Compact corrugated feedhorns with high Gaussian coupling efficiency and -60 dB sidelobes

Johannes E. McKay, Duncan A. Robertson, *Member, IEEE*,
Peter J. Speirs, *Member, IEEE*, Robert I. Hunter,
Richard J. Wylde and Graham M. Smith

***Abstract*—We demonstrate that very high performance, extremely compact, scalar corrugated feedhorns can be designed and constructed by optimizing the excitation and phasing of the HE_{11}, HE_{12} and HE_{13} modes near the throat of the horn whilst limiting excitation of higher order modes. We present the design and measurement of two families of dual-profiled horn, both with a directivity of 20 dBi that couple with very high efficiency to a fundamental Gaussian mode. The first was optimized for sidelobe performance and features sidelobes approaching -60 dB for a horn length of only 15.6λ. The second was designed to minimize horn length and achieves sidelobe levels below -35 dB for a horn which is only 4.8λ long. The horns exhibit excellent coupling to the fundamental free-space Gaussian mode, with LG_{00} power coupling of 99.92% and 99.75% respectively. We demonstrate excellent agreement between simulation and experiment at 94 GHz and simulate the performance over a 20% bandwidth. High performance compact scalar horns are of interest because they reduce manufacturing risk at high frequencies, and reduce size and weight at lower frequencies, which can be important in horn arrays and space applications, where horn arrays often have serious weight and size restrictions.**

***Index Terms*—Horn antennas, Gaussian beams, Quasi optics.**

I. INTRODUCTION

Corrugated feedhorns are high performance mode-converters that can transform waveguide modes into high directivity free-space modes with high efficiency, low sidelobe levels and low levels of cross-polarization. They are frequently used as high performance antennas in quasi-optical instruments, as feeds in large telescopes, communications or radar systems, or as part of imaging arrays.

The design and analysis of circular corrugated feedhorns has been the subject of many papers and monographs [1]–[3] and today, modern mode matching software allows the aperture fields and far-field beam patterns from any specific corrugated horn design to be predicted with high precision over a desired bandwidth, by calculating the hybrid HE_{1n} and EH_{1n} modes present at the aperture. In general, the role of the corrugated feedhorn designer is to specify the aperture field distribution that will produce the desired far field pattern and then calculate the combination of hybrid modes, and their dispersion characteristics, required to best create that aperture field distribution over a given bandwidth. In principle it is then possible to run optimization algorithms that will lead to an optimized horn profile to excite those modes.

One way to design a feedhorn with very low sidelobes is to maximize the coupling efficiency to a fundamental Gaussian beam. The optimal combination of HE_{1n} modes to maximize this coupling is easily calculated via mode coupling integrals and is given in [4]. If only the HE_{11} mode is excited, it will couple with no more than 98% power efficiency to a fundamental Gaussian [5], [6]. This typically leads to sidelobe levels around -27 dB.

In principle the coupling efficiency to a Gaussian beam (or 'gaussicity') can be improved, at the cost of aperture efficiency, if both the HE_{11} and HE_{12} modes can be excited in phase at the aperture with an optimum ratio. This can be achieved using shaped or dual-profiled horns to excite higher order modes with an optimal amplitude and phase relationship [7]–[9]. A wide variety of profiles and shapes to achieve this have been proposed including $\sin^2$, Gaussian, exponential flares or designs that are based on spline fits [3], [10]–[15].

We have previously described a $\sin^2$-parallel design [4], [16] whose profile is shown in Figure 1. In that design the $\sin^2$ section is used to excite the desired proportion of HE_{11} and HE_{12} modes and the parallel section brings them into phase. Analytical expressions for these lengths have been calculated for any desired aperture or directivity [15]. That so called "ultra-Gaussian" design has demonstrated sidelobe levels of -40 dB and gaussicity of 99.7%. Such horns have been used in a very high performance mm-wave electron paramagnetic resonance spectrometer [17], for cosmic microwave background detection [18] and for submm-wave 3D imaging radar [19].

