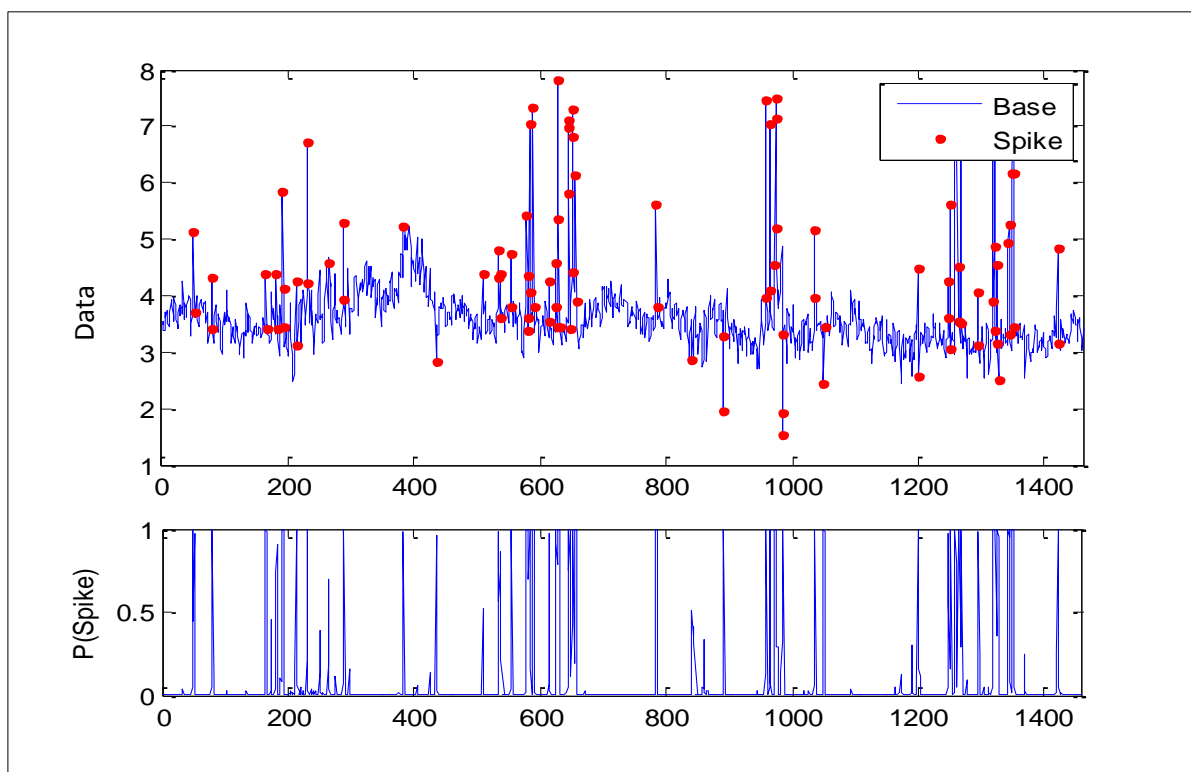
Source: Author's calculations

Figure 46 Regime Probabilities (SA), Log-Gaussian Spikes



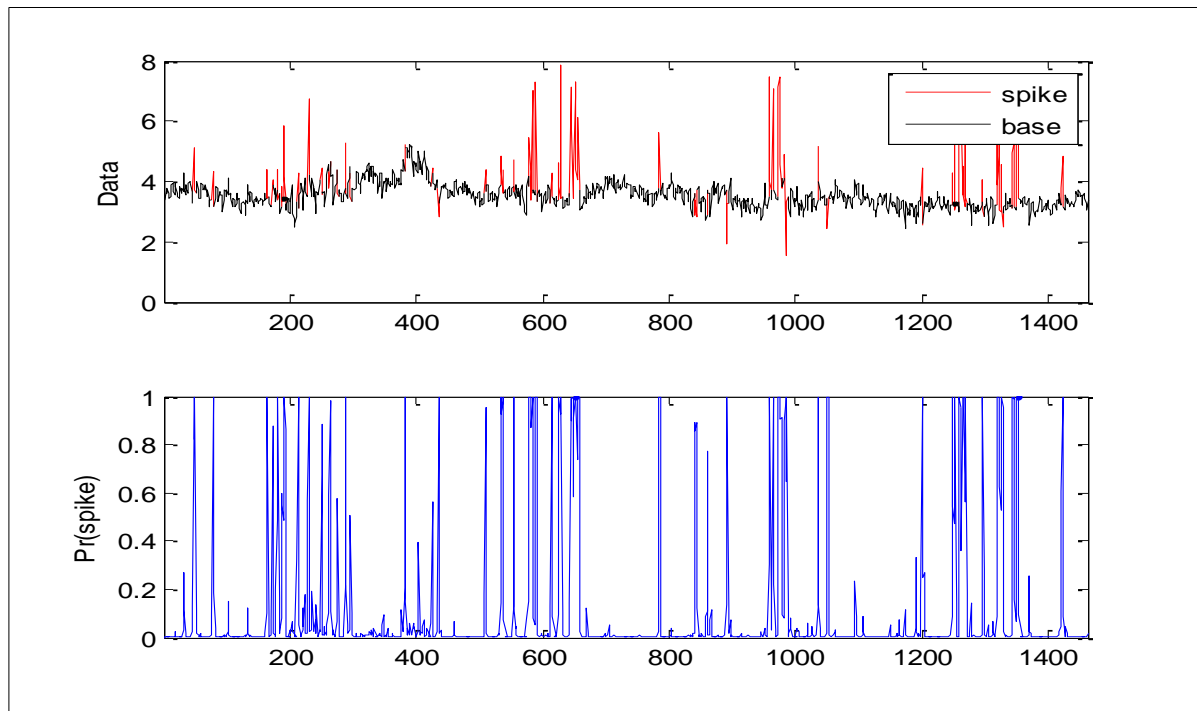
Source: Author's calculations

Figure 47 Regime Probabilities (SA), Pareto Spikes



Source: Author's calculations

Figure 48 Regime Probabilities (SA), Mean-Reverting Spikes



Source: Author's calculations

Two-regime model for TAS

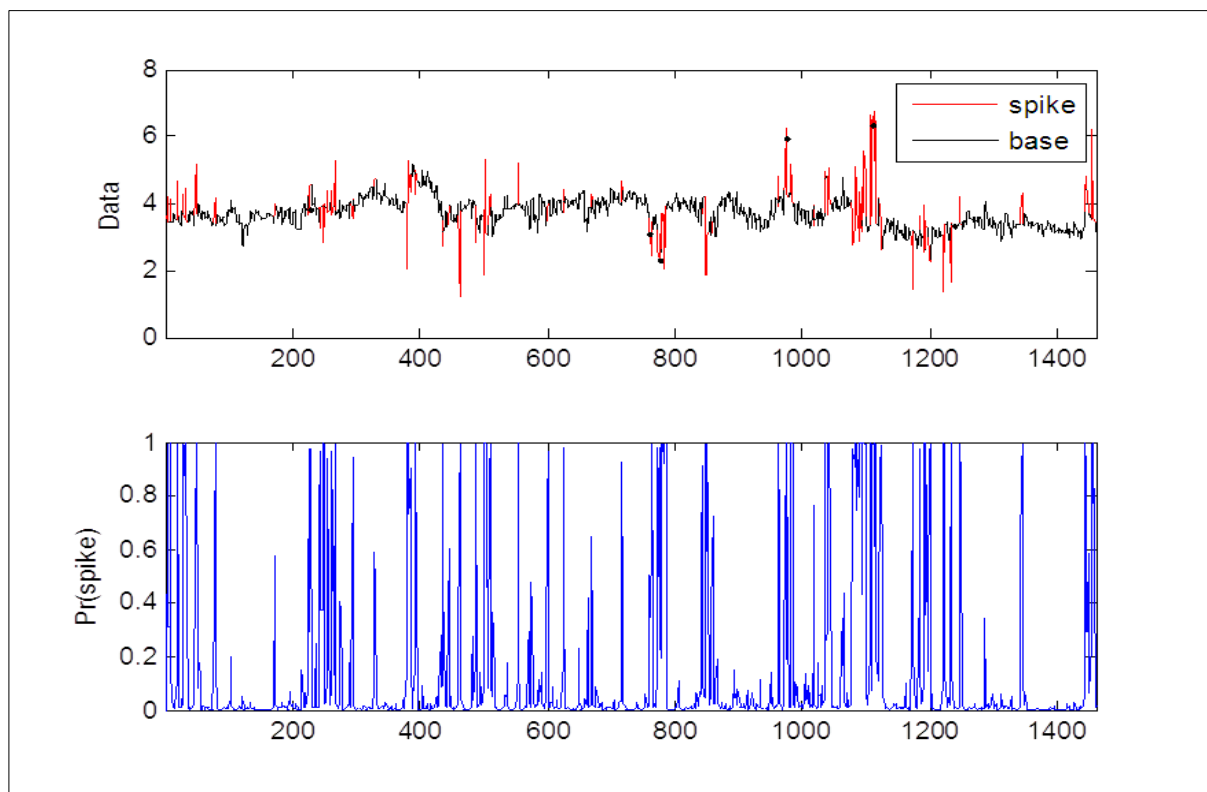
Table 19 has the same interpretation as previous tables of this section.

Table 19 Regime Probabilities, TAS Region

Panel A				Panel B			
Parameter estimates				Statistics			
Regime	β	c_i	σ_i^2	$E(Y_{t,i})$	$Var(Y_{t,i})$	q_{ii}	$P(R = i)$
Two regime model with Gaussian spikes							
Base	0.09533	0.35138	0.02745	3.68603	0.15120	0.94139	0.85936
Spike	-	3.79380	0.86474	3.79380	0.86474	0.64185	0.14064
Two regime model with Log-Gaussian spikes							
Base	0.09924	0.36730	0.02808	3.70112	0.14884	0.94609	0.86334
Spike	-	1.29108	0.07348	3.77280	1.08526	0.65942	0.13666
Two regime model with Pareto spikes							
Base	0.12349	0.45794	0.03709	3.70824	0.16004	0.96597	0.92214
Spike	-	0.91009	1.21788	12.32732	153.2012	0.59703	0.07786
Two regime model with mean-reverting process for spikes							
Base	0.09447	0.34840	0.02562	3.68789	0.14231	0.93976	0.82909
Spike	0.45932	1.74582	0.58090	3.80091	0.82087	0.70778	0.17091

Source: Author's calculations.

Figure 49 Regime Probabilities (TAS), Gaussian Spikes

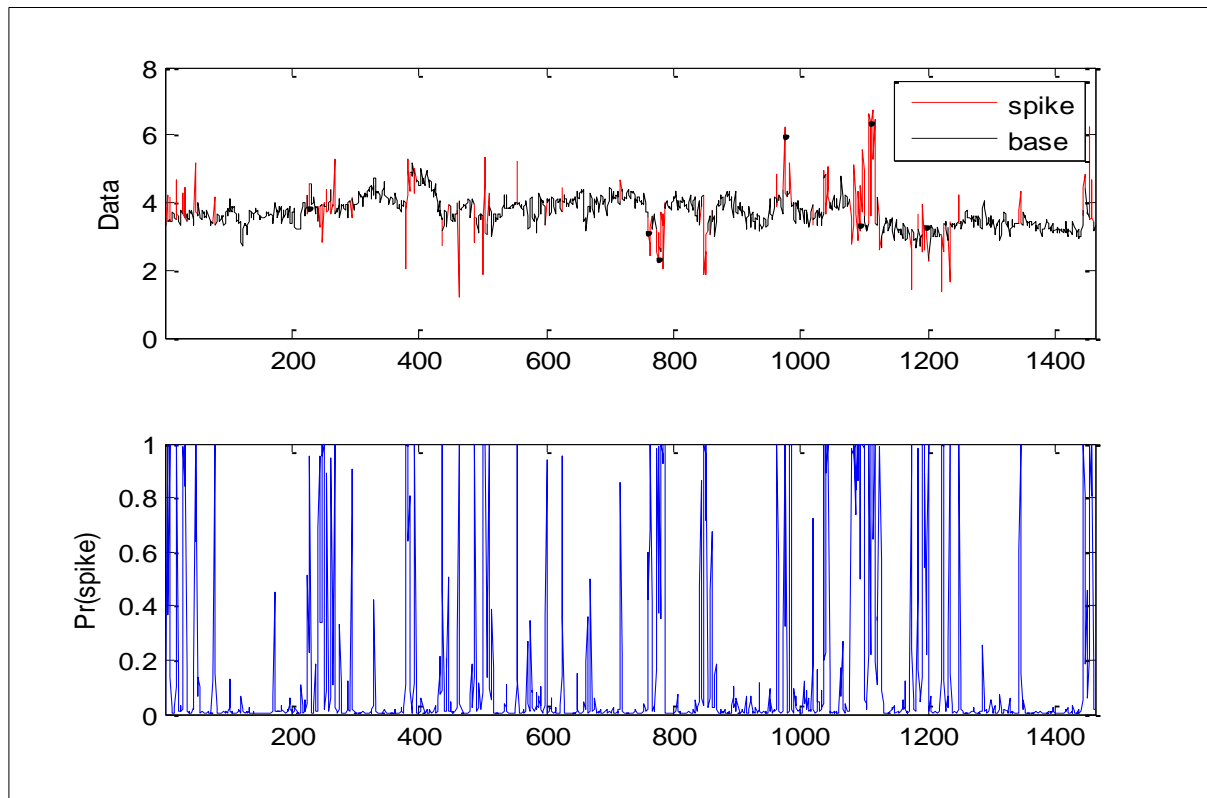


Source: Author's calculations.

The estimated regimes of electricity prices in the TAS region shows that the model with Gaussian spikes tends to underestimate the spike severity but it overestimates the number of spikes. It incorrectly identifies the data as being in the spike regime when in fact the data may be in a base regime. There are also instances where the model does not identify large movements in the prices as being in a spike regime.

Perhaps, the most important observation based on this model is that the mean price series in the base regime is greater than the mean price series in the spike regime. This is contrary to the stylised characteristics of electricity prices in NEM.

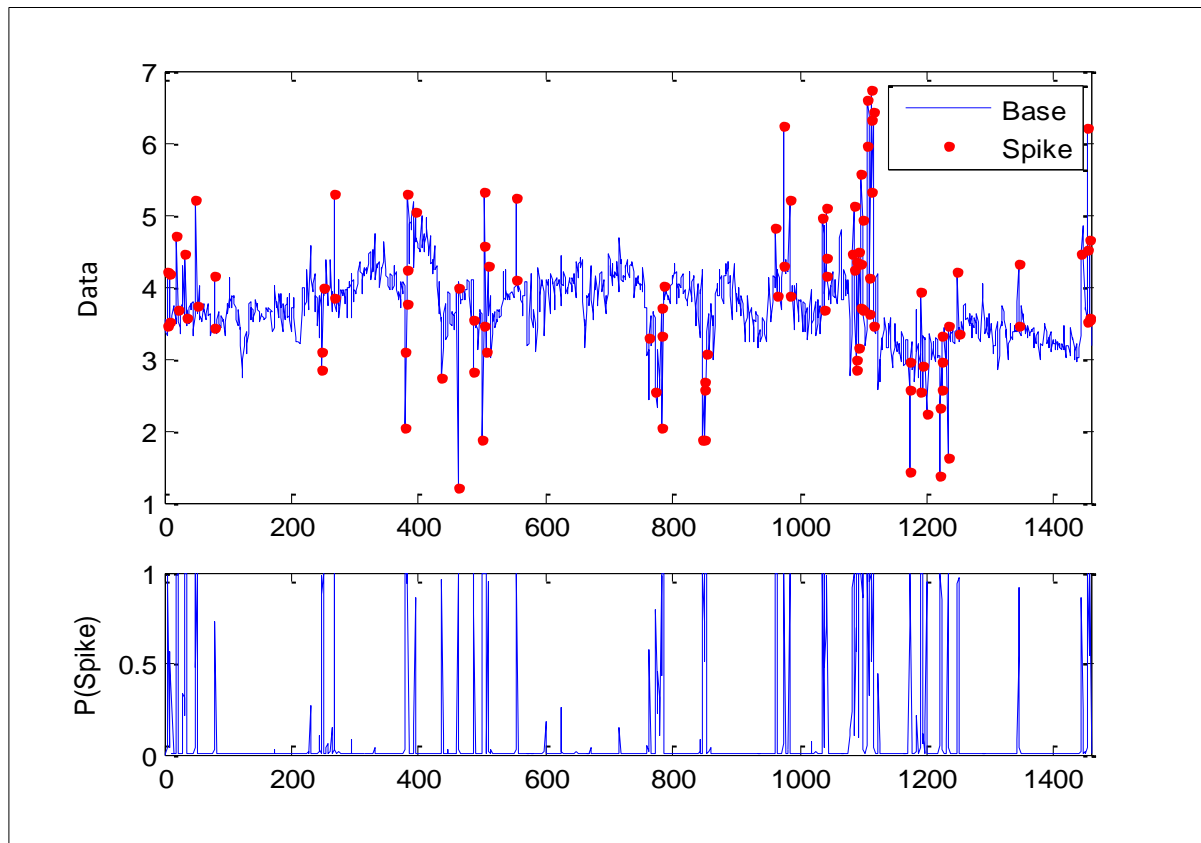
Figure 50 Regime Probabilities (TAS), Log-Gaussian Spikes



Source: Author's calculations.

While the Markov Regime-Switching model with Gaussian spikes generates a higher mean for the price series in the base regime than the spike regime, the model with Log-Gaussian spikes tends to be more accurate and produces a mean of the price series in spike regime higher than the mean of the price series in the base regime. This model also tends to identify large price moves as spikes in the TAS region more accurately than the previous model.

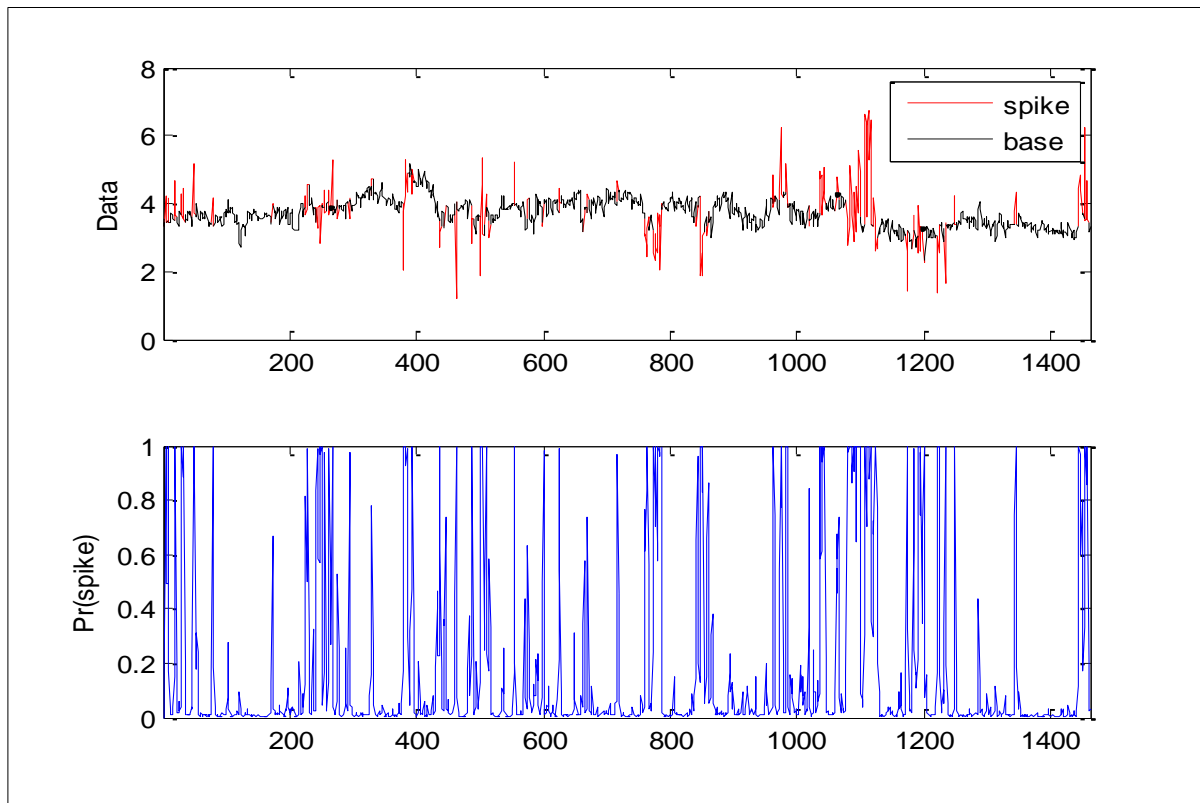
Figure 51 Regime Probabilities (TAS), Pareto Spikes



Source: Author's calculations

The Markov Regime-Switching model specification with Pareto spikes perform a relatively solid performance in identifying the large price moves as spikes and assign these observations as being in the spike regime. This model, by its nature, identifies less numbers of spikes as compared to the model specification with Log-Gaussian spikes. This is due to the fact that it identifies more extreme price moves as a spike. Therefore the mean of the price series identified as being in the spike regime is higher than the price series identified by the model specification with Log-Gaussian spikes.

Figure 52 Regime Probabilities (TAS), Mean-Reverting Spikes



Source: Author's calculations.

The estimated regimes of electricity prices in the TAS region shows that the model with mean-reverting spikes tends to identify the price series being in either of the base and spike regimes similar to the model specification with Log-Gaussian spikes. The model generates a mean of the price series in the spike regime higher than the mean of the prices series in the base regime. This model also tends to identify large price moves as spikes in the TAS region more accurately than the previous model specification with Gaussian spikes.

This section utilised Markov Regime-Switching models with two separate regimes (base and spike regimes) in modelling electricity prices in NEM. Whilst all of the SDE based models previously discussed in this study are useful in modelling spot prices, only the Markov Regime-

Switching model fully accounts for the high volatility, mean-reversion and consecutive spike-prone behaviour of electricity markets.

A number of salient features of this model are useful for understanding the price dynamics in NEM. The unconditional probabilities of a price spike on any particular day ranges between 10 to about 50 percent in NEM, depending on the chosen spike regime and region of the NEM. However, while these spikes are frequent, they are short-lived. In fact, prices generally revert faster when returning from spike periods than in normal periods. This is clearly seen in the top panel of the figures presented above. Secondly, price spikes account for much of the volatility in electricity spot prices. Third, there is great variation in the magnitude of spikes in NEM, with spikes being generally largest in SA and smallest in QLD. However, price spikes are less uniform in the QLD market, suggesting a higher degree of uncertainty in general price levels.

Estimated parameters based on empirical data are meaningful and can be interpreted as distinct between the different phases of volatility behaviour prevalent in electricity prices in NEM. Markov Regime-Switching specifications presented in this section lead to different results in terms of the probabilities of being in the base and spike regimes in the next steps and unconditional probabilities of being in those regimes. In all specifications except the model with Gaussian spikes, variance estimates for spike regimes $Var(Y_{t,i})$ are higher than the variance estimates for base regimes $Var(Y_{t,i})$ for all the regions of NEM. This indicates a variance level higher for spike regimes than for base regimes. This is an important finding from a price risk management perspective as it indicates that prices in spike regimes tend to have a higher variance, which in turn results in higher management costs.

The Markov Regime-Switching models applied in this section are useful tools in modelling electricity price dynamics in NEM as they provide logical paths to classify regime changes. These regime changes are of utmost importance for the NEM market players in managing their price risk. Market players' costs in managing their price risk will be reduced with better understanding of the price regimes. Accurate estimation of the regimes allows the generators to estimate their bids for production, which in turn contributes to more effective wholesale pricing in NEM.

SIMULATION OF MARKOV REGIME-SWITCHING MODEL

Higgs and Worthington (2010) modelled electricity prices in NEM with Markov Regime-Switching models. The main limitation of their study was the restrictive assumption regarding spike behaviour. The methodology they employed followed a three-regime structure proposed by Huisman and Mahieu (2003) where a normal regime, a jump regime created by the spike and a jump reversal regime where the price returns to the normal level were considered. Accordingly, there is no allowance for consecutive spikes that may arise.

The model of Markov Regime-Switching specifications with Gaussian and Log-Gaussian spikes developed in this study allows consecutive spikes in a natural way following the suggestions put forward by de Jong and Huisman (2002), Bierbrauer et al. (2004) and de Jong (2006). The consecutive spikes generated by the Markov Regime-Switching models of this study are realised in the lower panels of the regime probability figures, depicted earlier. In these figures, the probabilities of being in the spike regime are grouped together ($P\{R_t = 2\} > 0.5$).

The models of Markov Regime-Switching models described in this chapter are simulated following the approach taken in Janczura and Weron (2010). The simulation process generated a number of trajectories of a Markov Regime-Switching model with two independent regimes;

- (i) Mean-reverting process in the base regime and,
- (ii) Gaussian and a Log-Gaussian distributed spike regimes.

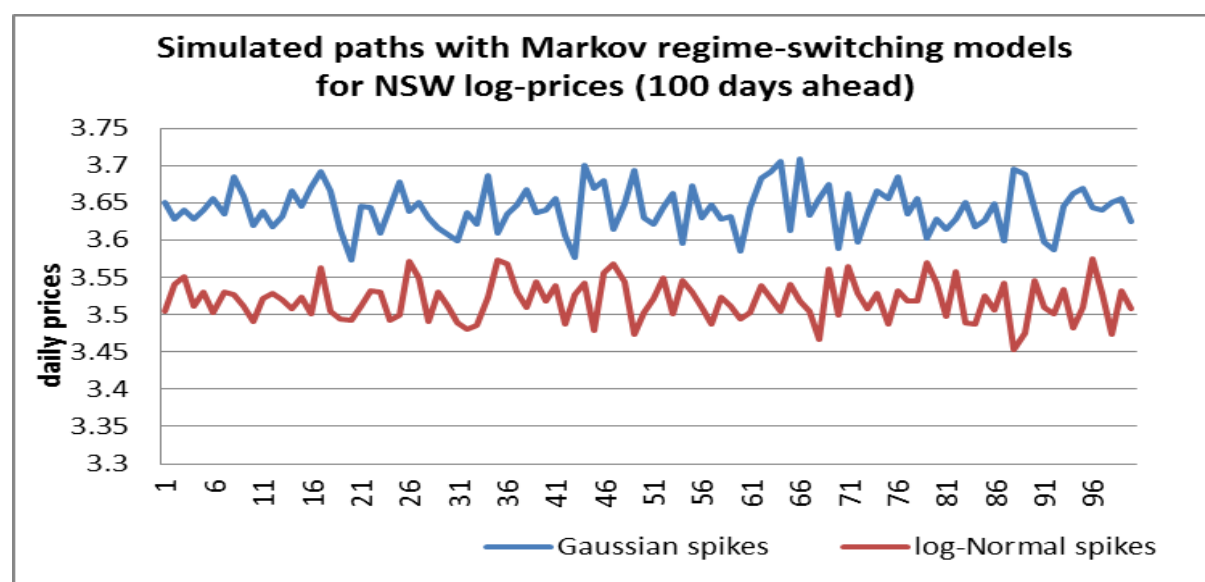
The simulation algorithm utilised consists of the transition matrix, model parameters and probabilities classifying the first observation to one of the regimes. The choice of the mean-reverting process in the base regime in simulating the price series reflects the fundamental characteristics of electricity prices in NEM. The spike regime of the model is simulated with Gaussian and Log-Gaussian distributions. The simulation of the spike regimes with the mean-reverting process is not considered as model parameters estimated for the model. The mean-reverting spikes are remarkably similar with the model with Log-Gaussian spikes as described previously. Similarly, simulation of the spike regime with Pareto distribution is not contemplated here as the parameter estimates of this model provide extremely high variance in all the regions of NEM.

The procedure followed in this section provided three months future point forecast values as the average of 10,000 simulations. The following charts illustrate an average of 10,000 sample paths generated via a Markov Regime-Switching model with Gaussian and Log-Gaussian spikes for the NSW region. As is seen, Markov Regime-Switching models generate random prices deviating from the long-run mean but on average these prices revert back to the long-run mean levels.

The noticeable difference in the mean of simulated values between the two specifications is due to moment characteristics of the distributions applied. The expected value of the Log-

Gaussian distribution is smaller than the expected value of the Gaussian distribution. The differences in the mean values of the simulations are attributed to this.

Figure 53 A Simulated Path with a Markov Regime Switching Model

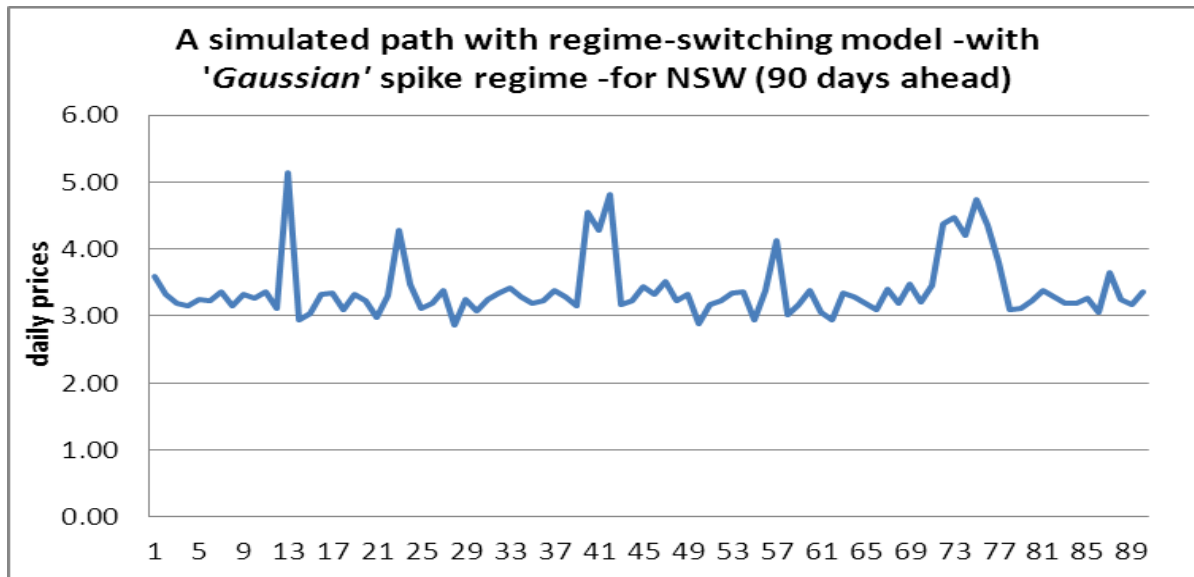


Source: Author's calculations.

The following charts illustrate a sample path generated via the Markov Regime-Switching model where the spike regimes are modelled with Gaussian and Log-Gaussian distributions for the NSW region. Figure 54 illustrates the sample path of a Monte-Carlo simulation of a Markov Regime-Switching specification with spikes distributed normally whilst Figure 55 illustrates the sample path of a Monte-Carlo simulation of a Markov Regime-Switching specification with spikes distributed log-normally for a period of 90 days. As is seen, the mean-reverting component of the model generates random prices deviating from the long-run mean but on average these prices revert back to the long-run mean levels. The spike component of the model generates spikes that last for a number of days with varying degrees of magnitude. As is evident from the charts below, unlike mean-reverting jump-diffusion models, the Markov Regime-Switching models also allow for consecutive spikes. It is this characteristic that makes Markov

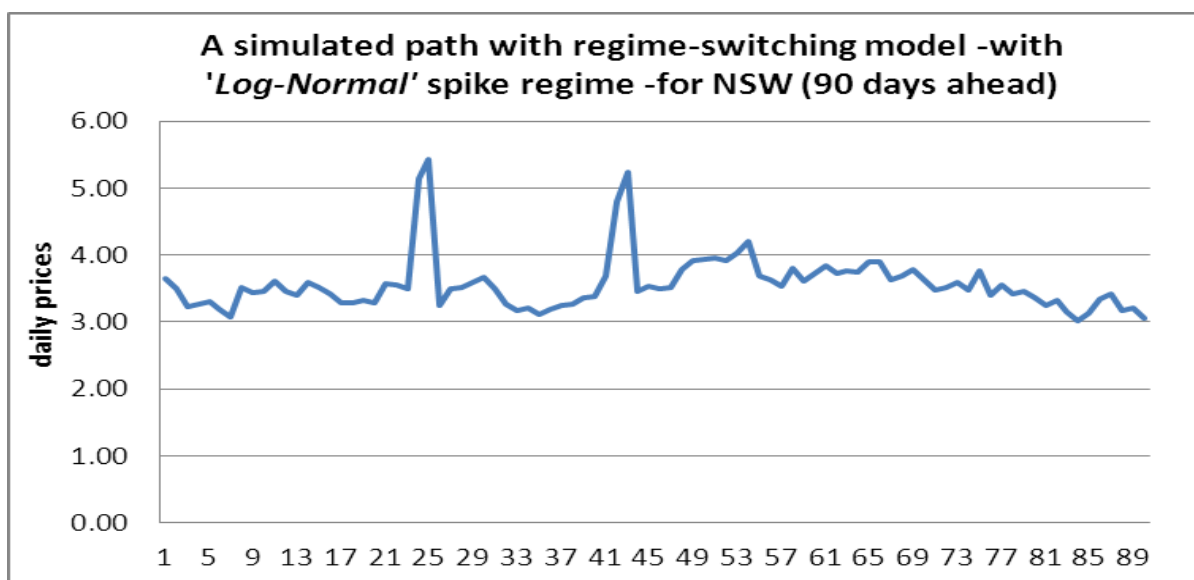
Regime-Switching models superior to simpler jump-diffusion specifications of previous sections of this thesis.

Figure 54 A Simulated Price Path with Spikes Distributed as Gaussian



Source: Author's calculations.

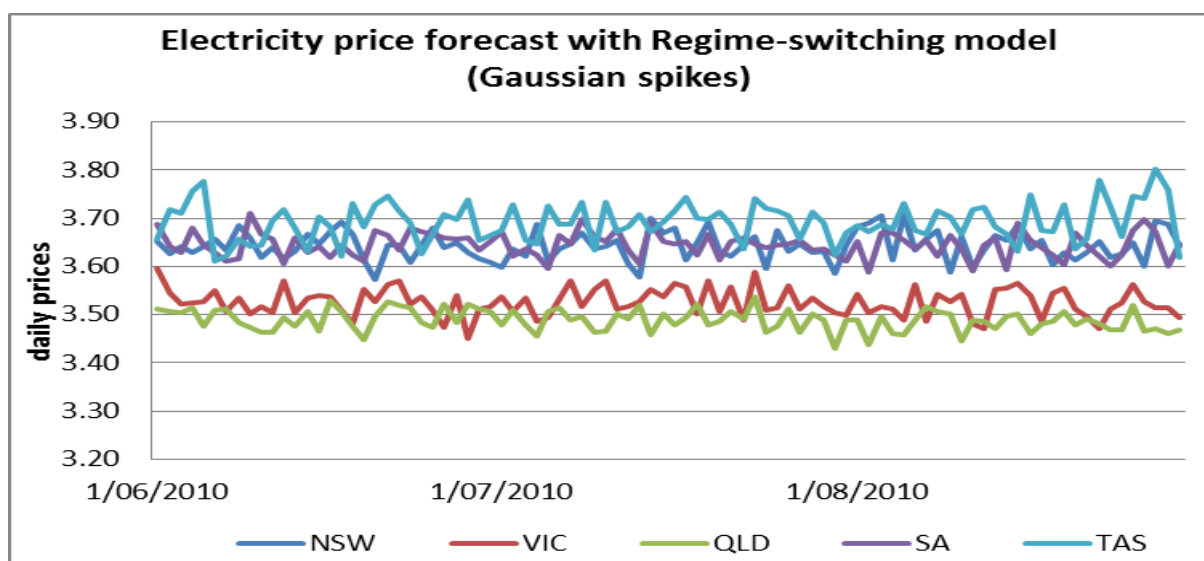
Figure 55 A Simulated Price Path with Spikes Distributed as Log-Normal



Source: Author's calculations.

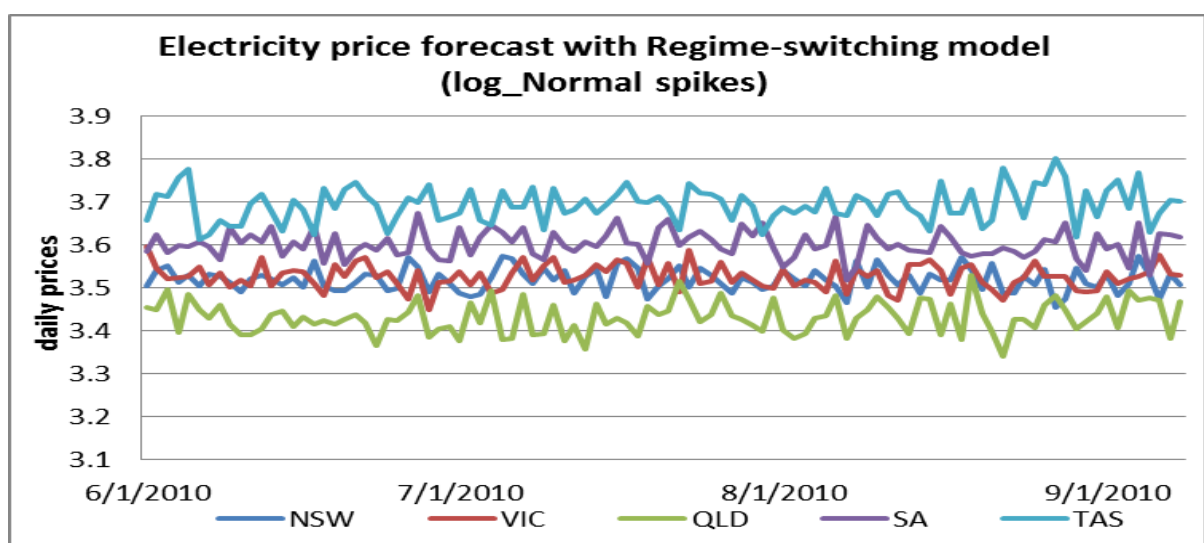
Electricity price forecasts generated by the Markov Regime-Switching model with Gaussian and Log-Gaussian spike processes for all regions of the NEM are presented in charts 56 and 57 for a forecast horizon of up to three months. The forecast values are an average of 10,000 simulations. As is seen, the simulated price series tend to be around the long-run mean levels. The spot price for the last day of the in-sample-period forms the starting value in the following figures.

Figure 56 Price Forecast for All Regions of NEM (Spikes dist. Gaussian)



Source: Author's calculations.

