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Approximate equations for the radiation impedance of a rectangular panel

John L. Davy[a)]
David J. Larner[b)]
School of Applied Sciences, RMIT University, GPO Box 2476V Melbourne, Victoria 3001
Australia

Robin R. Wareing[c)]
John R. Pearse[d)]
University of Canterbury, Mechanical Engineering, Private Bag 4800, Christchurch 8140
New Zealand

The one sided radiation impedance of a rectangular panel mounted in a plane infinite rigid baffle needs to be known in order to calculate its sound absorption, sound insulation and sound radiation directivity. This paper gives approximate equations for the azimuthally averaged one sided radiation impedance for both travelling plane transverse vibrational waves and simply supported transverse vibrational modes. Most previous papers have only considered one type of wave on the panel and have not covered the whole range of variables which are considered in this paper. The differences and similarities of the radiation impedances of the two wave types are discussed. The reduction of one level of integration in the numerical adaptive integration calculations has allowed the approximate equations to be compared with the numerical calculations over a bigger range of variables. There is reasonable agreement between the approximate equations and the numerical calculations. When a panel is experimentally excited, usually at least two different wave types are produced in the panel. This paper gives equations for the impedance of the combination of these two different wave types. Excitations by a diffuse sound field incident on one side, by transverse point forces and by transverse line forces are considered.

1. INTRODUCTION

The acoustical radiation impedance of one side of a finite rectangular panel mounted in an infinite rigid baffle is of importance for the prediction of sound insulation[1-5], sound absorption[1,6-8] and sound directivity[9]. It occurs naturally when variational techniques are used to solve these phenomena[1,2,7,8]. The normalized real part of the acoustical radiation

a) email: john.davy@rmit.edu.au
b) email: david.larner@rmit.edu.au
c) email: robin.wareing@pg.canterbury.ac.nz
d) email: john.pearse@canterbury.ac.nz

impedance of one side of a finite rectangular panel mounted in an infinite rigid baffle is also the panel's one sided acoustic radiation efficiency.

This paper gives approximate formulae for the radiation impedance of a rectangular panel for two types of wave. One wave type is a travelling plane transverse vibrational wave where the boundary conditions are assumed to be anechoic. This is the appropriate choice for waves for which are forced by incident plane acoustical waves. The other wave type is a simply supported transverse vibrational mode of the panel. This type is the appropriate choice for the resonant part of the vibrational field.

This paper also examines the difference in radiation impedance between the wave different types of waves. At first sight, it is surprising that there are differences in some cases between the radiation impedances of travelling plane waves and simply supported modes on a rectangular isotropic panel, because the simply supported modes can be expressed as a sum of travelling waves. The reason for the differences are that one wave on the panel can alter the impedance experienced by another wave. This also applies to the real part of the normalized radiation impedance of different modes on a panel, but Xie *et al.*[10] have shown that these modal interactions cancel out when the position of the transverse excitation point is averaged over the surface of the panel. The authors suspect that a similar cancellation of the interactions between different travelling waves or simply supported modes occurs when azimuthal averaging or incident diffuse field averaging is used. This is because the results of such averaged results have proved useful in making acoustical predictions. Such cancellation does not always occur when the travelling plane waves are summed to form a mode because the relative phase of the travelling plane waves is fixed by the boundary conditions of the panel. Hence these differences in impedance survive the azimuthal averaging.

When a panel is actually excited, there are usually at least two types of transverse vibrational fields excited in the panel. One is a freely propagating resonant field and the other is a forced non-resonant field or a near field. Equations are given for calculating the normalized radiation impedance of a panel in an infinite baffle which is excited by an incident diffuse sound field, by transverse point forces or by transverse line forces.

2. APPROXIMATE FORMULAE FOR RADIATION IMPEDANCE

In this paper, the sinusoidal variation with time is assumed to be proportional to exp($j\omega t$), where ω is the angular frequency, t is the time, j is the square root of -1. It should be noted that the assumption of exp($-j\omega t$) for the sinusoidal variation with time gives the opposite sign for the imaginary part of the impedance. The impedances in this paper are normalized by dividing by the characteristic impedance of the fluid medium Z_c, which is the product of the ambient density ρ_0 and the speed of sound c in the fluid medium.

The subscript t denotes the travelling wave case and the subscript s denotes the simply supported mode case. The subscripts r and i denote the real and imaginary parts of the impedance.