However, that horn has a relatively long phasing section, which adds to the horn's length. Detailed analysis also shows the gaussicity of the beam is limited by excitation of higher order modes, primarily the HE_{13} mode, which is difficult to phase with the HE_{11} and HE_{12} modes. Two questions arise: firstly whether the sidelobe performance can be improved by also optimizing the excitation and phasing of the HE_{13} mode relative to the HE_{11} and HE_{12} modes, and secondly could the horn be shortened by changing the phasing section? We previously suggested that this might be achieved by a dual-profile $\tanh$-linear design [4] and in this paper we show experimentally the advantages of this design for two types of compact horn at 94 GHz and compare it to a dual-profile $\sin^2$-parallel horn.

II. DESIGN

The $\tanh$-linear profile $r(z)$ can be described by equation 1

$$r(z) = r_{th} + (a_0 - r_{th}).[\frac{(1-A)z}{L_{profile}} + \frac{A}{2}(\tanh(\frac{B\pi z}{2L_{profile}} - \pi) + 1)] \tag{1}$$

where r_{th} is the throat radius, a_0 the aperture radius and $L_{profile}$ the length of the horn. A, B and $L_{profile}$ are then adjustable parameters. In this design most of the change in profile flare angle is near the throat of the horn, which is

D.A. Robertson, R.I. Hunter and G.M. Smith are with the Millimetre Wave and EPR Group, SUPA, School of Physics and Astronomy, University of St Andrews, North Haugh, St Andrews, Fife KY16 9SS, Scotland (e-mail: dar@st-and.ac.uk).

J.E. McKay is now with the National High Magnetic Field Laboratory (NHMFL) Tallahassee, FL USA (e-mail: mckay@magnet.fsu.edu).

P.J. Speirs is now with the Environmental Remote Sensing Laboratory (LTE), École Polytechnique Fédérale de Lausanne (ÉPFL) Lausanne, Switzerland (e-mail: peter.speirs@epfl.ch).

R.J. Wylde is with Thomas Keating Ltd., West Sussex RH14 9SH, U.K. and also with the Department of Physics and Astronomy, University of St. Andrews, Fife KY16 9SS, U.K. (e-mail: R.Wylde@terahertz.co.uk).

Manuscript received MMMM DD, YYYY; revised MMMM DD, YYYY.

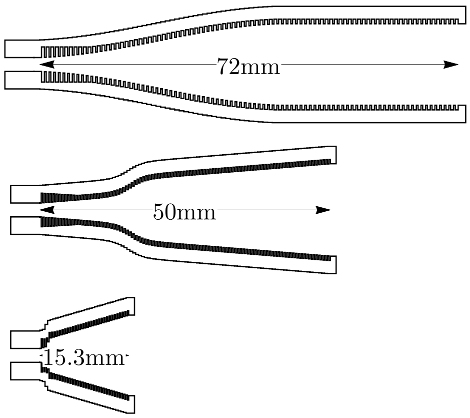

Fig. 1. Drawings of the profiled horns used in this study. Top to bottom; $\sin^2$-parallel, $\tanh$-linear Horn 1 and $\tanh$-linear Horn 2 for 94 GHz and 20 dBi gain. The lengths of the profiled sections are shown with double headed arrows. All profiles are shown on the same length scale.

closer to the cut-off radius of the $\geq$HE$_{13}$ modes. It now becomes possible to excite a near optimal proportion of hybrid waveguide mode amplitudes HE$_{11}$, HE$_{12}$ and HE$_{13}$. Having a linear taper then allows these modes to arrive with the desired phase relationships at the required aperture radius, whilst minimizing excitation of higher order modes. This leads to predictions of a very high power coupling to the LG$_{00}$ mode with highly compact designs. Using the mode matching software CORRUG[1], integrated with the MATLAB environment[2], it is possible to use global optimization routines to design a feedhorn quickly and methodically for a given set of design goals. In this work we sought a scalar design that would minimize both the level of sidelobes and horn length for a horn with 20 dBi directivity at a spot frequency of 94 GHz.