Figure 57 Price Forecast for All Regions of NEM (Spikes dist. Log-Normal)



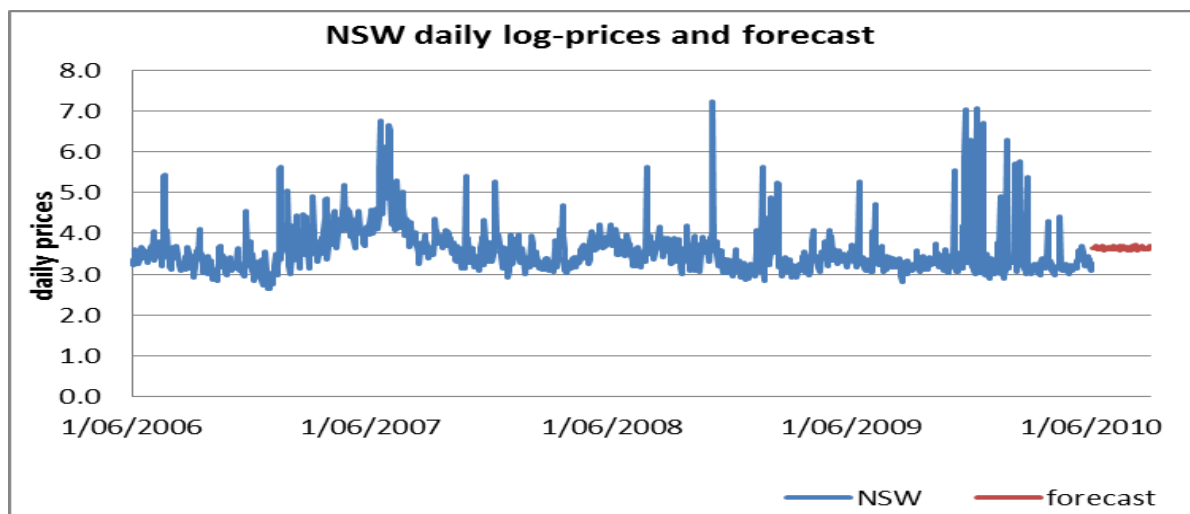
Source: Author's calculations

The simulations generated reflect the mean-reversion dynamics of electricity prices. This is achieved through modelling the base dynamics via a Mean-Reverting model. The distinction of modelling electricity prices with Markov Regime-Switching models as compared to the Mean-Reverting and Jump-Diffusion model is the spike formation mechanism. In the jump-diffusion model, it is the compound Poisson process that generates spikes whereas in the Markov Regime-Switching model, the specification itself is able to produce spikes that last for more than a day.

The Markov Regime-Switching model despite its complex calibration process and theoretic advantage does not perform better on face value than the simpler mean-reverting and mean-reverting jump-diffusion models. The superiority of the Markov Regime-Switching specifications over other previously examined continuous-time models is that it captures the consecutive price spikes prevalent in electricity prices. However, the magnitude and timing of these spikes does not align with the actual price spikes that occur.

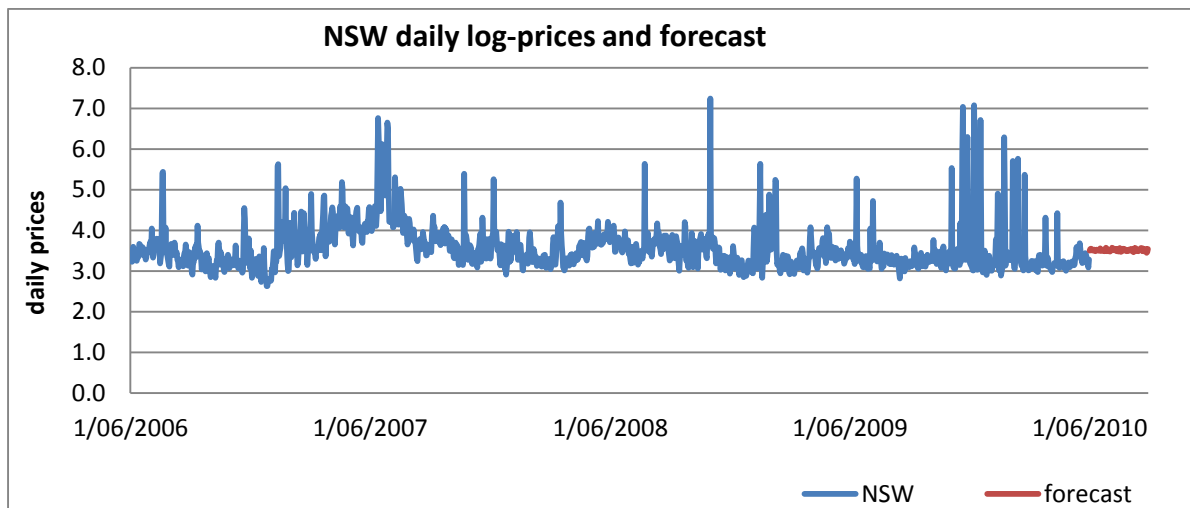
Figures 58 and 59 show the forecast fit to the in-sample data of the study whilst Figure 60 illustrates the actual versus simulated price trajectories of the Markov Regime-Switching models with spikes distributed both normally and log-normally. The empirical evidence of modelling with Mean-Reverting and Jump-Diffusion models shows that the jump component of this continuous-time model fails to capture the magnitude and timing of spikes in the electricity price series. Simulations based on this continuous-time specification over the 90 day forecast horizon do not represent the true price paths of the electricity prices. It is also acknowledged that the forecast prices are bias estimators.

Figure 58 Markov Regime-Switching Model with Spikes Dist. as Gaussian (NSW)



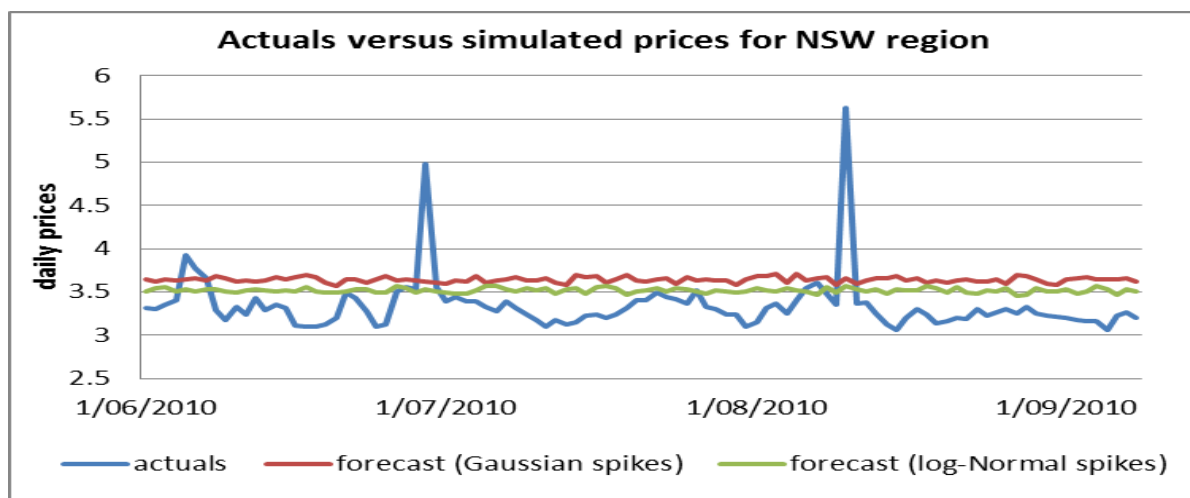
Source: Author's calculations.

Figure 59 Markov Regime-Switching Model with Spikes Dist. as Log-Gaussian for (NSW)



Source: Author's calculations.

Figure 60 Actual versus Simulated Price Paths for NSW



Source: Author's calculations.

The models investigated in this chapter produce estimates for transition probabilities that can be interpreted according to market behaviour. Simulated trajectories show similarity with real price data. However, it is found that the number of price spikes or extreme events produced by simulations of the estimated models is higher than what could be observed in real price data.

In the next section, a formal evaluation of the forecasting performance of the Markov Regime-Switching model is presented. The purpose of the next section is to see if the Markov Regime-Switching model's forecast errors are within reasonable limits or whether these errors are too large.

EVALUATION OF FORECAST PERFORMANCE

The following discussion evaluates the forecasting performance of the Markov Regime-Switching models. The purpose of this section is to see if this model's forecast errors are within the reasonable limit of expectations or whether these errors are unreasonably large and require an improvement in the statistical models and processes. The models built in this section possess two regimes; base regime and spike regime. The probabilities to stay in one regime or to move from one regime to another were derived.

To formally assess the accuracy of the forecast values generated by the mean-reverting jump-diffusion model, forecast accuracy statistics are produced for each region of the NEM in a similar fashion to previous sections.

The forecast performance measures are Mean Error (ME), Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Square Error (RMSE), Mean Percentage Error (MPE), Mean Absolute Percentage Error (MAPE) and Theil's U. Although, there are a number of forecast

accuracy measures produced here, the following discussion is based on the RMSE as this is focused on large errors.

As electricity prices spike and revert to long-run mean levels quickly, the forecast values generated by the Markov Regime-Switching models are subject to large errors. It is important to note that RMSE is the most widely used measure in the literature for its statistical properties as it places a greater penalty on large forecast errors than the MAE.

The empirical analyses of the models in terms of forecast performance provide encouraging results. As is seen in Table 20, the models generally provide better RMSE values compared to other SDE based models examined in this study. The model with Gaussian spikes has RMSE of 0.43 for NSW, 0.54 for VIC, 0.5 for QLD, 0.52 for SA and 0.61 for TAS regions. These RMSE values are generally better than previously examined continuous-time models.

The actual improvements in forecast performance emerge with the modelling of the spikes process with Log-Gaussian distribution. The Markov Regime-Switching model with Log-Gaussian spikes provide RMSE of 0.36 for NSW, 0.31 for VIC, 0.46 for QLD, 0.5 for SA and 0.61 for TAS. These values are significant improvements in forecast accuracies when compared to the Markov Regime-Switching model with Gaussian spikes and all other previously examined continuous-time models. It is important to note that the main aim of the simulated models in this thesis is to minimise the variance of the expected mean values. The Markov Regime-Switching specifications so far does this more effectively than previously examined continuous-time models as measured by RMSE.

Table 20 Forecast Accuracy Statistics for Markov Regime-Switching Model

	NSW	VIC	QLD	SA	TAS
Modeling with Log-Gaussian spikes					
Mean error	-0.17439	-0.22096	-0.38398	-0.25533	-0.33779
Mean square error	0.13165	0.09748	0.21570	0.25499	0.37934
Root mean square error	0.362846	0.312226	0.464438	0.504971	0.61591
Mean absolute error	0.26592	0.26503	0.38397	0.32826	0.48248
Mean percentage error	-1.96315	-7.13073	-14.4449	-37.8101	-11.7423
Mean absolute percentage error	2.36153	8.26027	14.4449	39.54846	14.2447
Theil's U	0.05274	0.04566	0.07156	0.07239	0.08688
Modelling with Gaussian spikes					
Mean error	-0.29777	-0.33613	-0.44163	-0.29947	-0.33188
Mean square error	0.19280	0.29653	0.25724	0.27763	0.38190
Root mean square error	0.43909	0.54455	0.50718	0.52690	0.61798
Mean absolute error	0.37268	0.37107	0.44162	0.36173	0.48125
Mean percentage error	-9.61508	-10.634	-16.2979	-39.3879	-11.586
Mean absolute percentage error	11.07722	11.51137	16.29794	40.86051	14.1878
Theil's U	0.06270	0.07811	0.07747	0.07506	0.08725

Source: Author's calculations.

Comparison of these models' forecast accuracy with the benchmark model however indicates that the benchmark model outperforms Markov Regime-Switching specifications for each market region of the NEM measured by root mean square errors (RMSE). The AR-GARCH model used in this study has RMSE scores of 0.328 for New South Wales (NSW), 0.238 for Victoria (VIC), 0.336 for Queensland (QLD), 0.475 for SA and 0.605 for Tasmania (TAS).

Markov Regime-Switching model fully accounts for the high volatility, mean-reversion and spike-prone behaviour that is so characteristic of electricity prices in NEM. A number of salient

features are found in this model and these are useful for understanding the price dynamics in NEM. First, the probability of a price spike on any particular day ranges between two per cent to nearly eight per cent in separate regions of NEM depending on the model specifications. Second, price spikes account for much of the volatility in electricity spot prices. Volatility measures in base regimes are actually quite low albeit varies between the regions of NEM and the specific Markov Regime-Switching specification considered. These volatilities seem to reflect the marginal cost of production. Third, there is great variation in the magnitude of spikes in NEM, with spikes being generally largest in the SA and smallest in the QLD regions.

The forecasting of electricity prices in NEM with Markov Regime-Switching model in this chapter is restricted to a simple specification for the base regime process. The base regime is modelled with a mean-reverting specification with constant variance. Several studies like Janczura and Weron (2010) highlight the importance of more complex specifications. Furthermore, the seasonalities prevalent in the electricity price series in NEM makes the incorporation of time varying transition probabilities a necessary property of the Markov Regime-Switching models.

CHAPTER 10 -MODELLING ELECTRICITY PRICES WITH EVT AND COPULA FUNCTIONS

Electricity spot prices are characteristically non-Gaussian. Importantly they are among the most volatile commodities in the world as electricity cannot be stored, has limited transportability and has restricted arbitrage transactions. Previous investigations, presented in earlier chapters, into modelling Australian electricity prices are generally limited in scope as they have primarily considered techniques that are characterised either by Gaussian assumptions or ignored the price dependencies amongst the interconnected regions of the National Electricity Market (NEM).

In this chapter this shortfall is addressed by using Extreme Value Theory (EVT) and Copula functions. The approach used in this chapter models electricity prices in NEM with EVT and generates price forecasts using a Copula method.

EVT shifts the focus to the heavy-tailed characteristics of electricity prices whereas Copula functions are suitable to generate multivariate forecast values as they reflect the dependencies present in electricity prices within the NEM. Essentially, the model developed in this chapter has two components:

1. Application of EVT to wholesale electricity prices
2. Simulation of the EVT treated model with Copula functions

As noted in Chapter 2, electricity in NEM is subject to transfers between the regions of the NEM (integrated electricity organisations are able to service their customers with electricity produced from different regions of the NEM) and this is believed to influence price formation. NEM consists of separate but interconnected regions, meaning wholesalers located in a specific region can import and/or exports loads from other regions of the NEM. In theory, a wholesaler

can purchase electricity from generators located in other regions when there is high demand in their region or when the prices are lower in other regions. This arrangement naturally implies interdependencies in electricity prices between the regions of the NEM.

The following sections demonstrate the need for a non-linear electricity price modelling approach that takes into account the price interdependencies between the regions of the NEM. Then, a brief introduction to EVT and copula functions is provided separately followed by an empirical investigation of electricity price modelling with EVT and Copula functions. Lastly, findings are discussed along with future research directions.

INTRODUCTION

Price Interrelations and Non-Linearity in NEM Dependence structure of price series in NEM

NEM comprises five interconnected regions with major generation and demand centres. According to the Australian Energy Management Organisation (AEMO), interconnectors⁵⁰ import electricity into a region when demand is higher than can be met by local generators, or when the price of electricity in an adjoining region is low enough to displace the local supply.

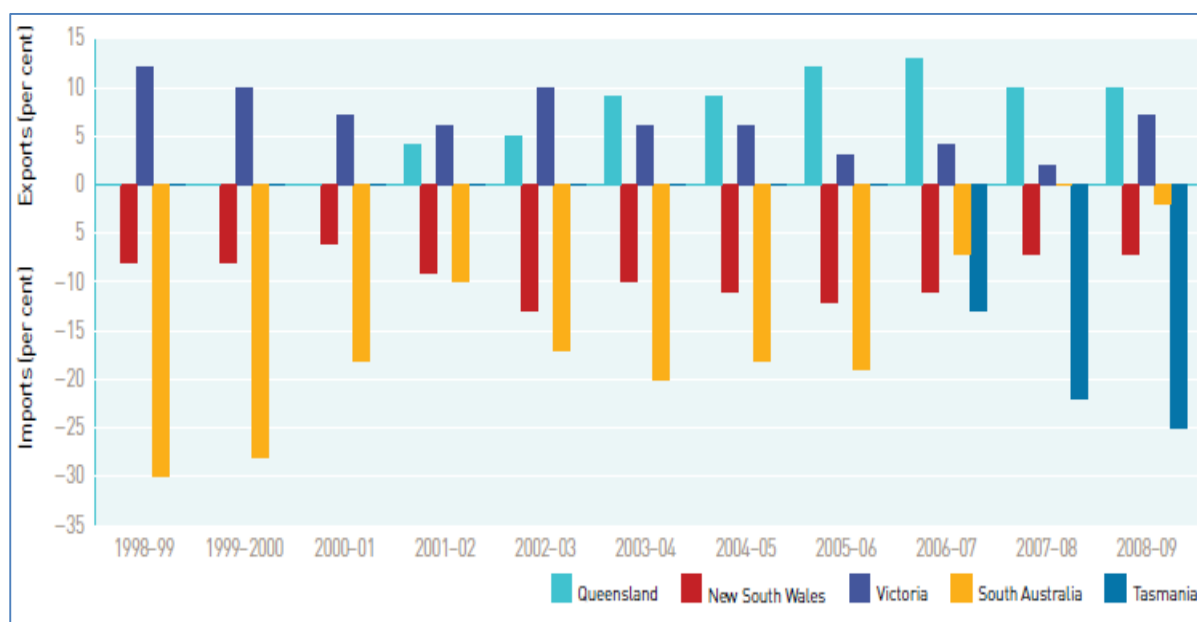
Figure 61 shows NEM's interregional trade relationships. The import and export data covers the period from 1998-99 to 2008-09. The left axis shows the imported/exported quantity of electricity as a proportion of total electricity demand for all the regions of the NEM.

As is seen in Figure 61, the New South Wales (NSW) region is a net importer of electricity. The NSW region imported over 10 per cent of its electricity requirements from 2002-03 to

⁵⁰ High-voltage transmission lines that transport electricity between adjacent NEM regions.

2006-07, but this rate fell to around seven per cent in 2007-08 and 2008-09. This high level of import dependency is due to the fact that the region relies on local base-load generation, and has limited peaking capacity at times of high demand. This naturally puts upward pressure on prices in peak periods, making imports a competitive alternative.

Figure 61 Intra-Regional Electricity Trade in NEM



Source: State of the Energy Market 2009, Australian Energy Regulator

The Victorian region is a net exporter of electricity because it has a substantially low cost base-load capacity. In 2008-09, Victoria's net electricity exports were equivalent to around 8 per cent of the region's consumption.

Queensland's region installed capacity exceeds its peak demand for electricity by around 3400 MW, making it a significant net exporter (AER, 2009). As Figure 60 shows net exports from Queensland rose steadily from 2001-02, reaching around 13 per cent of the state's electricity consumption in 2006-07. However, this figure fell to slightly below 10 per cent of consumption in 2008-09.

South Australia on the other hand historically is the most import dependent region in the NEM, which imported over 25 per cent of its energy requirements in the period of 1998-99 to 2008-09. This reflects the region's relatively higher fuel costs, which results in high cost generation. New investment in generation (mostly in wind capacity) has significantly reduced South Australia's net imports since 2005-06 (AER, 2009). The state was a net exporter for the first time in 2007-08, but recorded net imports of around two per cent of electricity consumption in 2008-09.

Lastly, Tasmania has been a net importer since its interconnection with the NEM in 2006. It imported over 25 per cent of its electricity requirements in 2008-09, partly because drought constrained its ability to generate hydroelectricity (AER, 2009).

This physical interconnection between the regions of the NEM suggests price and volatility interrelationships. Efficiency and effectiveness of inter-regional trade in facilitating energy price risk management has been a long standing focus for the NEM participants. The issues raised in relation to this aspect of energy trading within the NEM are complex due to the interrelationship between the physical market and operation of financial markets. Due to the regional structure of the NEM, inter-regional risks are assumed by market participants managing their load risks or using their contracting positions in one region to support retail activities in another region.

There are various risk management products and techniques available to NEM market participants to manage inter-regional risks i.e. inter-regional swaps and options, or intra-regional hedges. Energy trading entities within the NEM operate highly structured multi-region portfolios that utilise a combination of swaps and options to manage dynamically changing

inter-regional risk arising from price changes within the NEM (KPMG, 2006). Therefore, it is believed that a modelling approach that fails to capture the price inter-dependencies amongst the separate regions of the NEM would be subject to incorrect model specification. It is noteworthy to reiterate that the price differences between regions can arise due to numerous factors including:

- Generator (unplanned) and transmission outages;
- Physical limitations of the interconnecting links between regions; and
- Generators' bidding behaviour and interventions that affect the spot price setting process.

Correlation Analysis of Electricity Price Series in NEM

The following table demonstrates the existing dependence structure of price series between the regions of the NEM measured with Pearson correlation coefficients. The greatest coefficients are prevalent in between the regions of New South Wales (NSW) and Queensland (QLD) (0.751), NSW and Victoria (VIC) (0.703) and South Australia (SA) and VIC (0.776). These coefficients are found to be significant at one per cent significance.

This correlation structure is in line with the findings of Higgs (2009) who examined the inter-relationships of electricity prices and price volatility in the NEM regions of NSW, QLD, SA and VIC. This study examined eight years of half-hourly data and consisted of three different conditional correlation multivariate GARCH models. It demonstrated that price and price volatility inter-relationships in the regions of the NEM are best described by the dynamic conditional correlation multivariate GARCH specification.

Table 21 Pearson Correlation Coefficients

	NSW	QLD	SA	TAS	VIC
NSW	1.000	.751**	.490**	.455**	.703**
QLD		1.000	.389**	.383**	.587**
SA			1.000	.461**	.776**
TAS				1.000	.574
VIC					1.000

Source: Author's calculations.

Importantly, Higgs (2009) observed that higher conditional correlations exist between the well-connected markets, namely: New South Wales and Queensland; New South Wales and Victoria; and South Australia and Victoria. An important conclusion reached by this study was that the interconnectivity between the regions of the NEM have fostered a nationally integrated and stable spot electricity market, thus indicating that the interconnected markets are informationally efficient.

Further on the dependence structure of the price series, Table 22⁵¹ demonstrates the cross correlations at different lags between the interconnected regions of the NEM, namely between the regions of New South Wales and Queensland; New South Wales and Victoria; and South Australia and Victoria. Standard errors are derived based on the assumption that the series are not cross correlated and that one of the series is white noise. These cross correlations at different lags between the highly interconnected markets of the NEM further provide evidence that electricity prices amongst the regions of the NEM tend to be interconnected supporting the views of Higgs (2009). In all of the pairs, the greatest correlation is observed on the day (lag 0). This correlation declines as the lags increase.

⁵¹ The graphical representation of this table is also provided in the Appendix 11.

Table 22 Cross Correlations

	Series Pair: SA with VIC	Series Pair: NSW with QLD	Series Pair: NSW with VIC
Lag	Cross Correlation	Cross Correlation	Cross Correlation
7	0.406	0.33	0.426
6	0.354	0.324	0.377
5	0.326	0.27	0.343
4	0.342	0.277	0.371
3	0.349	0.299	0.413
2	0.395	0.355	0.481
1	0.547	0.506	0.633
0	0.751	0.776	0.703
1	0.58	0.578	0.521
2	0.423	0.376	0.41
3	0.388	0.333	0.374
4	0.368	0.3	0.4
5	0.355	0.301	0.406
6	0.401	0.345	0.401
7	0.427	0.362	0.409

Source: Author's calculations.

Correlation coefficients as presented above (between the prices of all regions of the NEM) and the recent literature show that there is a degree of price interdependency between the regions of the NEM. Therefore, electricity price forecast models that take this price interdependency into consideration are likely to overcome the incorrect model specification issues as compared to the price forecast models that ignore that price interdependencies exist within the NEM.

Furthermore, understanding the pricing relationships between the regions of the NEM would enable better understanding of the dynamics of electricity pricing. This is also likely to throw light on the efficiency of pricing in the NEM.

Non-Linearity of Electricity Prices in NEM

An examination of non-linearity in electricity prices was performed by Wild et al. (2010) who applied Portmanteau correlation, bicorrelation, and tricorrelation tests to detect nonlinear serial dependence in electricity price series in the NEM. They found strong evidence of non-linear serial dependence in electricity prices.

The finding that non-linearity was present could rule out many classes of linear models that are widely applied to electricity price modelling i.e. mean-reverting models. It is argued that the presence of third and fourth-order non-linear serial dependence in electricity prices makes models that employ a linear structure, or assume a pure noise input, such as the mean-reverting jump-diffusion models problematic. This dependence structure violates both the normality and Markovian assumptions underpinning conventional stochastic differential equation (SDE) based models.

INTRODUCTION TO EXTREME VALUE THEORY (EVT)

Extreme Value Theory is a powerful and fairly robust framework to study the tail behaviour of a distribution asymptotically⁵². The main purpose of this theory is to provide asymptotic models which can model the tails of a distribution (Bystrom, 2005). EVT reduces the focus from modelling the whole distribution to modelling of the tail behaviour. Hence the symmetry assumption of the Gaussian distribution is examined directly by estimating the left and right tails separately. A critical assumption of the EVT is that extreme prices are independently and identically distributed (Nystrom and Skoglund, 2002).

Fisher and Tippett (1928) were pioneers of EVT followed by Gnedenko (1943), Gumbel (1958), Balkema and de Haan (1974) and Pickands (1975). Most recently, Embrechts et al. (1997), Reiss and Thomas (1997), Beirlant et al. (1996) and McNeil and Frey (2000) applied EVT to insurance problems. Bystrom (2005) applied EVT to investigate the tails of the price distribution of hourly electricity spot prices in NordPool in an attempt to capture the extreme price behaviour better than conventional electricity price model. Rozario (2002) derived Value-at-risk (VaR) for Victorian half-hourly electricity returns using a threshold based EVT model

⁵² Asymptotic theory is a generic framework for assessment of properties of estimators and statistical tests. Within this framework it is typically assumed that the sample size grows indefinitely, and the properties of statistical procedures are evaluated in the limit as sample size approaches to infinity. EVT has a fundamental role when modelling the maxima of a random variable. This role is similar to the role of Central Limit Theorem when modelling the sums of random variables. In both cases, the theory states what the limiting distributions are.

whereas Chan and Gray (2006) used EVT to measure Value at Risk for daily spot electricity prices in five different electricity markets around the globe including Victoria.

There are two commonly used ways in identifying extremes in the data; block maxima and threshold methods. The limit law for the block maxima with the size of the subsample is given by the following theorem of Fisher and Tippet (1928) and Gnedenko (1943);

Let (X_n) be a sequence of *i. i. d.* random variables. If there exists constants $c_n > 0, d_n \in R$ and some non-degenerate distribution function H such that $\frac{M_n - d_n}{c_n} \xrightarrow{d} H$, then H belongs to one of following three extreme value distributions:

$$\text{Frechet, } \Phi_\alpha(x) = \begin{cases} 0, & x \leq 0 \\ e^{-(-x)^{-\alpha}}, & x > 0 \end{cases} \quad (54)$$

$$\text{Weibull, } \Phi_\alpha(x) = \begin{cases} e^{-(-x)^\alpha}, & x \leq 0 \\ 1, & x > 0 \end{cases} \quad (55)$$

$$\text{Gumbel, } \Lambda(x) = e^{-(-x)^{-\alpha}}, x \in R \quad (56)$$

for $\alpha > 0$

Von Mises (1936) and Jenkinson (1955) showed one-parameter generalisations of these standard distributions as;

$$H_\xi(x) = \begin{cases} e^{-(1+\xi x)^{-1/\xi}}, & \text{if } \xi \neq 0 \\ e^{-e^{-x}}, & \text{if } \xi = 0 \end{cases} \quad (57)$$

This generalisation is known as Generalised Extreme Value (GEV) distribution and obtained by setting $\xi = \alpha^{-1}$ for the Frechet distribution, $\xi = -\alpha^{-1}$ for the Weibull distribution and by interpreting the Gumbel distribution as the limit case for $\xi = 0$.

Secondly, the Peaks-over-Threshold (POT) method is a threshold method where in the absence of a known distribution function (F) of a random variable (X), the interest lies in estimating the distribution function $F(u)$ of values of x above a certain threshold (u).

The distribution function $F(u)$ is called the Conditional Excess Distribution Function (CEDF) and is defined as;

$$F_u(y) = P(X - u \leq y | x > u), 0 \leq y \leq x_F - u \quad (58)$$

where X is a random variable, u is a given threshold, $y = x - u$ are the excesses and $x_F \leq \infty$ is the right end point of F . This can be written in terms of F as;

$$F_u(y) = \frac{F(u+y)-F(u)}{1-F(u)} = \frac{F(x)-F(u)}{1-F(u)} \quad (59)$$

The realizations of the random variable X lie mainly between 0 and u and therefore the estimation of F in this interval is straightforward. The estimation of the portion $F(u)$ however might be difficult as there are in general very few observations in this area. Similar to the block maxima method, which provides a choice of an optimal block length, the POT method relies on a reasonable choice of threshold.

A threshold value that is too low and the asymptotic theory is no longer met, too high a threshold value and one does not have enough data points to estimate the parameters in the excess distribution (Bystrom, 2005).

EVT provides a powerful result about the CEDF. This is stated by the following theorem of Pickands (1975), Balkema and de Haan (1974);

For a large class of underlying distribution function (F) the conditional excess distribution function $F_u(y)$, for a large u , is well approximated by $F_u(y) \approx G_{\xi,\sigma}(y), u \rightarrow \infty$.

$$\text{where } G_{\xi,\sigma}(y) = \begin{cases} 1 - \left(1 + \frac{\xi}{\sigma}y\right)^{-1/\xi} & \text{if } \xi \neq 0 \\ 1 - e^{-y/\sigma} & \text{if } \xi = 0 \end{cases} \quad (60)$$

for $y \in [0, (x_F - u)]$ if $\xi \geq 0$ and $y \in \left[0, -\frac{\sigma}{\xi}\right]$ if $\xi < 0$. $G_{\xi,\sigma}$ is called the Generalised Pareto Distribution (GPD).

AN INTRODUCTION TO COPULA FUNCTIONS

Copulas are functions that join multivariate distributions⁵³ to their univariate margins. Copulas are useful when the form of the marginal distributions⁵⁴ are known (as in the case of electricity prices in the NEM), but the joint distributions are not. Copulas allow the creation of a joint distribution, permitting for modelling of dependence between electricity prices in all the regions of the NEM.

A d-dimensional copula is a d-dimensional distribution function on $[0,1]^d$ with standard uniform marginal distributions. Sklar's Theorem states that $F_1 \dots F_d$ can be written as;

$$F(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d)) \quad (61)$$

for some copula C , which is uniquely determined on $[0,1]^d$ for distributions F with absolutely continuous margins (Nelsen, 1999). Conversely any copula C may be used to join any collection of univariate distribution functions $F_1 \dots F_d$ using the above equation to create a multivariate distribution function F with margins $F_1 \dots F_d$ (Demarta and McNeil, 2005).

Copula approach to modelling of multivariate density functions have a number of advantages over traditional approaches based on the representation of such probabilities as $F(x, y)$. First of all, copulas are flexible, because they allow fitting the dependence structure separately from the marginal distributions. This is a superior modelling strategy, especially in situations where the dependence structure is the interest. With traditional representations, by contrast, one

⁵³ Given two random variables X and Y that are defined on the same probability space, the joint distribution for X and Y defines the probability of events defined in terms of both X and Y . In the case of only two random variables, this is called a bivariate distribution, but the concept generalizes to any number of random variables, giving a multivariate distribution. The equation for joint probability is different for both dependent and independent events.

⁵⁴ A marginal density ("marginal distribution" in the discrete case) is found by integrating (or summing in the discrete case) over the domain of one of the other variables in the joint distribution.

cannot get at the dependence structure without also having to make assumptions about marginal distributions, and there is the related problem that dependence parameters are sometimes present in the marginals.

Furthermore, Copula functions allow separating the modelling of dependence from the modelling of the marginals to fit different marginal distributions to different random variables. By comparison, traditional approaches require fitting the same marginals to all random variables.

Last but not least, copula approaches also make for greater flexibility as they provide greater choice over the type of dependence structure (Dowd, 2008). There are a number of Copula functions identified and used in modelling. Some of the most commonly used copulas are; Gaussian copula, Student's t-copula, Clayton copula, Frank copula, and Gumbel copula.

Student's t-copula captures the fat-tailed features of the price series therefore it is quite suitable in modelling electricity prices in the NEM. The multivariate t-copula is defined as;

$$C_p(u, v) = T_{p,n}(t_n^{-1}(u), t_n^{-1}(v)) \quad (62)$$

and

$$c_p(u, v) = \frac{1}{\sqrt{1-p^2}} \frac{\Gamma(\frac{n+2}{2})\Gamma(\frac{n}{2})(1+\frac{1}{n}\psi\Omega^{-1}\psi)^{-\frac{n+2}{2}}}{(\Gamma(\frac{n+2}{2}))^2 \prod_{i=1}^2 (1+\frac{1}{n}\psi_i^2)^{-\frac{n+1}{2}}} \quad (63)$$

where $\psi = (t_n^{-1}(u), t_n^{-1}(v))'$ and $T_{p,n}$ is the bivariate Student-t cumulative distribution function with n degrees of freedom and correlation p .

While Archimedean (Clayton, Frank or Gumbel) copulas are calculated over a closed-form solution and do not need to be represented by an application of multivariate distribution using

Sklar's theorem. Elliptical (Gaussian or Student-t) copulas are derived via simulations of these multivariate distributions. A caveat of elliptical copulas is that the upper and lower tail dependence, being informative on joint extreme realizations, is identical, due to their radial symmetric shape. In addition, a Gaussian copula has no tail dependence at all (Bradley and Taqqu, 2003), and this is the main argument against its use in this study.

Furthermore, a number of studies such as Mashaal and Zeevi (2002) and Breymann et al. (2003) showed that the empirical fit for the Student-t is superior to the Gaussian copula as Student-t captures the dependent extreme values better. A Student-t has uniform marginal distributions similar to a Gaussian copula and the rank correlations in a t-copula are also the same for a Gaussian copula. However, Student-t captures the dependence structure better even though its components have the same ranking correlation with the Gaussian copula.

MODELLING ELECTRICITY PRICES WITH EVT IN NEM

The development of the model characterised by EVT in this section follows a similar procedure as to Bystrom (2005) who investigated the tails of electricity spot returns from NordPool. Bystrom (2005) pre-filtered the returns data from NordPool with AR-GARCH model to achieve *i.i.d.* process before applying peaks-over-thresholds (POT) method to model the electricity return series.

Electricity prices in the NEM are characterized by volatility clustering and therefore a set of individually identically distributed *i.i.d.* residuals firstly need to be generated before applying EVT to data empirically. This procedure will meet an important assumption of EVT.

The dependency structure of the price series in each region of the NEM as indicated by the auto-correlation functions⁵⁵ (ACF) of the prices and squared prices reveals a degree of auto-correlation and persistence in variance. These configurations suggest that a GARCH approach is an appropriate means of conditioning the data to yield near *i.i.d.* residuals before applying EVT to the price series. Furthermore, an exponential GARCH (EGARCH) specification is thought to be a suitable specification in filtering the electricity prices in NEM. Higgs and Worthington (2005) earlier identified asymmetric volatility response in electricity prices in the NEM. They pointed out that the volatility tends to rise in response to ‘good news’ for traders (proxied by positive price spikes) and fall in response to ‘bad news’ (negative spikes), which is a perverse asymmetry that runs counter to the effects generally observed in conventional financial markets. This leverage effects can be captured by an EGARCH specification.

Filtering Residuals via AR-EGARCH

Parameter estimation of AR-EGARCH model

The following table summarizes the parameter estimates that pertain to the AR-EGARCH model with t-distributed errors. The rationale for this is due to the fact that the empirical evidence rejects the normality assumption of electricity prices (Bystrom, 2000).

Table 23 Parameter Estimates for the AR-EGARCH model

Conditional mean parameters of the AR-EGARCH model					
	NSW	QLD	SA	TAS	VIC
C	0.515	0.5321	0.6608	0.3324	0.5507
α	0.8481	0.8372	0.8111	0.9095	0.8396
Conditional variance parameters of the AR-EGARCH model					
ω	-1.0723	-0.4011	-0.5206	-0.3613	-1.2227
B	0.5015	0.7301	0.4837	0.8152	0.4866
α	0.5899	0.7062	1.0508	0.7445	0.6570
γ	0.3237	0.371	-0.0168	-0.0136	-0.0205

Source: Author’s calculations.

⁵⁵ See Appendix 11 for the ACFs.

Both mean and conditional variance parameters are positive and some of these parameters are not significantly lower than one. These findings are in line with previous study of Bystrom (2005).

Estimated parameters of the model further demonstrate that there is strong evidence of a GARCH effect in all regions of the NEM. Moreover, estimate of the leverage parameter suggests a strong positive leverage effect for the regions of NSW (0.3237), QLD (0.371) and negative leverage effects for SA (-0.0168), TAS (-0.0136) and VIC (-0.0205). The evidence of this inverse leverage effect indicates that the asymmetry parameter of the AR-EGARCH process was positive and significant. This indicates that positive shocks to prices amplify the conditional variance of the process more so than negative shocks.

Application of AR-EGARCH

To produce a series of *i.i.d.* observations, firstly, fitting a first order autoregressive model to the conditional mean of the prices of each region is performed as;

$$AR(1) = c + \alpha X_{(t-1)} + \varepsilon_t \quad (64)$$

where $\alpha_1, \dots, \alpha_p$ are the parameters of the model, c is constant and ε_t is the white noise with zero mean and variance σ_ε^2 .

Secondly, an EGARCH model to the conditional variance is also fitted as;

$$\log(\sigma_t^2) = \omega + \alpha(|z_{t-1}| - E(|z_{t-1}|)) + \gamma z_{t-1} + \beta \log(\sigma_{t-1}^2) \quad (65)$$

where $z_t \equiv \varepsilon_t \sigma_t^{-1}$ denotes the standardized residuals and γ is the leverage parameter.

The first order autoregressive component of the model compensates for autocorrelation, while the EGARCH component compensates for heteroskedasticity present in the price series. Then, modelling of the standardized residuals of each series is performed as a standardized Student's t-distribution to compensate for the fat tails associated with electricity prices in the NEM.

This filtering process allows for asymmetries in the relationship between prices and volatility. In particular for $\gamma > 0$ positive shocks will have a bigger impact on future volatility than negative shocks of the same magnitude. By parameterising the logarithm of the conditional variance as opposed to the conditional variance, the EGARCH model also avoids complexities from having to ensure that the process remains positive. However, this logarithmic transformation complicates the construction of unbiased forecasts for the level of future variances as pointed out by Bollerslev (1986).