Calculate

$$\mu = \frac{k_b}{k} = \sin\left(\theta\right), \tag{1}$$

where the second equality only applies if $k_b \le k$. μ is the ratio of k_b and k, which are respectively the transverse wave number in the rectangular panel and the sound wave number in the surrounding compressible fluid medium into which the panel is radiating sound. θ is the angle of incidence of an incoming three dimensional plane wave in the surrounding fluid medium which could generate the transverse wave in the rectangular panel.

Calculate

$$ke = 2kab/(a+b), \tag{2}$$

where $2a$ and $2b$ are the lengths of the sides of the rectangle and $2e$ is the length of the sides of a equivalent square.

If $\mu \le 1$, calculate

$$g = \sqrt{1-\mu^2} = \cos(\theta), \tag{3}$$

and

$$p = \sqrt{\pi/(2ke)}. \tag{4}$$

Set $w_{tr} = 1.29$, $w_{ti} = 0.9$, $w_{sr} = 1.3$, and $w_{si} = 1.77$ and calculate g_{xy} where x equals t for the travelling wave case or s for the simply supported mode case and y equals r or i.

$$g_{xy} = \min(w_{xy}p, 1) \text{ where } x \text{ equals } t \text{ or } s \text{ and } y \text{ equals } r \text{ or } i. \tag{5}$$

Set $\alpha_t = 2$ and $\alpha_s = 8$. Calculate

$$z_{xlr} = \alpha_x k^2 ab/\pi. \tag{6}$$

If $g_{xr} \le g \le 1$, where x equals t or s, calculate

$$z_{xhr} = 1/g. \tag{7}$$

Set $\beta_{tr} = -0.07$, $\beta_{ti} = 0.07$, $\beta_{sr} = 0.4$ and $\beta_{si} = -0.15$. If $g = 0$ calculate

$$z_{xhy} = z_{x0hy} = \max\{0.9616[2/(3p)] + \beta_{xy}, 0.001\}. \tag{8}$$

If $0 < g < g_{xr}$ calculate z_{x0hr} using Eqn. (8) and z_{xmhr} using Eqn. (7) with $g = g_{xr}$ for x equals both t and s. Interpolate in the inverse impedance domain as a function of g.

$$z_{xhr} = g_{xr}/[(g_{xr} - g)/z_{x0hr} + g/z_{xmhr}]. \tag{9}$$

Set $n_t = 2$ and $n_s = 3$. Calculate

$$z_{xr} = 1/\sqrt[n_x]{1/z_{xlr}^{n_x} + 1/z_{xhr}^{n_x}}, \tag{10}$$

where z_{xlr} is calculated using Eqn. (6) and z_{xhr} is calculated using Eqns. (7), (8), or (9).

Calculate

$$z_{tli} = 2k[bH(a/b) + aH(b/a)]/\pi, \tag{11}$$

where

$$H(q) = \ln\left(\sqrt{1+q^2} + q\right) - \left(\sqrt{1+q^2} - 1\right)/(3q). \tag{12}$$

Calculate

$$z_{sli} = 2k\left\{2a\ln\left[\sqrt{1+(b/a)^2} + b/a\right] - 2b\ln\left[\sqrt{1+(a/b)^2} - a/b\right]\right\}/\pi. \tag{13}$$

If $g_{xi} \le g \le 1$, calculate

$$z_{thi} = 2/(\pi keg^3), \tag{14}$$

$$z_{shi1} = 0.8/(ke), \tag{15}$$

$$z_{shi2} = \max[2\mu^3/(\pi keg^3), 0.000001], \tag{16}$$

and

$$z_{shi} = \begin{cases} z_{shi1} & \text{if } ke < 1.2 \\ z_{shi2} & \text{if } ke \geq 1.2 \end{cases}. \tag{17}$$

Set $n_t = 4$ and $n_s = 2$. Calculate

$$z_{xi} = 1 \Big/ \sqrt[n_x]{1/z_{xli}^{n_x} + 1/z_{xhi}^{n_x}} . \tag{18}$$

using Eqns. (8), (14) and (17).

If $g = 0$, calculate

$$z_{ti} = z_{t0i} = \min\left(z_{tli}, z_{t0hi}\right), \tag{19}$$

$$z_{si1} = 1 \Big/ \sqrt{1/z_{sli}^2 + 1/z_{shi1}^2} , \tag{20}$$

and

$$z_{si} = z_{s0i} = \max\left(z_{si1}, z_{s0hi}\right), \tag{21}$$

using Eqns. (8), (11) and (13).