Through exploration of the design space it was found that several optimal regimes exist for the $\tanh$-linear design; of particular note were the two regimes corresponding to profile lengths of $\sim 15.6\lambda$ and $\sim 4.8\lambda$, for a 20 dBi horn, and these were selected for manufacture at 94 GHz. The first $\sim 15.6\lambda$ design, Horn 1, was calculated to have extremely low sidelobe levels approaching -60 dB and a Gaussian coupling in excess of 99.9% whilst being reasonably compact. The second $\sim 4.8\lambda$ design, Horn 2, was chosen for its ultra-compact length whilst still offering very low sidelobe levels at -35 dB and Gaussian coupling in excess of 99.7%.

Dimensionally correct drawings of both $\tanh$-linear horn designs are compared with the $\sin^2$-parallel design in figure 1.

Table I shows design parameters and simulated performance characteristics for each of the three designs at the design frequency of 94 GHz. From this table it can be seen that for the $\sin^2$-parallel design the HE$_{11}$ and HE$_{12}$ modes arrive approximately in phase at the aperture but the HE$_{13}$ mode is considerably out of phase. However, for both $\tanh$-linear designs all three of these hybrid waveguide modes arrive in phase at the aperture at the center frequency. Performance across the bandwidth is limited by how quickly the modes dephase.

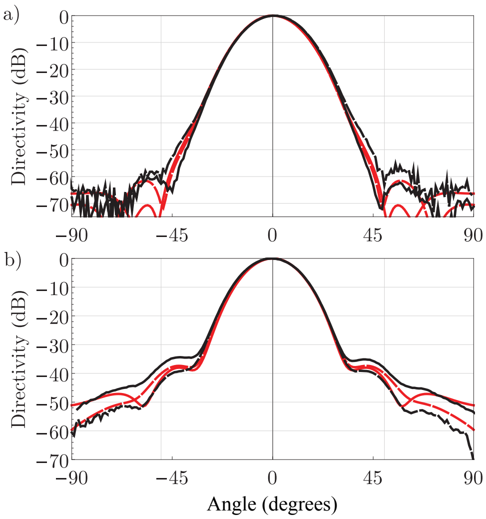

Fig. 2. Far field directivity plots for one example each of the $\tanh$-linear designs of a) Horn 1 and b) Horn 2. Solid lines show E-plane measurement and simulation in black and red respectively. Broken lines show H-plane measurement and simulation in black and red respectively.

All of the horns were manufactured by Thomas Keating Ltd.[3] using precision electroforming techniques that have previously given excellent results [4], [13], [20]. Two horns of each $\tanh$-linear type were manufactured to test repeatability and both gave comparable results. All the horns were manufactured with circular waveguide inputs with r_{th} = 1.194 mm so external smooth circular-to-rectangular (WR10) waveguide transitions were added for measurements.

III. FAR-FIELD MEASUREMENTS

Far field antenna measurements were conducted in a modified acoustic anechoic chamber which was adapted by adding additional convoluted millimeter wave absorbing material around the measurement area. The antennas under test were measured in receive in the far-field of an illuminating horn (at a distance of about 1.5 m). The source was a 94 GHz 20 mW Gunn oscillator with a high directivity (43 dBi) $\sin^2$-parallel horn, to maximize dynamic range. The antenna under test was attached to a heterodyne receiver whose 500 MHz IF signal was measured using a HP8593A spectrum analyzer interrogated by a computer via GPIB. The antenna under test and receiver were mounted on a computer controlled precision turntable to permit automatic recording of the far-field patterns. Care was taken to ensure the phase center of each horn lay above the center of rotation.