Table 24 Summary Statistics for the Raw and Standardized Residuals

Summary statistics on the AR-EGARCH residuals					
	NSW	QLD	SA	TAS	VIC
Median	-0.00129	0.00502	-0.05589	-0.0147	-0.03497
Mean	0.02483	0.03366	-0.00136	-0.00331	-0.00167
Standard deviation	0.01097	0.46964	0.01339	0.00950	0.00975
Skewness	0.81734	0.62135	2.99437	0.34199	1.96076
Kurtosis	20.83138	19.0961	21.82069	11.6469	22.67336
Ljung-Box Q(7)	127.851	145.534	92.460	131.65	103.733
Ljung-Box Q(15)	163.292	164.211	126.809	295.728	194.198
Ljung-Box Q(24)	200.651	190.041	154.271	389.694	251.812
Summary statistics on the standardized AR-EGARCH residuals					
	NSW	QLD	SA	TAS	VIC
Median	-0.0287	0.008912	-0.11382	-0.0299	-0.11922
Mean	0.12212	0.11086	-0.01326	-0.01556	-0.00998
Standard deviation	0.94243	0.75939	0.83371	1.01040	1.00588
Skewness	4.80218	2.73582	3.26500	0.06520	3.07418
Kurtosis	52.29163	45.17197	23.49221	16.99184	23.11349
Ljung-Box Q(7)	77.505	52.555	107.731	178.345	132.883
Ljung-Box Q(15)	119.888	78.8111	186.95	406.316	266.359
Ljung-Box Q(24)	181.368	125.968	239.843	505.604	354.271

Source: Author's calculations.

The filtering process developed aims to provide the data that meets the assumptions of EVT, mainly the precondition that the data should be *i. i. d.* before the application of EVT. Summary statistics of the AR-EGARCH filtered residuals are shown in Table 24 for both non-standardised and standardised residuals. The standardised residuals are better suited than the raw residuals for checking the assumptions on the random errors as they are designed to overcome the problem of different variances of the raw residuals.

Ljung-Box Q^{56} statistics at 0.1 significance levels are also reported in the table above. The test results suggest the acceptance of null hypothesis of model adequacy at significance level of 0.1. The hypothesis being tested in Ljung-Box Q is that the residuals have no autocorrelation. As is seen from the table on summary statistics of both raw and standardized residuals, standardizing the residuals caused declines in the test statistic's value for the regions of NSW and QLD whereas it increased the value of the test statistic in the other regions of the NEM. Ljung-Box Q test at different lags indicate that there is still some degree of autocorrelation present in the data. The standardisation of the residuals takes care of the issue of different variances, but nothing changes with regard to autocorrelation between the residuals. It can be seen that the dependence between the standardised residuals is exactly the same as the autocorrelation between the raw residuals.

⁵⁶ Ljung-Box Q is a test of the null hypothesis that the ACF does not differ from zero, up to lag k . It is evaluated as a chi-square with $k - m$ degrees of freedom, where k is the number of lags examined and m is the number of parameters estimated. More formally;

$$Q = N(N + 2) \sum_{k=1}^L \frac{r_k^2(\epsilon)}{N - k}$$

where N is the sample size, L is the number of autocorrelations included in the statistics, and r_k^2 is the squared sample autocorrelation of residual series at lag k . Under the null hypothesis of model accuracy, the test statistics are asymptotically distributed.

In light of this remaining autocorrelation (EVT pre-condition that the data should be *i.i.d.*), a robust method for testing independence is required before proceeding with the application of EVT to the electricity price series in the NEM. Appendix 7 describes the Brock-Dechert-Scheinkman (BDS) test for independence, as described in Brock et al. (1996).

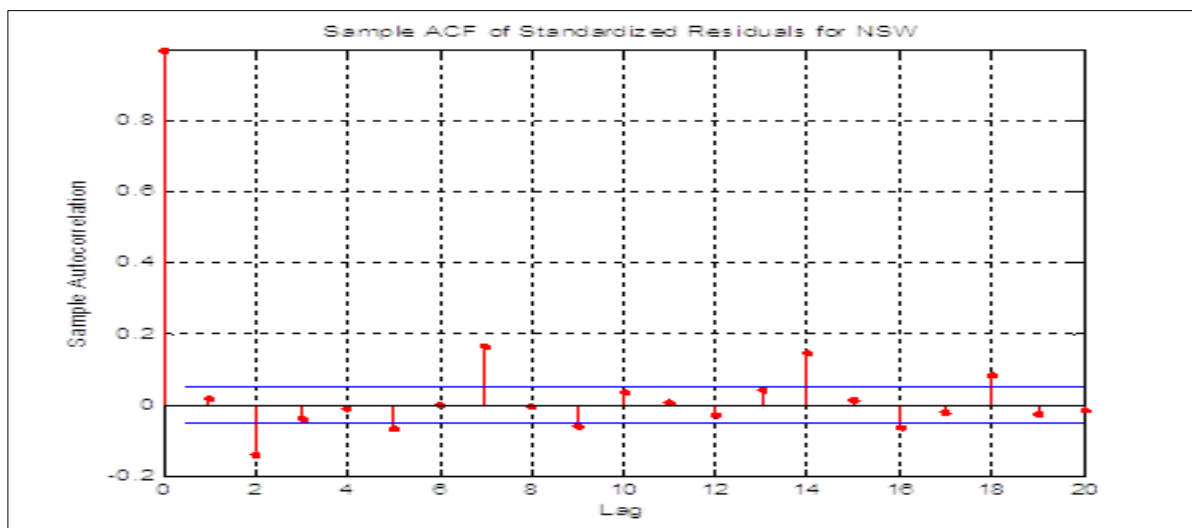
In short, the BDS test is a portmanteau test for time based dependence in a series and can be used for testing against a variety of possible deviations from independence including linear dependence, non-linear dependence, or chaos. This test can be applied to a series of estimated residuals to check whether the residuals are independently and identically distributed. BDS test rejects that the series is *i.i.d.*. The rejection of *i.i.d.* standardised residuals presents problems when applying EVT to electricity price series as one of the important assumptions of the EVT is violated. Therefore, the findings of this chapter will be interpreted cautiously.

Filtering and standardising the residuals from each series resulted in near zero-mean, unit variance, and near *i.i.d.* series upon which EVT estimation of the sample Cumulative Distribution Function (CDF) tails is based. This desirable but not perfect process can be examined in Appendix 8 that compares the model residuals and the corresponding conditional standard deviations filtered from the raw prices for all regions of the NEM.

Figures 62 to 71 display the ACFs of the standardized residuals to the corresponding ACFs for the series. These graphical representations reveal that the standardized residuals are now closer to be approximately *i.i.d.* and suitable for EVT modelling.

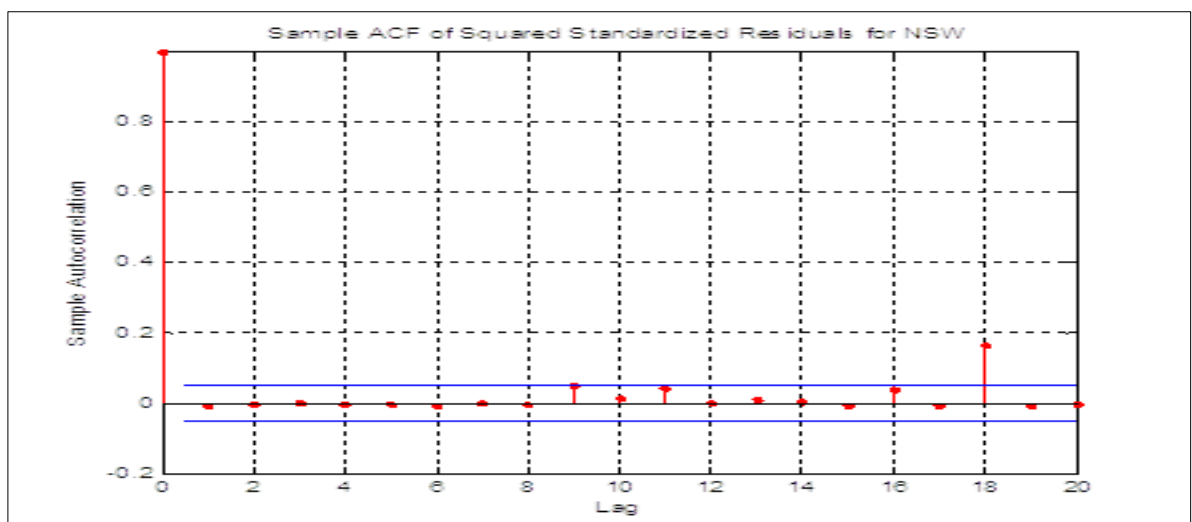
However, one realises that while the residuals seem statistically uncorrelated they are not identically distributed and a closer visual inspection of the figures shows that the residuals are not independent and identically distributed through time.

Figure 62 Sample ACF of Standardised Residuals (NSW)



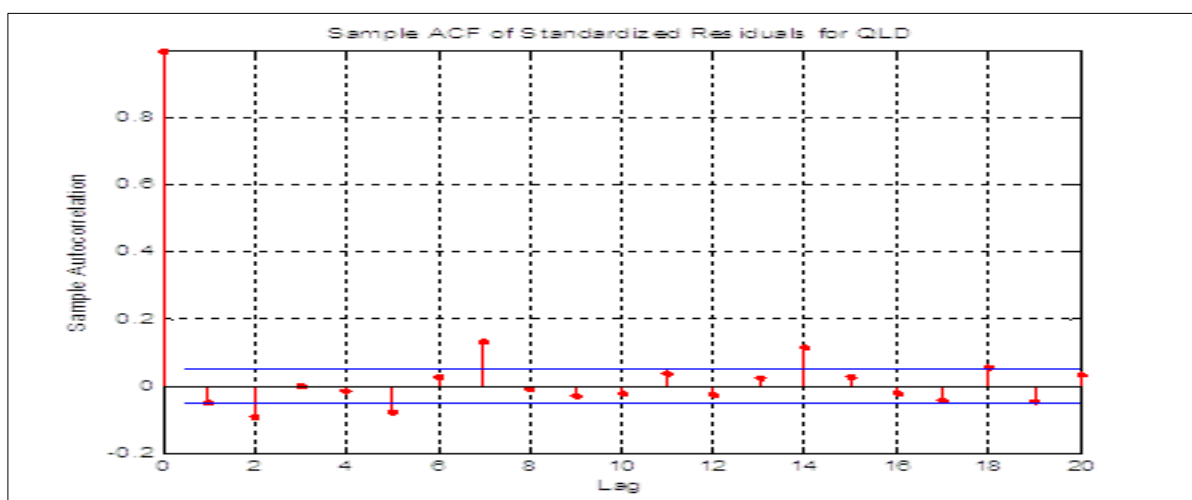
Source: Author's calculations.

Figure 63 Sample ACF of Squared Standardised Residuals (NSW)



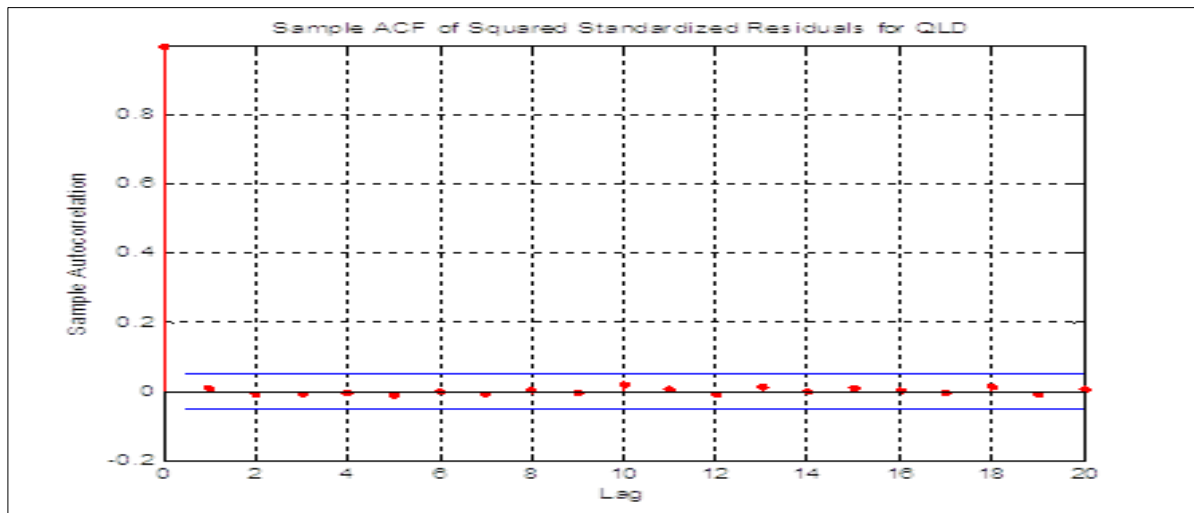
Source: Author's calculations.

Figure 64 Sample ACF of Standardised Residuals (QLD)



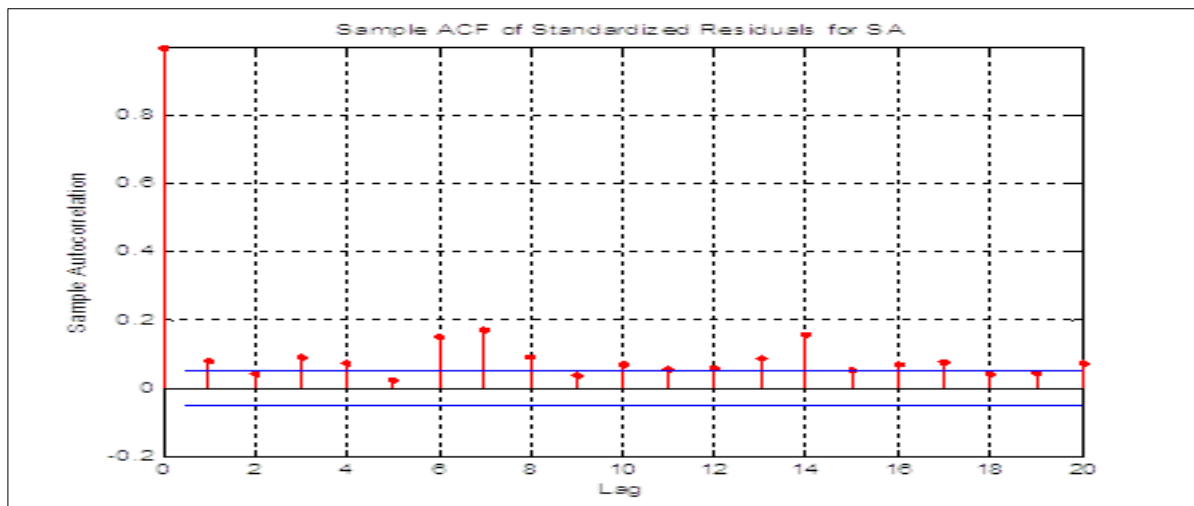
Source: Author's calculations.

Figure 65 Sample ACF of Squared Standardised Residuals (QLD)



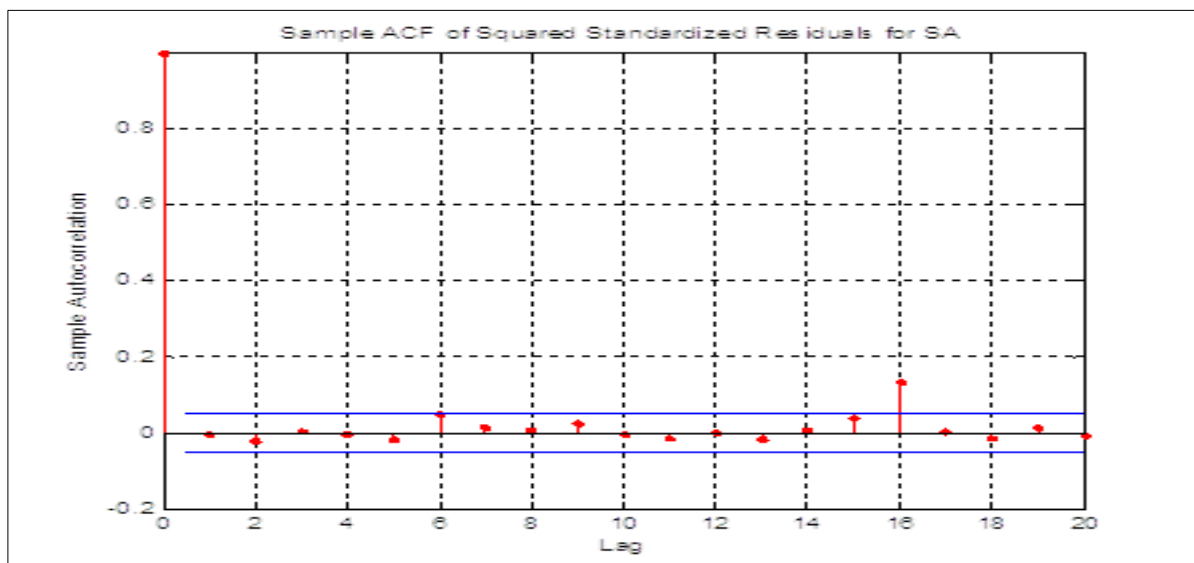
Source: Author's calculations.

Figure 66 Sample ACF of Standardised Residuals (SA)



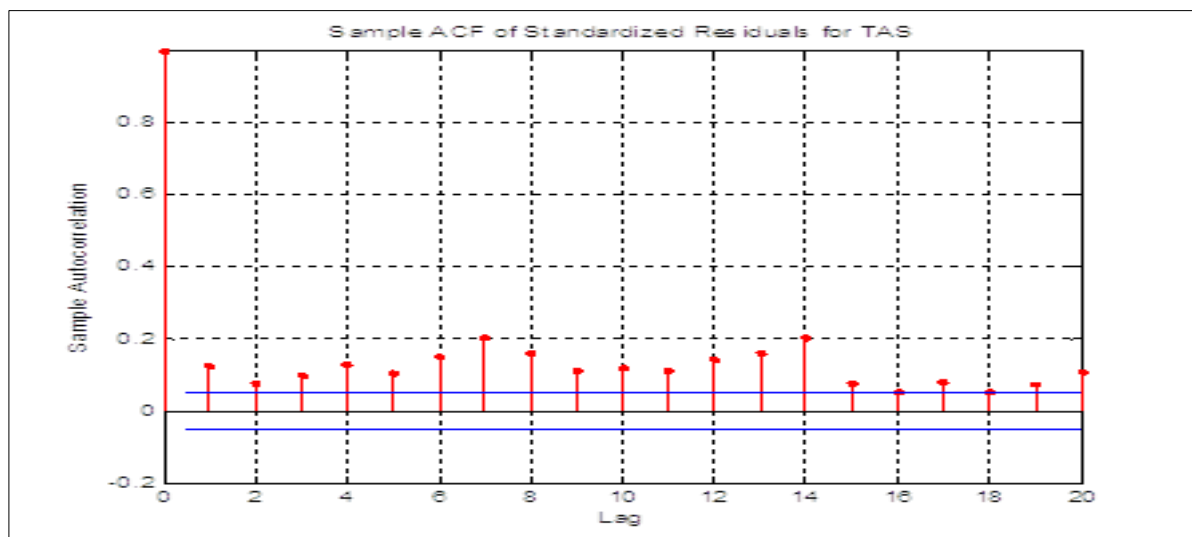
Source: Author's calculations.

Figure 67 Sample Autocorrelation Function of Squared Standardised Residuals for SA



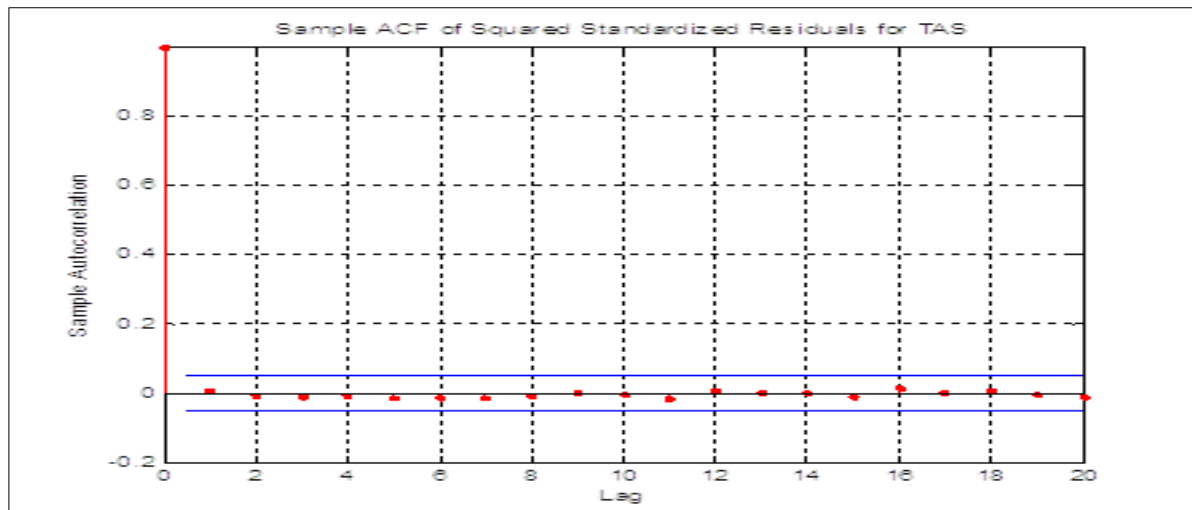
Source: Author's calculations.

Figure 68 Sample ACF of Standardised Residuals (TAS)



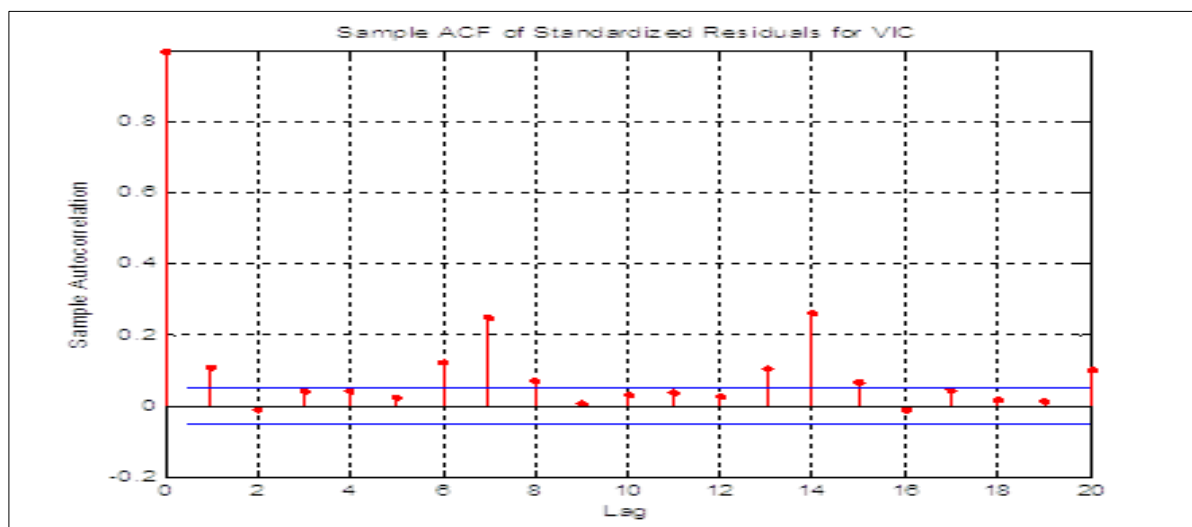
Source: Author's calculations.

Figure 69 Sample ACF of Squared Standardised Residuals (TAS)



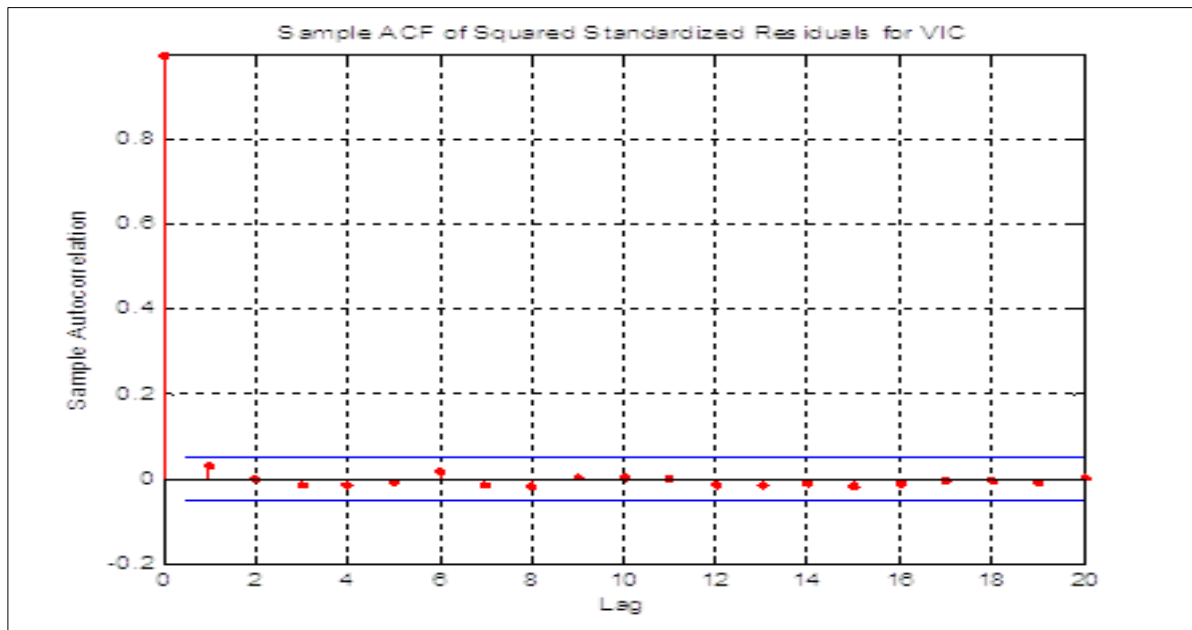
Source: Author's calculations.

Figure 70 Sample ACF of Standardised Residuals (VIC)



Source: Author's calculations.

Figure 71 Sample ACF of Standardised Residuals (VIC)



Source: Author's calculations.

Application of POT to Residuals

In applying EVT to the standardised residuals, the method adopted in this section is the Peak-over-Threshold (POT) method following McNeil and Frey (2000). This method identifies extreme standardised residuals that exceed a chosen threshold of 10 per cent⁵⁷ and models these extremes at each tail separately from non-extreme standardised residuals. This is in line with Chan and Grey's (2006) findings such that exceedances aggregate approximately 10 per cent of the sample in Victorian daily spot electricity prices.

However, it is important to note that there is a trade-off when choosing the size of the exceedances. If it is too small then the model will have too few observations in the tail resulting in large variance in estimators for the parameters in the Generalised Pareto Distribution (GPD) and if it is too large the fundamental model assumption of POT may be violated.

⁵⁷ McNeil and Frey (2001) had chosen approximately 10 per cent of the sample as exceedances. This study also uses ten per cent as the chosen threshold.

More specifically, let u denote the threshold beyond which observations of z are considered exceedances. The scale of the exceedances then is given by $y_i = z_i - u$, for $i = 1, \dots, N_y$, where N_y is the total number of exceedances in the sample. The distribution of y , for a given threshold u , is given by;

$$\begin{aligned}
 F_u(y) &= \Pr(z - u \leq y \mid z > u) \\
 &= \frac{\Pr(z - u \leq y, z > u)}{\Pr(z > u)} \\
 &= \frac{F_z(y + u) - F_z(u)}{1 - F_z(u)}
 \end{aligned} \tag{66}$$

This equation gives the probability that z exceeds the threshold u by an amount no greater than y , given z exceeds u . Balkema and de Haan (1974) and Picklands (1975) showed that, for a sufficiently high u , $F_u(y)$ can be approximated by the Generalised Pareto Distribution (GPD), which is defined as;

$$G_{\xi, \nu}(y) = \begin{cases} 1 - (1 + \frac{\xi y}{\nu})^{-1/\xi} & \text{if } \xi \neq 0 \\ 1 - \exp(-y/\nu) & \text{if } \xi = 0, \end{cases} \tag{67}$$

where ξ and $\nu > 0$ are shape and scale parameters, respectively.

Parameterisation of POT and GPD Fit

The estimation of the parameters of the POT method is implemented by following Nystrom and Skoglund (2002) where given the exceedances in each tail; optimisation of the negative log-likelihood function is performed to estimate the shape and scale parameters of the GPD. It

is important to note that the maximum likelihood estimator is based on the assumption that the tail under consideration follows a GPD.

The following table shows the maximum likelihood estimates of the shape and scale parameters, derived by fitting the GPD to the standardized residuals.

Table 25 Tail Parameter Estimates of the GPD

	<i>NSW</i>	<i>VIC</i>	<i>QLD</i>	<i>SA</i>	<i>TAS</i>
Upper tail parameters					
ξ	0.5507	0.4450	0.4317	0.8074	0.3072
ν	0.5222	0.6024	0.533	0.3411	0.4964
Lower tail parameters					
ξ	0.1016	0.0550	0.2366	0.2071	0.2869
ν	0.3202	0.4116	0.2134	0.2447	0.3835

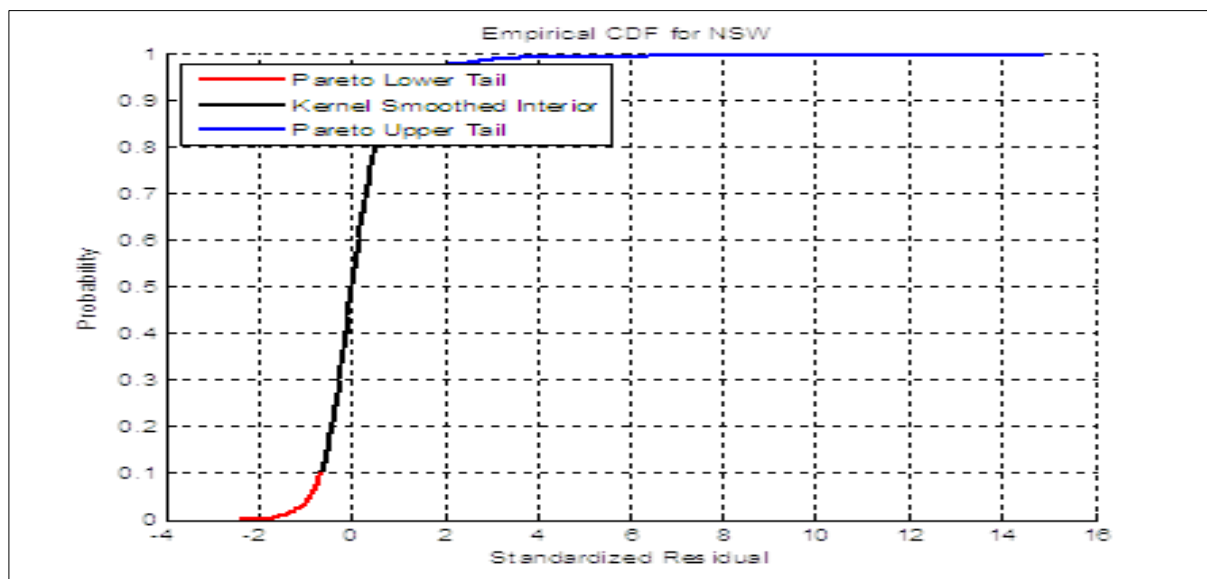
Source: Author's calculations.

As is seen, the shape parameters are positive in each region at both tails suggesting the standardized residuals are heavy tailed. When ξ is greater than zero then the distribution has heavy tails. For instance, Pareto and Student's t distributions fall in this category. For normally distributed variables the tail shape parameter equals zero where the distribution has thin tails and an infinite number of existing moments. If it is smaller than zero, the distribution has a finite upper limit, and therefore no long tail. For example, the uniform distribution is bounded and has a negative tail shape parameter.

Furthermore, the following graphs show the three distinct areas of the empirical cumulative distribution function (ECDF)⁵⁸ of prices for each region of the NEM. The empirical distribution function is the CDF associated with the empirical measure of the sample.

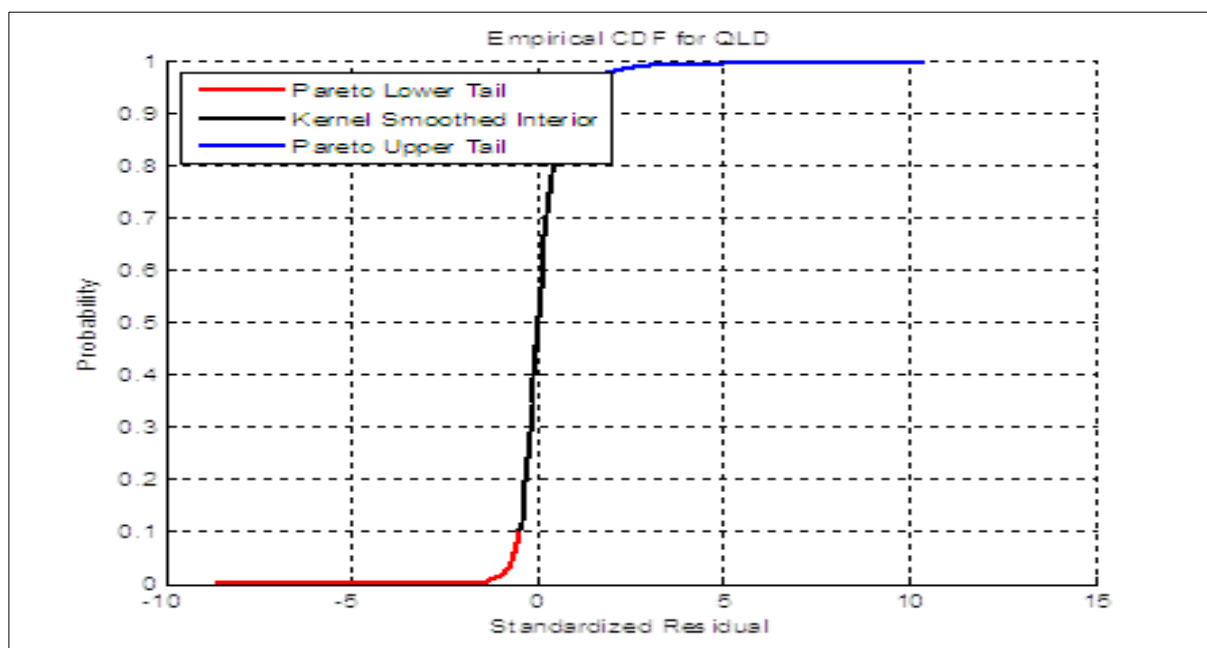
⁵⁸ The idea behind the empirical CDF is that it is a function that assigns probability to each of the observations in a sample. Its graph has a stair-step appearance. If a sample comes from a distribution in a parametric family (such

Figure 72 Empirical Cumulative Distribution Function (NSW)



Source: Author's calculations.

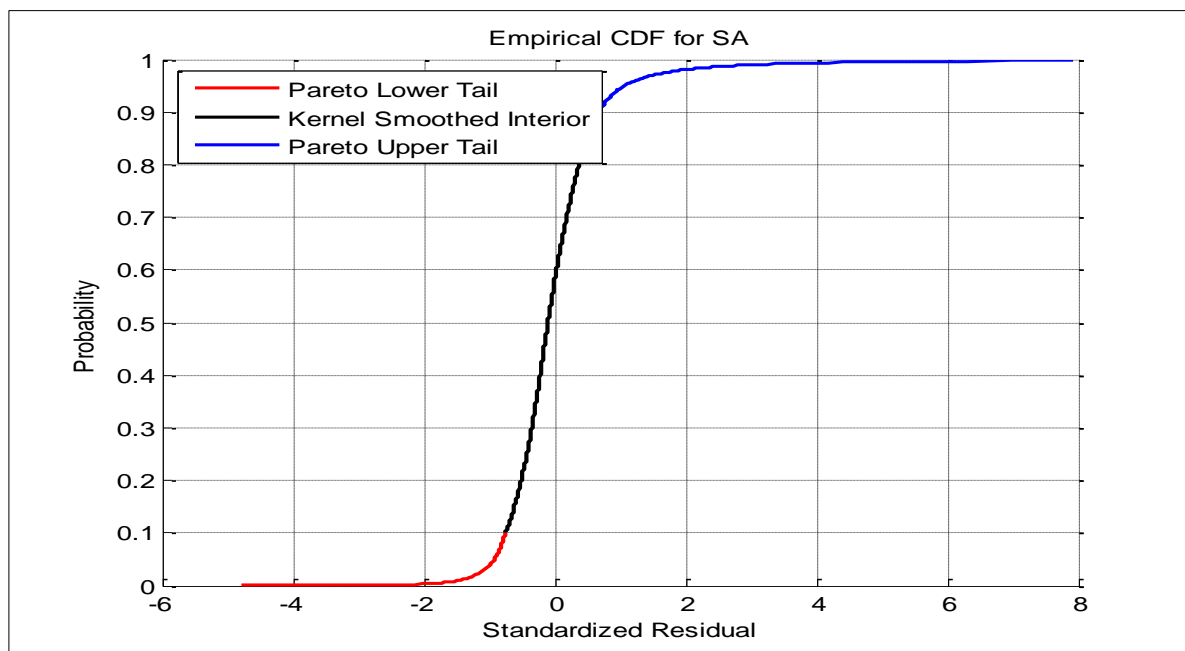
Figure 73 Empirical Cumulative Distribution Function (QLD)



Source: Author's calculations.

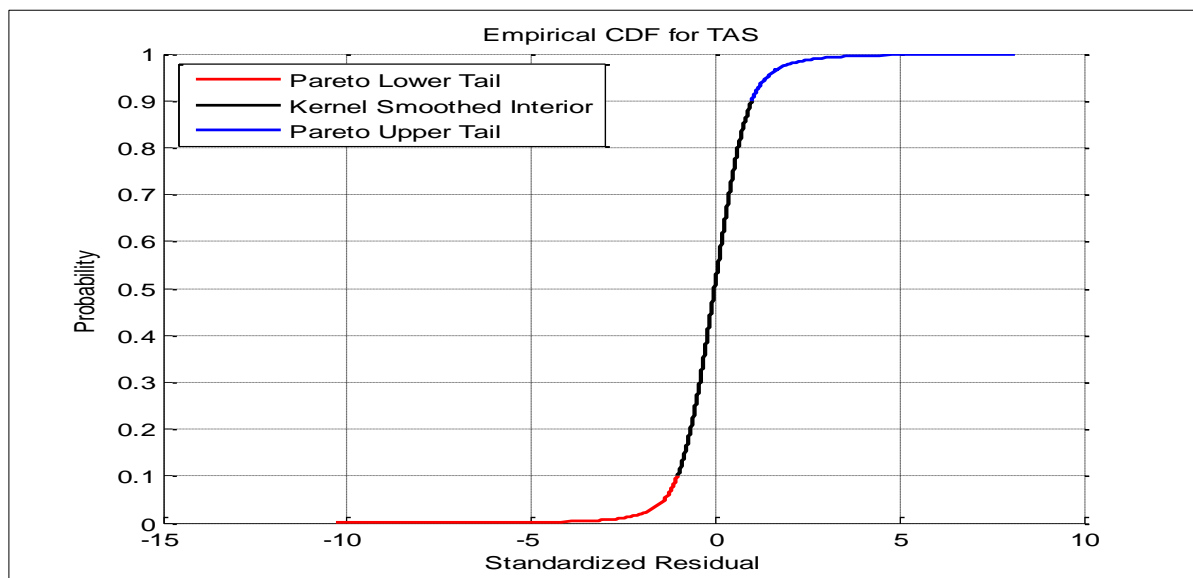
as a Gaussian distribution), its empirical CDF is likely to resemble the parametric distribution. If not, its empirical distribution still gives an estimate of the CDF for the distribution that generated the data.