If $0 < g < g_{xi}$, calculate z_{x0i} using Eqns. (19) and (21), and z_{xmi} using Eqn. (18) with $g = g_{xi}$. Interpolate in the impedance domain as a function of g.

$$z_{xi} = \left[\left(g_{xi} - g\right) z_{x0i} + g z_{xmi}\right] \Big/ g_{xi} . \tag{22}$$

If $\mu \leq 1$, calculate

$$z_x = z_{xr} + j z_{xi} , \tag{23}$$

where z_{xr} is given by Eqn. (10) and z_{xi} is given by Eqns. (18), (19), (21), or (22).

Else if $\mu > 1$, set $h_{tr} = 1.7$, $h_{ti} = 1.3$ $h_{sr} = 1.8$ and $h_{si} = 1.5$ and calculate μ_{xy} where *x* equals *t* or *s* and *y* equals *r* and *i*.

$$\mu_{xy} = \sqrt{1 + \pi h_{xy}^2 / \left(2ke\right)} \text{ where } x \text{ equals } t \text{ or } s \text{ and } y \text{ equals } r \text{ or } i . \tag{24}$$

If $\mu \geq \mu_{xr}$, calculate

$$z_{tr} = 2 \Big/ \left[\pi ke \left(\mu^2 - 1\right)^{3/2}\right], \tag{25}$$

$$z_{sr} = z_{tr} + \ln\left[\left(\mu + 1\right)/\left(\mu - 1\right)\right] \Big/ \left(\pi ke \mu \sqrt{\mu^2 - 1}\right). \tag{26}$$

If $1 < \mu < \mu_{xr}$ calculate the real part z_{xmr} using Eqn. (25) or (26) with $\mu = \mu_{xr}$. Calculate the real part z_{x1r} as described for the $\mu \leq 1$ case with $\mu = 1$ which implies $g = 0$.

Interpolate in the impedance domain as a function of μ.

$$z_{xr} = \left[\left(\mu_{xr} - \mu\right) z_{x1r} + \left(\mu - 1\right) z_{xmr}\right] \Big/ \left(\mu_{xr} - 1\right). \tag{27}$$

The calculation of the imaginary part for the travelling wave case when $\mu > 1$ depends on the value of ke. If $ke \leq \sqrt{2}$, calculate the imaginary part z_{t1i} as described for the $\mu \leq 1$ case with $\mu = 1$. Calculate

$$z_{tmi} = 1 \Big/ \sqrt{\mu^2 - 1} . \tag{28}$$

Calculate

$$z_{ti} = 1 \Big/ \sqrt[4]{1/z_{t1i}^4 + 1/z_{tmi}^4} . \tag{29}$$

Else if $ke > \sqrt{2}$ proceed as follows

If $\mu \geq \mu_{ti}$, calculate

$$z_{ti} = 1\Big/\sqrt{\mu^2 - 1}\,. \tag{30}$$

If $1 < \mu < \mu_{ti}$ calculate the imaginary part z_{tmi} using Eq. (30) with $\mu = \mu_{ti}$.

Calculate the imaginary part z_{t1i} as described for the $\mu \leq 1$ case with $\mu = 1$ which implies $g = 0$.

Interpolate in the impedance domain as a function of μ.

$$z_{ti} = \left[\left(\mu_{ti} - \mu\right) z_{t1i} + \left(\mu - 1\right) z_{tmi}\right]\Big/\left(\mu_{ti} - 1\right). \tag{31}$$

For the simply supported mode case, the imaginary part when $\mu > 1$ is calculated as follows.

If $\mu \geq \mu_{si}$, calculate

$$z_{si} = \left[1 + 1.3/\left(\mu ke\right)\right]\Big/\sqrt{\mu^2 - 1}\,. \tag{32}$$

If $1 < \mu < \mu_{si}$ calculate the imaginary part z_{smi} using Eqn. (32) with $\mu = \mu_{si}$.

Calculate the imaginary part z_{s1i} as described for the $\mu \leq 1$ case with $\mu = 1$ which implies $g = 0$.

Interpolate in the impedance domain as a function of μ.

$$z_{si} = \left[\left(\mu_{si} - \mu\right) z_{s1i} + \left(\mu - 1\right) z_{smi}\right]\Big/\left(\mu_{si} - 1\right). \tag{33}$$

If $\mu > 1$, calculate

$$z_x = z_{xr} + jz_{xi}\,. \tag{34}$$

where z_{xr} is given by Eqns. (25) or Eqns. (27) and z_{xi} is given by Eqns. (29), (30), (31), (32) or(33).