Far field E- and H-plane measurements for one example of the Horn 1 and Horn 2 designs are shown in figure 2 a) and b) respectively. There is excellent agreement with simulated far-field patterns down to around -57 dB for Horn 1 and -35 dB for Horn 2.

The corrugations at the throat of the horn were designed to minimize return loss, using well known design principles [1],

---

[1] http://www.smtconsultancies.co.uk/

[2] http://uk.mathworks.com/products/matlab/

[3] Thomas Keating Ltd., Station Mills, Billingshurst, Sussex, UK.

TABLE I

DESIGN PARAMETERS AND CALCULATED MODE AMPLITUDES & RELATIVE PHASES FOR THE THREE HORN DESIGNS AT 94 GHZ

Design	a_0	A, B	$L_{profile}$	Directivity	HE$_{11}$	HE$_{12}$	HE$_{13}$
sin^2-parallel	7.00 mm	-,-	72.3 mm	20.6 dBi	0.9890 (0.0°)	0.1463 (7.3°)	0.0169 (284.5°)
tanh-linear Horn 1	7.94 mm	0.44, 6.42	50.0 mm	20.0 dBi	0.9737 (0.0°)	0.2233 (3.2°)	0.0388 (7.6°)
tanh-linear Horn 2	6.93 mm	0.23, 23.16	15.3 mm	20.1 dBi	0.9265 (0.0°)	0.3739 (0.0°)	0.0350 (−4.1°)

[2]. The input return loss of the feedhorns was measured using an Anritsu VectorStar ME7838A Vector Network Analyser where the beam was directed towards a conical quasi-optical load made of conductive polymer, which exhibits better than -80 dB reflection at 94 GHz [21]. These measurements gave return losses for all the horns in the range of -30 to -40 dB at the center frequency.

IV. FULL BAND SIMULATIONS

For many applications it is also frequently important to understand how the beam waist size and position (average phase center), Gaussian beam coupling, cross-polar performance and sidelobe level change with frequency. These parameters were calculated, over a 20% bandwidth, from the HE$_{1n}$ aperture modes obtained with CORRUG and using mode coupling integrals to calculate the Gaussian beam performance and these results are shown in figure 3.

The beam waist radius of Horn 1 shows the least variation whilst the very short Horn 2 shows the most variation. Conversely, the effective position of the beam waist (or phase center) varies most for Horn 1 whilst Horn 2 shows less variation with a broad maximum at around 98 GHz. Simulations showed it could be designed to be centered at 94 GHz without significant loss of performance. In some wideband applications the variation of the phase center with frequency is an important parameter, although no attempt was made to minimize this variation in this work. Gaussian coupling efficiency is highest for Horn 1, achieving 99.92% at 94 GHz, due to the near-optimum mode content. Horn 2 achieves 99.75% around 92 GHz and the sin^2-parallel achieves 99.45%. Maximum cross-polar level is similar for all three designs being better than -25 dB over the band and achieving -50 dB at band centre. The sidelobe level for Horn 1 approaches -60 dB over 92 to 100 GHz and that of Horn 2 is comparable with the sin^2-parallel design despite its much shorter length.

V. DISCUSSION

To the best of our knowledge the sidelobe levels approaching -60 dB seen in the far field patterns of the tanh-linear Horn 1, represent the lowest levels ever reported experimentally for a corrugated horn, and -40 dB is achieved over a 13% bandwidth. They are also in close agreement with mode-matching predictions. The improved sidelobe performance in a compact design (of $\sim 4.8\lambda$ and $\sim 15.6\lambda$ for a 20 dBi design) is attributed to the controlled excitation and phasing of the HE$_{11}$, HE$_{12}$ and HE$_{13}$ modes.