Figure 74 Empirical Cumulative Distribution Function (SA)



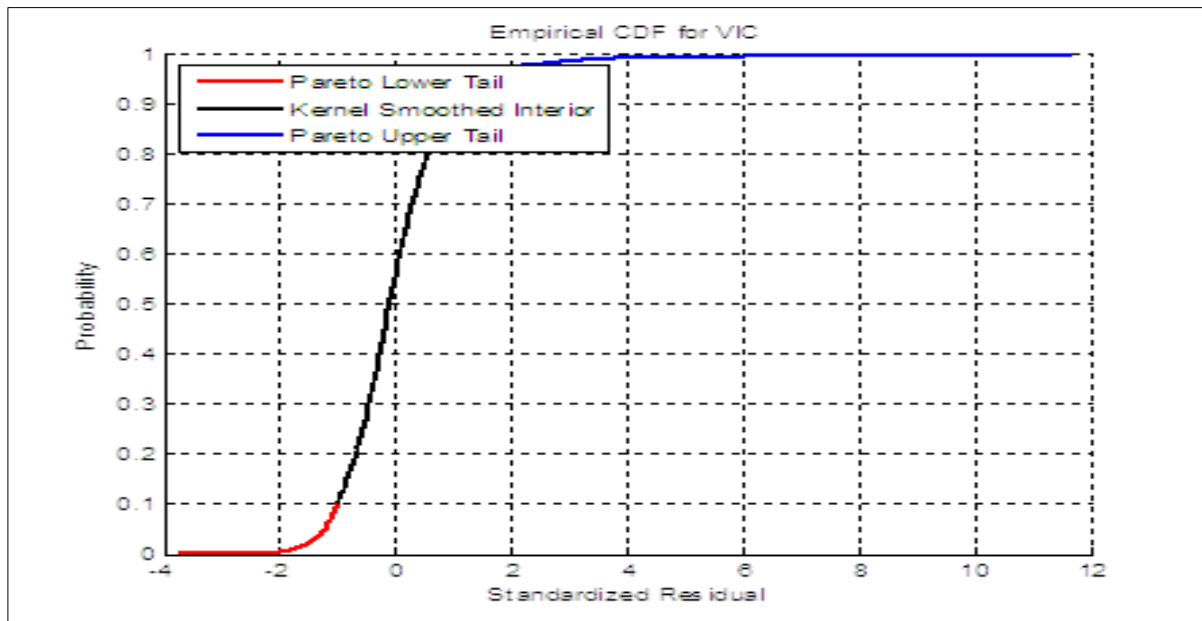
Source: Author's calculations.

Figure 75 Empirical Cumulative Distribution Function (TAS)



Source: Author's calculations.

Figure 76 Empirical Cumulative Distribution Function (VIC)



Source: Author's calculations.

As it was stated earlier, there are two approaches to fit the tail of a sample ECDF to one of the possible distribution functions. The first method relies on approximating a distribution from a block maxima series while the second method relies on sampling points from the data set that exceeds a certain threshold. This method is generally referred to as the POT method and is the method used in generating the ECDFs in this study. The ECDFs generated in this study provide a graphical representation of the probability distribution of a random vector without implying any prior assumption concerning the form of this distribution. The Pareto tails at both the lower and upper ends and the non-parametric kernel-smoothed interior⁵⁹ construct a composite semi-parametric CDF for each region of the NEM.

⁵⁹ Kernel smoothing is a non-parametric estimation method of the probability density function of a distribution. In dimension 1, the kernel smoothed probability density function \hat{p} has the following expression, where K is the univariate kernel, n the numerical sample size and $(X_1, \dots, X_n) \in \mathbb{R}$ the univariate random sample with $\forall_i, X_i \in \mathbb{R}$:

$$\hat{p}(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - X_i}{h}\right)$$

A central motivation for computing probability distribution functions from observed data is to understand how the distribution implied by market prices differs from a theoretical distribution. As is known, if the normal linear model holds, the standardised residuals will have approximately Gaussian distributions and approximately 95 per cent of the standardised residuals should therefore be between -2 and +2, and almost all should be between -3 and +3. However, as the above ECDFs depicts, this is not the case for each market region of the NEM as standardised residuals take values from -10 to 16. This analysis further strengthens the existence of fat tail behaviour in electricity price series.

Additionally, the figures in Appendix 10 assess the GPD fit for each region of the NEM, by plotting the empirical CDF of the upper tail exceedances of the residuals along with the CDF fitted by the GPD. The CDF of the GPD in these figures are parameterized as:

$$F(y) = 1 - \left(1 + \frac{\xi y}{\beta}\right)^{-\frac{1}{\xi}} \quad (68)$$

where $\gamma \geq 0$, $\beta > 0$, $\xi > -0.5$ for exceedances (y), tail index parameter (zeta), and scale parameter (beta).

The kernel K is a function satisfying $\int K(x)dx = 1$. Usually, K is chosen to be a unimodal probability density function that is symmetric about 0. The parameter h is called the bandwidth. In dimension $d > 1$, the kernel may be defined as a product kernel K_d , as follows where $\underline{x} = (x_1, \dots, x_d) \in \mathbb{R}^d$: $K_d(\underline{x}) = \prod_{j=1}^d K(x_j)$ which leads to the kernel smoothed probability density function in dimension d , where (X_1, \dots, X_n) is the d -variate random sample which components are denoted $(\underline{X}_i = X_{i1}, \dots, X_{id})$:

$$\hat{p}(\underline{x}) = \frac{1}{N \prod_{j=1}^d h_j} \sum_{i=1}^N K_d\left(\frac{x_1 - X_{i1}}{h_1}, \dots, \frac{x_d - X_{id}}{h_d}\right)$$

Although only 10 per cent of the standardised residuals are used, the fitted distribution closely follows the exceedances data, so the GPD model seems to be a good choice for modelling electricity spot prices in the NEM.

SIMULATION OF THE EVT MODEL WITH COPULA FUNCTIONS

The literature has earlier examples that pointed out the weaknesses in traditional Monte Carlo simulations in modelling electricity prices. For example, Lucia and Schwartz (2002) applied EVT to electricity prices to assess the level of tail fatness in series followed by comparison of the Monte Carlo simulation results based on Gaussian and Student-t distributions. Their simulation results improved upon the ones from the Gaussian distribution, as the Student-t price patterns resemble more closely the underlying price pattern of electricity prices.

This study showed the normality assumption that researchers and practitioners often make in their simulation or valuation method is not appropriate and prone to lead to erroneous conclusions. This further strengthens the view taken in this section in simulating the electricity prices data modelled with the assistance of EVT.

In this chapter, the residuals of the GPD model are simulated with Copula functions. As previously mentioned, Copula functions allow modelling of price and volatility dependencies, which exist in the regions of the NEM. The copula functions also provide a multivariate approach to modelling electricity prices in the NEM.

Copula approach is used to describe the dependence between multiple variables in this study. It is used to model the dependencies of electricity prices in the NEM. Dependence modelling with Copula functions is widely used in applications of credit risk assessment and actuarial

analysis but it is a fairly new approach to modelling electricity spot prices via copulas, which represent the dependence structure implicit in a multivariate t-distribution⁶⁰.

Parameterisation of the Student t-Copula

In estimating the parameters of the t-copula, this section follows Bouyé et al. (2000) where the parameter vector α of the t-copula is performed by first transforming the data (x'_1, \dots, x'_N) into uniform variates through CDFs (u'_1, \dots, u'_N) . Then, the estimation of the copula parameters are performed as;

$$\hat{\alpha} = \arg \max \sum_{t=1}^T \ln c(u_1^t, \dots, u_n^t, \dots, u_N^t; \alpha) \quad (69)$$

The above equation is defined as canonical maximum likelihood method (CML) by Bouyé et al. (2000) due to its basis on the empirical distributions rather than assumptions of the parametric form of the marginal distributions (α could be viewed as the maximum likelihood estimator given the observed margins).

Kendall's rank correlation provides a distribution free test of independence and a measure of the strength of dependence between two variables. Spearman's rank correlation is satisfactory for testing a null hypothesis of independence between two variables but it is difficult to interpret when the null hypothesis is rejected. Kendall's rank correlation improves upon this by reflecting the strength of the dependence between the variables being compared.

⁶⁰ It is important to note that a difficulty with the use of multivariate copulas such as the t-copula though is that there is only one parameter to control tail association (different pairs of variates might have different tail association) and it has symmetric tail dependency. This symmetry means that when the dependency parameter is assumed to be constant, large joint positive realisations have the same probability of occurrence than large joint negative realisations (Jondeau and Rockinger, 2002).

Kendall's tau measure of a pair (X,Y) can be defined as the difference between the probabilities of concordance and discordance for two independent pairs (X₁,Y₁) and (X₂,Y₂) each with distribution H. That is:

$$\tau_{XY} = \Pr\{(X_1 - X_2)(Y_1 - Y_2) > 0\} - \{(X_1 - X_2)(Y_1 - Y_2) < 0\} \quad (70)$$

These probabilities can be evaluated by integrating over the distribution of (X₁ - X₂). Therefore, in terms of copulas, Kendall's tau becomes;

$$\tau_C = 4 \int_0^1 \int_0^1 C(u, v) dC(u, v) - 1 \quad (71)$$

where C is the copula associated to (X, Y) (Trivedi and Zimmer, 2006).

The following tables comparatively show the estimated Kendall's tau values for all the regions of the NEM. Kendall's tau estimates indicate that the application of Student-t copula over Gaussian copula is more appropriate as Kendall's tau measures have greater values based on Student-t copula indicating a better dependency fit.

Table 26 Parameter Estimates of the Student t-Copula

	NSW	VIC	QLD	SA	TAS
NSW	1	0.944	0.900	0.882	0.689
VIC		1	0.824	0.954	0.768
QLD			1	0.750	0.614
SA				1	0.747
TAS					1

Source: Author's calculations.

Table 27 Parameter Estimates of the Gaussian Copula

	NSW	VIC	QLD	SA	TAS
NSW	1	0.853	0.821	0.730	0.599
VIC		1	0.692	0.896	0.692
QLD			1	0.565	0.499
SA				1	0.646
TAS					1

Source: Author's calculations.

Kendall's tau estimates between the regions of NSW and VIC, NSW and QLD, VIC and SA and NSW and SA are the greatest based on Student t-copula. It is important to note that the estimates suggest a similar pattern to the relationships suggested by Pearson correlation measures.

Simulation of the EVT Model with Student-T Copula

The simulation of t-copula involves generating samples from a multivariate t-distribution based on a pair from the Student t-copula and then inverting it to cumulative distribution samples. This section follows the same approach set by Bowman and Azzalini (1997) in simulating the EVT treated residuals for each region of the NEM. Then as the last step in generating forecast price series, the re-introduction of the autocorrelation and heteroskedasticity observed in the original price series was performed by using the simulated standardised residuals as near *i.i.d.* input noise process through another set of simulation processes. Ultimately, this simulation process aims at generating forecast future values of electricity prices that minimises the variance of the mean.

EVALUATION OF FORECAST PERFORMANCE

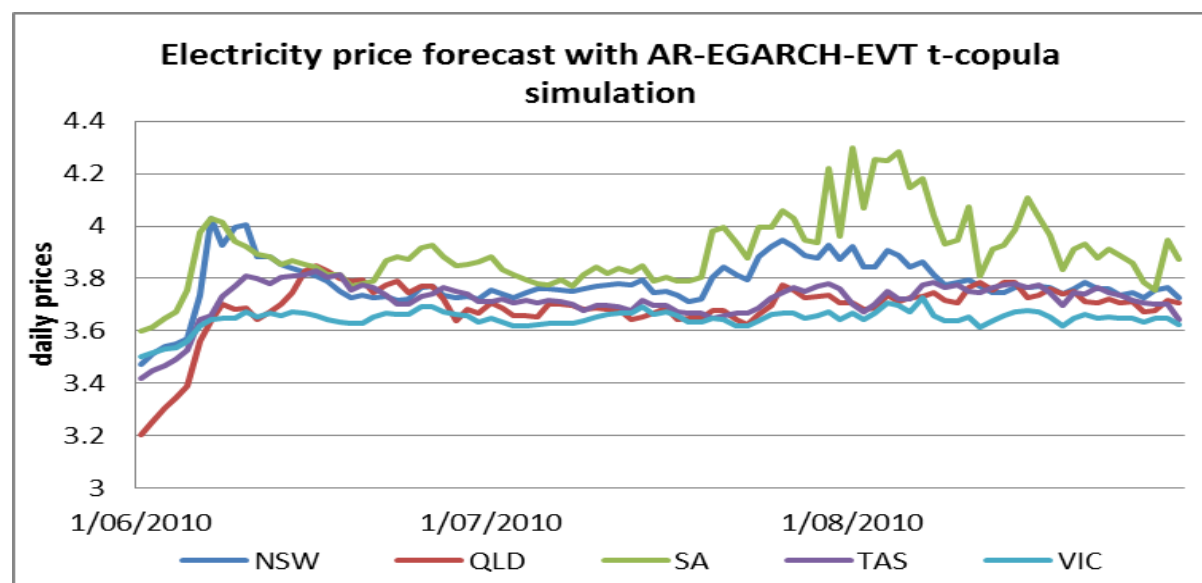
The results of the Student t-copula simulations over the three month horizon (90 days) for each region of the NEM are illustrated in the following graph for the period 01/06/2010 to

29/08/2010. The parameterisation of the model is based on the in-sample period chosen for this study and is the same as all other models previously discussed.

In this study, the price path is for ninety days horizon; the same horizon used as in diffusion type models earlier, allowing easy interpretation of performance comparisons between the various models constructed in this thesis. The price paths presented in the following charts are the result of mean of 10,000 simulated paths.

As is seen from Figure 77, simulated price series revert to long-run equilibrium price level. This long-run mean is a higher long-run mean than the one found by continuous-time models, which were discussed in earlier chapters of this thesis. This is due to the fact that EVT shifts the focus to the tails of the data.

Figure 77 Price Forecast with EVT-Copula Simulations for All Regions of NEM



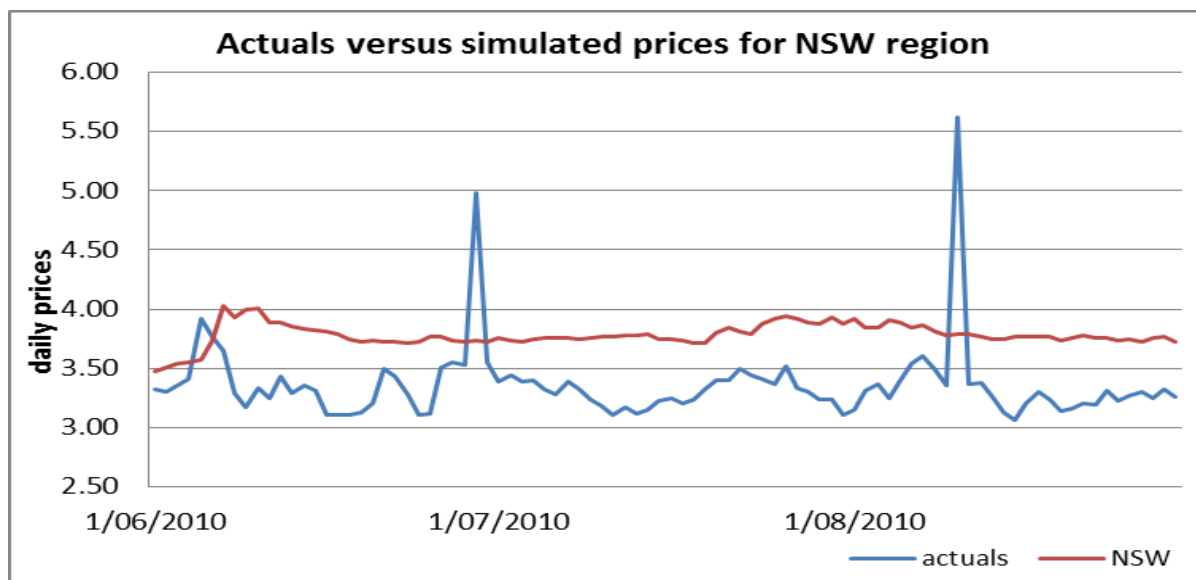
Source: Author's calculations.

Furthermore, the simulated price series for each region of the NEM tends to fluctuate in tandem most of the time throughout the forecast horizon. This is due to the fact that copula functions reflect the dependencies that exist amongst the regions of the NEM.

Whilst the figure above demonstrates the mean of 10,000 simulated paths for all regions of the NEM, the following figure illustrates the electricity price forecast generated by the Student-t copula simulations for NSW. As is seen, this modelling approach fails to replicate the observed time series' characteristics of the electricity prices in particularly, generating price spikes.

However, the model seems to capture the mean-reverting characteristics of the electricity prices (albeit with a higher long-run mean level) similar to mean-reverting jump diffusion and regime switching models.

Figure 78 A Simulated Price Path versus Actual Prices



Source: Author's calculations.

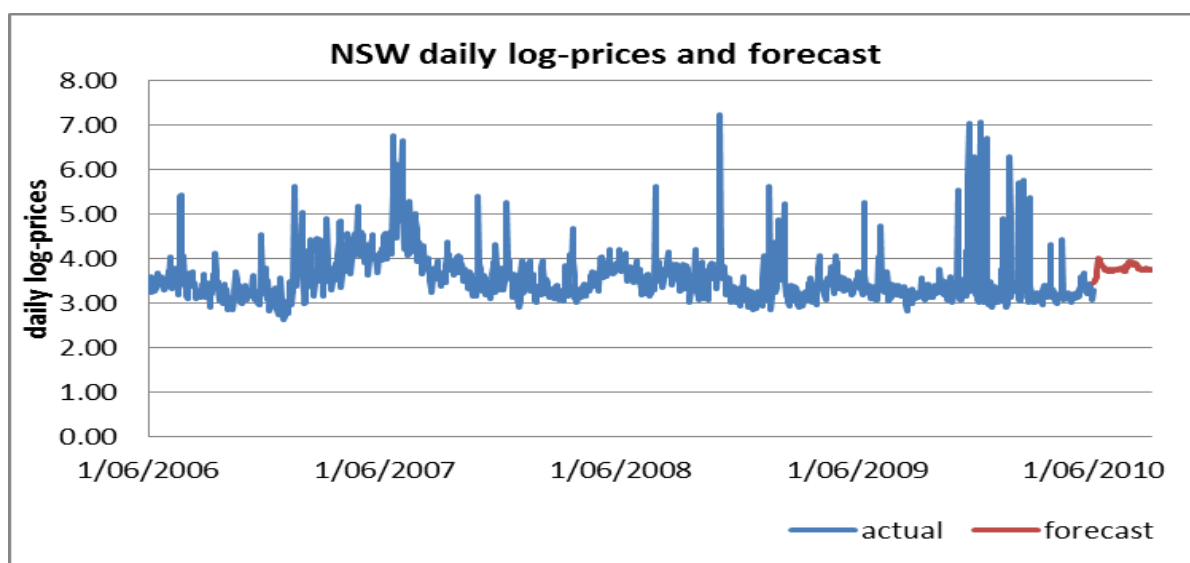
Despite the theoretical superiority of EVT and copula approaches to modelling electricity prices, graphical assessment of the out-of-sample forecast values seem to be underperforming

even the simpler mean-reverting jump-diffusion in mimicking the stylised facts of the electricity prices prevalent in the NEM. The simulation of the dynamics of the EVT treated prices certainly has desirable properties; however this approach fails to perform better than the previous two models investigated in this thesis, namely the Mean-Reverting Jump-Diffusion and Markov Regime-Switching models at face value.

This may be attributed to the structure of the NEM, i.e. it's not perfectly informationally efficient market structure. However, as the market matures each year and planned Government investments in electricity infrastructure projects roll-out with the capacity to enhance market interconnectivity, the AR-EGARCH-EVT specification simulated with copula functions is believed to be a robust tool in forecasting electricity prices in NEM. This is due to the fact that copula functions reflect the dependencies that exist amongst the regions of the NEM.

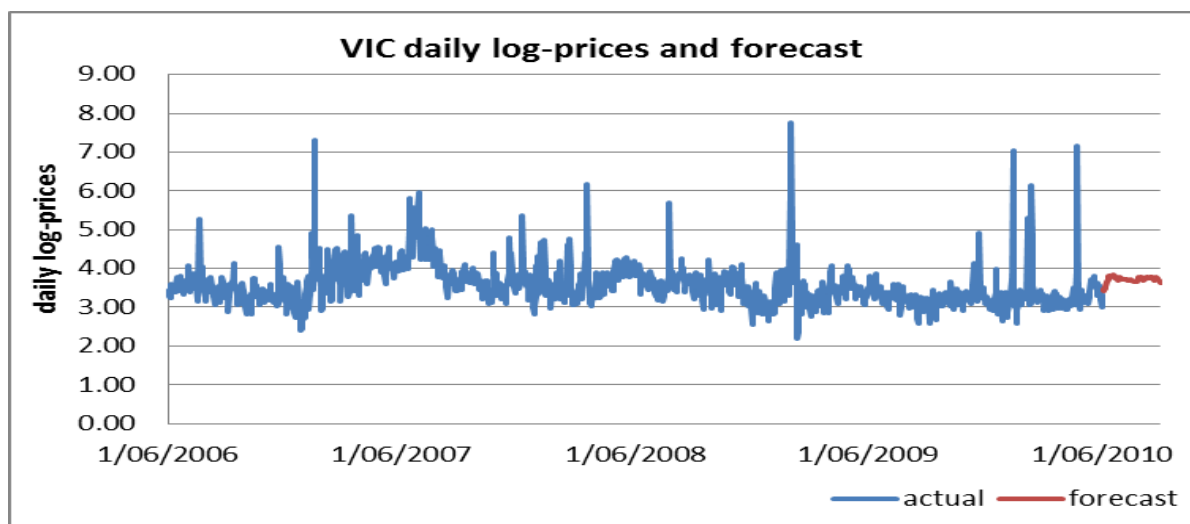
Following the presentation format of the previous models' performance evaluation, the subsequent information describes the in-sample data and forecast generated for each region of the NEM. The Figures 79 to 83 demonstrate the price forecast generated for each of the market regions of the NEM. In order to assess the forecast accuracy of the model, the average of 10,000 simulated paths are taken as the point forecast value similar to earlier models presented in this section.

Figure 79 Price Forecast with EVT-Copula Model (NSW)



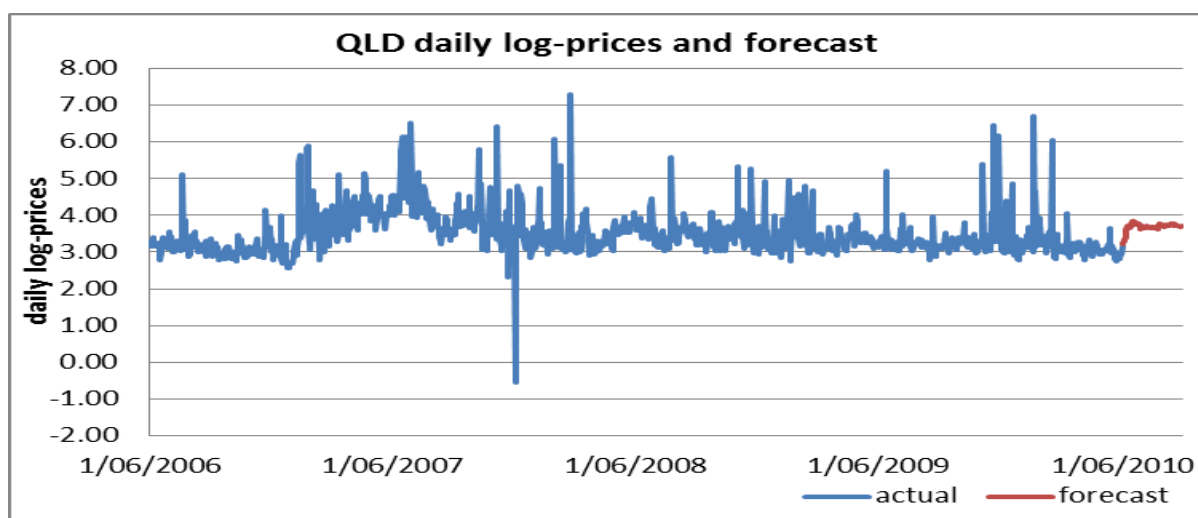
Source: Author's calculations.

Figure 80 Price Forecast with EVT-Copula Model for VIC



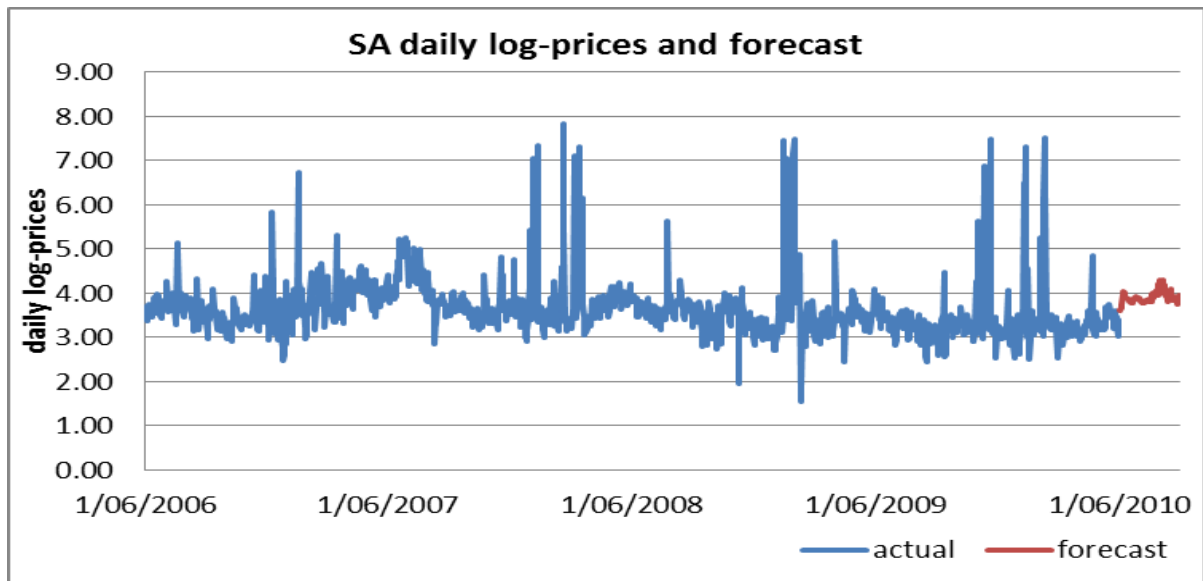
Source: Author's calculations.

Figure 81 Price Forecast with EVT-Copula Model (QLD)



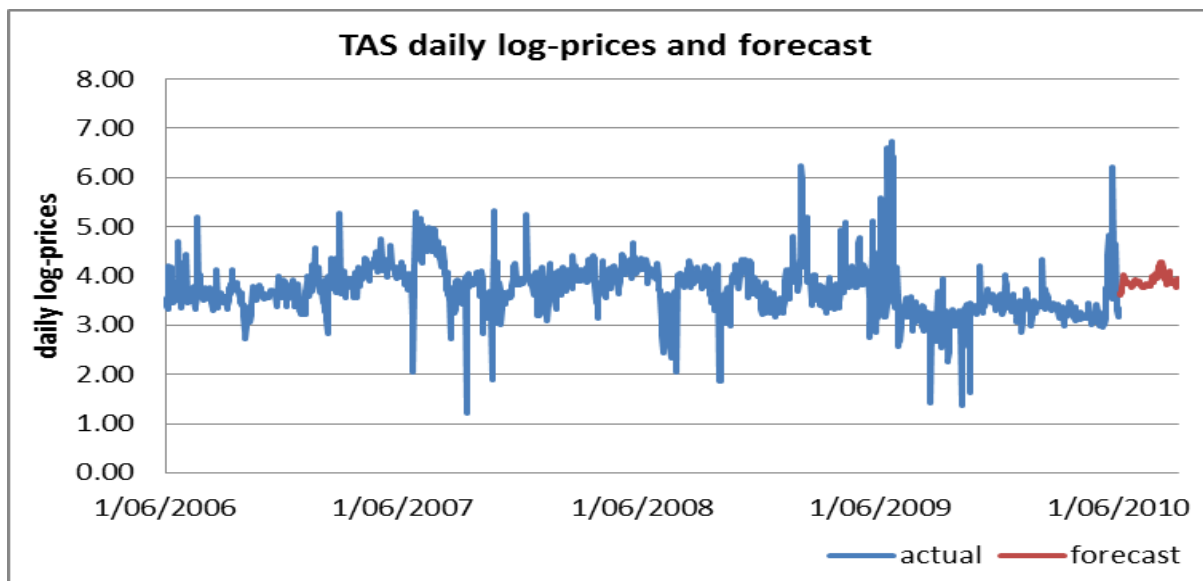
Source: Author's calculations.

Figure 82 Price Forecast with EVT-Copula Model (SA)



Source: Author's calculations.

Figure 83 Price Forecast with EVT-Copula Model (TAS)



Source: Author's calculations.

In the next section an evaluation of the forecasting performance of the EVT based Copula simulations is presented. The purpose of the next section is to see if the model's forecast errors are within reasonable limits or whether these errors are too large.

FORECAST PERFORMANCE STATISTICS

The following discussion evaluates the forecasting performance of the EVT based Copula simulations in forecasting electricity wholesale prices in the NEM. The purpose of this section is to see if this model's forecast errors are within the reasonable limit of expectations or whether these errors are unreasonably large and require an improvement in the statistical models and process of producing these forecasts. To assess the accuracy of the forecast generated by the modelling approach taken in this chapter, formal forecast accuracy statistics are produced for each region of the NEM.

Forecast accuracy statistics are produced for each region of the NEM in a similar fashion to previous sections. These measures are Mean Error (ME), Mean Absolute Error (MAE), Mean Squared Error (MSE), Root Mean Square Error (RMSE), Mean Percentage Error (MPE), Mean Absolute Percentage Error (MAPE) and Theil's U. Although, there is a number of forecast accuracy measures produced here, the following discussion is based on the RMSE as RMSE's focus is on large errors. As electricity prices spike and revert to its long-run mean levels quickly, the forecast values generated by the model are subject to large errors. It is important to note that RMSE is the most widely used measure in the literature for its statistical properties as it places greater penalty on large forecast errors than the MAE.

Table 28 demonstrates the forecast accuracy statistics of this model along with the accuracy statistics produced for the benchmarking model. The RMSE of this model shows increases in all the regions of NEM as compared to the benchmark model. Despite its theoretical superiority and complexity, this model does not produce out-of-sample forecast values that are better than the benchmark model. However, it is interesting to note that the simulation process with Student t-copula function results in best values of RMSE the electricity regions of NSW and

VIC suggesting a more integrated electricity market in these two regions. Comparison of these models' forecast accuracy with the benchmark model indicates that the benchmark model outperforms EVT based Copula simulations for each market region of the NEM as measured by RMSE. Benchmark model⁶¹ used in this study has RMSE scores of 0.328 for NSW, 0.238 for VIC, 0.336 for QLD, 0.475 for SA and 0.605 for TAS.

Table 28 Forecast Accuracy measures for EVT-Copula Model

	NSW	VIC	QLD	SA	TAS
Benchmark Model					
Mean error	-0.02573	-0.09435	-0.18307	-0.12713	-0.27293
Mean square error	0.10766	0.05666	0.1129	0.22619	0.36621
Root mean square error	0.32812	0.23804	0.33600	0.4756	0.60515
Mean absolute error	0.18430	0.19122	0.20202	0.26372	0.44279
Mean percentage error	-1.44263	-3.25139	-8.0017	-36.9556	-9.78207
Mean absolute percentage error	5.12366	5.83328	8.57096	40.55399	12.77883
Theil's U	0.04849	0.03525	0.05349	0.06926	0.08525
EVT-Copula Functions Simulation Model					
Mean error	-0.433	-0.343	-0.6437	-0.554	-0.368
Mean square error	0.300	0.169	0.4954	0.517	0.412
Root mean square error	0.548	0.411	0.7039	0.719	0.642
Mean absolute error	0.503	0.370	0.6437	0.589	0.515
Mean percentage error	-4.290	-10.855	-23.1056	-48.959	-12.730
Mean absolute percentage error	4.558	11.534	23.1056	49.743	15.306
Theil's U	0.076	0.059	0.1043	0.098	0.090

Source: Author's calculations.

⁶¹ See Appendix 5 for details

CONCLUSION

This chapter investigated modelling electricity spot prices in all five market regions of the NEM with a unique approach. The extreme nature of price changes in the NEM regions leads to prices that are non-Gaussian and highly volatile. In order to model extreme price changes, the methodology undertaken in this section is to filter the spot price series with an AR-EGARCH model and then apply EVT to standardised residuals. This technique allows the leverage effects in conditional volatility to be modelled and produce near identically independently distributed standardised model residuals. These residuals seem to behave better than raw residuals in terms of independence. Lastly, POT method was utilised in modelling extreme tails of daily electricity spot prices.

This chapter finds that the POT method accurately models the extreme values of the electricity prices with in-sample and out-of-sample evaluation of forecast providing support for EVT based modelling of electricity prices.

A unique approach to simulating electricity prices in NEM is also adopted in this chapter. Given the dependence structure both in prices and volatility in the NEM, copula approach is utilised to capture the extreme prices occurring simultaneously in different market regions of the NEM. This approach captures the non-linear measures of dependence present in electricity prices in the NEM. This approach allows modelling of the dependence between regional electricity markets as a time varying rather than a constant measure.

A particular use of this model is in estimating derivatives pricing. The most common over-the-counter derivative instrument in the NEM is Asian options. The Gaussian assumption of traditional option pricing models fails in accommodating for extreme observations in tails.

They also disregard the price and volatility interactions between the inter-connected regions of the NEM in generating forecast.

The model explored in this chapter provides an alternative solution to market participants in valuing derivative instruments that consider both the extreme values and time varying dependency widespread in electricity prices. With the assistance of this model, the participants in the NEM could generate long-term price forecasts that are a reflection of the extreme and fat tails of the data and the interconnectivity of the NEM regions.

This approach to modelling electricity spot prices will be of significant importance over the next decade as the regions of the NEM co-integrate further as planned infrastructure investments roll-out. This approach also avoids the estimation complexities present in mean-reverting jump-diffusion models. However, it fails to match the accuracy of the forecast generated with these techniques.

CHAPTER 11 -DISCUSSION AND CONCLUSION

The aim of this thesis was to assess whether there was any advantage in using forecast models that explicitly capture the stylised features of the data under investigation. This assessment was made in the context of the Australian National Electricity Market (NEM). It was believed that the accuracy of the price forecast models can be improved by explicitly modelling the stylised features of electricity wholesale spot prices. The stylised features modelled in this thesis were: mean-reversion, sudden short-lived, consecutive jumps and heavy tails.

When employing models that captured these stylised features they necessarily became more complex often containing a greater number of parameters which combine to mimic these non-trivial behaviours. Throughout this thesis adherence to the principle of parsimony was maintained, that is if two models generated effectively the same forecast performance then the simpler one was preferred whether it contained the stylised features or not.

This investigation was important as it was believed that a better understanding of what models are more useful has the potential to lead to more accurate price forecasts which will result in less volatility in market prices leading to more efficient markets. More efficient markets in turn will result in better outcomes for end users. Further, by assessing models that captured various stylised features it was also possible to infer the importance of particular features.

As is stated in the introductory chapter, the forecasting of electricity prices is important for both the physical and financial participants in the electricity industry for the following reasons:

1. Generators are required to bid in advance, thus accurate price forecasting is necessary if optimal bidding strategies are to be formulated. More accurate electricity price forecasting will allow market participants to develop more efficient and successful business strategies,

real and financial asset valuations and improve their price risk management. Modelling prices accurately is important for production assets such as generators. This is particularly true for peaking plants, whose value may be entirely dependent on the existence of price spikes that facilitate the recovery of high marginal costs and the recouping of fixed costs over very short running periods. Generators can also use the information on prices to measure the potential competition from other suppliers or the potential opportunity of servicing customers in other regions.

2. Generators need to plan ahead for capacity building purposes (peak and off-peak generators have varying input requirements for production) therefore efficient price forecasting allows efficiency in planning of supplies.

3. Traders need to take positions both at over the counter and at the Sydney Futures Exchange Board (SFEB) therefore accurate forecasting of spot prices are important for derivatives pricing.

4. The distribution of electricity is a public regulated monopoly; therefore there are also many government policy implications. For instance, having accurate electricity price forecasts is also of interest to government policy makers. Wholesale electricity prices influence the contract prices at the retail level, which in turn impacts upon the final prices for consumers. End-users are concerned with the better modelling of prices because of cost efficiencies associated with load shedding during peak periods, while retailers can benefit from improved forecasting of volatility to hedge against upside price risk. Also, potential benefits to households are expected to be realised via reduced prices at the retail level.

In summary, a better understanding of effective electricity price forecasting is believed to help all market participants to develop more efficient strategies, for example it will assist firms in formulating business strategies, real and financial asset valuations and improve their price risk management. Also, price forecasting is an important aspect of the industry as generators bid to

the market operator to be granted the rights to supply electricity to the grid. Ineffective bids results in revenue losses for generators. For instance, bidding a low price to get the electricity to the grid will result in lost revenue if the market price is above the bid price and vice versa.

An important feature of wholesale electricity spot prices is that they are highly volatile due to non-storability, limited transportability, restricted arbitrage transactions and imperfect price forecasting techniques. As such the nature of the electricity time series is not the same as traditional stock prices. In addition they are recognised as being spikier, showing extreme volatility and exhibiting more rapid mean-reverting behaviour than stock prices. These factors contribute to the imperfect forecast techniques commonly applied to electricity wholesale prices.

Mean-reversion, sporadic spikiness, and non-Gaussian manifesting in positive skewness and leptokurtosis as stylised features of electricity. Therefore any forecasting model that fails to capture these features likely result in relatively larger forecast errors (albeit on occasions). Consequently this thesis investigated the explicit incorporation of the stylised features of data into the forecasting models. Specifically, the stylised features of mean-reversion, sudden and short-lived jumps, occasional consecutive jumps and non-Gaussian manifested as heavy tails were examined.

It was expected that by explicitly modelling the stylised features of the electricity wholesale spot prices, forecast accuracy can be improved when compared to baseline models commonly used in quantitative finance. Engle's AR-GARCH model is chosen to be the standard approach in forecasting price series and was taken as the benchmark model in this thesis.