The azimuthally averaged one sided radiation impedance of a finite rectangular panel is given by Eqns. (23) or (34).

It is also possible to give approximate formulae for the real part of the normalized incident diffuse sound field forced radiation impedance in the travelling wave case[11]. Using Eqn. (5) define

$$f = g_{tr}\,. \tag{35}$$

Using Eqn. (6) define

$$q = 1\big/z_{t1r}\,. \tag{36}$$

Using Eqn. (8) define

$$h = 1\big/z_{t0hr}\,. \tag{37}$$

Define

$$\gamma = h\big/f - 1\,. \tag{38}$$

Then the real part of the normalized incident diffuse sound field forced radiation impedance in the travelling wave case is

$$\left\langle z_{tr}\right\rangle = \ln\left[\left(1+\sqrt{1+q^2}\right)\Big/\left(f+\sqrt{f^2+q^2}\right)\right] + \ln\left[\left(h+\sqrt{h^2+q^2}\right)\Big/\left(f+\sqrt{f^2+q^2}\right)\right]\Big/\gamma\,. \tag{39}$$

Note that this is also the incident diffuse sound field forced radiation efficiency of the panel.

3. SIMPLY SUPPORTED MODE

If the isotropic rectangular panel is simply supported in the infinite rigid baffle with its sides parallel to the x and y axes, each of its simply supported transverse velocity modes has m and n positive integer half wavelengths in the directions of the x and y axes respectively. Each mode is freely vibrating at its natural frequency or being forced to vibrate at a frequency which corresponds to a wave number of k in the fluid medium on one side of the panel into which the panel is radiating sound. In this case the variables α and β can only take the following discrete values.

$$\alpha = m\pi/(2ka). \tag{40}$$

$$\beta = n\pi/(2kb). \tag{41}$$

However for the purposes of calculating the azimuthally averaged impedance, it is convenient to regard them as continuous variables. Because the variables α and β are positive, azimuthal averaging only needs to be conducted over the range from 0 to $\pi/2$.

The minimum value of μ in this case is

$$\min(\mu) = \pi\sqrt{1/a^2 + 1/b^2}/(2k). \tag{42}$$

For a square, where a=b, the minimum is

$$\min(\mu) = \pi/(\sqrt{2}ka). \tag{43}$$

Thus, it does not really make sense to calculate the impedance for the simply supported standing wave case when μ is less than the minimum value given by Eqn. (42) or (43).

If the transverse velocity of the panel is due to freely propagating bending waves, then

$$\mu = k_b/k = \sqrt{\omega_c/\omega}, \tag{44}$$

where ω_c is the angular critical frequency of the panel. The angular critical frequency is the angular frequency at which the wave number of the freely propagating bending waves in the panel equals the wave number of the sound in the surrounding compressible medium into which the panel is radiating sound. If the value of μ calculated using Eqn. (44) is less than the minimum value of μ calculated using Eqn. (42) or (43), then the radiation impedance for the simply supported mode case should be calculated using the minimum value of μ calculated using Eqn. (42) or (43) rather than the value of μ calculated using Eqn. (44).

4. THE EFFECT OF DIFFERENT WAVE TYPES AND BOUNDARY CONDITIONS

For the travelling plane wave case, the normalized radiation impedance of an infinite panel is[12]

$$z = \begin{cases} 1/\sqrt{1-\mu^2} & \text{if } 0 \le \mu < 1 \\ \infty & \text{if } \mu = 1 \\ j/\sqrt{\mu^2 - 1} & \text{if } 1 < \mu \end{cases}. \tag{45}$$

Thus for large values of ke, the real part of the impedance when $0 \le \mu < 1$ and the imaginary part of the impedance when $\mu > 1$ are expected to be close to the values given by Eqn. (45) and thus easier to approximate. The travelling plane wave case and the simply supported mode case are expected to have similar values of the real or the imaginary parts of their impedance in these two cases. Examination of Eqns. (3), (7), (28), (30) and (32) shows that this is indeed the case. On the other hand, for large values of ke, the imaginary part of the impedance when $0 \le \mu < 1$ and the real part of the impedance when $\mu > 1$ are expected to be close to the zero values given by Eqn. (45) and thus harder to approximate. The travelling plane wave case and the simply supported mode case are expected to have different values of the imaginary or the real parts of their impedance in these two cases.