Whilst there may be only a few specialized applications that require sidelobe levels approaching -60 dB, Horn 1 is still a compact design and provides beams with high gaussicity over relatively wide bandwidths and is as easy to manufacture as any other corrugated horn design described in the literature. If horn length or weight is a major design consideration then the sidelobe level demonstrated by Horn 2 is comparable to the sin^2-parallel "ultra-Gaussian" design, but is achieved in a horn

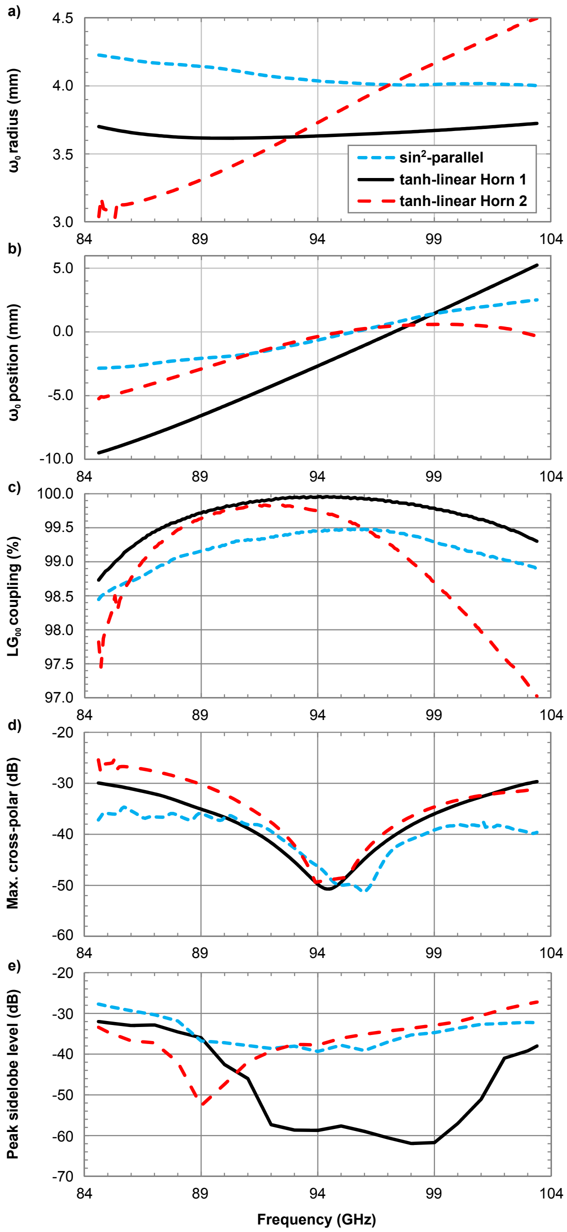

Fig. 3. Full band simulations of each horn design, as indicated in the legend, for a) LG$_{00}$ beam waist radius, b) LG$_{00}$ beam waist position relative to the aperture, c) power coupling to the LG$_{00}$ mode as a percentage of total power, d) maximum cross-polar level, and e) peak sidelobe level.

that is almost 5 times shorter. This design can be compared to the similar horn described in [22] which uses a step change at the input to excite the higher order modes followed by a $\sin$ transition. Both designs excite similar modes and have similar sidelobe performance, although the one described here is slightly shorter and has better return loss characteristics, and may operate over larger bandwidths. A major reduction in horn length becomes important for imaging or space applications where size and weight can be important criteria in horn arrays. At very low frequencies corrugated horns can simply become too large and heavy whilst at very high frequencies manufacturing becomes more challenging. In this particular study we were optimizing the design to minimize sidelobe level and horn length at a spot frequency of 94 GHz. We also restricted ourselves to a $\tanh$-linear profile, which partly restricted the range of solutions in terms of the horn length.