This thesis employed models from two distinct model classes which by construction captured the stylised characteristics of electricity prices: linear non-linear modelling methods to answer the following research questions:

1. Are forecast models generated by continuous-time models more accurate than traditional AR-GARCH model?
2. Are forecast models generated by non-linear models more accurate than traditional AR-GARCH model?

Continuous time models were consisted of Geometric Brownian Motion (GBM), Mean-Reverting, and Mean-Reverting and Jump-Diffusion models, which accounted for the mean-reversion and sudden, short-lived jumpy characteristics of the electricity price data as prevalent in the NEM. Non-linear models of Markov Regime-Switching and Extreme Value Theory (EVT) based models accounted for the consecutive jumps and non-Gaussian prevalent in NEM.

It was found that the benchmark model (AR-GARCH) outperformed all continuous-time and non-linear models in short-term (90 days) forecasts. This assessment was made by comparing the forecasts' Root Mean Square Error (RMSE) values across all regions of the NEM. This relatively superior performance of the benchmark model over other models investigated was mainly due to its ability to capture the serial dependence and varying degree of variance prevalent in the data.

The following sections will explain the findings of this thesis in greater detail. However, a brief overview of the data is believed to be useful before the presentation of forecast evaluations.

BRIEF BACKGROUND TO THE DATA

There are five market regions in the National Electricity Market (NEM) and this thesis investigated time series data from each region of the NEM. The regions of NEM considered in this thesis were New South Wales (NSW), Victoria (VIC), Queensland (QLD), South Australia (SA), and Tasmania (TAS). The data was collated from Australian Energy Market Operator (AEMO). AEMO collates and reports average daily observations for each price for the five market regions of NEM.

The price data used in this thesis were average hourly pool price observations sourced directly from AEMO for the period of 01/06/2006 to 29/08/2010 for the regions of NSW, VIC, SA, QLD and TAS. The data from 01/06/2006 to 31/05/2010 were used to estimate the parameters of the models while the period from 01/06/2010 to 29/08/2010 were used to derive out-of-sample forecast accuracy statistics. As stated in the introductory chapter, the rationale behind choosing this sample period was due to Tasmania's entry to AEMO towards the end of 2005, as such data prior to this date was not available.

Average daily reported price values for each region, expressed in Australian dollars per megawatt hour (MWh) for each day constituted the empirical data of this study. Although the highest frequency of electricity spot prices in the NEM are quoted as half-hourly, daily average prices are of significant importance to market players. The most common derivative instrument in electricity markets is Asian options, which is priced by average daily prices.

In fitting the data, logarithm to base prices was used due to the fact that in time series analysis this transformation is often considered to stabilise the variance of a series (Luetkepohl, 2012). Log specification was only a problem in the presence of non-positive prices. There were so few

instances of negative prices in the NEM. In these cases, the average of nearby points in the series replaced the data points in the in-sample data used where log prices were not defined. Specifically there was one instance of negative price occurrence in Victoria, 12 in Tasmania and two in South Australia.

Analysis of the descriptive statistics, as demonstrated in Chapter 3, showed that the distribution of prices is significantly non-Gaussian for all regions of the NEM as consistent with the existing literature. The price series in all of the electricity regions were positively skewed and leptokurtic. This extreme fat-tailed characteristic was consistent with the findings of earlier studies and was likely to be driven by the prevalence of extremely high prices.

Electricity price series in the NEM were found to exhibit extreme price spikes and were found to be prevalent even at the daily intervals. The formal normality and unit root tests as presented in Chapter 3 confirmed the non-Gaussian and stationary nature of the price series. Additionally, Wald-Wolfowitz Runs test indicated the importance of modelling electricity prices in Australia in an attempt to reduce risk management costs. One appreciates that in inefficient markets; traders can possibly make a difference with advanced models in managing their risks and exploring profit taking opportunities.

CONTRIBUTION TO THE LITERATURE

Wholesale electricity spot prices in the NEM are highly volatile due to non-storability, limited transportability, restricted arbitrage transactions and imperfect price forecasting techniques. As such the nature of the electricity time series is not the same as traditional stock prices. Typically, they are spikier, show extreme volatility and exhibit a rapid mean-reverting pattern (Bunn 2004). Mean-reversion, the presence of jumps, and non-Gaussian manifested as positive

skewness and leptokurtosis are the main stylised features of electricity. Therefore, it was believed that any forecasting model that fails to capture the stylised features of electricity prices will likely result in large forecast errors.

Consequently this thesis investigated the explicit incorporation of the stylised features of data in forecasting electricity prices in NEM. The stylised features of electricity price data that were explicitly accounted for in the models presented in this thesis were; mean-reversion, sudden, short-lived, occasional consecutive jumps and non-Gaussian manifested as heavy tails.

It was believed that by explicitly modelling the stylised features of electricity prices, forecast accuracy can be improved upon baseline models commonly used in quantitative finance. This thesis investigated the forecasting ability of two distinct modelling approaches which by construction capture the stylised characteristics of electricity prices. Namely, these were linear continuous time and non-linear modelling methods. The AR-GARCH model is chosen to be the standard approach in forecasting price series (Engle, 2001) and was taken as the benchmark model in this thesis.

More specifically, this thesis aimed to answer the following research questions:

- 1- Are forecast models generated by continuous-time models more accurate than traditional AR-GARCH model?
- 2- Are forecast models generated by non-linear models more accurate than traditional AR-GARCH model?

In both instances the models were chosen to reflect the stylised features of the data.

Continuous-Time Models

The overwhelming majority of electricity-pricing models are adaptations of popular models for price or returns from the financial econometrics literature that have been augmented to capture the idiosyncratic time-series properties of electricity prices, albeit with varying degrees of success (Weron, 2006). Evidence suggests that using models based on Stochastic Differential Equation (SDE) otherwise known as continuous-time models provide a much better fit to electricity prices than the Autoregressive models (Lucia and Schwartz 2002, Huisman and Mahieu 2003, Weron and Misiolek et al. 2006).

The continuous time-models examined in this thesis were; Geometric Brownian Motion (GBM), Mean-Reverting, and Mean-Reverting and Jump-Diffusion models. The inclusion of GBM in this thesis was mainly due to it being the foundation for the other continuous time models investigated in this study.

Another two continuous-time models captured some of the main stylised features of electricity prices. The Mean-Reverting model captured the mean-reversion (tendency of electricity prices to revert back to their long-term average over time) characteristics of electricity prices whilst Mean-Reverting and Jump-Diffusion model incorporated the sudden jumps prevalent in electricity wholesale prices.

Continuous-time models were ordered such that each successive model extended the one preceding it. Note that each extension addressed a stylised feature of the data therefore the a-priori expectation was that the forecasting performance will improve.

Non-Linear Models

The inclusion of the non-linear approach to forecasting Australian electricity prices was performed with the application of Markov Regime-Switching model and the combination of Extreme Value Theory (EVT) and Copula functions.

The Markov Regime-Switching model was a non-linear modelling tool that was able to capture consecutive spikes prevalent in Australian electricity prices that the Mean-Reverting and Jump-Diffusion models failed to capture. The EVT model on the other hand captured the heavy tails present in electricity price data. The forecasts based on the EVT model were built upon the application of Copula functions as these functions model the interdependence of prices within the separate regions of the Australian electricity markets.

Forecast Methodology

To summarise, in total six econometric models were applied in this thesis and their short-term forecast performances (90 days) were compared with each other and with the chosen benchmark model (AR-GARCH). The price data used in this study were average hourly pool price observations sourced directly from AEMO for the period of 01/06/2006 to 29/08/2010. The data from 01/06/2006 to 31/05/2010 (in-sample data) were used to estimate the parameters of the models while the period from 01/06/2010 to 29/08/2010 (out-of-sample data) were used to derive out-of-sample forecast accuracy statistics.

The models examined in this thesis were;

Table 29 List of Econometric Models

Benchmark Model	
<ul style="list-style-type: none"> • AR(1)-GARCH(1) 	
Continous-time Models	
<ul style="list-style-type: none"> • Geometric Brownian Motion • Mean-Reverting Model • Mean-Reverting Jump Diffusion Model 	
Non-linear Models	
<ul style="list-style-type: none"> • Markov Regime-Switching models with spike distributions modelled with <ul style="list-style-type: none"> -Gaussian distribution -Log-Gaussian distribution • Extreme Value Theory and Copula functions. 	

Each model under investigation mimicked a known characteristic of electricity prices. Mean-Reverting model replicated the mean reversion feature of price series whilst Mean-Reverting and Jump-Diffusion model incorporated jumpy features of price series along with mean-reversion. Markov Regime-Switching model on the other hand incorporated the consecutive jumps prevalent in the NEM in its formation. Finally EVT based model replicated the nonlinear, heavy tailed nature of the electricity price series.

In this study, each continuous-time and the Markov Regime-Switching model was simulated using the Euler approximation method. This method simulated sample paths of correlated state variables driven by Brownian motion sources of changes over consecutive observation periods and thus approximating continuous-time stochastic processes.

EVT based forecast models on the other hand were simulated with Copula functions, returning random vectors generated from a t-copula with linear correlation parameters. This method generated a set of simulations from a bivariate t-copula and each column of the simulation sets is a sample from a uniform marginal distribution.

The aim of these studies was to characterise the electricity prices use of Autoregressive and Mean-Reverting models and only employed data from one market region of the Australian National Electricity Market (NEM). However, the focus of these studies was limited to stylising the behaviours of electricity prices in the NEM hence the focus was never on the forecast ability of conventional models. The models as investigated in this thesis were more sophisticated than conventional models as they captured the non-linear and extreme features of electricity prices in a forecast application.

COMPARATIVE FORECAST PERFORMANCE ANALYSIS

This thesis investigated a number of models that represent the stylised characteristics of electricity prices in the NEM. These stylised characteristics were non-Gaussian, mean-reversion and the jumpy nature of prices. In addition, the evidence of co-dependence in prices between the separate regions of the NEM has been an emerging theme in the literature (Wild et al., 2010). Therefore this unique constituent of electricity prices was also incorporated in one of the modelling applications developed in this thesis.

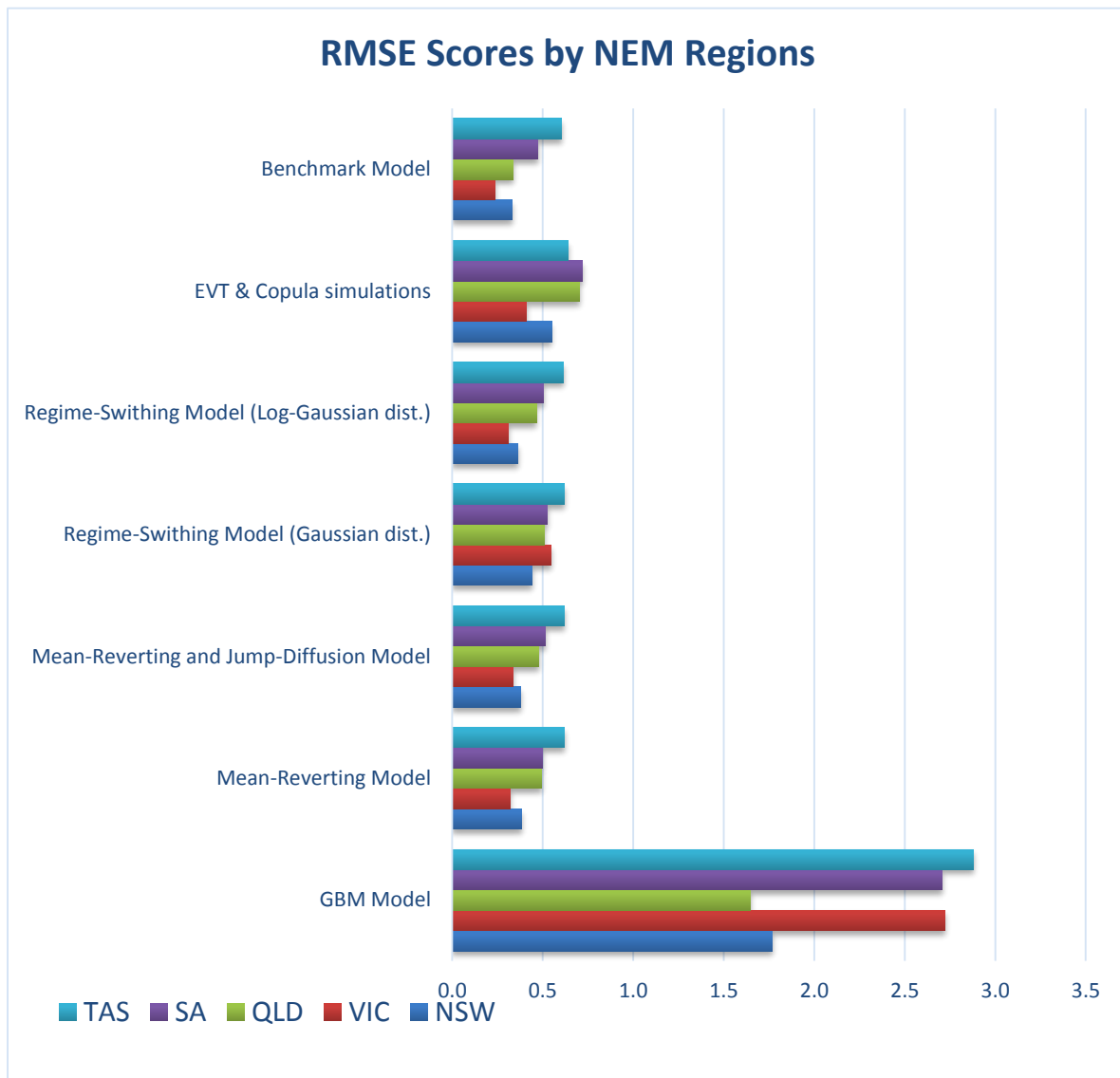
The inclusion of GBM in this thesis is mainly due to it being the foundation for the other continuous-time models considered in this study. The other two continuous-time models captured particular stylised features of electricity prices. Markov Regime-Switching model was presented as the fourth forecast model in this thesis. EVT based forecast approach to modelling

prices in the NEM (and its Copula centric simulation method) was another unique contribution of this thesis to the literature. This model allowed capturing the fat-tailed nature of electricity price series in addition to incorporating price co-dependencies between interconnected regions of the NEM.

As can be appreciated, it was essential to take a consistent view of what is to be forecast and to make sure that forecasts being compared were based on the same data. Therefore, formal forecast accuracy statistics were calculated and presented for each model investigated in this thesis across all regions of the NEM.

Figure 84 presents the RMSE values of each model investigated by separate regions of the NEM. As is seen, Markov Regime-Switching model with spike processes modelled as Log-Gaussian distribution had the lowest RMSE values across all continuous-time and non-linear models investigated in this thesis. The relative performance of the Markov-Regime Switching model is persistent in all NEM regions. Nevertheless, this model had slightly higher RMSE values than the benchmark model as is seen from Figure 84. The benchmark model of this study outperformed all models investigated in this study across all regions of the NEM.

Figure 84 RMSE Values of All Models by NEM Regions



Source: Author's calculations.

1-Short-term forecast accuracy of GBM Model

As is seen in Figure 84, forecast accuracy statistics for the GBM model as measured by RMSE indicate the following values; 1.767 for NSW, 2.722 for VIC, 1.648 for QLD, 2.706 for SA and 2.88 for TAS regions. These RMSE values indicate a relatively large error in forecasts, which suggests that the GBM model as investigated in this thesis was a relatively ineffective forecast model. This maybe because it did not account any stylised features of the price series.

The observed poor performance of the GBM model is in line with Johnson and Barz (1999) who concluded that arithmetic and geometric Brownian processes are unsuitable in modelling electricity prices. Johnson and Barz (1999) evaluated the effectiveness of a number of SDE based models in electricity price modelling, including arithmetic and GBM processes along with mean-reverting diffusion processes known as Ornstein-Uhlenbeck process (first proposed by Vasicek (1977)). Johnson and Barz (1999) concluded that the Geometric Mean-Reverting Jump-Diffusion model gave the best performance and all models without jumps (Arithmetic and Geometric Brownian Motion processes) were inappropriate in modelling electricity prices.

This was attributed to the fact that when a price spike occurred, the GBM would assume that the new price level is a normal event and it would proceed randomly via a continuous diffusion process with no consideration of prior price levels.

2-Short-term forecast accuracy of Mean-Reverting Model

The models investigated in this thesis were in an order such that each successive model extended the one preceding it. Consequently, the Mean-Reverting model was expected to perform better than the GBM model as it addressed a stylised feature of the data.

Subsequently, it was found that the Mean-Reverting model investigated in this thesis performed in a superior manner to the investigated GBM model in forecasting electricity prices. The RMSE values as presented in Figure 84 are significantly less with this model as compared to the GBM model in all regions of the NEM. RMSE statistics based on the Mean-Reverting model indicated the following values for each of the NEM regions; 0.382 for New South Wales (NSW), 0.318 for Victoria (VIC), 0.498 for Queensland (QLD), 0.499 for South Australia (SA) and 0.619 for Tasmania (TAS) regions. This decline in the values of RMSE is about eight times

in some instances (the RMSE value dropped about 4.6 times for NSW and about 8.5 times for VIC) when compared to RMSE values attained via the GBM model.

However, comparison of the Mean-Reverting model's forecast accuracy with the benchmark model chosen for this study indicated that the benchmark model outperformed the Mean-Reverting model for each region of the NEM as measured by RMSE. The AR-GARCH model used in this study had RMSE scores (in order of region sizes) of 0.328 for NSW, 0.238 for VIC, 0.336 for QLD, 0.475 for SA and 0.605 for TAS. These values are slightly lower than the values obtained via the Mean-Reverting model hence indicating a superior forecast performance for the benchmark model.

The relative poor performance of the forecast generated via the Mean-Reverting model as compared to the benchmark model is attributed to the fact that the benchmark model handles the serial dependence and the varying variance prevalent in electricity data better than the Mean-Reverting model. Despite that fact that the Mean-Reverting model mimicked the mean-reversion feature of the data well in generating short-term forecasts.

3-Short-term forecast accuracy of Mean-Reverting and Jump-Diffusion Model

It is believed that adding a jump component to Mean-Reverting model to capture the jumpy behaviour of electricity prices has theoretical superiority over the simpler Mean-Reverting and GBM models. Mean-Reverting and Jump-Diffusion model as investigated in this thesis was an extension of the previously studied continuous-time models of GBM and Mean-Reverting models. This model was shown to be superior as it captured some of the main stylised features of the data, namely; the sudden and short-lived jumps in data.

Nonetheless, a closer examination of the forecast prices revealed that the differences between the magnitude of jumps and timing were quite different when compared to actual price series. Figure 84 presents the RMSE values of the forecast prices, which indicated; 0.378 for NSW, 0.333 for VIC, 0.477 for QLD, 0.513 for SA and 0.62 for TAS regions. Forecast accuracy of this model revealed improvements in forecast made for the regions of NSW and QLD as compared to previously studied Mean-Reverting model by approximately 0.7 per cent and 3 per cent, respectively.

However, in the regions of VIC, SA and TAS, the application of Mean-Reverting and Jump-Diffusion models did not result in more accurate forecasts. RMSE values in these regions actually showed deterioration in the range of 0.3 per cent to 4 per cent. However, comparison of the Mean-Reverting model's forecast accuracy with the benchmark model chosen for this study indicated that the benchmark model outperformed the Mean-Reverting and Jump-Diffusion model for each region of the NEM as measured by RMSE. The benchmark model used in this study had RMSE scores (in order of region sizes) of 0.328 for NSW, 0.238 for VIC, 0.336 for QLD, 0.475 for SA and 0.605 for TAS.

In brief, it was found that forecasting electricity prices in the NEM with a Mean-Reverting and Jump-Diffusion model out-performed GBM but it provided mixed results when compared to Mean-Reverting model. The relative poor performance of the forecast generated via this model as compared to the benchmark model is attributed to the fact that the benchmark model handles the serial dependence and the varying variance prevalent in electricity data more accurately than the Mean-Reverting and Jump-Diffusion model. Also, this relative underperformance of

the Mean-Reverting and Jump-Diffusion model⁶² is partly attributed to its less than perfect ability to capture the timing and magnitude of spikes prevalent in the data.

4-Short-term forecast accuracy of Markov Regime-Switching Model

Modelling the mean-reverting and sudden, short-lived jumpy features of electricity prices in the NEM with a Mean-Reverting and Jump-Diffusion model was found to be less than perfect as the timing and magnitude of spikes are hard to be captured by these specifications. Markov Regime-Switching models have such specifications that capture both the mean-reverting and the jumpy features of electricity prices more accurate than Mean-Reverting and Jump-Diffusion models.

Markov Regime-Switching models as investigated in this thesis revealed a number of salient features, which are importantly useful for understanding the price dynamics of the NEM. First of all, it is found that the probability of a price spike on any particular day ranges between two per cent to nearly eight per cent⁶³ in separate regions of the NEM depending on the model specifications. Secondly, price spikes were found to account for much of the volatility in electricity spot prices. Hence, volatility measures in base regimes were found to be actually quite low albeit varying between the regions of NEM and the specific Markov Regime-Switching model considered. In forecasting electricity prices in the NEM with Markov Regime-Switching models as investigated in this thesis, base regime is modelled with a mean-reverting specification with constant variance whilst spike regimes are modelled with a number of different distributions. Accordingly, these volatilities seemed to reflect the marginal cost of

⁶² While the analytic fundamentals and features of the Jump-Diffusion models are well established, the issue of how accurately the model can be calibrated to historical data remains the weakness of the model. These models are challenging to calibrate also because of the large number of parameters that must be determined, many of which are time dependent.

⁶³ See Chapter 3.

production. Lastly, a great variation in the magnitude of spikes was found in the NEM, with spikes being generally largest in SA and smallest in QLD.

Figure 84 presents that Markov Regime-Switching model with Gaussian distributions had RMSE of 0.439 for NSW, 0.545 for VIC, 0.507 for QLD, 0.527 for SA and 0.618 for TAS regions. These RMSE values were found to be smaller than other SDE based models as previously examined, indicating a relatively superior performance of the Markov Regime-Switching model.

Nevertheless, tangible improvements in forecast performance with this model arose with the modelling of the spike process with a Log-Gaussian distribution. Markov Regime-Switching model with spike process was modelled with a Log-Gaussian distribution provided RMSE scores (in order of region sizes) of 0.363 for NSW, 0.312 for VIC, 0.464 for QLD, 0.505 for SA and 0.616 for TAS as is seen in Figure 84. These were significant improvements in forecast accuracies when compared to the Markov Regime-Switching model with Gaussian spikes and all other SDE based models as previously investigated in this thesis. It is important to note that the main aim of the simulated models in this thesis is to minimise the variance of the expected mean values. Markov Regime-Switching models do this well compared to other SDE specifications, as measured by RMSE.

However, comparison of the model's forecast accuracy with the benchmark model chosen for this study indicated that the Markov Regime-Switching model underperforms the benchmark model for each region of the NEM as measured by RMSE. The relative underperformance of the forecast generated via the Markov Regime-Switching model as compared to the benchmark

model is attributed to the fact that the benchmark model handles the serial dependence and the varying variance prevalent in electricity data better than the Markov Regime-Switching model.

5-Short-term forecast accuracy of Extreme Value Theory Model

The extreme nature of price changes, manifesting as sudden and short-lived jumps, in the NEM leads to prices that are non-Gaussian and highly volatile. This feature of the data under investigation in this thesis set the requirement for a model that captured the extremities at the long tail.

In order to model this feature, EVT was applied to the data. It was found that the EVT method accurately modelled the extreme values of the electricity prices with evaluation of forecast providing support for EVT based modelling of electricity prices. To generate forecast electricity price values, this thesis applied a unique approach to the simulation of data, which was modelled with the aid of EVT. Given the dependence structure both in prices and volatility in the NEM, it was hypothesised that Copula approach captures the non-linear measures of dependence present in electricity prices. This approach also allowed modelling of the dependence between the different NEM regions as a time varying rather than a constant measure.

Figure 84 shows the RMSE values for the forecast model based on a combination of EVT and Copula functions. As is seen from Figure 84, these values are slightly worse as compared to all the models investigated in this study (except GBM model) across all regions of the NEM. RMSE scores (in order of region sizes) of this model were found as; 0.548 for NSW, 0.41 for VIC, 0.704 for QLD, 0.719 for SA and 0.642 for TAS regions.

Finally, comparison of the models' forecast accuracy with the benchmark model indicates that the benchmark model outperformed the forecast generated via EVT-Copula model for each market region of the NEM. The AR-GARCH model used in this study had RMSE scores (in order of region sizes) of 0.328 for NSW, 0.238 for VIC, 0.336 for QLD, 0.475 for SA and 0.605 for TAS.

This relative underperformance of the EVT- Copula model was attributed to two underlying facts as discussed in the previous chapter. First of all, the filtering procedure applied in this study failed to achieve perfectly identical and independent distributed errors. Secondly, even though the regional interconnectors work towards an interconnected electricity market in the NEM, the market is not fully efficient in terms of price and volatility interdependencies therefore the application of Copula functions in simulating price series were thought to result in less accurate than hypothesised. Moreover, it was believed that the benchmark model handled the serial dependence and the varying variance prevalent in electricity data better than the EVT-Copula model.

OVERALL SUMMARY OF THE COMPARATIVE ACCURACIES

Comparative performance evaluations of each model investigated in this thesis showed that the benchmark model provides superior short-term forecasting ability.

Continuous-time and the Markov Regime-Switching models were found to be capable of modelling the mean-reversion and sudden but short-lived jumps prevalent in electricity prices. Markov Regime-Switching models were found to be successful in generating spikes and mean-reversion features of electricity prices but they failed to capture the exact timing of the spike occurrences. Having said that, in generating accurate short-term forecast, they did not

outperform the benchmark model. Similarly, the forecast generated by the combination of EVT and Copula functions performed quite poorly as compared to the benchmark model.

It is believed that more accurate short-term forecast ability of Markov Regime-Switching models over all other models examined in this thesis is due to its ability to model spike regimes as separate processes to the base regime. This property also overcomes the parameter estimation problems of Jump-Diffusion models, which model the spikes and the base regimes distinctly.

These results are also in line with the findings of Higgs and Worthington (2008) who applied three different mean-reverting jump diffusion models to electricity price series in an attempt to determine the best spot price model applicable in all of the NEM regions. The models they utilised were a basic stochastic model, a mean-reverting model and a Markov Regime-Switching model. Their results showed that the Markov Regime-Switching model outperforms the basic stochastic and mean-reverting models.

Table 30 presents a number of forecast accuracy statistics by NEM regions for each model investigated in this thesis. These statistics as shown in this chapter aimed to provide a comprehensive view of the short-term forecast accuracy of the models investigated.

Table 30 Comparative Forecast Accuracy Measures of All Models by Regions of NEM

		ME	MSE	RMSE	MAE	MPE	MAPE	Theil's U
NSW	GBM	-1.421	3.122	1.767	1.446	-51.102	43.998	0.214
	Mean-reverting	-0.185	0.146	0.382	0.281	-6.249	8.17	0.055
	Mean-reverting jump-diffusion	-0.198	0.143	0.378	0.286	-6.608	8.396	0.055
	Markov Regime-Switching(Gaussian spikes)	-0.298	0.193	0.439	0.373	-9.615	11.077	0.063
	Markov Regime-Switching(Log-Gaussian spikes)	-0.174	0.132	0.363	0.266	-1.963	2.362	0.053
	AR-EGARCH-EVT	-0.434	0.301	0.548	0.503	-4.29	4.558	0.077
	Benchmark Model	-0.02	0.107	0.328	0.184	-1.442	5.123	0.048
VIC	GBM	-2.482	7.407	2.722	2.482	-75.794	75.794	0.295
	Mean-reverting	-0.228	0.101	0.318	0.269	-7.31	8.349	0.046
	Mean-reverting jump-diffusion	-0.248	0.111	0.333	0.286	-7.95	8.913	0.048
	Markov Regime-Switching(Gaussian spikes)	-0.336	0.297	0.545	0.371	-10.634	11.511	0.078
	Markov Regime-Switching(Log-Gaussian spikes)	-0.221	0.097	0.312	0.265	-7.131	8.26	0.046
	AR-EGARCH-EVT	-0.341	0.168	0.41	0.368	-10.78	11.46	0.059
	Benchmark Model	-0.094	0.056	0.238	0.191	-3.251	5.833	0.035
QLD	GBM	-1.389	2.717	1.648	1.389	-48.612	48.612	0.218
	Mean-reverting	-0.414	0.243	0.493	0.414	-15.629	15.629	0.076
	Mean-reverting jump-diffusion	-0.398	0.227	0.477	0.398	-14.899	14.899	0.073
	Markov Regime-Switching(Gaussian spikes)	-0.442	0.257	0.507	0.442	-16.298	16.298	0.077
	Markov Regime-Switching(Log-Gaussian spikes)	-0.384	0.216	0.464	0.384	-14.445	14.445	0.072
	AR-EGARCH-EVT	-0.644	0.495	0.704	0.644	-23.106	23.106	0.104
	Benchmark Model	-0.183	0.112	0.336	0.202	-8.001	8.570	0.053
SA	GBM	-2.224	7.323	2.706	2.252	-135.72	136.42	0.297
	Mean-reverting	-0.259	0.249	0.499	0.31	-41.385	42.649	0.071
	Mean-reverting jump-diffusion	-0.266	0.263	0.513	0.336	-38.497	40.153	0.073
	Markov Regime-Switching(Gaussian spikes)	-0.299	0.278	0.527	0.362	-39.388	40.861	0.075
	Markov Regime-Switching(Log-Gaussian spikes)	-0.255	0.255	0.505	0.328	-37.81	39.548	0.072
	AR-EGARCH-EVT	-0.544	0.518	0.719	0.589	-48.96	49.743	0.099
	Benchmark Model	-0.127	0.226	0.475	0.263	-36.955	40.553	0.069
TAS	GBM	-2.281	8.293	2.88	2.286	-70.415	70.548	0.308
	Mean-reverting	-0.313	0.383	0.619	0.475	-10.958	13.782	0.087
	Mean-reverting jump-diffusion	-0.33	0.385	0.62	0.484	-11.556	14.135	0.088
	Markov Regime-Switching(Gaussian spikes)	-0.332	0.382	0.618	0.481	-11.586	14.188	0.087
	Markov Regime-Switching(Log-Gaussian spikes)	-0.338	0.379	0.616	0.482	-11.742	14.242	0.087
	AR-EGARCH-EVT	-0.369	0.412	0.642	0.515	-12.731	15.307	0.09
	Benchmark Model	-0.272	0.366	0.605	0.442	-9.782	12.778	0.085

Source: Author's calculations

The RMSE based on forecast with GBM process is the lowest for the Queensland (QLD) region (1.648) followed by the NSW region (1.767). The values of RMSE for other regions of the electricity network are found to be 2.721 for Victoria (VIC), 2.706 for South Australia (SA)

and 2.879 for TAS. These RMSE values are much higher than the forecast accuracy statistics based on the benchmark model. The benchmark model outperforms basic GBM model for each market region of the NEM measured by RMSE.

Significant improvements can be noticed in RMSE values based on the Mean-Reverting model as compared to GBM based forecasts. For instance, the RMSE statistics showed significant drops in mean-reverting model as compared to GBM model in all regions of the NEM. The drop in the values of this statistic was about eight times in some instances (the RMSE value dropped about 4.6 times for NSW and about 8.5 times for VIC).

As it can be observed from Table 30, the comparative performance analysis of the Mean-Reverting and Mean-Reverting Jump-Diffusion models shows a mixed picture. The RMSE of the Mean-Reverting Jump-Diffusion model indicate declines in NSW and QLD as compared to the RMSE of the Mean-Reverting model by about 0.7 per cent and 3 per cent, respectively. However, in VIC, SA and TAS, the application of Mean-Reverting Jump-Diffusion model does not result in any improvements in RMSE values. Furthermore, RMSE of the model are found to have poor performance as compared to the benchmark model.

The Markov Regime-Switching models generally provide better RMSE values compared to other models examined in this study. The Markov Regime-Switching model with Gaussian spikes has RMSE of 0.43 for NSW, 0.54 for VIC, 0.5 for QLD, 0.52 for SA and 0.61 for TAS regions. The noteworthy improvements in forecast performance emerge with the modelling of the spikes process with Log-Gaussian distribution though. The Markov Regime-Switching model with Log-Gaussian spikes result in RMSE of 0.36 for NSW, 0.31 for VIC, 0.46 for QLD, 0.5 for SA and 0.61 for TAS. It is important to note that the main aim of the simulated models

in this thesis is to minimise the variance of the expected mean values. The Markov Regime-Switching specifications so far does this more effective than previously examined continuous-time models. Comparison of these models' forecast accuracy with the benchmark model however indicates that the benchmark model outperforms Markov Regime-Switching specifications for each market region of the NEM measured by root mean square errors (RMSE). The AR-GARCH model used in this study has RMSE scores of 0.328 for New South Wales (NSW), 0.238 for Victoria (VIC), 0.336 for Queensland (QLD), 0.475 for SA and 0.605 for Tasmania (TAS).

In terms of the forecast accuracy of the Copula model, the RMSE values show deterioration in all regions of NEM. Despite its theoretical superiority and complexity, this model does not produce out-of-sample forecast values that are better than the benchmark model. However, it is interesting to note that the simulation process with Student t-copula function results in best values of RMSE the electricity regions of NSW and VIC suggesting a more integrated electricity market in these two regions.

AREAS OF FUTURE RESEARCH

Areas of future research in relation to enhancing the forecast accuracy of the models investigated in this thesis can be described under a number of broad areas. The following sections describe the broad issues under each section and give directions for future researchers to investigate those. It is believed that more accurate electricity price forecasts can mostly be done with improvements in these areas.

Simulations

The simulation processes performed in this thesis for both continuous-time and Markov Regime-Switching models were based on the Euler-Maruyama method. Although this method

for ordinary differential equations (ODE) has order one, the strong order for the Euler-Maruyama method for stochastic differential equations is $1/2$. This was proved in Gikhman and Skorokhod (1972). This difference in strong order indicates that the Euler method has larger mean errors compared to Milstein method. Hence, simulation of SDE based models with the Milstein method rather than the Euler method may result in more accurate forecast values. This is an area of future research that may be conducted in an effort to enhance the forecast accuracy of the models investigated in this thesis.

Parameter estimation

Models estimated in this thesis have a large number of parameters. The accurate estimate of these parameters provides an important area for researchers. As is appreciated, parameter estimates must be interpreted in light of the experience and knowledge of the conditions in the market at the time. All continuous-time models as studied in this thesis apart from GBM⁶⁴ utilised some complex parameter estimation methods that may be subject to interpretation and may be enhanced at the discretion of the researcher.

Mean-Reverting Model

The parameters of the Mean-Reverting model as were estimated via an ordinary least squares (OLS) method where the speed of mean reversion and long-run mean level are calculated from the coefficients of a linear fit between the log prices and their first difference scaled by the time interval parameter. This was rather a less complex way of estimating the parameters of the mean-reverting model.

⁶⁴ The mean and standard deviation of the in-sample data are used as the parameters of the GBM specification in this study. This implies constant mean and variance even though electricity prices in the NEM present heteroskedasticity.

The literature has examples of slightly more complex estimation procedures like Maximum Likelihood Estimator (MLE) and MLE with Jack-knife techniques. For instance, Geman and Roncoroni (2006) estimated the parameters of their model by an estimator based on the exact likelihood of the unknown process with respect to a prior process chosen as a reference within the same class. The estimator is provided by the parameter vector maximising this process over a suitable domain.

It is argued that this method has two major advantages:

- analytical form of the exact likelihood function under continuous time observations can be computed for nearly all semi-martingales through a generalised version of the Girsanov theorem and
- discrete sample estimator converges to the continuous sample one and a well-established estimation theory exists in this latter case.

Equally, incorporating the heteroskedasticity present in the data into these estimates is important. It is expected that more complex parameter estimation methods are likely to increase the forecast performance for the Mean-Reverting model and this area presents opportunities for further research.

Mean-Reverting and Jump-Diffusion Model

Mean-Reverting and Jump-Diffusion model as investigated in this thesis had a large number of parameters. The parameter estimation process poses the greatest challenge in estimating the model. The chosen parameter estimation method for this model was a hybrid approach as suggested by Weron (2006). Furthermore, another undesirable property of the MLE method in calibrating jump-diffusion processes was that it tends to converge on the smallest and most

frequent spike component of the price series whereas the aim of this thesis is to capture the lower frequency, large jump components.

Secondly, another difficulty in estimating the parameters of the Mean-Reverting and Jump-Diffusion model arose from the difficulty in estimating the continuous time jump processes from discretely sampled data. The literature has some examples of innovative methods in estimating the model parameters such as the application of partial maximum likelihood estimation based on Fourier inversion of the conditional characteristic function (CCF) or the quasi-maximum likelihood estimation based on conditional moments captured from derivatives of the CCF evaluated at zero (Weron, 2006). Hence, application of these more complex parameter estimation methods are likely to improve the forecast accuracy of the Mean-Reverting and Jump-Diffusion model and this area poses challenges for future researchers.

Markov Regime-Switching Model

Parameter estimation of Markov Regime-Switching models as investigated in this thesis were performed via the application of Expectation-Maximization (EM) algorithm where the whole set of parameters was estimated by an iterative two-step procedure. The variable transformation and the derivation of the maximum likelihood estimates are not straightforward in an EM algorithm. A solution to this could be the implementation of an extension proposed by Kim (1994). This extension casts the Markov Regime-Switching model into a state space model and presents a new filtering and smoothing algorithm. The major contribution of this extension would be allowing a broader class model to be estimated and increase computing smoothed probabilities.

Furthermore, an EM algorithm allows setting the initial parameters differently for each time the algorithm is run. Therefore, it is accepted that the starting values of the transition probabilities impact on the final outcome of the model. Hence, experimenting with starting values of the initial transition probabilities may assist finding the optimal starting values and hence improve forecast accuracy of Markov Regime-Switching models.

Extreme Value Theory and Copula functions

Despite the theoretical superiority of the forecast model that combines EVT with Copula functions suggested that this model fails to outperform the SDE based techniques except the GBM model. This relatively poor performance of the model is attributed to its failure to obtain identically and independently (*i.i.d.*) errors when the data were treated with the AR(1)-EGARCH(1) model.

Therefore, a filtering procedure that achieves *i.i.d.* errors would add significant value to the forecast accuracy of this model. Starting a formal lag specification inclusion into the filtering procedure could be the first step towards achieving *i.i.d.* errors. Also, employing an efficient estimator such as Hill estimator to determine the threshold value levels⁶⁵ could potentially enhance the forecast performance of the model. Therefore, it is suggested here that further research should aim at addressing these areas.

Times Series Approach

The approach taken in this thesis in forecasting electricity price series is in essence a time-series model with no exogenous variables. In other words, models investigated in this thesis

⁶⁵ This study uses a 10 per cent threshold value in determining the extremes of the tail data.

did not incorporate any non-price data. Therefore, models used in this thesis may benefit from the introduction of fundamental variables as exogenous input to the models.