Examination of Eqn. (6) shows that for small values of ke and values of μ that are not too large, the simply supported mode case has a real part of its impedance that is four times or six decibels greater than the real part of the impedance for the travelling wave case. Comparison of Eqn. (25) and the term that is added to it in Eqn. (26), show that they become equal for large values of μ. This means that the real part of the impedance for the simply supported mode case tends to twice or 3 decibels greater than the real part of the impedance for the travelling wave case for large values of ke and μ. Comparison of Eqns. (11), (12) and (13) shows that for small values of ke and values of μ that are not too large, the simply supported mode case has an imaginary part of its impedance that is 2.37 times or 3.75 dB greater than the imaginary part of the impedance for the travelling wave case.

The real part of the radiation impedance of a clamped panel is twice as great as that of a simply supported panel for $\mu > 1$ and ke large[13,14]. More generally if the boundary conditions of the panel are zero displacement and a rotational stiffness, then zero rotational stiffness corresponds to the simply supported case and infinite rotational stiffness corresponds to clamped case. As the rotational stiffness varies from zero to infinity, the real part of the radiation impedance varies between the simply supported case and the clamped case and always lies in between these two cases.

Squicciarini *et al.*[15] have shown that real part of the normalized radiation impedance (radiation efficiency) of a point excited panel mounted in an infinite baffle depends on the edge conditions. In particular they have shown that when $\mu > 1$ a freely supported panel has lower radiation efficiency than a simply supported panel and that a clamped panel has higher radiation efficiency than a simply supported panel except at very low values of $k_b e = \mu ke$ where the radiation is dominated by the fundamental drum mode.

5. ACCURACY OF THE APPROXIMATE FORMULAE

In 2009, the first author[11] compared the approximate formulae for the real part of the azimuthally averaged normalized impedance (this is also the radiation efficiency) of a square panel mounted in an infinite rigid baffle for the travelling wave case when $0 \le \mu \le 1$ with numerical calculations made by Sato[16]. The comparisons were made for 15 values of ke from 0.5 to 64 and for seven values of $\mu = \sin(\theta)$ from θ equals 0° to 90° in 15° increments. The differences are shown in dB in table 1 of Davy[11]. Although the constants in the approximate equations for this case have been slightly changed in this paper in order to be more consistent with the other approximate equations and as a result of having a wider range of values to compare against, the differences given in table 1 of Davy[11] still give a good indication of the typical accuracy of the approximate equations in this case. The first author[11] also gave the mean, the standard deviation, the maximum and the minimum of the differences between Sato's[16] numerical calculations and the approximate equations given in Davy[11] and the approximate equations given in four other publications. The first author[11] compared his own approximate equations and those of three other authors with Sato's[16] numerical calculations for the diffuse sound field excited case.

In 2014, because the first author's approximate equations only predicted the real part of the impedance for μ less than or equal to one, the authors[17] combined Thomasson's[8] high and low frequency equations, for μ less than or equal to one, for both the real and imaginary parts so that they covered the whole frequency range. These combined equations were compared with Thomasson's[8] tabulated numerical values for a square mounted in an infinite rigid baffle. The values of ke ranged from 0.25 to 64 in half octave steps and the values of $\mu = \sin(\theta)$ ranged from θ equals 0° to 90° in 15° increments. There were also comparisons with five extra numerical values which were read from Thomasson's[8] graphs. The authors[17] also compared the first author's[11] approximate formulae for the real part of the impedance with Thomasson's[8] numerical calculations and discovered that they were a better approximation than the combined version of the real part of Thomasson's[8] formulae.

The authors[18] extended their combined version of Thomasson's approximate formula to cover the case when μ was greater than one. These extended approximate formulae were compared with numerical calculations made by the authors for a square in an infinite rigid baffle for values of ke ranging from 0.25 to 11.31 in half octave steps and for values of μ ranging from 1 to 10 in one tenth of a decade steps. The highest value of ke was limited to 11.31 rather than 64 because of the increase in calculation time with increasing values of ke when numerically evaluating the double integral version of the exact equation.