VI. CONCLUSION

We have demonstrated that highly compact corrugated feedhorns can be designed and manufactured to give extremely low sidelobe levels by optimising the HE_{1n} mode amplitudes and phases up to n = 3. Such designs are inherently scalable and can be applied at much lower or higher frequencies where the manufacture of long horns is more challenging, and where weight and size can be important criteria, particularly for space applications. The results presented for the $\tanh$-linear designs show that one can design 20 dBi horns less than 5 wavelengths long, with sidelobe levels better than -35 dB, or horns less than 16 wavelengths long with sidelobe levels approaching -60 dB. We suggest that achieving the highest performance from corrugated feedhorns is almost entirely determined by the excitation, correct phasing and dispersion of the first three HE_{1n} modes. The amplitudes and phases of these modes can be chosen to maximize the gaussicity of the beam, which is a proxy for low sidelobe and cross-polarization levels. They thus represent excellent target criteria in optimization routines and as a means of estimating how close a given design is to ideal, across a given frequency range. Finally, we have also explored solutions using more general optimization algorithms where there are no restrictions on the profile. This is a more time-consuming approach, but offers more flexibility in optimizing beam parameters for a given length of horn. Such algorithms have given optimized designs very close to the $\tanh$-linear design, with similar performance, for the lengths of horn used in this study and this is the subject of ongoing work.

ACKNOWLEDGMENT

The authors would like to thank Paul Holes of Anritsu EMEA Ltd. for the loan of the Anritsu VectorStar ME7838A Vector Network Analyser.

REFERENCES

[1] P. J. B. Clarricoats and A. D. Olver, *Corrugated Horns for Microwave Antennas*. Peter Peregrinus Ltd, 1984.

[2] A. D. Olver, P. J. B. Clarricoats, A. A. Kishk, and L. Shafai, *Microwave Horns and Feeds*. Piscataway, N.J.: IEEE Press, 1994.

[3] T. S. Bird and C. Granet, "Profiled horns and feeds," in *Handbook of Reflector Antennas and Feed Systems Volume II: Feed Systems*, ser. Antennas and Propagation, L. Shafai, S. K. Sharma, and S. Rao, Eds. Artech House, 2013.

[4] J. E. McKay, D. A. Robertson, P. A. S. Cruickshank, R. I. Hunter, D. R. Bolton, and G. M. Smith, "Compact wideband corrugated feedhorns with ultra-low sidelobes for very high performance antennas and quasi-optical systems," *IEEE Trans. Antennas Propag.*, vol. 61, no. 4, pp. 1714–1721, 2013.

[5] C. Aubry and D. Bitter, "Radiation-pattern of a corrugated conical horn in terms of Laguerre-Gaussian functions," *Electronics Letters*, vol. 11, no. 7, pp. 154–156, 1975.

[6] R. J. Wylde, "Millimetre-wave gaussian beam-mode optics and corrugated feed horns," *IEE Proc.-H Microwaves Antennas and Propagation*, vol. 131, no. 4, pp. 258–262, 1984.

[7] T. Graubner, W. Kasparek, and H. Kumric, "Optimization of coupling between HE(11)-wave-guide mode and Gaussian-beam," *18th Int. Conf. Infrared and Millimeter Waves*, vol. 2104, pp. 477–478, 1993.

[8] ——, "Synthesis and applications of special power distributions by HE1n mixtures," in *Int. Conf. Infrared and Millimeter Waves*, 1994, pp. 238–239.

[9] C. Del Rio, R. Gonzalo, and M. Sorolla, "High purity Gaussian beam excitation by optimal horn antenna," in *Proc. ISAP'96*, 1997, pp. pp.1133–1136.

[10] C. D. Bocio, R. Gonzalo, M. S. Ayza, and M. Thumm, "Optimal horn antenna design to excite high-order Gaussian beam modes from TE0m smooth circular waveguide modes," *IEEE Trans. Antennas Propag.*, vol. 47, no. 9, pp. 1440–1448, 1999.

[11] L. Lucci, R. Nesti, G. Pelosi, and S. Selleri, "Design of an improved profiled corrugated circular horn at 320 GHz," *Journal of Electromagnetic Waves and Applications*, vol. 18, no. 3, pp. 387–396, 2004.