For instance, the jump parameters may be included as functions of load, generation capacity or reserve margins as spikes occur when the system is significantly constrained. As was explored in an earlier chapter on price formation, weather plays an important role in determining electricity prices. Under certain weather (i.e. heat-waves) conditions when the majority of the generation dispatched, transmission congestion or outages lead to price spikes. This suggests that temperature could be incorporated as a fundamental variable into the models in future work.

Data Frequency

Through the use of daily data, the set of models investigated in this thesis set the shortest duration of a spike to one day. In many instances, short-duration spikes may also occur in half-hourly prices, but these are often averaged away in daily prices. This is especially important because the spiking behaviour in electricity markets appears to exhibit strong time variation, with spikes being relatively more common in peak daylight times. Specification of intraday data may provide a logical resolution to these as yet unexplored features and sets an area of further research.

APPENDIX 1 KOLMOGOROV–SMIRNOV TEST

Kolmogorov–Smirnov test

Kolmogorov–Smirnov test (K–S test) is a nonparametric test for the equality of continuous, one-dimensional probability distributions that can be used to compare a sample with a reference probability distribution. The Kolmogorov–Smirnov statistic quantifies a distance between the empirical distribution function of the sample and the cumulative distribution function of the reference distribution, or between the empirical distribution functions of two samples.

Kolmogorov–Smirnov test is defined by letting x_1, \dots, x_m be observations on continuous i.i.d. random values X_1, \dots, X_m with a cumulative distribution function F . Then $H_0: F(x) = F_0(x)$ for all x , where F_0 is a known cumulative distribution function. The Kolmogorov–Smirnov test relies on the fact that the value of the sample cumulative density function is an asymptotically normally distributed.

Kolmogorov–Smirnov test indicates that the electricity price series in Australia are non-Gaussian as the test statistics are greater than the critical value, which then results in rejecting the null hypothesis that the distribution is of the expected form. The results from both Jarque-Bera and Kolmogorov-Smirnov tests support the idea that non-conventional modelling techniques that do not rely on the assumption of normal distribution should be used to forecast electricity prices in NEM.

Table 31 One-Sample Kolmogorov-Smirnov Tests

	NSW	VIC	QLD	SA	TAS
Kolmogorov-Smirnov Z statistic	5.018	2.182	3.610	4.848	5.409
Probability	0.000	0.000	0.000	0.000	0.000

APPENDIX 2- OVERVIEW OF THE FORECAST ACCURACY MEASURES

The literature has a number of widely accepted forecast accuracy measures as there are several statistical methods available to evaluate forecast performance. The following section lists the commonly used forecast accuracy measures. It is important to note that root mean squared error (RMSE) is the most widely used measure in the literature for its statistical properties. MSE places a greater penalty on large forecast errors than the mean absolute error (MAE) and the MSE and MAE may overcome the cancellation of positive and negative errors limitation of the mean error (ME), but they fail to provide information on forecasting accuracy relative to the scale of the series examined.

Mean Squared Error

The mean squared error is an accuracy measure computed by squaring the individual error for each item in a data set and then finding the average or mean value of the sum of those squares.

It is represented by the following equation:

$$MSE = \frac{1}{n} \sum_{t=1}^n (a_t - f_t)^2 \quad (72)$$

where;

a = actual end of year outcome

f = forecast outcomes

t = time reference [i.e. t is now, (t-1) is last year, (t+1) is next year] usually reported as a subscript reference (e.g. t, t-1)

n = the number of time periods.

Mean Percentage Error

The average of percentage errors by which forecasts differ from outcomes.

$$MPE = \frac{1}{n} \sum_{t=1}^n \frac{(a_t - f_t)}{a_t} * 100 \quad (73)$$

Mean Absolute Error

The mean absolute percentage error is the mean or average of the sum of all of the percentage errors for a given data set taken without regard to sign so as to avoid the problem of positive and negative values cancelling one another. It is represented by the following equation:

$$MAE = \frac{1}{n} \sum_{t=1}^n |(a_t - f_t)| \quad (74)$$

Mean Absolute Percentage Error

$$MAPE = \frac{1}{n} \sum_{t=1}^n \frac{|(a_t - f_t)|}{a_t} * 100 \quad (75)$$

The average of absolute percentage amount by which forecasts differ from outcomes

Theil's U statistics

The U-statistic is an accuracy measure that emphasises the importance of large errors (as well as providing a relative basis for comparison with naïve forecasting methods. Theil's U-statistic can be interpreted as dividing the RMSE of the proposed forecasting method by the RMSE of a no-change (naïve, U=1) model. If U is equal to 1, it means that the proposed model is as good as the naïve model. If U is greater than 1, there is no point in using the proposed forecasting model since a naïve method would produce better results. It is represented as:

$$U = \frac{\sqrt{\sum_{t=1}^n (a_t - f_t)^2}}{\sqrt{\sum_{t=1}^n a_t^2 + \sum_{t=1}^n f_t^2}} \quad (76)$$

All of the forecast accuracy measures described are subject to interpretation. For example, a simple dollar amount of mean or mean squared error would provide some useful information for a particular variable, however, the mean percentage error means the relative errors can be compared across a number of variables. Ignoring the sign of the error term by adopting absolute changes, one gets an idea of the magnitude of the errors generated by the forecasting techniques.

APPENDIX 3- OVERVIEW OF STOCHASTIC CALCULUS

Let S_t be the time of a particular asset. It is known that if $S_t \sim GBM(\mu, \sigma^2)$, then

$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma \beta_t} \quad (77)$$

where β_t is the Brownian motion driving the asset price. An alternative possibility is to use a stochastic differential equation (SDE) to describe the evolution of S_t . In this case it is written

$$S_t = S_0 + \int_0^t \mu S_u du + \int_0^t \sigma S_u d\beta_u \quad (78)$$

Or shortly,

$$dS_t = \mu S_t dt + \sigma S_t d\beta_t$$

A number of observations are in order. The SDE defined by (78) can be shown to be well defined. In particular, while the first integral on the right-hand-side of (78) is a regular Riemann integral, the second integral is a stochastic integral. Without going into any technical details, it is interpreted as:

$$\int_0^t \sigma S_u d\beta_u = \lim_{h \rightarrow 0} \sum \sigma S_{t_{i-1}} (\beta_{t_i} - \beta_{t_{i-1}}) \quad (79)$$

where $h = \max_i |t_i - t_{i-1}|$ is the width of the partition. The important feature (79) is that the S_t terms are evaluated at the left-hand point of the intervals. This feature is extremely important in finance as it may be interpreted as modelling the inability of people to see into the future. In general, it can be interpreted the stochastic integral, $\int X(u, \beta_u) d\beta_u$, so that

$$\int_0^t X(u, \beta_u) d\beta_u = \lim_{h \rightarrow 0} \sum X(t_{i-1}, \beta_{t_{i-1}})(\beta_{t_i} - \beta_{t_{i-1}}) \quad (80)$$

In general it is often convenient to model asset prices and interest rates as SDEs. Another example is given by the assumption that $X_t = \log(S_t)$ is an Ornstein-Uhlenbeck (OU) process.

In particular this means that $dX_t = -\gamma(X_t - \alpha)dt + \sigma d\beta_t$

where γ , α and σ are non-negative constants.

Ito's Lemma: Suppose x_t is an Ito process with $dx_t = \alpha(x, t)dt + b(x, t)d\beta_t$.

$$\text{Let } y_t = F(x, t) \text{ then } dy_t = \left(\frac{\partial F}{\partial x} \alpha + \frac{\partial F}{\partial t} + \frac{1}{2} \frac{\partial^2 F}{\partial x^2} b^2 \right) dt + \frac{\partial F}{\partial x} b d\beta_t \quad (81)$$

APPENDIX 4- SIMULATION OF STOCHASTIC DIFFERENTIAL EQUATIONS

Defining an SDE of the form: $dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dB_t$

To simulate values of X_t (as there is no explicit solution for it or its distribution can not be determined), what is meant actually is to simulate a discretised version of the SDE. In particular, a discretised process, $\{X_h, X_{2h}, \dots, X_{mh}\}$, where m is the number of time steps, h is a constant and $mh=T$. The smaller the value of h , the closer our discretised path will be to the continuous-time path in the simulation. The Euler scheme satisfies;

$$X_{kh} = X_{(k-1)h} + \mu((k-1)h, X_{(k-1)h})h + \sigma((k-1)h, X_{(k-1)h})\sqrt{h} Z_k$$

where Z_k are identically and independently distributed $N(0,1)$. Estimation of $\theta := E[f(X_T)]$ using the Euler scheme for a fixed number of paths, n , and discretization interval, h , is performed with the following algorithm:

```

     $t = 0; \mathbf{X} = X_0$ 

    for  $j = 1$  to  $n$ 

        for  $k = 1$  to  $\frac{T}{h} = m$ 

            generate  $Z \sim N(0,1)$ 

            set  $\mathbf{X} = \mathbf{X} + \mu(t, \mathbf{X})h + \sigma(t, \mathbf{X})\sqrt{h} Z$ 

            set  $t = t + h$ 

        end for

        set  $f_j = f(\mathbf{X})$ 

    end for

    set  $\theta_n = (f_1 + \dots + f_n)/n$ 

```

$$\text{set } \sigma_n^2 = \sum_{j=1}^n (f_j - \theta_n)^2 / (n - 1)$$

$$\text{set approx. } 100(1 - \alpha)\% \text{ CI} = \theta_n \pm z_{1-\alpha/2} \frac{\sigma_n}{\sqrt{n}}$$

To minimise the discretisation error, there is a need for generating intermediate values X_{ih} .

Also, to estimate $\theta = E[f(X_{t_1}, \dots, X_{t_p})]$ in general, one needs to keep track of $(X_{t_1}, \dots, X_{t_{p1}})$ inside the inner for-loop of the algorithm above.

Discretisation error

The discretisation error is defined by $D := |E[f(X_T)] - E[f(\mathbf{X}_T)]|$ and it is very important when simulating SDEs to ensure that D is sufficiently small. Otherwise, the estimate of θ_n will be a biased estimate of $E[f(X_T)]$, the quantity of interest.

Simulating a multidimensional SDE

In the multidimensional case, \mathbf{X}_t , B_t , and $\mu(t, \mathbf{X}_t)$ in the above equation are vectors, and $\sigma(t, \mathbf{X}_t)$ is a matrix. This arises when there are multiple SDEs in the model.

Simulating SDEs: Jump-Diffusion models

The following discusses how to approximately simulate certain types of jump-diffusion processes when exact simulation is impossible. This discussion is based on Glasserman (2003).

First discretise time and utilise an Euler-type scheme. Let N_t be a Poisson process, W_t a standard Brownian motion and $Y = f\{Y_1, Y_2, \dots\}$ a sequence of IID random variables. Further, assume N_t , W_t and Y are all independent of one another. Consider now a jump-diffusion model of the form

$$dX_t = \mu(X_t)dt + \sigma(X_t) + \sigma W_t + c(X_t, Y_{N_{t+1}})dN_t \quad (82)$$

X_t represents the time t value of an underlying state variable. Then the approximate simulation of a path of X_t on $[0, T]$ is performed as follows:

1. Define an initial grid $0, h, 2h, \dots, mh = T$
2. Since the Poisson process, N_t is independent of W_t , we can imagine that we first simulate the jump times of the process in $[0, T]$. Let these times be $\tau_1, \dots, \tau_{N_T}$ noting of course that N_t will vary from sample path to sample path.
3. We now create a combined time grid, $0 = t_0, t_1, \dots, t_M = T$ consisting of the original $mh + 1 + N_T$
4. We then approximately simulate X_t at points on the combined grid.

APPENDIX 5- EXPECTATION MAXIMIZATION ALGORITHM

Given the initial parameter values of:

$$\theta^{(0)} = (\alpha_i^{(0)}, \beta_i^{(0)}, \sigma_i^{(0)}, P^{(0)}) \quad (83)$$

In the first step of the iterative procedure (the E-step) inferences about the state processes are derived. Since R_t is latent and not directly observable, only the expected values of the state process, given the observation vector $E(R_{t=i}|x_1, x_1, \dots, x_T; \theta)$, can be calculated. These expectations result in the so called smoothed inferences, i.e. the conditional probabilities $P(R_t = j|x_1, x_1, \dots, x_T; \theta)$ for the process being in regime j at time t .

In the second step (the M-step) new maximum likelihood estimates of the parameter vector θ , based on the smoothed inferences obtained in the E-step, are calculated. Both steps are reoperated until the local maximum of the likelihood function is reached.

i. The E-step

Let $x_t = (x_1, \dots, x_t)$. The E-step consists of the following steps:

1-filtering: based on Bayes rule for $t = 1, 2, \dots, T$ iterate on equations:

$$P(R_t = i|x_t; \theta^{(n)}) = \frac{P(R_t=i|x_{t-1}; \theta^{(n)})f(x_t|R_t=i; x_{t-1}; \theta^{(n)})}{\sum_{i=1}^2 P(R_t=i|x_{t-1}; \theta^{(n)})f(x_t|R_t=i; x_{t-1}; \theta^{(n)})} \quad (84)$$

where $f(x_t|R_t = i; x_{t-1}; \theta^{(n)})$ is the density of the underlying process at time t conditional that the process was in regime $(i \in 1, 2)$, and

$$P(R_{t+1} = i | x_t; \theta^{(n)}) = \sum_{j=1}^2 p_{ji}^{(n)} P(R_t = j | x_t; \theta^{(n)}) \quad (85)$$

Until $P(R_T = i | x_T; \theta^{(n)})$ is calculated.

2-Smoothing: for $t = T - 1, T - 2, \dots, 1$ iterate on

$$P(R_t = i | x_T; \theta^{(n)}) = \sum_{j=1}^2 \frac{P(R_t = i | x_t; \theta^{(n)}) P(R_{t+1} = j | x_T; \theta^{(n)}) p_{ij}^{(n)}}{P(R_{t+1} = i | x_t; \theta^{(n)})} \quad (86)$$

The above procedure requires the derivation of $f(x_t | R_t = i; x_{t-1}; \theta^{(n)})$ used in the filtering part. This model definition implies that X_t given X_{t-1} has a conditional Gaussian distribution with mean $\alpha_i + (1 - \beta_i)X_{t-1}$ and standard deviation $\sigma_i | X_{t-1}|^{\nu_i}$. Given by the following probability density function:

$$f(x_t | R_t = i; x_{t-1}; \theta^{(n)}) = \frac{1}{\sqrt{2\pi\sigma_i^{(n)} | x_{t-1}|^{(n)}}} \quad (87)$$

$$\exp\left\{-\frac{(x_t - (1 - \beta_i^{(n)})x_{t-1} - \alpha_i^{(n)})^2}{2(\sigma_i^{(n)})^2 | x_{t-1}|^{2(n)}}\right\} \quad (88)$$

ii. The M-step

In the second step of the EM algorithm, new and more exact ML estimates $\theta^{(n+1)}$ for all model parameters are calculated. Compared to standard ML estimation, where for a given probability distribution function (f) the log-likelihood function ($\sum_{t=1}^T \log f(x_t, \theta^{(n)})$) is maximised, here each component of this sum has to be weighted with the corresponding smoothed inference, since each observation (x_t) belongs to the i^{th} regime with probability, ($P(R_t = i | x_{t-1}; \theta^{(n)})$).

In particular, for the model defined by Equation (88), explicit formulas for the estimates are provided in the following Lemma.

Lemma 1. *The ML estimates for the parameters of the model defined by equation above are given by the following formulas:*

$$\alpha_i = \frac{\sum_{t=2}^T [P(R_t=i|x_T;\theta^{(n)})|x_{t-1}|^{-2}(x_t-(1-\beta_i)x_{t-1})]}{\sum_{t=2}^T [P(R_t=i|x_T;\theta^{(n)})|x_{t-1}|^{-2}]} \quad (89)$$

$$\beta_i = \frac{\sum_{t=2}^T \{P(R_t=i|x_T;\theta^{(n)})x_{t-1}|x_{t-1}|^{-2}B_1\}}{\sum_{t=2}^T [P(R_t=i|x_T;\theta^{(n)})x_{t-1}|x_{t-1}|^{-2}B_2]} \quad (90)$$

$$B_1 = x_t - x_{t-1} - \frac{\sum_{t=2}^T P(R_t=i|x_T;\theta^{(n)})|x_{t-1}|^{-2}(x_t-x_{t-1})}{\sum_{t=2}^T P(R_t=i|x_T;\theta^{(n)})|x_{t-1}|^{-2}} \quad (91)$$

$$B_2 = \frac{\sum_{t=2}^T P(R_t=i|x_T;\theta^{(n)})x_{t-1}|x_{t-1}|^{-2}}{\sum_{t=2}^T P(R_t=i|x_T;\theta^{(n)})|x_{t-1}|^{-2}} - x_{t-1} \quad (92)$$

$$\sigma_i^2 = \frac{\sum_{t=2}^T \{P(R_t=i|x_T;\theta^{(n)})|x_{t-1}|^{-2}(x_t-\alpha_i-(1-\beta_i)x_{t-1})\}^2}{\sum_{t=2}^T P(R_t=i|x_T;\theta^{(n)})} \quad (93)$$

Finally, in the last part of the M-step, the transition probabilities are estimated according to the following formula:

$$p_{ij}^{(n+1)} = \frac{\sum_{t=2}^T P(R_t=j, R_{t-1}=i|x_T;\theta^{(n)})}{\sum_{t=2}^T P(R_{t-1}=i|x_T;\theta^{(n)})} = \quad (94)$$

$$= \frac{\sum_{t=2}^T P(R_t=j|x_T;\theta^{(n)}) \frac{p_{ij}^{(n)} P(R_{t-1}=i|x_t;\theta^{(n)})}{P(R_{t-1}=j|x_t;\theta^{(n)})}}{\sum_{t=2}^T P(R_{t-1}=i|x_T;\theta^{(n)})} \quad (95)$$

where $p_{ij}^{(n)}$ is the transition probability from the previous iteration. All values obtained in the M-step are then used as a new parameter vector $\theta^{(n+1)} = (\alpha_i, \beta_i, \sigma_i, P^{(n+1)})$, $i = 1, 2$, in the next iteration of the E-step.

APPENDIX 6- BROCK-DECHERT-SCHEINKMAN (BDS) TEST FOR INDEPENDENCE

This part carries out the BDS test for independence, as described in Brock, Dechert, Scheinkman and LeBaron (1996). The BDS test is a portmanteau test for time based dependence in a series and can be used for testing against a variety of possible deviations from independence including linear dependence, non-linear dependence, or chaos. This test can be applied to a series of estimated residuals to check whether the residuals are independently and identically distributed. In particular, when the BDS test is applied to the residuals from a fitted linear time series model, the BDS test can be used to detect the remaining dependence and the presence of an omitted nonlinear structure.

The BDS test takes its roots from the concept of correlation integral. The correlation integral at embedding dimension m can be estimated by:

$$C_{m,\epsilon} = \frac{2}{T_m(T_m - 1)} \sum_{m \leq s < t \leq T} I(x_t^m, x_s^m; \epsilon)$$

where $T_m = T - m + 1$ and $I(x_t^m, x_s^m; \epsilon)$ is an indicator function which is equal to one if $|x_{t-i} - x_{s-i}| < \epsilon$ for $i = 0, 1, \dots, m - 1$ and zero otherwise.

Correlation integral estimates the probability that any two m -dimensional points are within a distance of ϵ of each other. That is, it estimates the joint probability; $\Pr(|x_t - x_s| < \epsilon, |x_{t-1} - x_{s-1}| < \epsilon, \dots, |x_{t-m+1} - x_{s-m+1}| < \epsilon)$

If x_t are *i.i.d.* this probability should be equal to the following in the limiting case: $C_{1,\epsilon}^m = \Pr(|x_t - x_s| < \epsilon)^m$

Brock et. al. (1996) defines the BDS statistic as follows: $V_{m,\epsilon} = \sqrt{T} \frac{C_{m,\epsilon} - C_{1,\epsilon}^m}{s_{m,\epsilon}}$, where $s_{m,\epsilon}$ is the standard deviation of $\sqrt{T}(C_{m,\epsilon} - C_{1,\epsilon}^m)$ and can be estimated consistently. Under moderate regularity conditions, BDS statistic converges in distribution to $N(0,1)$ so the null hypothesis of *i. i. d.* is rejected at the 5% significance level whenever $|V_{m,\epsilon}| > 1.96$. The results of the BDS test for each region of the NEM at different dimensions are provided in the following table. The z-statistic is the BDS-statistic divided by the standard error. It is the final result that is used for the hypothesis test and its probability values that the given z-statistic would be observed from *i. i. d.* data.

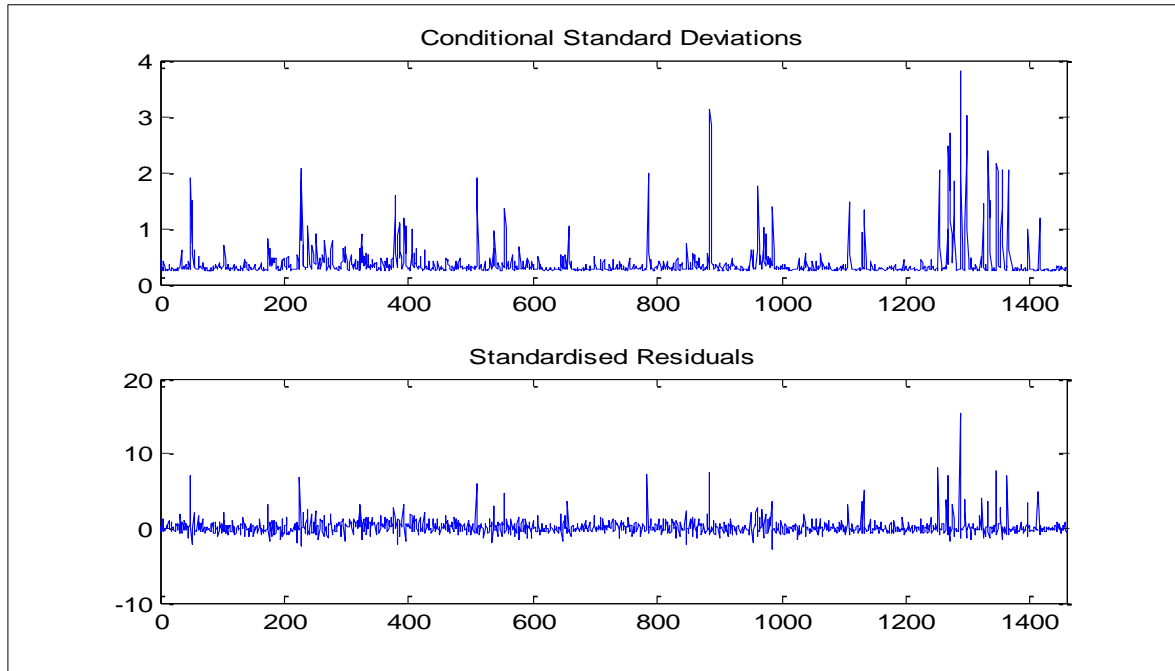
Table 32 BDS Parameter Estimates

NEM Region	Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
NSW	2	0.007	0.003	2.805	0.005
	3	0.012	0.004	3.001	0.003
	4	0.016	0.005	3.344	0.001
	5	0.018	0.005	3.489	0.001
	6	0.018	0.005	3.738	0.000
VIC	2	0.016	0.002	7.525	0.000
	3	0.020	0.003	5.795	0.000
	4	0.019	0.004	4.694	0.000
	5	0.016	0.004	3.838	0.000
	6	0.014	0.004	3.316	0.001
QLD	2	0.006	0.003	2.078	0.038
	3	0.011	0.004	2.693	0.007
	4	0.014	0.005	2.826	0.005
	5	0.016	0.005	3.011	0.003
	6	0.016	0.005	3.171	0.002
SA	2	0.020	0.002	8.357	0.000
	3	0.025	0.004	6.674	0.000
	4	0.024	0.004	5.475	0.000
	5	0.023	0.005	4.936	0.000
	6	0.020	0.004	4.446	0.000
TAS	2	0.017	0.002	7.671	0.000
	3	0.025	0.004	7.221	0.000
	4	0.029	0.004	6.889	0.000
	5	0.031	0.004	7.104	0.000
	6	0.029	0.004	6.947	0.000

Source: Author's calculations.

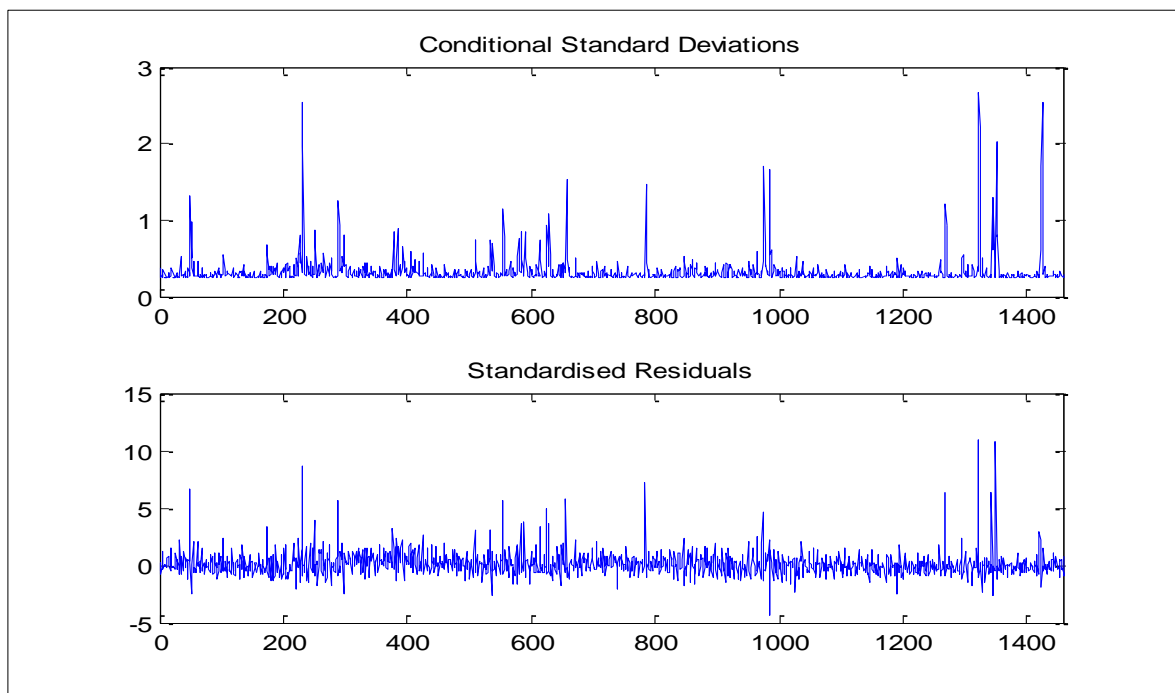
APPENDIX 7- CONDITIONAL STANDARD DEVIATIONS AND STANDARDISED RESIDUALS

Figure 85 Conditional Standard Deviations, NSW



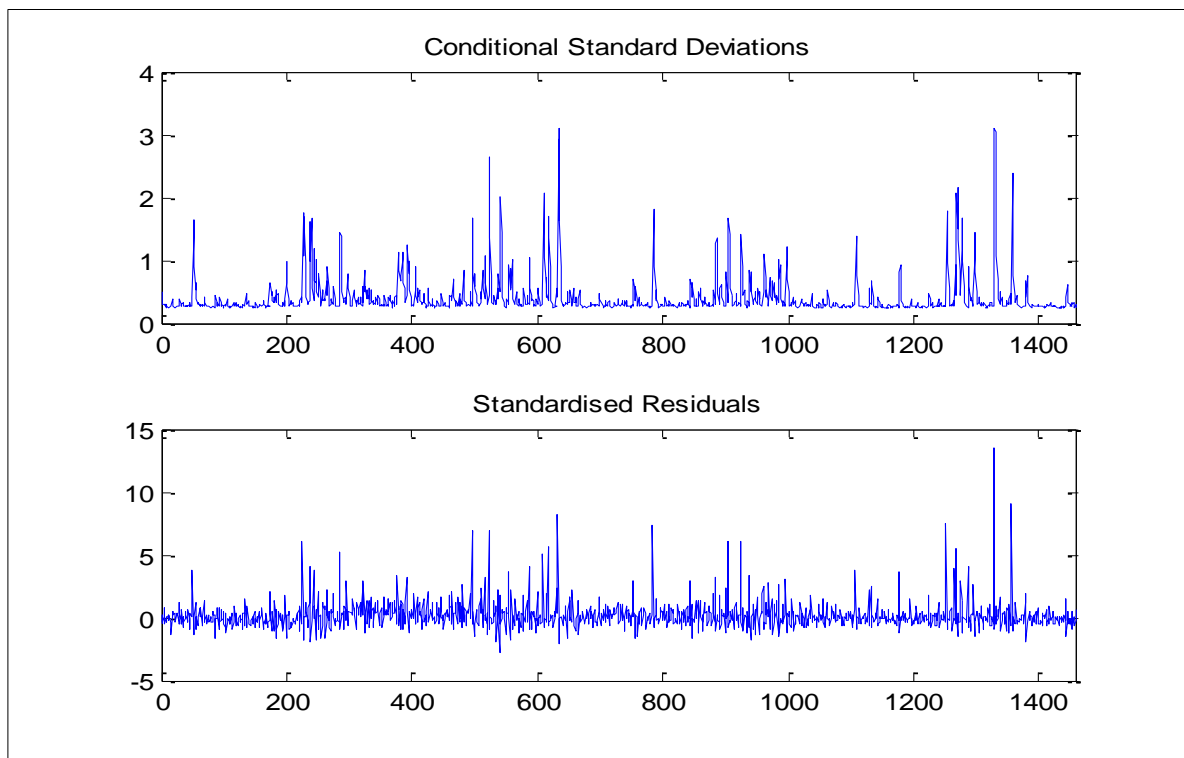
Source: Author's calculations.

Figure 86 Conditional Standard Deviations, VIC



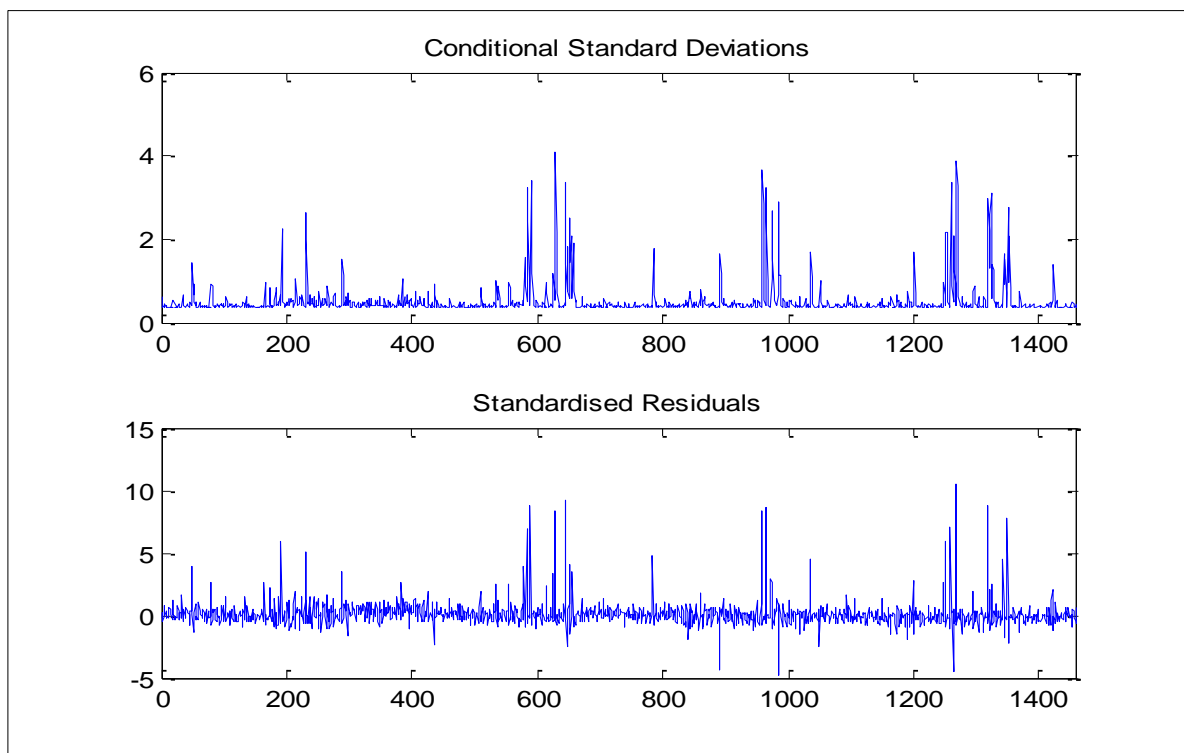
Source: Author's calculations.

Figure 87 Conditional Standard Deviations, QLD



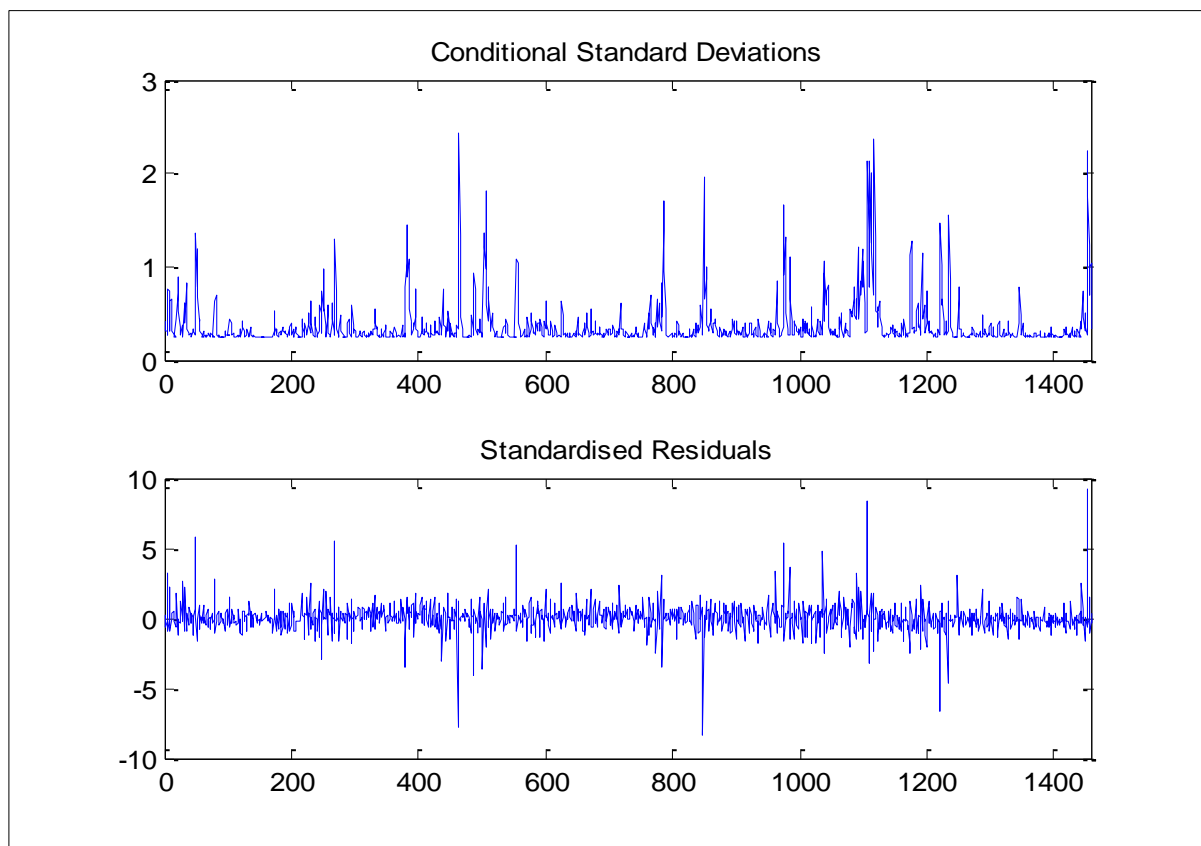
Source: Author's calculations.

Figure 88 Conditional Standard Deviations, SA



Source: Author's calculations.

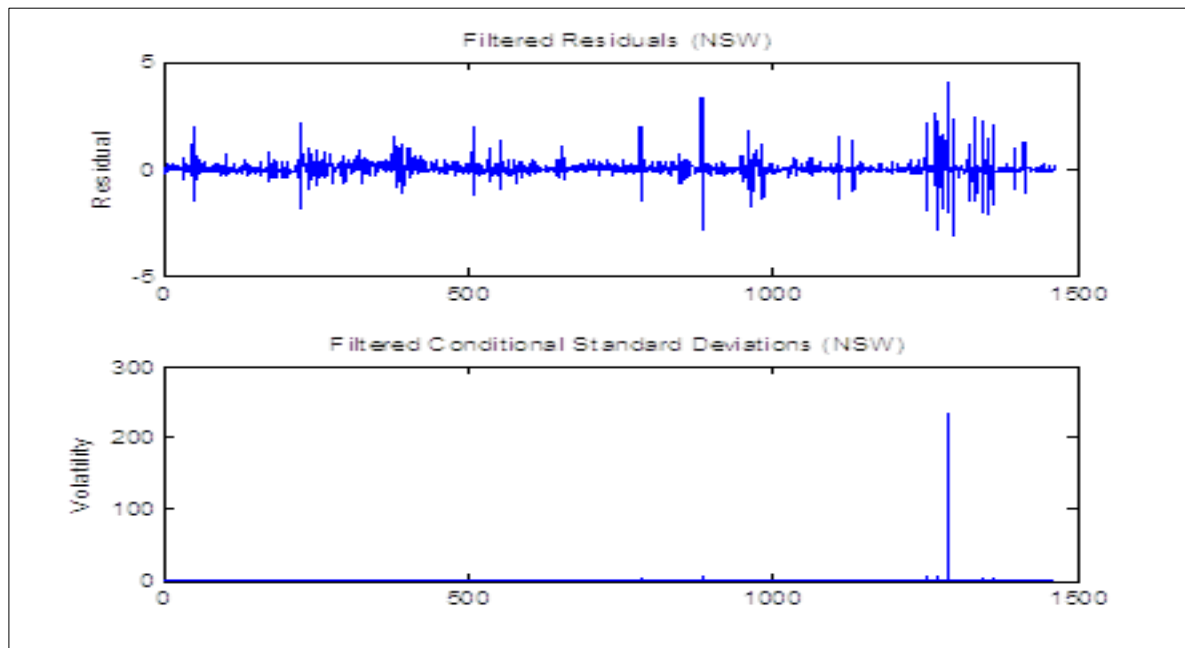
Figure 89 Conditional Standard Deviations, TAS



Source: Author's calculations.