Also in 2014, the authors[19] replaced their combined version of Thomasson's approximate formulae, for the case when μ is less than or equal to one, with the first author's[11] better performing approximate equations for the real part and developed new better approximate formulae for the imaginary part. Because the μ equals one values are used when interpolating to calculate some of the impedances when μ is greater than one, this change also changed some of the values when μ is greater than one. Comparisons between the approximate formulae and the numerically calculated impedances for a square panel in an infinite rigid baffle were made for the same values of variables as in Davy *et al.*[18]. In addition, because the authors' own numerically calculated values were used instead of Thomasson's tabulated values, comparisons were also made for the same ke values when θ equals 70°, 80° and 85°.

In this paper, the reduction of one level of integration[20] has enabled the authors to extend the range of ke to be from 0.25 to 1024 in half octave steps while maintaining the same values of μ that were used in Davy *et al.*[19]. The one exception was the real part of the impedance when μ is greater than one. In this case the upper limit of ke was only increased from 11.31 to 64 because of long calculation times even with the reduction of one level of integration.

All the authors' previous publications have been only for the travelling wave case. In this paper the authors present approximate formulae for the simply supported mode case. The introduction of a second case and the significant extension of the range of ke means that the comparison tables of individual decibel differences would have doubled in number and increased more than a third in size. Thus in this paper only the means, the standard deviations, the root mean square (rms) sums of the mean and standard deviation, the maxima and the minima of the differences are presented. Readers who are interested in the difference in impedance for individual values of the variables ke and μ should consult the authors' previous papers on this topic to obtain a rough idea of the actual differences. The differences obtained with the versions of the formulae obtained in this paper are generally smaller. Some of these papers also give graphs of the numerically calculated values of the impedance.

For the simply supported wave case, the evaluation only occurs for values of ke and μ which satisfy Eqn. (43) where a equals e. For the simply supported mode case, some of the numerically calculated imaginary impedances for μ corresponding to values of θ less than 45° were negative while the results produced by the approximate formulae were always positive. The decibel difference for these negative values could not be calculated unless the modulus of ratio was taken first. It was considered that taking modulus of the ratio would have been misleading. Thus in this one case all values for θ less than 45° were excluded from the evaluation.

The results are shown in table 1. The most obvious thing is that, as speculated immediately after Eqn. (45), the real part of the impedance when $0 \le \mu < 1$ and the imaginary part of the impedance when $\mu > 1$ are more accurately predicted than the other cases. The agreement between the approximate formulae and the numerical calculations for the real part of the impedance when $\mu > 1$ is good for large values of ke. The disagreement shown in table 1 for this case is due to ripple in the numerically calculated results as a function of μ for small values of ke. The agreement in the travelling wave case for the imaginary part when $\mu < 1$ is also very good apart from some ripple in the numerically calculated results when μ is close to zero and ke is of the order of two. For the imaginary part of the impedance for the simply supported mode case when $\mu < 1$, it needs to be remembered that only differences when μ is greater 0.71 are analyzed. This is because some of the numerically calculated values are negative when μ is less than 0.71. In this region the impedance oscillates between positive and negative values. The agreement for the μ equals one case is good. This is slightly surprising since the infinite panel value is infinite in this case. Table 1 also shows the results for the real part of the impedance for the incident diffuse sound field forced travelling wave case. As would be expected, the results are slightly better than for the real parts of the azimuthally averaged impedance when $\mu \le 1$ for the travelling wave case and for the simply supported mode case.

6. CALCULATION OF RADIATION EFFICIENCY

One of the main uses of the formulae given in this paper is to calculate the radiation efficiency which is just the real part of the normalized radiation impedance. If a thin isotropic panel, mounted in an infinite rigid baffle is excited by an airborne incident diffuse sound field, then the transverse vibration of the panel consists of a forced non-resonant field and a freely propagating resonant field. The ratio r of the resonant vibrational energy to the non-resonant vibrational energy level of a panel which has been excited by a diffuse sound field is[21]

$$r = \pi\omega_c\sigma_r / (4\omega\eta), \tag{46}$$

where η is the total in situ damping loss factor of the panel. σ_r is the radiation efficiency of the resonant transverse vibrational field. This is calculated using the equations for the real part of the normalized radiation impedance for the simply supported case given in this paper with the value of μ given by Eqn. (44) and limiting μ to be greater than or equal to the minimum value given by Eqn. (42) or (43). σ_{nr} is the radiation efficiency of the non-resonant vibration field. This is calculated using the approximate Eqn. (39) for the real part of the normalized diffuse field incident radiation impedance for the travelling wave case. The radiation efficiency of the airborne diffuse sound field excited panel is the weighted average.

$$\sigma_a = \begin{cases} (r\sigma_r + \sigma_{nr})/(r+1) & \text{if } \omega < \omega_c \\ \sigma_r & \text{if } \omega \geq \omega_c \end{cases}. \tag{47}$$

Above the angular critical frequency, it is not possible to distinguish between the resonant and the non-resonant transverse vibrational fields in the panel.