[12] ——, "Phase centre optimization in profiled corrugated circular horns with parallel genetic algorithms," *Progress in Electromagnetics Research*, vol. 46, pp. 1227–142, 2004.

[13] F. Villa, M. Sandri, N. Mandolesi, R. Nesti, M. Bersanelli, A. Simonetto, C. Sozzi, O. D'Arcangelo, V. Muzzini, A. Mennella, P. Guzzi, P. Radaelli, R. Fusi, and E. Alippi, "High performance corrugated feed horns for space applications at millimetre wavelengths," *Experimental Astronomy*, vol. 14, no. 1, pp. 1–15, 2002.

[14] G. G. Gentili, R. Nesti, G. Pelosi, and V. Natale, "Compact dual-profile corrugated circular waveguide horn," *Electronics Letters*, vol. 36, no. 6, pp. 486–487, 2000.

[15] B. Maffei, F. Noviello, J. A. Murphy, P. A. R. Ade, J. M. Lamarre, F. R. Bouchet, J. Brossard, A. Catalano, R. Colgan, R. Gispert, E. Gleeson, C. V. Haynes, W. C. Jones, A. E. Lange, Y. Longval, I. McAuley, F. Pajot, T. Peacocke, G. Pisano, J. L. Puget, I. Ristorcelli, G. Savini, R. Sudiwala, R. J. Wylde, and V. Yurchenko, "Planck pre-launch status: HFI beam expectations from the optical optimisation of the focal plane," *Astronomy and Astrophysics*, vol. 520, 2010.

[16] P. A. S. Cruickshank, D. R. Bolton, D. A. Robertson, R. J. Wylde, and G. M. Smith, "Reducing standing waves in quasi-optical systems by optimal feedhorn design," *Joint 32nd Int. Conf. Infrared and Millimeter Waves and 15th Int. Conf. Terahertz Electronics*, pp. 922–923, 2007.

[17] P. A. S. Cruickshank, D. R. Bolton, D. A. Robertson, R. I. Hunter, R. J. Wylde, and G. M. Smith, "A kilowatt pulsed 94 GHz electron paramagnetic resonance spectrometer with high concentration sensitivity, high instantaneous bandwidth, and low dead time," *Review of Scientific Instruments*, vol. 80, no. 10, pp. –, 2009. [Online]. Available: http://scitation.aip.org/content/aip/journal/rsi/80/10/10.1063/1.3239402

[18] P. A. Ade, R. J. Wylde, and J. Zhang, "Ultra-Gaussian horns for CLOVER - a B-Mode CMB experiment," in *20th Int. Symp. on Space Terahertz Technology*, April 2009, pp. 128–137.

[19] D. A. Robertson, P. N. Marsh, D. R. Bolton, R. J. C. Middleton, R. I. Hunter, P. J. Speirs, D. G. Macfarlane, S. L. Cassidy, and G. M. Smith, "340-GHz 3D radar imaging test bed with 10-Hz frame rate," pp. 836 206–836 206–11, May 2012. [Online]. Available: http://dx.doi.org/10.1117/12.918581

[20] A. Murk, N. Kampfer, R. J. Wylde, J. Inatani, T. Manabe, and M. Seta, "Characterization of various quasi-optical components for the submillimeter limb-sounder SMILES," in *14th Int. Symp. on Space Terahertz Technology*, 2001, pp. 217–226.

[21] D. A. Robertson, D. R. Bolton, P. A. S. Cruickshank, R. I. Hunter, and G. M. Smith, "Technologies for sub-ns pulse coherent mm-wave radar," in *3rd European Radar Conference, EuRAD*, Sept 2006, pp. 150–153.

[22] C. Granet, T. S. Bird, and G. L. James, "Compact multimode horn with low sidelobes for global earth coverage," *IEEE Trans. Antennas Propag.*, vol. 48, no. 7, pp. 1125–1133, Jul 2000.