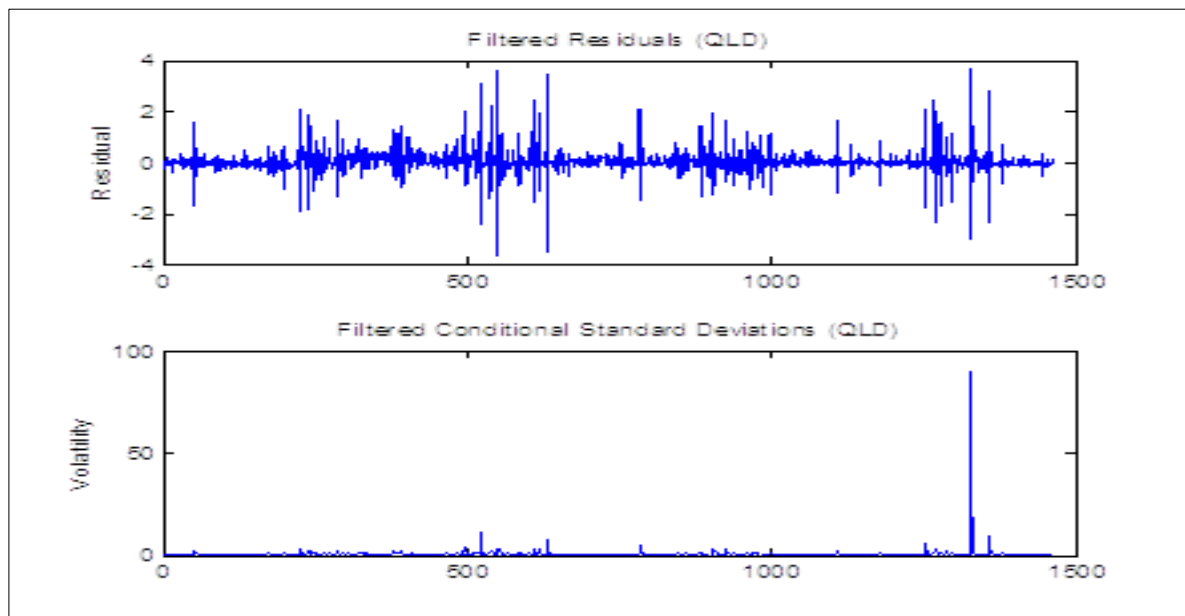
APPENDIX 8- FILTERED RESIDUALS AND CONDITIONAL STANDARD DEVIATIONS

Figure 90 Filtered Residuals, NSW



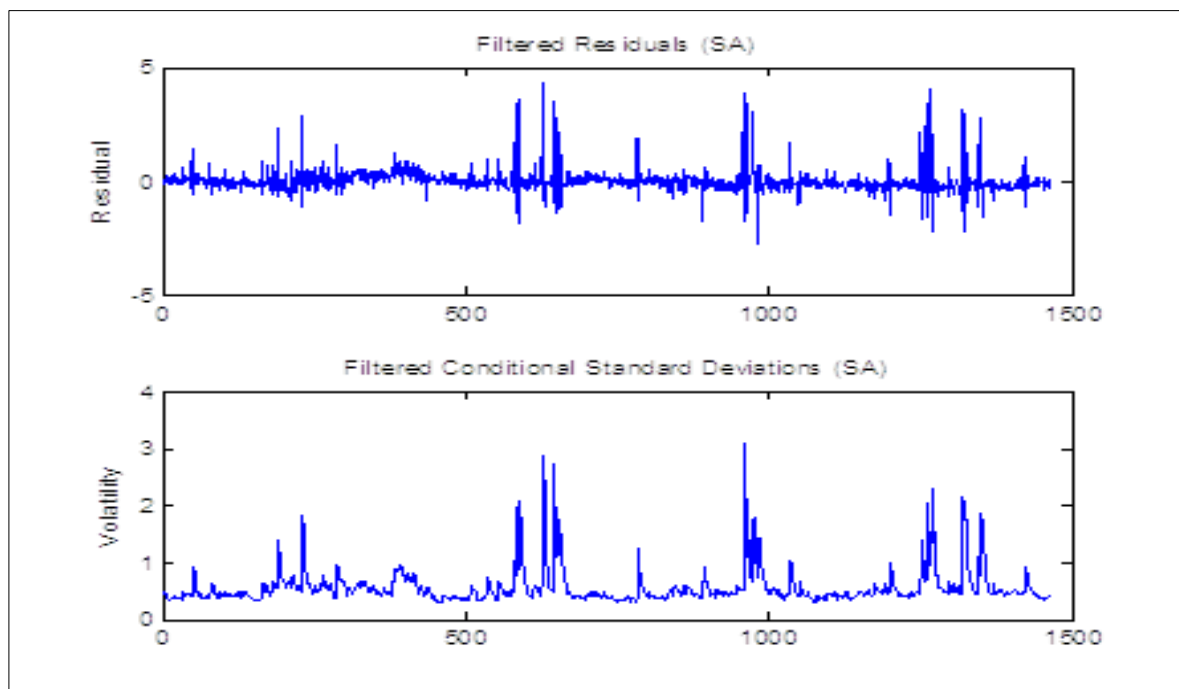
Source: Author's calculations.

Figure 91 Filtered Residuals, VIC



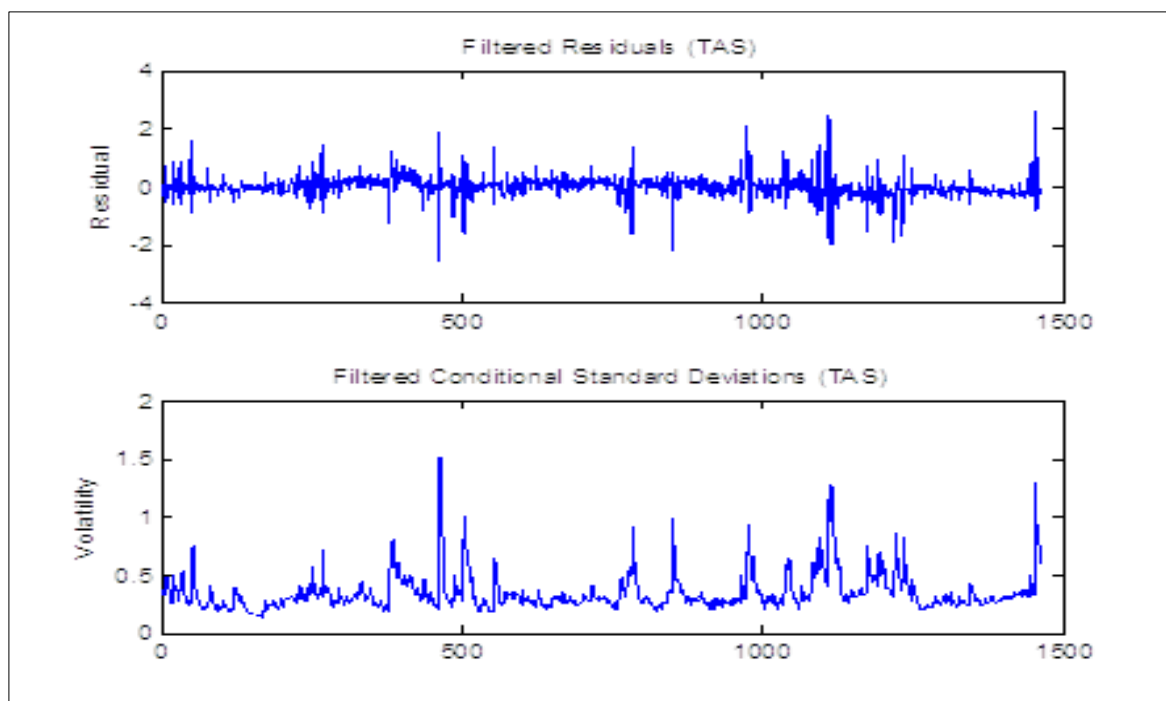
Source: Author's calculations.

Figure 92 Filtered Residuals, QLD



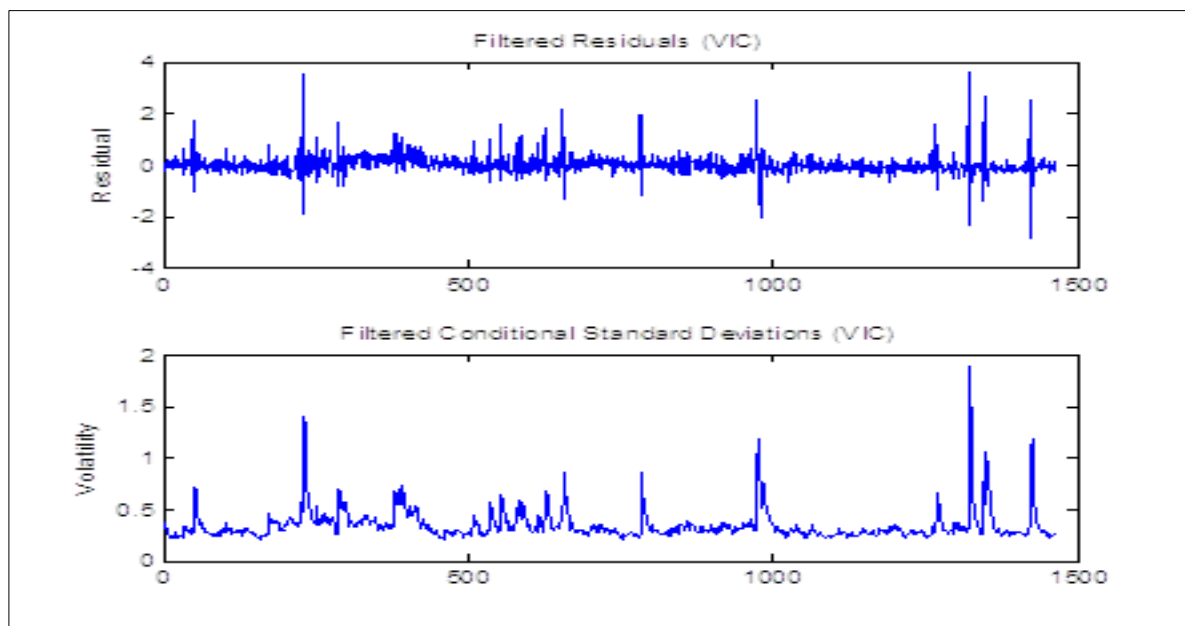
Source: Author's calculations.

Figure 93 Filtered Residuals, SA



Source: Author's calculations.

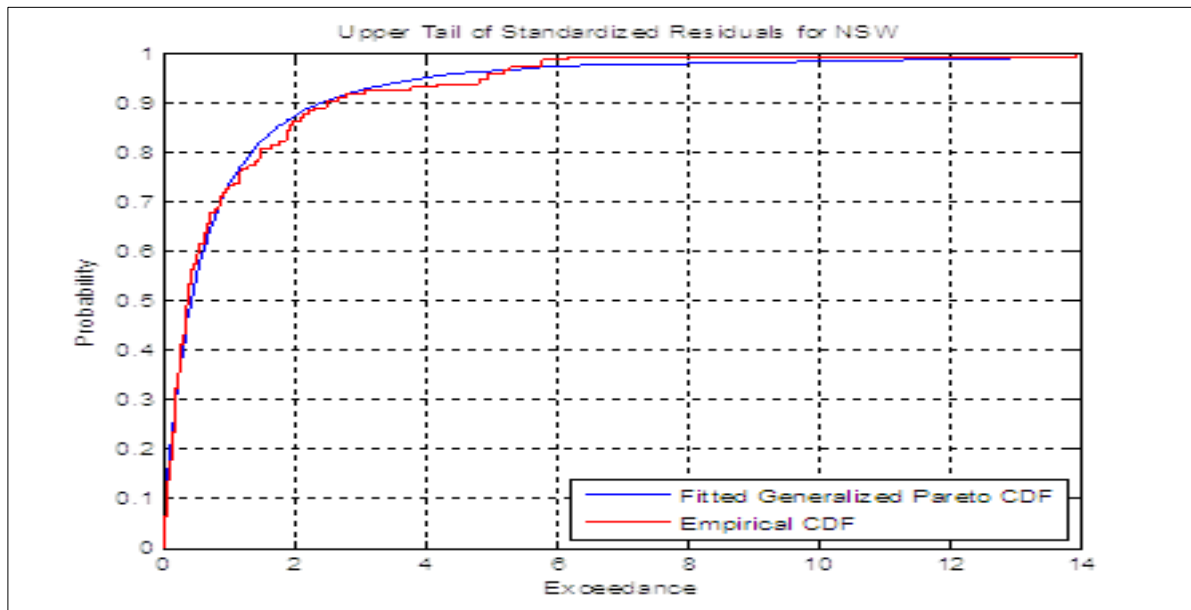
Figure 94 Filtered Residuals, TAS



Source: Author's calculations.

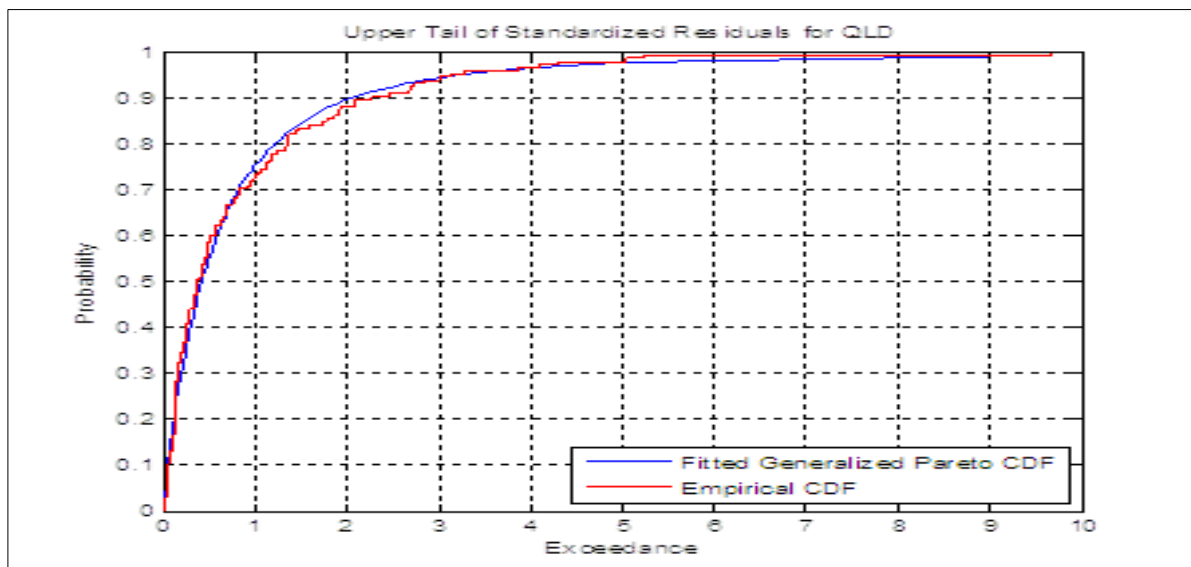
APPENDIX 9- COMPARISON OF GPD FITTED CDF WITH ECDF

Figure 95 Empirical Cumulative Distribution Function, NSW



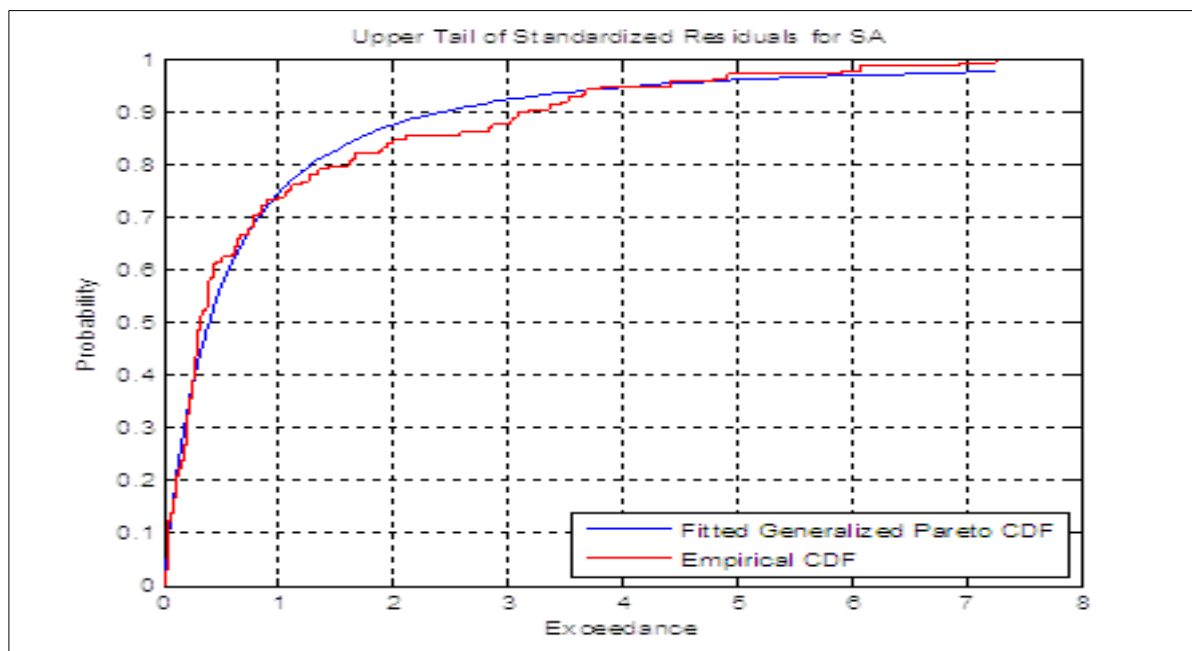
Source: Author's calculations.

Figure 96 Empirical Cumulative Distribution Function, VIC



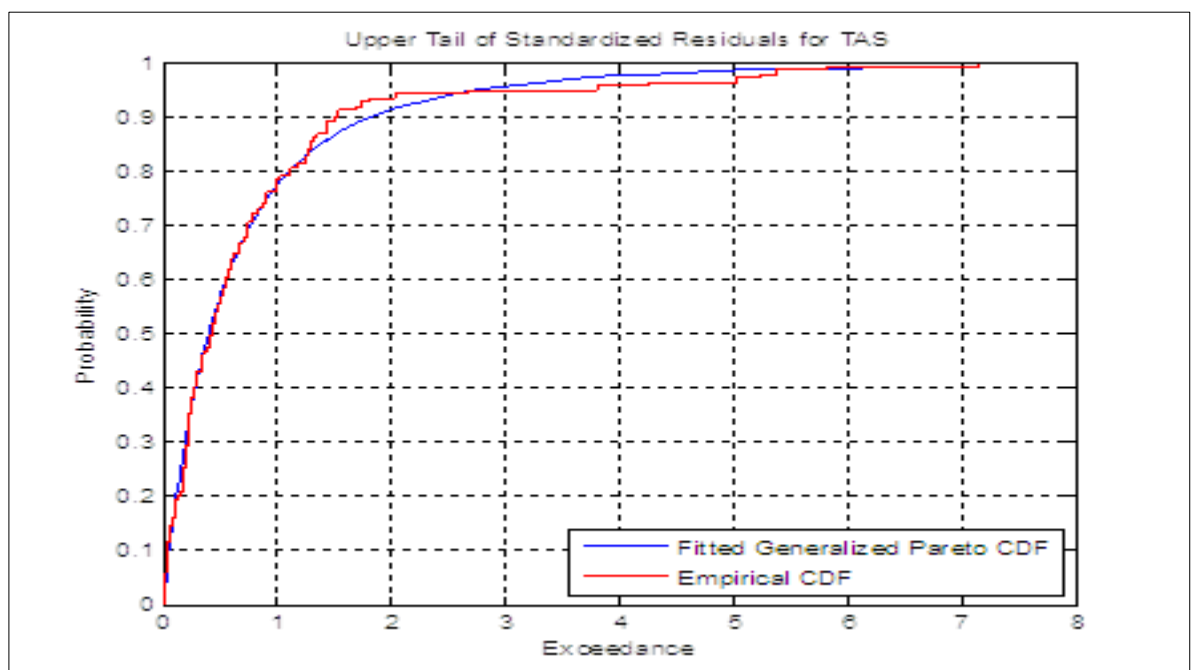
Source: Author's calculations.

Figure 97 Empirical Cumulative Distribution Function, QLD



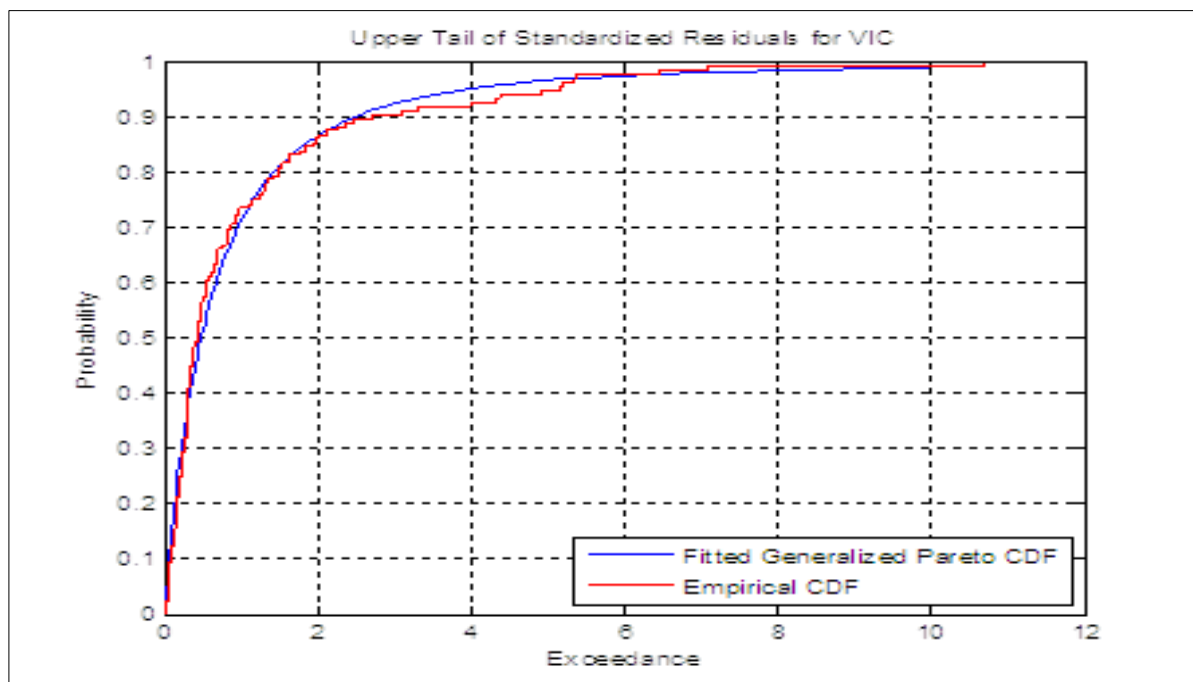
Source: Author's calculations.

Figure 98 Empirical Cumulative Distribution Function, SA



Source: Author's calculations.

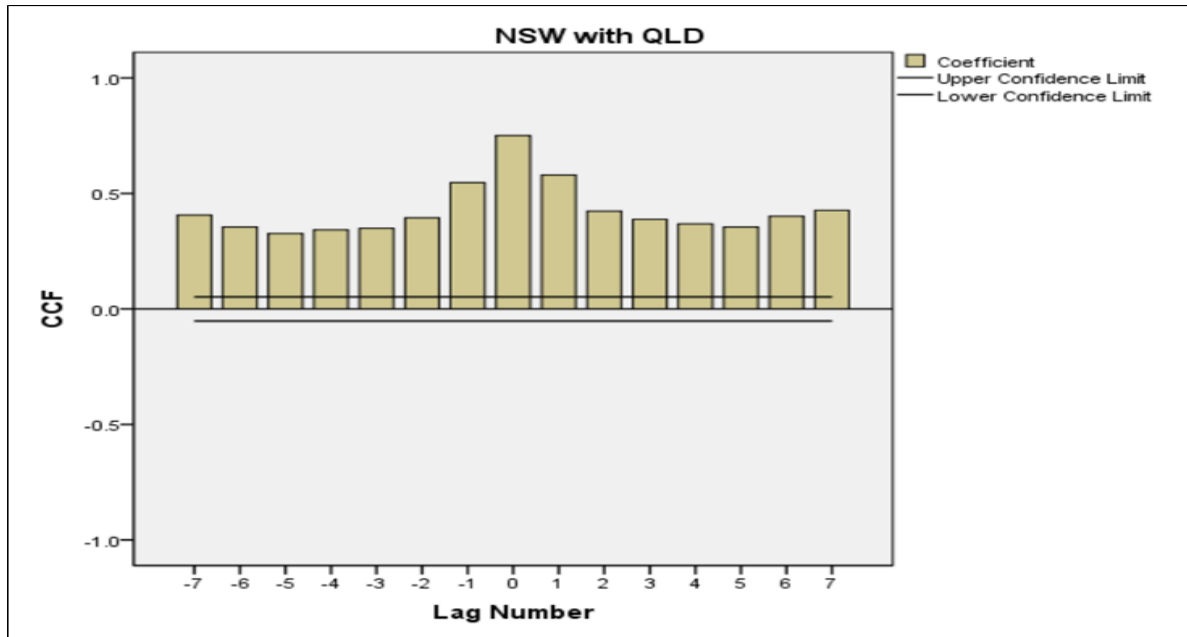
Figure 99 Empirical Cumulative Distribution Function, TAS



Source: Author's calculations.

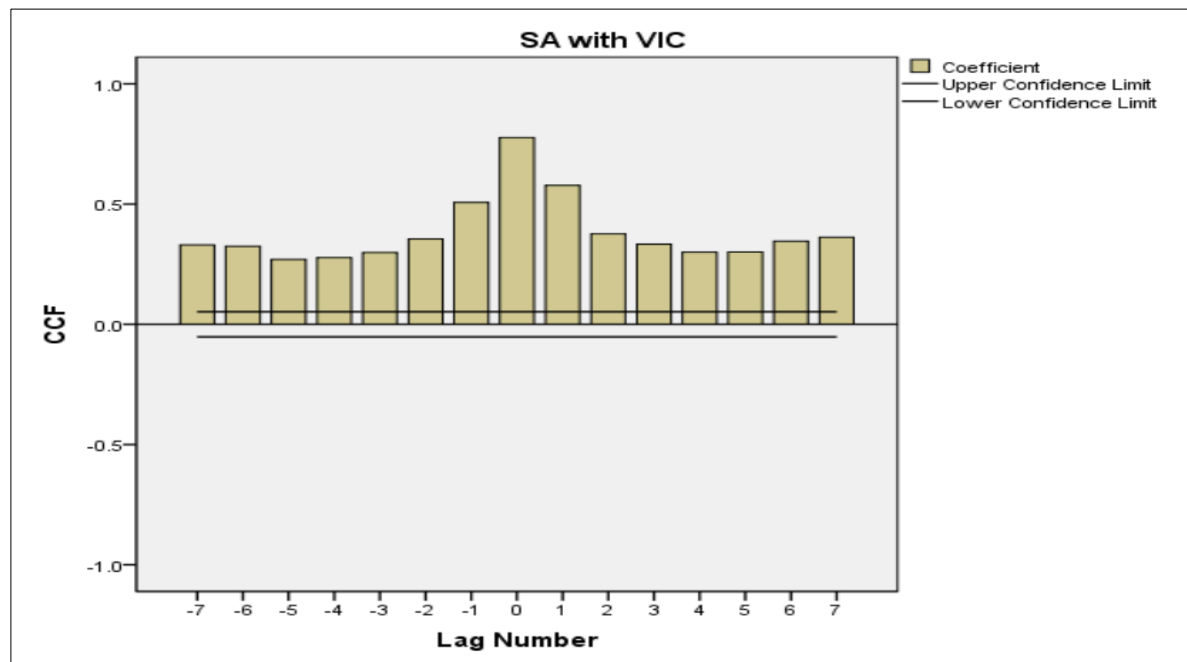
APPENDIX 10- CROSS CORRELATION FUNCTIONS

Figure 100 Cross Correlation Functions, NSW with QLD



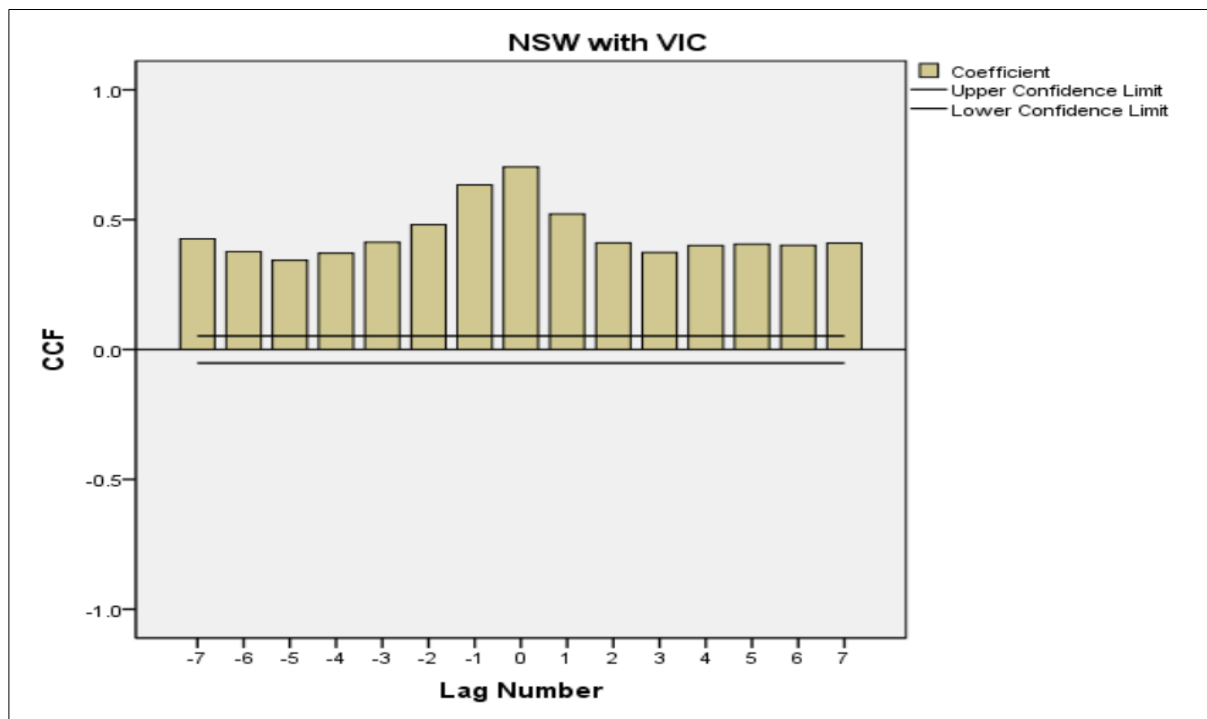
Source: Author's calculations.

Figure 101 Cross Correlation Functions, SA with VIC



Source: Author's calculations.

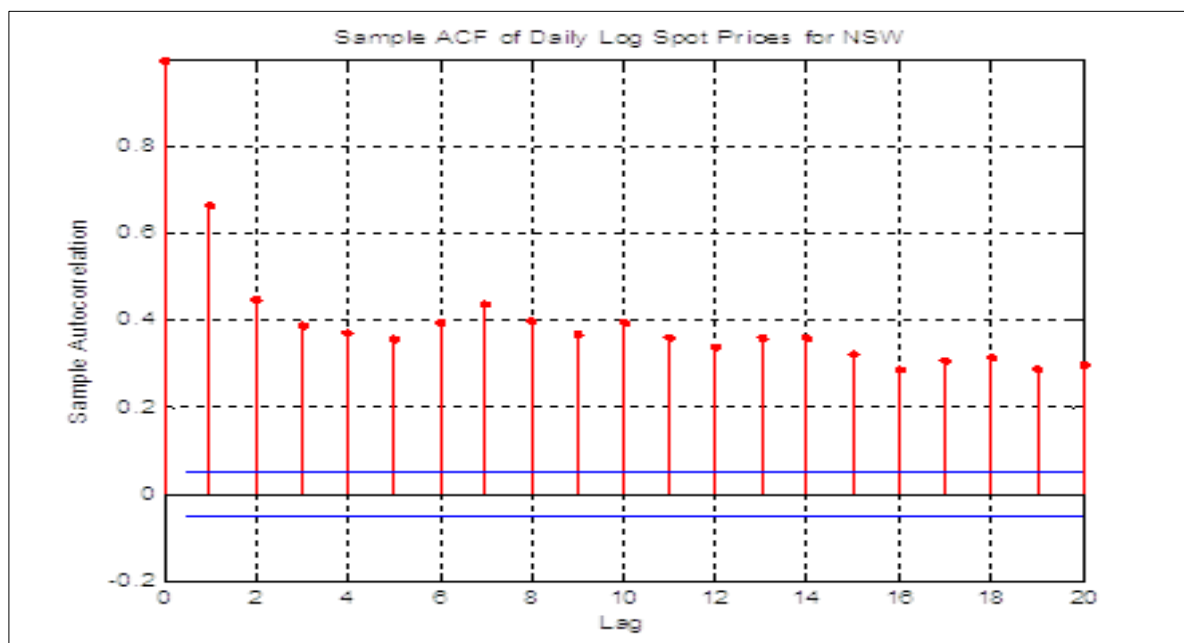
Figure 102 Cross Correlation Functions, NSW with VIC



Source: Author's calculations.

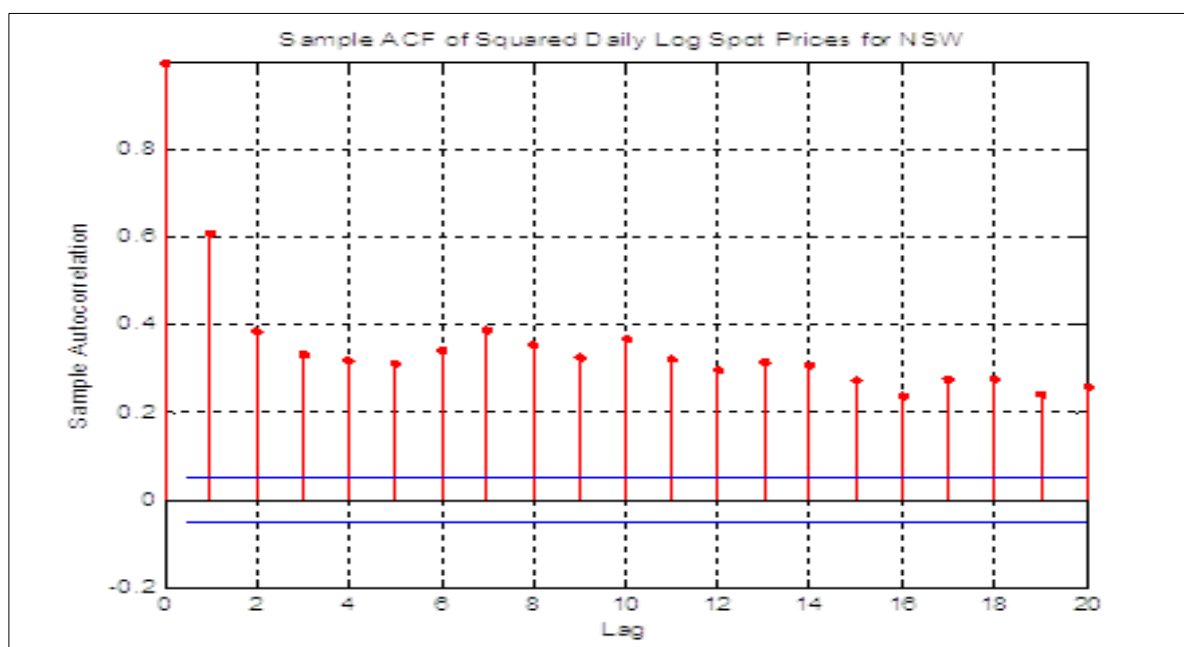
APPENDIX 11- SAMPLE ACFs, DAILY AND SQUARED PRICES

Figure 103 Autocorrelation Functions (Daily log prices), NSW



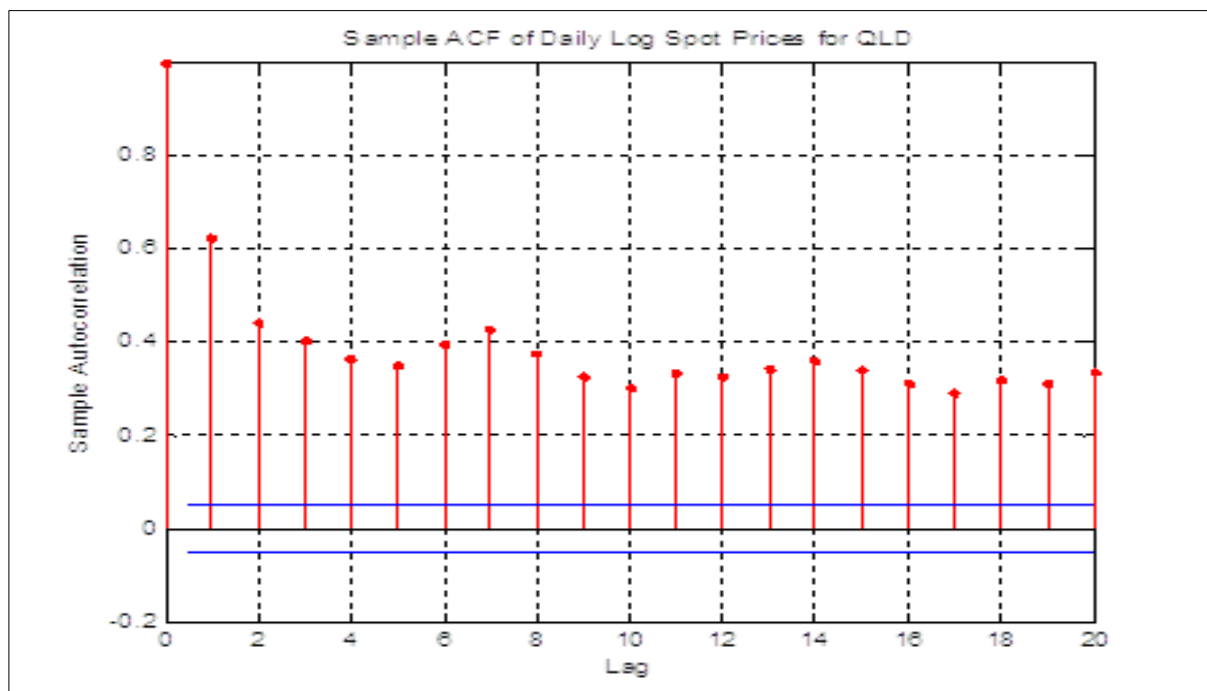
Source: Author's calculations.