The variable r is also the ratio of the power radiated by the resonant vibrational fields to the power radiated by the vibrational near fields for a panel excited by point forces acting at right angles to the panel[5]. The radiation efficiency of a panel excited by point forces acting at right angles to the panel is (see Eqn. (28) of Davy *et al.*[22])

$$\sigma_p = \begin{cases} \sigma_r (1 + 1/r) & \text{if } \omega < \omega_c \\ \sigma_r & \text{if } \omega \geq \omega_c \end{cases}. \tag{48}$$

The ratio of the power radiated by the resonant vibrational fields to the power radiated by the vibrational near fields for a panel excited by line forces acting at right angles to the panel[5] is

$$r_l = \sqrt{\omega_c/\omega}\,\sigma_r/(2\eta). \tag{49}$$

The radiation efficiency of a panel excited by line forces acting at right angles to the panel is

$$\sigma_l = \begin{cases} \sigma_r (1 + 1/r_l) & \text{if } \omega < \omega_c \\ \sigma_r & \text{if } \omega \geq \omega_c \end{cases}. \tag{50}$$

Thus although the fundamental radiation efficiencies do not depend on the damping loss factor, the above equations show that the radiation efficiency of an excited panel does depend on the damping loss factor when $\omega < \omega_c$. This has also been shown by other authors[15,23].

7. CONCLUSIONS

This paper gives approximate equations for the azimuthally averaged normalized radiation impedance of a rectangular panel set in an infinite baffle. These equations are given for the real and imaginary parts for both the travelling plane wave case and for the simply supported mode case. These two cases have usually been treated separately in the literature and often only the real part has been considered. Prior to the authors' research, most attempts to produce approximate formulae have not covered the whole range of variables. This paper gives a uniform treatment of both cases and covers the whole range of variables. An approximate formula for the real part of the normalized impedance for the incident diffuse sound field excited case is also given.

These approximate formulae are compared with numerical calculations and reasonable agreement is obtained in most situations. The faster speed of the single integral version of the exact equations, compared to the previous double integral versions used previously by the authors, has enabled the comparisons to be made over a bigger range of variables.

The effect of different wave types and boundary conditions on the radiation impedance is discussed. Because at least two different types of transverse vibrational fields are usually established in a finite panel when it is excited, formulae are given for calculating the radiation impedance of the panel due to the combined effect of these two different types of fields when the panel is excited by an incident diffuse sound field or by transverse point or line forces.

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Table 1. The mean, the standard deviation, the root mean square (rms) sum of the mean and standard deviation, the maxima and the minima of the differences in decibels between the approximate formulae and the numerical calculations for the real and imaginary parts of the azimuthally averaged radiation wave impedance of an isotropic square panel mounted in an infinite rigid baffle. The travelling wave case is denoted by TW and the simply supported wave case is denoted by SS. μ *is the ratio of the wave number of the transverse vibrational wave in the panel to the wave number of sound in the medium into which the panel is radiating. Also shown are the results for the real part of the incident diffuse sound field excited case.*

Case	Mean (dB)	Standard Deviation (dB)	Root Mean Square (dB)	Maximum (dB)	Minimum (dB)
Real TW $\mu \le 1$	0.00	0.13	0.13	0.36	-0.61
Real SS $\mu \le 1$	0.01	0.14	0.14	0.35	-0.58
Imag TW $\mu \le 1$	-0.02	0.41	0.41	2.11	-1.56
Imag SS $\mu \le 1$	-0.25	0.73	0.77	2.73	-1.38
Real TW $\mu \ge 1$	-0.02	0.91	0.91	6.23	-2.44
Real SS $\mu \ge 1$	-0.67	1.22	1.39	3.21	-5.25
Imag TW $\mu \ge 1$	0.01	0.09	0.09	0.42	-0.35
Imag SS $\mu \ge 1$	0.02	0.09	0.09	0.35	-0.49
Real Diffuse	0.00	0.11	0.11	0.25	-0.26