Figure 104 Autocorrelation Functions (Squared daily log prices), NSW



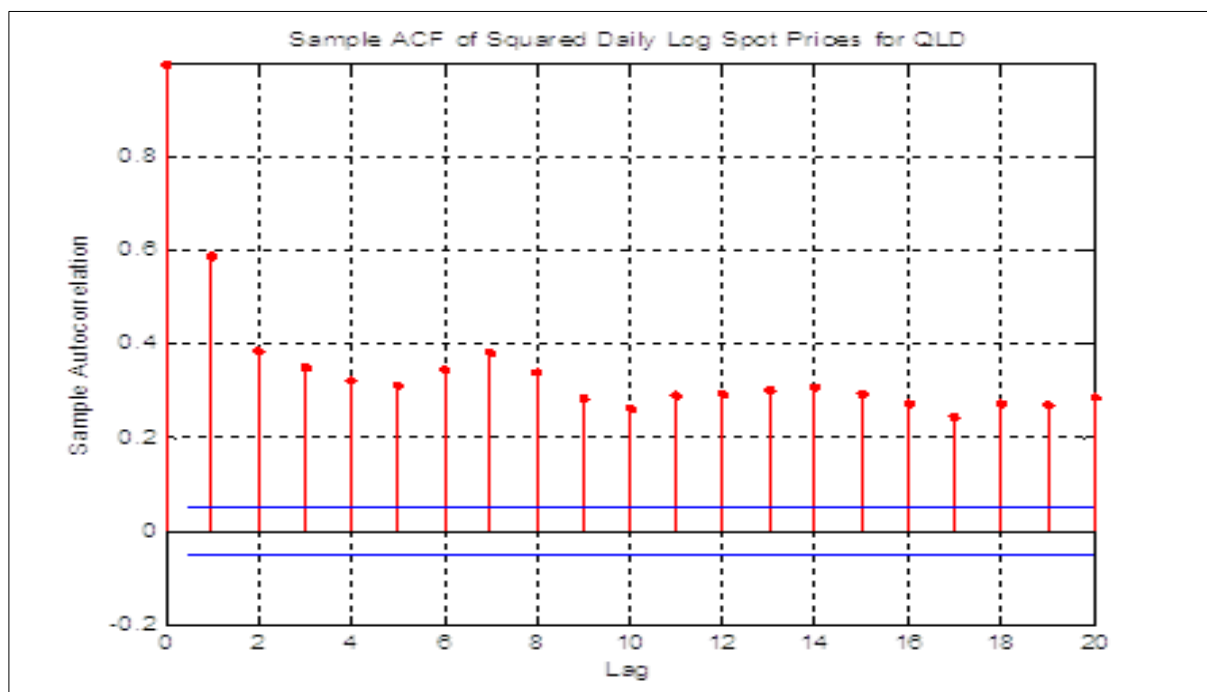
Source: Author's calculations.

Figure 105 Autocorrelation Functions (Daily log prices), QLD



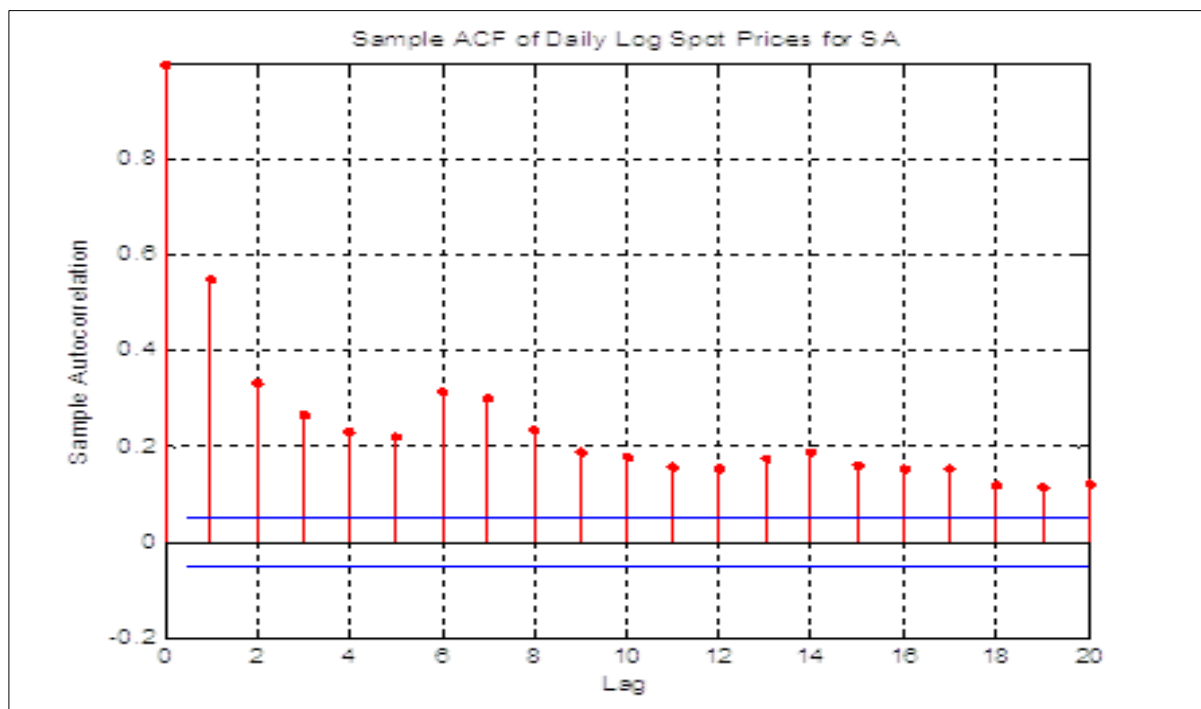
Source: Author's calculations.

Figure 106 Autocorrelation Functions (Squared daily log prices), QLD



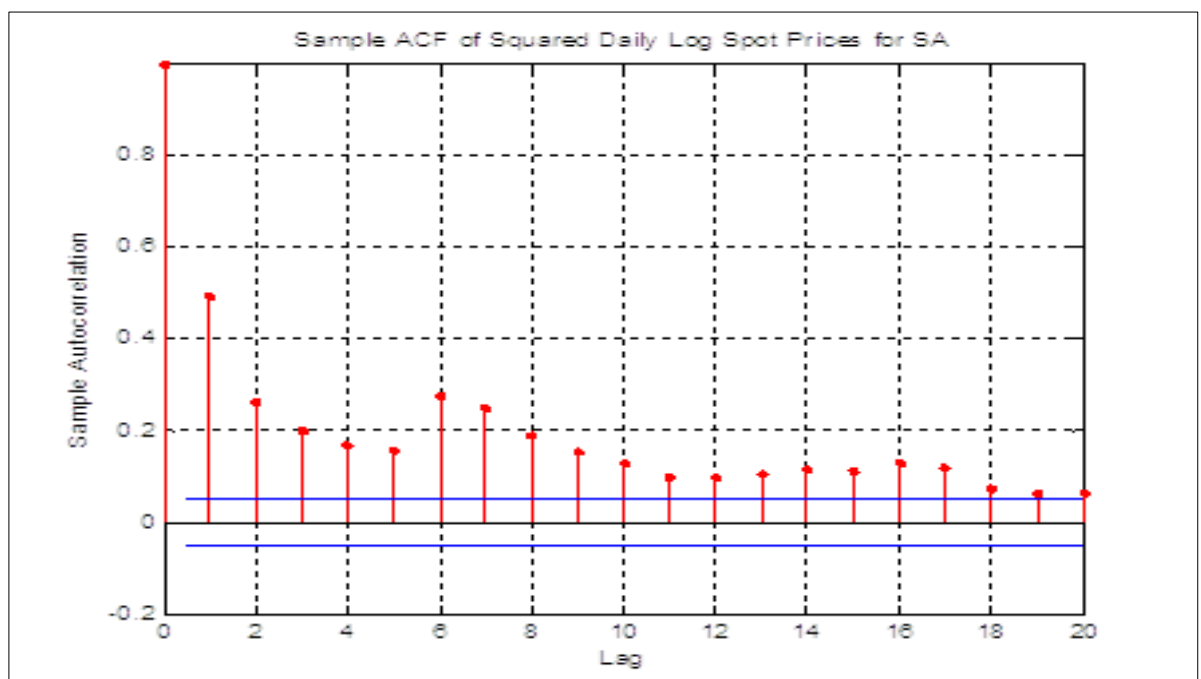
Source: Author's calculations.

Figure 107 Autocorrelation Functions (Squared daily log prices), SA



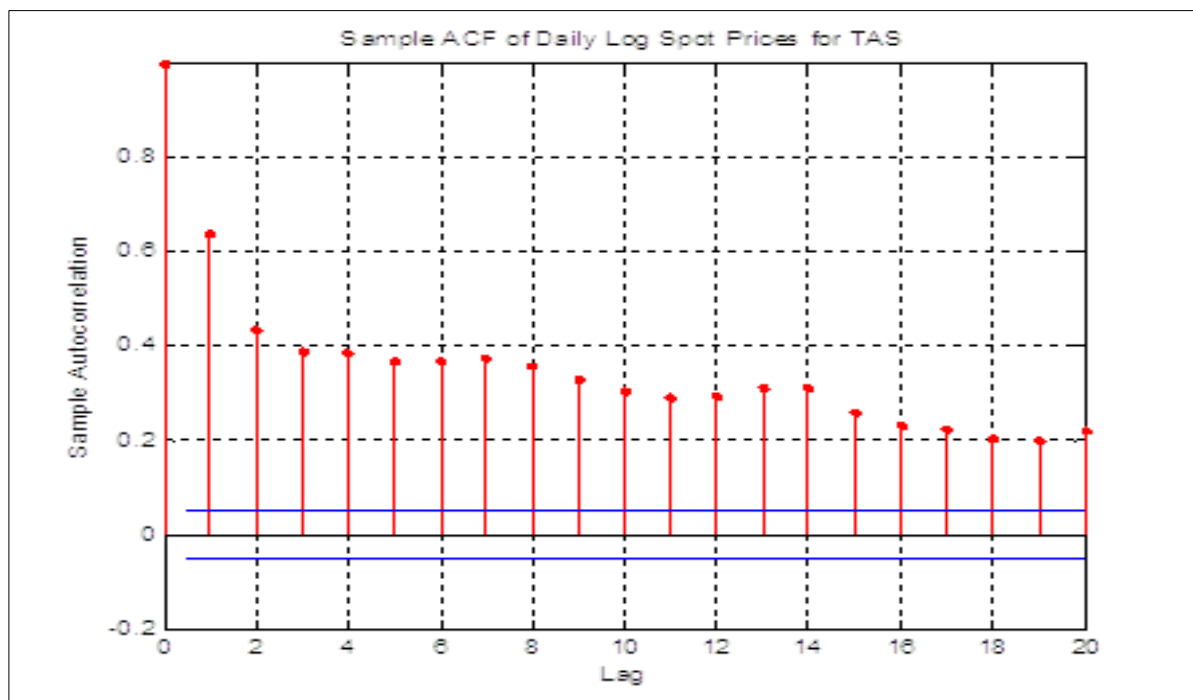
Source: Author's calculations.

Figure 108 Autocorrelation Functions (Squared daily log prices), SA



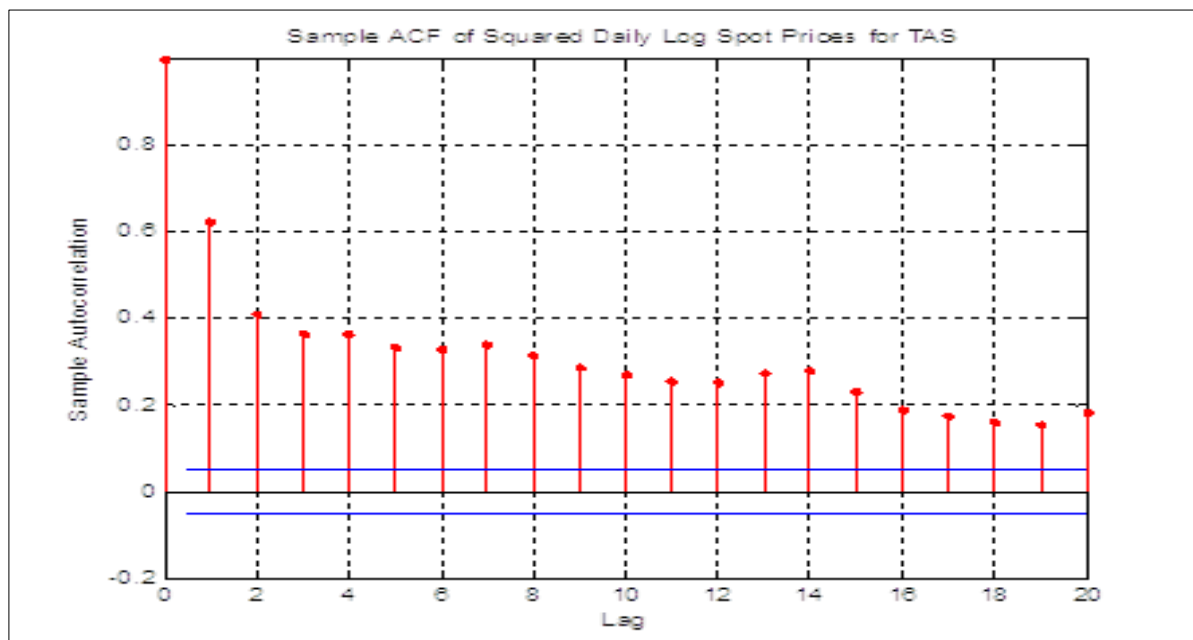
Source: Author's calculations.

Figure 109 Autocorrelation Functions (Daily log prices), TAS



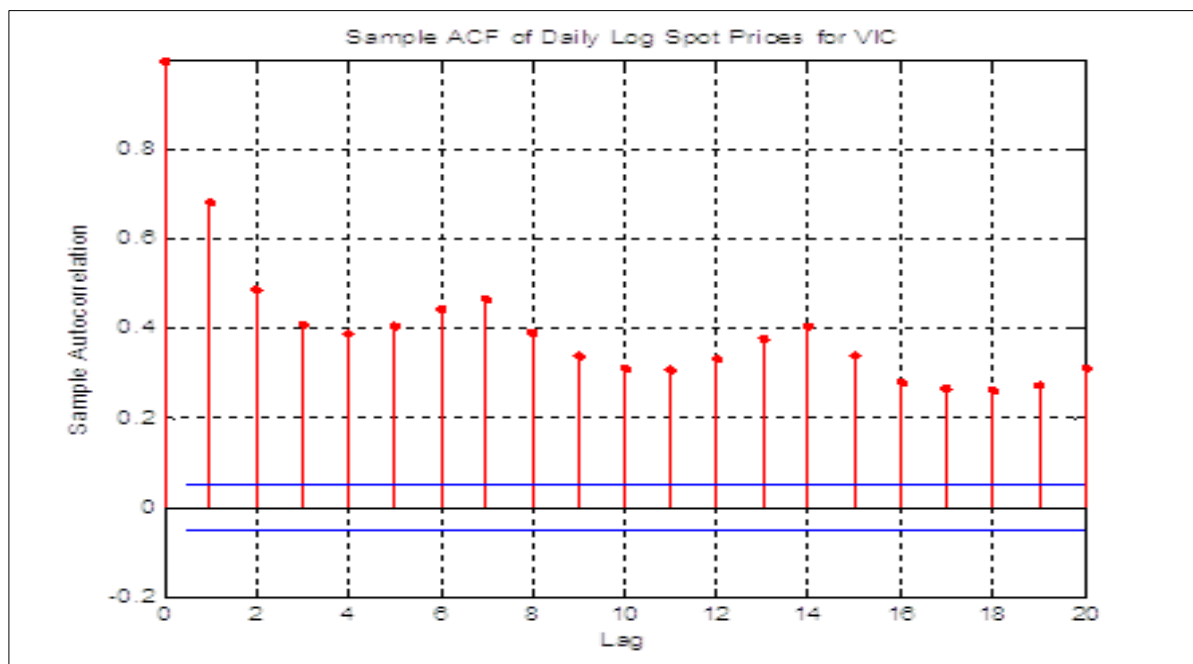
Source: Author's calculations.

Figure 110 Autocorrelation Functions (Squared daily log prices), TAS



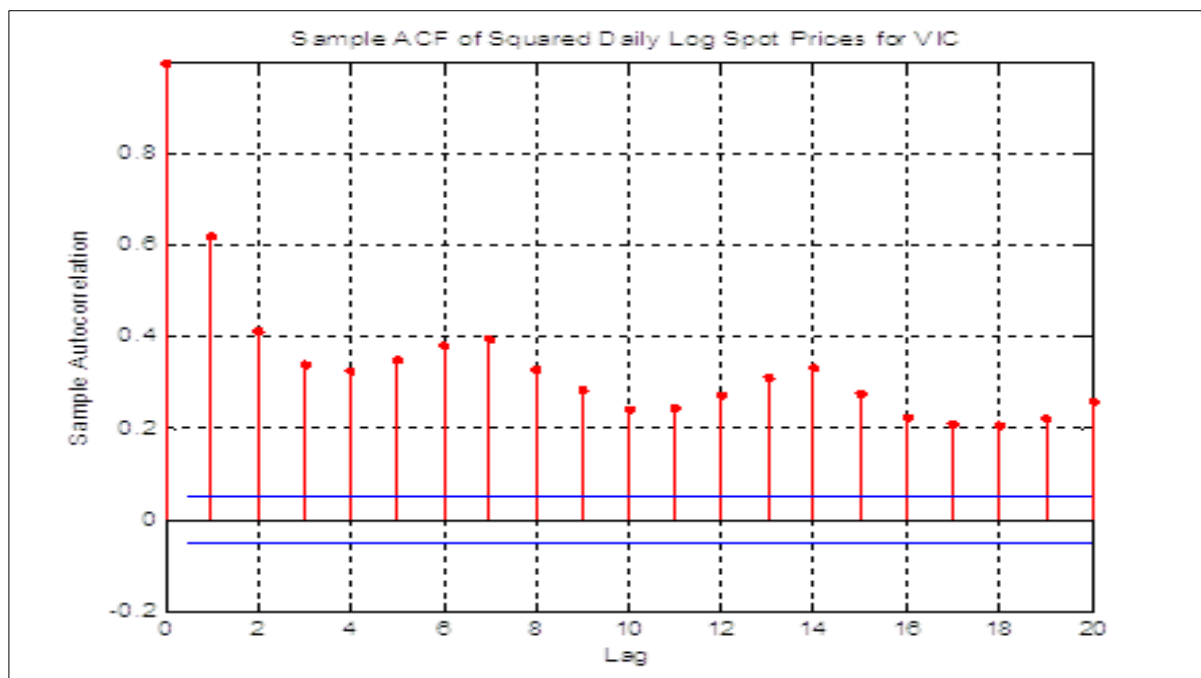
Source: Author's calculations.

Figure 111 Autocorrelation Functions (Daily log prices), VIC



Source: Author's calculations.

Figure 112 Autocorrelation Functions (Squared daily log prices), VIC



Source: Author's calculations.

REFERENCES

- Ait-Sahalia, Y. (2004). Disentangling diffusion from jumps. *Journal of Financial Economics*, 74, 487-528.
- Alexander, C. (2008). *Practical Financial Econometrics: Market Risk Analysis*, John&Wiley, England.
- Australian Energy Market Operator, (2011), *Annual Report*. AEMO, Melbourne.
- Atkins, F. J., & Chen, J. (2002). Some statistical properties of deregulated electricity prices in Alberta. Working Paper, University of Calgary, Department of Economics.
- Baillie, R. T. (1996). Long memory processes and fractional integration in econometrics. *Journal of Econometrics*, 73(1), 5-59.
- Ball, C. A., & Torous, W. N. (1983). A Simplified Jump Process for Common Stock Returns. *Journal of Financial & Quantitative Analysis*, 18(1), 53-65.
- Balkema A. and Laurens de Haan (1974) Residual life time at great age, *Annals of Probability*, 2, 792–804.
- Baltagi, B. H., & Li, D. (2004). Prediction in the Panel Data Model with Spatial Correlation. In L. Anselin, R. J. G. M. Florax & S. J. Rey (Eds.), *Advances in Spatial Econometrics: Methodology, Tools and Applications* (pp. 283-295): Advances in Spatial Science. Heidelberg and New York: Springer.
- Beder, S. (2003). *The Fight for Control of the World's Electricity*. Melbourne: Scribe.
- Beenstock, M., Goldin, E., & Nabot, D. (1999). The demand for electricity in Israel. *Energy Economics*, 21, 168-183.
- Beirlant, J., Teugels, J.L., Vynckier, P., (1996). *Practical Analysis of Extreme Values*. Leuven University Press, Leuven.

- Benth, F. E., Kallsen, J., & Meyer-Brandis, T. (2007). A non-Gaussian Ornstein-Uhlenbeck process for electricity spot price modeling and derivatives pricing. *Applied Mathematical Finance*, 14 (2), 153-169.
- Beran, J. (1994). *Statistics for Long-Memory Processes / Jan Beran*: New York : Chapman & Hall.
- Bierbrauer, M., Truck, S. and Weron, R, (2003). Modelling electricity prices with regime switching models, available at: <http://www.econwpa.wustl.edu> accessed November 2010.
- Bierbrauer, M., S. Trueck, and R. Weron, (2004), “Modelling electricity prices with regime switching models, Lecture Notes in Computer Science 3039, p. 859-867
- Bohi, D. R., & Zimmerman, M. B. (1984). An Update on Econometric Studies of Energy Demand Behavior. *Annual Review of Energy*, 9(1), 105-154.
- Borovkova, S., & Permana, F. J. (2009). Implied volatility in oil markets. *Computational Statistics & Data Analysis*, 53(6), 2022-2039.
- Bosco, B., Parisio, M., & Pelagatti, M. (2006). Deregulated wholesale electricity prices in Italy: International Advances in Economic Research.
- Bouyé, E., Durrleman, V., Nikeghbali, A., Riboulet, G., and Roncalli, T. (2000). Copulas for Finance: A reading guide and some applications. Groupe de Recherche Opérationnelle, Crédit Lyonnais, Paris, France.
- Bradley, B. and M. Taqqu (2003) Financial Risk and Heavy Tails, in Handbook of Heavy-Tailed Distributions in Finance, Elsevier-Science, Amsterdam
- Breymann, W., Dias, A., Embrechts, P., (2003). Dependence structures for multivariate high-frequency data in finance. Quantitative Finance.

- Brock, W.A., W.D. Dechert, J.A. Scheinkman and B. LeBaron (1996). "A Test for Independence Based on the Correlation Dimension", *Econometric Reviews*, 15, 197-235.
- Bowman, A.W., and Azzalini, A. (1997), *Applied Smoothing Techniques for Data Analysis*, London: Oxford University Press.
- Bollerslev, T. (1986), "Generalized autoregressive conditional heteroskedasticity", *Journal of Econometrics* 31, 327-337.
- Bunn, D. and Karakatsani, N. (2003), "Forecasting Electricity Prices", v1, Working Paper, London Business School.
- Byström, H., (2000), Taking an Extreme View at Electricity Prices at the Nordic Power Exchange, unpublished manuscript, Dept. of Economics, Lund University, Sweden
- Bystrom, H. N. E. (2005). Extreme value theory and extremely large electricity price changes. *International Review of Economics and Finance*, 14, 41-55.
- Carnero, M. A., Koopman, S. J., & Ooms, M. (2003). Periodic heteroskedastic RegARFIMA models for daily electricity spot prices. Tinbergen Institute Discussion Paper No. TI 03-071/4.
- Chan, K. F., & Gray, P. (2006). Using extreme value theory to measure value-at-risk for daily electricity spot prices. *International Journal of Forecasting*, 22(2), 283-300.
- Chester, L. (2006). The conundrums facing australia's national electricity market. *Economic Papers: A journal of applied economics and policy*, 25(4), 362-377.
- Christensen, T., Hurn, A. S., & Lindsay, K. (2009). It never rains but it pours: modeling the persistence of spikes in electricity prices. *The Energy Journal*, 30(1), 25-48.
- Clewlow, L., & Strickland, C. (2000). *Energy Derivatives-Pricing and Risk Management*. London: Lacima Publications.

- Conejo, A. J., Plazas, M. A., Espinola, R., & Molina, A. B. (2005). Day-ahead electricity price forecasting using the wavelet transform and ARIMA models. *IEEE Transactions on Power Systems*, 20(2), 1035-1042.
- Contreras, J., Espinola, R., Nogales, F.J., Conejo, A.J., (2003). ARIMA models to predict next-day electricity prices, Transactions on Power systems. *Environment and Electrical Engineering IEEE 18*(3), 1014-1020.
- Cuaresma, J. C., Hlouskova, J., Kossmeier, S., & Obersteiner, M. (2004). Forecasting electricity spot-prices using linear univariate time-series models. *Applied Energy*, 77(1), 87-106.
- Culot, M., Goffin, V., Lawford, S., De Menten, S, & Smeers, Y. (2006 Working paper). *An affine jump diffusion model for electricity: working paper*. Leuven: Catholic University of Leuven.
- de Jong, C. (2006). The Nature of Power Spikes: A Regime-Switch Approach. *Studies in Nonlinear Dynamics and Econometrics*, 10(3), 1-28.
- de Jong, C., & Huisman, R. (2002). Option formulas for mean-reverting power prices with spikes Energy Global Research Paper; and ERIM Report Series Reference No. ERS-2002-96-F&A.
- Demarta, S. and McNeil, A. J., (2005). The t copula and related copulas. *International Statistical Review*, 73, 111-129 .
- Dickey, D. A. and Fuller, W. A. (1979). Distribution of the estimates for autoregressive time series with a unit root. *Journal of the American Statistical Association*, 74, 427-431.
- Diongue, A. K., Guegan, D. and Vignal, B., (2009), Forecasting electricity spot market prices with a k-factor GIGARCH process. Université Paris1 Panthéon-Sorbonne (Post-Print and Working Papers), HAL.

- Dowd, K. (2008), "Copulas in Macroeconomics", *Journal of International and Global Economic Studies*.
- Embrechts, P., Kluppelberg, C. and Mikosch, T. (1997), *Modelling Extremal Events*, Berlin Heidelberg, Springer
- Escribano A.J., Peaea J. and P. Villaplana, P., (2002), "Modelling Electricity Prices: International Evidence". Working paper, Universidad Carlos III de Madrid.
- Ethier, R., & Mount, T. (1998). Estimating the volatility of spot prices in restructured electricity markets and the implication for option values. Power Systems Engineering Research Center (PSerc), Cornell University.
- "Electricity Generation Industry in Australia D3611." IBISWorld Market Research. April 2012. IBISWorld. RMIT Lib., Melbourne.
- "Electricity Transmission Industry in Australia D3612." IBISWorld Market Research. April 2012. IBISWorld. RMIT Lib., Melbourne.
- "Electricity Distribution Industry in Australia D3613." IBISWorld Market Research. April 2012. IBISWorld. RMIT Lib., Melbourne.
- Faruqui, A., & George, S. (2005). Quantifying Customer Response to Dynamic Pricing. *The Electricity Journal*, 18, 53-63.
- Filippini, M. (1999). Swiss Residential Demand for Electricity. *Applied Economics Letters*, 6(8), 533-538.
- Fisher, R. and Tippett, L. (1928), "Limiting forms of the frequency distribution of the largest and smallest member of a sample", *Proceedings of Cambridge Philosophy Society* 24, 180-190.
- Francq, C., and J.-M. Zakoïan (2001), "Stationarity of Multivariate Markov-Switching ARMA Models," *Journal of Econometrics*, 102, 339-364.

- Geman, H., & Roncoroni, A. (2006). Understanding the fine structure of electricity prices. *Journal of Business* 79, 1225-1261.
- Geweke J. and S. Porter-Hudak (1983), The estimation and application of long memory time series models, *Journal of Time Series Analysis*, 4, 221-238.
- Glosten, L., Jagannathan, R., & Runkle, D. E. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance*, 48(5), 1779-1801.
- Gnedenko, B. (1943), “Sur la distribution limite du terme maximum d’une série aléatoire”, *Annals of Mathematics* 44, 423-453.
- Goto, M., & Karolyi, G. A. (2004). Understanding electricity price volatility within and across markets: Dice Center Working Paper No. 2004-12. Retrieved 24/1, 2012
- Granger, C. W. J., & Joyeux, R. (1980). An Introduction To Long-Memory Time Series Models And Fractional Differencing. *Journal of Time Series Analysis*, 1(1), 15-29.
- Gray, S. F. (1996). Modeling the conditional distribution of interest rates as a Markov Regime-Switching Process. *Journal of Financial Economics*, 42(1), 27-62.
- Guegan, D. (2003). A Prospective Study of the k-Factor Gegenbauer Processes with Heteroscedastic Errors and an Application to Inflation Rates. *Finance India*, 17(1), 165-197.
- Gumbel, E. (1958), *Statistics of Extremes*, New York: Columbia University Press.
- Guthrie, G., & Videbeck, S. (2007). Electricity spot price dynamics: beyond financial models. *Energy Policy*, 35(11), 5614-5621.
- Hadsell, L., Marathe, A., & Shawky, H. A. (2004). Estimating the volatility of wholesale electricity spot prices in the US. *Energy Journal*, 25(4), 23-40.
- Haldrup, N., & Nielsen, M. O. (2006). A Regime-Switching long memory model for electricity prices. *Journal of Econometrics*, 135(1-2), 349-376.

- Hamilton, J. D. (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, 57(2), 357-384.
- Hamilton, J. D. (1990). Analysis of time series subject to changes in regime. *Journal of Econometrics*, 45(1-2), 39-70.
- Hamilton, J. D. and R. Susmel (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics*, 64, 307-333.
- Higgs, H. (2009). Modelling price and volatility inter-relationships in the Australian wholesale spot electricity markets. *Energy Economics*, 31(5), 748-756.
- Higgs, H., & Worthington, A. C. (2005). Systematic features of high frequency volatility in the Australian electricity market: Intraday patterns, information arrival and calendar effects. *The Energy Journal*, 26 (4), 1-20.
- Higgs, H & Worthington, A.C. (2010), 'Modelling spot prices in deregulated wholesale electricity markets: A selected empirical review, *Energy Studies Review*, vol. 17, no. 1, pp. 1-25.
- Higgs, H., & Worthington, A. (2008). Stochastic price modeling of high volatility, mean-reverting, spike-prone commodities: The Australian wholesale spot electricity market. *Energy Economics*, 30(6), 3172-3185.
- Huisman, R., & de Jong, C. (2003). Option pricing for power prices with spikes. *Energy Power Risk Management*, 7(11), 12-16.
- Huisman, R., Huurman, C., & Mahieu, R. (2007). Hourly electricity prices in day-ahead markets. *Energy Economics*, 29 (2), 240-248.
- Huisman, R., & Mahieu, R. (2003). Regime jumps in electricity prices. *Energy Economics* 25(1-2), 425-434.
- Imkeller, P., & Pavlyukevich, I. (2006). First exit times of SDEs driven by stable Lévy processes. *Stochastic Processes and their Applications* 116(4), 611-642.

- Janczura, J., & Weron, R. (2010). An empirical comparison of alternate Markov Regime-Switching Models for electricity spot prices. *Energy Economics*, 32(5), 1059-1073.
- Jenkinson, A. F. (1955) "The frequency distribution of the annual maximum (or minimum) values of meteorological elements". Quarterly Journal of the Royal Meteorological Society, 87, 158-171.
- Johnson, B., & Barz, G. (1999). *Selecting stochastic processes for modelling electricity prices: Energy Modelling and Management of Uncertainty*. London: Risk Books.
- Jondeau E. & Rockinger M. (2002). Entropy Densities with an Application to Autoregressive Conditional Skewness and Kurtosis. *Journal of Econometrics*, 106(1), 119-142.
- Kaminski, V. (1997). The challenge of pricing and risk managing electricity derivatives *The US power market* (pp. 149-171). London: Risk Publications.
- Kim, C.-J., & Nelson, C. R. (1999). State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications (Vol. 1): The MIT Press.
- Kim, J. H., & Powell, W. B. (2011). An hour-ahead prediction model for heavy-tailed spot prices. *Energy Economics*, 33(6), 1252-1266.
- King, K., & Shatrawka, P. (1994). American Council for an Energy Efficient Economy (ACEEE) 1994 Summer Study on Energy Efficiency in Buildings. Panel 2 Demand and Load Shapes (pp. 2194-2203).
- Klaassen, F. (2002). Improving GARCH volatility forecasts with regime-switching-garch. *Empirical Economics*, 27, 363-394.
- Knittel, C. R., & Roberts, M. R. (2001). An empirical examination of deregulated electricity prices: Working Paper PWP-087. Retrieved 19/1, 2012
- Knittel, C. R., & Roberts, M. R. (2005). An empirical examination of restructured electricity prices. *Energy Economics* 27, 791-817.

- KPMG, (2006) Energy Reform Implementation Group, Review of Energy Related Financial Markets.
- Kosater, P. and Mosler, K. (2006). 'Can Markov Regime-Switching Models improve power-price forecasts? Evidence from German daily power prices', *Applied Energy*, 83, 943-958.
- Lee, C., & Shie, F.-S. (2004). Fractional Integration and the Phillips-Perron Test. *Academia Economic Papers*, 32(2), 273-312.
- Leon, A., & Rubia, A. (2002). Forecasting time-varying covariance matrices in intradaily electricity spot prices: Working Paper Series: Serie AD, N°. 10, 2002. Retrieved 19/1, 2012.
- Lucia, J. J. & Schwartz, E. S, (2002). Electricity prices and power derivatives: Evidence from the Nordic Power Exchange. *Review of Derivatives research*, 5, 5-50.
- McNeil, A. and Frey, R. (2000), "Estimation of tail-related risk measures for heteroskedastic financial time series: an extreme value approach", *Journal of Empirical Finance*, 7, 271-300.
- Merton, R. C. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics*, 3(1-2), 125-144.
- Meyer-Brandis, T., & Tankov, P. (2008). Multi-factor Jump-Diffusion Models of Electricity Prices. *International Journal of Theoretical and Applied Finance*, 11(5), 503-528.
- Misiorek, A., Trueck, S., & Weron, R. (2006). Point and Interval Forecasting of Spot Electricity Prices: Linear vs. Non-linear Time Series Models. *Studies in Nonlinear Dynamics and Econometrics*, 10(3), 1-36.
- Ning, Y. (2001). *Modeling Spot Markets for Electricity and Pricing Electricity Derivatives*. Cornell University.

- Nystrom, K., & Skoglund, J. (2002). Univariate Extreme Value Theory, GARCH and Measures of Risk. Working Paper, Swedbank.
- Pilipovic, D. (1998). *Energy Risk: Valuing and Managing energy Derivatives*. New-York: McGraw-Hill.
- Quiggin, J. (2004). *Power Progress: An Audit of Australia's Electricity Reform Experiment B2 - Power Progress: An Audit of Australia's Electricity Reform Experiment* (pp. 109-123). Melbourne: Australian Scholarly Publishing.
- Rambharat, B. R., Brockwell, A. E., & Seppi, D. J. (2005). A threshold autoregressive model for wholesale electricity prices. *Journal of the Royal Statistical Society, Series C* 54(2), 287-300.
- Reiss R. and M. Thomas, (1997), *Statistical Analysis of Extreme Values* (Birkhäuser, Basel)
- Reiss, P. C., & White, M. W. (2005). Household Electricity Demand, Revisited. *Review of Economic Studies*, 72(3), 853-883.
- Rozario, D. R. (2002). Estimating value at risk for the electricity market using a technique from extreme value theory School of Banking and Finance, University of NSW.
- Schwartz, E. S., & Smith, J. E. (2000). Short-term variations and long-term dynamics in commodity prices. *Management Science*, 46(7), 893-911.
- Serletis, A., & Bianchi, M. (2007). Informational Efficiency and Interchange Transactions in Alberta's Electricity Market. *Energy Journal*, 28(3), 121-143.
- Shephard, N. (1996). Statistical aspects of ARCH and stochastic volatility. *Monographs On Statistics And Applied Probability*, 65, 1-68.
- Solibakke, P. (2002). Efficient estimated mean and volatility characteristics for the Nordic spot electricity power market. *International Journal of Business*, 7 (2), 17-35.

- Taylor, T. N., Schwarz, P. M., & Cochell, J. E. (2005). 24/7 Hourly Response to Electricity Real-Time Pricing with Up to Eight Summers of Experience. *Journal of Regulatory Economics*, 27(3), 235-262.
- Thomas, S. (2007), *Modelling commodity prices in the Australian National Electricity Market*, PhD Thesis, School of Economics, Finance and Marketing, RMIT University.
- Thomas, S., Ramiah, V., Mitchell, H., & Heaney, R. (2011). Seasonal factors and outlier effects in rate of return on electricity spot prices in Australia's National Electricity Market. *Applied Economics*, 43(3), 355-369.
- Tjostheim, D. (1986). Estimation in nonlinear time series models. *Stoch. Proc. Appl.*, 21, 251-273.
- Tong, H., & Lim, K. S. (1980). Threshold Autoregression, Limit Cycles and Cyclical Data. *Journal of the Royal Statistical Society. Series B (Methodological)*, 42(3), 245-292.
- Trivedi, P. K., Zimmer, D. M., (2006). Copula modeling: an introduction for practitioners, *Foundations and Trends in Econometrics*, 11: 1-110.
- Uritskaya, O. Y., & Serletis, A. (2008). Quantifying multiscale inefficiency in electricity markets. *Energy Economics*, 30(6), 3109-3117.
- von Mises, R. (1936). "La Distribution de la Plus Grande de n Valeurs." In Selected Papers of Richard von Mises, 2, 271-294. Providence, RI: American Mathematical Society.
- Weron, R., Simonsen, I., & Wilman, P. (2004). Modeling highly volatile and seasonal markets: Evidence from the Nord Pool electricity market. In H. Takayasu (Ed.), *The application of Econophysics*. Tokyo: Springer.
- Weron, R., Bierbrauer, M. and Trauck, S. (2004a). 'Modelling electricity prices: jump diffusion and regime switching', *Physica A*, 336, 39-48.
- Weron, R (2006) Modeling and Forecasting Electricity Loads and Prices: A Statistical Approach Volume 403 of The Wiley Finance Series

- Weron, R., (2009). "Forecasting wholesale electricity prices: A review of time series models," MPRA Paper 21299, University Library of Munich, Germany.
- Weron, R., & Misiorek, A. (2008). Forecasting spot electricity prices: A comparison of parametric and semiparametric time series models. *International Journal of Forecasting*, 24(4), 744-763.
- Whittle, P. (1953). "The analysis of multiple stationary time series". *J. Roy. Statist. Soc. Ser. B*. 15 (1): 125–139.
- Wild, P., Hinich, M. J., & Foster, J. (2010). Are daily and weekly load and spot price dynamics in Australia's National Electricity Market governed by episodic nonlinearity? *Energy Economics*, 32(5), 1082-1091.
- Wilkinson, L., & Winsen, J. (2002). What We Can Learn from a Statistical Analysis of Electricity Prices in New South Wales? *The Electricity Journal*, 15(3), 60-69.
- Yang, M. X. (2000), "Some Properties of Vector Autoregressive Processes with Markov-Switching Coefficients," *Econometric Theory*, 16, 23-43.